Lecture 12
Statistics

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Last Lecture Review

- Probability:
  - Probability Limits
  - Independence
  - Law of Total Probability
  - Conditional Probability
  - Cumulative Distribution Function
  - Probability Distribution Function
  - Joint & Marginal Distributions
  - Gaussian (Normal) Distribution
  - Bayes Rules
  - Moments of a Distribution
  - Covariance & Correlation

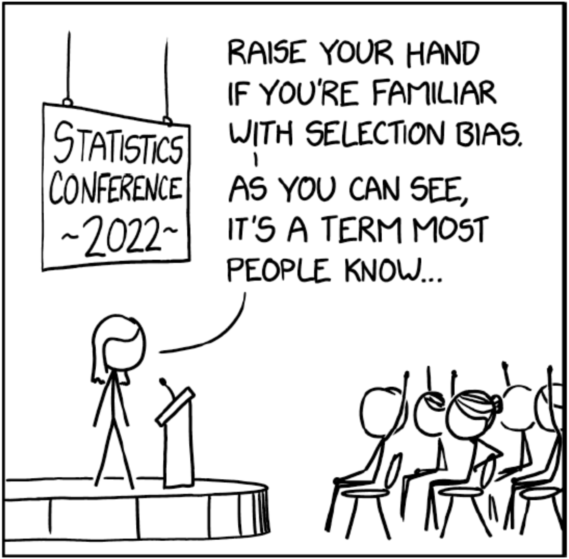
Review Assignment

1. Problem Set 11 solutions are available on Github.
2. Any issues or problems **You** would like to discuss?

Daily Icebreaker

- Attendance via prompt:
  - Name
  - Daily Icebreaker: You can time travel forward or backward in time. When and where do you go? What do you do?

SELECTION BIAS...



Statistics

Motivation

- General background
  - This is how we map real world data into the world of probabilities.
- Why do economists' care?
  - This is one of the primary tools of economists.
  - We use statistics to determine what information we choose to believe is 'true'.
- Application in this career
  - Throughout applied research.
  - A core component of econometrics.

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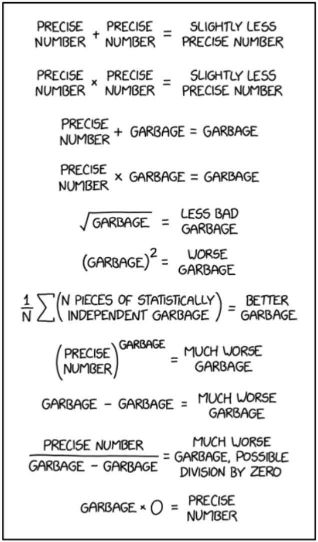
Overview

1. Population, Parameters, and Distributions
2. Random Variables
3. Discrete & Continuous Variables
4. Law of Iterated Expectations
5. Sampling
6. Data Types
7. Data Structure
8. Randomization
9. Estimate, Estimator, & Estimand
10. Parametric vs. Non-parametric
11. Expectations
12. Conditional Expectation Function
13. Law of Large Numbers
14. Central Limit Theorem
15. Continuous Mapping Theorem
16. Delta Method
17. Standardizing Units
18. Hypothesis Testing
19. Causation
20. Prediction
21. Descriptive Analysis
22. Inferential Analysis

STATS: GARBAGE IN → GARBAGE OUT



1. Population, Parameters, and Distributions

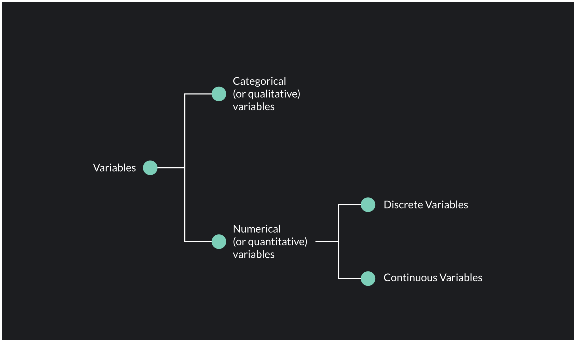
- Population
  - A well-defined group of subjects of interest.
  - Theoretically, an infinite sized group of all members fitting the definition.
- Parameters
  - A constant, unknown value describing the true relationship of variables in the population.
  - Parameters are defined, but never known. You estimate to approximate their true value.
- Distributions
  - The frequency of values (outcomes) about the range of the sample space for a population.

