Lecture 14
Ancillary Material

Ryan McWay[†]

[†] *Applied Economics,*
University of Minnesota

Mathematics Review Course, Summer 2023
University of Minnesota
August 24th, 2023

Last Lecture Review

- Time Series:
  - Stochastic Processes
  - Discrete & Continuous Time Markov Chain
  - Poisson Processes
  - Stationarity
  - Ergodicity
  - Unit Root
- Dynamic Programming:
  - Dynamic Programming Problem
  - Theory of the Maximum
  - Bellman's Principle of Optimality
  - Backward Induction
  - Bellman Equation

Daily Icebreaker

- Attendance via prompt:
  - Name
  - Daily Icebreaker: To your surprise, your distance relative of immense wealth (whom you did not know existed) has left you the inheritance of their estate. What is the first purchase you make?

Ancillary Material

Motivation

- These are topics that will be touched on in microeconomics and econometrics.
- Many of the topics are discussed in passing with little elaboration on the concept themselves.
- Some of these topics will be discussed at length in your coursework, but are worth priming now so they are not novel or new ideas.

Overview

1. Positive vs. Normative
2. Ex-Ante & Ex-Post
3. Cardinal vs. Ordinal
4. Extensive vs. Intensive Margin
5. Preference Relations
6. Bernoulli Functions
7. Homogeneity
8. Monotonicity
9. Elasticity
10. Local Non-satiation
11. Contour Sets
12. Gorman Form
13. Simplex
14. Singleton Set
15. Mean Preserving Spread
16. Independence of Irrelevant Alternatives
17. Exogenous vs. Endogenous
18. Sigma Fields
19. Jensen Inequality
20. Algebraic vs. Geometric Means
21. Analog Principle & Plug-in Estimators
22. Extreme Value Distribution
23. Inverse Mills Ratio

1. Positive vs. Normative

- Positive Analysis:
  - Objective statements.
  - Descriptions of the possible states of the world.
  - E.g., X% of people are poor.
- Normative Analysis:
  - Subjective statements.
  - A value judgement on the state of the world.
  - E.g., We should provide cash transfers to the bottom X% of the income distribution.

2. Ex-Ante & Ex-Post

- Ex-Ante: Before the event
  - What you expect to occur.
  - Expectation, forecasting, prediction, etc.
  - Suffers from post-hoc logical fallacies or type I errors.
- Ex-Post: After the fact
  - Understanding what has already occurred.
  - Causal analysis and ex-post counterfactuals

3. Cardinal vs. Ordinal

- Cardinal: Indicate quantities.
- Ordinal: Indicate rank or order in a set.
- Nominal: Indicate an identity (e.g., a zip code or a player's jersey number).

4. Extensive vs. Intensive Margin

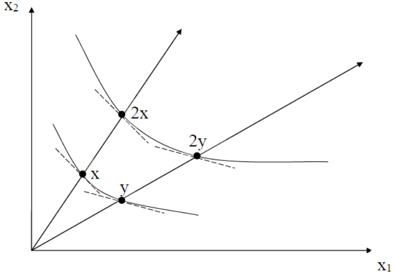
- Extensive Margin: Dichotomous switch of selecting into or out of an activity (e.g., $0 \rightarrow 1$).
  - i.e., Labor force participation.
- Intensive Margin: Continuous intensity at which a person selecting into the activity participates.
  - i.e., Number of hours worked.

5. Preference Relations

- Preferences are are ordinal relationships between commodities or bundles.
- This describes how individuals and firms make choices based on a ranking of desires.
- Preferences are expressed cardinally through utility functions.
    - Strongly prefer x to y: $x \succ y$
    - Weakly prefer x to y: $x \succsim y$
    - Indifferent between x and y: $x \sim y$

5. Preference Relations

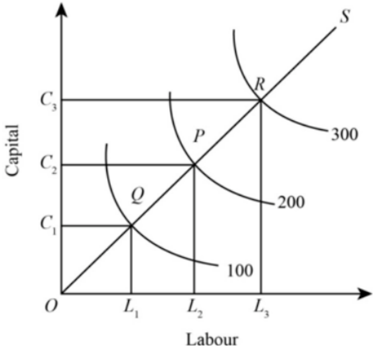
- Homothetic preferences: For any ray drawn from the origin, the slopes of all indifference sets at the points crossed by that ray are equal.
- If $x \sim y$ then $\alpha x \sim \alpha y \forall \geq 0$.
- E.g., Consumers with **different** incomes facing the **same** prices with identical preferences will demand goods in the **same proportions**.



