Network Generation Models using concept of Friends of Friends

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Three Important Characteristics of real World network

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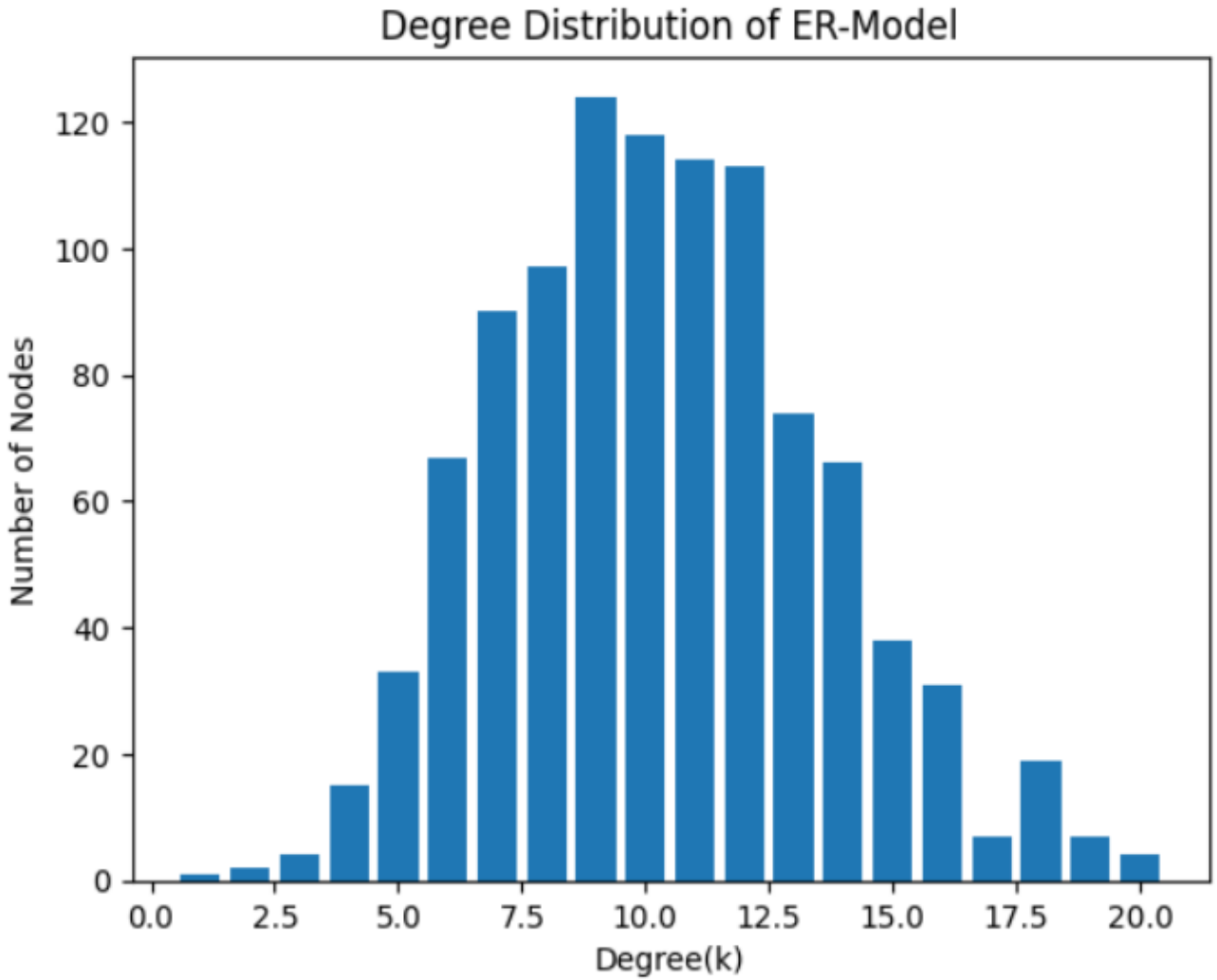
- **Small world effect:** small Diametre and radius
- **Scale free propertiy:** folow power law Degree distribution $p_k \sim k^{-\alpha}$
- **high clustering coefficient:** High link between Neighbour of nodes $C_i = \frac{2L}{k_i(k_i-1)}$

