Lecture 14
Ancillary Material

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Last Lecture Review

- Time Series:
  - Stochastic Processes
  - Discrete & Continuous Time Markov Chain
  - Poisson Processes
  - Stationarity
  - Ergodicity
  - Unit Root or Random Walk
- Dynamic Programming:
  - Dynamic Programming Problem
  - Theory of the Maximum
  - Bellman Equation with Finite Horizon
  - Bellman's Principle of Optimality
  - Backward Induction
  - Bellman Equation with Infinite Horizon

Review Assignment

1. Problem Set 13 solutions are available on Github.
2. Any issues or problems **You** would like to discuss?

Daily Icebreaker

- Attendance via prompt:
  - Name
  - Program and track
  - Daily icebreaker subject...

Ancillary Material

Motivation

- These are topics that will be touched on in microeconomics and econometrics.
- Many of the topics are discussed in passing with little elaboration on the concept themselves.
- Some of these topics will be discussed at length in your coursework, but are worth priming now so they are not novel or new ideas.

Overview

1. Positive vs. Normative
2. Ex-Ante & Ex-Post
3. Cardinal vs. Ordinal
4. Extensive vs. Intensive Margin
5. Preference Relations
6. Bernoulli Functions
7. Homogeneity
8. Monotonicity
9. Elasticity
10. Local Non-satiation
11. Contour Sets
12. Gorman Form
13. Simplex
14. Singleton Set
15. Mean Preserving Spread
16. Independence of Irrelevant Alternatives
17. Exogenous vs. Endogenous
18. Sigma Fields
19. Jensen Inequality
20. Law of Iterated Expectations
21. Algebraic vs. Geometric Means
22. Analog Principle & Plug-in Estimators
23. Extreme Value Distribution
24. Inverse Mills Ratio

1. Positive vs. Normative

- Positive Analysis:
  - Objective statements.
  - Descriptions of the possible states of the world.
  - E.g., X% of people are poor.
- Normative Analysis:
  - Subjective statements.
  - A value judgement on the state of the world.
  - E.g., We should provide cash transfers to the bottom X% of the income distribution.

2. Ex-Ante & Ex-Post

- Ex-Ante: Before the event
  - What you expect to occur.
  - Expectation, forecasting, prediction, etc.
  - Suffers from post-hoc or type I errors.
- Ex-Post: After the fact
  - Understanding what as already occurred.
  - Causal analysis

3. Cardinal vs. Ordinal

- Cardinal: Indicate quantities.
- Ordinal: Indicate rank or order in a set.
- Nominal: Indicate an identity (e.g., a zip code or a player's jersey number).

4. Extensive vs. Intensive Margin

- Extensive Margin: Dichotomous switch of selecting into or out of an activity (e.g., 0 – 1).
  - i.e., Labor force participation.
- Intensive Margin: Continuous intensity at which a person selecting into the activity participates.
  - i.e., Number of hours worked.

