
Computer Vision 1-Assignment 3

Michael Mo
University of Amsterdam
michael.mo@uva.student.nl

Changxin Miao
University of Amsterdam
changxin.miao@uva.student.nl

1 Introduction

In this assignment, we are going to explore different techniques to extract important features from an image. We start with experimenting with harris corner algorithms to detect corners in an image. Subsequently, we implement Lucas-Kanade algorithm to estimate Optical Flow of an image. At last, we combine both algorithms into a feature-tracking algorithm, with which we could detect important features such as corners in an image and track its movements.

2 Harris Corner Detector

2.1 Question 1

1

Following the instruction, we build up the algorithm according to harris corner detector to locate corners in the image. At first, edges of image are also classified as corners. This could be due to the zero padding option in the *conv2* function. As a result, the drastic change on the edge for "Ix" or "Iy" might contribute to high gradient. As long as there is also certain change on the other axis, the pixel might be identified as a corner.

2

We implement the Harris corner detector on the *'persontoy/00000001.jpg'* and *'pingpong/0000.jpg'*. During the experiment, we found that as the sigma value of the Gaussian filter increases, the threshold we set for detecting the corner becomes lower. This is due to the fact that as the image become more blurred, the eigen values also decrease. The threshold that we set up for these two images are different. For the person toy image, we set the threshold as the 10 times mean value of the Harris matrix. Results of these images will be displayed in the figure 1.

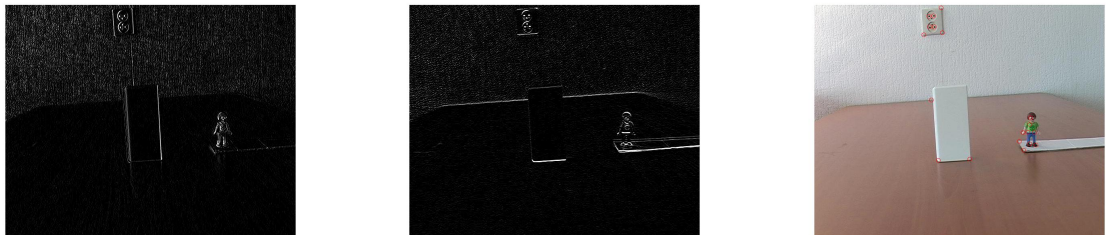


Figure 1: The left, middle and right image correspond to the Ix, Iy and original image with corners

The ping pong image requires higher threshold than the toy image, it will be 30 times the mean value of the matrix. It could be visualized as the images in the figure 2.



Figure 2: The left, middle and right image correspond to the I_x , I_y and original image with corners

24 **3**
 25 The image is rotated with the "imrotate" function in matlab with the option "bilinear". As we could
 26 observe from the below images, harris corner detector is rotation-invariant. Harris corner detector
 27 works on images with different angles are displayed by figure 3. The edge still causes some problem
 28 on the rotated image, we assume that is due to the black background added to the original image,
 29 which cause the instant change on the edges of images.