Analyze the Properties of Existing Model

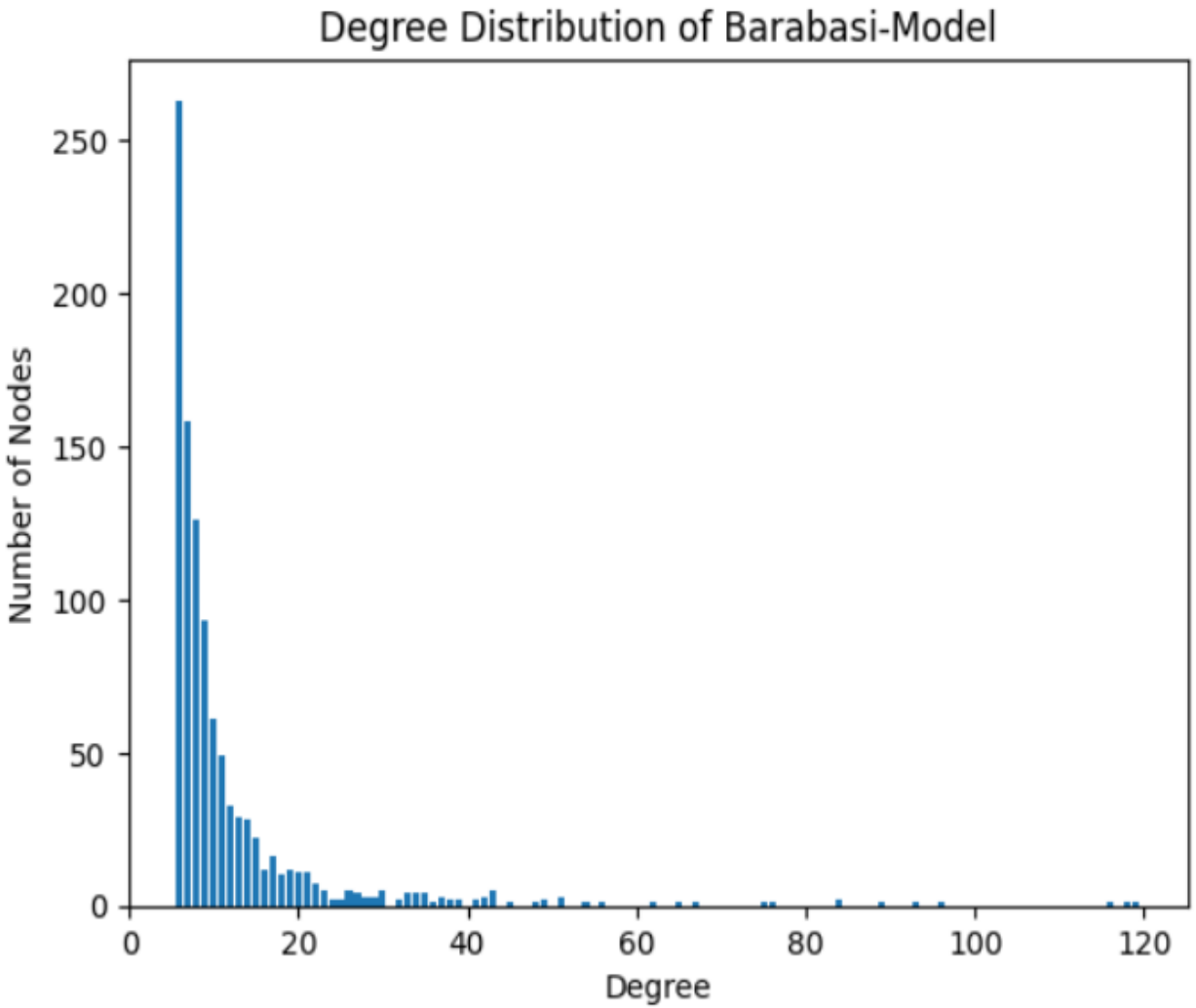
```
number of nodes : 1024
number of edges : 5291
Average degree: 10.333984375
Average clustering coefficient: 0.011224258360708035
```