2. Random Variables

- Experiment: Procedure that can be infinitely repeated and has well-defined set of outcomes. But, the actual outcome may be of unknown certainty.
- Random variable: $x \in X$ is a numerical value outcome in a set of possible outcomes where the outcome is determined by an experiment.
- Binary (Bernoulli) random variables: Takes values 0 or 1 such that $Pr(X = 1) = \theta : 0 \leq \theta \leq 1$.

3. Discrete & Continuous Variables

- Discrete Variables: Random variable that has a finite or countably infinite domain of values.
- Continuous Variables: Random variable with infinite domain of values.
  - Can take on any real value with $Pr(X) \geq 0$.



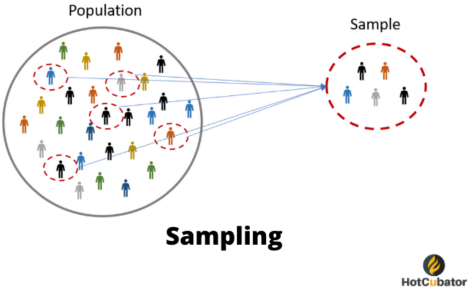
5. Law of Iterated Expectations

- The expected value of the conditional expected value of X given Y is the same as the expected value of X.
- The average of the group averages is the average of the whole distribution.
- E.g., you can recover the population average from aggregated (group) averages.

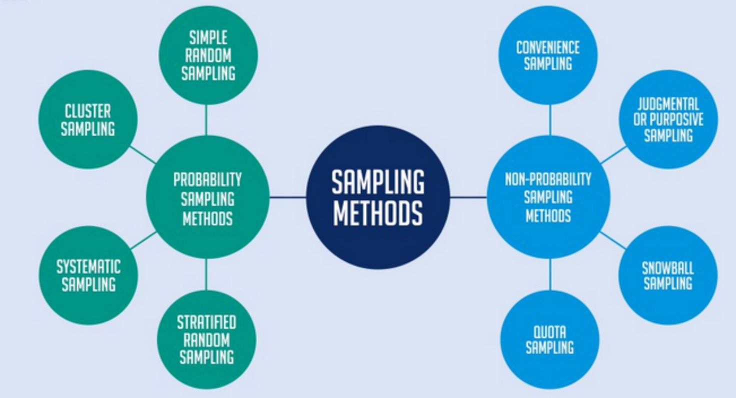
$$\mathbb{E}(X) = \mathbb{E}(\mathbb{E}(X|Y))$$

6. Sampling

- Sampling pulls units from the population distribution as the data used to estimate population parameters.
- Random Sampling
  - A sample of independently drawn units.
  - Independence means that no individual is more likely to be sampled than another from the population.



6. Sampling

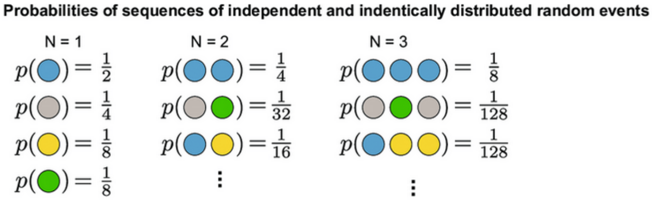

SIMPLE RANDOM SAMPLING
CLUSTER SAMPLING
PROBABILITY SAMPLING METHODS
SYSTEMATIC SAMPLING
STRATIFIED RANDOM SAMPLING
SAMPLING METHODS
CONVENIENCE SAMPLING
JUDGMENTAL OR PURPOSIVE SAMPLING
NON-PROBABILITY SAMPLING METHODS
SNOWBALL SAMPLING
QUOTA SAMPLING

7. Data Types

- The data you gather may come in many forms – qualitative and quantitative.
- Most of the data you will work with are samples formatted into tabular arrays (viz., matrix).
- Types:
  - Float: Fractions
  - Boolean: True/False
  - Integer: $\mathcal{Z}$
  - Categorical: Cardinally ranked integers
  - Character: Letter or special character
  - String: Combination of characters
  - Date: Formatted string or integer
  - Null: Empty Set
  - N/A: Missing value