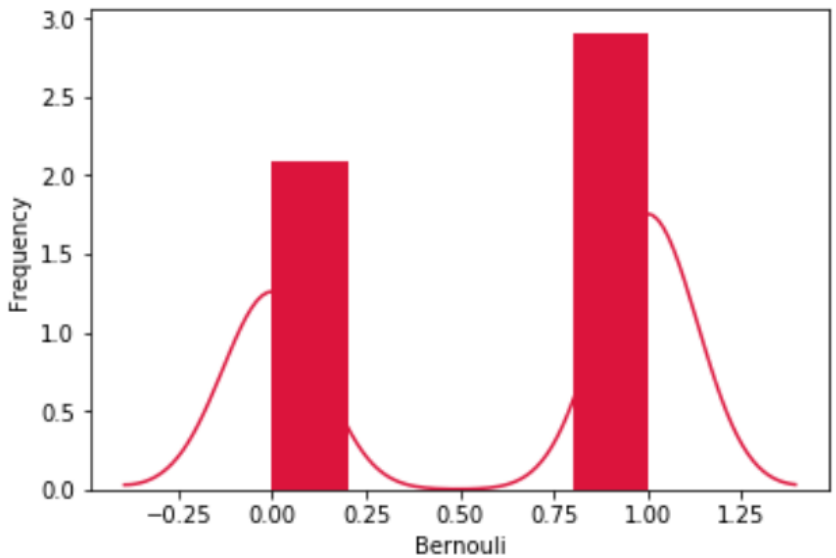
5. Preference Relations

CB Lecture 4 and 5 Define preferences Homothetic, Quasilinear, Lexiographic, Leontiff, Cobb-Douglas Welfare: Stone Geary

6. Bernoulli Functions

- Discrete probability distribution taking only values $k \in \{0, 1\}$ at given probabilities p.

$$f(k, p) = pk + (1 - p)(1 - k)$$



7. Homogeneity

- Homogeneous Function: When the arguments of a function are multiplied by a scalar, then the value of the function is a power (i.e., degree) of this scalar.
- Homogenous of degree 0

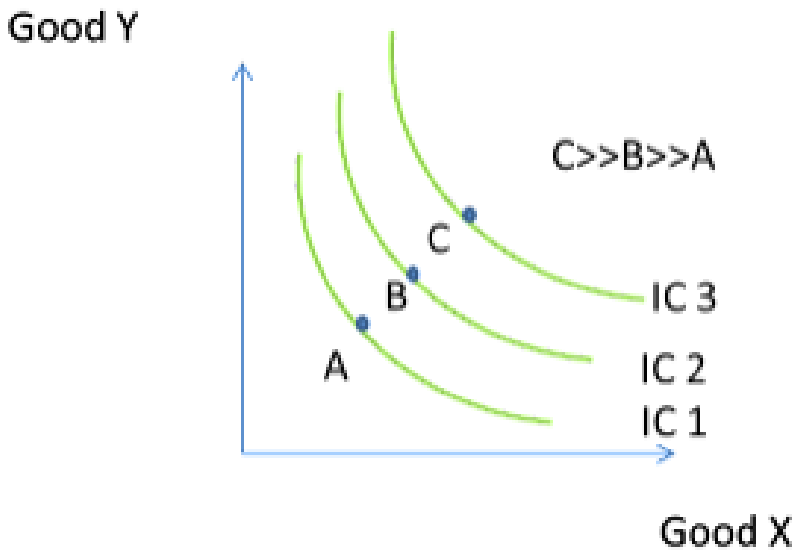
$$x(\alpha p, \alpha w) = x(p, w) \forall \alpha > 0$$

- Homogenous of degree 1

$$x(\alpha p, \alpha w) = \alpha x(p, w) \forall \alpha > 0$$

8. Monotonicity

- E.g., More is better and we like variety.
- Suppose there are n commodities in x_1 and x_2.
- Weak monotonic preferences if $x_1 \geq x_2 \implies x_1$ (i.e., at least one more in quantity of any good n means you prefer that bundle).
- Strong if you replace with $>$.
- Important to making a $log(\cdot)$ transformation.



9. Elasticity

- Relative change in demand for good l in response to a percentage change in the parameter.
- Price Elasticity:

$$\varepsilon_{lk}(p,w) = \frac{\partial x_l(p,w)}{\partial p_k} \frac{p_k}{x_l(p,w)}$$

- Wealth Elasticity:

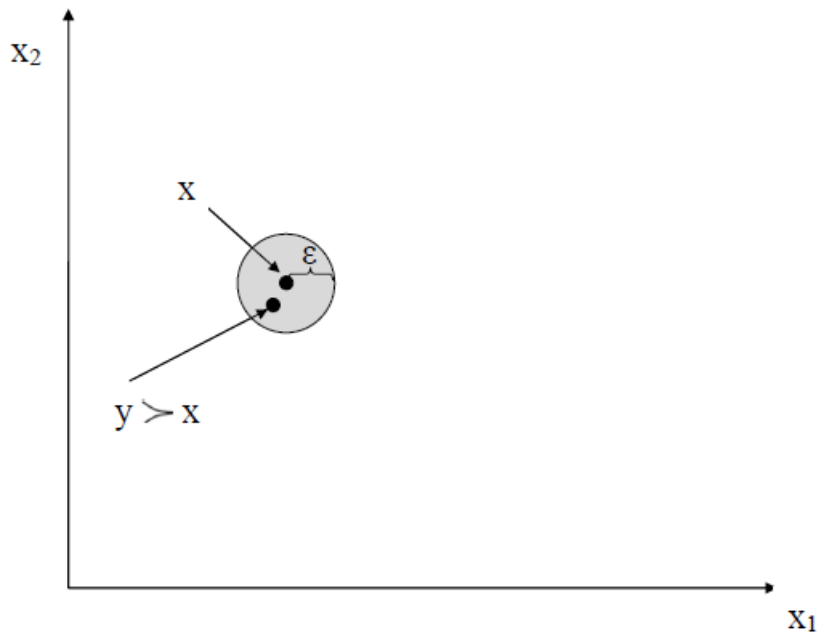
$$\varepsilon_{lw}(p,w) = \frac{\partial x_l(p,w)}{\partial w} \frac{w}{x_l(p,w)}$$

10. Local Non-satiation

- For a point x, there is some very close point y which is strictly preferred.

$$\forall x \in X, \varepsilon > 0, \exists y \in X : ||y - x|| \leq x \wedge y \succ x$$

$$||y - x|| = \left(\sum_{l=1}^{L} (y_l - x_l)^2 \right)^{1/2}$$



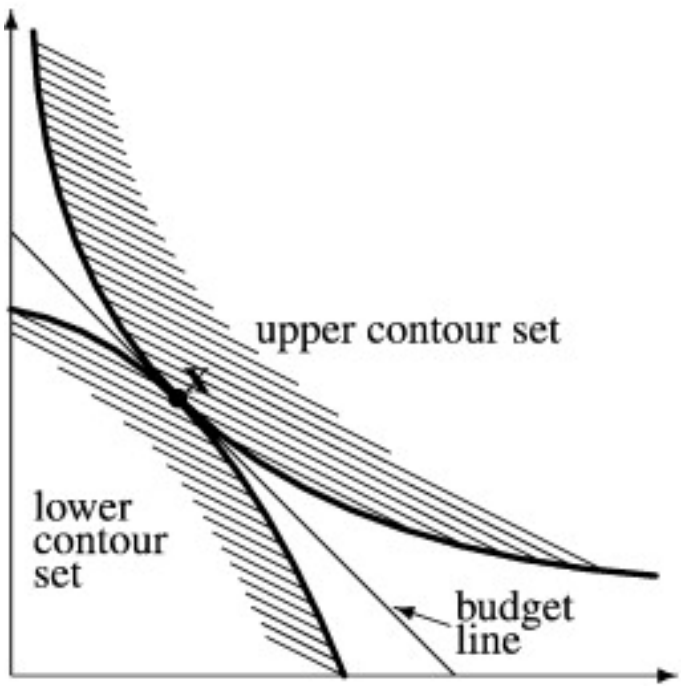
11. Contour Sets

- Upper Contour Set:

$$\{y \in y \succsim x\}$$

- Lower Contour Set:

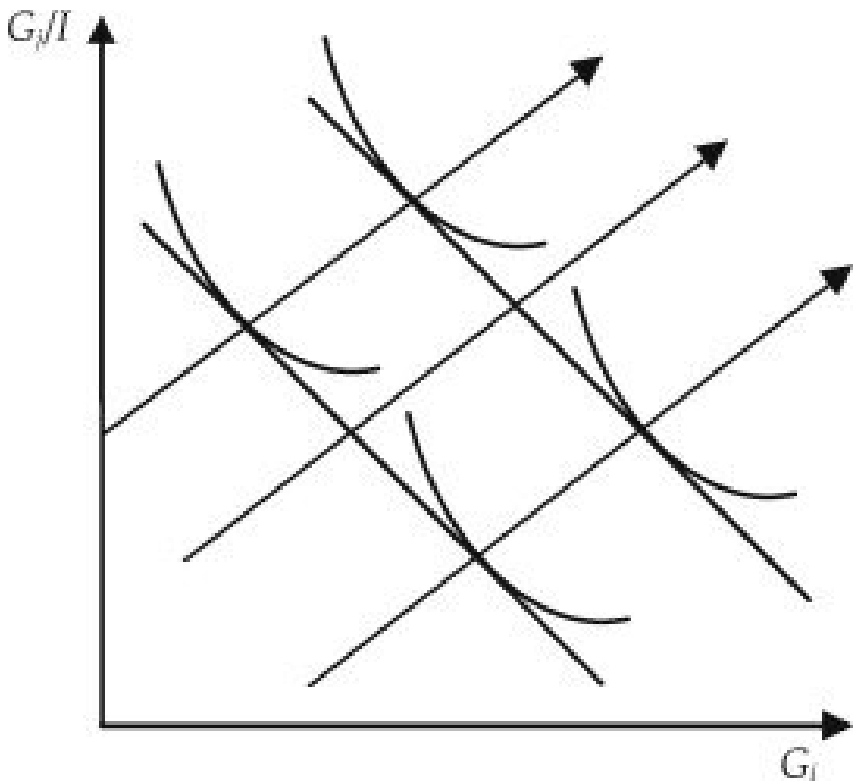
$$\{y \in x \succsim y\}$$



12. Gorman Form

- Indirect utility function allows you to aggregate utilities
- $a_i(p)$: Reference utility at zero for each individual.
- $b(p)$: Parallel wealth expansion paths for all individuals

