Trade and Carbon Taxes: An Analytic General Equliibrium Model

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Abstract

Blah blah blah

1 Introduction

Leakage is a central concern to design of climate change policy.

Originally, concern about developed countries adopting carbon prices while developing do not. Even without such extreme differentiation, still concern about differential carbon prices (whether explicit, such as through a tax, subsidies, or cap and trade, or implicit, such as through regulations).

As a result, large number of studies. Mostly CGE. Small handful of analytic models.

CGE have the advantage of detailed representations of the economy, often with gret detail about the energy sector. Disadvantage is that they are non-transparent.

Basic idea is to develop an analytic model that captures drivers of leakage. Use DFS structure to model specialization in manufacturing across countries. See how taxes change patterns of extraction, production, and consumption of energy and goods created with energy.

Derive a number of analytic results. In other cases, calibrate and numerically solve model. In these cases, look for results that are robust to choices of parameters.

1.1 Prior literature

1.1.1 CGE approaches

Key findings:

- Leakage rates most often in the range of 5% to 20%, with some outliers.
- The larger the taxing coalition, the lower the leakage.
- BA's reduce leakage substantially
- Most important variable in determining effects of unilateral carbon price is the energy supply elasticity.
- Distinguish two drivers of leakage: the fuel price effect (in which lower demand for fossil fuels in the taxing region suppresses prices, increasing demand on the non-taxing regions) and the competitiveness effect [different name?] (in which increased costs for industry in the taxing region causes a shift to the non-taxing region).

1.1.2 Analytic approaches

Small body of work has used analytic models to understand leakage. Markusen (1975), which focuses on environmental harms more generally, is earliest example.

Jakob, Marschinski and Hubler (2013) use a version of Markusen (1975), modified to allow different sectors in the economy to have different emissions intensities. Their key finding is that the relative intensities of the exporting and non-exporting sectors in the non-taxing region affect whether BA's reduce leakage.

Bohringer Lange and Rutherford (2014) use an analytic model to consider differential carbon price, such as lower taxes on trade-exposed sectors. They decompose the effects of differential taxes into leakage effects and terms of trade effects. They incorporate this decomposition into a CGE model to produce guidelines for carbon pricing.

Fischer and Fox (2012).

Fullerton, Karney and Baylis (2014) use an analytic general equilibrium model to analyze how capital used in abatement of emissions effects leakage.

They find that if abatement is resource intensive (such as requiring substantial capital), a domestic carbon tax can produce negative leakage because it increases global resource costs and hence investment in non-taxing regions.

2 Model structure

There are two countries or regions, Home ($\mathcal{H}$) and Foreign ($\mathcal{F}$). Each is endowed with energy deposits (E, and E^*, in H and F respectively) and labor (L and L^*, measured in efficiency units). (Throughout, variables related to $\mathcal{F}$ are denoted with a *.) Representative individuals or firms in each country produce goods, receive income from the use of their endowments in production, and use the income to purchase and consume the goods. The countries differ only in their endowments of deposits and labor, in the parameters of the utility functions of individuals in that country, and in their labor share of energy extraction.

[expand the above paragraph to give better description]

2.1 Production

Each country produces extracts deposits using labor to produce usable energy. Each country also produces two types of consumption goods: a good produced only with labor, the l-good or services, and manufactured goods, the m-goods, which are produced using energy and labor. All goods are costlessly traded (but labor and deposits are immobile). We parameterize production as follows.

2.1.1 Energy extraction

Firms in $\mathcal{H}$ extract energy using labor L_e and desposits E in a Cobb-Douglas production function with labor share β:

$$Q_e = \left(\frac{L_e}{\beta}\right)^{\beta} E^{1-\beta}.$$

(We derive this production function from more primitive assumptions about extraction in the Appendix.). As a result, firms hire labor:

$$L_e = \beta \left(\frac{p_e}{w}\right)^{1/(1-\beta)} E$$

where w is the wage rate in H, and p_e is the global price of energy. Firms extract a quantity:

$$Q_e = \left(\frac{p_e}{w}\right)^{\beta/(1-\beta)} E.$$

The problem is the same in $\mathcal{F}$, replacing β with β^*, w with w^*, E with E^*. The solution in $\mathcal{F}$ is L_e^* and Q_e^*. Differences in β and β^* capture differences in the labor share of energy extraction, such as between a country like Saudi Arabia where energy is cheap to extract (low β) and Canada, where energy is costly to extract (high β).

