# ICTS Graduate Course PHY-404.5: Physics of Compact Objects
# Tutorials

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## I. STELLAR STRUCTURE

1. Assuming the equations of hydrostatic, radiative and thermal equilibria, derive the following equations describing the structure of homologous stars.

$$\frac{dp}{dm} = -\frac{m}{x^4}, \quad \frac{dx}{dm} = -\frac{t^b}{x^2 p^a}, \quad \frac{dt}{dm} = -\frac{p^{an} l}{x^4 t^{3+s+bn}}, \quad \frac{dl}{dm} = A p^{a\lambda} t^{\nu - b\lambda}, \tag{1.1}$$

where the dimensionless quantities $p, x, t$ and $l$ are defined in terms of the central pressure $P_c$, central temperature $T_c$ and the scaling variables $M, R$ and $L$.

$$P = P_c p, \quad T = T_c t, \quad r = Rx, \quad L_r := L(r) = Ll, M_r := M(r) = Mm, \tag{1.2}$$

and assuming the following power-law approximations for the density $\rho$, opacity $\kappa$ and energy density density $\epsilon$:

$$\rho = \rho_0 P^a T^{-b}, \quad \kappa = \kappa_0 \rho^n T^{-s}, \quad \epsilon = \epsilon_0 \rho^\lambda T^\nu. \tag{1.3}$$

The scaling variables $M, R$ and $L$ are defined by the following conditions

$$\frac{GM^2}{4\pi R^4 P_c} = 1, \quad \frac{M}{4\pi R^3 \rho_0} \frac{T_c^b}{P_c^a} = 1, \quad \frac{3\kappa_0 \rho_0^n}{64\pi^2 ac} \frac{P_c^{an} ML}{T_c^{4+s+bn} R^4} = 1, \tag{1.4}$$

while $A$ is given by

$$A = \frac{3\kappa_0 \rho_0^{n+\lambda}}{16\pi Gac} \frac{P_c^{a(n+\lambda)+1}}{T_c^{4+s+b(n+\lambda)-\nu}}, \tag{1.5}$$

2. Numerically solve the set of equations Eq. (1.1) assuming the following power-law indices:

- *Ideal gas equation of state*: $a = b = 1, \rho_0 = \mu/\mathcal{R}$ where $\mu \simeq 0.5X^{-0.57}$ is the mean molecular weight, $\mathcal{R} = 8.3 \times 10^7$ ergs/mol/K is the gas constant and $X$ is the mass fraction of hydrogen (assume $X \simeq 0.75$).
- *Electron scattering opacity*: $n = s = 0, \kappa_0 = 0.2(1 + X)$ cm$^2$/g.
- *Energy generation by $p - p$ chain*: $\lambda = 1, \nu = 4$.

You can start by solving the dimensionless equations Eq.(1.1) by setting $A = 1$. Plot $p(m), x(m), l(m)$ and $t(m)$. Identify the value of $m$ at which the $p(m)$ goes to zero, denoted by $m_\star$; i.e. $p(m = m_\star) = 0$. Physically we require that $t(m)$ should go to zero at the same value of $m$; i.e., $t(m_\star) = 0$. This is satisfied only for a specific value of $A$, say $A_\star$. Setting $A = A_\star$ in Eq.(1.5), we can express $P_c$ fully in terms of $T_c$. This allows us to express $R, T_c$ and $L$ in terms of $M$ using Eqs.(1.4). Now, the only free parameter is $M$. Now plot the structure of a star of mass $1 M_\odot$.

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