Mathematics Review Course
Summer 2023
Problem Set 09
Solutions

Ryan McWay

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First Order Conditions

1. Max. $f(x) = log(x^2 + 2x - 3)$.

Solution: While the critical points are $x = \{-1, 1, -3\}$, none of these have a defined function for these values on the domain. So there is no maximum values for this FOC.

2. Consider a firm is a monopoly. They have the following revenue function $R(p, Q) = pQ$ with a price of $p = 8 - Q$ subject to a cost function of $c(Q) = \frac{Q}{3}$. Write the profit function and maximize profit to determine the monopoly quantity supplied Q^*.

Solution: The profit function:

$$\pi(Q) = (8 - Q)Q - \frac{Q}{3}$$

And quantity supplied is $Q^* = \frac{23}{6}$.

Lagrangian Method

3. Max $g(a, b) = e^a b^3$ s.t. $2a + b \leq 10$.

Solution:

$$\begin{aligned} a^* &= 2 \\ b^* &= 6 \end{aligned}$$

4. Max $U(x_1, x_2, x_3) = e^{x_1} x_2^{\alpha} x_3^{\beta}$ s.t. $w \geq p_1 x_1 + p_2 x_2 + p_3 x_3$.

Solution:

$$\begin{aligned} x_1^* &= \frac{w}{p_1} - (\alpha + \beta) \\ x_2^* &= \frac{\alpha p_1}{p_2} \\ x_3^* &= \frac{\beta p_1}{p_3} \end{aligned}$$