Computed Tomography (CT)

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Introduction

- A computerized tomography (CT) scan
  - combines a series of X-ray images taken from different angles around your body and
  - uses computer processing to create cross-sectional images (slices) of the bones, blood vessels and soft tissues inside your body.
- CT scan images provide more-detailed information than plain X-rays do.

Introduction: Uses

- Diagnose muscle and bone disorders, such as bone tumors and fractures
- Pinpoint the location of a tumor, infection or blood clot
- Guide procedures such as surgery, biopsy and radiation therapy
- Detect and monitor diseases and conditions such as cancer, heart disease, lung nodules and liver masses
- Monitor the effectiveness of certain treatments, such as cancer treatment
- Detect internal injuries and internal bleeding

Introduction: Risks

- Radiation Exposure [x-rays]
- Harm to unborn babies
- Reactions to contrast material [Radiation dose]

Basic principles

- Mathematical principles of CT were first developed in 1917 by Radon
- Proved that an image of an unknown object could be produced if one had an infinite number of projections through the object

Basic Principles

- CT Acquisition
  - A single transmission measurement through the patient made by a single detector at a given moment in time is called ray.
  - A series of rays that pass through the patient at the same orientation is called projection.

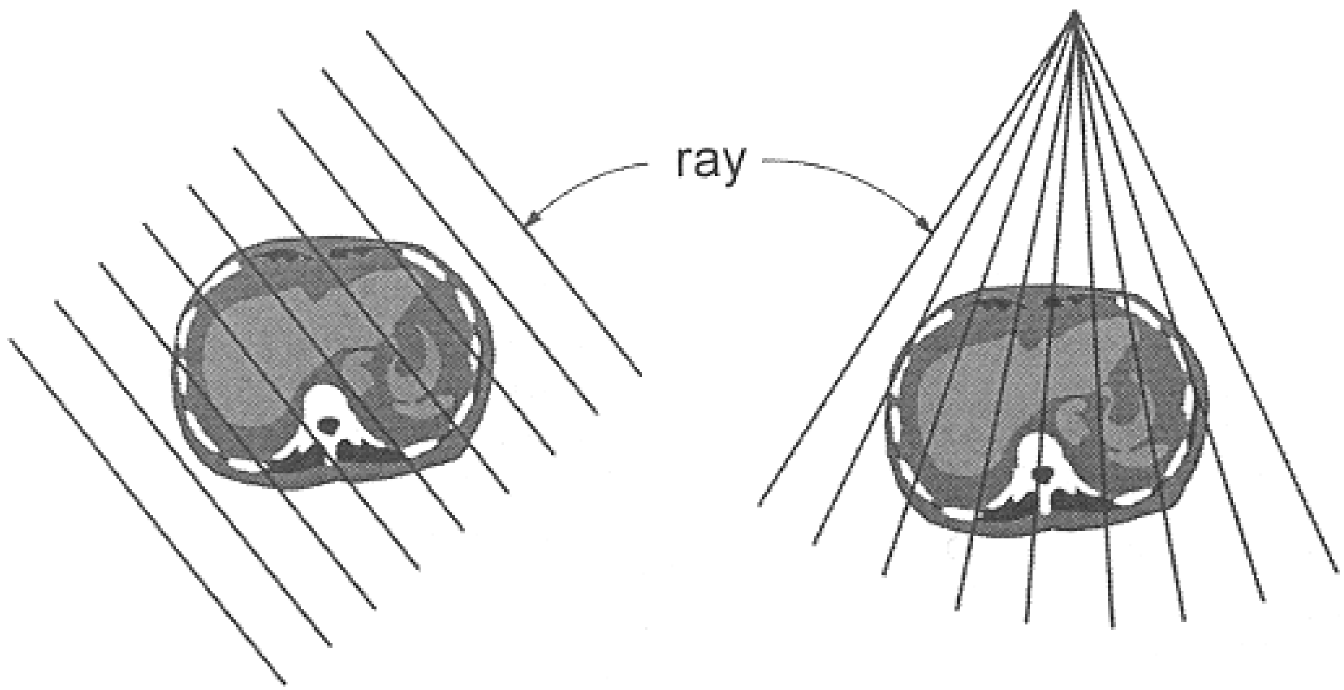
Basic principles

- With a conventional radiograph, information with respect to the dimension parallel to the x-ray beam is lost
- Limitation can be overcome, to some degree, by acquiring two images at an angle of 90 degrees to one another
- For objects that can be identified in both images, the two films provide location information

Acquisitions

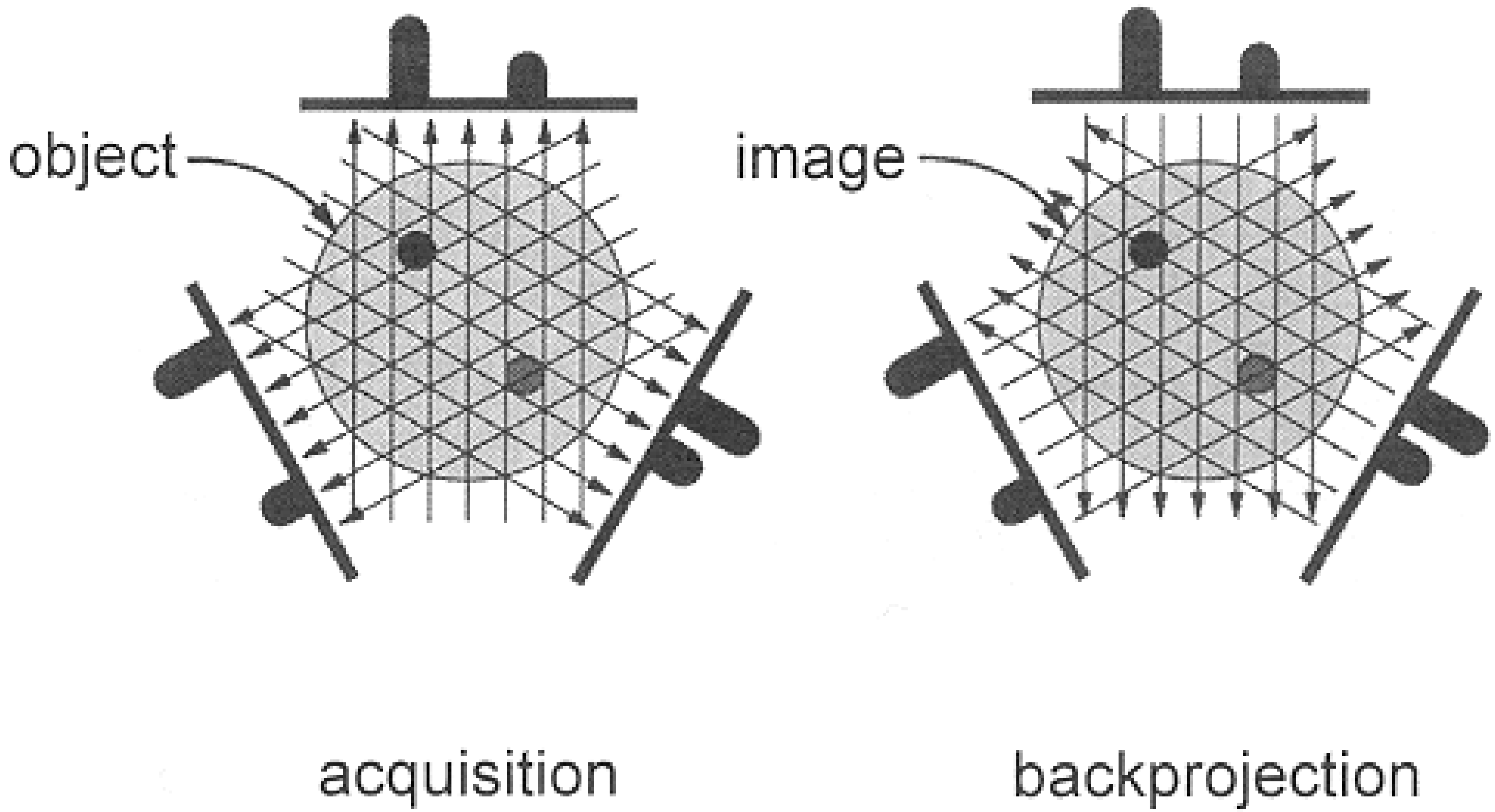
- Types of Projection
  - Parallel beam geometry- in which all of the rays in a projection are parallel to each other.
  - Fan beam geometry – the rays at a given projection angle diverge and have the appearance of a fan.
- All modern CT scanners incorporate fan beam geometry in the acquisition and reconstruction process.

