Chapter 1

DISCRETE CHOICE MODELS IN PREFERENCE SPACE AND WILLINGNESS-TO-PAY SPACE

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Abstract In models with unobserved taste heterogeneity, distributional assumptions can be placed in two ways: (1) by specifying the distribution of coefficients in the utility function and deriving the distribution of willingness to pay (*WTP*), or (2) by specifying the distribution of *WTP* and deriving the distribution of coefficients. In general the two approaches are equivalent, in that any mutually compatible distributions for coefficients and *WTP* can be represented in either way. However, in practice, convenient distributions, such as normal or log-normal, are usually specified, and these convenient distributions have different implications when placed on *WTP*'s than on coefficients. We compare models that use normal and log-normal distributions for coefficients (called models in preference space) with models using these distributions for *WTP* (called models in *WTP* space). We find that the models in preference space fit the data better but provide less reasonable distributions of *WTP* than the models in *WTP* space. Our findings suggests that further work is needed to identify distributions that either fit better when applied in *WTP* space or imply more reasonable distributions of *WTP* when applied in preference space.

Keywords: Mixed logit, random parameters, random willingness to pay.

1. Introduction

In many applications of discrete choice models with random coefficients, the price coefficient is held constant, especially when the goal is to estimate the distribution of consumers' willingness to pay for alternative attributes (e.g., Revelt and Train, 1998; Goett *et al.*, 2000; Layton and Brown, 2000; Scarpa *et al.*, 2002; Hensher *et al.*, 2004) and/or to infer the willingess to pay of individual consumers from their observed choices and the population distribution (Train, 2003, Ch. 11; Scarpa *et al.*, 2005; Greene *et al.*, 2005.) This restriction allows the distributions of willingness to pay (*WTP*) to be calculated easily from the distributions of the non-price coefficients, since the two distributions take the same form. For example, if the coefficient of an attribute is distributed normally, then *WTP* for that attribute, which is the attribute's coefficient divided by the price coefficient, is also normally distributed. The mean and standard deviation of *WTP* are simply the mean and standard deviation of the attribute coefficient scaled by the inverse of the (fixed) price coefficient. The restriction also facilitates estimation. As Ruud (1996) points out, a model with all random coefficients, including the price coefficient, can be practically unidentified empirically, especially in datasets with only one observed choice for each decision-maker.

A fixed price coefficient,[1] however, implies that the standard deviation of unobserved utility, which is called the scale parameter, is the same for all observations. Louviere (2003) discusses the importance of recognizing that the scale parameter can, and in many situations clearly does, vary randomly over observations and that ignoring this variation in estimation can lead to erroneous interpretation and conclusions. For example, if the price coefficient is constrained to be fixed when in fact scale varies over observations, then the variation in scale will be erroneously attributed to variation in *WTP*.

In this paper we investigate alternative ways to specify random coefficients and *WTP* when the price coefficient varies. Cameron and James (1987) and Cameron (1988) introduced the concept of parameterizing a fixed-coefficient model in terms of *WTP* rather than coefficients. We extend their analysis to models with random coefficients, where distributional assumptions and restrictions can be placed on the coefficients or on the *WTP*'s. The two approaches are formally equivalent, in the sense that any distribution of coefficients translates into some derivable distribution of *WTP*'s, and vice-versa. However, the two approaches differ in terms of numerical convenience under any given distributional assumptions. For example, a model with an attribute coefficient that is normally distributed and a price coefficient that is log-normal implies that *WTP* for the attribute is distributed as the ratio of a normal to a log-normal.

[1] Or, more generally, any fixed coefficient, or uncorrelated random coefficients.

A researcher working directly in *WTP* space is unlikely to choose this inconvenient distribution for *WTP*'s. Conversely, a model with normal *WTP* and log-normal price coefficient implies that the attribute coefficient is the product of a normal and log-normal, which is a distribution that has never, to our knowledge, been applied in preference space. Restrictions are also asymmetric. For example, uncorrelated preference coefficients translate into *WTP*'s that are correlated in a particular way that would be hard to implement and test in the context of *WTP* distributions, and vice-versa.

We estimate and compare models that are parameterized in terms of coefficients, called "models in preference space," and models parameterized in terms of *WTP*, called "models in *WTP* space." For the models in preference space, a convenient distribution is specified for the coefficients, and the parameters of this distribution (such as its mean and variance) are estimated. The distribution of *WTP*'s is then derived from the estimated distribution of coefficients. This is currently the standard practice for application of choice models. For the models in *WTP* space, convenient distributions are specified for the *WTP*'s and the price coefficient. The parameters of this distribution are estimated, from which the estimated distribution of utility coefficients is derived.

