Unilateral Carbon Taxes and International Trade: An Analytic General Equlibrium Model

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Abstract

Blah blah blah

1 Introduction

Leakage, defined roughly as an increase in emissions abroad in response to a carbon tax at home, is a central concern for domestic climate change policy. Originally, in response to the dichotomy introduced by the Kyoto Protocol, leakage was a concern because developed countries were to adopt carbon prices while developing were not. The concern was that in response to this dichotomy, heavy industry would flee developed countries, leading to inefficiencies in the location of production and making carbon prices ineffective. Even without such extreme differentiation, leakage is still concern because nations will inevitably have different carbon prices (whether explicit, such as through a tax, subsidies, or tradable permits, or implicit, such as through regulations). For example, under the Paris Agreement, nations have different emissions reductions goals, implying different shadow carbon prices, generating the possibility of leakage.

Because of its centrality to climate change policy, there have been a large number of studies of leakage. Most studies use computable general equilbirium models to study the issue. CGE models have the advantage of detailed representations of the economy, often including particularly detailed representations of the energy sector. Their disadvantage is that they are non-transparent. Moreover, validiation of CGE results is not straightforward, which means that the additional detail these models offer may not produce meaningful benefits.

There are also a smaller number of analytic approaches to studying leakage (described below). [What should we say about these?]

We study unilateral carbon prices using a stylized general equilibrium model of international trade. The model allows nations to impose carbon taxes at different stages of production and on consumption and allows nations to impose border adjustments. Border adjustments are taxes on the importation of goods produced using fossil fuels and rebates of any prior carbon taxes paid on the export of goods. They are the primary policy tool suggested to address the distortions from unilateral pricing. We derive a number of analytic results. In other cases, calibrate and numerically solve model. In these cases, look for results that are robust to choices of parameters.

The policy tradeoffs are surprisingly subtle in this context. While leakage is touted as a prominent indicator of distortions from unilateral carbon pricing, we find that leakage has little relationship to welfare. A policy minimizing leakage is rarely optimal (for a given objective of global carbon reduction). [We also show that full border adjustments are unlikely to be optimal, even leaving aside their considerable administrative costs (for a discussion of these costs, see Kortum and Weisbach 2017))

1.1 Prior literature

1.1.1 CGE approaches

The CGE literature is too large to summarize in detail. The major studies are cited in the notes.[1] There are five findings that seem to be robust across

[1] Work includes Alexeeva-Talebi et al. (2012), Babiker (2005), Balistreri and Rutherford (2012), Bednar-Friedl, Schinko, and Steininger (2012), Boeters and Bollen (2012), Böhringer, Balistreri, and Rutherford (2012), Böhringer et al. (2012), Böhringer, Carbone, and Rutherford (2012), Böhringer et al. (2012),Branger and Quirion (2014a), Branger and Quirion (2014a), Caron (2012), Dong and Walley (2012), Dröge (2009), Elliott et al. (2010), Felder and Rutherford (1993), Jakob, Marschinski, and Hübler (2013), Jakob, Steckel, and Edenhofer (2014), Lockwood and Whalley (2010), Richels, Blanford, and Rutherford (2009), van Asselt and Brewer (2010), Weitzel, Hübler, and Peterson (2012),

most of the CGE models. They are:

- Leakage rates are most often in the range of 5% to 20% (with leakage defined as the increase in emissions in non-taxing regines as a percent of the reduction in emisisons in the taxing region). There are some outliers.
- The larger the taxing coalition, the lower the leakage.
- Border adjustments reduce leakage substantially, with leakage rates under border adjustments ranging from __ to __.
- The most important variable in determining effects of unilateral carbon price is the elasticity of energy supply.
- The models distinguish two drivers of leakage: the fuel price effect (in which lower demand for fossil fuels in the taxing region suppresses prices, increasing demand on the non-taxing regions) and the competitiveness effect (in which increased costs for industry in the taxing region causes a shift to the non-taxing region).

1.1.2 Analytic approaches

Small body of work has used analytic models to understand leakage. Markusen (1975), which focuses on environmental harms more generally, is earliest example.

Jakob, Marschinski and Hubler (2013) use a version of Markusen (1975), modified to allow different sectors in the economy to have different emissions intensities. Their key finding is that the relative intensities of the exporting and non-exporting sectors in the non-taxing region affect whether BA's reduce leakage.