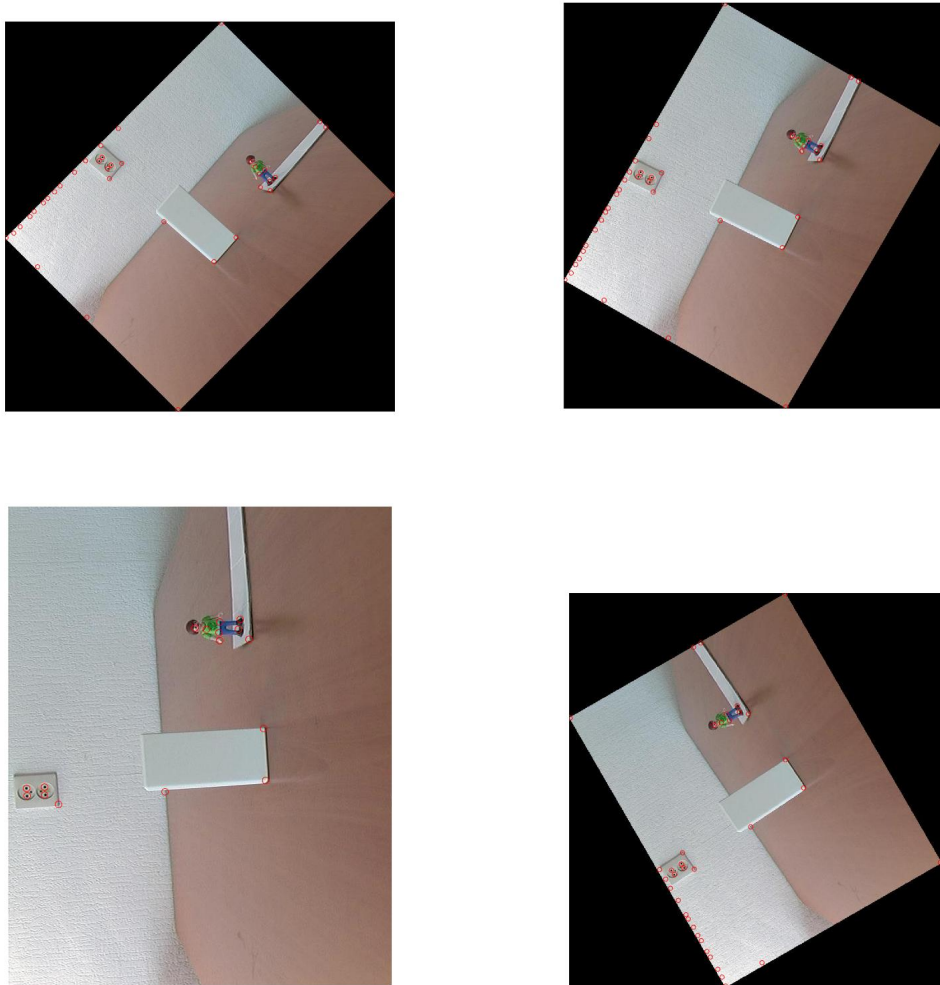


Figure 3: Up: left-45 degree right-60 degree Down: left-90 degree right-120 degree

30 The rotation-invariant property of harris corner algorithm could be explained by the eigen value used
 31 for calculating the Harris matrix. $H = \lambda_1\lambda_2 - 0.04(\lambda_1 + \lambda_2)^2$ Although "Ix" and "Iy" of the corner
 32 changes(ellipse rotate), the eigen value remains the same. Therefore, H will also stay the same.

33 2.2 Question 2

34 **1**

35 In the original harris corner algorithm, the H matrix is calculated by the mathematical expression
 36 $H = \lambda_1\lambda_2 - 0.04(\lambda_1 + \lambda_2)^2$. However, for Shi-Tomasi, it is calculated as $H = \min(\lambda_1, \lambda_2)$. The
 37 corner window will only be identified when H is larger than a threshold value λ . They performed
 38 several experiments and found out that when the smaller eigen value is larger than threshold value,
 39 the image matrix is well conditioned. As the larger eigenvalue could not be arbitrarily larger due to
 40 similar intensity ?.

41 **2**

42 If we implement Harris corner method, we do not necessarily need to calculate the two eigen values.
 43 It could be expressed as $H = \det(Q) - 0.04(\text{trace}(Q))^2$. Nevertheless, if we use Shi-Tomasi
 44 approach, it is essential for us to calculate both eigen values to determine the smaller one.

45 **3**

- 46 • (a) If both eigen values are near 0, the region has no interesting features.
- 47 • (b) If one eigen value is positive while the other is near 0. Then it could be the edge.
- 48 • (c) If both eigen values are big, then it could be defined as a corner.

49 3 Lucas-Kanade

50 3.1 Question 1

51 Given a sequence of images made in consecutive time, it can be possible to determine the motion of
 52 objects within those images. There are multiple algorithms which try to estimate that "optical flow",
 53 and the Lucas-Kanade algorithm is considered here.

54 **1**

55 We start with two pairs of images of a scene which were assumed to be made consecutive in some
 56 small time (see Figures 4 and 5).

