Day 4 Notes

Thursday, August 12, 2021 9:04 AM

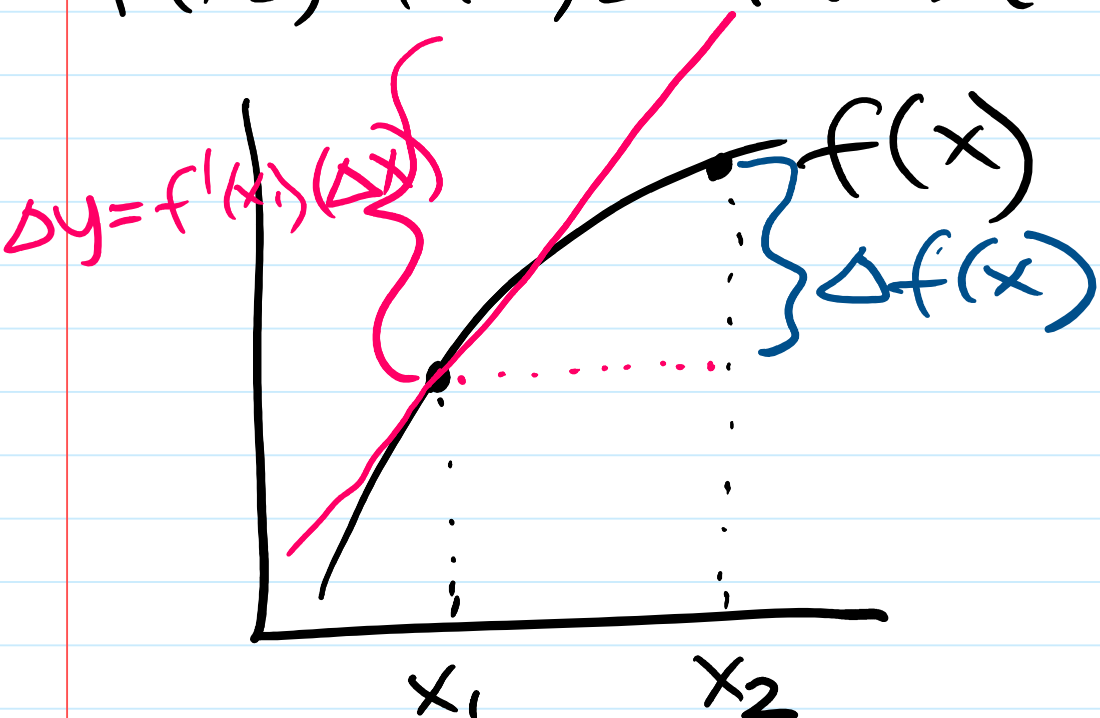
Convexity w/ Calcus

f is concave on

$I \subseteq \mathbb{R}$ iff

$\forall x_1, x_2 \in I$

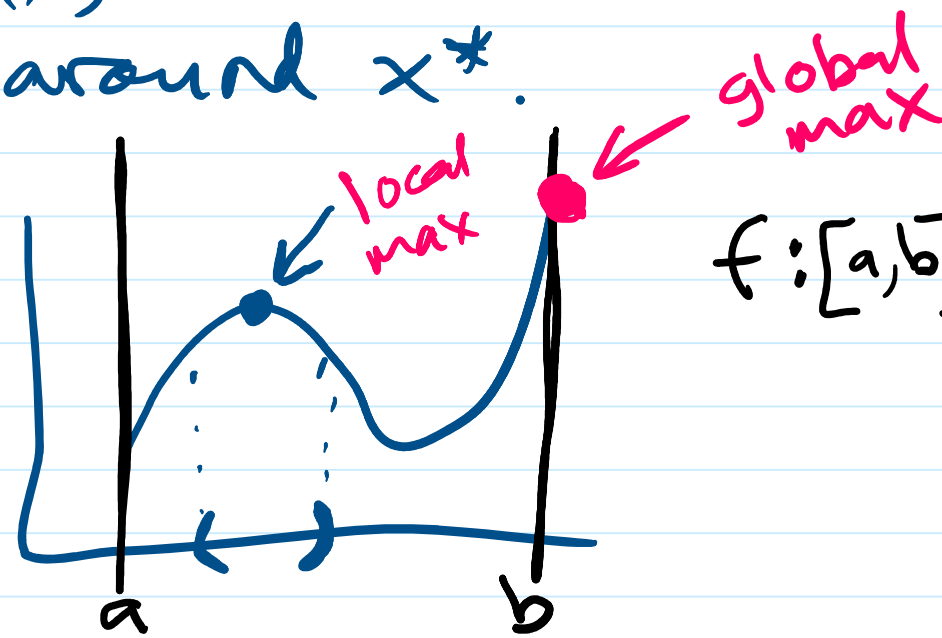
$$f(x_2) - f(x_1) \leq f'(x_1)(x_2 - x_1)$$



Local Max

Local Max

x^* local max if $f(x^*)$ is the largest value of $f(x)$ in some interval around x^*.