Acquisitions



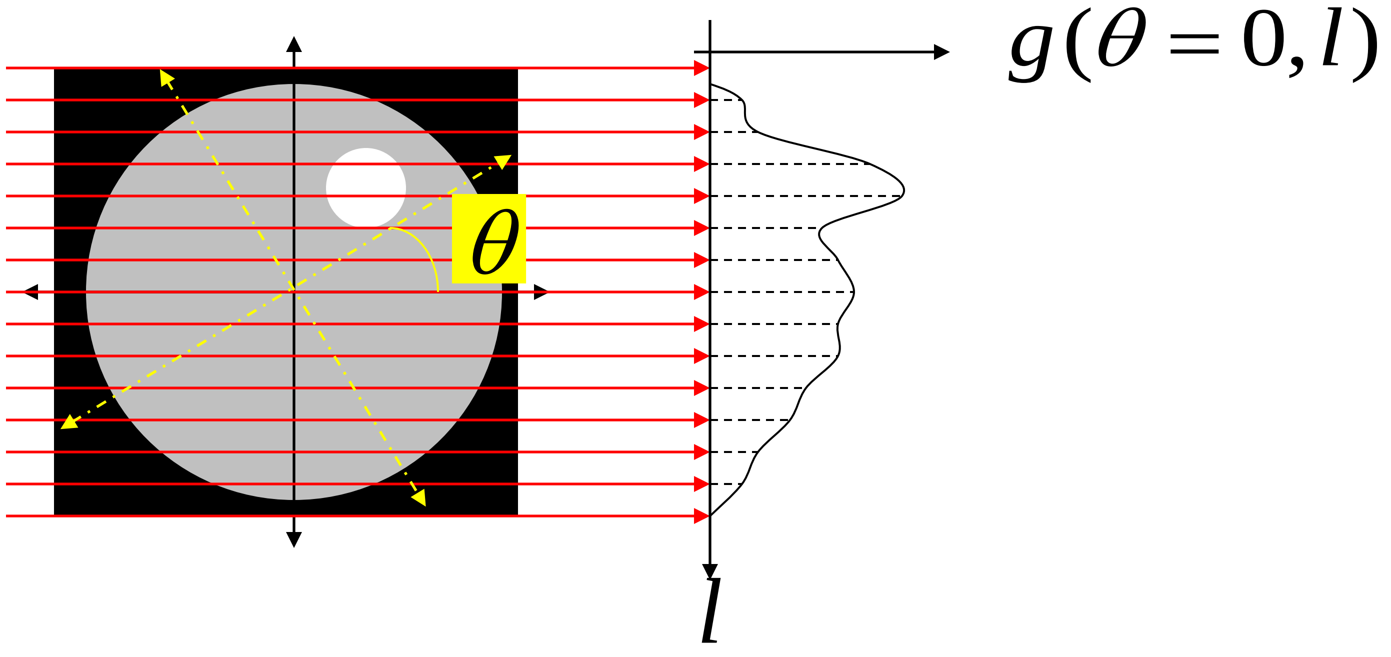
parallel beam projection

fan beam projection

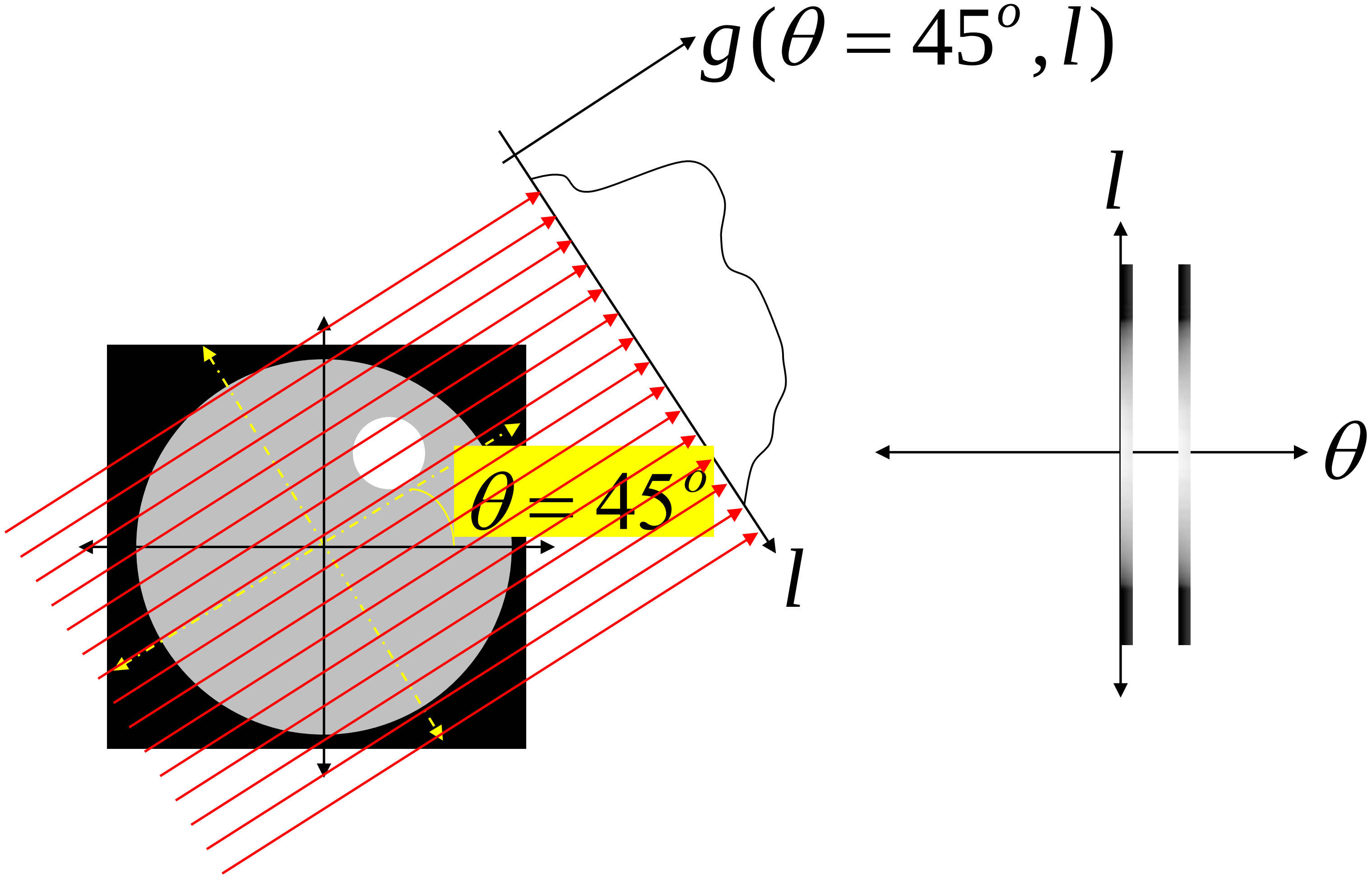


Radon -→ Projections; projections-→ backprojection → Image;