World supply of energy is the energy produced in H and F:

$$Q_e^W = Q_e + Q_e^* = \left(\frac{p_e}{w}\right)^{\beta/(1-\beta)} E + \left(\frac{p_e}{w^*}\right)^{\beta^*/(1-\beta^*)} E^*.$$

2.1.2 Manufacturing

Firms in each country use energy and labor to produce manufactured goods or m-goods. To model specialization in manufacturing and trade, we use a version of the Ricardian model introduced by Dornbusch, Fisher, and Samuelson (1977). In particular, let there be a continuous variety of m-goods indexed by $j \in [0, 1]$. The production function for variety j in $\mathcal{H}$ is:

$$Q_m(j) = A(j) \left(\frac{L_m(j)}{\gamma}\right)^{\gamma} \left(\frac{M(j)}{1-\gamma}\right)^{1-\gamma},$$

where $A(j)$ is $\mathcal{H}$'s productivity in variety j, $M(j)$ is energy, and γ is labor's share. There are many price-taking producers have access to the technology to produce each variety j. In $\mathcal{F}$ the production function has the same form, with productivity $A^*(j)$.

Productivity for each variety is

$$A(j) = \frac{A}{j^{1/\theta}}$$

in $\mathcal{H}$ and

$$A^*(j) = \frac{A^*}{(1-j)^{1/\theta}}$$

in $\mathcal{F}$. The parameters A and A^* capture absolute advantage in $\mathcal{H}$ and $\mathcal{F}$. The relative productivity of the two countries in producing variety j is:

$$R(j) = \frac{A(j)}{A^*(j)} = \frac{A}{A^*} \left(\frac{j}{1-j}\right)^{-1/\theta}. \tag{1}$$

For $j < j'$, $\mathcal{H}$ has a comparative advantage in variety j and $\mathcal{F}$ in j'. The parameter θ captures (inversely) the strength of comparative advantage. As $\theta \to \infty$ relative productivity does not vary across varieties.

Specialization in m-goods Individual varieties of the m-good are costlessly traded, which means that specialization in production of manufactured goods is detached from country-level demand. Due to trade in energy, prices p_e are the same in each country. A bundle of inputs costs $w^\gamma p_e^{1-\gamma}$ in $\mathcal{H}$ and $w^{*\gamma} p_e^{1-\gamma}$ in $\mathcal{F}$. Hence $\mathcal{H}$ will produce varieties j for which $R(j) \geq (w/w^*)^\gamma$ and $\mathcal{F}$ will produce the rest. In other words, $\mathcal{H}$ produces all varieties in the interval $[0, \bar{j}]$ and $\mathcal{F}$ in the interval $(\bar{j}, 1]$ where:

$$\bar{j} = \frac{A^\theta w^{-\gamma\theta}}{A^\theta w^{-\gamma\theta} + A^{*\theta} w^{*-\gamma\theta}}. \tag{2}$$

Given specialization, we can solve for the competitive price of each manufactured variety. For $j \leq \bar{j}$, the variety is produced in $\mathcal{H}$, so that:

$$p_m(j) = \frac{w^\gamma p_e^{1-\gamma}}{A(j)}, \tag{3}$$

while for $j > \bar{j}$, the variety is produced in $\mathcal{F}$, with:

$$p_m(j) = \frac{w^{*\gamma} p_e^{1-\gamma}}{A^*(j)}.$$

With no trade cost we have the law of one price; these variety-level prices apply for consumers in either country.

