

$$\begin{aligned}
 \textcircled{5} \quad & \ln(x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}) \\
 &= \ln(x_1^{\alpha_1}) + \ln(x_2^{\alpha_2}) + \dots + \ln(x_n^{\alpha_n}) \\
 &= \alpha_1 \ln(x_1) + \alpha_2 \ln(x_2) + \dots + \alpha_n \ln(x_n) \\
 &= \sum_{i=1}^n \alpha_i \ln(x_i)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad & \frac{1}{x^{\alpha_1}} \cdot \frac{1}{x^{\alpha_2}} \dots \frac{1}{x^{\alpha_n}} \\
 &= \prod_{i=1}^n \frac{1}{x^{\alpha_i}} = \prod_{i=1}^n x^{-\alpha_i}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{7} \quad & \ln\left(\prod_{n=1}^N x_n\right) \\
 &= \ln(x_1 x_2 \dots x_N) \\
 &= \ln(x_1) + \ln(x_2) + \dots + \ln(x_N)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \quad & \frac{\prod_{n=1}^N (x_n^2 - 4)}{\prod_{n=1}^N (x_n - 2)} \\
 &= \frac{\prod_{n=1}^N (x_n - 2)(x_n + 2)}{\prod_{n=1}^N (x_n - 2)} \\
 &= \prod_{n=1}^N (x_n + 2) = (x_1 + 2)(x_2 + 2) \dots (x_N + 2)
 \end{aligned}$$

$$1) \ b) \frac{d \ln(x)}{dx} \frac{1}{6ax^3}$$

$$= \frac{1}{6a} \cdot \ln(x) \cdot x^{-3}$$

Product Rule

$$\frac{d}{dx} f(x)g(x)$$

$$= fg' + f'g$$

$$= \frac{1}{6a} (\ln x \cdot -3x^{-4} + \frac{1}{x} \cdot x^{-3})$$

$$= \frac{1}{6a} x^{-4} (-3 \ln x + 1)$$

$$= \frac{1 - 3 \ln x}{6ax^4}$$

② Find f_x and f_y

a) $a \ln x + by$

$$f_x = \frac{a}{x}$$

$$a \ln x \cdot by$$

$$\ln(a \ln x \cdot by)$$

$$f_y = b = \ln(ax) + \ln(b)$$

$$b) \underline{x^\alpha y^\beta}$$

$$f_x = y^\beta \cdot \alpha \cdot x^{\alpha-1}$$

$$f_y = x^\alpha \cdot \beta \cdot y^{\beta-1}$$

$$\text{III. } \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ 4 & 4 & 5 \end{bmatrix} = \text{DNE}$$

$$\begin{bmatrix} 3 & 3 \\ 2 & 2 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 8 & 6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -6 & -4 \\ -3 & 3 \end{bmatrix}$$

$$5 \cdot \begin{bmatrix} -1 & 2 \\ 3 & x \end{bmatrix} = \begin{bmatrix} -5 & 10 \\ 15 & 5x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 2 + 3 \cdot 4 & 2 \cdot 4 + 3 \cdot 3 \\ 4 \cdot 2 + 6 \cdot 4 & 4 \cdot 4 + 6 \cdot 3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 17 \\ 32 & 34 \end{bmatrix}$$

$$AB \neq BA$$

$$\begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \\ 13 & 14 & 15 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{identity}} = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \\ 13 & 14 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 5 & 12 \\ 2 & 0 & 7 \\ 3 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & 3 \\ 5 & 0 & 2 \\ 12 & 7 & -1 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

IV.

$$2) \quad Q^S = -5 + 2P$$

$$Q^D = 20 - 3P$$

$$Q^S = Q^D$$

$$-5 + 2P = 20 - 3P$$

$$\begin{array}{l} SP = 25 \\ \boxed{P = 5} \end{array}$$

$$\Rightarrow U(x, y) = a \log x + b \log y$$

$$\text{Max}_{x, y} a \log x + b \log y$$

$$\text{s.t. } P_x \cdot x + P_y \cdot y = W$$

$$\text{FOC's: } \nabla U(x, y) = \lambda \nabla g$$

$$g(x, y) = P_x x + P_y y - W$$

$$\left. \begin{array}{l} \frac{a}{x} = \lambda P_x \\ \frac{b}{y} = \lambda P_y \end{array} \right\} \rightarrow \frac{\frac{a}{x}}{\frac{b}{y}} = \frac{P_x}{P_y}$$