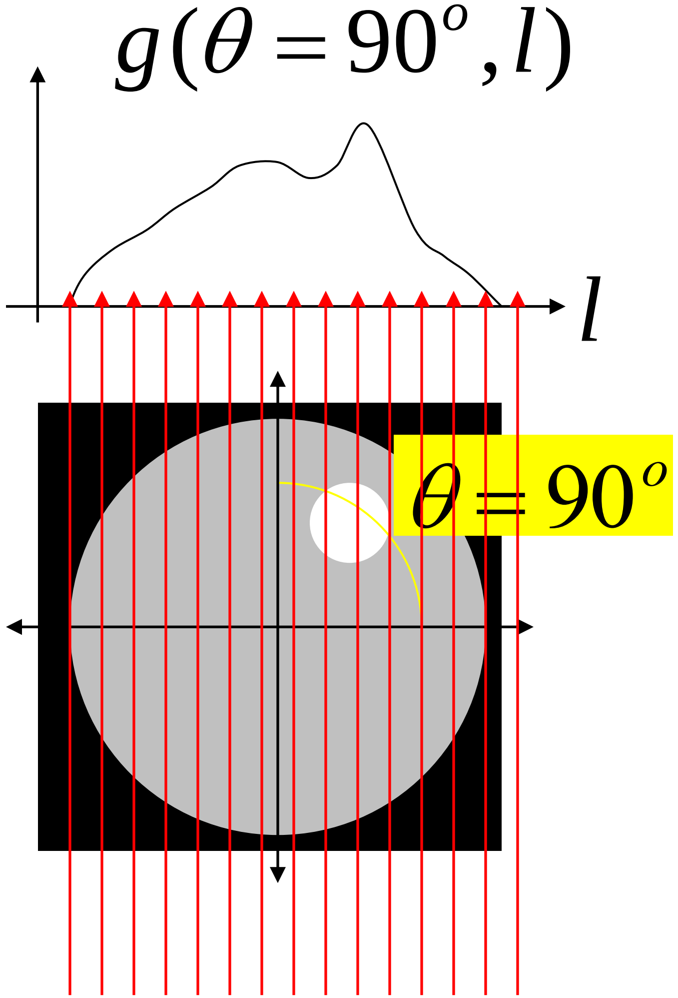
CT: First Projection

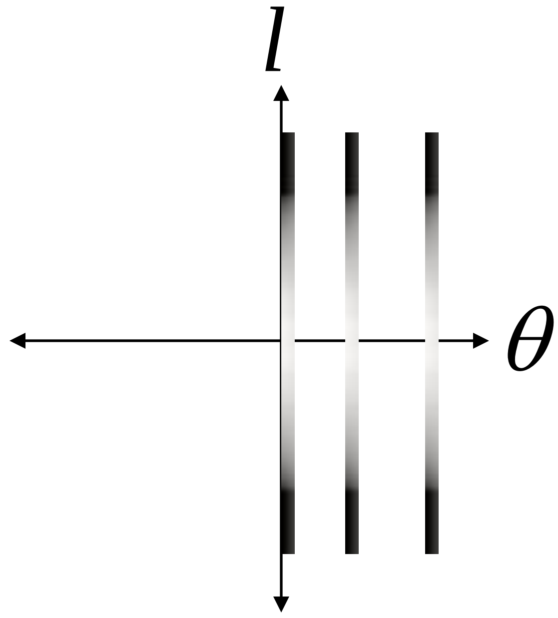


Rotate and Take Another Projection

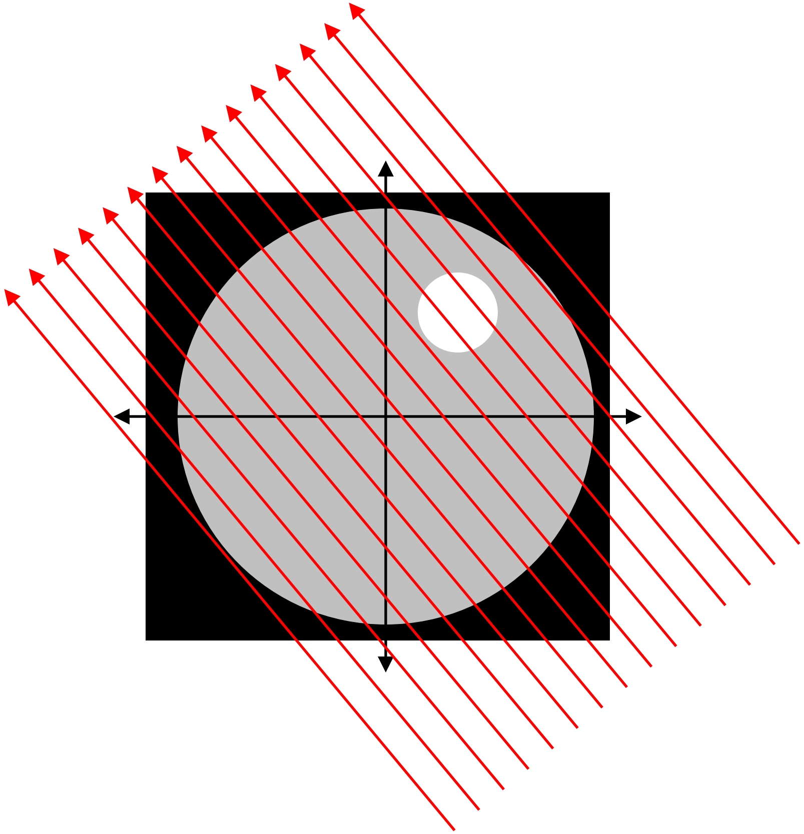


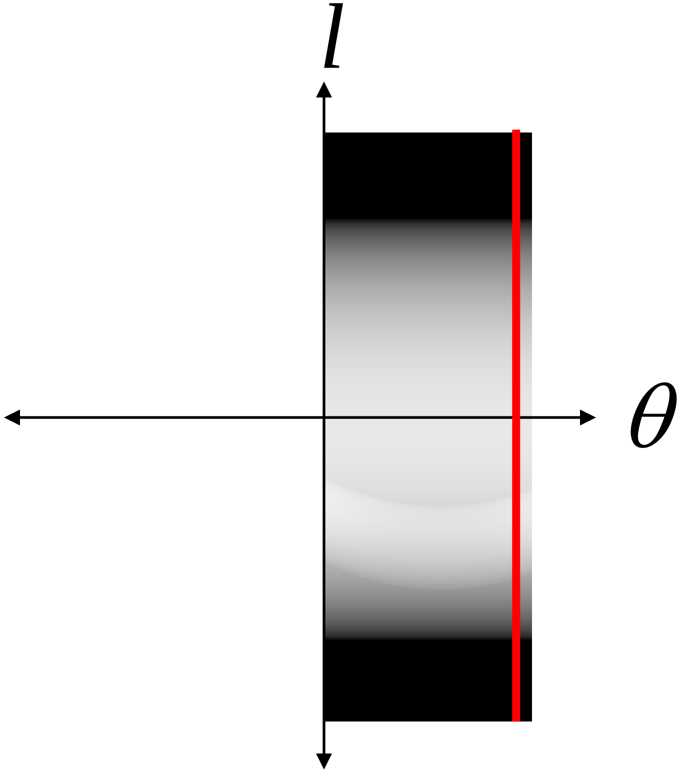
Rotate and Take Another Projection





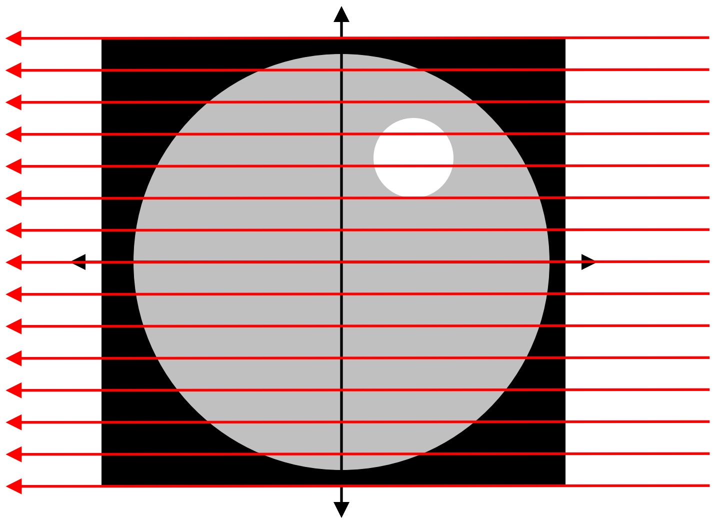
Projection: 135

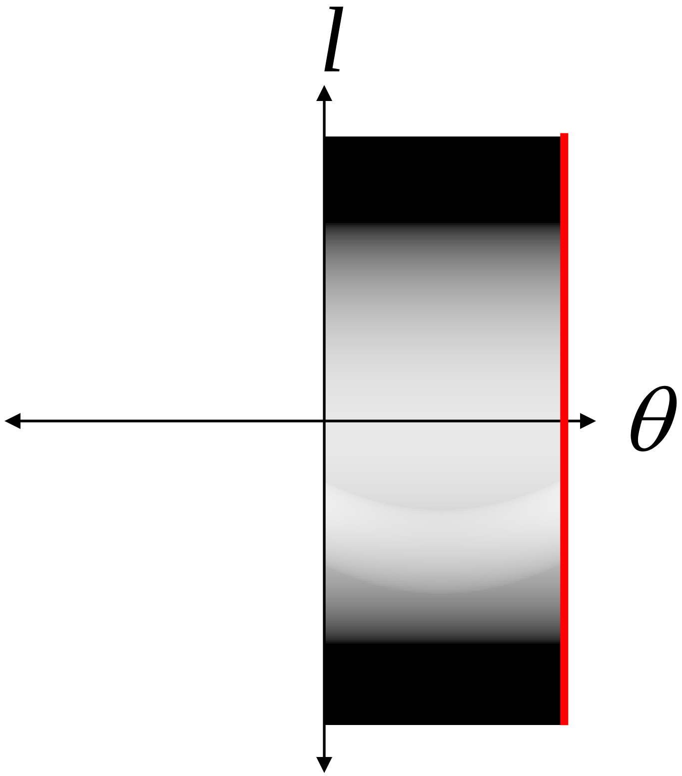




Sinogram

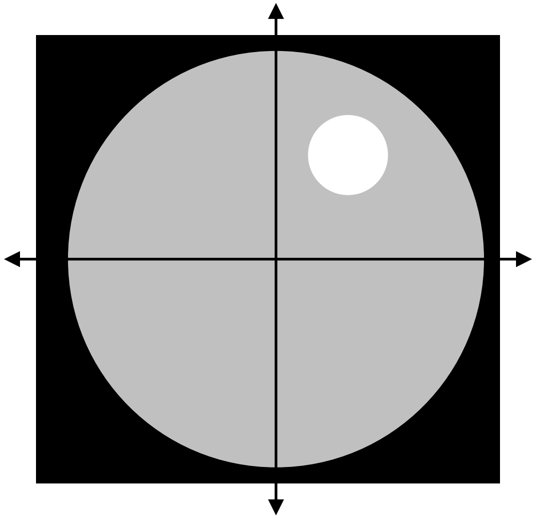
Projection: 180

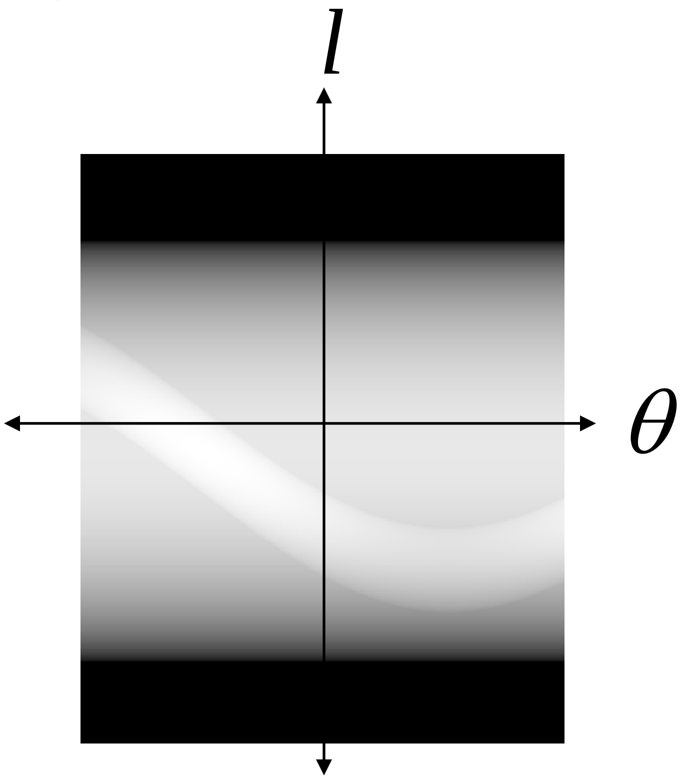




Sinogram

Projections





The sinogram is what is measured by a CT machine.

Radon Transform

- The Radon transform is the integral transform
  - which takes a function **f** defined on the plane to a function **Rf** defined on the (two-dimensional) space of lines in the plane,
  - whose value at a particular line is equal to the line integral of the function over that line.

Radon Transform

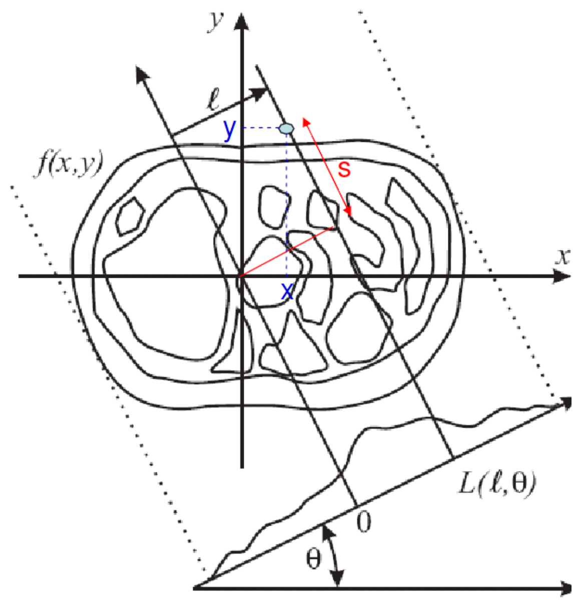
- The Radon transform data is often called a sinogram because the Radon transform of an off-center point source is a sinusoid.
- Consequently, the Radon transform of a number of small objects appears graphically as a number of blurred sine waves with different amplitudes and phases.

Radon Transform

In CT we measure $g(l,\theta)$

and need to find $f(x,y)$

$$l = x cos\theta + y sin\theta$$



