Mathematics Review Course
Summer 2023
Problem Set 05
Solutions

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Partial Derivatives

1. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = \frac{ln(xy)}{x+2y}$.

Solution:

$$f_x = \frac{-1}{xy(x+2y)^2}$$
$$f_y = \frac{-2}{xy(x+2y)^2}$$

2. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$ for $f(x,y,z) = \frac{9^z}{x^2+zy}$.

Solution:

$$f_x = \frac{-2x9^z}{(x^2+zy)^2}$$
$$f_y = \frac{-z9^z}{(x^2+zy)^2}$$
$$f_z = \frac{9^z(ln(9)(x^2+zy)-y}{(x^2+zy)^2}$$

Total Differentiation

3. Differentiate $f(x,y,z) = 13x + 2y^2 + e^z$

Solution:

$$df = 13dx + 4ydy - e^z dz$$

4. Differentiate $f(x,y,z) = (x + y^{1/2} + z^2)^3$

Solution:

$$df = 3(x + y^{1/2} + z^2)^2 dx + \frac{3}{2}y^{1/2}(x + y^{1/2} + z^2)^2 dy - 6z(x + y^{1/2} + z^2)^2 dz$$

Gradients

5. Find ∇ for $f(x,y) = e^{5yx} + \frac{x}{y}$

Solution:

$$\nabla f = \langle 5ye^{5yx} + \frac{1}{y}, 5xe^{5yx} - \frac{x}{y^2} \rangle$$

6. Find ∇ for $f(x,y) = ln(x+y^2) - 8^x$

Solution:

$$\nabla f = \langle \frac{1}{x+y^2} - ln(8)8^x, \frac{2y}{x+y^2} \rangle$$

7. Find ∇ for $f(x,y,z) = xy^2z^3 + 4xe^{y^2} - ln(x-z)$

Solution:

$$\nabla f = \langle y^2z^3 + 4e^{y^2} - \frac{1}{x-z}, 2yxz^3 + 8yxe^{y^2}, 3xz^2y^2 + \frac{1}{x-z} \rangle$$

Implicit Partial Differentiation

8. $\frac{\partial}{\partial x} f(x, y(x)) = xy + x^2 - ln(y)$

Solution:

$$f_x = \frac{-1-2x}{x - \frac{1}{y}}$$

9. $\frac{\partial}{\partial x} f(x, y(x)) = e^x y^{3/2} x^y$

Solution:

$$f_x = \frac{-y(1+\frac{1}{x}}{\frac{3}{2} + yln(x)}$$