Day 2 Notes

Tuesday, August 10, 2021 9:05 AM

$ax^2+bx+b-a$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

and here $c=b-a$

so $$x=\frac{-b\pm\sqrt{b^2-4a(b-a)}}{2a}$$

$$=\frac{-b\pm\sqrt{b^2-4ab+4a^2}}{2a}$$

$$=\frac{-b\pm\sqrt{(2a-b)^2}}{2a}$$

$$=\frac{-b\pm(2a-b)}{2a}$$

So $$x=\frac{-b+2a-b}{2a}=\frac{-2b+2a}{2a}$$

$$=\boxed{\frac{-b}{a}+1}$$ not an integer

or $$x=\frac{-b-2a+b}{2a}=-1$$

$\mathbb{R}$ real numbers

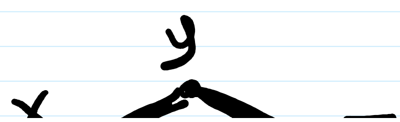
$\mathbb{R}^n$

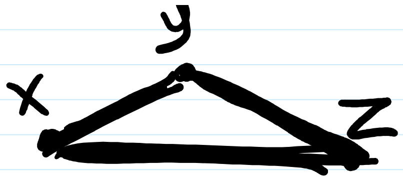
$(2,3,10)\in\mathbb{R}^3$

$\mathbb{R}^n_+$ non-negative

$\mathbb{R}^n_{++}$ strictly positive

Metric spaces





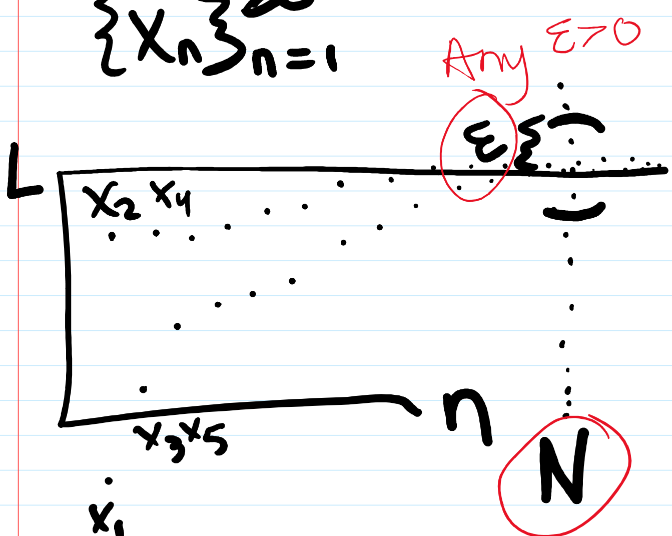
$$\mathbb{R}^2 = \sqrt{(x_1^2 - x_1^1)^2 + (x_2^2 - x_2^1)^2}$$