We use

$$g(l,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\cdot\delta(x\cos\theta + y\sin\theta - l)dxdy$$

$$g(l,\theta) = \sum_{0}^{M-1}\sum_{0}^{N-1} f(x,y).\,\delta(x cos\theta + y sin\theta - l)$$

Radon Transform

- Projections can be computed along any angle theta (θ). In general, the Radon transform of $f(x,y)$ is the line integral of f parallel to the y´-axis.

$$R(x',\theta) = \int_{-\infty}^{\infty} f(x'cos\theta\text{-}y'sin\theta, x'sin\theta+y'cos\theta)dy'$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Projections

1	2	3	4	5
2	3	5	9	7
4	6	5	8	7
0	2	3	1	9
8	6	2	0	4

Original Image

0	0.364	0	0.182
1.875	6.9758	2.5	4.5642
13.63	9.7924	16.875	12.2374
18.38	13.7166	17.5	25.1863
18.63	22.3072	27.625	35.8877
22.75	29.2487	25.125	18.805
26.75	17.7076	14.5	7.6369
4	5.6603	1.875	1.455
0	0.2275	0	0.0455

Projections: 0, 45, 90 and 135 degree

Reconstruction

- The process of reconstruction produces the image **f** from its projection data. Reconstruction is an inverse problem
  - Back-Projection
  - Filtered Back-Projection
  - Algebraic Reconstruction Algorithm

Reconstruction: Back Projection

Step #1: Generate a complete image for each projection (e.g. for each angle θ)

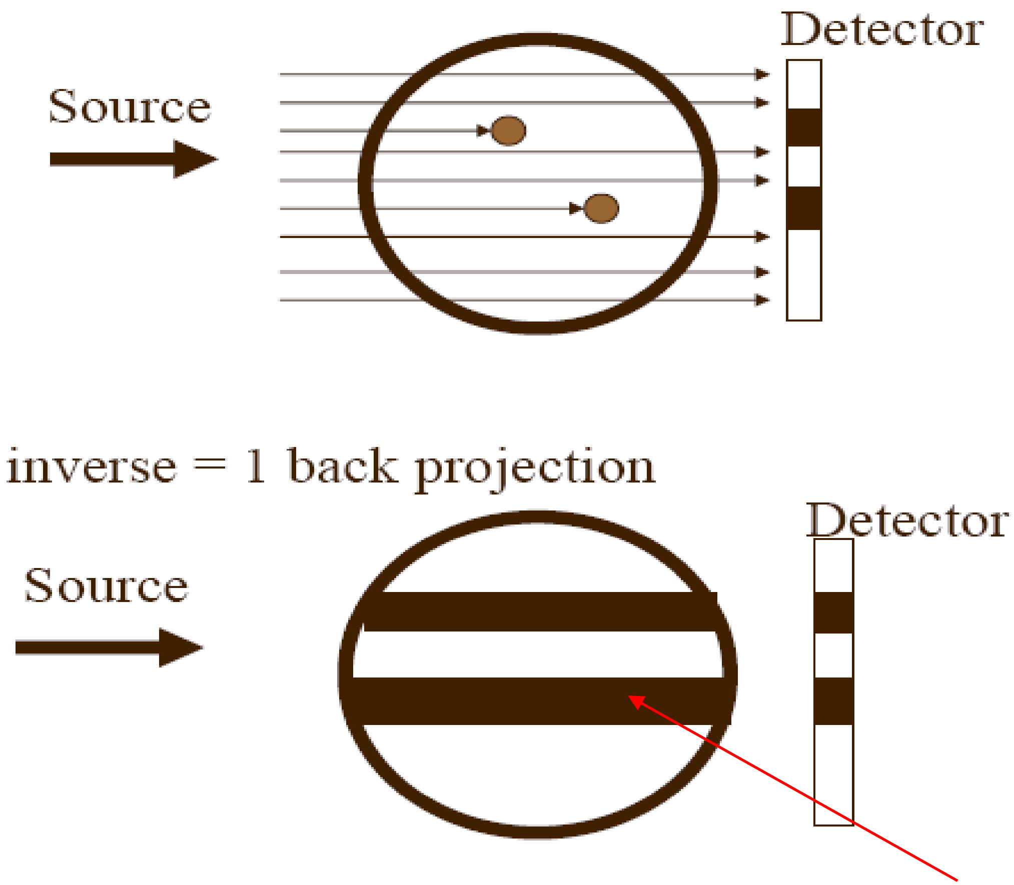
$$b_\theta(x, y) = g(x\cos(\theta) + y\sin(\theta), \theta)$$