Price index for manufactured goods The price index for the m-good is the average price of varieties of the m-good. We show in the online appendix that we can express this as:

$$p_m = \phi p_e^{1-\gamma} \left(A^\theta w^{-\gamma\theta} + A^{*\theta} w^{*-\gamma\theta} \right)^{1/\theta},$$

where

$$\phi = \left(\frac{\theta}{\theta - (\rho - 1)} \right)^{-1/(\rho-1)}.$$

Because the relevant average price is independent of where a variety is produced, $\bar{j}$ given in (2) is also the share of world spending on the m-good devoted to producers in $\mathcal{H}$ and $1-\bar{j}$ the share devoted to producers in $\mathcal{F}$. Note that this share does not depend on on ρ . The so-called trade elasticity, giving the response of trade shares to factor costs, is θ.

Energy use in manufacturing Firms in H manufacture varieties $[0, \bar{j}]$ and for each variety, use $M(j)$ of energy. Hence, total energy used by firms in H is:

$$M_W = \int_0^{\bar{j}'} M(j)\, dj,$$

Similarly, firms in F use M_W^*, integrating from $(1-\bar{j})$ to 1. Total energy used in the manufacturing sector is: $M_W + M_W^* = M$.

2.1.3 Services (the l-good)

Production of the l-good in a given country is given by

$$Q_l = L_l.$$

Labor is measured efficiency units, so this formulation can capture differences in l-sector productivity across countries.

2.2 Consumption

A representative individual in each country receives rental income from the ownership of the energy deposits and receives wages w and w^* in exchange for services. The individuals use this income to purchase manufactured goods and services, maximizing a constant-elasticity of substitution utility function.

2.2.1 Income

Income comes from labor and rents on energy deposits:

$$Y = wL + rE = wL + (1-\beta)\, p_e Q_e = wL + (1-\beta)\, p_e^{1/(1-\beta)} E.$$

If H imposes taxes, income also includes net tax revenue, which is rebated lump sum. A similar equation holds for $\mathcal{F}$ and world income is the sum of the two: $Y^W = Y + Y^*$.

2.2.2 Utility

Preferences are represented by a CES utility function:

$$U\left(C_c, C_l\right) = \left(\alpha^{1/\sigma} C_c^{(\sigma-1)/\sigma} + (1-\alpha)^{1/\sigma} C_l^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}. \tag{4}$$

where the c-good is a composite of the direct consumption of energy C_e and of the manufactured good C_m:

$$C_c = \left(\frac{C_m}{\eta}\right)^{\eta} \left(\frac{C_e}{1-\eta}\right)^{1-\eta},$$

and consumption nof the m-good is an aggregate of the individual varieties:

$$C_m = \left(\int_0^1 C_m(j)^{(\rho-1)/\rho} dj\right)^{\rho/(\rho-1)}.$$

To simplify the presentation, in what follows, we set $\eta = 0$, (in which case, $C_c = C_m$, and we use C_m as our convention). This assumption means that that individuals do not directly consume energy. The qualitative results do not change with this simplification.

Define the demand function for consumers in H for the m-good:

$$D\left(p_m\right) = \frac{\alpha p_m^{-(\sigma-1)}}{a p_m^{-(\sigma-1)} + (1-\alpha)\, w^{-(\sigma-1)}}$$

With this utility function (and no taxes), spending in H on the m-goods is:

$$p_m C_m = D\left(p_m\right) Y.$$

A parallel result holds in F.

2.3 Equilibrium without taxes

An equilibrium is an allocation of labor in H across the l-good, energy extraction,and the composite m-good:

$$L = L_l + L_e + \int_0^1 L_m(j)\,dj,$$

a parallel condition in F, and a price of energy that clears the market. We set labor in F as the numeraire and only consider equilibria where both countries produce the l-good, so we have $w^* = w = 1$.