ER-Model

Barabasi-Model

```
number of nodes : 1024
number of edges : 6108
Average degree: 11.9296875
Average clustering coefficient: 0.04372936342936496
```



Our Proposed Model

- Work on the basis of these two common observations ginenumerate
  1. A person makes new connection via someone he/she already knows i.e., chances to make new connection among friends of friends is more.
  2. A person does not make all of its friends at once, rather it is done in multiple steps in iterative fashion

Approach

- In first iteration, at each timestep, a new node is added to the network with one link that connects new node to any one of the existing nodes with uniform probability.
- In subsequent iterations, each node is allowed to make one new connection with friends of friends either randomly or based on the degree or based on the common neighborhood
- Repeat second step till average degree of network reaches to the average degree provided by user

1. Random Selection among friends of friends

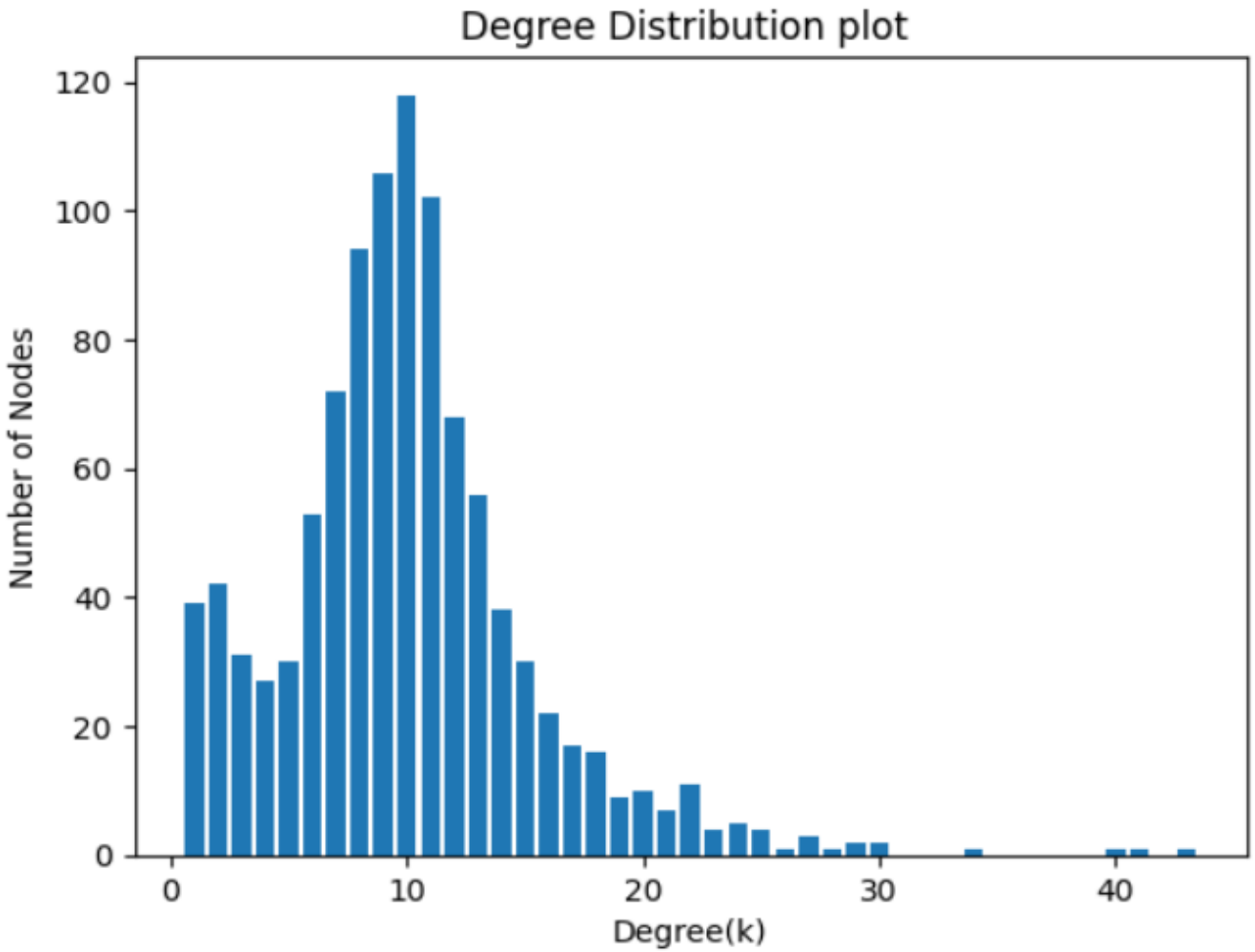
The probability for a node i to connect to a node j is given by