8. Data Structure

- The data generating process (DGP)
  - The way that the data is constructed in the real world.
  - Typically unknown – often an assumption.
- Independently Identically Distributed (IID)
  - Each random variable has the same probability distribution as the others and are mutually independent of one another.
  - The sample is ‘representative’ of the population.
  - Identically distributed: Distribution does not fluctuate by sample.
  - Independent: Sample items (outcomes) are independent events



8. Data Structure

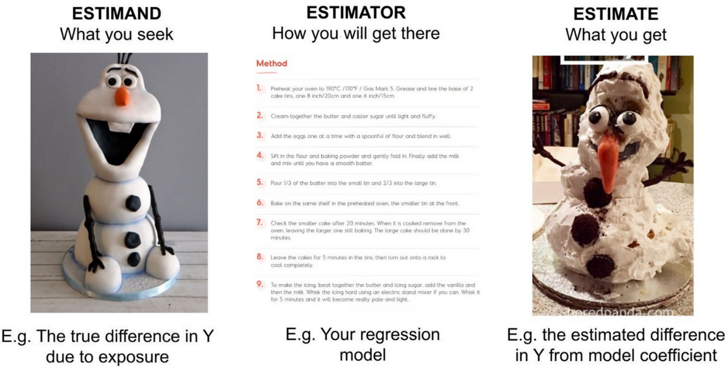
- Cross-sectional
  - A sample of individuals (units) at a given point in time (period) with many variables (characteristics).
- Time Series
  - Potentially several units on a single variable over many periods.
- Pool Cross-section
  - A series of cross-sections with some overlapping units at different time periods.
- Panel Datasets
  - A time series of many periods for many variables across many units.

9. Randomization

- A random process is a random sequence that does not follow a deterministic process.
- Random Sample: Individuals have a known probability of sampling from the population.
- Ensure the data is I.I.D. and that the estimates are representative for the parameters in the population.

10. Estimate, Estimator, & Estimand

- Estimate: Approximation of the population parameter.
- Estimator: The function or algorithm doing the estimation.
- Estimand: The parameter in the population you aim to estimate.



11. Parametric vs. Non-parametric

- Parametric: Imposes some distribution on the underlying population (e.g., the distribution of the parameters)
- Non-parametric: Is agnostic to how the distribution of the population might look
- Parametric estimators rely on asymptotic properties of the population (e.g., Assume to know the underlying distribution in large samples).
- Non-parametric estimators are driven by distributional properties of the sample.
- Can often use non-parametric estimators when the moments of the distribution do not converge to a known distribution.

12. Expectations

- Expected Values (Mean): A weighted average of all possible values of X, with weights equal to the probability of each outcome occurring.
- Discrete:

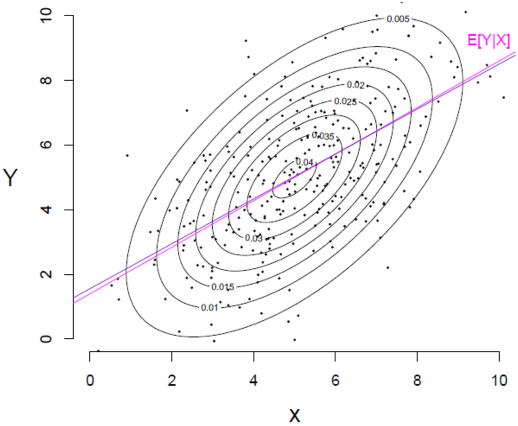
$$\mathbb{E}(X) = \sum_{j}^{k} x_j f(x_j) = \mu_x$$

- Continuous:

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x) dx$$

13. Conditional Expectation Function

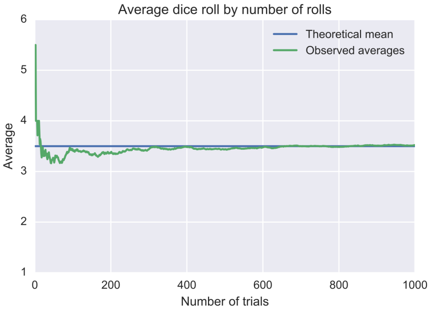
$$\mathbb{E}[Y|X=x] = \int_{-\infty}^{\infty} y f_{Y|X}(y|x)dy$$