Bohringer Lange and Rutherford (2014) use an analytic model to consider differential carbon price, such as lower taxes on trade-exposed sectors. They decompose the effects of differential taxes into leakage effects and terms of trade effects. They incorporate this decomposition into a CGE model to produce guidelines for carbon pricing.

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and Winchester, Paltsev, and Reilly (2011), Kuik and Gerlagh (2003), Kuik and Hofkes (2010), Kuik and Verbruggen (2002), Monjon and Quirion (2011), Helm, Hepburn, and Ruta (2012), de Cendra (2006), Fischer and Fox (2011), Fischer and Fox (2012a), Fischer and Fox (2012b), and Elliott et al. (2013).

Fischer and Fox (2012).

Fullerton, Karney and Baylis (2014) use an analytic general equilibrium model to analyze how capital used in abatement of emissions effects leakage. They find that if abatement is resource intensive (such as requiring substantial capital), a domestic carbon tax can produce negative leakage because it increases global resource costs and hence investment in non-taxing regions.

2 Model structure

Our goal is to develop a relatively simple and transparent general equilibrium model that captures the effects of carbon taxes on trade. We purposefully suppress many elements commonly found in CGE models so our model is analytically tractable.

In particular, we assume that the world is divided into only two countries or regions, which we call Home and Foreign, and that there are only two factors of production, labor and deposits of energy both of which are immobile. Each country has a fixed endowment of each factor, and the countries may differ in their endowments.

Production takes place in two stages. First, firms in each country use labor to extract deposits, producing usable energy, which is costlessly traded. Although they use the same extraction technology, the countries may differ in their cost of extraction (as measured in units of labor). In our base model, there is only one type of energy, fossil fuel energy, which produces carbon dioxide when used. In an extension, we add renewable energy.

In the second stage of production, firms in each country use energy (creating emissions) and labor to produce manufactured goods. One of our goals is to investigate how taxes affect the location of production. To allow for these affects, we assume that each country has a comparative advantage in particular varieties of manufactured goods, determined by a version of the Ricardian model introduced by Dornbush, Fisher and Samuelson (1977).

Each country also produces services using only labor. Therefore, labor in each country is used in three ways: to extract deposits, to work in manufacturing, and to produce services.

Services play an important role in the model. We assume trade balances, which means that if taxes shift where extraction or production takes place (which is a central concern with unilateral carbon taxes), the labor used to produce services effectively acts as a residual. For example, if the Home country imposes a tax on the use of energy in production and, because of the resulting change in comparative advantage, some production moves offshore to the Foreign country, labor in the Home country shifts to producing services (and likewise, labor in the Foreign country shifts away from producing services). Taxes, in this case, effectively shift the energy intensity of the two countries by shifting where manufacturing takes place and where the production of services takes place.

Consumers in each country earn income by working in one of the three sectors and by receiving rents from deposits. Consumers use their income to purchase manufactured goods and services to maximize utility, closing the model. Consumers can purchase goods produced in either country without trade costs.

Our goal is to consider how taxes affect elements of the model, including where extraction, production, and consumption take place, how emissions change in each of the two countries (including a measure of leakage), and how taxes affect welfare. To do this, we consider a very general set of taxes: each country can impose taxes on energy at each stage of production or on the consumption of goods created using energy. In particular, each country can impose a tax on the extraction of energy (an extraction tax), on the use of energy in production (a production tax), and on the purchase of goods that were produced using energy (a consumption tax).

We also allow the Home country to impose border adjustments, which are taxes on the energy used in the production of imported goods and rebates of taxes paid for the use of energy on exported goods. In most models, border adjustments are imposed when a country has a tax on production, and the border adjustment tax rate is equal to the production tax rate. One of our goals is to determine whether and the extent to which border adjustments are optimal, so we allow border adjustments to be imposed at any rate between zero (i.e., no border adjustments) and the production tax rate. In general, we find that a corner solution is not optimal.

To make the model tractable, we assume relatively simply functional forms: Cobb-Douglas production functions and a CES utility function. In particular, we parameterize the model as follows.