$$p_{ij} = \begin{cases} \frac{1}{|\Gamma_2(i)|} & \textbf{if } j \in \Gamma_2(i) \\ 0 & \textbf{otherwise} \end{cases}$$

Here, $|\Gamma_2(i)|$ is cardinality of the set of nodes, which are at distance 2 from node i

Result

```
avrage degree : 10.001953125
Number of nodes: 1024
Number of edges: 5121
Average clustering coefficient: 0.48970958293158273
Number of triangles: 7562.0
```



2. Preferential Attachment among friends of friends

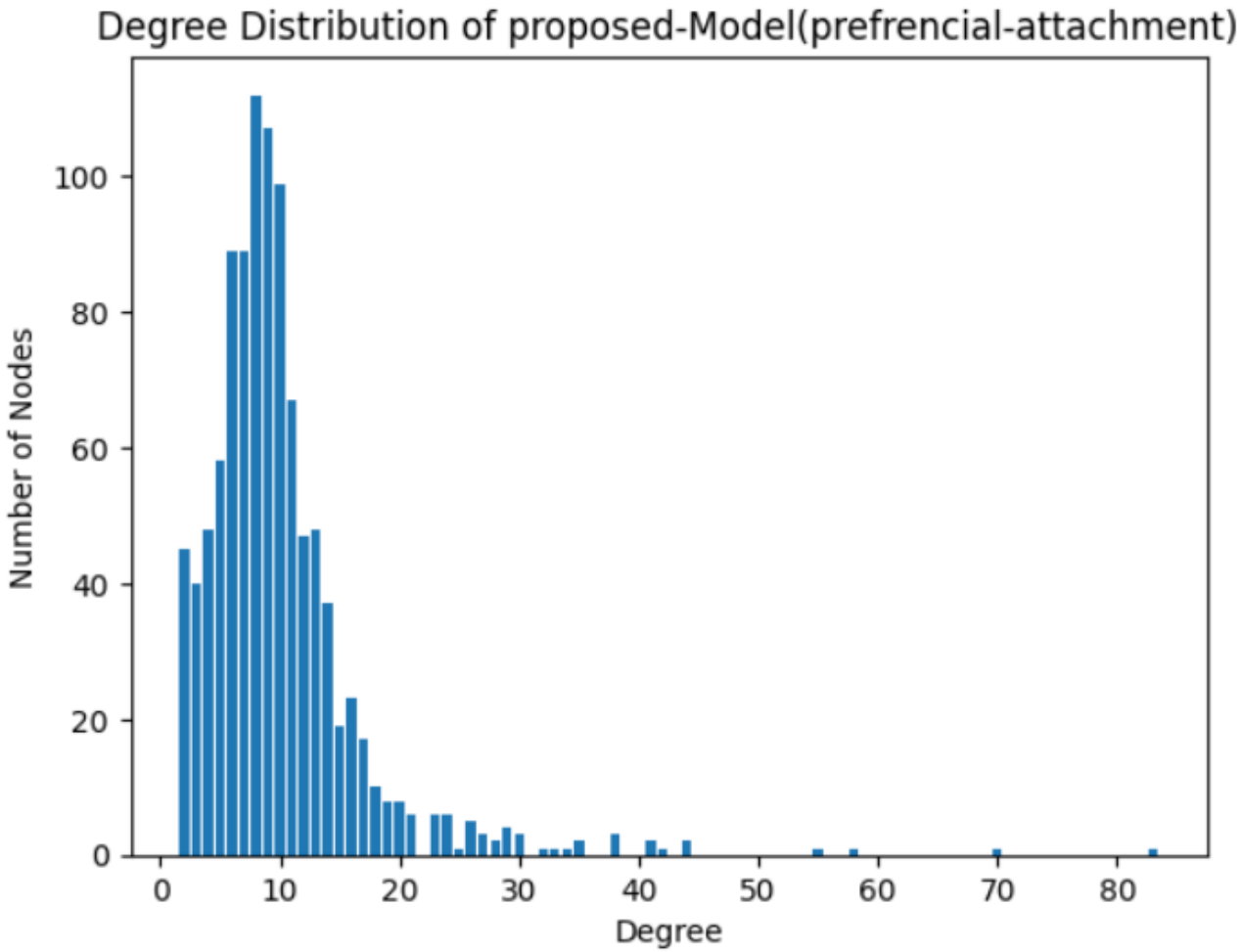
- The probability for a node i to connect to a node j is given by

