

```
In[ ]:= << GeneralRelativityTensors`
```

```
In[ ]:= g = ToMetric[{"g", "g"}, {v, r, θ, ϕ}, {{- (1 -  $\frac{2 \text{ G M}}{r}$ ), 1, 0, 0},
      {1, 0, 0, 0}, {0, 0, r2, 0}, {0, 0, 0, r2 Sin[θ]2}}, "Greek"]
```

```
Out[ ]:=  $g_{\alpha\beta}$ 
```

```
In[ ]:= g // TensorValues // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} -1 + \frac{2 \text{ G M}}{r} & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2[\theta] \end{pmatrix}$$

```
In[ ]:= χ = ToTensor["χ", g, {1, 0, 0, 0}]
```

```
Out[ ]:=  $\chi^\alpha$ 
```

```
In[ ]:= dχ = MergeTensors[CovariantD[χ[-α], -γ], ActWith → Simplify]
```

```
Out[ ]:= ((-1) · (Γ · χ)) + (∂χ)αγ
```

```
In[ ]:= dχ // TensorValues // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & -\frac{\text{G M}}{r^2} & 0 & 0 \\ \frac{\text{G M}}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= ε = ToTensor["ε", g, Normal[LeviCivitaTensor[4]]]
```

```
Out[ ]:=  $\epsilon^{\alpha\beta\gamma\delta}$ 
```

```
In[ ]:=  $\epsilon$  // TensorValues
```

```
Out[ ]:= {{ {{ {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 0, 0}, {0, 1, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, 0}, {0, 0, 0, 0}}} },
{{ {{ {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {-1, 0, 0, 0}},
  {{0, 0, -1, 0}, {0, 0, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}} },
{{ {{ {0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}, {0, -1, 0, 0}},
  {{0, 0, 0, -1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {1, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}} },
{{ {{ {0, 0, 0, 0}, {0, 0, -1, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 1, 0}, {0, 0, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, -1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}} }
```

```
In[ ]:=  $\Omega$  = MergeTensors[ $\epsilon$ [ $\delta$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ ] (d $\chi$ [- $\alpha$ , - $\beta$ ] - d $\chi$ [- $\beta$ , - $\alpha$ ])  $\chi$ [- $\gamma$ ]]
```

```
Out[ ]:= (( (-1) · ((( (-1) · (Γ ·  $\chi$ )) + (∂ $\chi$ )) ·  $\epsilon$ ) ·  $\chi$ )) + ((( (-1) · (Γ ·  $\chi$ )) + (∂ $\chi$ )) ·  $\epsilon$ ) ·  $\chi$ ))δ
```

```
In[ ]:=  $\Omega$  // TensorValues
```

```
Out[ ]:= {0, 0, 0, 0}
```