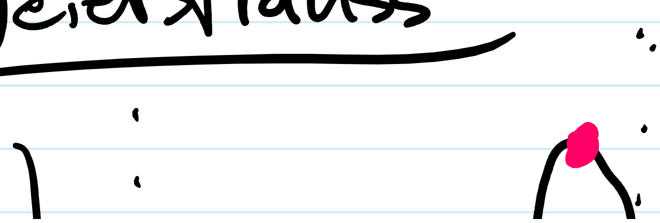


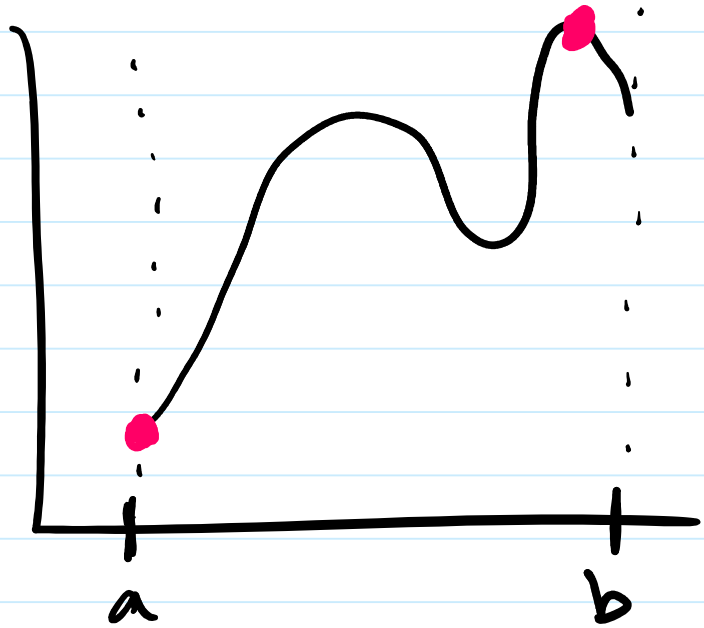
$f : [a,b] \to \mathbb{R}$

Global Max

$f(x^*)$ must be the largest value of f on the entire domain ~~of~~ f.

