MATH221 – Mathematics for Computer Science – Autumn 2022

Assignment One – Due Week 6 Friday 5:00pm

*Student Name:*_____ *Student Number:*_____

*Tutorial Day & Time:*_____

Question 1. [4 marks]

(a) Let p and q be statements. Write down a compound statement that uses only $\{\wedge, \vee, \sim\}$ (not necessarily all of them) and is true only when both p and q have the same truth value. Justify your answer using a truth table.

(b) Is $\sim q \Rightarrow q \wedge (p \vee \sim q)$ a tautology, fallacy or contingent statement? Justify your answer.

Question 2. [2 marks] Prove that for every natural number n, the number $4 + n + n^2$ is not prime.

Question 3. [3 marks] Using the substitution and logical equivalence laws, prove the following equivalence. Do not use a truth table.

$$p \leftrightarrow q \equiv (\sim q \vee p) \wedge (\sim p \vee q)$$

Question 4. [2 marks] Prove or disprove the validity of the following argument.

If Scott Morrison is re-elected, the majority of Australians will not be happy.
Scott Morrison is not re-elected.
Therefore, most Australians are happy.

Question 5. [4 marks] Prove by mathematical induction that $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{4n^3 - n}{3}$ for all $n \in \mathbb{N}$.