We find that models using convenient distributions in preference space fit the data better, both within sample and out-of-sample, than models using convenient distributions in *WTP* space. However, the distributions of *WTP* that are derived from these models have unreasonably large variance, which translates into an untenable implication that many people are willing to pay an enormous amount of money to have or avoid an attribute. Stating the conclusions in combination: the models that fit better give less reasonable distributions for *WTP*. These results suggests that alternative distributional specifications are needed that either fit the data better when applied in *WTP* space or imply more reasonable *WTP* distributions when applied in preference space.

Our analysis and findings mirror those of Sonnier, Ainslee, and Otter (2003), with one exception. In similar comparisons as ours they find that their models in preference space fit the within-sample data better than their models in *WTP* space but provide unreasonably large variances in *WTP*. In these regards, their results match ours. However, they find that their models in *WTP* space attain better out-of-sample fit than their models in preference space, which is opposite of what we find. Sonnier *et al.* (2003) use a different method for evaluating out-of-sample fit than we do, which might account for the difference. However, differences like this one are to be expected over different datasets, since the issue under investigation is the performance of various distributional specifications and the appropriate distribution is necessarily situation-dependent.

2. Specification

In this section we describe the two types of models. Decision-makers are indexed by n, alternatives by j, and choice situations by t. To facilitate discussion, we specify utility as separable in price, p, and non-price attributes, x:

$$U_{njt} = -\alpha_n p_{njt} + \beta_n' x_{njt} + \epsilon_{njt} \tag{2.1}$$

where α_n and β_n vary randomly over decision-makers and ϵ_{njt} is i.i.d. We assume ϵ_{njt} is distributed extreme value, though the analysis is the analogous for other distributions. The variance of ϵ_{njt} can be different for different decision-makers: $Var(\epsilon_{njt}) = k_n^2(\pi^2/6)$, where k_n is the scale parameter for decision-maker n.

Though the utility specification is not yet normalized, the current formulation allows us to clarify the circumstances under which the scale parameter can be expected to vary over decision-makers. A random scale parameter is conceptually different from random values for α and β. α_n and β_n represent the tastes of person n, and these parameters vary over decision-makers because different people have different tastes. In contrast, the scale parameter does not represent a term within the utility function in any given choice situation but rather the standard deviation of utility over different choice situations. By allowing the scale parameter to be random, the researcher gives a variance to a variance. The question arises: what would cause the variance of ϵ to vary? Two prominent situations arise:

1 The unobserved term ϵ might reflect factors that are actually random or quixotic from the decision-maker's perspective, rather than, as in the usual derivation, factors that are known to the decision-maker but unknown by the researcher. In this situation, the variance of ϵ reflects the degree of randomness in the decision-maker's process, which can be expected to differ over decision-makers. This concept of randomness is particularly relevant with stated preference data, where respondents differ in their attention to the task and in their constructs of unlisted attributes. However, randomness in behavior can arise in revealed preference data as well.

2 In panel data settings, each decision-maker faces a sequence of choice situations with unobserved factors differing in each choice situation. It is reasonable to believe in this situation that the variance of these unobserved factors over choice situations for each decision-maker is different for different decision-makers, even when the unobserved factors are known to the decision-maker and unobserved only by the researcher.

These two situations also clarify the converse: When ϵ represents factors that are known to the decision-maker but unknown by the researcher, and only one choice situation is observed for each decision-maker such that each observation represents a different decision-maker, there is perhaps little need or meaning to allowing the scale parameter to vary over decision-makers. In this circumstance, the scale parameter captures variance over observations in factors that the researcher does not observe; this variance is defined on the researcher, not the decision-maker, and takes a given (i.e., fixed) value for the researcher.

Dividing utility (2.1) by the scale parameter does not affect behavior and yet results in a new error term that has the same variance for all decision-makers:

$$U_{njt} = -(\alpha_n/k_n)p_{njt} + (\beta_n/k_n)'x_{njt} + \varepsilon_{njt} \tag{2.2}$$

where ε_{njt} is i.i.d. type-one extreme value, with constant variance $\pi^2/6$. The utility coefficients are defined as $\lambda_n = (\alpha_n/k_n)$ and $c_n = (\beta_n/k_n)$, such that utility is written:

$$U_{njt} = -\lambda_n p_{njt} + c_n' x_{njt} + \varepsilon_{njt} \tag{2.3}$$

Note that if k_n varies randomly, then the utility coefficients are correlated, since k_n enters the denominator of each coefficient. Specifying the utility coefficients to be independent implicitly constrains the scale parameter to be constant. If the scale parameter varies and α_n and β_n are fixed, then the utility coefficients vary with perfect correlation. If the utility coefficients have correlation less than unity, then α_n and β_n are necessarily varying in addition to, or instead of, the scale parameter.

