Day 1 Warm-up problems

Monday, August 9, 2021 9:38 AM

⑤ $\ln(x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n})$

$= \ln(x_1^{\alpha_1}) + \ln(x_2^{\alpha_2}) + \cdots + \ln(x_n^{\alpha_n})$

$= \alpha_1 \ln(x_1) + \alpha_2 \ln(x_2) + \cdots + \alpha_n \ln(x_n)$

$= \sum_{i=1}^{n} \alpha_i \ln(x_i)$

⑥ $\frac{1}{x^{\alpha_1}} \cdot \frac{1}{x^{\alpha_2}} \cdots \frac{1}{x^{\alpha_n}}$

$= \prod_{i=1}^{n} \frac{1}{x^{\alpha_i}} = \prod_{i=1}^{n} x^{-\alpha_i}$

③ $\ln\left(\prod_{n=1}^{N} x_n\right)$

$= \ln(x_1 x_2 \cdots x_N)$

$= \ln(x_1) + \ln(x_2) + \cdots + \ln(x_N)$

⑤ $\frac{\prod_{n=1}^{N} (x_n^2 - 4)}{\prod_{n=1}^{N} (x_n - 2)}$

$= \frac{\prod \cancel{(x_n-2)}(x_n+2)}{\prod \cancel{(x_n-2)}}$

$= \prod_{n=1}^{N} (x_n+2) = (x_1+2)(x_2+2)\cdots(x_N+2)$

1) b) $\frac{d}{dx} \frac{\ln(x)}{6ax^3}$

$= \frac{1}{6a} \cdot \ln(x) \cdot x^{-3}$

$$\boxed{\begin{array}{l}\text{Product Rule} \\ \frac{d}{dx} f(x)g(x) \\ = fg' + f'g\end{array}}$$

$= \frac{1}{6a}\left(\ln x \cdot -3x^{-4} + \frac{1}{x} \cdot x^{-3}\right)$

$= \frac{1}{6a} x^{-4}(-3\ln x + 1)$

$= \frac{1-3\ln x}{6ax^4}$

② Find f_x and f_y

a) $\underbrace{a\ln x}_{} + \underbrace{by}_{}$ $\qquad a\ln x \cdot by$

$f_x = \frac{a}{x}$ $\qquad \ln(a\ln x \cdot by)$

$f_y = b \qquad = \ln(a\ln x) + \ln(by) \geq$

b) $\underline{\underline{x}}^{\alpha} y^{\beta}$

$f_x = y^{\beta} \cdot \alpha \cdot x^{\alpha - 1}$

$f_y = x^{\alpha} \cdot \beta \cdot y^{\beta - 1}$

III. $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ 4 & 4 & 5 \end{bmatrix} = \text{DNE}$

$$\begin{bmatrix} 3 & 3 \\ 2 & 2 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 8 & 6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -6 & -4 \\ -3 & 3 \end{bmatrix}$$

$$5 \cdot \begin{bmatrix} -1 & 2 \\ 3 & x \end{bmatrix} = \begin{bmatrix} -5 & 10 \\ 15 & 5x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2\cdot2+3\cdot4 & 2\cdot4+3\cdot3 \\ 4\cdot2+6\cdot4 & 4\cdot4+6\cdot3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 17 \\ 32 & 34 \end{bmatrix}$$

$$AB \neq BA$$

$$\begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \\ 13 & 14 & 15 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Identity}} = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \\ 13 & 14 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 5 & 12 \\ 2 & 0 & 7 \\ 3 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & 3 \\ 5 & 0 & 2 \\ 12 & 7 & -1 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

IV.

2) $Q^S = -5 + 2P$

$Q^D = 20 - 3P$

$Q^S = Q^D$

$-5 + 2P = 20 - 3P$

$5P = 25$

$\boxed{P = 5}$

5) $U(x,y) = a\log x + b\log y$

$$\max_{x,y} \; a\log x + b\log y$$
$$\text{s.t. } P_x \cdot x + P_y \cdot y = W$$