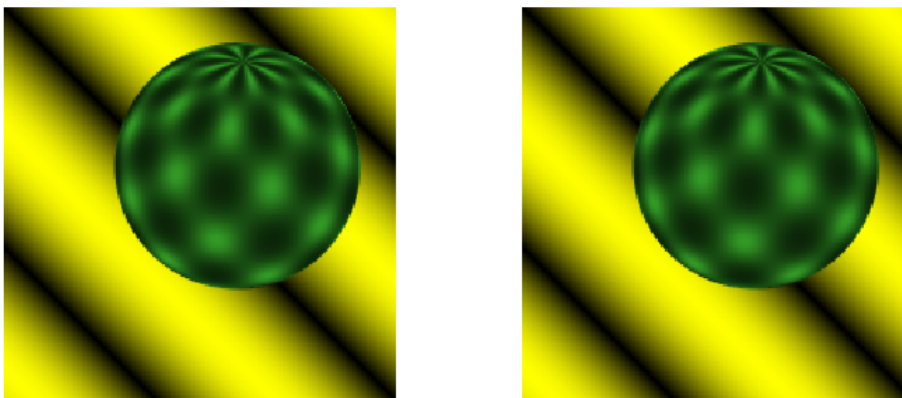


Figure 4: Pair of images: sphere1.ppm (left), sphere2.ppm (right).

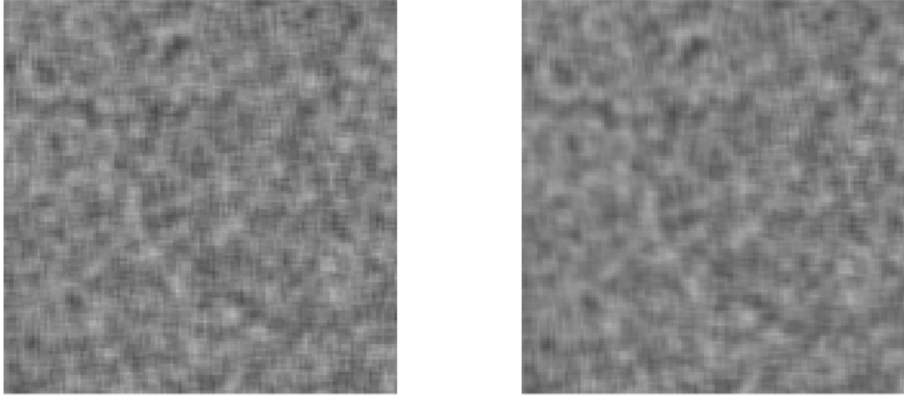


Figure 5: Pair of images: synth1.pgm (left), synth2.pgm (right).

57 We first divide the image in consecutive non-overlapping regions of a 15x15 size where we start at
 58 the top-left corner of the image. After this we will use the Lucas-Kanade algorithm to estimate the
 59 optical flow vector at the centre of each region. This means that when we estimate the optical flow at
 60 a point, the local neighbourhood of which we make use of is a square with the point at its centre.

61 2

62 We implemented a function which can estimate the optical flow at a point p with its local neighbour-
 63 hood given. In the following experiments we always let a 15x15 sized square with p as its centre
 64 be the local neighbourhood of that point. And if p is for example located near the top border of the
 65 image and the square does not fit inside the image, we move the square a bit downwards so that it
 66 just fits. After determining the local neighbourhood, we then use the Lucas-Kanade algorithm to
 67 solve the optical flow equations using least squares (where each pixel of the local neighbourhood
 68 gives rise to one linear equation). Note that when we calculate the image derivatives we convolve the
 69 image with the derivative of a gaussian. The sigma for this experiment was set to 2, and different
 70 values of sigma give different results. A bigger sigma blurs the image more, and ignores smaller
 71 details. So the value of sigma which is best should depend on what scale you want to calculate the
 72 movement, but note that this in turn also depends on the size of the local region.