Equation (2.3) is called the model in preference space. Willingness to pay for an attribute is the ratio of the attribute's coefficient to the price coefficient: $w_n = c_n/\lambda_n$. Using this definition, utility can be rewritten as

$$U_{njt} = -\lambda_n p_{njt} + (\lambda_n w_n)' x_{njt} + \varepsilon_{njt}, \tag{2.4}$$

which is called utility in *WTP* space. Under this parameterization, the variation in *WTP*, which is independent of scale, is distinguished from the variation in the price coefficient, which incorporates scale.[2]

The utility expressions are equivalent of course. Any distribution of λ_n and c_n in (2.3) implies a distribution of λ_n and w_n in (2.4), and vice-versa. The general practice has been to specify distributions in preference space, estimate the parameters of those distributions, and derive the distributions of *WTP* from these estimated distributions in preference space (e.g., Train, 1998.) While fully general in theory, this practice is usually limited in implementation by the use of convenient distributions for utility coefficients. Convenient distributions for utility coefficients do not imply convenient distributions for *WTP*, and

[2]Any coefficient can be used as the base that incorporates scale, with each other coefficient expressed as the product of this coefficient and a term that is independent of scale. The only reason to use the price coefficient as the base is that the scale-free terms become *WTP*'s, which are easy to interpret.

vice-versa. As stated above, if the price coefficient is distributed log-normal and the coefficients of non-price attributes are normal, then *WTP* is the ratio of a normal term to a log-normal term. Similarly, normal distributions for *WTP* and a log-normal for the price coefficient implies that the utility coefficients are the product of a normal term and a log-normal term. The placement of restrictions is similarly asymmetric. It is fairly common for researchers to specify uncorrelated utility coefficients; however, this restriction implies that scale is constant, as stated above, and moreover that *WTP* is correlated in a particular way. It is doubtful that a researcher in specifying uncorrelated coefficients is actually thinking that *WTP* is correlated in this way. Similarly, uncorrelated *WTP*, which the researcher might want to assume or test, implies a pattern of correlation in utility coefficients that is difficult to implement in preference space.

The issue becomes: does the use of convenient distributions and restrictions in preference space or *WTP* space result in more accurate and reasonable models? The answer is necessarily situationally dependent, since the true distributions differ in different applications. However, some insight into the issue can be obtained by comparisons on a given dataset. This is the topic of the next section.

3. Data

We use the stated-preference data collected by Train and Hudson (2000) on households' choice among alternative-fueled vehicles, including gas, electric, and hybrid gas-electric vehicles. 500 respondents were presented with 15 choice situations apiece. For each choice situation, the respondent was given a card that described three vehicles and was asked to state which of the vehicles he/she would choose to buy. Each vehicle was described in terms of the following variables:

- Engine type (gas, electric, or hybrid),
- Purchase price, in dollars,
- Operating cost, in dollars per month,
- Performance (grouped into three levels, which we call "low," "medium," and "high," [3]
- Range between recharging/refueling, in hundreds of miles,

[3] Performance was described on the card in terms of top speed and seconds required to reach 60 mph. However, these two components were not varied independently, and only three combinations of the two components were utilized.

- Body type (10 types ranging from mini car to large van).

Each of the attributes varied over choice situations and over respondents. Range varied for electric vehicles but was constant for gas and hybrid vehicles, since the purpose of this variable was to determine consumers' response to the relatively restricted range of electric vehicles. All but a few respondents completed the fifteen choice tasks, giving a total of 7,437 observations for estimation. These data have been previously used by Hess *et al.* (2003) and Train and Sonnier (2005) for other purposes. We use the data to compare specifications in preference and *WTP* space.

4. Estimation

4.1 Uncorrelated coefficients in preference space

Our first model is specified in preference space with a random coefficient for each variable and no correlation over coefficients. As discussed above, uncorrelated coefficients implies that the scale parameter is fixed. This model can therefore be seen as a version that does not allow for random scale. It is compared with models, described below, that allow random scale.