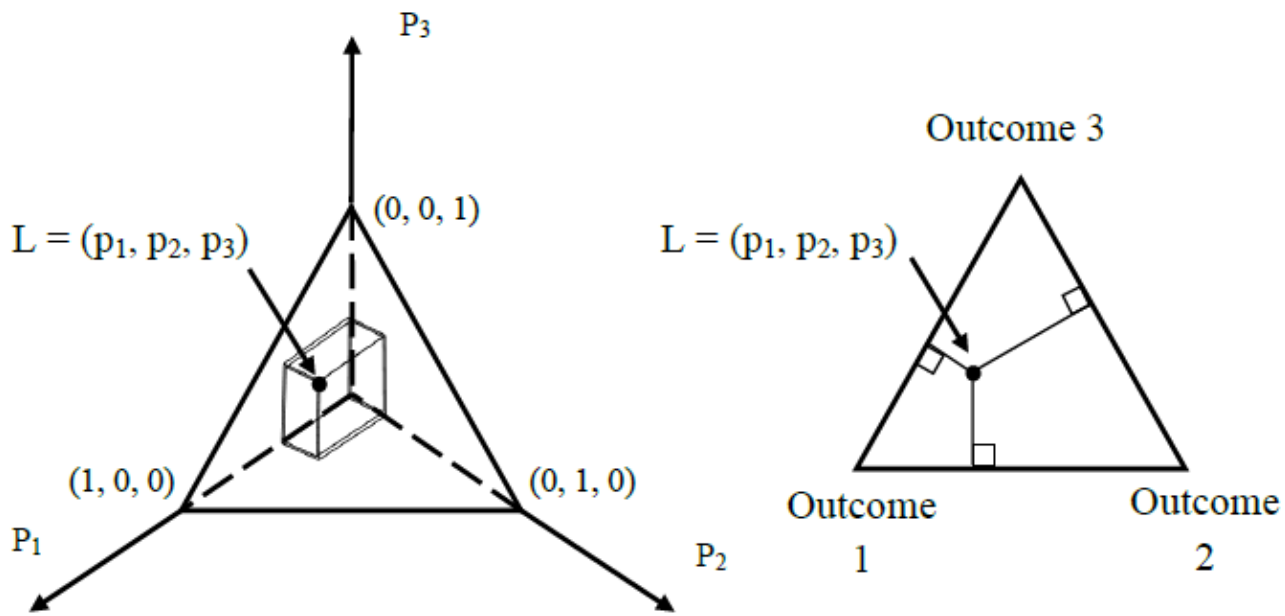
$$v_i(p, w_i) = a_i(p) + b(p)w_i$$



13. Simplex

- A symmetric triangle of n-dimensions.
- Used in probability to represent events.

$$\Delta = \{p \in \mathbb{R}_+^N : p_1 + p_2 + \cdots + p_N = 1\}$$



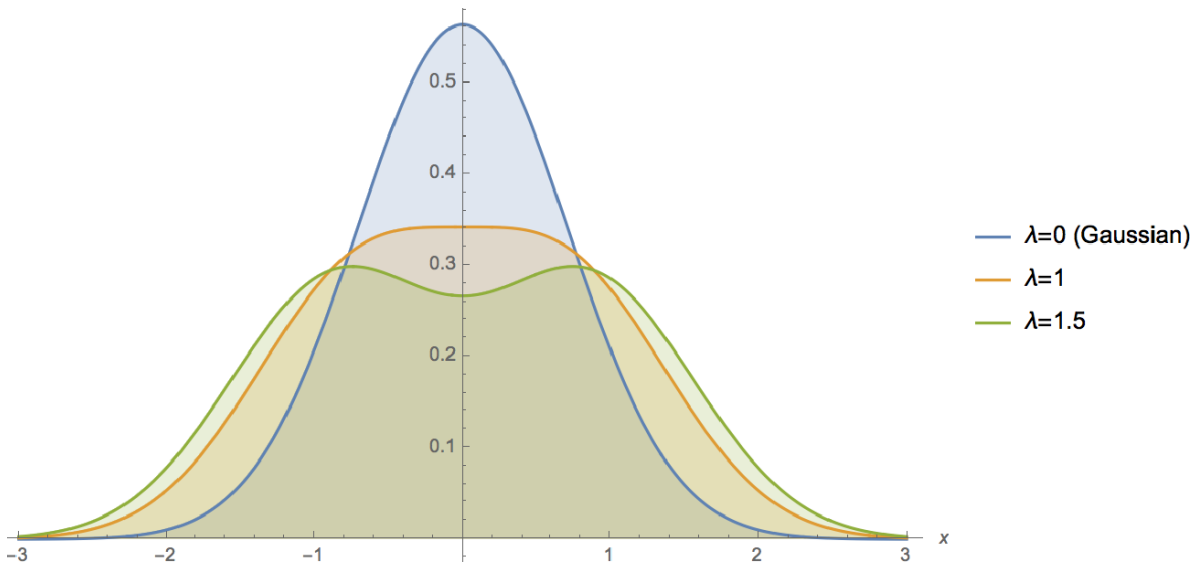
14. Singleton Set

- A set with exactly one element.
- E.g., $\{0\} \rightarrow$ the only element is 0.
- Let S be a class of indicator function such that $b : X \rightarrow \{0, 1\}$.
- Then S is a singleton *iff* $\exists y \in X : \forall x \in X$

$$b(x) = (x = y)$$

15. Mean Preserving Spread

- When you change from one distribution A to another distribution B, the expected value (i.e., mean) remains unchanged.
- Variance by vary.



16. Independence of Irrelevant Alternatives

- The social preferences between alternatives x and y depend only on the individual preferences x and y.
- That is to say if we added z to the mix, preference ordering would remain the same.
- Corollary: If you remove an option, it will not change the rank order of selection.