73 3

74 Now choosing the set of points p as mentioned in Q1.1, we get the following results for the two pairs
 75 of images 'sphere1.ppm', 'sphere2.ppm' and 'synth1.pgm', 'synth2.pgm' (Figure 6).

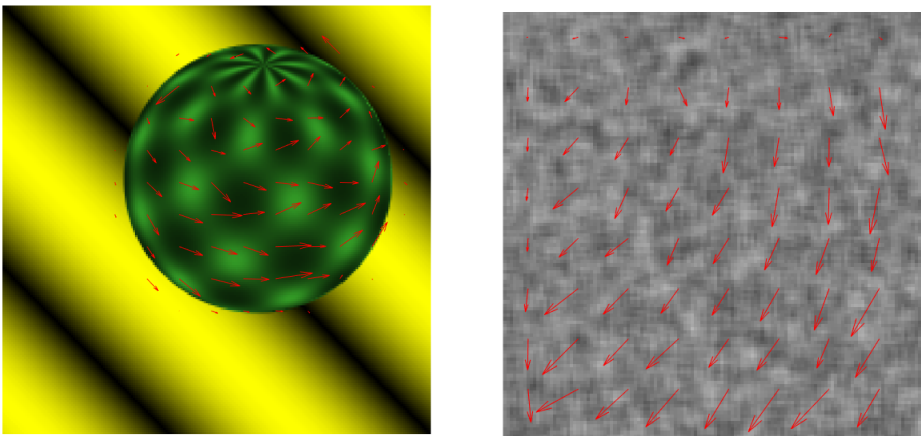


Figure 6: Estimates of optical flow vector at set of points in 'sphere1.ppm' (left) and 'synth1.pgm' (right).

76 We see that all the estimated optical flow vectors point towards the direction we expect the object
77 really moved. In regions where there is no change between the two images, the optical flow vector is
78 also indeed zero.

79 3.2 Question 2

80 1

81 There are more algorithms available than just the Lucas-Kanade algorithm. One of them is known as
82 the Horn-Schunck method. There is a big difference between those two methods, which has to do on
83 whether they operate on a local or global level.

84 The Lucas-Kanade algorithm operates on a local level. This is because when we want to estimate
85 the optical flow for a point p , the Lucas-Kanade only uses information from a local neighbourhood
86 around p . This means that the estimate for the optical flow at point p can not make use of any
87 information outside that neighbourhood.

88 In contrast, the Horn-Schunck method is a method that works on a global level. This means that to
89 estimate the optical flow at a point p , it also uses information outside a local neighbourhood around p .
90 The idea of this method is based on a smoothness constraint on the optical flow vectors. The method
91 also works in an iterative manner. The smoothness constraint says that the optical flow vectors will
92 not abruptly change but in a smooth manner. So when this method estimates an optical flow vector
93 at a point p , it does not only take the local neighbourhood into account, but also takes into account
94 that the estimation for a point p has to be similar to the optical flow vectors around it (which were
95 estimated in the previous iteration).

96 To see how this can have a big difference is answered in the next question.

97 2

98 There could be a big difference in the two methods when we look at how they work at flat regions of
99 the images which are uniform valued. To see why, consider we need to estimate the optical flow at a
100 point p which in both images lies in a uniform valued region. (i.e. We have images of a rectangle
101 bar which moved to the left. In the first image the point p is at the centre of the bar, in the second
102 (because the bar moved to the left) the point p lies more to the right of the bar.)

103 Lucas-Kanade will only makes use of local neighbourhood around the point p . It will see that in both
104 images the neighbourhood around p is identical, therefore the algorithm regards it as a zero movement.
105 So in the example of the rectangle bar: Even when we can see that the bar moved to the left, this
106 method will not notice that.

107 Horn-Schunck on the other hand, operates on a global level with a smoothness constraint and in
108 an iteration based way. So when this method is used to estimate the optical flow at a point p , it
109 does not only look at the neighbourhood around p , but also takes the optical flow of its surrounding
110 (which were estimated in previous iteration) into account. The smoothness constraint, says that the
111 optical flow can not abruptly change, meaning the optical flow at point p should not be much different
112 than the optical flow vectors estimated around p . So in the example with the rectangle it means this
113 method will estimate the optical flow at p to be towards the left. The reason is that it will estimate at
114 the bar edges to be movement towards the left, and (after some iterations) therefore because of the
115 smoothness constraint say that the optical flow at p should also be towards the left.