Demand for energy is $(1-\gamma)$ of the demand for m-goods, $(1-\gamma)\left(D\left(p_m\right)Y + D^*\left(p_m\right)Y^*\right).$

With no taxes, the equilibrium is straighforward. Setting the supply of energy of equal to that demand, we get:

$$p_e^{1/(1-\beta)}E + p_e^{1/(1-\beta^*)}E^* = (1-\gamma)\left(D\left(p_m\right)Y + D^*\left(p_m\right)Y^*\right). \tag{5}$$

Solving for the price of energy determines the labor used in extraction. In H, we get:

$$L_e = \beta p_e^{1/(1-\beta)}E,$$

and likewise for F. The price of energy also determines the demand for m-goods and, because of Cobb-Douglas production, we know the labor share. A fraction $\bar{j}$ is produced in H, giving us:

$$L_m = \bar{j}\gamma\left(D(p_c)Y + D^*(p_c)Y^*\right)$$

A similar expression holds for F, except using $(1-\bar{j})$. Labor used to produce the l-good is the residual.

3 Taxes

Our goal is to consider how this equilibrium changes with taxes. We allow taxes to be imposed at each level of production. In particular, we consider taxes (1) on the extraction of energy, (2) on the use of energy in production, and (3) on the energy content of consumption. We call these three taxes an extraction tax, t_e, a production tax, t_p, and a consumption tax t_c. Taxes can be imposed in both H and F or just in H. All taxes are ad valorem. We also allow H to impose border tax adjustments (defined below).

Without loss of generality, we assume that emissions are one-for-one with energy use, which means that taxes on emissions are simply taxes on energy. We do not have any non-polluting sources of energy in the model, which means we cut off that source of substitution in response to taxation. Taxes on energy will, however, cause poducers to substitute toward non-polluting labor, so a similar effect is seen in the model if we think of labor as the set of non-polluting inputs.

We first define each type of tax and border tax adjustments. Then we consider the effects of each tax on the location of activities, on leakage rates, and on welfare. In the next section, we provide simulations that allow us to compute the optimal set of taxes and border adjustments to achieve a given emisisons goal.

[Notation: we need to sort this out. Should we use a prime for pretty much everything below to indicate that prices and quantities are tax-adjusted? Without some difference in notation between no-tax and tax, I get confused. But carrying primes around everywhere might not be ideal.]

3.1 Extraction tax

An extraction tax is an ad valorem tax on the extraction of energy. If the market price of energy is p_e, energy extraction firms receive only $p_e / (1 + t_e)$. Firms in H extract:

$$Q_e = \left(\frac{p_e}{1+t_e}\right)^{\beta/(1-\beta)} E.$$

Similar expressions hold for F if F imposes an extraction tax. After-tax revenue in the extraction sector is

$$R_e = \frac{p_e}{1+t_e} Q_e = \left(\frac{p_e}{1+t_e}\right)^{1/(1-\beta)} E,$$

and tax revenue is $T_e = t_e R_e$.

3.2 Production tax

A production tax is a tax on energy used in manufacturing. Energy used in manufacturing now costs $p_e (1 + t_p)$. As a result, producers shift away from energy and toward labor. The cost of manufactured goods rises by the factor $(1 + t_p)^{1-\gamma}$.

3.3 Consumption tax

A consumption tax is a tax imposed on the direct consumption of energy and on the consumption of a good produced using energy based on the energy (or emissions) from the production of the good. For example, a consumption tax on an automobile is a tax on the energy used to produce the steel in the automobile as well as on any fuel used to drive the automobile. It is a tax on the total energy used to obtain transportation services.

A consumption tax will affect how goods are manufactured because it raises the relative price of energy inputs. Therefore, to derive an expression for a consumption tax, we must determine how manufacturers will respond to the tax in equilibrium.

Consider a firm's sales to customers in H. The firm's problem is to choose inputs of labor l and energy e to maximize revenue per unit, taking as given the competitive price fo the good $p\left(j\right)$ in H and $p^{*}\left(j\right)$ in F, inclusive of ad valorem taxes t_c and t_{c^*} on consumption:

$$\max_{l,e}\left\{(p_m(j)-t_c p_e e)-wl-p_e e\right\}$$

subject to:

$$A(j)\left(\frac{l}{\gamma}\right)^{\gamma}\left(\frac{e}{1-\gamma}\right)^{1-\gamma}=1.$$

Solving this problem gives a price of:

$$p_m(j)=(1+t_c)^{1-\gamma}\,\frac{p_e^{1-\gamma}}{A(j)}.$$

(setting $w=1$). This looks like a simple ad valorem tax because the after-tax price of the m-good is the pre-tax price multipled by $(1+t_c)^{1-\gamma}$. Note, however, that this feature is a result of our assumption of Cobb-Douglas production, and is not general.