2.1 Consumption

Consumers in the two countries, Home (H) and Foreign (F) consume services (the l-good) and a composite manufactured good (the m-good). These goods enter utility via a CES utility function with an elasticity of substitution σ:

$$U(C_m, C_l) = \left(\alpha^{1/\sigma} \int_0^1 C_m(j)^{(\sigma-1)/\sigma} dj + (1-\alpha)^{1/\sigma} C_l^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}. \quad (1)$$

[somewhere in the notes, we say that the parameters of the utility function can be different across countries. Are we allowing this? It doesn't seem like it in the analytic section but maybe in the simulations?] We use $C_m = \left(\int_0^1 C_m(j)^{(\sigma-1)/\sigma} dj\right)^{\sigma/(\sigma-1)}$ to denote the composite m-good. C_l is services or the l-good.[2]

Consumer optimization gives a demand function for the m-good of:

$$D(p_m) = \frac{\alpha p_m^{-(\sigma-1)}}{a p_m^{-(\sigma-1)} + (1-\alpha) w^{-(\sigma-1)}},$$

where p_m is the aggregate price of the m-good and w is the wage rate. Spending in H on the m-good is:

$$p_m C_m = D(p_m) Y.$$

A parallel result holds in F. We denote all quantities in F using an asterisk, giving us $p_m^* C_m^* = D(p_k^*) Y^*$.

2.1.1 Consumption taxes

Each country can impose an ad valorem tax, t_c and t_c^*, on the energy content of the m-good (and, in our extended model in the simulations, on the direct consumption of energy, such as for home heating). The energy content of a good is the total energy used in production of the good. For example, a consumption tax on an automobile includes a tax on the energy used to produce the steel in the automobile even if the consumer does not directly use that energy or produce the resulting emissions.

Firms that sell m-goods will have an incentive to shift their mix of inputs because the use of energy as an input is taxed while the use of labor is not. As a result, the price of a good that bears a consumption tax on its energy content must be determined based in equilibrium behavior of firms. To derive

[2] We could allow the exponent in the Dixit Stiglitz aggregator to have an arbitrary value but this value plays no role in the model, so we let it be σ without loss of generality.]

this expression we must first specify the production technology, so we defer this to Part _.

2.2 Production

Upstream, the two final goods are produced as follows.

2.2.1 Services (the l-good)

Production of the l-good in a H is

$$Q_l = L_l.$$

and similarly for $F: Q_l^* = L_l^*$. Labor is measured efficiency units, so this formulation can capture differences in labor productivity across countries. We only consider equilibria where each country supplies services, and services are freely traded. We set the l-good in F to be the numeraire, which allows us to set $w = w^* = 1$.

2.2.2 The m-good

As noted, production of the m-good takes place in two stages.

Extraction First, firms extract energy using labor L_e and desposits E in a Cobb-Douglas production function with labor share β. In H, we have:

$$Q_e = \left(\frac{L_e}{\beta}\right)^\beta E^{1-\beta}.$$

(We derive this production function from more primitive assumptions about extraction in the Appendix.) Extraction of deposits in F uses the same production function, replacing β with β^*, E with E^*. Differences in β and β^* capture differences in the labor share of energy extraction, such as between a country like Saudi Arabia where energy is cheap to extract (low β) and Canada, where energy is costly to extract (high β).

Each country can impose an ad valorem tax on extraction, t_e and t_e^*. If the market price of energy is p_e, firms in H engaging in extraction receive only $p_e/(1+t_e)$. With this tax and recalling that $w=1$, we get an energy supply curve in H of

$$Q_e = \left(\frac{p_e}{1+t_e}\right)^{\beta/(1-\beta)} E.$$

The supply curve for energy in F is similar, givng us output of Q_e^*.

World supply of energy is the sum of the energy produced in H and F:

$$Q_e^W = Q_e + Q_e^* = \left(\frac{p_e}{1+t_e}\right)^{\beta/(1-\beta)} E + \left(\frac{p_e}{1+t_e^*}\right)^{\beta^*/(1-\beta^*)} E^*. \quad (2)$$

Production Firms in each country use energy and labor to produce varieties of the m-good with Cobb-Douglas technology and labor share γ. To model specialization in manufacturing and trade, we use a version of the Ricardian model introduced by Dornbusch, Fisher, and Samuelson (1977). In particular, let there be a continuous variety of m-goods indexed by $j \in [0,1]$. The production function for variety j in $\mathcal{H}$ is:

$$Q_m(j) = A(j)\left(\frac{L_m(j)}{\gamma}\right)^{\gamma}\left(\frac{M(j)}{1-\gamma}\right)^{1-\gamma},$$

where $A(j)$ is $\mathcal{H}$'s productivity in variety j, $M(j)$ is energy, and γ is labor's share. There are many price-taking producers have access to the technology to produce each variety j. In $\mathcal{F}$ the production function has the same form, with productivity $A^*(j)$.

