SET THEORY

Definition : A set S, is a collection of objects. The objects are called members of the set.

The elements of a set are found in the universal set $\boldsymbol{U}$. The elements of a set S, are denoted by s. we describe the set S by the following way:

$$S = \{s \in U: P\}$$

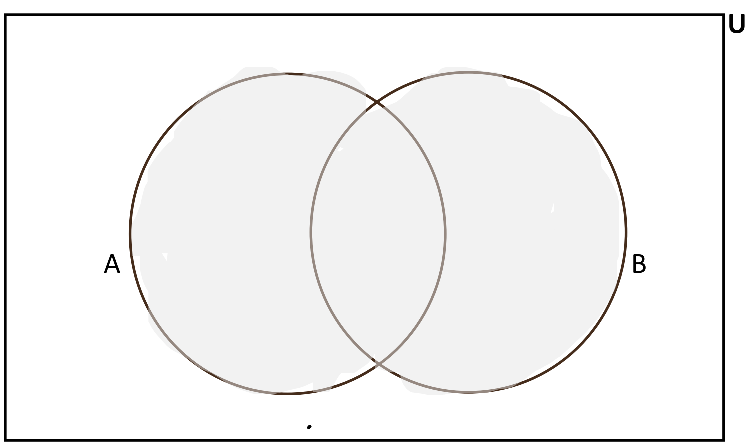
Where P denotes a property.

You can consider **U** to be the University of Minnesota students and P to be students in the Math Review class.

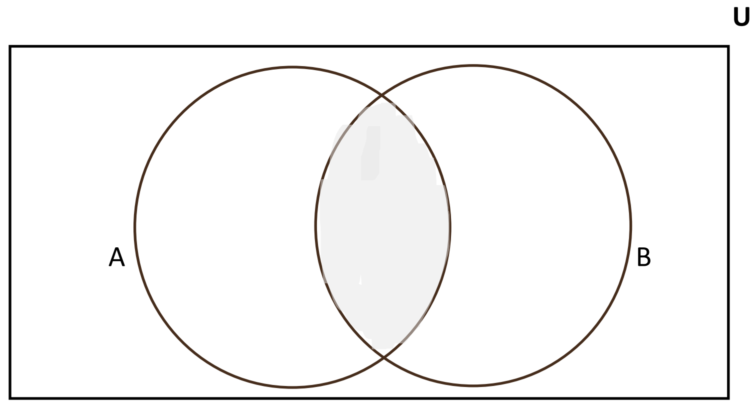
A set can be described by listing elements in the set. For example the set of the first 4 positive integers is : S = {1,2,3,4}

Union, Intersection and Complement

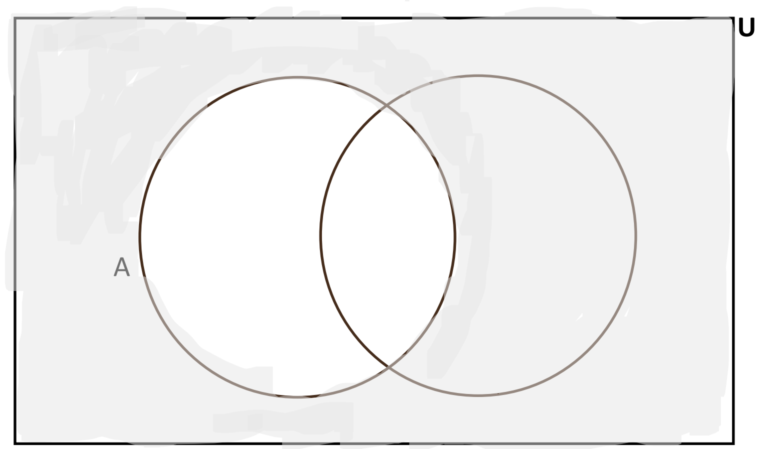
Union: $A \cup B = \{x \in U: x \in A\ or\ x \in b\}$



