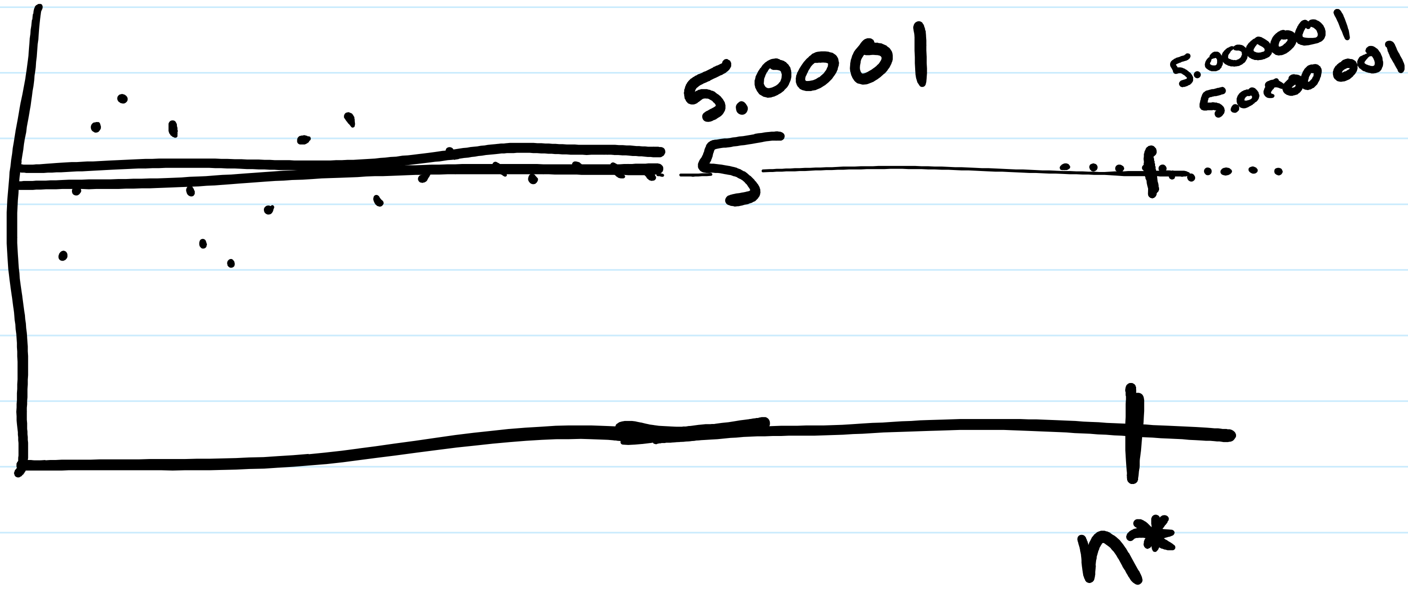
Day 3 Notes

Wednesday, August 11, 2021 8:25 AM



A sequence $\{x_n\}$ converges to L if $\forall \varepsilon > 0$, $\exists n^*$ such that for all $n > n^*$

$$|x_n - L| < \varepsilon$$

In $\mathbb{R}^n$, because it is a <u>complete</u> metric space, a sequence converges iff it is a Cauchy sequence

Cauchy convergence criterion.

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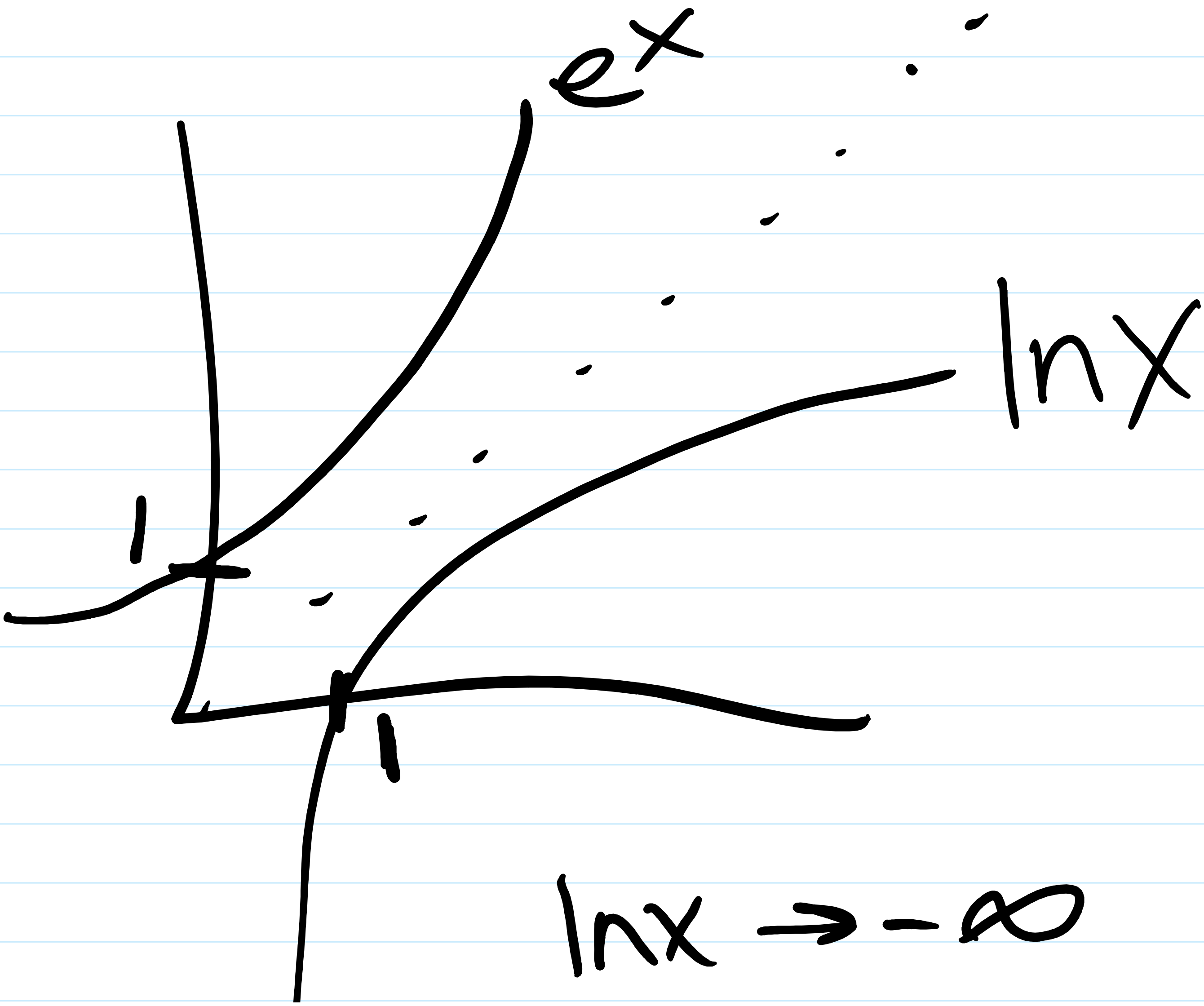
$\forall x, y \in IRS(q)$ and for all $t \in (0,1)$ we have:

$$z = tx + (1-t)y \in IRS(q)$$



$\ln x \to -\infty$

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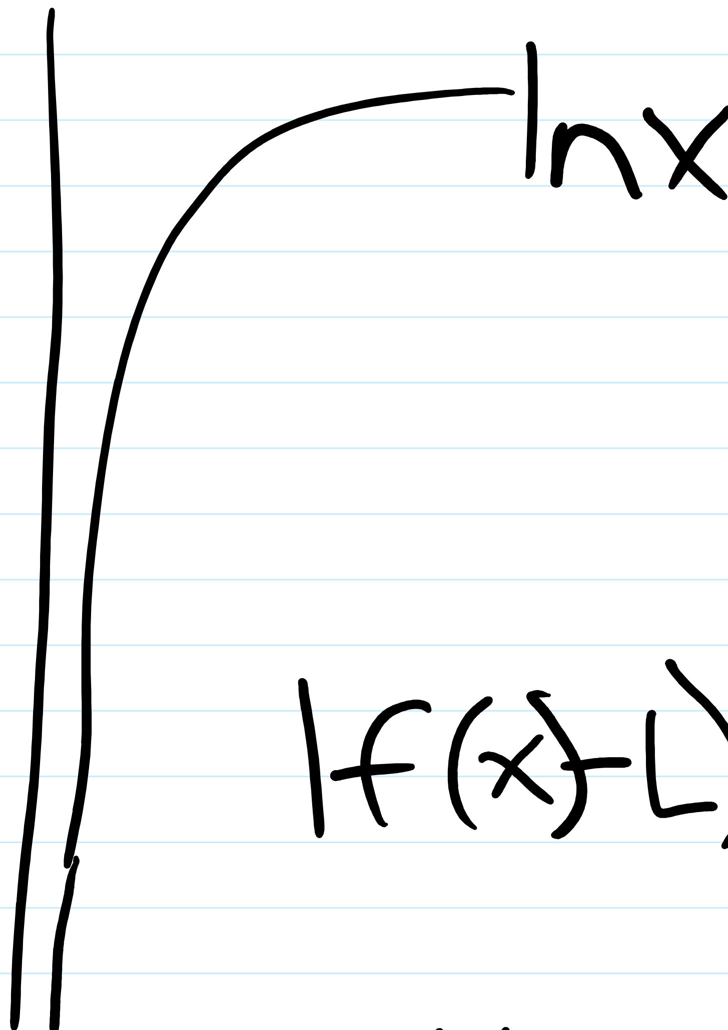
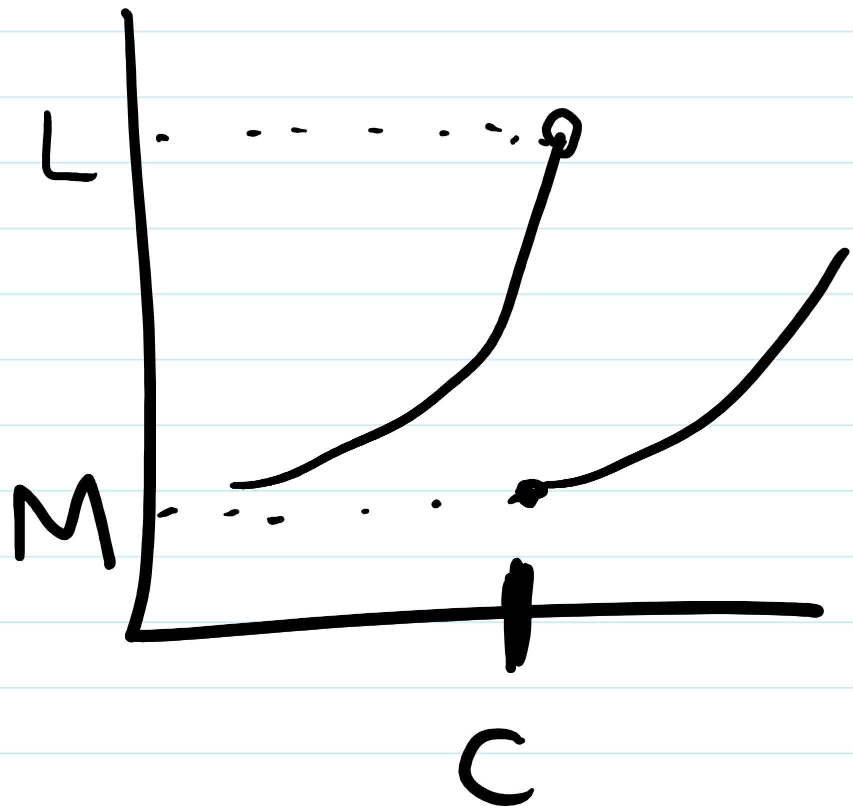
as $x \to 0$