5. Preference Relations

- Cobb-Douglas Preferences: Type of homothetic preference that are 'well-behaved' and have a monotonic relationship.
- Historically observed relationship of labor and capital $\rightarrow$ production function.

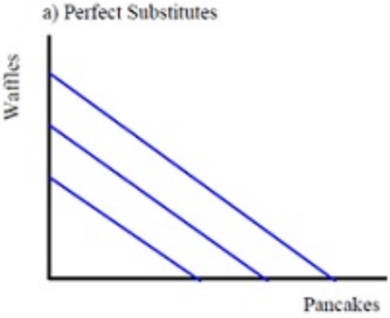
$$U(x,y) = x^{\alpha}y^{1-\alpha}\forall 0 < \alpha < 1$$



5. Preference Relations

- Linear Preferences: Type of homothetic preference representing perfect substitutes.

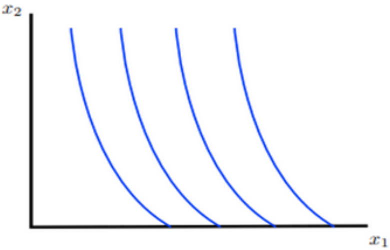
$$U(x, y) = \alpha x + \beta y$$



5. Preference Relations

- Quasilinear Preferences: Sometimes homothetic preferences in which one good has a linear relationship and the other does not.
- E.g., Demand for one good (the linear good) has a limit (up to a certain amount in their basket).
- Non-linear transformation: $h(\cdot)$

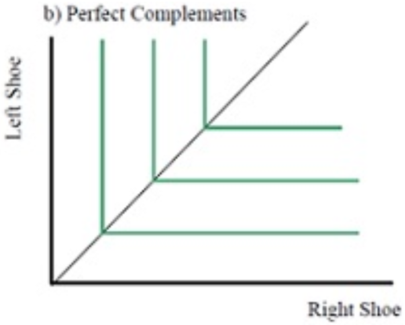
$$U(x, y) = \alpha x + h(y)$$



5. Preference Relations

- Leontief Preferences: Type of homothetic preference representing perfect compliments.
- Often involves some $min(\cdot)$ or $max(\cdot)$ operator.

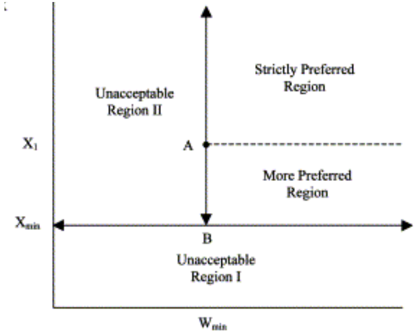
$$U(x, y) = \min\{x, y\}$$



5. Preference Relations

- Lexiographic Preferences: Comparative preferences where any amount of one good is preferred to any amount of another.
- E.g., they only care about one of the goods and does not consider the other good.

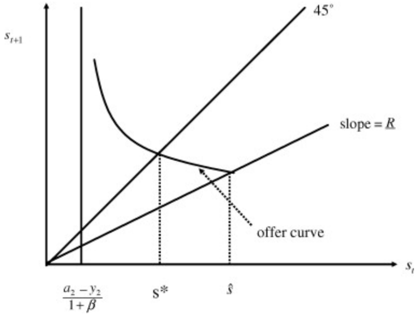
$$x \succsim y \text{ if either } x_1 > y_1 \vee (x_1 = y_1 \wedge x_2 \geq y_2)$$



5. Preference Relations

- Stone-Geary Preferences: Denies homothetic preferences leading to non-linear demands
- E.g., Doubling income does not double demands.