14. Law of Large Numbers

- The expected value of the distribution converges to the population parameter as the sample becomes larger.
- If X_i are I.I.D. and $\mathbb{E}[X] < \infty$, then as $n \to \infty$:

$$\bar{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{p} \mathbb{E}[X]$$

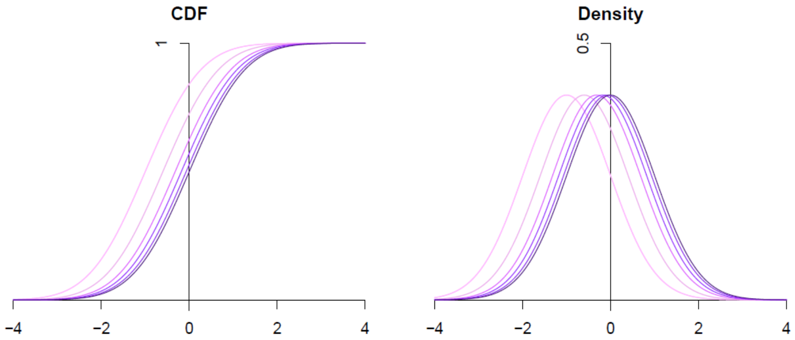


15. Central Limit Theorem

- The distribution of the sample approximates a normal distribution as the sample becomes larger – **regardless of the population's underlying distribution**.
- Let $X_i \in \mathbb{R}^k$ be I.I.D. with $\mathbb{E}[||X_i||^2] < \infty$.
- Define $\mu = \mathbb{E}[X]$.
- Define $\Sigma = \mathbb{E}[(X - \mu)(X - \mu)^T]$
- Then, as $n \to \infty$:

$$\sqrt{n}(\bar{X} - \mu) \xrightarrow{d} N(0, \Sigma)$$

15. Central Limit Theorem



16. Continuous Mapping Theorem

- If a sequence converges, then you can apply a function/transformation to the sequence and it converges to the function of that limit.
- Idea: $g(Z_n) = g(plimZ_n)$
- So: $\hat{\beta} \xrightarrow{p} \beta$
- If $Z_n \xrightarrow{p} c$ as $n \to \infty$, and $h(\cdot)$ is continuous at c, then:

$$h(Z_n) \xrightarrow{p} h(c) \text{ as } n \to \infty$$

17. Delta Method

- Variance will be approximately normal.
- Let $\theta \in \mathbb{R}^k$ and $h : \mathbb{R}^k \to \mathbb{R}$ be C^1 in the neighborhood of θ.
- Let $h = \nabla h(\theta)$.
- If $\sqrt{n}(\hat{\theta} - \theta) \xrightarrow{d} \eta$, then as $n \to \infty$,
$$\sqrt{n}(h(\hat{\theta}) - h(\theta)) \xrightarrow{d} h^T \cdot \eta$$
- If $\eta \sim N(0, V)$ (by CLT), then
$$\sqrt{n}(h(\hat{\theta}) - h(\theta)) \xrightarrow{d} N(0, h^T V h)$$

18. Standardizing Units

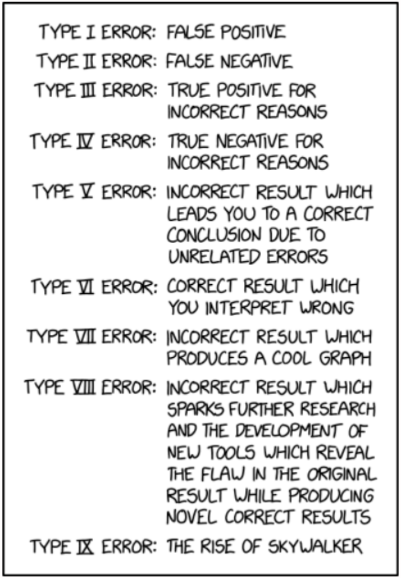
- Commonly referred to as 'taking a z-score'.
- De-meaning the random variable and weighting it by the standard error to create a new random variable measured in standard deviations from the mean.
- When the units are standard deviations, it is now easy to compare many types of outcomes with various ranges and distributions.
- $\mathbb{E}[Z] = 0$
- $Var(Z) = 1$

$$Z = \frac{x - \mu_x}{\sigma_x}$$

19. Hypothesis Testing

	Null Hypothesis is TRUE	Null Hypothesis is FALSE
Reject null hypothesis	Type I Error (False positive)	Correct Outcome! (True positive)
Fail to reject null hypothesis	Correct Outcome! (True negative)	Type II Error (False negative)