Productivity for each variety is

$$A(j) = \frac{A}{j^{1/\theta}}$$

in $\mathcal{H}$ and

$$A^*(j) = \frac{A^*}{(1-j)^{1/\theta}}$$

in $\mathcal{F}$. The parameters A and A^* capture absolute advantage in $\mathcal{H}$ and $\mathcal{F}$. The relative productivity of the two countries in producing variety j is:

$$R(j) = \frac{A(j)}{A^*(j)} = \frac{A}{A^*}\left(\frac{j}{1-j}\right)^{-1/\theta}. \quad (3)$$

For $j < j'$, $\mathcal{H}$ has a comparative advantage in variety j and $\mathcal{F}$ in j'. The parameter θ captures (inversely) the strength of comparative advantage. As $\theta \to \infty$ relative productivity does not vary across varieties.

Production tax Each country can, if it chooses, impose an ad valorem tax on the use of energy in production, raising the price from p_e to $(1+t_p)\,p_e$ in H and $\left(1+t_p^*\right)p_e$ in F. Because the energy share of production is $(1-\gamma)$, this raises the cost of production by $(1+t_e)^{1-\gamma}$ in H and correspondingly in F if F imposes a production tax.

Specialization in m-goods Individual varieties of the m-good are costlessly traded, which means that specialization in production of manufactured goods is detached from country-level demand. Due to trade in energy, prices p_e are the same in each country. A bundle of inputs costs $(1+t_p)^{1-\gamma}\,p_e^{1-\gamma}$ in $\mathcal{H}$ and $\left(1+t_p^*\right)^{1-\gamma}p_e^{1-\gamma}$ in $\mathcal{F}$. Hence $\mathcal{H}$ will produce varieties j for which $R(j)\geq\frac{(1+t_p)^{1-\gamma}}{\left(1+t_p^*\right)^{1-\gamma}}$ and $\mathcal{F}$ will produce the rest. This gives us an expression for the good produced in $\mathcal{H}$: $j\in[0,\bar{j}]$ where:

$$\bar{j}=\frac{A^{\theta}\,(1+t_p)^{-\theta(1-\gamma)}}{A^{\theta}\,(1+t_p)^{-\theta(1-\gamma)}+A^{*\theta}\left(1+t_p^*\right)^{-\theta(1-\gamma)}}. \tag{4}$$

with goods $j\in(\bar{j},1]$ produced in F.

Price index for manufactured goods The price index for the m-good is the average price of varieties of the m-good. We show in the appendix that we can express this as:

$$p_m=\phi p_e^{1-\gamma}\left(A^{\theta}\,(1+t_p)^{-\theta(1-\gamma)}+A^{*\theta}\left(1+t_p^*\right)^{-\theta(1-\gamma)}\right)^{1/\theta},$$

where

$$\phi=\left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-1/(\sigma-1)}.$$

Because the relevant average price is independent of where a variety is produced, $\bar{j}$ given in (4) is also the share of world spending on the m-good devoted to producers in $\mathcal{H}$ and $1-\bar{j}$ the share devoted to producers in $\mathcal{F}$. Note that this share does not depend on on σ . The so-called trade elasticity, giving the response of trade shares to factor costs, is θ.

Consumption tax With this statement of production technology, we can now give an expression for a consumption tax. A firm producing variety j and facing a consumption tax on sale, will choose an input mix to maximize revenue, taking prices as fixed. Solving this problem gives a final price$p_m(j)$ of:

$$p_m(j) = (1 + t_c)^{1-\gamma} \frac{p_e^{1-\gamma}}{A(j)}.$$

(recalling that $w = 1$). Although this looks like a simple ad valorem tax because the after-tax price of the m-good is the pre-tax price multipled by $(1 + t_c)^{1-\gamma}$,. this a result of our assumption of Cobb-Douglas production, and is not general.

[I find this notation confusing because there is nothing to tell the reader that this is an after-tax price. Can we say something like $p_m'(j) = (1 + t_c) p_m(j)$, where the prime indicates an after-tax price?]

For the H firm's sales to consumers in $\mathcal{F}$, the expression is the same, substituting the consumption tax, if any, charged to consumers in F, t_c^*:

$$p_m^*(j) = (1 + t_c^*)^{1-\gamma} \frac{p_e^{1-\gamma}}{A(j)}.$$

Note that $p_m(j)$ may not equal $p_m^*(j)$ because the consumption tax rates in H and F may not be the same (for example, with a unilateral consumption tax in H, t_c^* would be 0). A similar derivation gives the consumption tax for firms in F for sales to consumers in H and in F.