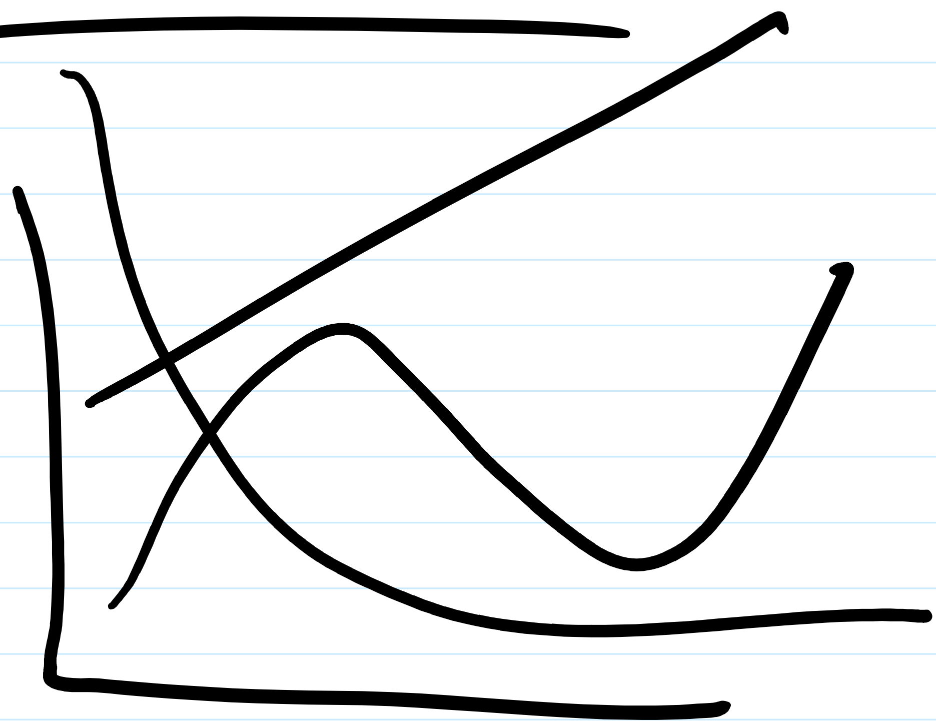
$|f(x) - L| < \varepsilon$

$f(x) < \varepsilon$

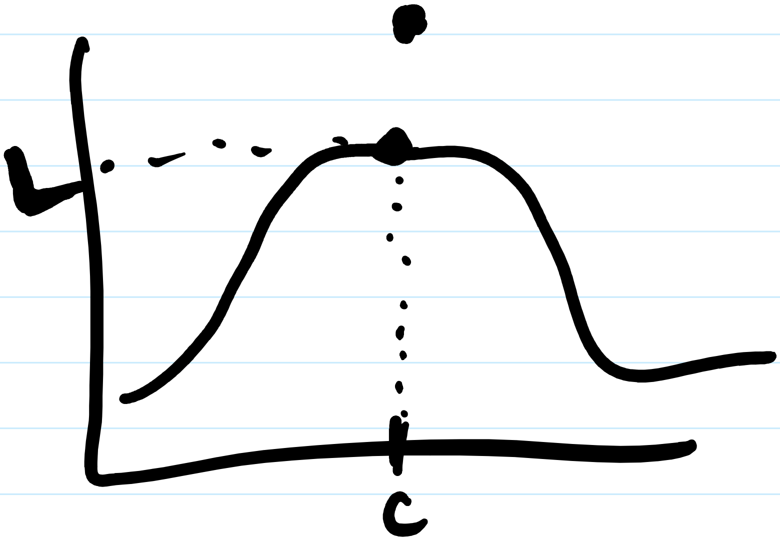
No limit;

