Mathematics Review Course
Summer 2023
Problem Set 02

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Note: [Source] at the start of each problem denotes the source of the question. If there is no source, it is an original problem of my creation.

Sets

1. [S&B] Let A be the set of even integers, and B the set of odd integers. Describe both $A \cap B$ and $A \cup B$.
2. Show that the Cartesian product $A \times B \times C \neq (A \times B) \times C$. Let $A = \{1, 2\}, B = \{3, 4\}, C = \{5, 6\}$.
3. Attempt a proof for De Morgan's Law: $\left[\bigcup_{i=1}^{k} A_i\right]^c = \bigcap_{i=1}^{k} A_i^c$. Note: You will want to prove this by considering two sets A and B and proving both directions (1) $(A \cap B)^c \subseteq A^c \cup B^c$ and (2) $A^c \cup B^c \subseteq (A \cap B)^c$.
4. Let $S = (1, 2, 3, 4, 5)$. Show that the set $\{s \in \mathbb{R}^5 | s \cdot r \leq 25\}$ is a convex set.
5. Let c_1 and c_2 be convex sets. And let $c = c_1 \cap c_n$. Show that c must also be a convex set.

Topology

6. Prove that $\inf S = \{x \in \mathbb{R} : 0 < x < 1\} = 0$.
7. [UC Davis] Show that if functions $f, g : A \to \mathbb{R}$ are bounded functions such that $|f(x) - f(y)| \leq |g(x) - g(y)| \forall x, y \in A$ then $\sup_A f - \inf_A f \leq \sup_A g - \inf_A g$.
8. [S&B] Prove that $|xy| = |x| \cdot |y| \forall x, y$.
9. [UC Davis] Show that $\lim \left(\frac{2n+1}{5n+4}\right) \to \frac{2}{5}$.
10. [S&B] Prove that a set of real numbers $\mathbb{R}$ can have at most one least upper bound (i.e., sup).