2.3 Income

Income comes from labor, rents on energy deposits, and tax revenue:

$$Y = wL + rE + T_e + T_c + T_p.$$

where T_e, T_c, and T_p, are tax revenues from extraction, consumption, and production taxes respectively. A similar equation holds for $\mathcal{F}$ and world income is the sum of the two: $Y^W = Y + Y^*$.

2.4 Equilibrium

Equilibrium consists of a price of energy p_e and an allocation of labor across the three sectors in each country so that markets clear. In particular, we need the global demand for energy $(1-\gamma)(D(p_m)Y + D^*(p_m)Y^*)$ [make sure this is okay with taxes] to equal global supply as given in expression (2).

In many cases below and particularly in our simulations, we are interested in how the equilibrium changes when we change taxes. We use the following notation.

Initial position (which may include taxes) uses notation above.

New taxes, always has a prime, so might start with a production tax t_p and change to a tax of $t_p^{'}$. Note that this includes the case where we start with no taxes because we can let $t_p = 0$.

Change notation for any variable is $\hat{x} = x^{'}/x$.

3 Effects of global taxes

Our goal is to understand the effects of taxes in H. Before turning to that analysis, we first consider global taxes. Deriviations of these propositions are in the Appendix.

Start by considering as a base case, a globally harmonized carbon tax, which means that both countries impose the same tax at the same level of production.

Proposition 1 *If the parameters of the utility function, α and σ are the same in H and F, a global production tax at rate t_p has the same effect on the price of energy (and, therefore, emissions) as a global consumption tax and as a global extraction tax at the same rates $(t_c = t_p = t_e)$. These taxes differ in the location of tax revenue, with the tax revenue tracking the location of the taxed activity. With a global tax, each country prefers the tax to be imposed where it has relatively more of the activity.*

This result arises because all energy that is extracted is used in production (in manufacturing in the basic model or directly consumed, and therefore, in home production in the more general model) and all production is consumed. The difference in the three taxes when imposed globally is where in the chain of prodution they are imposed.

Identical demand parameters are needed because the location of the tax revenue creates an income effect: without identical demand parameters, the income effect would alter consumption choices.Note that this means that with global taxes, the place that taxes are imposed, can, in theory, be chosen based entirely on adminsitrative considerations, with offsetting revenue transfers made between countries determining the distributive effects.

Note that with any of the global taxes, we have $\bar{j}' = \bar{j}$, which can be see by examining the expression for $\bar{j}$ (expression 4). This means that there is no change in the location of production.

Define a global emissions reduction goal $G = \frac{Q_e^{w\prime}}{Q_e^w} < 1$, [where $Q_e^w = Q_e + Q_e^*$ is global energy production under a baseline policy and $Q_e^{w\prime}$ is global energy production under a proposed tax; also on notation, are we going to use G or $\hat{Q}_e$? We use hats for all other changes].

Proposition 2 *Energy use, and, therefore, emissions reductions, G, depends only on the price of energy, p_e. With a production or consumption tax, the change in energy price $\hat{p}_e = \frac{p_e'}{p_e}$ to meet an global emissions goal must satisfy:*

$$G = \frac{(p_e')^{\beta/(1-\beta)} E + (p_e')^{\beta^*/(1-\beta^*)} E^*}{p_e^{\beta/(1-\beta)} E + p_e^{\beta^*/(1-\beta^*)} E^*} \left(1 + \hat{t}_e\right)^{-\beta/(1-\beta)}.$$

where$1+\hat{t}_e = \frac{1+t_e'}{1+t_e}$. In the special case where H and F have the same labor share of extraction β, we get

$$\frac{\hat{p}_e}{1+\hat{t}_e} = G^{(1-\beta)/\beta}$$

This result follows from the global supply curve in expression (2), the definition of $\hat{Q}_e$ and because , in the model, energy is costlessly traded, so there is a single global price of energy.

This result (which also holds for taxes imposed only by H) means that for any tax, we can determine its effects on emissions by determining the equilibrium price of energy under the tax. Moreover, all taxes operate to reduce emissions by lowering the global price of energy. With production and consumption taxes, the nominal price goes down. With an extraction tax, the after-tax price received by extractors goes down in exactly the same amount.