For this and other models in preference space, the attributes that are desirable, or undesirable, for everyone are given log-normally distributed coefficients. These attributes are: price, operating cost, range, a dummy for medium performance or higher, and a dummy for high performance. The coefficient for the first of the performance variables captures the extra utility associated with increasing performance from low to medium, while the coefficient for the second performance variable reflects the extra utility associated with increasing performance from medium to high. Price and operating cost are entered as negative, since the log-normal distribution implies positive coefficients. The other attributes can be either desirable or undesirable, depending on the views and tastes of the consumer. These attributes are: dummies for electric and hybrid engines, whose coefficients reflect the value of these engine types relative to gas; and dummies for each body type except mid-sized car, whose coefficients reflect the value of these body types relative to a mid-sized car (holding other attributes constant, of course.) The coefficients of these variables are given normal distributions.

The model, and all the ones which follow, was estimated by Bayesian MCMC procedures, using diffuse priors. These procedures for mixed logit models are described by Train (2003) in general and by Train and Sonnier (2005) in relation to these particular data. 10,000 iterations were used as "burn-in" after which every tenth draw was retained from 10,000 additional iterations, providing a total 1,000 draws from the posterior distribution of the parameters. Previous analysis of these data by Train and Sonnier, as well as our own analysis, indicates that the MCMC sequences converged within the burn-in period.

Table 1.1. Model in Preference Space with Uncorrelated Coefficients

Attribute	*Parameter*	*Estimate*	*St. error*
Price in $10,000's	Mean of ln(–coeff.)	–0.2233	0.0508
	Variance of ln(–coeff.)	0.5442	0.0635
Operating cost in $/month	Mean of ln(–coeff.)	–3.5540	0.0993
	Variance of ln(–coeff.)	0.7727	0.1449
Range in 100's of miles	Mean of ln(coeff.)	–0.7272	0.1298
	Variance of ln(coeff.)	0.3317	0.1209
Electric engine	Mean of coeff.	–1.9453	0.1354
	Variance of coeff.	1.6492	0.2820
Hybrid engine	Mean of coeff.	0.8331	0.1102
	Variance of coeff.	1.4089	0.1797
High performance	Mean of ln(coeff.)	–3.0639	0.3546
	Variance of ln(coeff.)	3.3681	0.8493
Medium or high performance	Mean of ln(coeff.)	–1.3030	0.2630
	Variance of ln(coeff.)	1.4041	0.5204
Mini car	Mean of coeff.	–3.0325	0.1767
	Variance of coeff.	3.5540	1.0535
Small car	Mean of coeff.	–1.3966	0.1240
	Variance of coeff.	1.3086	0.4290
Large car	Mean of coeff.	–0.4008	0.1272
	Variance of coeff.	1.3084	0.7080
Small SUV	Mean of coeff.	–0.8499	0.1072
	Variance of coeff.	0.7032	0.3655
Midsize SUV	Mean of coeff.	0.2490	0.1449
	Variance of coeff.	0.9772	0.3548
Large SUV	Mean of coeff.	–0.1295	0.1765
	Variance of coeff.	2.4334	0.9578
Compact pickup	Mean of coeff.	–1.3201	0.1507
	Variance of coeff.	1.3209	0.4484
Full-sized pickup	Mean of coeff.	–0.7908	0.1544
	Variance of coeff.	3.1370	0.8326
Minivan	Mean of coeff.	–0.5219	0.1441
	Variance of coeff.	2.6569	0.6334
Log likelihood at convergence		–6,297.81	

The Bernstein-von Mises theorem states that, under fairly benign conditions, the mean of the Bayesian posterior is a classical estimator that is asymptotically equivalent to the maximum likelihood estimator. Also, the variance of the posterior is the asymptotic variance of this estimator. See Train (2003) for an explanation with citations. Therefore, even though the model is estimated

Table 1.2. Mean and standard deviations of coefficients and *WTP*, implied by estimated parameters of model in preference space (Table 1.1)

Attribute	*Coefficient Mean*	*Coefficient Std. dev.*	*WTP Mean*	*WTP Std. dev.*
Price in $10,000's	–1.0499	0.8948		
Operating cost in $/month	–0.0421	0.0453	–0.0690	0.1130
Range in 100's of miles	0.5701	0.3576	0.9365	1.1077
Electric engine	–1.9453	1.2842	–3.1957	3.8605
Hybrid engine	0.8331	1.1870	1.3703	2.8062
High performance	0.2518	1.1829	0.4164	2.7611
Medium or high performance	0.5483	0.9581	0.9004	2.1917
Mini car	–3.0325	1.8852	–4.9773	5.8563
Small car	–1.3966	1.1439	–2.2938	3.1446
Large car	–0.4008	1.1439	–0.6598	2.5314
Small SUV	–0.8499	0.8386	–1.3952	2.1607
Midsize SUV	0.2490	0.9885	0.4060	2.1527
Large SUV	–0.1295	1.5599	–0.2120	3.3620
Compact pickup	–1.3201	1.1493	–2.1702	3.0874
Full-sized pickup	–0.7908	1.7712	–1.3032	3.9653
Minivan	–0.5219	1.6300	–0.8621	3.5859

by Bayesian procedures, the results can be interpreted from a purely classical perspective.