116 4 Feature tracking

117 4.1 Question 1

118 1

119 We now combine the Harris corner detector and the Lucas-Kanade for a feature-tracking algorithm.
120 Assuming we are given for example a sequence of video frames of some object moving, we thus like
121 the algorithm to be able to detect some features of the object and also show in which direction it
122 moves.

123 The method for N frames then can be described as follows:

124 // Choose tracked points as the points of interest of first image.

```

125 points = detect_cornerpoints(frame(1))
126
127 For k = 1 to (N-1)
128     // Only for the tracked points calculate the optical flow.
129     opt_flow_vectors = calc_opt_flow(frame(k),frame(k+1),points)
130     // Save a plot for the video
131     save_plot(frame(k),points,opt_flow_vectors)
132     // Update the location of where the point will be in next frame
133     points = update_new_positions(points,opt_flow_vectors)

```

134 2

135 To see how the feature tracking works, we have performed the algorithm on two different sequences
136 of videoframes. We have created two demos 'pingpong' and 'person_toy'. For the "pingpong" demo,
137 we see that the tracking works reasonable. For the first frame we see some distinctive corner points,
138 and almost all the optical flow vectors seem to be in the correct direction. The positions of the tracked
139 points move towards the optical flow vector and seem reasonable close to the real position of where it
140 should be. We note that for big movement the estimation is much less accurate, than when there is
141 smaller movement. This happens because one of the assumptions of the Lucas-Kanade algorithm
142 is that the movement between a pair of images is small. So if there is a sudden big movement, the
143 estimation will work worse. For the "person_toy" demo, the direction of the tracked points are also
144 mostly in the correct direction. Since this video only contains small movements, we see that at the
145 end of the video all tracked points are still quite close to the position where they should be.

146 4.2 Question 2

147 If we want to do a motion tracking given a sequence of images consecutive over time, we need to
148 know for how much the object has moved per image. Even if for each image of the sequence we can
149 detect features, that does not automatically tell us how the object has moved. This is because having
150 only the features themselves, does not necessarily give information to know where each feature went
151 to in the next image. (i.e. taking the closest feature does not necessarily work.) However, knowing
152 the optical flow might help the match of features. Since for each point of where the feature lies, the
153 estimation of the optical flow at that point will give you an estimation of where that point will move
154 to. In the next image with the new position, you can then draw the conclusion that you expect there
155 to be the feature point.

156 5 Conclusion

157 For corner detection, we implement the Harris corner algorithm. We found that as the sigma of
158 Gaussian filter increases, we have to decrease the threshold in order to make the algorithm performs
159 well. Additionally, for different images, we might need different thresholds, a good metric could be
160 multiples of the mean of Harris matrix. Harris corner detector works sufficiently to detect corners
161 in the image. Nonetheless, it gets confused with corners in the shadow. The Shi-Tomasi algorithm
162 might improve this point by comparing the minimum eigen value with the threshold. This method
163 might have its own drawback that it requires the eigen value calculation every time. While for Harris
164 corner, we could also implement the determinant and trace to compute the Harris matrix.

165 To estimate the optical flow at points we can use the Lucas-Kanade algorithm. It is based on the
166 assumptions that the movement of a local neighbourhood around the point is constant and that
167 between consecutive images is small.

168 The Harris corner detector can also be combined with a optical flow estimator to work as a feature
169 tracker. From experiments, we conclude that the biggest problem for this is that the more motion
170 there is, the less accurate the feature tracker was. Using other (global) methods (i.e. Horn-Schunck
171 method) might give more accurate performance.

172 References

173 [1] Shi, J. & Tomasi, C. (1994) Good features to track, Technical Report. Cornell University, Ithaca,
174 NY, USA.