APEC Math Review

Part 1 Logic

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Logic stretch: necessity and sufficiency

Source: R.I.S.E. Physical Therapy

Logic stretch: necessity and sufficiency

- A is **necessary** for B
  - If B is true, A must be true: $B \implies A$
  - What if B is not true?
  - If A is not true, B is not either: $\neg A \implies \neg B$
- Example?

Logic stretch: necessity and sufficiency

- A is **sufficient** for B
  - If A is true, B must be true: $A \implies B$
  - What if A is not true?
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- Example?

Logic stretch: necessity and sufficiency

- If A is sufficient for B, B is necessary for A.
- If $A \implies B$ and $B \implies A$, then $A \iff B$
    - A is necessary and sufficient for B.
    - A and B are equivalent.
    - A is true if and only if B is true: A iif B

Vocabulary

- Axiom: statements we assume to be true
  - e.g. $a = b, b = c \implies a = c$
- Theorem: a statement that has been proven to be true.
- Corollary: a theorem that follows on from another theorem.
- Lemma: a less important theorem that is used to prove another theorem.

Ways to prove

1. Direct proof: show $A \implies B$

Example: Let m be an even integer and p be any integer. Then $m * p$ is an even integer.

Proof:
m is an even integer so $\exists$ an integer q such that $m = 2 * q$ by the definition of even integer.
$m * p = (2 * q) * p = 2 * (q * p)$ so $m * p$ is an even integer.

Ways to prove

2. Proof by contradiction: if $\neg B \implies \neg A$, then $A \implies B$.

Example: Walras' law
$\forall \mathbf{x} \in \mathbf{x}(\mathbf{p}, w)$ that maximizes consumer utility, $\mathbf{x} * \mathbf{p} = w$.

Proof:
Suppose $\exists \mathbf{x} \in \mathbf{x}(\mathbf{p}, w)$ that $\mathbf{x} * \mathbf{p} < w$, $(\neg B)$
there must be another $\mathbf{y} \in \mathbf{x}(\mathbf{p}, w)$ that is also affordable and $\mathbf{y} \succ \mathbf{x}$ by the "local non-satiation" assumption.
So $\mathbf{x}$ does not maximize utility. $(\neg A)$

Ways to prove

3. Mathematical induction: only used on propositions about integers or proposition indexed by integers.

Example:

$$P(n):\ 1+2+3+...+n=\frac{n(n+1)}{2}$$

Proof:
First, $P(1)$ is true because $1=\frac{1\times 2}{2}$.
Assume $P(n)$ is true for some integer k:

$$1+2+3+...+k=\frac{k(k+1)}{2}$$

Adding (k+1) to both sides:

$$1+2+3+...+k+(k+1)=\frac{k(k+1)}{2}+(k+1)=\frac{(k+1)(k+2)}{2}$$

This is exactly $P(k+1)$.

Micro Drill

A rational preference has two properties:
1. Completeness: $\forall x, y$ in a set of possible alternatives, either $x \succsim y$ or $y \succsim x$, or both.
2. Transitivity: $\forall x, y, z$ in a set of possible alternatives, $x \succsim y$ and $y \succsim z \implies x \succsim z$.

Prove that $x \succ y \succsim z \implies x \succ z$.