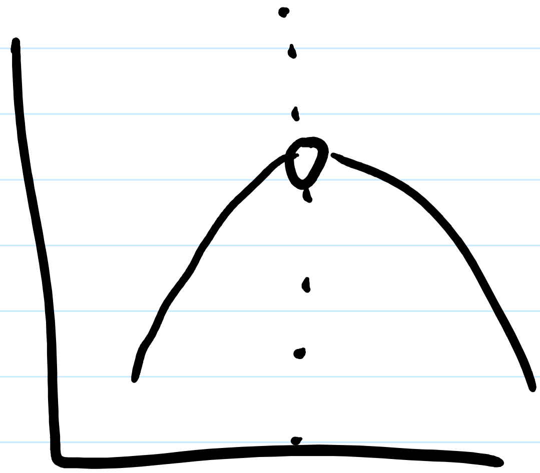


continuous



Not continuous





$$f(x) = \begin{cases} -(x-c)^2 & \forall x \neq c \\ 10 & x = c \end{cases}$$

Sum Law

$$\lim_{x \to a} \left(f(x) + g(x) \right) = \lim f(x) + \lim g(x)$$

Product Law

$$\lim f(x) \cdot g(x) = \lim f(x) \cdot \lim(g(x))$$

Quotient Law

$$\lim \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)} \text{ if } \lim g(x) \neq 0$$

Show that $f(x) = ax+b$ is continuous.

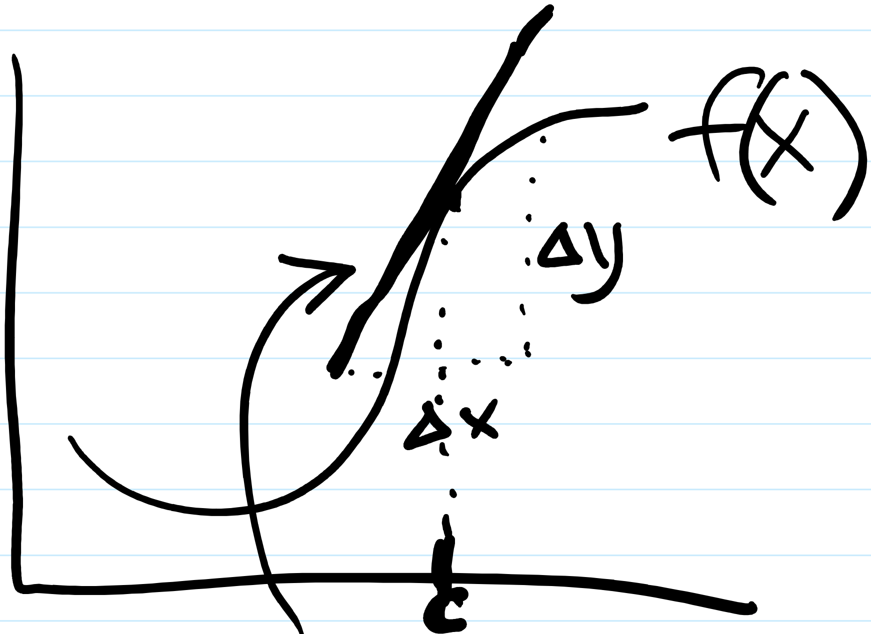
$$\lim_{x \to c} ax+b = \lim ax + \lim b$$

$$= a \lim x + b$$

$$= ac + b$$

$$= f(c)$$

So f is continuous.



slope of
linear approximation
is the derivative

$$f'(c) = \frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx} = f'(x)$$

$$f'(a) = \lim_{h \to 0} \frac{\overbrace{f(a+h) - f(a)}^{\Delta Y}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\quad}{\underbrace{h}_{\Delta x}}$$

Compute $f'(x)$ for $f(x) = \frac{1}{2}x - \frac{3}{5}$ using limits.

$$f'(x) = \lim_{h\to 0} \frac{\left(\frac{1}{2}(x+h) - \frac{3}{5}\right) - \left(\frac{1}{2}x - \frac{3}{5}\right)}{h}$$

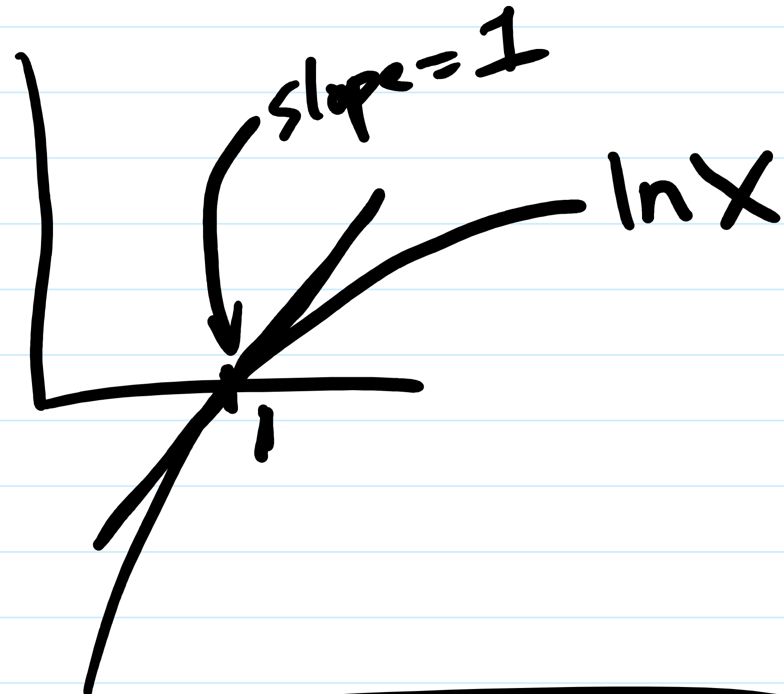
$$= \lim_{h\to 0} \frac{\cancel{\frac{1}{2}x} + \frac{1}{2}h - \cancel{\frac{3}{5}} - \cancel{\frac{1}{2}x} + \cancel{\frac{3}{5}}}{h}$$