17. Exogenous vs. Endogenous

- Endogenous: From within in the system (e.g., parametrically determined).
- Exogenous: From outside the system (e.g., deterministic).
- Meaning varies between use in structural econometrics (viz., GMM) and causal inference (viz., RCT)

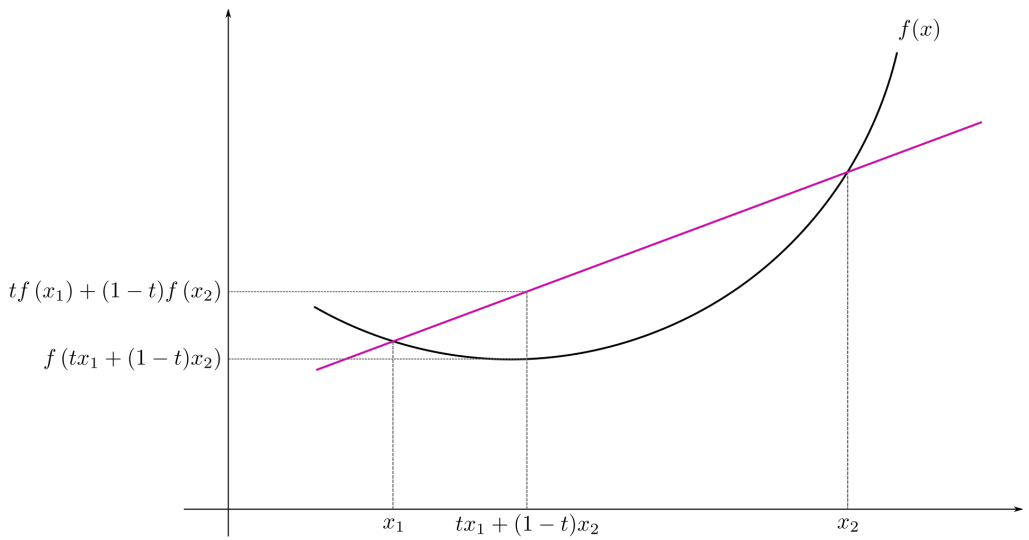
18. Sigma Fields

- Sigma Field: A collection of subsets $\mathcal{B}$ of the sample space S excluding weird sets.
  - $\emptyset \in \mathcal{B}$
  - $A^c \in \mathcal{B}$
  - $\mathcal{B}$ is closed under countable unions
- Borel σ-algebra: The smallest sigma algebra on the real line containing all open intervals.
- Borel Sets: Sets in Borel σ-algebra.
- Helps us to limit outcomes to real-values.

19. Jensen Inequality

- The parts in the sum are less than the sum of the parts.

$$f(\alpha x_1 + (1-\alpha)x_2) \leq \alpha f(x_1) + (1-\alpha)f(x_2)$$



20. Law of Iterated Expectations

- The expected value of the conditional expected values of X given Y is the same as the expected value of X.
- E.g., The mean of the 'group means' is the same as the overall mean.

$$\mathbb{E}[X] = \mathbb{E}[\mathbb{E}[X|Y]]$$

21. Algebraic vs. Geometric Means

- Algebraic Mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Geometric Mean:

$$\bar{x} = (\Pi_{i=1}^{n} x_i)^{\frac{1}{n}} = exp\left(\frac{1}{n}\sum_{i=1}^{n} ln(x_i)\right)$$