Intersetion: $A \cap B = \{x \in U : x \in A \; and \; x \in b\}$



Complement: $A^c = \{x \in U : x \notin A\}$

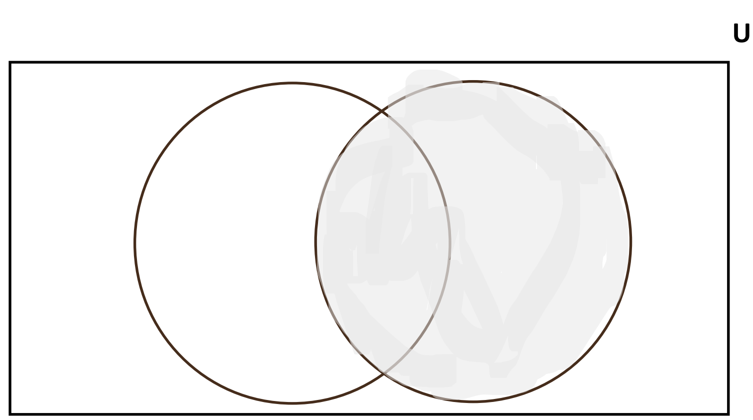


Set difference, subset, disjoint and partition

Union

Given two sets A and B in U, their difference, denoted A\B, is

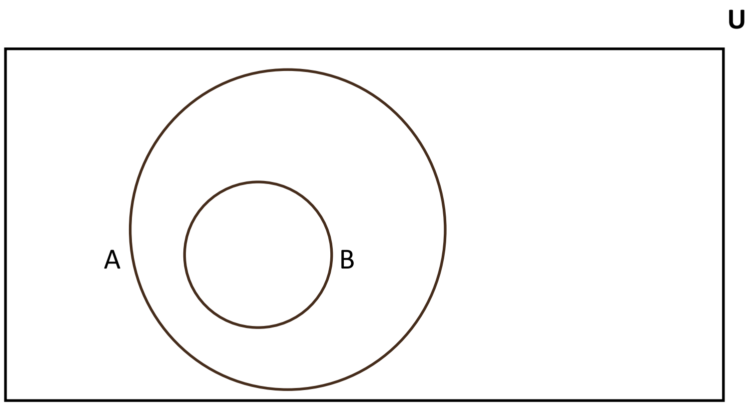
$A \setminus B = \{x \in U : x \in A \; and \; x \notin B\}.$



A B A\B

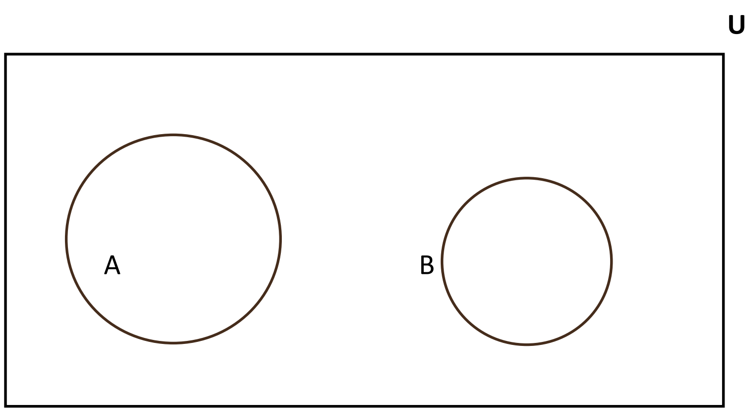
Subsets

If A and B are sets in U, then B is a subset of A, denoted $B \subset A, if\ [x \in B] \Rightarrow [x \in A]$. B is a proper subset of A if $B \subset A\ and\ B \neq A$.



Disjoint sets

Sets A and B are disjoint sets if $A \cap B = \emptyset$



The union and intersection operators are commutative. For any sets A and B,

$$A \cup B = B \cup A\ and\ A \cap B = B \cap A.$$

The are also associative. For any sets A, B, and C,

$$(A \cup B) \cup C = A \cup (B \cup C)\ and\ (A \cap B) \cap C = A \cap (B \cap C)$$

Exercise 1 : Prove the commutative and associative properties described above.

de Morgan's laws and Cartesian Product

de Morgan's laws : If $A_1, A_2, \dots., A_k$ are subsets, then:

$$\left[\bigcup_{i=1}^{k} A_i\right]^c = \bigcap_{i=1}^{k} A_i^c \quad \text{and} \quad \left[\bigcap_{i=1}^{k} A_i\right]^c = \bigcup_{i=1}^{k} A_i^c$$

Exercise: Verify that this is true for the 3 subsets of all real numbers :

$$A_1 = [0,1], A_2 = [0.5,3],\ and\ A_3 = [4,5]$$

Cartesian product

For 2 sets $A\ and\ B$, their cartesian product is $\{(a,b): a \in A\ , b \in B\}$

Exercise : What is the Cartesian product of a = [1,2] and b = [3,5]? Illustrate it on a diagram.

Cardinality and countability

Cardinality of a set A, denoted by $\#|A|$ is the number of elements in the set. For example, if $A = \{1,2,3\}$, $then\ r\#|A| = 3$.

What is #|A| if A = {2,4,6,8,……..} ?

Indeed, some sets have infinite elements. There are 3 cases of cardinality: finite, countably infinite and uncountable.

Set A is finite if $\#|A| < \infty$

The set of counting integers or positive integers N = {1,2,3,4,…….}.

A **countable set** is (a finite set or) any infinite set that can be placed in one-to-one correspondence with N.

The set of integers Z = {…..,-2,-1,0,1,2,….} is countable. How can we justify this?

Convex sets

Consider a set $X \subset R^n$, the n-dimensional real numbers. X is convex if for all t ∈ [0, 1] and all $x, y \in X$, the element $xt = tx + (1 - t)y\ is\ in\ X$.

Bounded sets in Rn

A set in R^n is bounded if it is entirely contained within some ϵ-ball. The ball can either open or closed. That is, S is bounded if there exists some ε>0, such that $S \subset B_\epsilon(x)$ for some $x \in R^n$. You can think of boundedness as meaning that a set if finite in size.

Compact sets

A set S in Rn is called compact if and only if it is closed and bounded.

The separating hyperplane theorem

Suppose the $B \subset R^n$ is a convex and closed set and that $x \notin B$. Then there is $p \in R^n$ and a value $c \in R$ such that $p \cdot x > c$ and $p \cdot y < c$ for every $y \in B$.

The separating hyperplane theorem is important in the **Second Welfare Theorem**.