These are called back projected images

Step #2: Add all the back projected images together

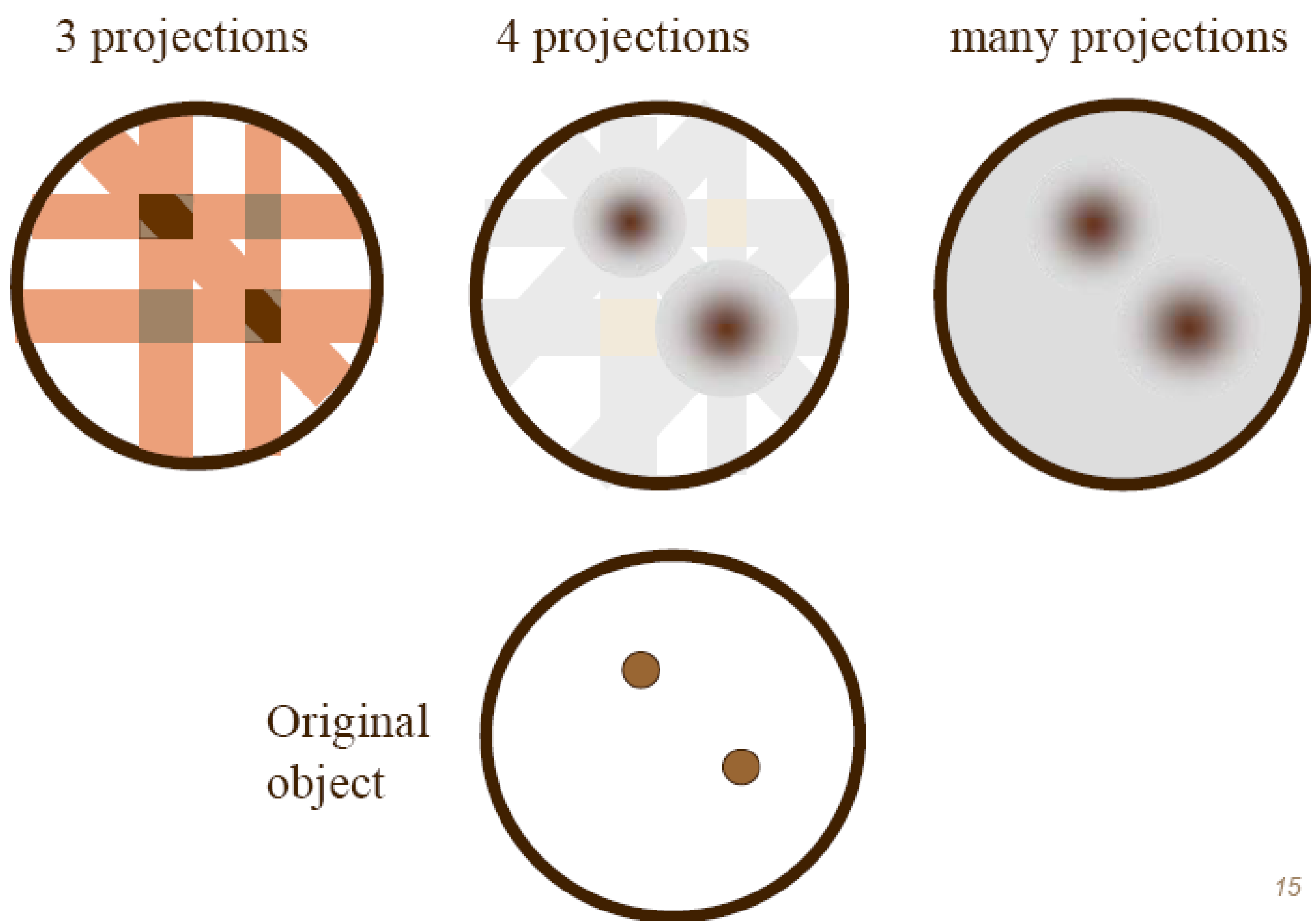
$$f_b(x, y) = \int_0^\pi b_\theta(x, y)\,d\theta$$

Reconstruction: Back projection



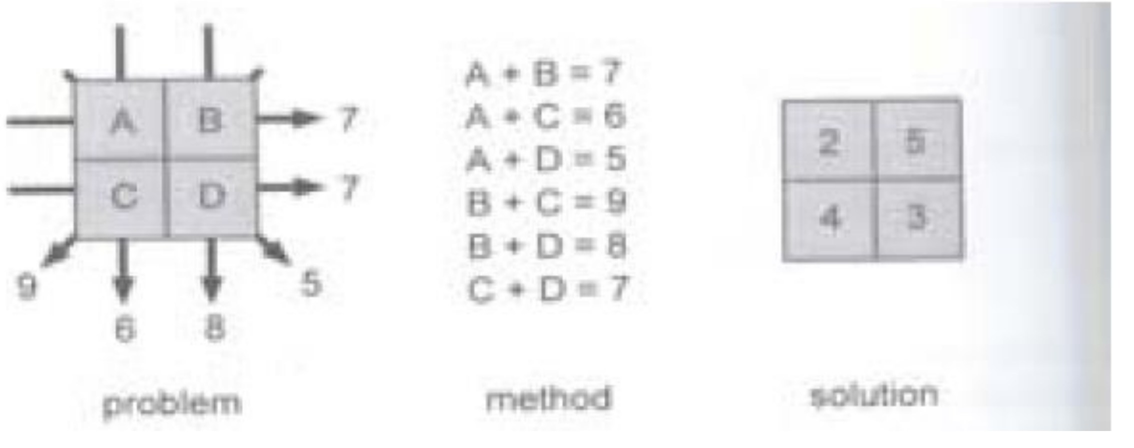
We know that objects are somewhere here in black stripes, but where?

Reconstruction: Back projection



Reconstruction: Back projection

- Consider the basic 2x2 image known only by projection values.



- Given are sums, we have to reconstruct values of pixels A, B, C and D

Reconstruction: ART

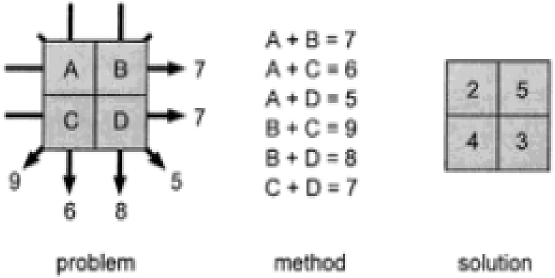


FIGURE 13-27. The mathematical problem posed by computed tomographic (CT) reconstruction is to calculate image data (the pixel values—A, B, C, and D) from the projection values (arrows). For the simple image of four pixels shown here, algebra can be used to solve for the pixel values. With the six equations shown, using substitution of equations, the solution can be determined as illustrated. For the larger images of clinical CT, algebraic solutions become unfeasible, and filtered backprojection methods are used.

$$\begin{pmatrix} 1\,1\,0\,0 \\ 1\,0\,1\,0 \\ 1\,0\,0\,1 \\ 0\,1\,1\,0 \\ 0\,1\,0\,1 \\ 0\,0\,1\,1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 5 \\ 9 \\ 8 \\ 7 \end{pmatrix}$$