FOC's: $\nabla U(x,y) = \lambda \nabla g \qquad g(x,y) = P_x x + P_y y - W$

$$\left.\begin{aligned} \frac{a}{x} &= \lambda P_x \\ \frac{b}{y} &= \lambda P_y \end{aligned}\right] \rightarrow \frac{\frac{a}{x}}{\frac{b}{y}} = \frac{P_x}{P_y}$$

$$\Rightarrow \frac{a}{b} \cdot \frac{y}{x} = \frac{P_x}{P_y}$$

$$\Rightarrow y = \frac{P_x}{P_y} \cdot \frac{b}{a} x \quad (*$$

(B.C) $P_x X + P_y Y = W$
$\downarrow (*)$

$$P_x X + \cancel{P_y} \cdot \left(\frac{P_x}{\cancel{P_y}} \frac{b}{a} X \right) = W$$

$$X \left(P_x + P_x \frac{b}{a} \right) = W$$

$$X = \frac{W}{P_x \left(1 + \frac{b}{a} \right)}$$

$$\boxed{X^* = \frac{W}{P_x} \cdot \frac{a}{b+a}}$$

$$\boxed{Y^* = \frac{W}{P_y} \cdot \frac{b}{b+a}}$$

Day 1 Lecture

Monday, August 9, 2021 10:36 AM

$\forall x \in \mathbb{R}_+, x \geq 0$

$\exists x \in \mathbb{R}, x > 1$

$\Longrightarrow$

$\wedge \quad \vee \quad \neg$

$\boxed{A \Rightarrow B}$

$\boxed{\neg A \vee B}$

$\Longleftrightarrow$

means

$A \Rightarrow B$

AND

$B \Rightarrow A$

$A \Leftrightarrow B$

$\mathbb{N}$

$\mathbb{Z}$

$\mathbb{Q}$ $\quad q \neq 0$

$\mathbb{R}$

[ ] closed

( ) open

$[a, \infty)$

$(-\infty, b]$

3 Types of Proofs

1. Direct
2. Indirect
3. Induction

$A \Rightarrow B$

Indirect Proof

assume A

show that $\neg B$

implies a contradiction

$\swarrow$

$A \wedge \neg A$

Prove that $\sqrt{2}$ is irrational

Rational: $\frac{p}{q}$

where $p, q \in \mathbb{Z}$

$q \neq 0$

If $\sqrt{2}$ rational, then:

$$\sqrt{2} = \frac{p}{q}$$ for some integers in lowest terms

$$\Rightarrow \sqrt{2}^2 = \left(\frac{p}{q}\right)^2$$

$$\Rightarrow 2 = \frac{p^2}{q^2}$$

$$\Rightarrow 2q^2 = p^2$$

$\Rightarrow p^2$ is even

$\Rightarrow p$ is even

$\therefore p = 2n$ for some integer n

$$\Rightarrow 2q^2 = (2n)^2 = 4n^2$$

$\Rightarrow q^2 = 2n^2$

$\Rightarrow q$ is even

if p and q are both even, then they are not in lowest terms. This is a contradiction.

A

assume $\neg A$

$\Rightarrow$ contradiction

Direct Proofs

$A \Rightarrow B$

Assume A
show it implies B

$A \Rightarrow C \Rightarrow D \Rightarrow B$

Ex 1

If a, b consecutive integers, then a+b is odd.

$b = a+1$

$\Rightarrow a+b = a+(a+1)$
$= 2a+1$

Since a is an integer, 2a is even and 2a+1 is odd, by definition.

$\therefore$ a+b is odd.

$$\blacksquare$$

Induction

Ex 1) Prove that

$$1+2+3+\dots+n = \frac{n(n+1)}{2} \quad \forall n \in \mathbb{N}$$

Suppose $n=1$.

$$1 \text{ should} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

✓

Suppose for some integer k, it's true that $1+2+\dots+k$

$$= \frac{k(k+1)}{2}$$

Then $1+2+\cdots+k+(k+1)$

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$\therefore \frac{n(n+1)}{2}$ holds for $n=k+1$

① True for $n=1$

② if true for k, then true for $k+1$

$\therefore 1 \Rightarrow$ true for 2

$\Rightarrow$ true for 3

$\ldots$

$\Rightarrow$ true for ALL $n \geq 1$

Even: can be divided by 2 to get an integer.

Suppose $a, b \in \mathbb{N}$

Then $2a, 2b$ are even.

Then $2a + 2b$

$= 2(a+b)$

And $a+b \in \mathbb{N}$

So $2(a+b)$ is even.

a and b are integers, with $a \neq 0$.

If a does not divide b (this means $\frac{b}{a} \notin \mathbb{Z}$) then the equation

$$ax^2 + bx + b - a = 0$$

has no <u>positive</u> integer solution.
