Lecture 04
Integration

Ryan McWay†

† *Applied Economics,*
University of Minnesota

Mathematics Review Course, Summer 2023
University of Minnesota
August 10th, 2023

Last Lecture Review

- Derivatives:
  - Continutity & Differentiability
  - First & Second Derivatives
  - Derivative Rules
  - Implicit Function
  - l'Hopital's Rule
  - Taylor Series Approximation
  - Mean Value Theorem
  - Convexity

Review Assignment

1. Problem Set 03 solutions are available on Github.
2. Any issues or problems **You** would like to discuss?

Daily Icebreaker

- Attendance via prompt:
  - Name
  - Program and track
  - Daily icebreaker subject...

Topic: Integration

Motivation

- General background
  - The total area under the curve.
  - Understanding the accumulation of the parts as a whole.
  - A core component of calculus alongside derivatives.
- Why do economists' care?
  - A tool to aggregate effects and approximate sums.
- Application in this career
  - Evaluating surplus or total welfare.

Motivation

- General background
  - The total area under the curve.
  - Understanding the accumulation of the parts as a whole.
  - A core component of calculus alongside derivatives.
- Why do economists' care?
  - A tool to aggregate effects and approximate sums.
- Application in this career
  - Evaluating surplus or total welfare.

Motivation

- General background
  - The total area under the curve.
  - Understanding the accumulation of the parts as a whole.
  - A core component of calculus alongside derivatives.
- Why do economists' care?
  - A tool to aggregate effects and approximate sums.
- Application in this career
  - Evaluating surplus or total welfare.

Overview

1. Definite Integral
2. Reimann Sum
3. Fundamental Theorem of Calculus
4. Integration Rules
5. Integration by Substitution
6. Integration by Parts
7. Leibnz's Rule

1. Definite Integral

- Consider the anti-derivative $F(x)$.

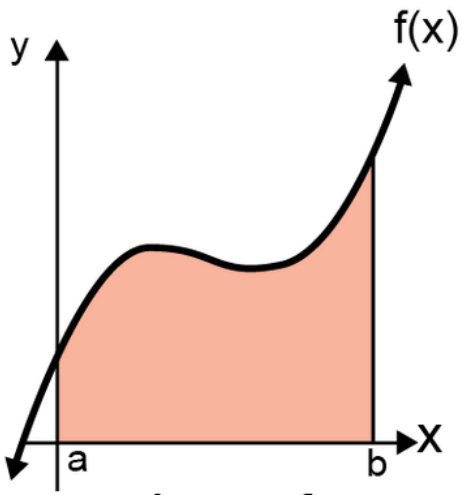
$$\int_a^b f(x)dx = \lim_{x \to \Delta x} \sum_{x=a}^{b} f(x)\Delta x = F(b) - F(a) : F'(x) = f(x)$$

2. Reimann Sum

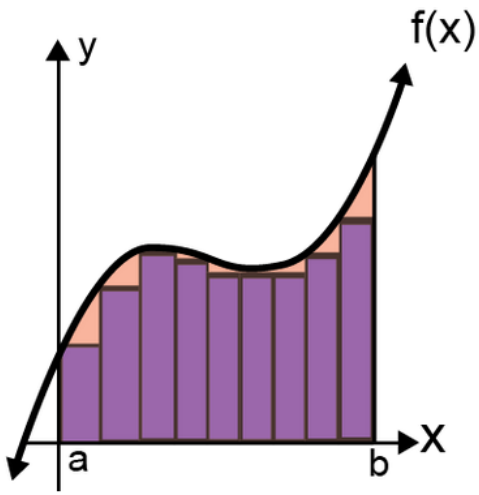
- Subdivide the interval (a, b) into N sub-intervals with endpoints x_i.