Over-determined non-square matrix K
Multiply by K^T first
Invert square matrix

$$K\ x = b$$

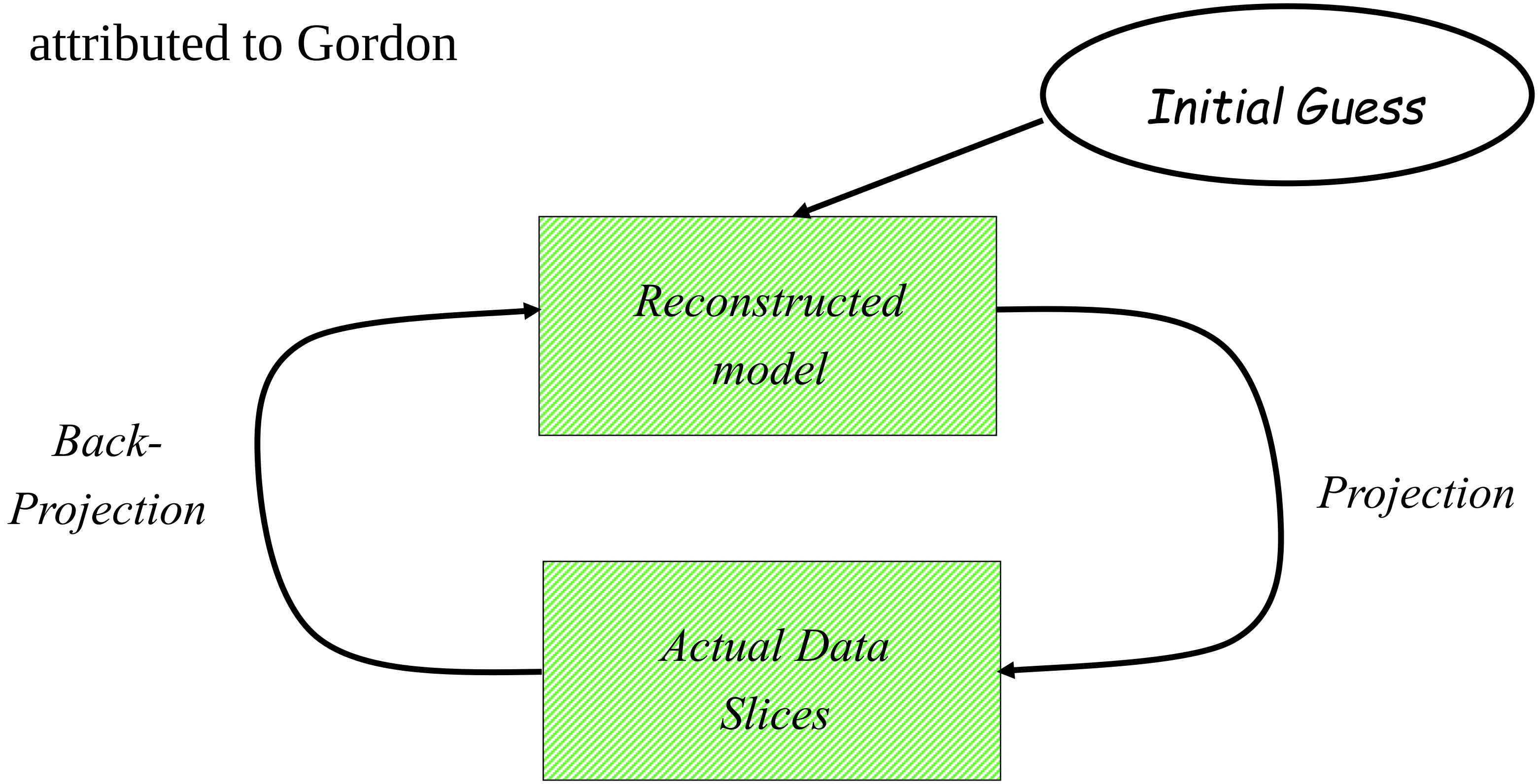
$$K^TK\ x = K^Tb$$

$$x = [K^TK]^{-1}\ K^Tb$$

Larger problems must be solved iteratively using standard methods for solving large matrix operation problems.

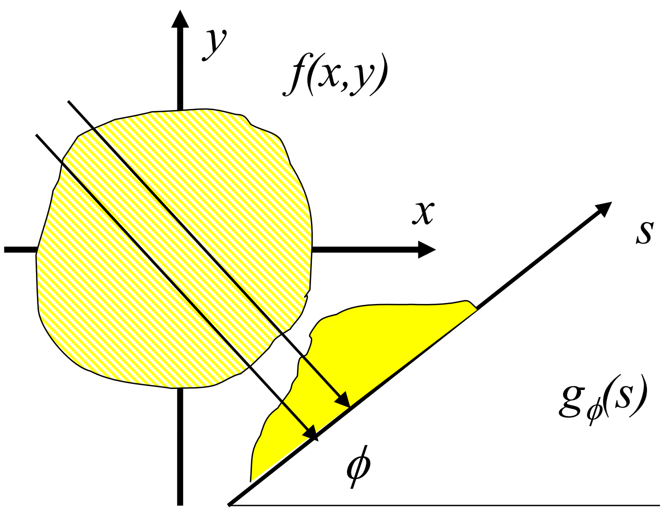
Reconstruction: ART

- METHOD: Algebraic Reconstruction Technique
  - iterative technique
  - attributed to Gordon

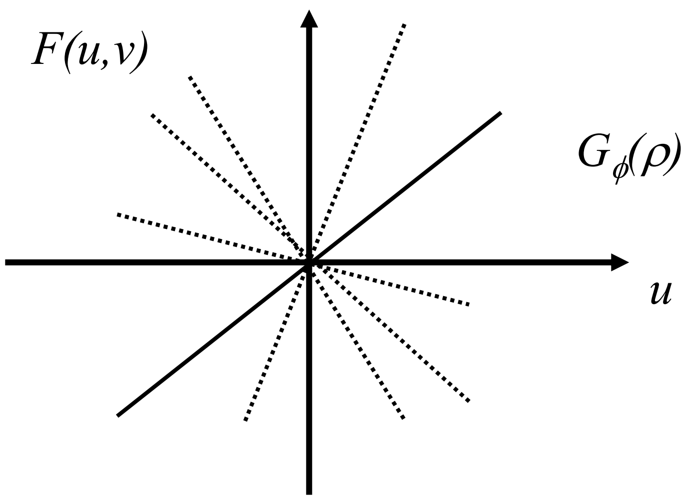


Reconstruction: Filtered Back Projection

- Method: Filtered Back Projection
  - common method
  - uses Radon transform and Fourier Slice Theorem



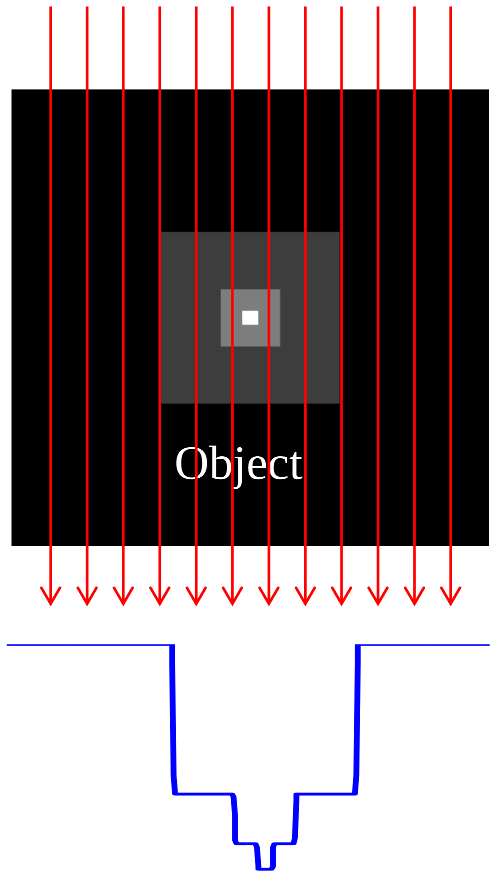
Spatial Domain

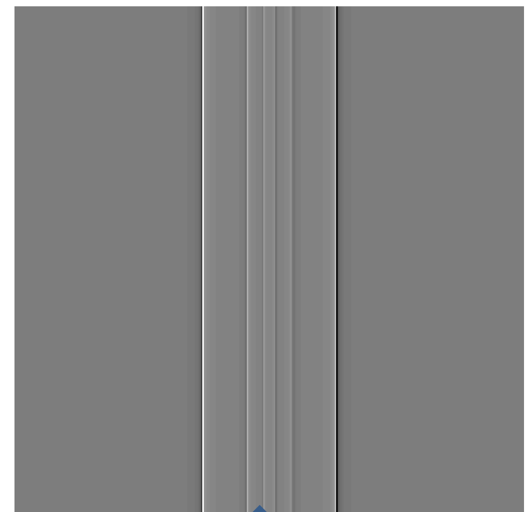


Frequency Domain

Reconstruction: Filtered Back Projection

This always works!





