

Mathematics Review Course
Summer 2023
Problem Set 03
Solutions

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First Derivatives

1. [Xmath Project] Find $\frac{d}{dx}$ for $f(x) = 2x(x^2 + 3x)$

Solution:

$$f'(x) = 6x^2 + 12x$$

2. [Xmath Project] Find $\frac{d}{dx}$ for $g(x) = (\frac{1}{x} + 1)(x - 1)$.

Solution:

$$g'(x) = \frac{x^2 + 1}{x^2}$$

3. [Paul Dawkins] Find $\frac{d}{dt}$ for $V(t) = \frac{6(t)^{1/3}}{4t+1}$

Solution:

$$V'(t) = \frac{-16t^{1/3} + 2t^{-2/3}}{(4t + 1)^2}$$

4. [Paul Dawkins] Find $\frac{d}{dw}$ for $h(w) = e^{w^4 - 3w^2 + 9}$

Solution:

$$h'(w) = (4w^3 - 6w)e^{w^4 - 3w^2 + 9}$$

5. Find $\frac{d}{dx}$ for $f(x) = \left(\frac{x^4}{x+3}\right)(\ln(x) - 3x)^{-2}$

Solution:

$$f'(x) = \frac{3x^2}{(\ln(x) - 3x)^2} - \frac{2(\frac{1}{x} - 3)(x^3 + 3)}{(\ln(x) - 3x)^3}$$

Implicit Functions

6. Find $\frac{dy}{dx}$ for $(x - y)^2 = x + y - 1$

Solution:

$$\frac{dy}{dx} = \frac{1 - 2x + 2y}{-2x + 2y - 1}$$

7. Find $\frac{dy}{dx}$ for $xe^y = x - y$

Solution:

$$\frac{dy}{dx} = \frac{1}{e^y + 1}$$

8. Find $\frac{dy}{dx}$ for $x^3 + y^3 = 7$

Solution:

$$\frac{dy}{dx} = \frac{-y^2}{x^2}$$

9. Find $\frac{dy}{dx}$ for $\frac{y}{1-x^2} = 6y + 7x$

Solution:

$$\frac{dy}{dx} = \frac{13(1 - x^2)^2 - 2xy}{-x^2 + 1}$$

10. Find $\frac{dy}{dx}$ for $\frac{3xy}{x^5 + y^2} = 1$

Solution:

$$\frac{dy}{dx} = \frac{15x^5y^5 - 3yx^5 + 3y^2}{3x^6 + 3xy - 6xy^2}$$

Critical Values

11. [Paul Dawkins] Critical values for $A(t) = 3t - 7\ln(8t + 2)$

Solution:

$$t^* = \left\{\frac{25}{12}\right\}$$

12. [Paul Dawkins] Critical values for $f(x) = 5xe^{9-2x}$

Solution:

$$x^* = \left\{\frac{1}{2}\right\}$$

13. [Paul Dawkins] Critical values for $f(z) = \frac{z+4}{2z^2+z+8}$

Solution:

$$x^* = \{-4 \pm 3\sqrt{2}\}$$