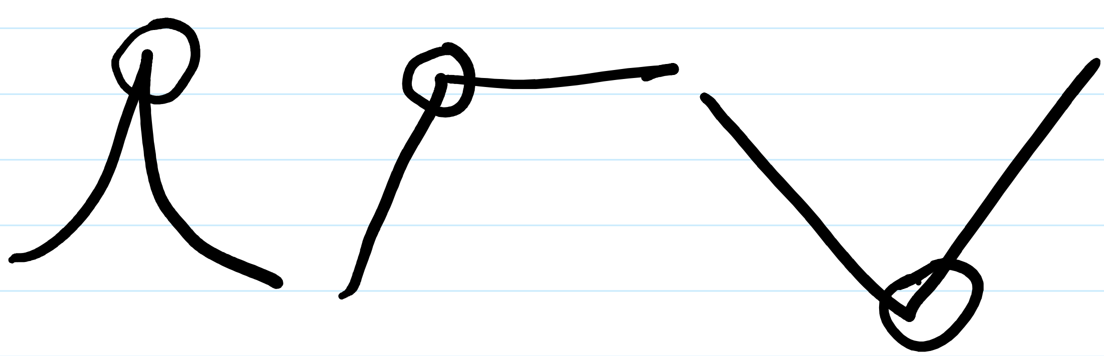
$$= \lim_{h\to 0} \frac{\frac{1}{2}\cancel{h}}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{1}{2} = \frac{1}{2}$$

In groups, find the derivative of x^2 using derivative of x^2 using limits.

$\ln(x+1) \approx x$ for small x





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if $f' > 0$, f is increasing

if $f' > 0$, f is increasing

< 0 decreasing

critical point:

all have $f'(x) = 0$

$$\frac{1}{g(x)} = [g(x)]^{-1}$$

Chain rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$e\left(\frac{x+6}{x+5}\right)^{1/4}$$

$$= (x+6)^{1/4} \cdot (x+5)^{-1/4}$$

$$= (x+6)^{1/4} \cdot \left(-\tfrac{1}{4}\right)(x+5)^{-5/4} \cdot 1$$

$$+ \tfrac{1}{4}(x+6)^{-3/4} \cdot (x+5)^{-1/4}$$

If $h = \ln(g(x))$

then $\dfrac{dh}{dx} = \dfrac{g'(x)}{g(x)}$

$$\frac{d\ln(g(x))}{dx} = \frac{1}{g(x)} \cdot g'(x)$$

chain rule

Elasticity

$$\frac{d\ln x}{d\ln y} = \frac{dx}{dy}\cdot\frac{y}{x} = \varepsilon_{xy}$$

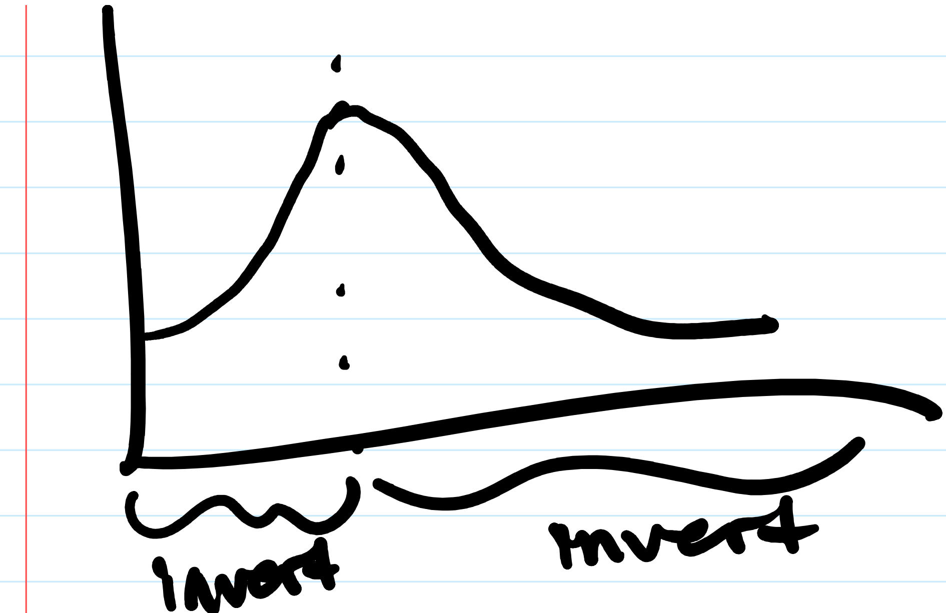
ex $\frac{d}{dx} 2\ln(3x^2-1)$

$$= 2\cdot\frac{1}{3x^2-1}\cdot 6x$$

$$= \frac{12x}{3x^2-1}$$

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\frac{dy}{dx} = \frac{1}{dx/dy}$$



$$y = f(x)$$

$$\boxed{dy = f'(x)dx}$$

$$\frac{dy}{dx} = f'(x)$$

$$dy(x, dx)$$

dx is just a new

dx is just a new independent variable.