[I find this notation confusing because there is nothing to tell the reader that this is an after-tax price. Can we say something like $p_m'\left(j\right)=\left(1+t_c\right)p_m\left(j\right)$, where the prime indicates an after-tax price?]

For the firm's sales to consumers in $\mathcal{F}$ the same derivation applies [where t_c^* is the consumption tax, if any in F], so that:

$$p_m^*(j)=(1+t_c^*)^{1-\gamma}\,\frac{w^{\gamma}p_e^{1-\gamma}}{A(j)}.$$

Note that $p_m(j)$ may not equal $p_m^*(j)$ because the consumption tax rates in H and F may not be the same (and with a unilateral consumption tax in H, t_c^* would be 0). A similar derivation gives the consumption tax for producers in F for sales to consumers in H and F.

3.3.1 Border adjustments

Border adjustments are (1) a tax (at rate t_b) on the value of energy embodied in H's imports of the m-good and (2) a tax rebate (also at rate t_b) on the value of energy embodied in H's exports of the m-good. The border tax adjustment tax rate need not be the same as the tax rate on production, Instead, we allow $t_b' \in [0, t_p']$. [We do, however, restrict the tax rate on imports to be the same as the rate of rebate on exports.]

With no border adjustments ($t_b' = 0$), the tax is a pure production tax (at rate t_p'). We refer to the case where $t_b' = t_p'$ as full border taxes and $t_b' \in \left(0, t_p'\right)$ as partial border taxes. Below, we show that a production tax with a full border tax is the same as a pure consumption tax (at rate t_b').

It is often helpful to parameterize taxes in terms of the border adjustment t_b' and the effective production tax $\tilde{t}_p'$, satisfying:

$$1 + \tilde{t}_p = \frac{1 + t_p}{1 + t_b}.$$

In this space, there are no constraints on the tax rates, other than $t_b \geq 0$ and $\tilde{t}_p \geq 0$.

Something about BAs under an extraction tax? Can tax extraction and border adjust for imports and exports of energy to produce tax on energy used in H, so a producdtion tax. Or could border adjust for energy content of the m-goods, to produce a consumption tax.

4 Effects of global taxes

Our goal is to understand the effects of taxes in H. Before turning to that analysis, we first consider global taxes. Deriviations of these propositions are in the Appendix.

Suppose that considering as a base case, a globallly harmonized carbon tax, which means that both countries impose a tax at the same level of production and at the same rate.

Proposition 1 *If the parameters of the utility function, α and σ are the same in H and F, a global production tax at rate t_p has the same effect on the price of energy (and, therefore, emissions) as a global consumption tax and as a global extraction tax at the same rates $(t_c = t_p = t_e)$. These taxes differ in the location of tax revenue, with the tax revenue tracking the location of the taxed activity. With a global tax, each country prefers the tax to be imposed where it has relatively more of the activity.*

This result arises because all energy that is extracted is used in production (in manufacturing in the basic model or directly consumed, and therefore, in home production in the more general model) and all production is consumed. The difference in the three taxes when imposed globally is where in the chain of prodution they are imposed.

Identical demand parameters are needed because the location of the tax revenue creates an income effect: without identical demand parameters, the income effect would alter consumption choices.Note that this means that with global taxes, the place that taxes are imposed, can, in theory, be chosen based entirely on adminsitrative considerations, with offsetting revenue transfers made between countries determining the distributive effects.

Proposition 2 *The equilibrium with global taxes is the the same as specified by (5) substituting $p_e/(1+t_e)$ for p_e.In particular, with a global tax, $\bar{j}' = \bar{j}$. This follows immediately from the equivalence of the three taxes, the expression for energy supply with an extracton tax, and the equilibrium conditions.*

Define a global emissions reduction goal $G = \frac{Q_e^{w\prime}}{Q_e^w} < 1$, [where $Q_e^w = Q_e + Q_e^*$ is global energy production under a baseline policy and $Q_e^{w\prime}$ is global energy production under a proposed tax; also on notation, are we going to use G or $\hat{Q}_e$? We use hats for all other changes].