Table 1.1 gives estimation results for our model in preference space with uncorrelated coefficients. The estimate for each parameter is the mean of the 1,000 draws from the posterior, and the standard error of the estimate is the standard deviation of these draws. Presenting the results in this way facilitates interpretation by researchers who maintain a classical perspective: the estimates and standard errors can be interpreted the same as if they had been obtained by maximum likelihood procedures. The results can also, of course, be interpreted from a Bayesian perspective, with the mean and standard deviation of the draws providing summary information about the posterior. The log-likelihood value given at the bottom of table 1.1 is calculated in the classical way at the parameter estimates.[4]

For the log-normally distributed coefficients, the estimates in Table 1.1 are the mean and variance of the log of coefficient, which are difficult to interpret directly. Table 1.2 gives the estimated mean and standard deviation of the co-

[4]A Bayesian log-likelihood would be calculated by integrating the log-likelihood over the posterior or, as described by Sonnier *et al.* (2003), by integrating the inverse of the log-likelihood over the posterior and then taking the inverse.

efficients themselves, derived from the estimated parameters in Table 1.1. The estimates seem generally reasonable. Electric vehicles are considered worse than gas vehicles by the vast majority of the population, even if the two types of vehicles could cost the same and have the same range. The mean and standard deviation of the electric vehicle coefficient imply that 94 percent of the population place a negative value of electric vehicles relative to gas. Hybrid vehicles, on the other hand, are preferred to gas vehicles by most consumers, if they were to cost the same. The estimated mean and standard deviation imply that 75 percent have a positive coefficient for the hybrid dummy. Performance is valued at a decreasing rate, as expected. The average utility associated with moving from low to medium performance is greater than that for moving from medium to high performance (0.5483 and 0.2518 respectively.) The standard deviation of the range coefficient is much lower than of the two performance variables. This difference indicates that consumers are more similar in their desire for extra range than in their value for higher top speed and acceleration. The body type coefficients seem reasonable, with mid-sized cars and SUVs being preferred, on average, to either smaller or larger versions (holding price and operating cost constant). And pickups are valued less, on average, than comparably sized SUVs.

The estimated parameters in preference space imply distributions of *WTP*. A draw from the estimated distribution of *WTP* for an attribute is simulated by taking a draw from the estimated distribution of the attribute's coefficient and dividing by a draw from the estimated distribution of the price coefficient. Statistics for the distribution of *WTP* are obtained by taking numerous such draws and calculating the requisite statistic for these draws. The estimated mean and standard deviation of the *WTP* for each attribute is given in the final two columns of Table 1.2.

The most distinguishing aspect of the estimated distributions of *WTP* is the prevalence of large standard deviations. The standard deviation exceeds the mean for all *WTP*'s, and are more than twice the means for eight of the fifteen. These large standard deviations imply that a nontrivial share of people are willing to pay enormous amounts of money to obtain/avoid some attributes. For example, ten percent the population is estimated to have a *WTP* for range that exceeds 2. Given the units for price and range, a *WTP* over 2 means that the consumer is willing to pay more than \$20,000 to have an extra 100 miles of range. Similarly, ten percent of the population is estimated to be willing to pay over \$20,000 to move from low to medium performance. We return to this issue after presenting results of a model estimated in *WTP* space, where the distribution of *WTP* is estimated directly rather than derived from estimated coefficient distributions.