$$p_{ij} = \begin{cases} \frac{|N(j)|}{\sum_{k \in \Gamma_2(i)} |N(k)|} & \forall j \in \Gamma_2(i) \\ 0 & \text{otherwise.} \end{cases}$$

Here, $|\Gamma_2(i)|$ is cardinality of the set of nodes, which are at distance 2 from node i

Result

```
average degree initionally  1.998046875
avrage degree: 10.001953125
Number of nodes:  1024
Number of edges:  5121
Average clustering coefficient: 0.5802815269981706
Number of triangles: 9723.0
```



3. Common Neighborhood Based Selection among friends of friends

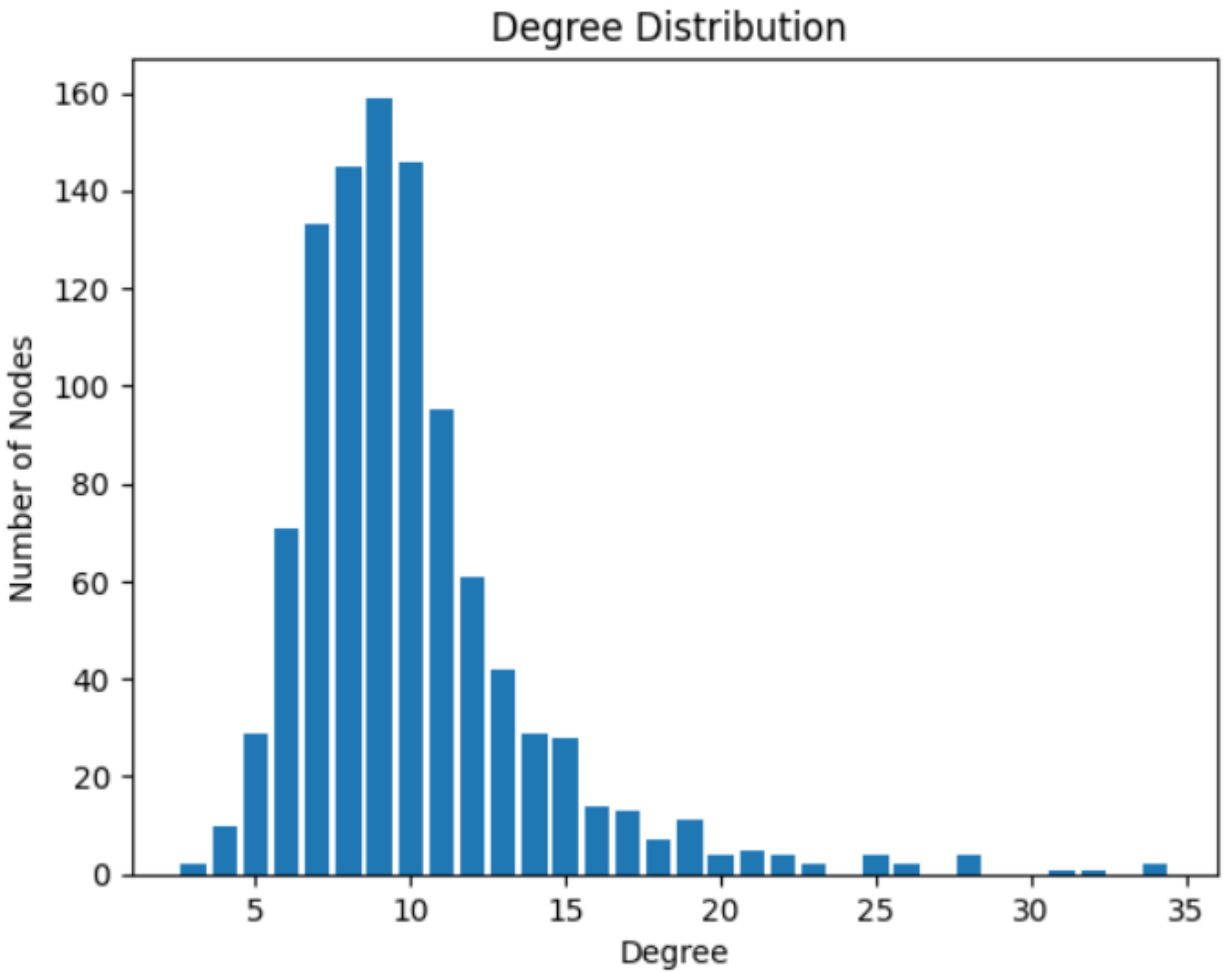
- The probability for a node i to connect to a node j is given by

$$p_{ij} = \begin{cases} \frac{|\boldsymbol{N}(i) \cap \boldsymbol{N}(j)|}{\sum_{k \in \boldsymbol{\Gamma_2}(i)} |\boldsymbol{N}(i) \cap \boldsymbol{N}(k)|} & \forall j \in \boldsymbol{\Gamma_2}(i) \\ 0 & \text{otherwise.} \end{cases}$$

Here, $|\Gamma_2(i)|$ is cardinality of the set of nodes, which are at distance 2 from node i

Result

```
avrage degree : 10.001953125
number of nodes: 1024
number of edges : 5121
Average clustering coefficient: 0.6496832071191555
Number of triangles: 9724.0
```