[Probably not worth mentioning, but global taxes have to be coordinated on where in the chain of production they are imposed as well as the tax rate. if H imposes an extraction tax and F imposes a production tax, we don't get the same answer as when both impose one or the other.]

4 Effects of unlateral taxes

4.1 Choice of tax rate

Although we now examine taxes unilaterally imposed by H, we compare taxes that have the same effect on global emissions. Reason is that purpose of the tax is to reduce emisisons and the resulting harms. Because temperature changes are the same regardless of which country pollutes, we assume that H cares about global emissions.

This assumption may only partially capture H's incentives. While H will care about global emissions because the harms to H from climate change are the same regardless of where the emissions come from, climate treaties often focus on emissions from each country. If H is imposing the tax primarily to comply with a treaty, H will care about domestic emissions.

With unilateral tax, extraction still depends on p_e, so the level of emissions still depends only on p_e (taking into account that with extraction taxes, extractors only keep an after-tax amount).

4.2 Location of activities

Central concern with unilateral carbon prices is shift in where activities occur. Note that this is not the same as leakage which is normally defined based on changes in emissions. For example, consumption tax (we show) has the potential to reduce consumption in H which is partially offset by an increase in consumption in F. This shift in the location of cousmption does not necessarily impliy that the location of production will change. Because emissions arise from production, there may be no leakage under a consumption tax. But even if no leakage, this shift in consumption might be a concern. Same for location of extraction.

[Is there a general statement we can make about location: each of the taxes only directly affects the location of the taxed activity (extraction, production or consumption). All other shifts occur either because of the change in p_e or an income effect due to tax revenue?]

[the below seems kind of obvious given the expressions above. Leave this out and just refer to the expressions already given?]

4.2.1 Extraction tax in H

With extraction tax in H, we get

$$Q_e = \left(\frac{p_e}{1+t_e}\right)^{\beta/(1-\beta)} E$$

[Or in hat notation]

$$\hat{Q}_e = \left(\frac{\hat{p}_e}{1+\hat{t}_e}\right)^{\beta/(1-\beta)}.$$

Q_e^* is unchanged. For a given price of energy, we see relatively lower extraction in H than previously. Should push up the price of energy (less supply), which means that in equilibrium, we see higher extraction in F, partially offsetting reduction in H.

[Worth specifying this - compare $\hat{Q}_e$ to $\hat{Q}_e^*$?]

Given an equilibrium price of energy, an extraction tax in H does not affect the location of production ($\bar{j}$ does not change) because energy is costlessly traded so the location of the extraction of energy has no effect on its use in manufacturing. The tax affects consumption only through income effects.

The value of world supply of energy with an extraction tax in H is:

$$p_e Q_e^W = (1+t_e)^{-\beta/(1-\beta)} p_e^{1/(1-\beta)} E + p_e^{1/(1-\beta)} E^* \tag{5}$$

[also put this in hat notation - doesn't simplify so we get a fraction similar to the global tax case but without the extraction tax in H.]

4.2.2 Production tax in H

No change in location of extraction (other than because p_e changes).

$\bar{j}$ becomes:

$$\bar{j}' = \frac{A^\theta \left(1+t_p'\right)^{-\theta(1-\gamma)}}{A^\theta \left(1+t_p'\right)^{-\theta(1-\gamma)} + A^{*\theta}}.$$

[Notation - it would be helpful to specify when $\bar{j}$ is with or without tax. I added the "prime" to do this.]

Changes p_m.

$$p_m = \phi p_e^{1-\gamma} \left(A^{\theta} \left(1 + t_p\right)^{-\theta(1-\gamma)} + A^{*\theta} \right)^{-1/\theta}.$$

Changes Y. Given this, consumption is as above.

Could write demand equation here and note that it equals supply in equilibrium.

4.2.3 Consumption tax in H

Price of the m-goods is now different in H and F. In H, we get

$$p_m = \phi \left(\left(1 + t_c\right) p_e \right)^{1-\gamma} \left(A^{\theta} + A^{*\theta} \right)^{-1/\theta}.$$

In F, it is unchanged from above. We get corresponding changes in demand for the m-good (adjusting the income in H for taxes.) But for any level of emissions reduction and hence p_e, the location of extraction and of manufacturing ($\bar{j}$) are unchanged.

4.3 Relationship between taxes

A key policy proposal to accompany uinilateral carbon taxes is border adjustments. In this section, we seek to understand the relationship between border adjustments and the three types of taxes considered so far.