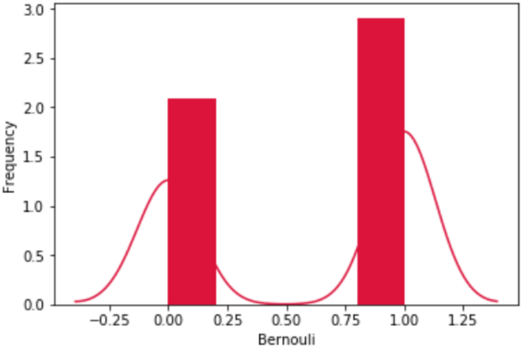
$$U(x,y) = (x-\gamma_x)^{\alpha}(y-\gamma_y)^{\beta}$$



6. Bernoulli Functions

- Discrete probability distribution taking only values $k \in \{0, 1\}$ at given probabilities p.

$$f(k, p) = pk + (1-p)(1-k)$$



7. Homogeneity

- Homogeneous Function: When the arguments of a function are multiplied by a scalar, then the value of the function is a power (i.e., degree) of this scalar.
- Homogenous of degree 0 (H.D.0)

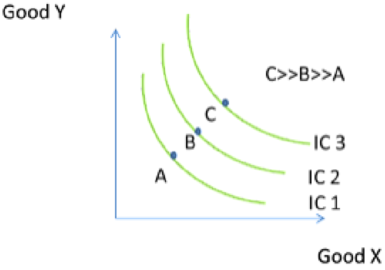
$$x(\alpha p, \alpha w) = x(p, w) \forall \alpha > 0$$

- Homogenous of degree 1 (H.D.1)

$$x(\alpha p, \alpha w) = \alpha x(p, w) \forall \alpha > 0$$

8. Monotonicity

- E.g., More is better and we like variety.
- Suppose there are n commodities in x_1 and x_2.
- Weak monotonic preferences if $x_1 \geq x_2 \implies x_1 \succsim x_2$ (i.e., at least one more in quantity of any good n means you prefer that bundle).
- Strong if you replace with $>$.
- Important to making a $log(\cdot)$ transformation.



9. Elasticity

- Relative change in demand for good l in response to a percentage change in the parameter.
- Price Elasticity:

$$\varepsilon_{lk}(p,w) = \frac{\partial x_l(p,w)}{\partial p_k} \frac{p_k}{x_l(p,w)}$$

- Wealth Elasticity:

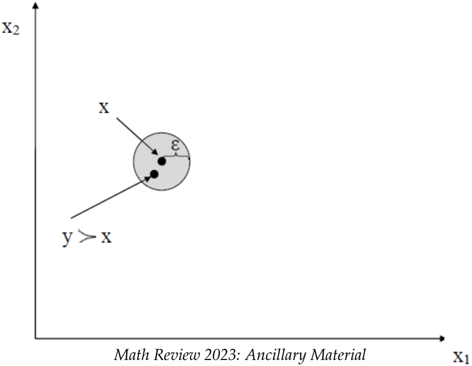
$$\varepsilon_{lw}(p,w) = \frac{\partial x_l(p,w)}{\partial w} \frac{w}{x_l(p,w)}$$

10. Local Non-satiation

- For a point x, there is some very close point y which is strictly preferred.

$$\forall x \in X, \varepsilon > 0, \exists y \in X : ||y - x|| \leq \varepsilon \wedge y \succ x$$

$$||y - x|| = \left(\sum_{l=1}^{L} (y_l - x_l)^2 \right)^{1/2}$$



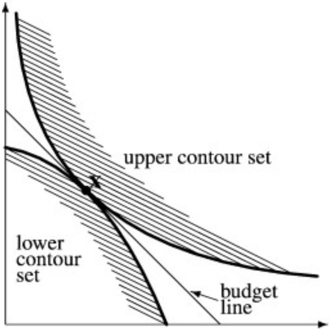
11. Contour Sets

- Upper Contour Set:

$$\{y \in y \succsim x\}$$

- Lower Contour Set:

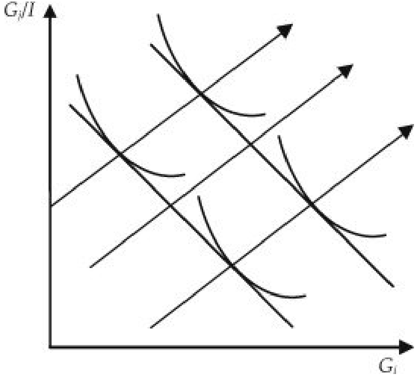
$$\{y \in x \succsim y\}$$



12. Gorman Form

- Indirect utility function allows you to aggregate utilities
- $a_i(p)$: Reference utility at zero for each individual.
- $b(p)$: Parallel wealth expansion paths for all individuals

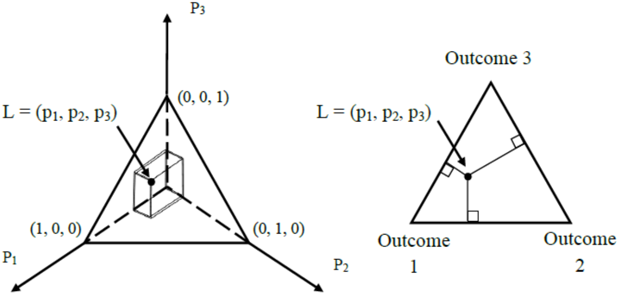
$$v_i(p, w_i) = a_i(p) + b(p)w_i$$



13. Simplex

- A symmetric triangle of n-dimensions.
- Used in probability to represent events.

$$\Delta = \{p \in \mathbb{R}^N_+ : p_1 + p_2 + \cdots + p_N = 1\}$$

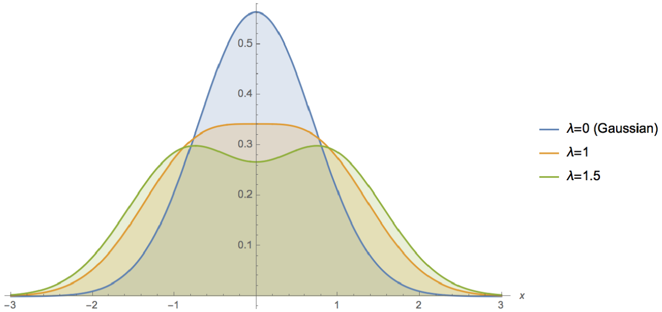


14. Singleton Set

- A set with exactly one element.
- E.g., $\{0\} \rightarrow$ the only element is 0.
- Let S be a class of indicator function such that $b : X \rightarrow \{0, 1\}$.
- Then S is a singleton iff $\exists y \in X : \forall x \in X$
$$b(x) = (x = y)$$

15. Mean Preserving Spread

- When you change from one distribution A to another distribution B, the expected value (i.e., mean) remains unchanged.
- Variance may vary.



16. Independence of Irrelevant Alternatives

- The social preferences between alternatives x and y depend only on the individual preferences x and y.
- That is to say if we added z to the mix, preference ordering would remain the same.
- Corollary: If you remove an option, it will not change the rank order of selection.

17. Exogenous vs. Endogenous

- Endogenous: From within in the system (e.g., parametrically determined).
- Exogenous: From outside the system (e.g., deterministic).
- Meaning varies between use in structural econometrics (viz., GMM) and causal inference (viz., RCT) as to what is 'deterministic'.