linear approx.

$$f(a+dx) \approx f(a) + \underbrace{f'(a)dx}_{dy}$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

but $\lim f(x) = \infty$

and $\lim g(x) = \infty$

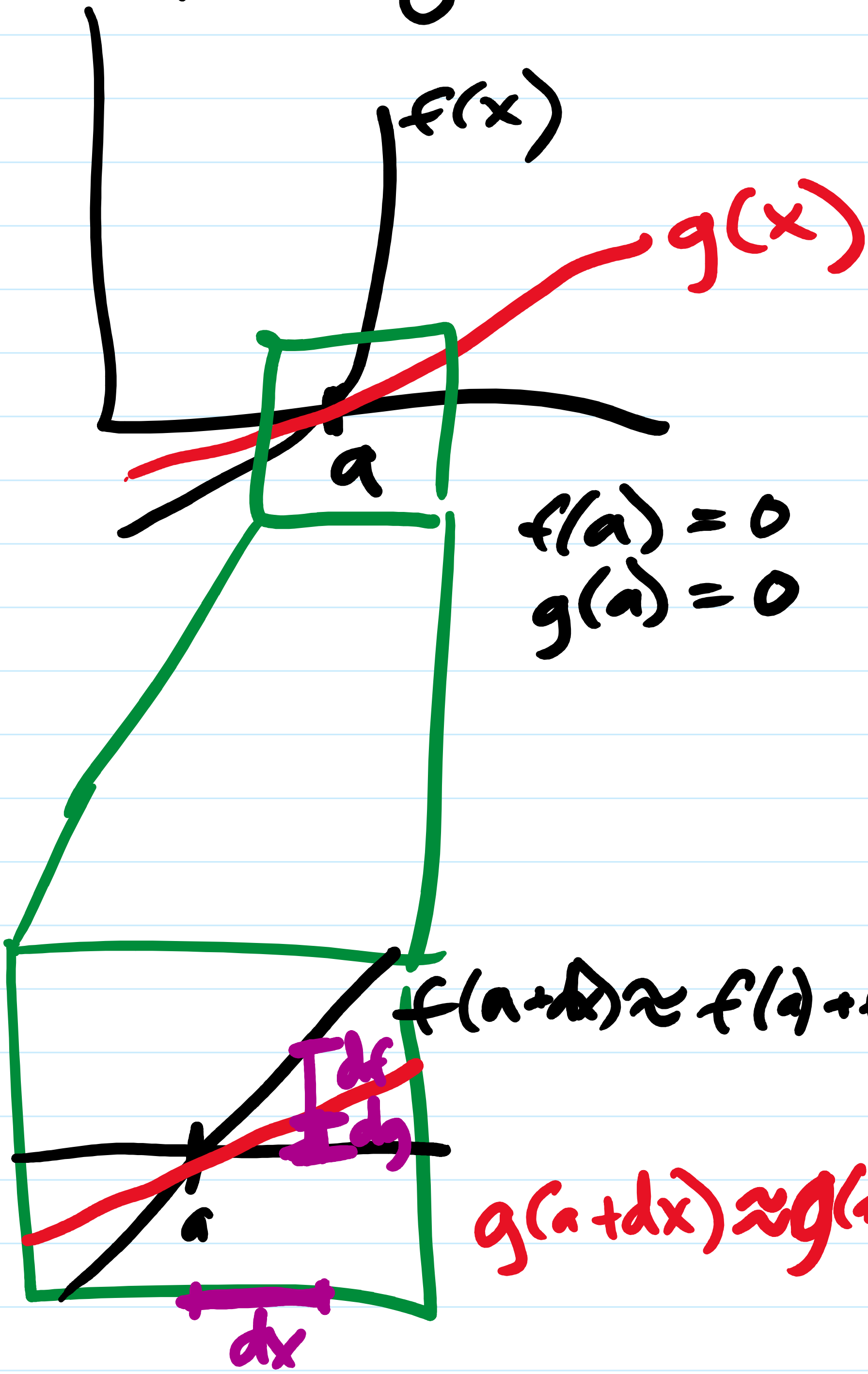
so we have $\lim \frac{\infty}{\infty} = ?$

Indeterminate form:

$\frac{\infty}{\infty}$ or $\frac{0}{0}$

$\frac{\infty}{\infty}$ or $\frac{0}{0}$

for $\frac{0}{0}$



$f(a) = 0$

$g(a) = 0$

$f(a+dx) \approx f(a) + f'(a)dx$

$g(a+dx) \approx g(a) + g'(a)dx$

CES $\rightarrow$ Cobb-Douglas

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$$f''(x) = \frac{d^2 f}{dx^2}$$

$$= \frac{d}{dx}\left[\frac{d}{dx} f(x)\right]$$

$$f''' = f^{(3)}$$

$$f^{(k)}(x)$$

C^0 continuous

C^1 continuously differentiable

C^2 twice cont. diff.

