Mathematics Review Course
Summer 2023
Problem Set 03
Solutions

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First Derivatives

1. [Xmath Project] Find $\frac{d}{dx}$ for $f(x) = 2x(x^2 + 3x)$

Solution:

$$f'(x) = 6x^2 + 12x$$

2. [Xmath Project] Find $\frac{d}{dx}$ for $g(x) = (\frac{1}{x} + 1)(x - 1)$.

Solution:

$$g'(x) = \frac{x^2 + 1}{x^2}$$

3. [Paul Dawkins] Find $\frac{d}{dt}$ for $V(t) = \frac{6(t)^{1/3}}{4t+1}$

Solution:

$$V'(t) = \frac{-16t^{1/3} + 2t^{-2/3}}{(4t + 1)^2}$$

4. [Paul Dawkins] Find $\frac{d}{dw}$ for $h(w) = e^{w^4-3w^2+9}$

Solution:

$$h'(w) = (4w^3 - 6w)e^{w^4-3w^2+9}$$

5. Find $\frac{d}{dx}$ for $f(x) = \left(\frac{x^4}{x+3}\right)(ln(x) - 3x)^{-2}$

Solution:

$$f'(x) = \frac{3x^2}{(ln(x) - 3x)^2} - \frac{2(\frac{1}{x} - 3)(x^3 + 3)}{(ln(x) - 3x)^3}$$

Implicit Functions

6. Find $\frac{dy}{dx}$ for $(x-y)^2 = x+y-1$

Solution:

$$\frac{dy}{dx} = \frac{1-2x+2y}{-2x+2y-1}$$

7. Find $\frac{dy}{dx}$ for $xe^y = x-y$

Solution:

$$\frac{dy}{dx} = \frac{1}{e^y+1}$$

8. Find $\frac{dy}{dx}$ for $x^3+y^3 = 7$

Solution:

$$\frac{dy}{dx} = \frac{-y^2}{x^2}$$

9. Find $\frac{dy}{dx}$ for $\frac{y}{1-x^2} = 6y+7x$

Solution:

$$\frac{dy}{dx} = \frac{13(1-x^2)^2 - 2xy}{-x^2+1}$$

10. Find $\frac{dy}{dx}$ for $\frac{3xy}{x^5+y^2} = 1$

Solution:

$$\frac{dy}{dx} = \frac{15x^5y^5 - 3yx^5 + 3y^2}{3x^6+3xy-6xy^2}$$

Critical Points

11. [Paul Dawkins] Critical points for $A(t) = 3t - 7ln(8t+2)$

Solution:

$$t^* = \{\frac{25}{12}\}$$

12. [Paul Dawkins] Critical points for $f(x) = 5xe^{9-2x}$

Solution:

$$x^* = \{\frac{1}{2}\}$$

13. [Paul Dawkins] Critical points for $f(z) = \frac{z+4}{2z^2+z+8}$

Solution:

$$x^* = \{-4 \pm 3\sqrt{2}\}$$