$$\lim_{\Delta \to 0} \sum_{i=1}^{N} f(x_i)\Delta = \int_a^b f(x)dx$$

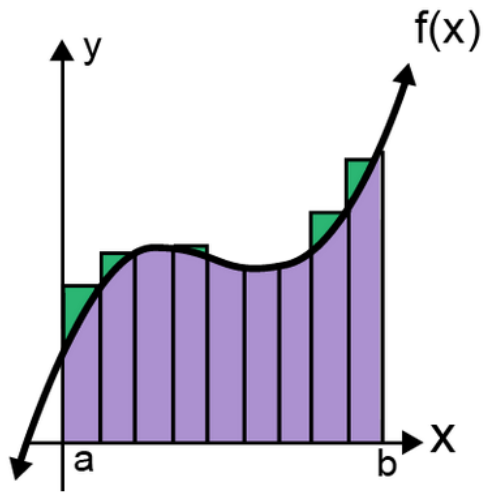
2. Reimann Sum



Area of region

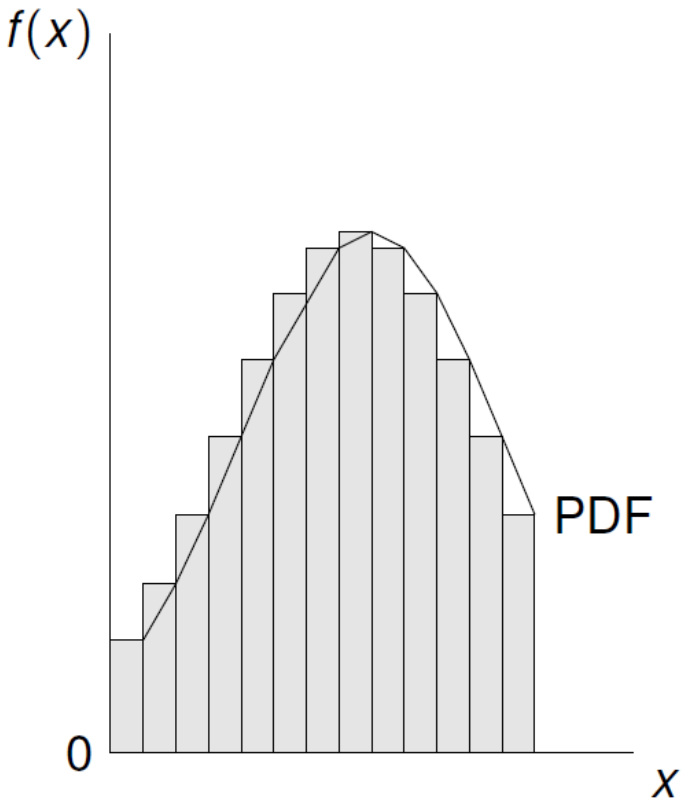


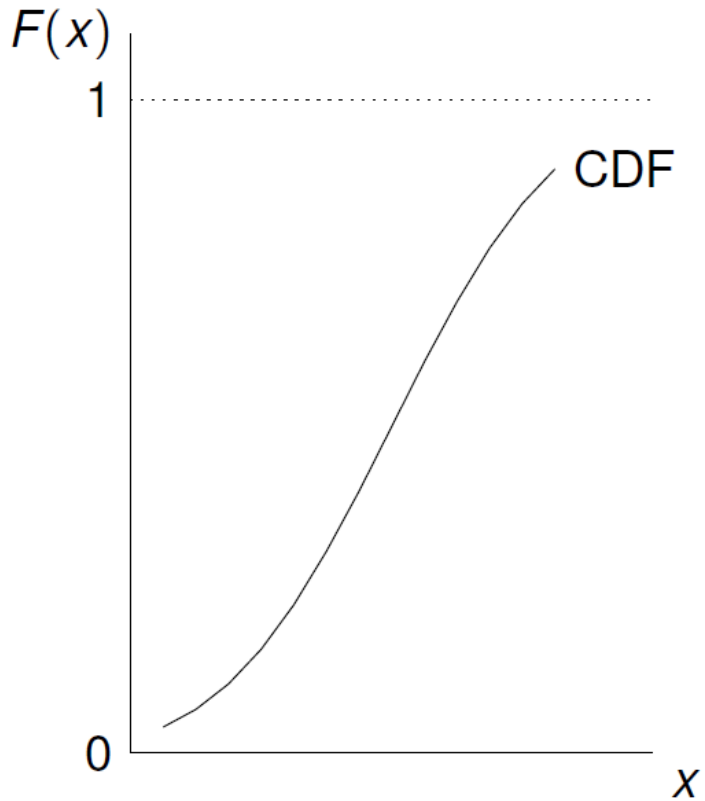
Lower Sum



Upper Sum

Probability and Cumulative Density Functions





3. Fundamental Theorem of Calculus

- Theorem that connects differentiation to integration.
- Let f be a continuous function on the open interval $[a, b]$. If $f(x) = F'(x)$, then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

4. Integration Rules

- Constant:

$$\int a dx = ax + C$$

- Constant Multiplication:

$$\int cf(x)dx = c\int f(x)dx$$

- Reciprocal:

$$\int \frac{1}{x}dx = ln(x) + C$$

- Exponential:

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{ln(a)} + C$$

4. Integration Rules

- Logarithm:

$$\int ln(x)dx = xln(x) - x + C$$

- Power Rule:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

- Sum/Difference Rule:

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$

5. Integration by Substitution

- Reverse chain rule from differentiation.
- Commonly referred to as u substitution.

$$\int f(g(x))g'(x)dx$$

$$\int f(u)du$$

6. Integration by Parts

- Reverse product rule from differentiation.
- Rarely used in economic applications, but important to know.

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

$$\int udv = u \cdot v - \int vdv$$

7. Leibnz's Rule

- A general rule for differentiating integrals.

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(x,t)dx = \frac{db(t)}{dt}f(b(t),t) - \frac{da(t)}{dt}f(a(t),t) + \int_{a(t)}^{b(t)} \frac{\partial f(x,t)}{\partial t}dx$$

Practice: Integrals

1.

Review of Integrals

1. Definite Integral
2. Reimann Sum
3. Fundamental Theorem of Calculus
4. Integration Rules
5. Integration by Substitution
6. Integration by Parts
7. Leibnz's Rule

Assignment

- Readings on Multi-variate Calculus before Lecture 05:
  -
- Assignment:
  - Problem Set 04 (PS04)
  - Solution set will be available following end of Lecture 05
- Struggling?
  1. Read the 'Encouraged Reading'
  2. Review 'Supplementary material'
  3. Reach out directly