$$\Rightarrow \frac{a}{b} \cdot \frac{y}{x} = \frac{P_x}{P_y}$$

$$\Rightarrow y = \frac{P_x}{P_y} \cdot \frac{b}{a} x \quad (*)$$

$$(B.C) \quad P_x X + P_y Y = W$$

$\downarrow (a)$

$$P_x X + P_y \cdot \left(\frac{P_x}{P_y} \cdot \frac{b}{a} X \right) = W$$

$$X(P_x + P_x \frac{b}{a}) = W$$

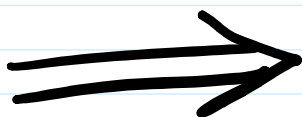
$$X = \frac{W}{P_x(1 + \frac{b}{a})}$$

$$X^* = \frac{W}{P_x} \cdot \frac{a}{b+a}$$

$$Y^* = \frac{W}{P_y} \cdot \frac{b}{b+a}$$

$$\forall x \in \mathbb{R}_+, x \geq 0$$

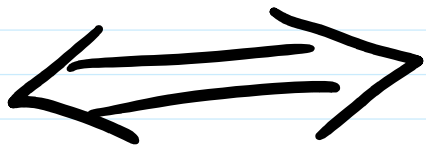
$$\exists x \in \mathbb{R}, x > 1$$



$\wedge \vee \neg$

$$A \Rightarrow B$$

$$\neg A \vee B$$



means

$$A \Rightarrow B$$

AND

$$B \Rightarrow A$$

$$A \Leftrightarrow B$$

\mathbb{N}

\mathbb{Z}

\mathbb{Q}

$$q \neq 0$$

\mathbb{R}

$[]$ closed

$()$ open

$[a, \infty)$

$(-\infty, b]$

3 Types of Proofs

1. Direct

2. Indirect

3. Induction

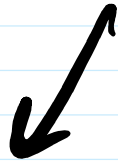
$$A \Rightarrow B$$

Indirect Proof

assume A

show that $\neg B$

implies a
contradiction



$$A \wedge \neg A$$

Prove that $\sqrt{2}$
is irrational

Rational: $\frac{p}{q}$

where $p, q \in \mathbb{Z}$
 $q \neq 0$

If $\sqrt{2}$ rational,
then:

$$\sqrt{2} = \frac{p}{q} \quad \text{for some integers in lowest terms}$$

$$\Rightarrow \sqrt{2}^2 = \left(\frac{p}{q}\right)^2$$

$$\Rightarrow 2 = \frac{p^2}{q^2}$$

$$\Rightarrow 2q^2 = p^2$$

$$\Rightarrow p^2 \text{ is even}$$

$$\Rightarrow p \text{ is even}$$

$$\therefore p = 2n \text{ for some integer } n$$

$$\Rightarrow 2q^2 = (2n)^2 = 4n^2$$

$$\Rightarrow q^2 = 2n^2$$

$\Rightarrow q$ is even

if p and q are both even, then they are not in lowest terms. This is a contradiction.

A

assume $\neg A$

\Rightarrow contradiction

Direct Proofs

$$A \Rightarrow B$$

Assume A
show it implies B

$$A \Rightarrow C \Rightarrow D \Rightarrow B$$

Ex 1

If a, b consecutive integers, then $a+b$ is odd.

$$b = a+1$$

$$\Rightarrow a+b = a+(a+1) \\ = 2a+1$$

Since a is an integer, $2a$ is even and $2a+1$ is odd, by definition.

$\therefore a+b$ is odd.



Induction

Ex 1 Prove that

$$1+2+3+\dots+n$$

$$= \frac{n(n+1)}{2} \quad \forall n \in \mathbb{N}$$

Suppose $n=1$.

$$1 \text{ should } = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

✓

Suppose for some integer k , it's true that $1+2+\dots+k$

$$= \frac{k(k+1)}{2}$$

$$\text{Then } 1+2+\dots+k+(k+1)$$

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$\therefore \frac{n(n+1)}{2} \text{ holds for } n=k+1$$

① True for $n=1$

② if true for k ,
then true for
 $k+1$

$$\therefore 1 \Rightarrow \text{true for } 2 \\ \Rightarrow \text{true for } 3$$

...

\Rightarrow true for
ALL $n \geq 1$

Even: can be
divided by 2 to
get an integer.

Suppose $a, b \in \mathbb{N}$

Then $2a, 2b$ are even.

Then $2a + 2b$

$$= 2(a+b)$$

And $a+b \in \mathbb{N}$

So $2(a+b)$ is even.

a and b are
integers, with $a \neq 0$.

If a does not divide
 b (this means

$\frac{b}{a} \notin \mathbb{Z}$) then

the equation

$$ax^2 + bx + b - a = 0$$

has no positive
integer solution.