$\vdots$

C^∞ smooth function

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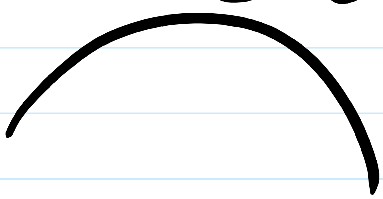
concave fn



$f''(x) < 0$

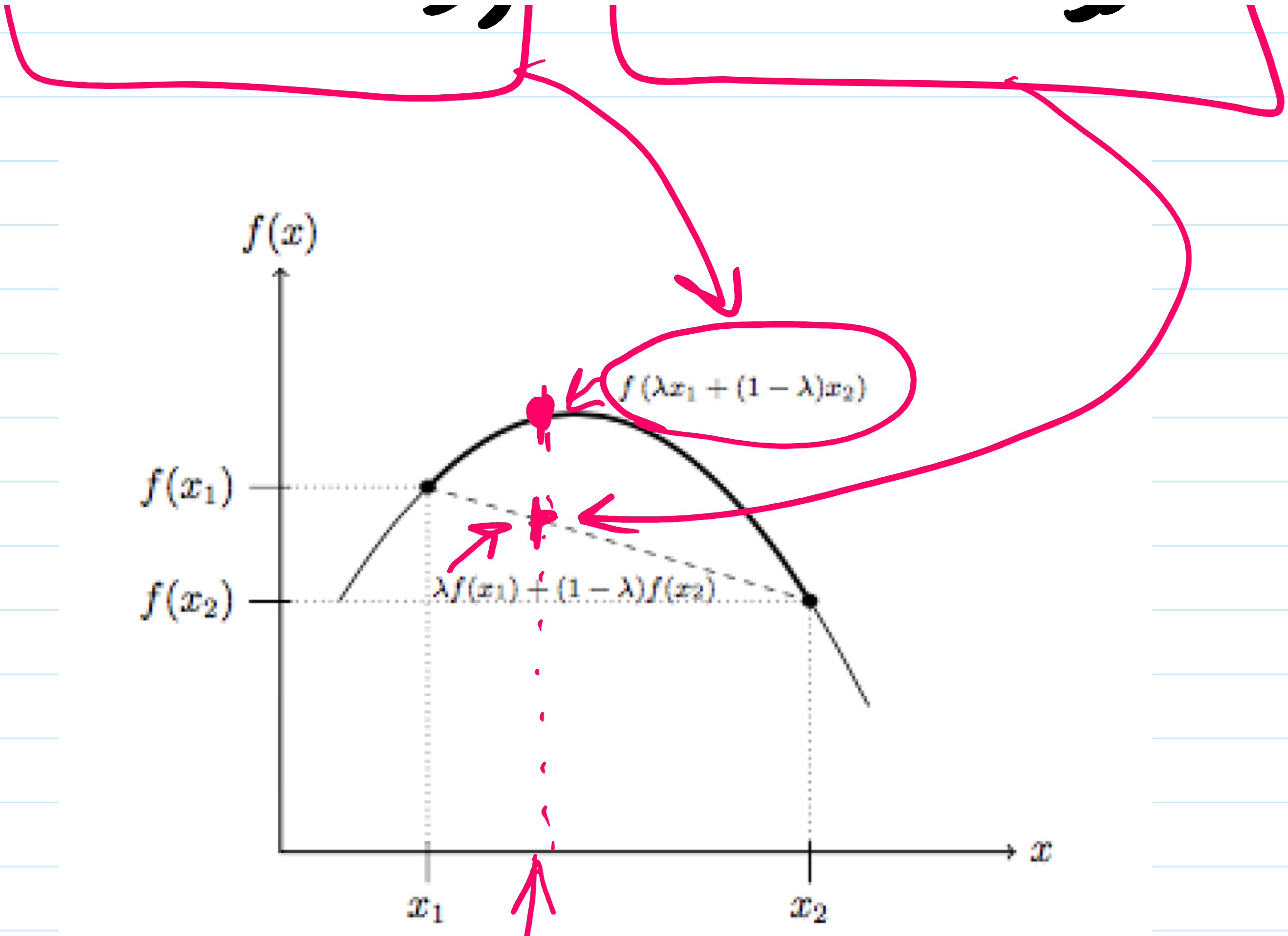
convex fn

$f''(x) > 0$

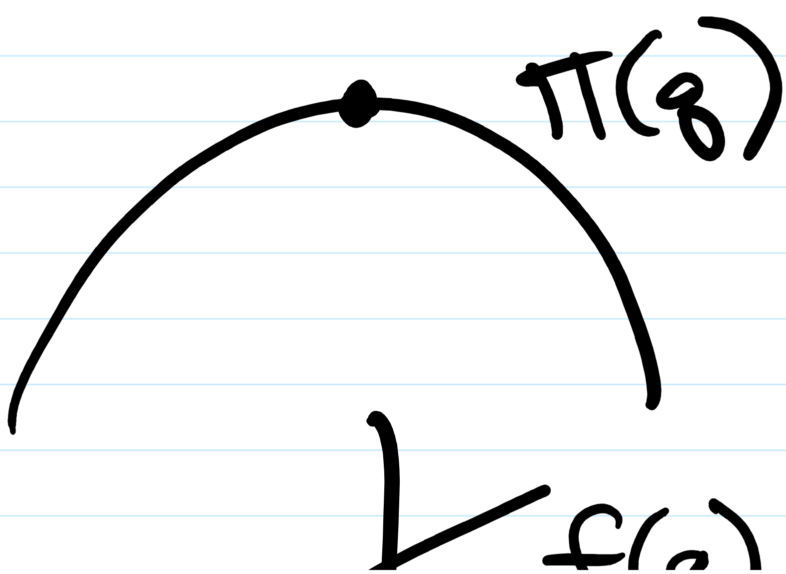
A fn is concave iff

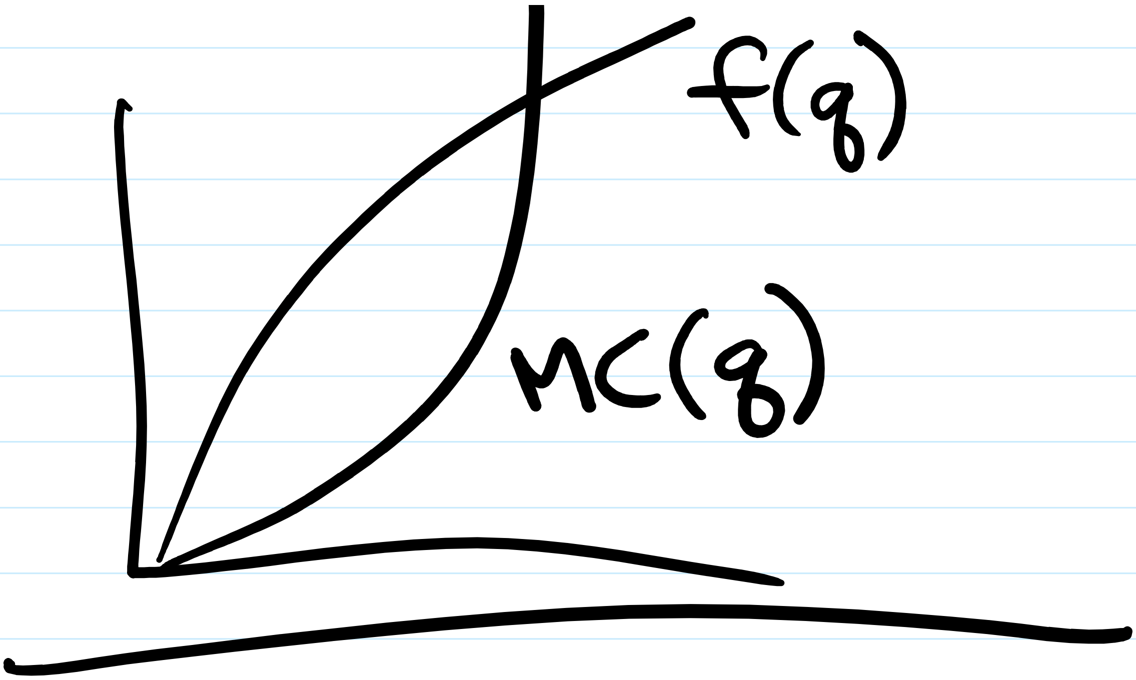