As stated above, a model with uncorrelated coefficients in preference space implies correlated *WTP*, with the correlation being the fairly arbitrary outcome

Table 1.3. Correlations between *WTP* for attributes, implied by estimated parameters of model in preference space (Table 1.1)

Attribute	*Op. cost*	*Range*	*Electric*	*Hybrid*	*Hi Perf*	*Med Perf*
Operating cost	1.0000	0.3687	–0.3627	0.2129	0.0679	0.1784
Range	0.3687	1.0000	–0.5029	0.2965	0.0958	0.2496
Electric	–0.3627	–0.5029	1.0000	–0.2855	–0.0929	–0.2411
Hybrid	0.2129	0.2965	–0.2855	1.0000	0.0584	0.1433
High perf	0.0679	0.0958	–0.0929	0.0584	1.0000	0.0439
Med–hi Perf	0.1784	0.2496	–0.2411	0.1433	0.0439	1.0000

(in the sense that the researcher does not specify it directly) of the estimated means and variances of the coefficients themselves. The correlation of *WTP* over attributes is given in Table 1.3. To conserve space, the correlation matrix does not contain the body types. As the table indicates, correlations among *WTP*'s are fairly large; researchers assuming uncorrelated coefficients might not be aware that they are implicitly assuming fairly large correlations among *WTP*'s.

4.2 Uncorrelated *WTP*'s in *WTP* space

We estimated a model with utility specified as in equation (2.4), where the coefficient of each non-price attribute is the product of the *WTP* for that attribute times the price coefficient. This model allows for random scale. If only scale varies, then the correlation between each pair of coefficients is one; correlations below one in coefficients imply that *WTP* varies as well as scale.

The price coefficient $-\lambda_n$ is given a log-normal distribution. The elements of ω_n (*WTP*'s) associated with operating cost, range, and the two performance variables are also specified to be log-normal, while the elements of ω_n associated with engine and body types are, instead, normal. The *WTP*'s are assumed to be uncorrelated over attributes. Note, of course, that when *WTP* for an attribute is normally distributed and the price coefficient is log-normal, the coefficient of the attribute is not normal (as in the previous model). Also, as stated above, uncorrelated *WTP* implies correlated coefficients (unlike the previous model), due to the common influence of the price coefficient on each other coefficient. The current model differs from the previous one in both of these ways.

Table 1.4 gives the estimation results. The log-likelihood is considerably lower than that for the model in Table 1.1. However, the distributions of *WTP* seem more reasonable. Comparing Table 1.5 with Table 1.2, the main dis-

Table 1.4. Model in *WTP* Space with Uncorrelated *WTP*'s

Attribute	*Parameter*	*Estimate*	*St. error*
Price in $10,000's	Mean of ln(–coeff.)	–0.0498	0.0602
	Variance of ln(–coeff.)	0.9014	0.1234
Operating cost in $/month	Mean of ln(*WTP*)	–3.4106	0.1100
	Variance of ln(*WTP*)	0.7847	0.1530
Range in 100's of miles	Mean of ln(*WTP*)	–0.4045	0.1286
	Variance of ln(*WTP*)	0.2706	0.0939
Electric engine	Mean of *WTP*	–2.5353	0.2369
	Variance of *WTP*	1.9828	0.4443
Hybrid engine	Mean of *WTP*	0.8738	0.1090
	Variance of *WTP*	2.1181	0.2745
High performance	Mean of ln(*WTP*)	–1.8854	0.2840
	Variance of ln(*WTP*)	1.7172	0.5898
Medium or high performance	Mean of ln(*WTP*)	–1.7380	0.2917
	Variance of ln(*WTP*)	2.4701	0.7310
Mini car	Mean of *WTP*	–3.4645	0.1894
	Variance of *WTP*	6.5767	1.3889
Small car	Mean of *WTP*	–1.5992	0.1451
	Variance of *WTP*	1.7010	0.5337
Large car	Mean of *WTP*	–0.6148	0.1716
	Variance of *WTP*	1.9353	0.6750
Small SUV	Mean of *WTP*	–1.0671	0.1287
	Variance of *WTP*	0.8203	0.5776
Midsize SUV	Mean of *WTP*	0.2173	0.1611
	Variance of *WTP*	1.8544	0.4389
Large SUV	Mean of *WTP*	–0.7559	0.2923
	Variance of *WTP*	8.2263	2.3072
Compact pickup	Mean of *WTP*	–1.4752	0.1398
	Variance of *WTP*	1.2675	0.5266
Full-sized pickup	Mean of *WTP*	–1.1230	0.1843
	Variance of *WTP*	5.7762	1.2558
Minivan	Mean of *WTP*	–0.7406	0.1827
	Variance of *WTP*	3.9847	0.9252
Log likelihood at convergence		–6,362.13	

tinction is that the means and especially the standard deviations of *WTP*'s are smaller for the model in *WTP* space than the model in preference space. This difference means that there is a smaller share with unreasonably large *WTP*'s. For example, the model in *WTP* space implies that 1.7 percent are estimated to be willing to pay more than $20,000 for 100 miles of extra range, while, as stated above, the model in preference space implies over 10 percent. Sim-