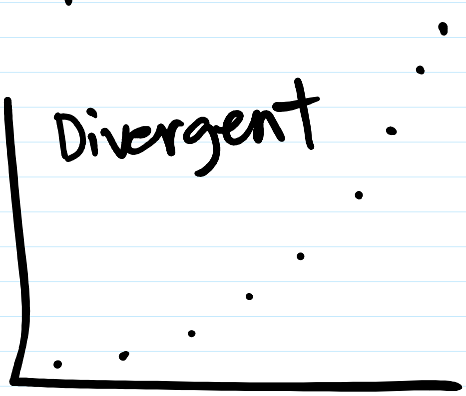
Sequence

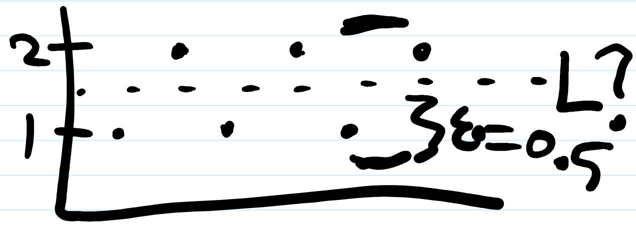
$x_1, x_2, x_3, \dots$

$\{X_n\}$

$\{X_n\}_{n=1}^{\infty}$







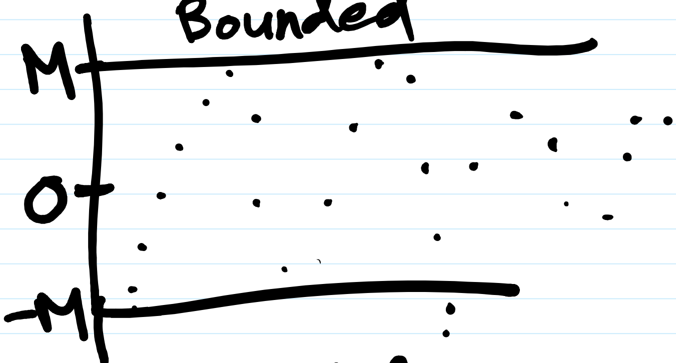
$\varepsilon = 0.5 \rightarrow$ OK

$\varepsilon = \frac{1}{100000} \rightarrow$ not OK

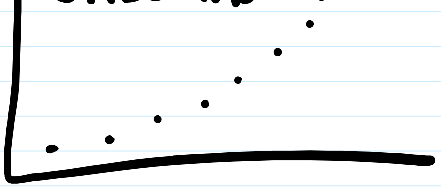
$\Rightarrow$ Divergent

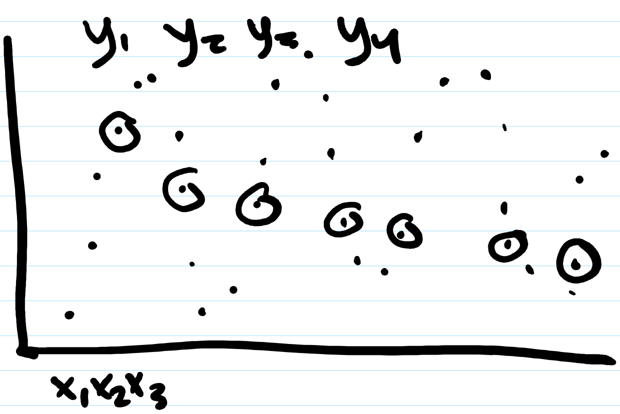
Bolzano-Weiterstrauss

Bounded



Unbounded

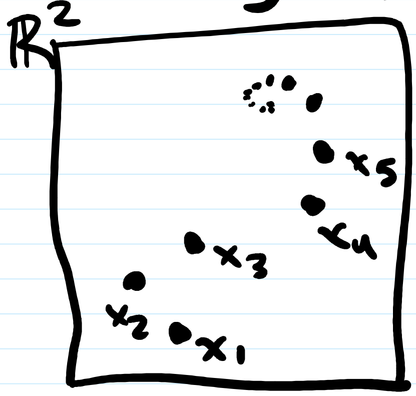




$\{x_n\}$ divergent

$\{y_n\}$ is a convergent sub-sequence

Cauchy Sequence



Sequences

In $\mathbb{R}^2$

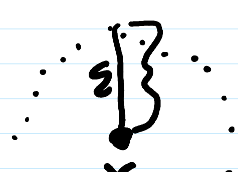
$d(x_n, L) < \varepsilon$

whenever $n > N$

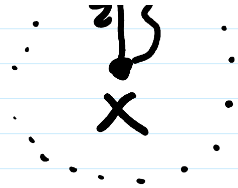
$$\sqrt{(x_n^1 - L^1)^2 + (x_n^2 - L^2)^2} < \varepsilon$$