$\forall x, y \in \mathbb{R}$, and $\forall \lambda \in (0,1)$

$$f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda) f(y)$$



$\lambda = 0.25$

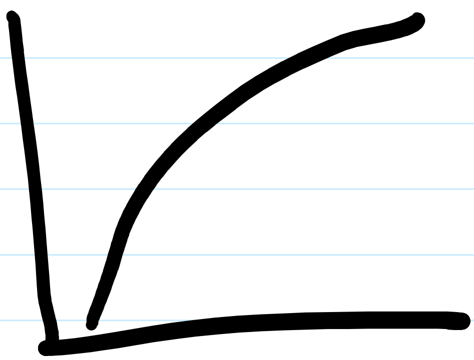




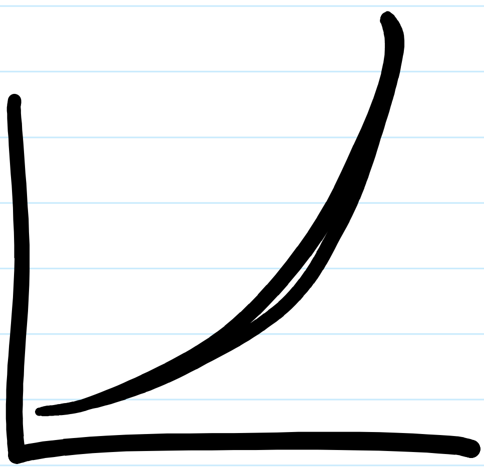
$f(x)$

$f'(x) > 0$

$f''(x) < 0$



$$\boxed{\begin{array}{l} f'(x) > 0 \\ f''(x) > 0 \end{array}}$$



Let $C(q)$ be a convex cost function.