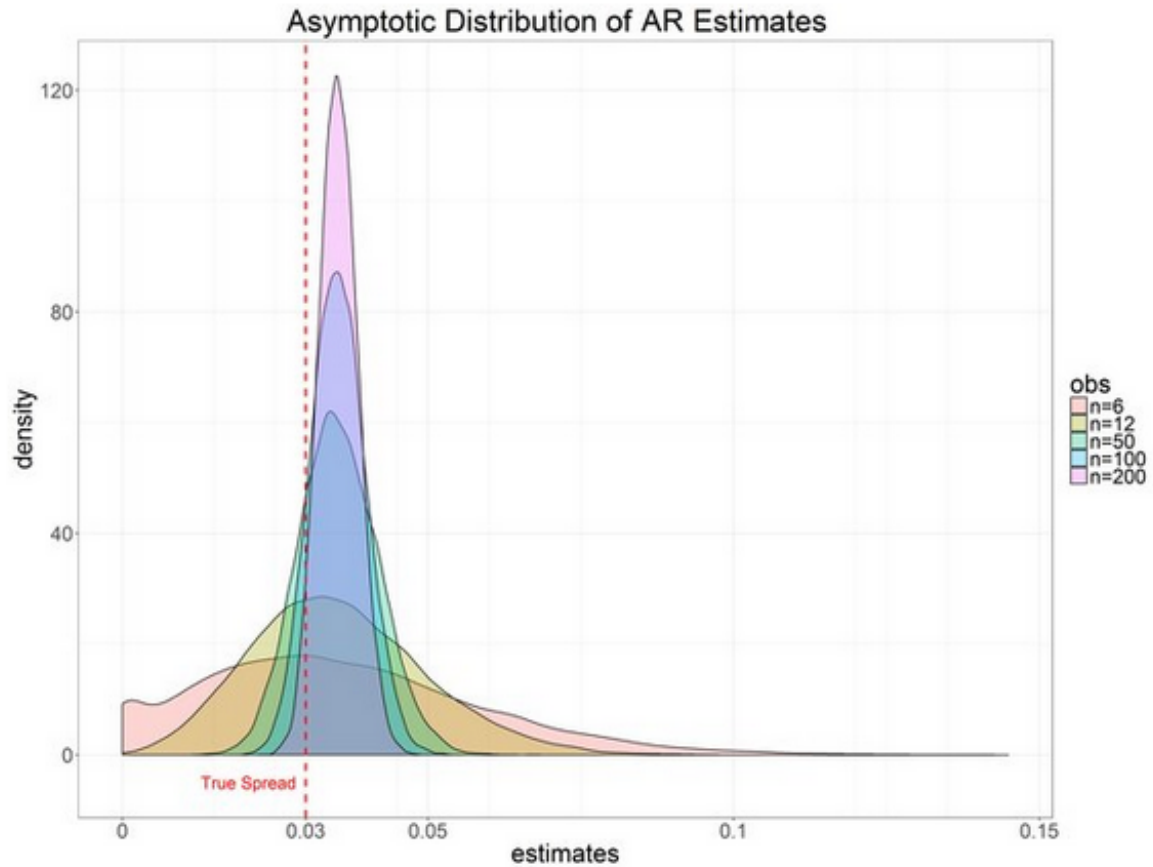
22. Analog Principle & Plug-in Estimators

- Analog Principle: Design an estimator of a parameter by mimicking the parameter.
- E.g., Create a function that looks like the parameter.
- I.e., If you want to mimic the distribution, apply a function the produces that distribution.
- Plug-in estimator:

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} g(X_i)$$

23. Asymptotic Distribution

- The limit of the distribution.
- The 'limiting' distribution of a sequence of distributions.



24. Extreme Value Distribution

- Used when outcomes are a rare occurrence (other than 0).
- I.e., Death by a murderous clown.
- τ: Shape of skew (i.e., rareness)
- Generalized Extreme Value (joint distribution):

$$F(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_J) = exp(-(\sum_{l=1}^{J} exp(\frac{-\varepsilon_j}{\tau}))^{\tau})$$

- Type I Extreme Value (uni-variate distribution):

$$F(\varepsilon) = exp(-exp(-\varepsilon)$$

25. Inverse Mills Ratio

- Ratio of PDF to CDF above a certain value α
- PDF: $\phi(\frac{\alpha-\mu}{\sigma}$
- CDF: $\Phi(\frac{\alpha-\mu}{\sigma}$

$$\mathbb{E}[X|X > \alpha] = \mu + \sigma \left(\frac{\phi(\frac{\alpha-\mu}{\sigma}}{1 - \Phi(\frac{\alpha-\mu}{\sigma}} \right)$$