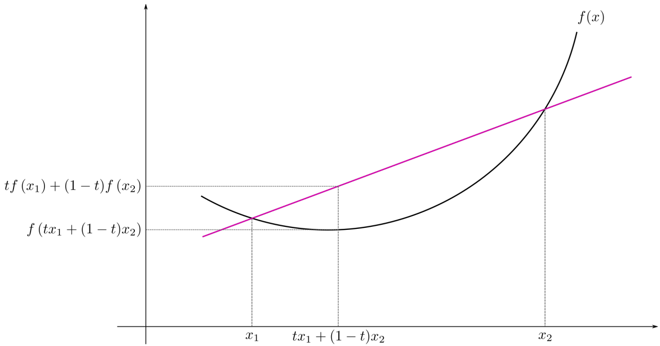
18. Sigma Fields

- Sigma Field (or σ-algebra): A collection of subsets $\mathcal{B}$ of the sample space S excluding weird sets.
  - $\emptyset \in \mathcal{B}$
  - $A^c \in \mathcal{B}$
  - $\mathcal{B}$ is closed under countable unions
- Borel σ-algebra: The smallest σ-algebra on the real number line containing all open intervals.
- Borel Sets: Sets in Borel σ-algebra.
- Helps us to limit outcomes to real-values (e.g., we need not consider the complex plane).

19. Jensen Inequality

- The parts in the sum are less than the sum of the parts.
- E.g., The mean of the payoffs will always be larger than or equal to the payoff of the mean outcome

$$f(\alpha x_1 + (1-\alpha)x_2) \leq \alpha f(x_1) + (1-\alpha)f(x_2)$$



20. Algebraic vs. Geometric Means

- Algebraic Mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Geometric Mean:

$$\bar{x} = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}} = exp\left(\frac{1}{n}\sum_{i=1}^{n} ln(x_i)\right)$$

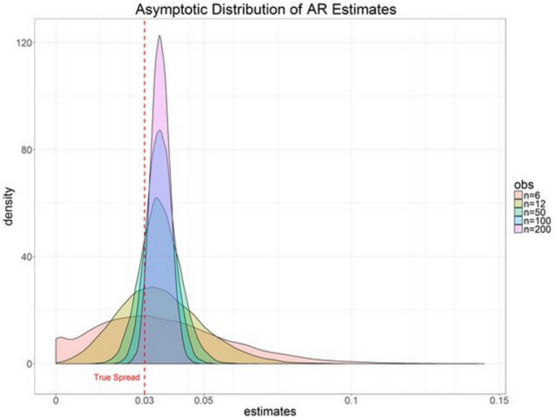
21. Analog Principle & Plug-in Estimators

- Analog Principle: Design an estimator of a parameter by mimicking the parameter.
- E.g., Create a function that looks like the parameter.
- I.e., If you want to mimic the distribution, apply a function the produces that distribution.
- Plug-in estimator:

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} g(X_i)$$

22. Asymptotic Distribution

- The limit of the distribution.
- The ‘limiting’ distribution of a sequence of distributions.



23. Extreme Value Distribution

- Used when outcomes are a rare occurrence (besides 0).
- I.e., Death by a murderous clown.
- τ: Shape of skew (i.e., rareness)
- Generalized Extreme Value (joint distribution):

$$F(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_J) = exp(-(\sum_{l=1}^{J} exp(\frac{-\varepsilon_j}{\tau}))^\tau)$$

- Type I Extreme Value (uni-variate distribution):

$$F(\varepsilon) = exp(-exp(-\varepsilon))$$

24. Inverse Mills Ratio

- Ratio of PDF to CDF above a certain value α
- PDF: $\phi(\frac{\alpha-\mu}{\sigma})$
- CDF: $\Phi(\frac{\alpha-\mu}{\sigma})$

$$\mathbb{E}[X|X > \alpha] = \mu + \sigma \left(\frac{\phi(\frac{\alpha-\mu}{\sigma})}{1 - \Phi(\frac{\alpha-\mu}{\sigma})} \right)$$

Tomorrow

- Final day of this course.
- We will review the course material. Come with your questions.
- We will preview what the next year will look like
- And then you will be free until the semester starts.