

Classifying Integral Affine Structures on Compact Surfaces

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Overview

- 1 Basic Constructions
- 2 Hilberts Theorem
- 3 Extensions of Hilberts Theorem

Affine Manifolds

Definition

An *affine manifold* is a differentiable manifold such that the transition maps lie in the affine group $\text{Aff}(\mathbb{R}^n)$

Remark

The group $\text{Aff}(\mathbb{R}^n) := \text{GL}(\mathbb{R}^n) \times \mathbb{R}^n$. An element $g \in \text{Aff}(\mathbb{R}^n)$ acts on $x \in \mathbb{R}^n$ by $g(x) = Ax + b$ where $A \in \text{GL}(\mathbb{R}^n)$ and $b \in \mathbb{R}^n$.

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