Weierstrauss



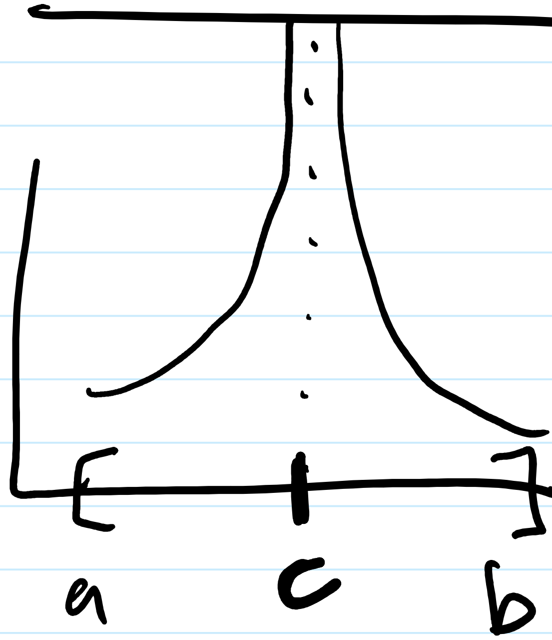


[a,b] closed

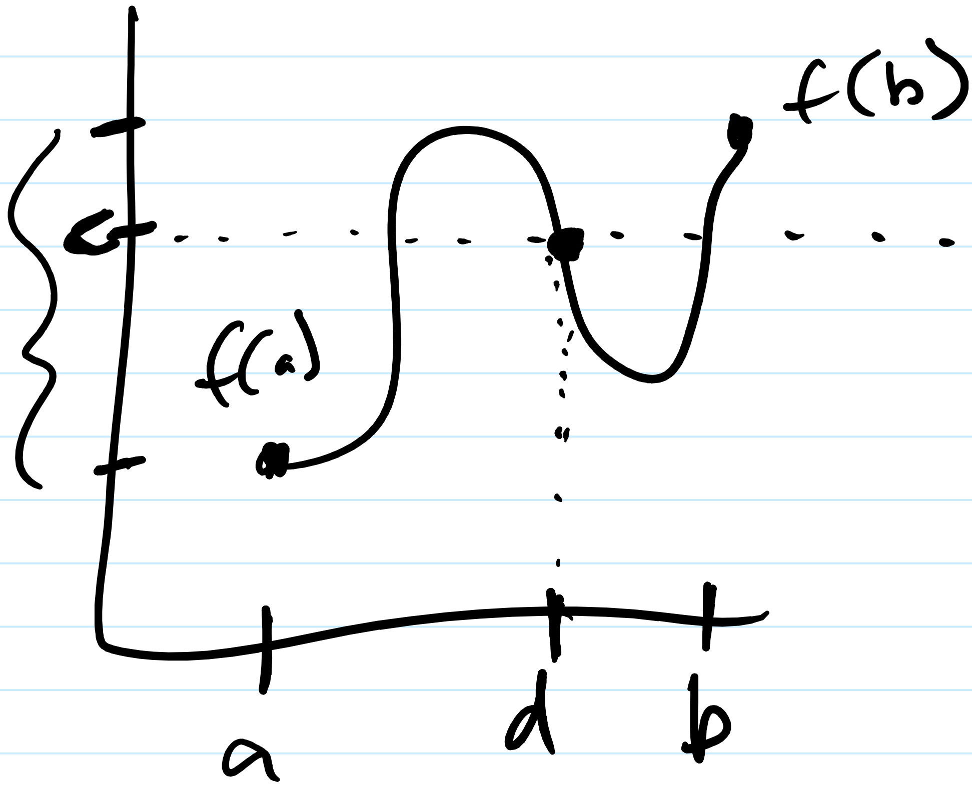
f continuous

Not closed

Not continuous

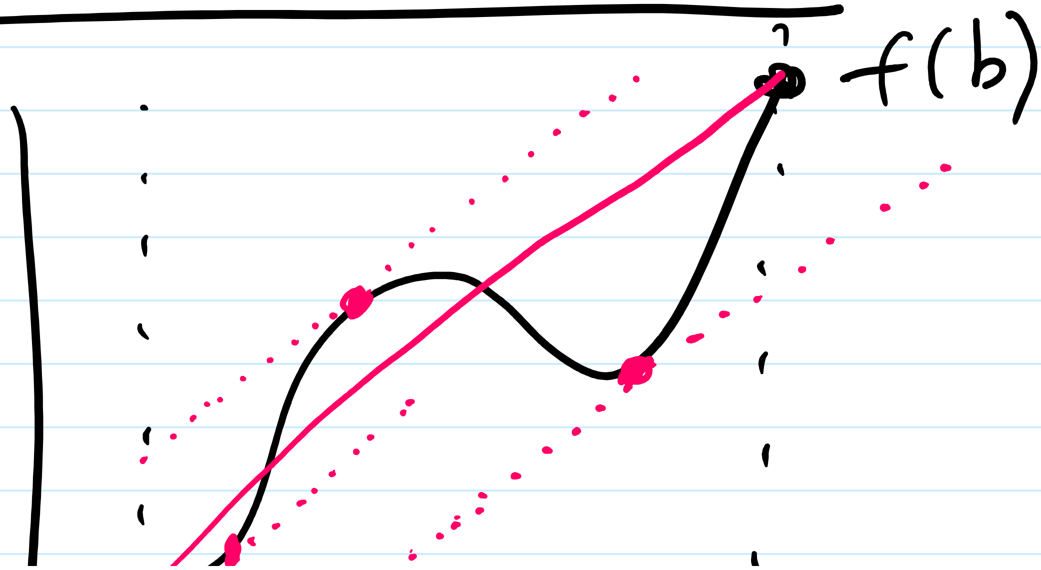


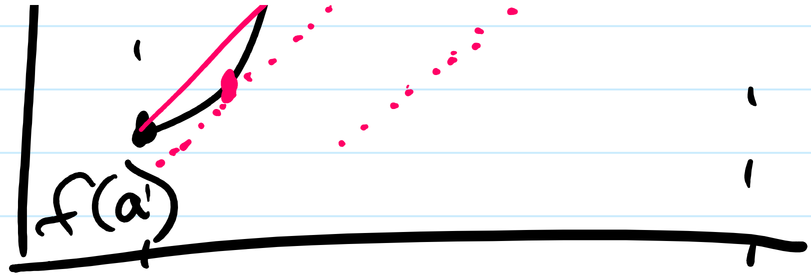
Intermediate Value Thm



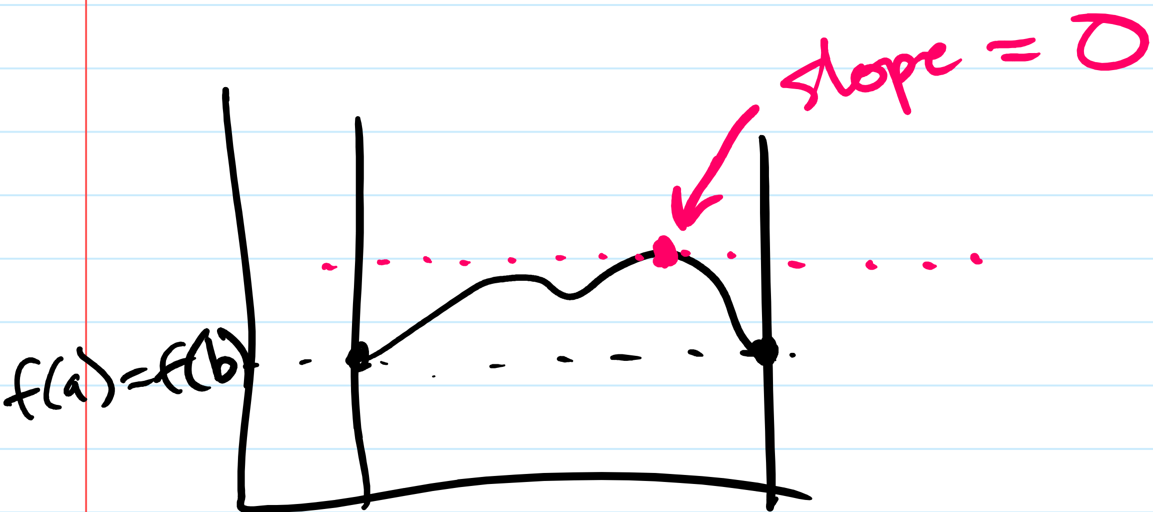
$$f(d) = c$$

Mean Value Thm



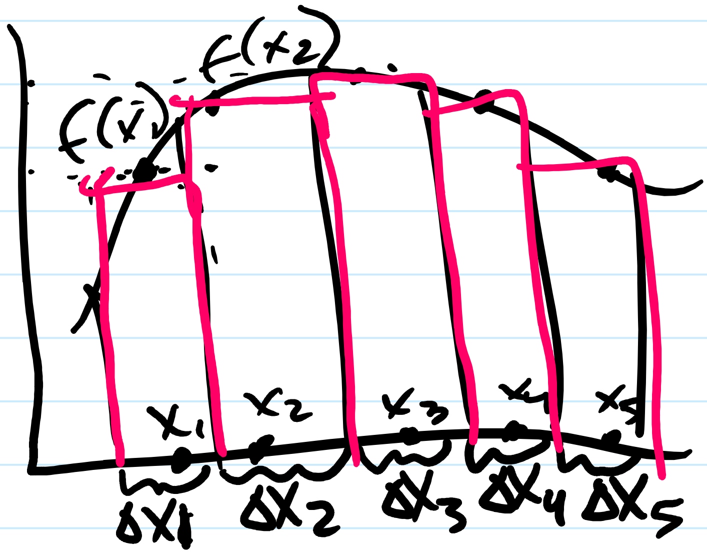


Rolle's Thm



$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

where Δx_i is the i'th subinterval obtained by subinterval obtained by dividing [a,b] into n even-length subintervals.



and $x_i \in \Delta x_i$

Fundamental Thm

$$g(x) = \int_a^x f(t)\,dt$$

$$g'(x) = f(x)$$

2nd Part

$$F(b) = F(a) + \int_a^b F'(x)\,dx$$

$\int_a^b f(x)\,dx$ definite

$\rightarrow$ a number

$\int f(x)\,dx$ indefinite

$\rightarrow$ a function of x

Let F be an antiderivative of f. Then:

$$\int f(x)\,dx = F(x) + C$$

Ex) calculate

$$\int\left(4x^2+x-\frac{3}{x}\right)dx$$

where $F(1)=0$

$$=4\int x^2\,dx+\int x\,dx-3\int\frac{1}{x}\,dx$$

$$=4\cdot\frac{x^3}{3}+\frac{x^2}{2}-3\ln|x|+C$$

Set $F(1)=0$

$$4\cdot\frac{1}{3}+\frac{1}{2}-3\cdot 0+C=0$$

$$\frac{4}{3}+\frac{1}{2}+C=0$$

$$C=\frac{-11}{6}$$

$$\int f g' \, dx = fg - \underbrace{\int g f' \, dx}_{\text{simpler}}$$

Ex Find $\int \ln x \, dx$

$u = \ln x \quad du = \frac{1}{x} dx$

$v = x \quad dv = dx$

$$\int u \, dv = uv - \int v \, du$$

$$= x \ln x - \int x \cdot \frac{1}{x} dx$$

$$= x \ln x - \int dx$$

$= x\ln x - x + C$

Substitution Rule

$$\int_a^b f(g(x))g'(x)\,dx$$

$g(x)$ becomes the variable of integration.

differentials:

$u = g(x)$

$du = g'(x)dx$

$$\int_{g(a)}^{g(b)} f(u)\,du$$

$$\int_{g(a)} f(u)\,du$$

$\boxed{\text{Ex}}$ Find $\int_0^1 e^{2x}\,dx$

$u = 2x \quad du = 2dx$

$x = 0, \; u = 0$

$x = 1, \; u = 2$

$dx = 2dx \cdot \frac{1}{2}$

Re-write:

$$\int_0^2 \frac{1}{2} \cdot e^u\,du$$

$$= \frac{1}{2} e^u \Big]_0^2$$

$$= \frac{e^2 - 1}{2}$$

Leibniz's Rule

$$\int_a^b f(x, \theta)\, dx$$

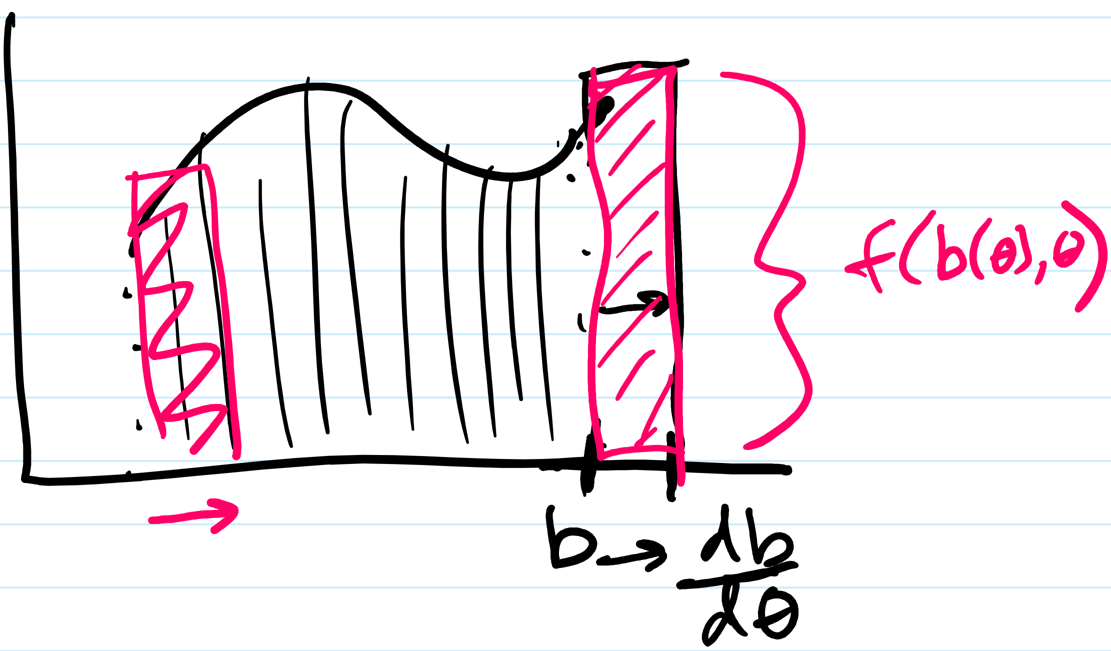
how does the value of the integral change when we change θ?

Simplest form: endpoints are constant.