Border adjustments are (1) a tax (at rate t_b) on the value of energy embodied in H's imports of the m-good and (2) a tax rebate (also at rate t_b) on the value of energy embodied in H's exports of the m-good. The border tax adjustment tax rate need not be the same as the tax rate on production, Instead, we allow $t_b \in [0, t_p]$. [We do, however, restrict the tax rate on imports to be the same as the rate of rebate on exports.]

With no border adjustments ($t_b = 0$), the tax is a pure production tax (at rate t_p). We refer to the case where $t_b = t_p$ as full border taxes and $t_b \in (0, t_p)$ as partial border taxes.

Define the following:

$$1 + \tilde{t}_p = \frac{1 + t_p}{1 + t_b}.$$

[Define two taxes as equal if they (1) produce the same equilibrium price of energy and allocation of labor in each country and (2) the same utility for all individuals in each country.]

We can characterize the relationship between production taxes, consumption taxes, and border adjustments as follows:

Proposition 3 *The combination of a production tax at rate t_p and border adjustments at rate t_b is equivalent to a consumption tax at rate $t_c = t_b$ and a residual production tax at rate $\tilde{t}_p$.*

To see why this result arises, consider a consumption tax imposed only in H. Under this tax, consumers in H pay a tax on all manufactured goods that they purchase regardless of where they were produced. Compare that to a production tax in H with border adjustments. Under a production tax, producers in H must pay a tax on their production. If they sell the good to consumers in H, there is no border adjustment, and the tax remains. If they sell the good to consumers in F, the tax is removed, so of the goods produced in H, only goods consumed in H continue to bear a tax. Similarly, producers in F do not initially pay a tax when they produce a good, but if, and only if, they sell it to consumers in H, border adjustments impose a tax. Therefore, under a production tax with border adjustments, all goods consumed in H and only those goods bear a tax

Proposition 4 ***Corollary 5*** *A consumption tax in H at rate t_c is equal to a production tax in H at rate $t_p = t_c$ plus full border adjustments (i.e., $t_b = t_p$).*

This follows immediately from the proposition by setting $t_b = t_p$.

[I thought we had a similar result for extraction taxes but I don't see it in the current notes.]

Put expressions for $R(j)$ and $\bar{j}$ here using this notation (expression 39 in notes): these just replace t_p with $\tilde{t}_p$ (and because the tax is unilateral, set $t_p^* = 0$) but are otherwise identical. Just say that?

Something about BAs under an extraction tax? Can tax extraction and border adjust for imports and exports of energy to produce tax on energy used in H, so a produc tion tax. Or could border adjust for energy content of the m-goods, to produce a consumption tax.

4.4 Leakage

Conventional definition of leakage is $-\Delta F/\Delta H$. This can be hard to interpret because if we hold global emissions fixed, the both the numerator and the denominator change when we change the type of tax.If ΔF changes, then ΔH must change too if emissions are held fixed. Often, leakage measures are compared holding the nominal tax rate fixed, which makes them very hard to compare.

Propose an alternative measure of leakage, which we call modified leakage: $-\Delta F / (\Delta H + \Delta F)$. With this definition, the denominator is the decline in global emissions, so if we compare two taxes set to be equally effective, we know that the denominator is the same. Can easily be converted into the conventional measure if desired.

Also leads to simplification because the denominator is determined by p_e: all leakage measures will assume the same assumptions about price of energy.

Leakage often said to include two effects: a fuel price effect and a location effect. Fuel price effect arises because carbon tax in H lowers the global price of energy, leading to increased consumption in F. With modified leakage, the change in fuel price is held constant across all scenarios. Depending on tax, consumers in F see its effect differently, so behavior of those consumers is not held fixed. E.g., with production tax, consumers in F see a tax on the goods that they purchase from H while with a consumption tax they do not. But because price of energy is held constant in all scenarios, we do not try to isolate a fuel price effect.

Emissions in our model come only from manufacturing. Convenient to have terms for emissions from manufacturing.