Table 1.5. Mean and standard of preference coefficients and *WTP*, implied by estimated parameters of model in *WTP* space (Table 1.4)

Attribute	*Coefficient Mean*	*Coefficient Std. dev.*	*WTP Mean*	*WTP Std. dev.*
Price in $10,000's	–1.4934	1.8123		
Operating cost in $/month	–0.0732	0.1616	–0.0489	0.0531
Range in 100's of miles	1.1406	1.7027	0.7636	0.4257
Electric engine	–3.7870	5.6565	–2.5353	1.4081
Hybrid engine	1.3053	3.7585	0.8738	1.4554
High performance	0.5335	1.7974	0.3584	0.7563
Medium or high performance	0.8951	4.5679	0.6047	1.9542
Mini car	–5.1712	8.6579	–3.4645	2.5645
Small car	–2.3849	4.1887	–1.5992	1.3042
Large car	–0.9180	3.4259	–0.6148	1.3912
Small SUV	–1.5914	2.8561	–1.0671	0.9057
Midsize SUV	0.3151	3.1997	0.2173	1.3618
Large SUV	–1.1336	6.8725	–0.7559	2.8682
Compact pickup	–2.2029	3.7700	–1.4752	1.1258
Full-sized pickup	–1.6858	5.9893	–1.1230	2.4034
Minivan	–1.1161	4.8729	–0.7406	1.9962

ilarly, but not as dramatically, the share who are willing to pay over $20,000 to move from low to medium performance is estimated to be 6 percent in the model in *WTP* space, which is less than the 10 percent implied by the model in preference space.

In conclusion, for both preference coefficients and *WTP* values, the indirect way of estimating the distributions results in larger means and standard deviations than when the distributions are estimated directly. As discussed above, the larger standard deviations in *WTP* imply implausible shares of the population willing to pay large amounts for an attribute. The meaning of larger means and standard deviations of coefficients is not clear.

Table 1.6 gives the correlations between coefficients that are implied by the estimated distributions of *WTP* and the price coefficient. The correlations are fairly high, due to the fact that each *WTP* is multiplied by the common price coefficient. These high correlations suggest that models with uncorrelated coefficients in preference space are incompatible empirically (as well as theoretically, of course) with independent *WTP*'s and price coefficient. Researchers, when considering independence over attributes, must be careful in distinguishing whether they want to assume that *WTP*'s are independent or

Table 1.6. Correlations between preference coefficients of attributes, implied by estimated parameters of model in *WTP* space (Table 1.4)

Attribute	*Price*	*Op. cost*	*Range*	*Electric*	*Hybrid*	*Hi Perf*	*Med Perf*
Price	1.0000	0.5526	0.8117	–0.8080	0.4157	0.3570	0.2242
Op cost	0.5526	1.0000	0.4481	-0.4456	0.2322	0.2087	0.1281
Range	0.8117	0.4481	1.0000	-0.6532	0.3375	0.2895	0.1796
Electric	–0.8080	–0.4456	–0.6532	1.0000	–0.3343	–0.2853	–0.1857
Hybrid	0.4157	0.2322	0.3375	-0.3343	1.0000	0.1439	0.0945
Hi perf	0.3570	0.2087	0.2895	-0.2853	0.1439	1.0000	0.0794
Med/Hi Perf	0.2242	0.1281	0.1796	–0.1857	0.0945	0.0794	1.0000

that utility coefficients are independent, since independence of one implies non-independence of the other.

4.3 Correlated coefficients and *WTP*

In general, neither coefficients nor *WTP*'s are independent. We estimated a model in preference space with correlated coefficients and a model in *WTP* space with correlated *WTP*'s. The model in preference space incorporates random scale, since it allows correlation between all coefficients. The two models (in preference space and *WTP* space) are therefore the same in allowing for random scale and differ only in the distributional assumptions for coefficients and *WTP*. Both models assume a log-normal price coefficient. The model in preference space assumes normal and log-normal non-price coefficients, which implies that *WTP*'s are distributed as the ratio of a normal or log-normal to a log-normal. The model in *WTP* space assumes normal and log-normal *WTP*'s, which implies coefficients that are the product of a log-normal with a normal or log-normal.

