Mathematics Review Course
Summer 2023
Problem Set 10
Solutions

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1. Max $U = xy$ s.t. $100 \geq x + y$ and $x \leq 40$.

Solution:

$$\mathcal{L} = xy + \lambda(100 - x - y) + \mu(40 - x)$$

$$\begin{aligned}
\mathcal{L}_x =& y - \lambda - mu = 0, x \geq 0, x \cdot \mathcal{L}_x = 0 \\
\mathcal{L}_y =& x - \lambda = 0, y \geq 0, y \cdot \mathcal{L}_y = 0 \\
\mathcal{L}_\lambda =& 100 - x - y = 0, \lambda \geq 0, \lambda \cdot \mathcal{L}_\lambda = 0 \\
\mathcal{L}_\mu =& 40 - x = 0, \mu \geq 0, \mu \cdot \mathcal{L}_\mu = 0
\end{aligned}$$

$$\begin{aligned}
x^* =& 40 \\
y^* =& 60 \\
\lambda^* =& 40 \\
\mu^* =& 20
\end{aligned}$$

2. Max $U = xy^2$ s.t. $100 \geq x + y$ and $120 \geq 2x + y$.

Solution:

$$\mathcal{L} = xy^2 + \lambda(100 - x - y) + \mu(120 - 2x - y)$$

$$\begin{aligned}
\mathcal{L}_x =& y^2 - \lambda - 2\mu = 0, x \geq 0, x \cdot \mathcal{L}_x = 0 \\
\mathcal{L}_y =& 2xy - \lambda - \mu = 0, y \geq 0, y \cdot \mathcal{L}_y = 0 \\
\mathcal{L}_\lambda =& 100 - x - y = 0, \lambda \geq 0, \lambda \cdot \mathcal{L}_\lambda = 0 \\
\mathcal{L}_\mu =& 120 - 2x - y = 0, \mu \geq 0, \mu \cdot \mathcal{L}_\mu = 0
\end{aligned}$$

$$\begin{aligned}
x^* =& 20 \\
y^* =& 80 \\
\lambda^* =& 0 \\
\mu^* =& 3,200
\end{aligned}$$

Comparative Statics

3. [Uni. Cape Town] Suppose you have found the equilibrium quantity as $Q^* = \frac{ad - bc}{b + d}$. Sign the comparative static changes for the non-negative variables a, b, c, d.

Solution:

$$\begin{aligned}
\frac{\partial Q^*}{\partial a} =& \frac{d}{b + d} > 0 \\
\frac{\partial Q^*}{\partial b} =& \frac{d(a - c)}{(b + d)^2} > 0 \mid a > c \\
\frac{\partial Q^*}{\partial c} =& \frac{-b}{b + d} < 0 \\
\frac{\partial Q^*}{\partial d} =& \frac{b(c - a)}{(b + d)^2} < 0 \mid a > c
\end{aligned}$$

4. [Uni. Cape Town] Suppose you have determined market equilibrium as $Q^* = \frac{\theta(\alpha + \gamma G) - \beta(\delta + \lambda N)}{(\beta + \theta)}$ and $P^* = \frac{\delta + \lambda N + \alpha + \gamma G}{(\beta + \theta)}$. Sign the comparative statics if the substitution good price G changes and if the input price N changes.

Solution:

$$\begin{aligned}
\frac{\partial Q^*}{\partial G} =& \frac{\theta\gamma}{\beta + \theta} > 0 \\
\frac{\partial P^*}{\partial G} =& \frac{\gamma}{\beta + \theta} > 0 \\
\frac{\partial Q^*}{\partial N} =& \frac{-\beta\lambda}{\beta + \theta} < 0 \\
\frac{\partial P^*}{\partial N} =& \frac{\lambda}{\beta + \theta} > 0
\end{aligned}$$