Proposition 3 *[need to assume common β across countries for this, right?] Energy use, and, therefore, emissions reductions, G, depends only on the price of energy, p_e. With a production or consumption tax, the change in energy price $\hat{p}_e = \frac{p_e'}{p_e}$ to meet an global emissions goal must satisfy:*

$$\hat{p}_e = G^{(1-\beta)/\beta}$$

[With an extraction tax, we need $\frac{\hat{p}_e}{1+t_e} = G^{(1-\beta)/\beta}$, right?] [Better to express this allowing the β's to be different?]

This result arises because, in the model, energy is costlessly traded, so there is a single global price of energy regardless of whether there are global taxes or only taxes in H. In addition, given the production function for energy, extraction depends only on p_e and w. Because we are assuming that $w = w^* = 1$, the price of energy determines extraction. The nominal price of energy, p_e, differs with an extraction tax than with a production or consumption tax because with an extraction tax, the price includes taxes paid by extractors while with production or consumption taxes it does not.

This result (which also holds for taxes imposed only by H) means that for any tax, we can determine its effects on emissions by determining the equilibrium price of energy under the tax. Moreover, all taxes operate to reduce emissions by lowering the global price of energy. With production and consumption taxes, the nominal price goes down. With an extraction tax, the after-tax price received by extractors goes down in exactly the same amount.

[Probably not worth mentioning, but global taxes have to be coordinated on where in the chain of production they are imposed as well as the tax rate. if H imposes an extraction tax and F imposes a production tax, we don't get the same answer as when both impose one or the other.]

5 Effects of taxes only in H

5.1 Preliminaries

Although we now assume that only H imposes a carbon tax, we compare taxes that have the same effect on global emissions. Reason is that purpose of the tax is to reduce emisisons and the resulting harms. Because temperature changes are the same regardless of which country pollutes, we assume that H cares about global emissions.

This assumption may only partially capture H's incentives. While H will care about global emissions because the harms to H from climate change are the same regardless of where the emissions come from, climate treaties often focus on emissions from each country. If H is imposing the tax primarily to comply with a treaty, H will care about domestic emissions.

With unilateral tax, extraction still depends on p_e, so the level of emissions still depends only on p_e (taking into account that with extraction taxes, extractors only keep an after-tax amount). Therefore, Proposition x still holds.

5.2 Location of activities

Central concern with unilateral carbon prices is shift in where activities occur. Note that this is not the same as leakage which is normally defined based on changes in emissions. For example, consumption tax (we show) has the potential to reduce consumption in H which is partially offset by an increase in consumption in F. Does not mean any shift in where production, and, therefore, emissions occur. Even if no leakage, might be a concern. Same for location of extraction.

[Is there a general statement we can make about location: each of the taxes only directly affects the location of the taxed activity (extraction, production or consumption). All other shifts occur either because of the change in p_e or an income effect due to tax revenue?]

5.2.1 Extraction tax in H

With extraction tax in H, we get

$$Q_e = \left(\frac{p_e}{1+t_e}\right)^{\beta/(1-\beta)} E$$

while Q_e^* is unchanged. For a given price of energy, we see relatively lower extraction in H than previously. Should push up the price of energy (less supply), which means that in equilibrium, we see higher extraction in F, partially offsetting reduction in H.

[Worth specifying this - compare $\hat{Q}_e$ to $\hat{Q}_e^*$?]

Given an equilibrium price of energy, an extraction tax in H does not affect the location of production ($\bar{j}$ does not change) because energy is costlessly traded so the location of the extraction of energy has no effect on its use in manufacturing. The tax affects consumption only through income effects.

The value of world supply of energy with an extraction tax in H is:

$$p_e Q_e^W = (1+t_e)^{-\beta/(1-\beta)} p_e^{1/(1-\beta)} E + p_e^{1/(1-\beta)} E^* \tag{6}$$

5.2.2 Production tax in H

No change in location of extraction (other than because p_e changes).