To save space, we do not present the estimates of these model; they are available to interested readers upon request. The results are consistent with those obtained above, namely: (1) the model in preference space obtains a higher log-likelihood, but (2) the estimated distribution of *WTP* is more reasonable (with smaller means and variances) for the model in *WTP* space. In addition, several conclusions can be drawn concerning correlations:

- The hypothesis that coefficients in preference space are uncorrelated can be rejected. The model in preference space attains a log-likelihood of -6,178.12 with correlated coefficients, compared to -6,297.81 for the model given in Table 1.1 with uncorrelated coefficients. The likelihood ratio test statistic is therefore 239.4 for the hypothesis that all 120 covariances are zero, which is greater than the 99-percentile value of the chi-square with 120 degrees of freedom.

- The estimated correlations among coefficients are generally small or moderate in size. 47 of the 160 correlations are below 0.1 in magnitude, and only 12 are above .4 in magnitude.
- The model in *WTP* space attains a log-likelihood of -6,228.31 when the *WTP*'s and price coefficient are all allowed to be correlated and -6,362.13 when they are constrained to be uncorrelated. The hypothesis of no correlation can be rejected.
- The estimated correlations between *WTP*'s for the model in *WTP* space are generally small or moderate, similar to the estimated correlations between coefficients for the model in preference space.
- The correlations among coefficients that are derived from the model in *WTP* space are considerably larger in magnitude than those estimated directly in the model in preference space. Similarly, the correlations among *WTP*'s that are derived from the model in preference space are considerably larger than those estimated directly in the model in *WTP* space. These findings are similar to those given above for variances, i.e., that larger variances in coefficients are obtained when they are estimated indirectly instead of directly, and larger variances in *WTP*'s are obtained when estimated indirectly than directly. It seems that the process of combining estimated distributions (dividing a normal by a log-normal for *WTP* or multiplying a normal by a log-normal for a coefficient) tends to inflate the estimated variances and covariances.

Sonnier *et al.* (2003) estimated models in preference space and *WTP* space, using the terms "linear models" and "nonlinear models" instead of our terminology to denote that the random customer-level parameters enter utility linearly in the former and nonlinearly in the later. Their results are consistent with our main conclusions, in that they obtained better within-sample fit for their model in preference space but more reasonable *WTP* distributions for their model in *WTP* space. However, their results differ from ours in one regard. They performed out-of-sample analysis and concluded that their model in *WTP* space fits better out-of-sample, even though it fits worse in-sample.

To examine this issue, we divided our sampled respondents into two equal-sized sub-samples, estimated each model on one sub-sample, and evaluated the log-likelihood of the estimated models on the other sub-sample. In each comparison (estimation on first half with evaluation on the second half, and estimation on the second half with evaluation on the first half), the model in preference space obtained a higher log-likelihood than the model in *WTP* space on the out-of-estimation sub-sample.

Our results therefore differ in this regard from those of Sonnier *et al.* (2003). The difference can perhaps be explained by the fact that we used a somewhat different method to evaluate out-of-sample fit than they did. We estimated on half the respondents using all of their choice situations and then calculated the log-likelihood for all the choice situations for the other half of the respondents, while they estimated the model on all but one choice situation for each respondent and then calculated the log-likelihood for this one "hold-out" choice situation for each respondent.

However, there is no reason to expect the same results in different settings, since the answer to the question "Which distributions fit better?" is necessarily situation-dependent. The purpose of the explorations is to focus our attention on the relation between distributions of coefficients and distributions of *WTP*, rather than to attempt to identify the appropriate distributions to use in all situations.

5. Conclusions

This paper examines consumer choice among alternative-fueled vehicles, including gas, electric, and gas/electric hybrids. The empirical results indicate that the vast majority of consumers would need to be compensated through a lower price (i.e., have a negative willingness to pay) for electric vehicles relative to gas vehicles, even if operating cost, performance, and range were the same. In contrast, most consumers are willing to pay extra for a hybrid relative to a gas vehicle with the same non-price attributes. This result is consistent with the market experience in the U.S. The few electric cars that have been introduced in the U.S. have fared poorly in the market, and models are being discontinued. In contrast, the initial offerings of hybrids have been relatively popular, and more models, such as hybrid SUVs, are being launched.

Discrete choice models were estimated with convenient distributions (normal and log-normal) in preference space and in willingness-to-pay *WTP* space. The models in preference space were found to fit the data better, both within-sample and out-of-sample, than the models in *WTP* space. However, the models in *WTP* space provided more reasonable distributions of *WTP*, with fewer consumers having untenably large *WTP*'s, than the models in preference space. This comparison implies that research is needed to identify distributions that fit the data better when applied in *WTP* space and/or provide more reasonable distributions of *WTP* when applied in preference space.