

Mathematics Review Course

Summer 2023

Problem Set 01

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Note: [Source] at the start of each problem denotes the source of the question. If there is no source, it is an original problem of my creation (an unlikely occurrence).

Necessary and Sufficient Conditions

1. [Khan Academy] Identify any necessary or sufficient conditions to falsify the statement, “My car needs gas in order to run. Therefore, if I put some gas in the tank, my car will run.”
2. [Khan Academy] Identify any necessary or sufficient conditions to falsify the statement, “Plagiarizing would have given Michael a high score on his history paper. Since Michael just received the highest score possible on the history paper, we can be reasonably confident that he plagiarized it.”
3. [Kaplan LSAT] Identify any necessary or sufficient conditions to falsify the statement, “If the lawnmower starts, then the key must be in the ignition.”
4. [Hayden Economics] Identify any necessary or sufficient conditions to falsify the statement, “Those who passed the course must have had a Grade A in the class.”
5. Identify any necessary or sufficient conditions to falsify the statement, “When a market clears, it must be in a Walrasian equilibrium.”

Mathematical Proofs

6. [Hammack] If $n \in \mathbb{Z}$, then $n^2 + 3n + 4$ is even.
7. [Hammack] Suppose $x, y \in \mathbb{Z}$. If $x^2(y + 3)$ is even, then x is even or y is odd. Note: It may be helpful to use De Morgan’s Law – $\left[\bigcup_{i=1}^k A_i\right]^c = \bigcap_{i=1}^k A_i^c$.
8. [Hammack] Prove that $\sqrt{3}$ is irrational.
9. [Ling Yao; 2020] A rational preference has two properties. 1. Completeness: $\forall x, y$ in a set of possible alternatives, either $x \succsim y$ or $y \succsim x$, or both. 2. Transitivity: $\forall x, y, z$ in a set of possible alternatives, $x \succsim y$ and $y \succsim z \implies x \succsim z$. Prove that $x \succ y \succsim z \implies x \succ z$. **Note:** \succ is a strong preference similar to $>$, and \succsim is a weak preference similar to \geq . \sim is an indifferent preference similar to \equiv .