Mathematics Review Course
Summer 2023
Problem Set 11
Solutions

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Bayes Rule

1. [Uni. Penn] Suppose we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side is colored black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

Solution:

$$Pr(RB|R) = \frac{1}{3}$$

2. Uni. Penn A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighborhood have the flu, while the other 10% are sick with measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that there no other maladies in that neighborhood. A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is $Pr(R|M) = 0.95$. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is $Pr(R|F) = 0.08$. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

Solution:

$$Pr(M|R) \approx 0.57$$

3. [MathsNET] Consider a test to detect a disease that 0.1% of the population have. The test is 99% effective in detecting an infected person. However, the test gives a false positive result in 0.5% of cases. If a person tests positive for the disease what is the probability that they actually have it?

Solution:

$$P(D = 1|T = 1) \approx 0.165$$

4. [MathsNET] Two production lines produce the same part. Line 1 produces 1,000 parts per week of which 100 are defective. Line 2 produces 2,000 parts per week of which 150 are defective. If you choose a part randomly from the stock what is the probability it is defective? If it is defective what is the probability it was produced by line 1?

Solution:

$$Pr(L_1|D) = \frac{2}{5}$$