

# A Tale of Knots with Many Twists and Turns

Michał Czapliński

Advisor: Dr. Enrico Au-Yeung

## Objective

- Two players alternate moves on a fixed knot diagram
- Unknotter wins by reaching the unknot; Knotter wins by preventing it

## Setup

- Pick a knot diagram and start from a diagram where no flips have been applied

## Turn Structure

- On your turn, choose any region  $R$
- Perform a region flip; toggle every crossing that is incident to  $R$
- Play then passes to the other player

## What a Region Flip Changes

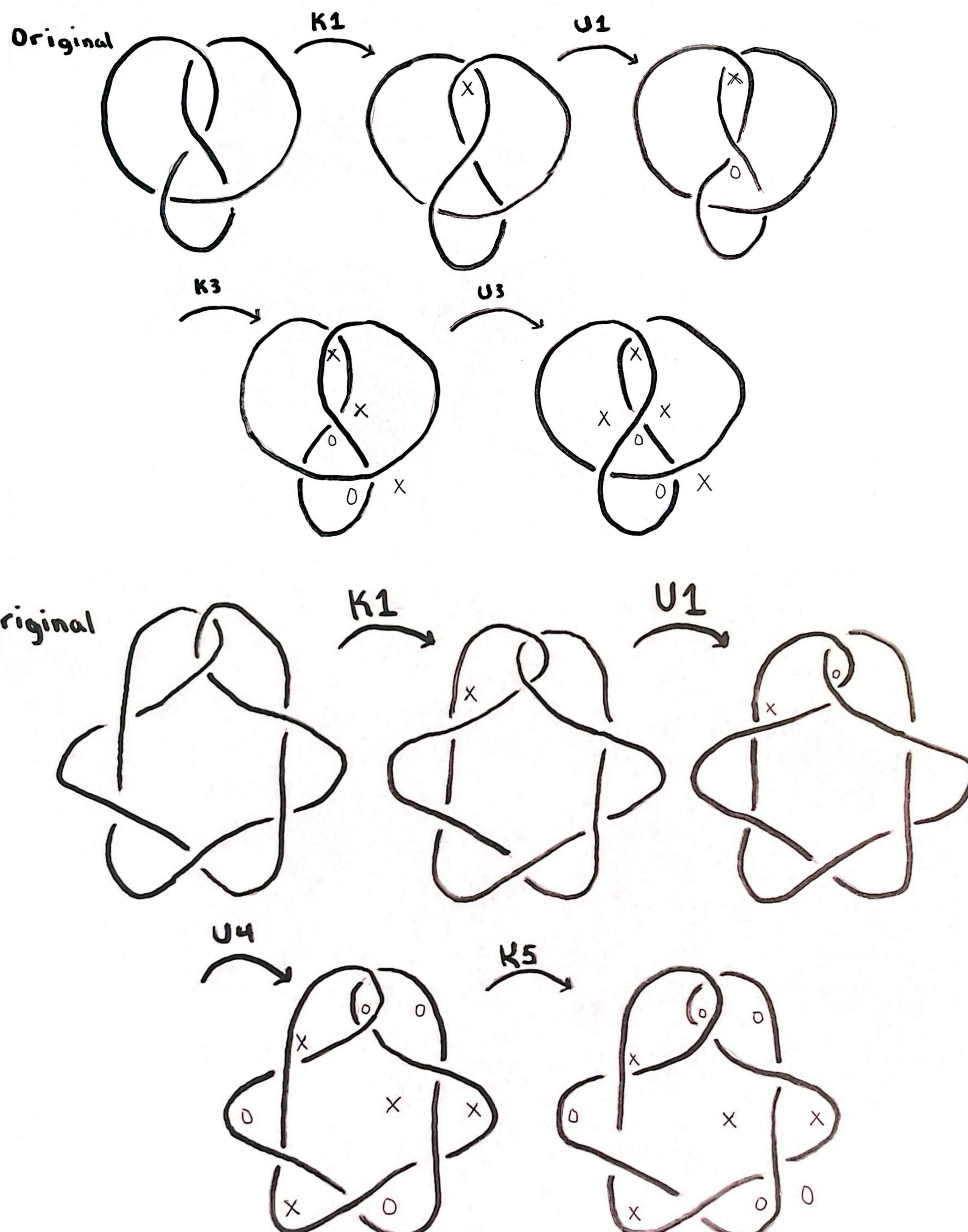
- Crossings touching  $R$  toggle (over/under switches); all others stay the same
- If a crossing is toggled multiple times, it switches each time

## Parity Principle (key idea)

- A crossing toggled an even number of times ends unchanged
- A crossing toggled an odd number of times ends unchanged
- Tracking parity helps predict if the unknot is reachable

## Victory Conditions

- Win for Unknotter when the diagram is the unknot and all regions are played
- If all regions are played and no sequence can reach the unknot, Knotter succeeds



## Core Ideas Of Strategy

- Use parity steering – make unwanted flips happen an even number of times (they cancel) and aim to end with one key crossing flipped

## For 4<sub>1</sub>

- Split Region into two groups (trios)  
Trio A = {R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>} with identity R<sub>1</sub> + R<sub>3</sub> = R<sub>5</sub>  
Trio B = {R<sub>2</sub>, R<sub>4</sub>, R<sub>6</sub>} with identity R<sub>2</sub> + R<sub>4</sub> = R<sub>6</sub>
- Precompute easy targets (flip exactly one crossing)  
e.g., a-only = R<sub>5</sub> + R<sub>6</sub>  
b-only = R<sub>4</sub> + R<sub>5</sub>  
c-only = R<sub>1</sub> + R<sub>2</sub>  
d-only = R<sub>2</sub> + R<sub>3</sub>

## For 7<sub>2</sub>

- Map Regions with Crossings  
Precompute a few one-crossing targets  
e.g., a-only = R<sub>1</sub> + R<sub>5</sub> + R<sub>6</sub> + R<sub>7</sub>  
b-only = R<sub>1</sub> + R<sub>2</sub> + R<sub>5</sub> + R<sub>6</sub> + R<sub>7</sub>
- Rule of thumb  
Whenever Knotter plays inside one of those groups, reply in the same group so the running sum moves toward one of your targets