4. Models allowing connection among friends of friends as well as others

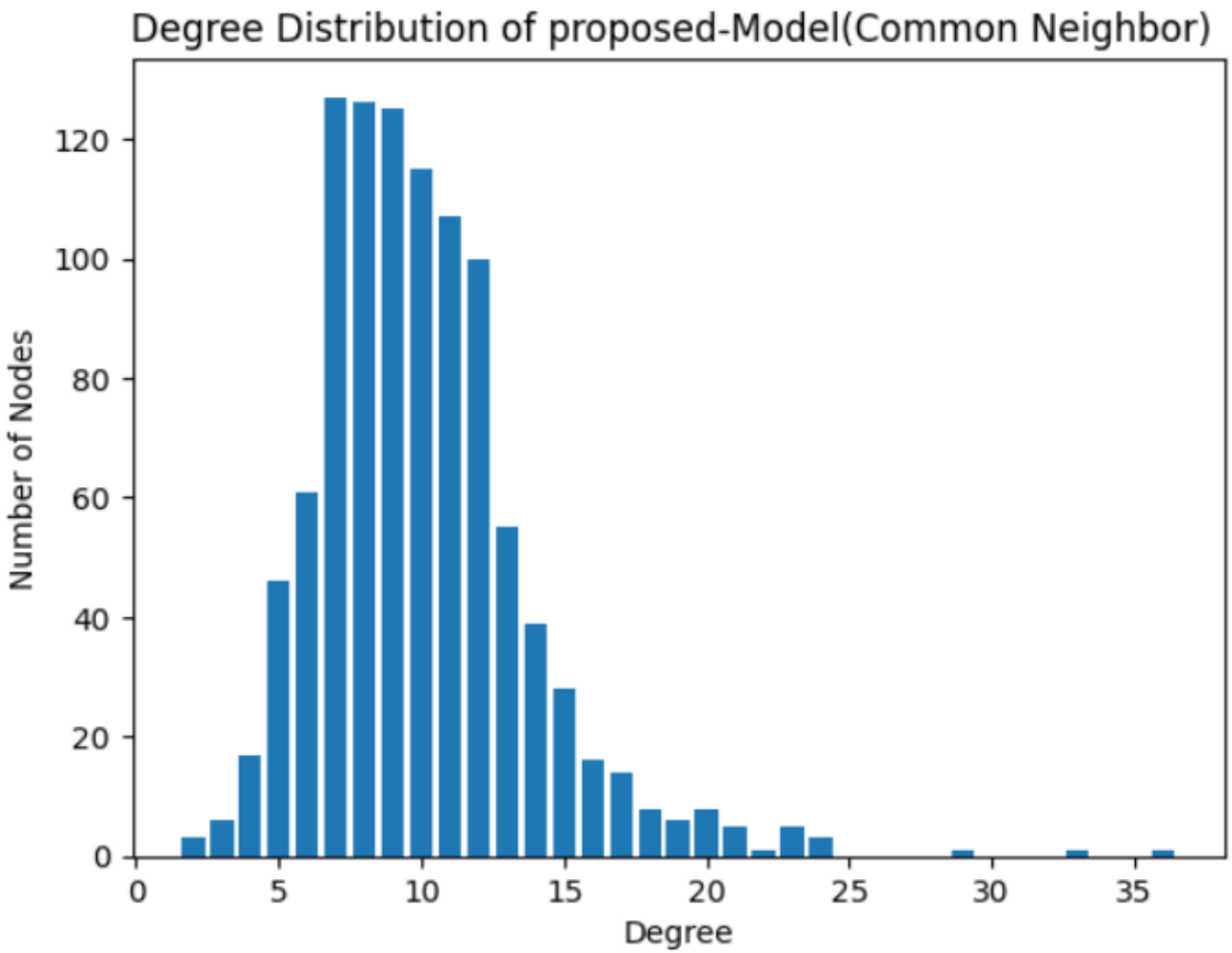
with fraction f make connection within Friends of Friends and with fraction (1-f) connect to the other nodes which are at distance more than 2

- This method can controll the clustring coefficient.
- if fraction f $\rightarrow$ 0 , this model follow almost ER-Model
- if f $\rightarrow$ 1 , this model follow Friends of Friends Model
- The probability for a node i to connect to a node j is given by

$$p_{ij} = \begin{cases} f \times \frac{|\boldsymbol{N}(i) \cap \boldsymbol{N}(j)|}{\sum_{k \in \boldsymbol{\Gamma_2}(i)} |\boldsymbol{N}(i) \cap \boldsymbol{N}(k)|} & \forall j \in \boldsymbol{\Gamma_2}(i) \\ (1-f) \times \frac{1}{\mathrm{N}-1-|\boldsymbol{N}(i) \cup \boldsymbol{\Gamma_2}(i)|} & \forall j \in \boldsymbol{S}(i) \\ 0 & \text{otherwise.} \end{cases}$$

Result

```
avrage degree 10.001953125
number of nodes: 1024
number of edges : 5121
Average clustering coefficient: 0.35113335224515657
Number of triangles: 5170.0
```



Thank You