Problem Set 2

APEC Math Review

August 2020

Approximation

1. (Simon & Blume Example 14.13) In a two commodity world, consider the pair of constant-elasticity demand functions

$$q_1 = 6p_1^{-2}p_2^{3/2}y \quad and \quad q_2 = 4p_1p_2^{-1}y^2$$

in the vicinity of the current prices and income

$$p_1^* = 6, \quad p_2^* = 9, \quad y^* = 2$$

How would the demand change when both prices rise by 0.1 and income falls by 0.1?

2. (Simon & Blume Example 15.9) Consider $3x^2yz + xyz^2 = 30$ as defining x as an implicit function of y and z around the point $x = 1$, $y = 3$, $z = 2$. If y increases to 3.2 and z remains at 2, estimate the corresponding x.

Homogeneous and homothetic functions

3. (Simon & Blume Exercise 20.7) Is the zero function $f(x) \equiv 0$ homogeneous? If so, of what degree?

4. (Simon & Blume Exercise 20.5) If $y = f(x_1, x_2)$ in C^2 and homogeneous of degree r, show that

$$(x_1)^2 f''_{x_1x_2} + 2x_1x_2 f''_{x_1x_2} + (x_2)^2 f''_{x_2x_2} = r(r-1)f$$

5. (MWG Exercise 3.C.5) A monotone preference relation $\succsim$ on $X = \mathbb{R}^n$ is homothetic if $\mathbf{x} \sim \mathbf{y}$ implies $\alpha\mathbf{x} \sim \alpha\mathbf{y}$ for any $\alpha \geq 0$. Show that the preference relation is homothetic if and only if it admits a utility function that is homogeneous of degree one.

Concavity and quasiconcavity

6. (Simon & Blume Example 21.1) Show that $f(x_1, x_2) = (x_1)^2 + (x_2)^2$ is convex on $\mathbb{R}^n$.

7. (Simon & Blume Theorem 21.8) Let $f_1, ..., f_k$ be concave (convex) functions, each defined on the same convex subset U of $\mathbb{R}^n$. Let $a_1, ..., a_k$ be positive numbers. Show that $a_1 f_1 + \cdots + a_k f_k$ is a concave (convex) function on U.

8. Show that f is quasiconcave on D iif

$$f(\mathbf{y}) \geq f(\mathbf{x}) \, implies \, that \, Df(\mathbf{x})(\mathbf{y} - \mathbf{x}) \geq 0$$

9. Show that any monotonic transformation of a concave(convex) function is a quasiconcave(quasiconvex) function.

10. (Simon & Blume Example 21.10) Consider the CES function

$$Q(x_1, x_2) = (a_1 x_1^{\gamma} + a_2 x_2^{\gamma})^{1/\gamma}, \quad where \, 0 < \gamma < 1$$

Is it quasiconcave?