Shift in j-bar:

$$\bar{j}' = \frac{A^{\theta}\left(1+t_p\right)^{-\theta(1-\gamma)}}{A^{\theta}\left(1+t_p\right)^{-\theta(1-\gamma)} + A^{*\theta}}.$$

[Notation - it would be helpful to specify when $\bar{j}$ is with or without tax. I added the "prime" to do this.]

Changes p_m.

$$p_m = \phi p_e^{1-\gamma}\left(A^{\theta}\left(1+t_p\right)^{-\theta(1-\gamma)} + A^{*\theta}\right)^{-1/\theta}.$$

Changes Y. Given this, consumption is as above.

Could write demand equation here and note that it equals supply in equilibrium.

5.2.3 Consumption tax in H

Price of the m-goods is now different in H and F. In H, we get

$$p_m = \phi\left(\left(1+t_c\right)p_e\right)^{1-\gamma}\left(A^{\theta} + A^{*\theta}\right)^{-1/\theta}.$$

In F, it is unchanged from above. We get corresponding changes in demand for the m-good (adjusting the income in H for taxes.) But for any level of emissions reduction and hence p_e, the location of extraction and of manufacturing ($\bar{j}$) are unchanged.

5.3 Relationship between taxes

[Define two taxes as equal if they (1) produce the same equilibrium price of energy and allocation of labor in each country and (2) the same utility for all individuals in each country.]

Proposition 4 *A consumption tax in H at rate t_c is equal to a production tax in H at rate $t_p = t_c$ plus full border adjustments.*

To see why this result arises, consider a consumption tax imposed only in H. Under this tax, consumers in H pay a tax on all manufactured goods that they purchase regardless of where they were produced. Compare that to a production tax in H with border adjustments. Under a production tax, producers in H must pay a tax on their production. If they sell the good to consumers in H, there is no border adjustment, and the tax remains. If they sell the good to consumers in F, the tax is removed, so of the goods produced in H, only goods consumed in H continue to bear a tax. Similarly, producers in F do not initially pay a tax when they produce a good, but if, and only if, they sell it to consumers in H, border adjustments impose a tax. Therefore, under a production tax with border adjustments, all goods consumed in H and only those goods bear a tax.

[I thought we had a similar result for extraction taxes but I don't see it in the current notes.]

5.4 Leakage

Conventional definition of leakage is $-\Delta F/\Delta H$. This can be hard to interpret because if we hold global emissions fixed, the both the numerator and the denominator change when we change the type of tax.If ΔF changes, then ΔH must change too if emissions are held fixed. Often, leakage measures are compared holding the nominal tax rate fixed, which makes them very hard to compare.

Propose an alternative measure of leakage, which we call modified leakage: $-\Delta F/\left(\Delta H+\Delta F\right)$. With this definition, the denominator is the decline in global emissions, so if we compare two taxes set to be equally effective, we know that the denominator is the same. Can easily be converted into the conventional measure if desired.

Also leads to simplification because the denominator is determined by p_e: all leakage measures will assume the same assumptions about price of energy.

Leakage often said to include two effects: a fuel price effect and a location effect. Fuel price effect arises because carbon tax in H lowers the global price of energy, leading to increased consumption in F. With modified leakage, the change in fuel price is held constant across all scenarios. Depending on tax, consumers in F see its effect differently, so behavior of those consumers is not held fixed. E.g., with production tax, consumers in F see a tax on the goods that they purchase from H while with a consumption tax they do not. But because price of energy is held constant in all scenarios, we do not try to isolate a fuel price effect.

Emissions in our model come only from manufacturing. Convenient to have terms for emissions from manufacturing. Define the spending in H on energy used during production in H, and similary for production in F and spending in F

$$
\begin{aligned}
p_e M_H &= (1-\gamma)\,\bar{j} D\,(p_m)\,Y \\
p_e M_F &= (1-\gamma)\,(1-\bar{j})\,D^*\,(p_m^*)\,Y^* \\
p_e M_H^* &= (1-\gamma)\,\bar{j} D\,(p_m)\,Y \\
p_e M_F^* &= (1-\gamma)\,(1-\bar{j})\,D^*\,(p_m^*)\,Y^*
\end{aligned} \tag{7}
$$

We then have $M_H + M_F = M_W$, and $M_H^* + M_F^* = M_W$, and $M_W + M_F^* = M_W = Q_e^W$.