Types of Errors



19. Hypothesis Testing

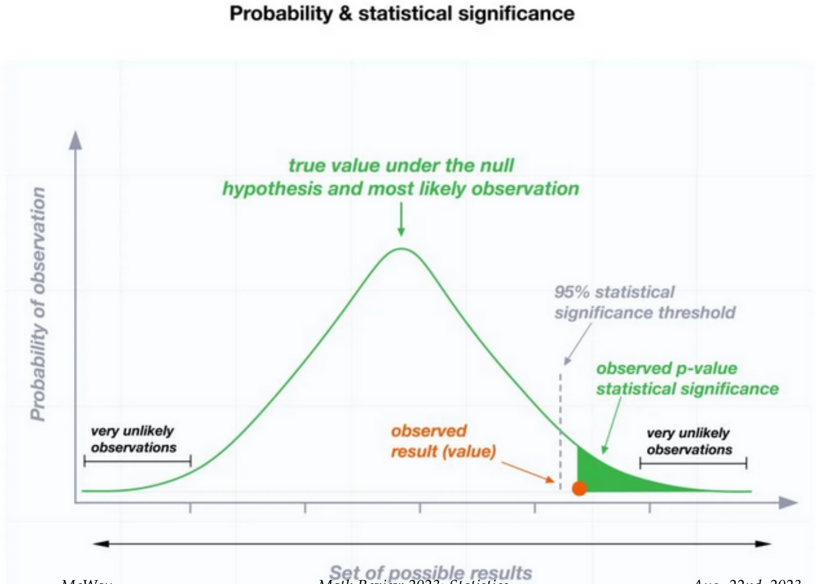
- Student's T-Test: Creates a measure of the distance a standardized value is from the mean weighted by the size of the standard error.

$$t = \frac{Z}{s} = \frac{\bar{X} - \mu}{\hat{\sigma}/\sqrt{n}}$$

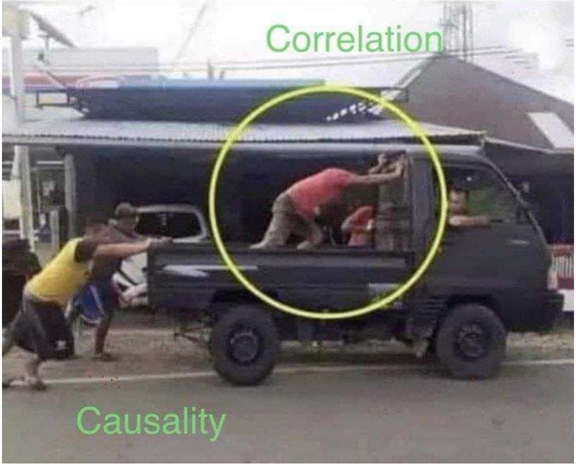
- P-Value: The probability of obtaining the outcome as compared to the null hypothesis of a given distribution (i.e., standard normal).

$$p = Pr(T \geq t|H_0)$$

19. Hypothesis Testing



Causation $\neq$ Correlation



20. Causation

- Considering the 'counterfactual' if a treatment D had not occurred.
- Using randomization to determine the average treatment effect (ATE), since we cannot recover the individual effect.

$$\begin{aligned} \delta_i =& y_i^1 - y_i^0 \\ Y =& DY^1 + (1-D)Y^0 \\ \mathbb{E}[\delta] =& \mathbb{E}[Y^1] - \mathbb{E}[Y^0] \end{aligned}$$