"Just sum up the change in the function for the whole interval."

$$\frac{d}{d\theta}\int_a^b f(x,\theta)\,dx$$

$$= \int_a^b \frac{\partial f(x,\theta)}{\partial \theta}\,dx$$



$\Delta\theta \to$

$\sim$

$\frac{db}{d\theta}$

Ex | Calculate

$$\frac{d}{dy}\int_{2+y}^{y^2}(x+y)^2\,dx$$

$$= \frac{d}{dy}y^2 \cdot (x+y)^2\Big]_{y^2}$$

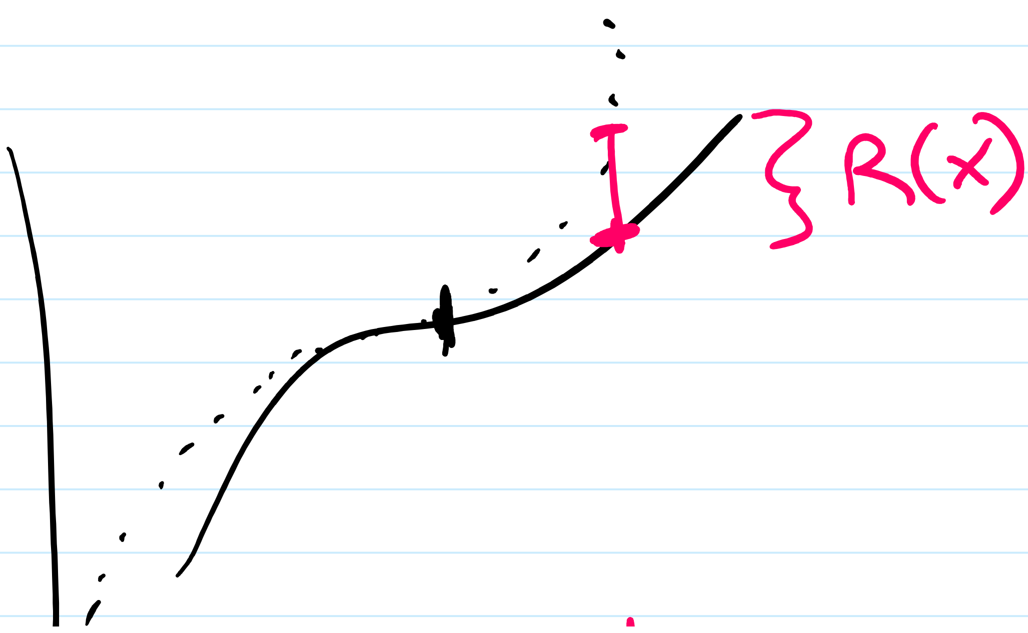
$$- \frac{d}{dy}(2+y) \cdot (x+y)^2\Big]_{2+y}$$

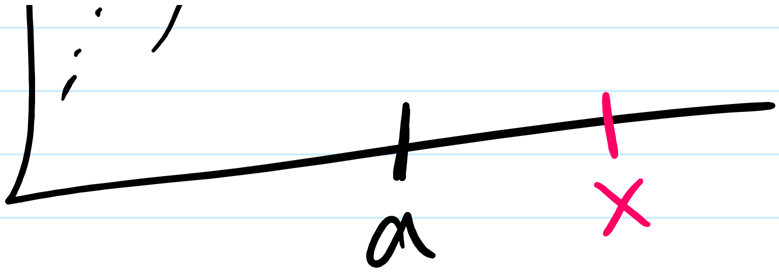
$$+ \int_{2+y}^{y^2} 2(x+y)\,dx$$

$$= 2y \cdot 2(y^2+y)(2y+1)$$

$$-2(2+y+y)\cdot 2$$

$$+\underbrace{\int_{2+y}^{y^2} 2x\,dx}_{\underbrace{x^2\Big]_{2+y}^{y^2}}_{(y^2)^2-(2+y)^2}} + \underbrace{\int_{2+y}^{y^2} 2y\,dx}_{\underbrace{2yx\Big]_{2+y}^{y^2}}_{2y(y^2-2-y)}}$$





$$P_k(x) = f(a) + f'(a)(x-a) + \dots + f^{(k)}(a)(x-a)^k$$

Taylor polynomial of order k.

$$f(x) = P_k(x) + \underbrace{R(x)}$$

Econometrics:

"Delta method" approximates the variance of a random variable or a statistic using a 2nd-order Taylor polynomial.

Ex e^x. Find P_5 at $a=0$.

$$P_5(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{3!} + f^{(4)}(0)\frac{x^4}{4!} + f^{(5)}(0)\frac{x^5}{5!}$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

$$\boxed{e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}}$$

Econometrics:

polynomials in linear regression.

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

$$y_i = \alpha + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \beta_4 x_i^4 + \varepsilon_i$$