We want an expression for the total use of energy in H for goods consumed in H, which we denote M_H. We know from the demand function that firms in H receive $\bar{j} D(p_m) Y$ by selling m-goods in H. The energy share is $(1-\gamma)$, giving us $p_e M_H = (1-\gamma) \bar{j} D(p_m) Y$. We can similar define the following:

$$
\begin{aligned}
p_e M_H &= (1-\gamma) \bar{j} D(p_m) Y \\
p_e M_F &= (1-\gamma)(1-\bar{j}) D^*(p_m^*) Y^* \\
p_e M_H^* &= (1-\gamma) \bar{j} D(p_m) Y \\
p_e M_F^* &= (1-\gamma)(1-\bar{j}) D^*(p_m^*) Y^*
\end{aligned} \tag{6}
$$

We then have $M_H + M_F = M_W$, and $M_H^* + M_F^* = M_W$, and $M_W + M_F^* = M_W = Q_e^W$.

Modified leakage for a given global reduction in emissions G is:

$$\tilde{l}_P = \frac{\left(M_e^{HF'} + M_e^{FF'}\right) - \left(M_e^{HF} + M_e^{FF}\right)}{\left(M_e^{HH} + M_e^{FH} + M_e^{HF} + M_e^{FF}\right)(1-G)}$$

Proposition 6 *Leakage with a production or consumption tax is:*

$$\tilde{l}_P = \frac{1-\bar{j}}{1-G}\left(\frac{G}{\bar{j}\left(1+\tilde{t}_p'\right)^{-\theta(1-\gamma)-1}+1-\bar{j}} - 1\right)$$

Recalling that $\tilde{t}_p'$ is the residual production tax if there are partial border tax adjustments, we had previously that if $\tilde{t}_p' = 0$, the tax is a pure consumption tax. We therefore get:

[This proposition assumes $\beta = \beta^*$, right?]

Corollary 7 *With a pure consumption tax, leakage is equal to* $-\left(1-\bar{j}\right) < 0$.

Need to explain why we get negative leakage when nobody else does (other than DF who gets it for entirely different reasons). Basic idea here: with a consumption tax, producers in both H and F can sell fewer m-goods to consumers in H. Energy prices go down, which means producers in both countries sell more to consumers in F but to generate a net reduction, producers have to produce overall less. Producers in F will be affected the same way producers in H will. Home bias could offset this somewhat and most other models have home bias built in, although it is not clear that this is the full explanation for the difference.

4.5 Welfare

Design of a carbon tax, including choice to impose border adjustments must be based on welfare, not on a measure of leakage or of the location of activities.

Define welfare as effective consumption: $\frac{Y}{p}$, where p is the price index for the m-good and the l-good: $p = \left(\alpha p_m^{-(\sigma-1)} + (1-\alpha)\right)^{-1/(1-\sigma)}$.

4.5.1 F's welfare

Proposition 8 *F is better off with a consumption tax in H. That is, if H is going to impose a unialteral production tax that acheives a desired reduction in global emissions, F is bette off if H imposes border adjustments as well.*

The reasoning is straightforward, Holding emissions constant means holding p_e fixed and by construction, the price of the l-good is fixed at 1. Because income in F is from wages and rents from energy extraction, income in F is fixed. This leaves only the price of the m-good to vary. With a production tax, any varieties of the m-good produced in H that consumers in F purchase will bear the tax while with a border adjustment, the tax is removed. Therefore, F is better off with border adjustments.

4.5.2 H's welfare

Proposition 9 *If H sufficiently dominates manufacturing, H is better off without border adjustments.*

Reasoning:

5 Simulations

We only need the share notation (the π's) here, because their use is for calibration, right? No need to introduce earlier. A few derivations use it but I think all can easily be restated without. But needed for calibration of the simulations, so introduce here.

Possible simulations:

- Beta v. leakage under a production tax (and possibly, under a consumption tax).
- leakage under a production tax with variable border adjustments
- H's welfare under production tax plus variable border adjustments (i.e., BA's on x axis, welfare on y axis).
- leakage v. relative size of H
- Welfare gains v. relative size of H (under production tax?): tells us the benefits of increasing the size of the taxing coalition.

6 Comparisons to CGE results

Want to compare our results to the summary of the CGE results given above;

- Leakage rates most often in the range of 5% to 20%, with some outliers.
- The larger the taxing coalition, the lower the leakage.
- BA's reduce leakage substantially
- Most important variable in determining effects of unilateral carbon price is the energy supply elasticity.
- Distinguish two drivers of leakage: the fuel price effect (in which lower demand for fossil fuels in the taxing region suppresses prices, increasing demand on the non-taxing regions) and the competitiveness effect [different name?] (in which increased costs for industry in the taxing region causes a shift to the non-taxing region).