Digital Filter

1) take 1D FFT of projection
2) multiply by ramp filter
3) take 1D inverse FFT
4) make a back projection

Comparison: ART Vs FBP

FBP

Filtered Back Projection

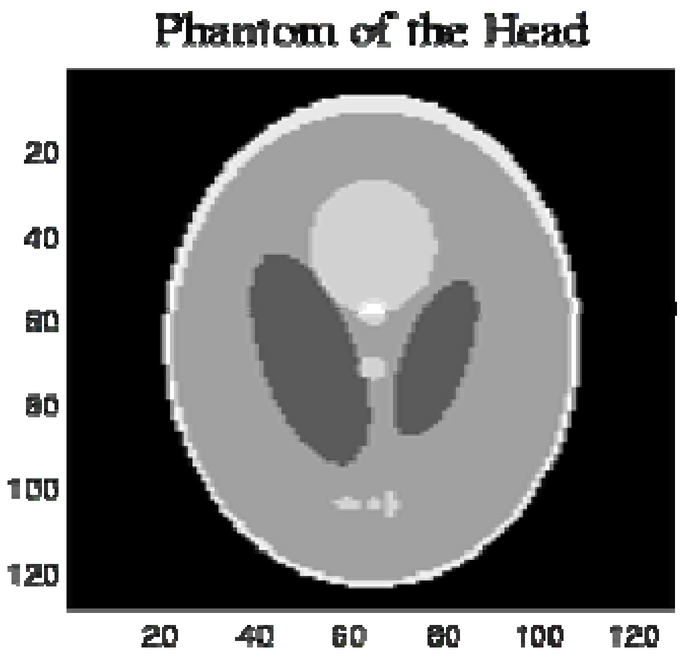
- Computationally cheap
- Clinically usually 500 projections per slice
- problematic for noisy projections

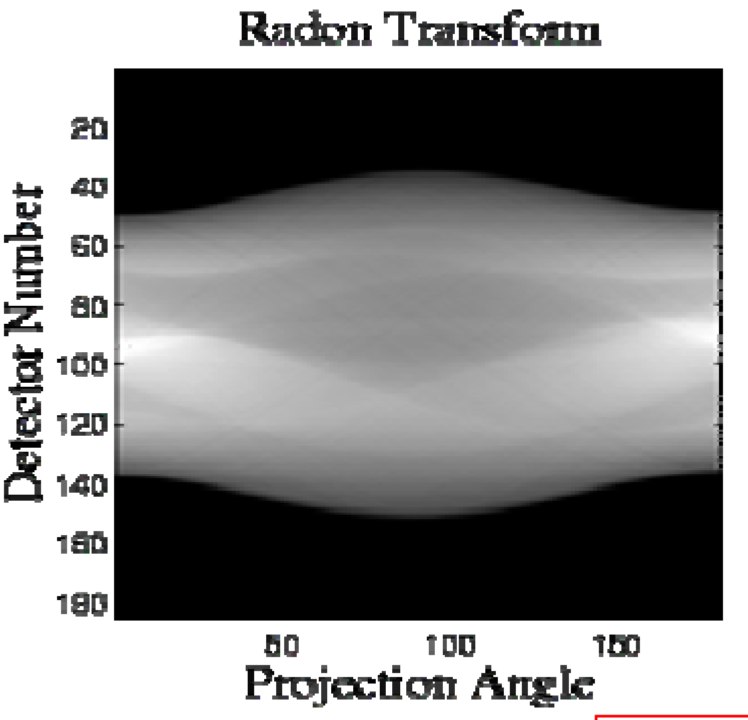
ART

Algebraic Reconstruction Technique

- Still slow
- better quality for fewer projections
- better quality for non-uniform project.
- “guided” reconstruct. (initial guess!)

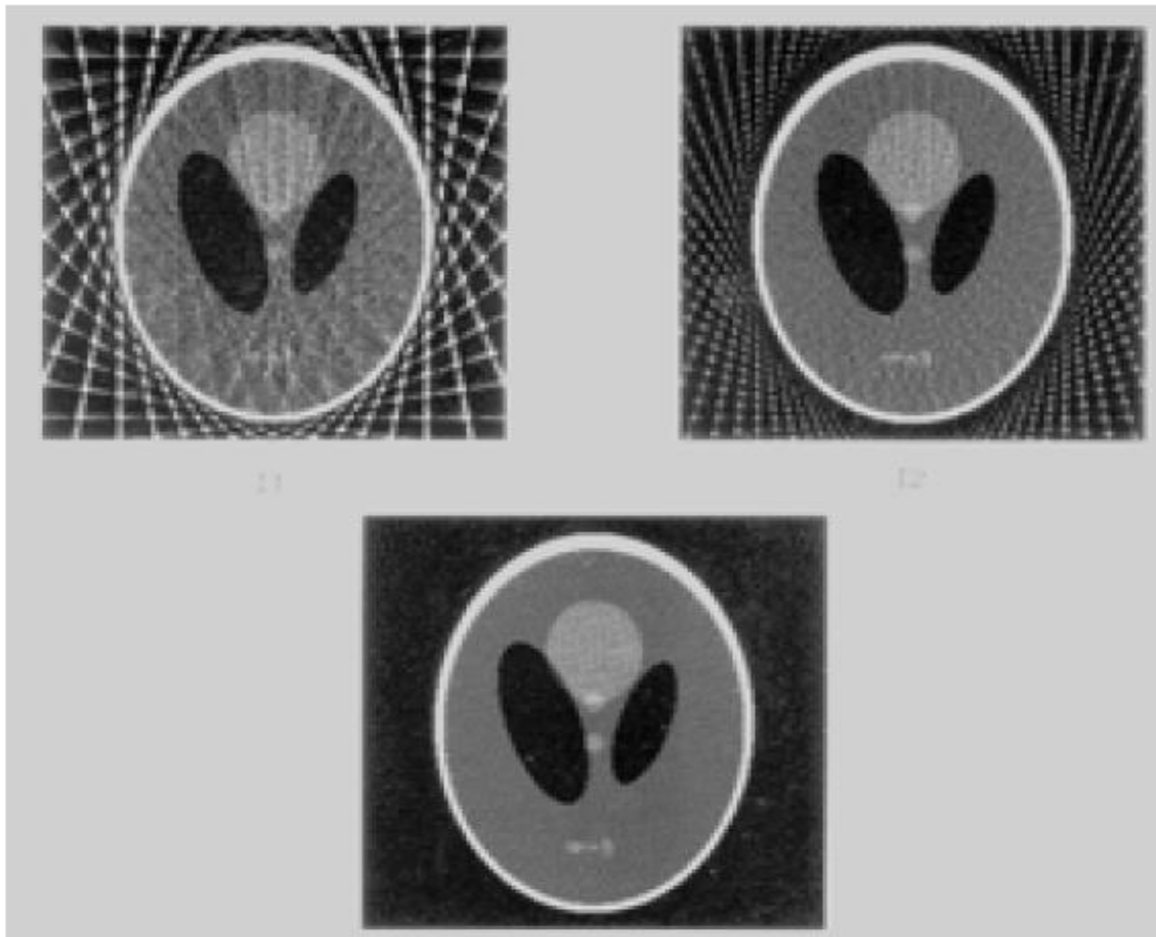
CT Reconstruction: Matlab





The Shepp-Logan head phantom and its radon transform. θ is the counterclockwise angle from the horizontal to the line on which the detector array is located.

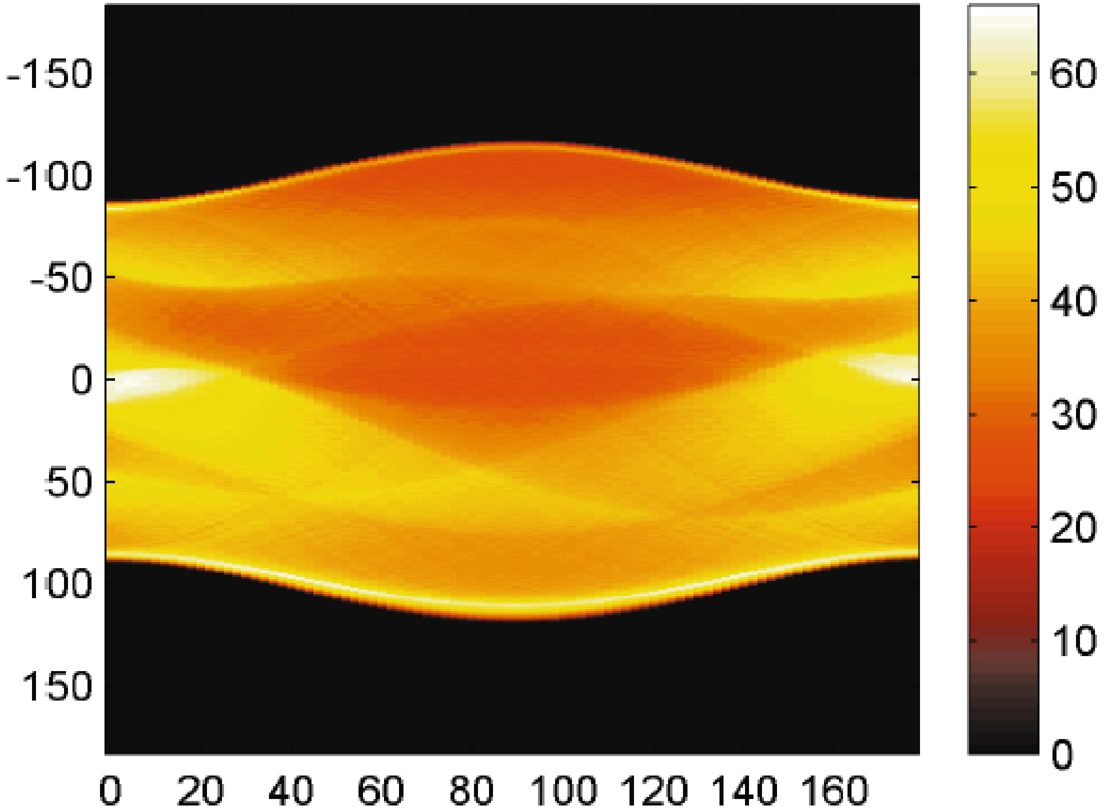
CT Reconstruction: Matlab



Reconstructing with
more and more rays

CT Reconstruction: Matlab

[Y-axis] distance, [X-axis] angle



(From Matlab Image Processing Toolbox Documentation)