Open ε-ball

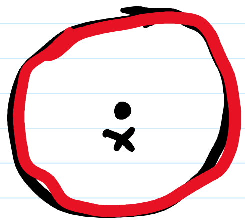
$B_\varepsilon(x)$



$x \in (X)$

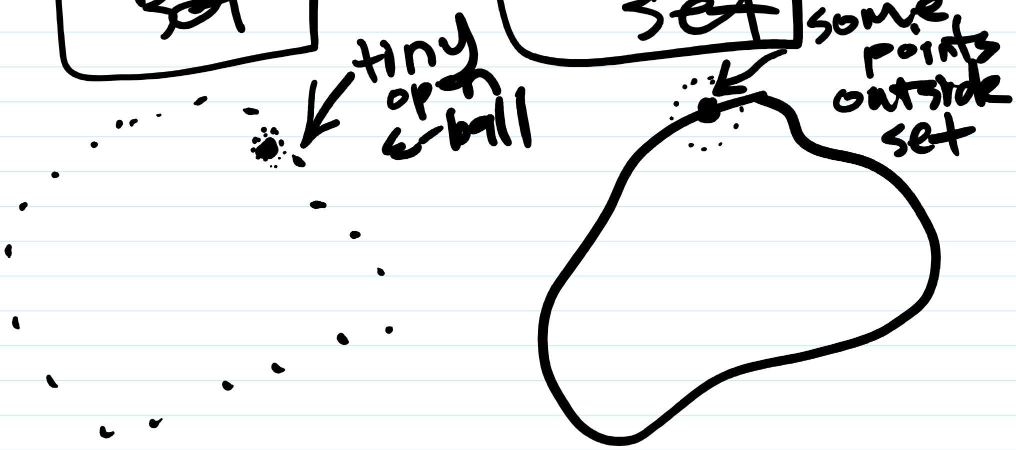


Closed:

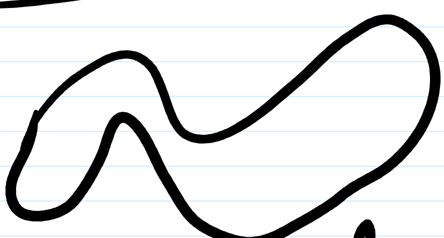


Open set

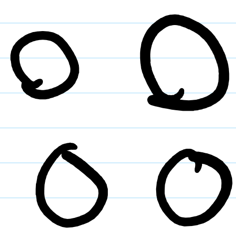
Closed set



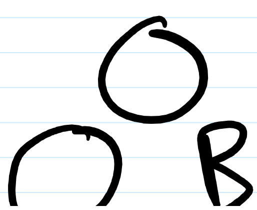
Connected Sets

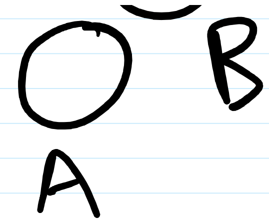


connected



not connected

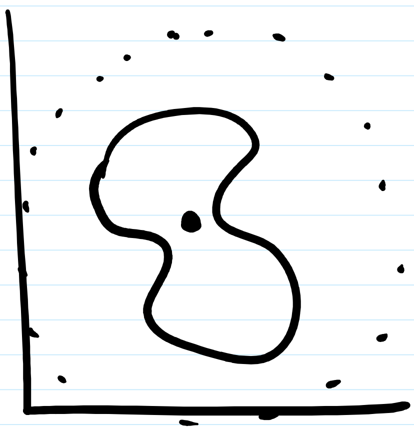




A and B disjoint

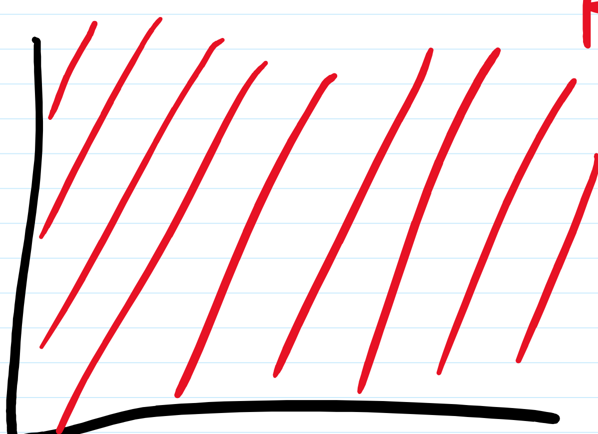
$\iff A \cap B = \emptyset$

bounded



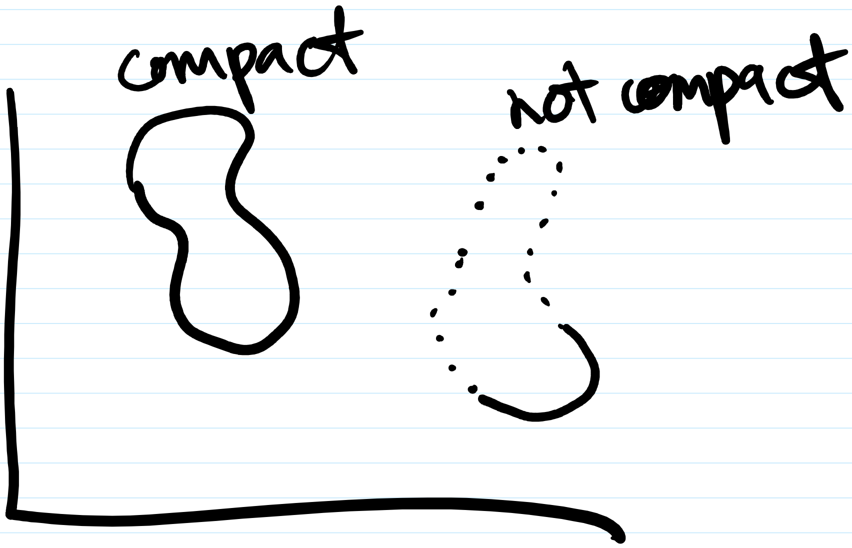
unbounded

$\mathbb{R}^2_+$

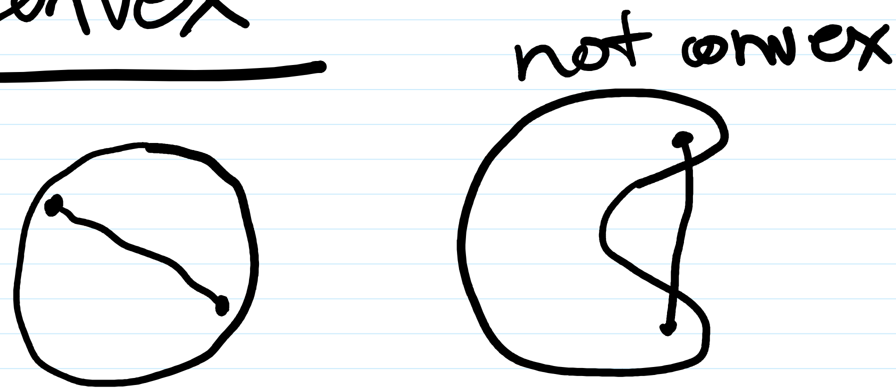


E would need to be infinite, which

infinite, which
is not permitted



Convex



Let $x_1, x_2 \in S$.

Then S is convex if

$\forall x_1, x_2 \in S$, and $t \in [0,1]$,

$$\underbrace{t x_1 + (1-t) x_2}_{\substack{\text{convex} \\ \text{combination} \\ \text{of } x_1 \text{ and } x_2}} \in S$$

<u>example</u>

Let $x_a = (x_a^1, x_a^2)$
$\quad x_b = (x_b^1, x_b^2)$

Convex combination:

$$\theta x_a + (1-\theta) x_b$$

$$= \left(\theta x_a^1 + (1-\theta) x_b^1, \theta x_a^2 + (1-\theta) x_b^2\right)$$

Thm

If S, T are convex, then $S \cap T$ is also.

Proof

Let S and T be convex.

Let x^1 and $x^2 \in S \cap T$

So $x^1, x^2 \in S$ and $x^1, x^2 \in T$

Let $z = tx^1 + (1-t)x^2$.

Because S is convex and x^1 and x^2 are in S, $z \in S$.

Bec. T convex and $x^1, x^2 \in T$, then $z \in T$.

But that means

$z \in S \cap T$.

So since x^1 and x^2 were arbitrary, this shows

$S \cap T$ is convex.

$\blacksquare$

Example

PPS is set of all feasible combinations of inputs and outputs. Generalization of a production function.

You are <u>told</u> the IRS is convex.

$a = (q_1, z_1)$ $b = (q_2, z_2)$

$ta + (1-t)b \in IRS$

$(tz_1 + (1-t)z_2, tq_1 + (1-t)q_2) \in IRS$