Modified leakage for a given global reduction in emissions G is:

$$\tilde{l}_P = \frac{\left(M_e^{HF'} + M_e^{FF'}\right) - \left(M_e^{HF} + M_e^{FF}\right)}{\left(M_e^{HH} + M_e^{FH} + M_e^{HF} + M_e^{FF}\right)(1-G)}$$

Proposition 5 *Leakage with a production or consumption tax is:*

$$\tilde{l}_P = \frac{1-\bar{j}}{1-G}\left(\frac{G}{\bar{j}\left(1+\tilde{t}_p'\right)^{-\theta(1-\gamma)-1} + 1 - \bar{j}} - 1\right)$$

Recalling that $\tilde{t}_p'$ is the residual production tax if there are partial border tax adjustments, we had previously that if $\tilde{t}_p' = 0$, the tax is a pure consumption tax. We therefore get:

Corollary 6 *With a pure consumption tax, leakage is equal to* $-(1-\bar{j}) < 0$.

Need to explain why we get negative leakage when nobody else does (other than DF who gets it for entirely different reasons). Basic idea here: with a consumption tax, producers in both H and F can sell fewer m-goods to consumers in H. Energy prices go down, which means producers in both countries sell more to consumers in F but to generate a net reduction, producers have to produce overall less. Producers in F will be affected the same way producers in H will. Home bias could offset this somewhat and most other models have home bias built in, although it is not clear that this is the full explanation for the difference.

5.5 Welfare

Design of a carbon tax, including choice to impose border adjustments must be based on welfare, not on a measure of leakage or of the location of activities.

Define welfare as effective consumption: $\frac{Y}{p}$, where p is the price index for the m-good and the l-good: $p = \left(\alpha p_m^{-(\sigma-1)} + (1-\alpha)\right)^{-1/(1-\sigma)}$.

5.5.1 F's welfare

Proposition 7 *F is better off with a consumption tax in H. That is, if H is going to impose a unialteral production tax that acheives a desired reduction in global emissions, F is bette off if H imposes border adjustments as well.*

The reasoning is straightforward, Holding emissions constant means holding p_e fixed and by construction, the price of the l-good is fixed at 1. Because income in F is from wages and rents from energy extraction, income in F is fixed. This leaves only the price of the m-good to vary. With a production tax, any varieties of the m-good produced in H that consumers in F purchase will bear the tax while with a border adjustment, the tax is removed. Therefore, F is better off with border adjustments.

5.5.2 H's welfare

Proposition 8 *If H sufficiently dominates manufacturing, H is better off without border adjustments.*

Reasoning:

6 Simulations

Possible simulations:

- Beta v. leakage under a production tax (and possibly, under a consumption tax).
- leakage under a production tax with variable border adjustments

- H's welfare under production tax plus variable border adjustments (i.e., BA's on x axis, welfare on y axis).
- leakage v. relative size of H
- Welfare gains v. relative size of H (under production tax?): tells us the benefits of increasing the size of the taxing coalition.

Note to remember that we might want to present some of these results using the carbon matrices from the old paper. These are, I think, just the four $\hat{M}$'s so they should be easy to compute.

7 Comparisons to CGE results

Want to compare our results to the summary of the CGE results given above;

- Leakage rates most often in the range of 5% to 20%, with some outliers.
- The larger the taxing coalition, the lower the leakage.
- BA's reduce leakage substantially
- Most important variable in determining effects of unilateral carbon price is the energy supply elasticity.
- Distinguish two drivers of leakage: the fuel price effect (in which lower demand for fossil fuels in the taxing region suppresses prices, increasing demand on the non-taxing regions) and the competitiveness effect [different name?] (in which increased costs for industry in the taxing region causes a shift to the non-taxing region).