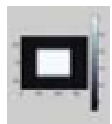
Figure 8-18: Radon Transform of Head Phantom Using 90 Projections

CT Reconstruction: Matlab

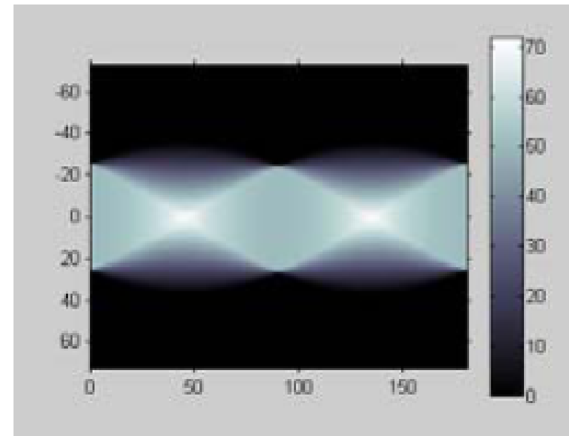
R = radon(I,theta)
If you omit theta, it defaults to 0:179.

Imaging a square

iptsetpref('ImshowAxesVisible','on')
I = zeros(100,100);
I(25:75,25:75) = 1;

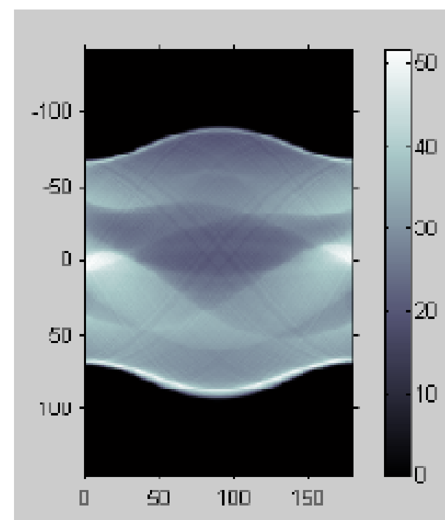


theta = 0:180;
[RI,xp] = radon(I,theta);
imshow(theta,xp,RI,[],'notruesize'), colormap(bone), colorbar



Imaging the Modified Shepp-Logan phantom

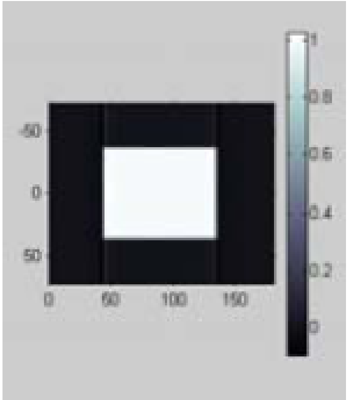
P = phantom('Modified Shepp-Logan',200); imshow(P)
[RP,xp] = radon(P,theta);
imshow(theta,xp,RP,[],'notruesize'), colormap(bone), colorbar



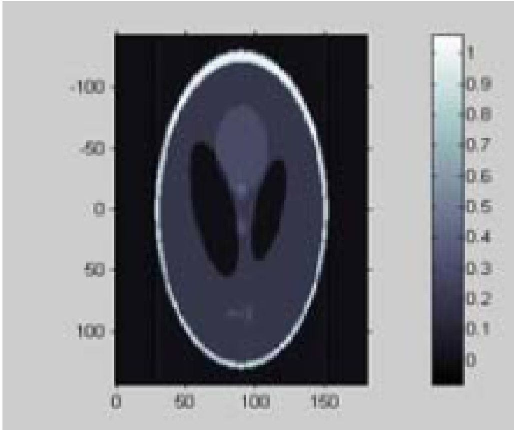
Ref: Matlab Help Guide

CT Reconstruction: Matlab

```
I = iradon(P,theta)
I = iradon(P,theta,interp,filter,d,n)
```

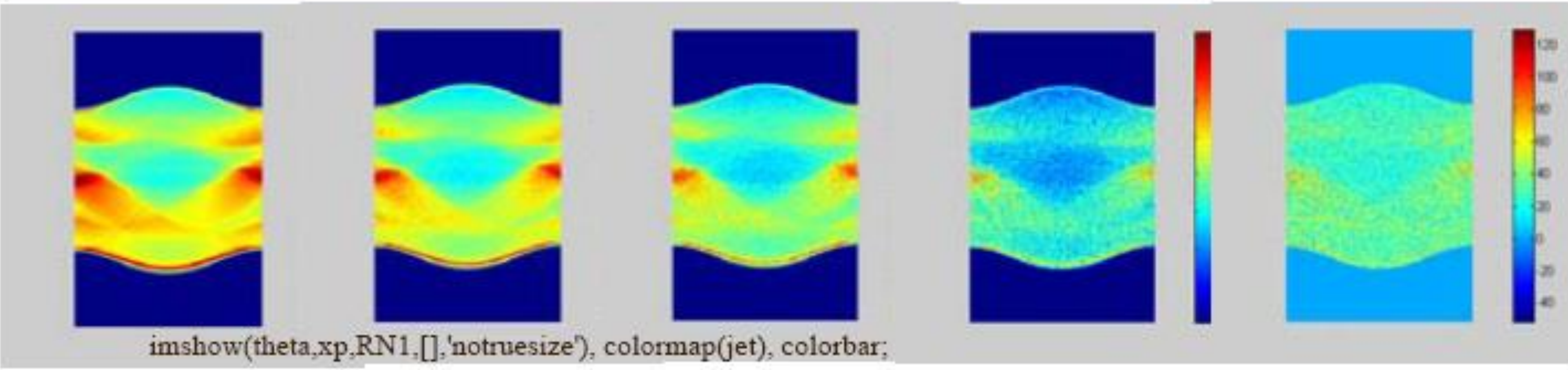


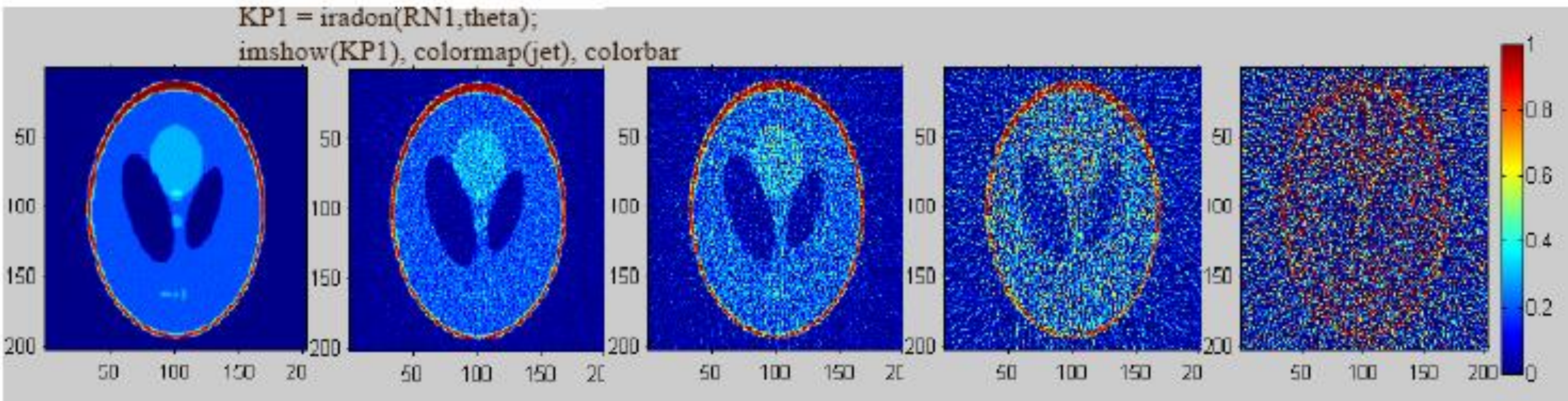

```
KI = iradon(PI,theta)
imshow(theta,xp,KI,[],'notruesize'), colormap(bone), colorbar
```




```
KP = iradon(P,theta)
imshow(theta,xp,KP,[],'notruesize'), colormap(bone), colorbar
```

CT Reconstruction: Matlab





sd=0 sd=0.05 sd=0.10 sd=0.20 sd=0.50

No noise big noise

Project

- Reconstruction from fewer projections
- Reconstruction from low dose projections

Thank you

Questions?