Table 2.1 The Fundamental Problem of Causal Inference

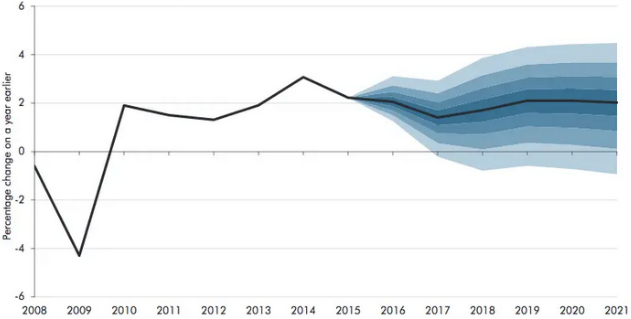
Group	Y^1	Y^0
Treatment group ($D=1$)	Observable as Y	Counterfactual
Control group ($D=0$)	Counterfactual	Observable as Y

Control Group

21. Prediction

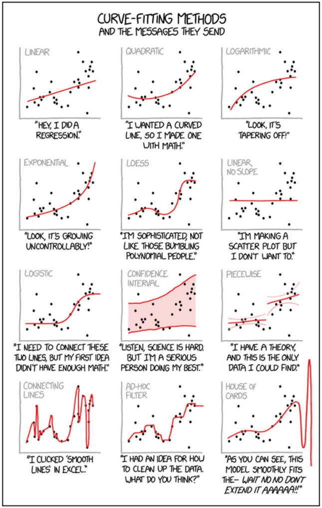
- Sometimes called 'forecasting'.
- The goal is to create a model that will predict the outcome $\hat{Y}$ outside of the sample using some measure of predictiveness (e.g., out-of-sample R^2).
- Determining which group of predictors X have the highest combined correlation with Y.

Chart 1.3: Real GDP growth fan chart



Source: ONS, OBR

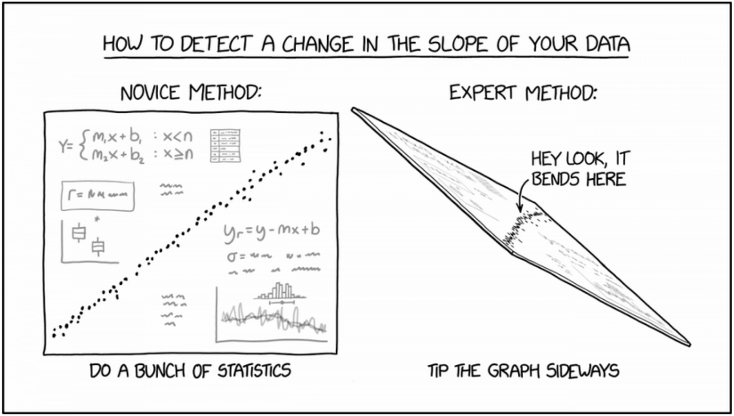
Try to Not Over-fit



22. Descriptive Analysis

- Rather than determine cause and effect or forecasting the future, we are creating a description of the the current phenomenon.
- Can we estimate moments of the distribution for important variables that are representative of the population?
- Can we fit data to a model that approximates the phenomenon? What happens if we change the variables in this model?

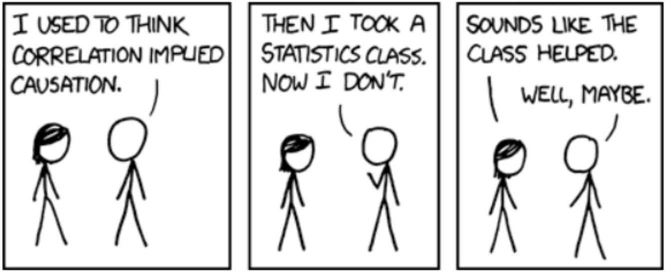
Analysis Can Be Hard



23. Inferential Analysis

- Drawing conclusions about cause and effect given a 'treatment' or program's effect on the sample.
- What can we infer this will do the larger population (e.g., how does it shift the distribution of our dependent variables).
- Requires a comparison of statistical significance against a known (or theorized) distribution of the underlying population.

CORRELATION $\neq$ CAUSATION



Review

Review: Statistics

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Assignment

- Readings on Time Series & Dynamic Programming before Lecture 13:
  - MWG Appendix M.N.
  - Stokey Ch. 3, 4, & 10
- Assignment:
  - Problem Set 12 (PS12)
  - Solution set will be available following end of Lecture 12
- Struggling?
  1. Read the 'Encouraged Reading'
  2. Review 'Supplementary material'
  3. Reach out directly