Compressed Sampling in Image Processing

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Compressed Sensing

- Compressive sensing (CS) theory asserts that one can recover certain signals and images from far fewer samples or measurements than traditional methods use.
- CS relies on two principle
  - Sparsity: which pertains to the signal of interest
  - In coherence: which pertains to the sensing modality

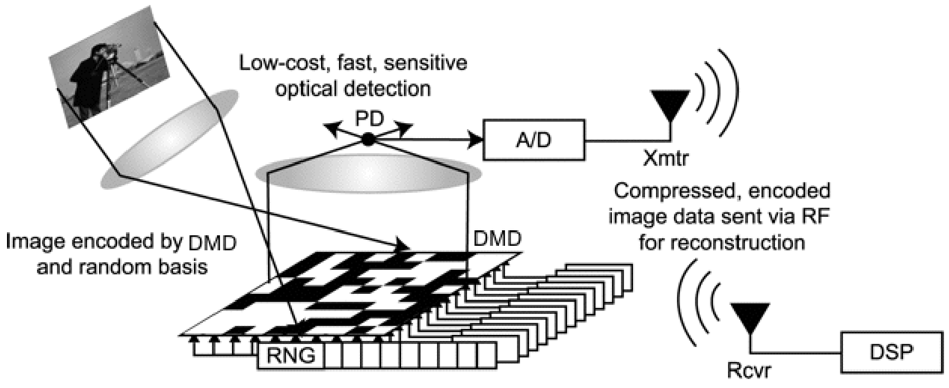
Compressed Sensing

- Signal acquisition/processing framework:
  - Want to acquire a signal $x=[x_1 \ldots x_n]$
  - Acquisition proceeds by computing Ax of dimension $m<<n$
  - From Ax we want to recover an approximation x^* of x
    - Note: x^* does not have to be k-sparse
    - Method: solve the following program:

$$\begin{aligned} &\text{minimize } \|x^*\|_1 \\ &\text{subject to } Ax^*=Ax \end{aligned}$$

Signal acquisition

- Measurement:
  - Image x reflected by a mirror a (pixels randomly off and on)
  - The reflected rays are aggregated using lens
  - The sensor receives ax
- Measurement process repeated k times → sensor receives Ax
- Now we want to recover the image from the measurements



Solving the program

- Recovery:
  - minimize $||x^*||_1$
  - subject to $Ax^*=Ax$
- This is a linear program:
  - minimize $\sum_i t_i$
  - subject to
    - $-t_i \le x^*_i \le t_i$
    - $Ax^*=Ax$

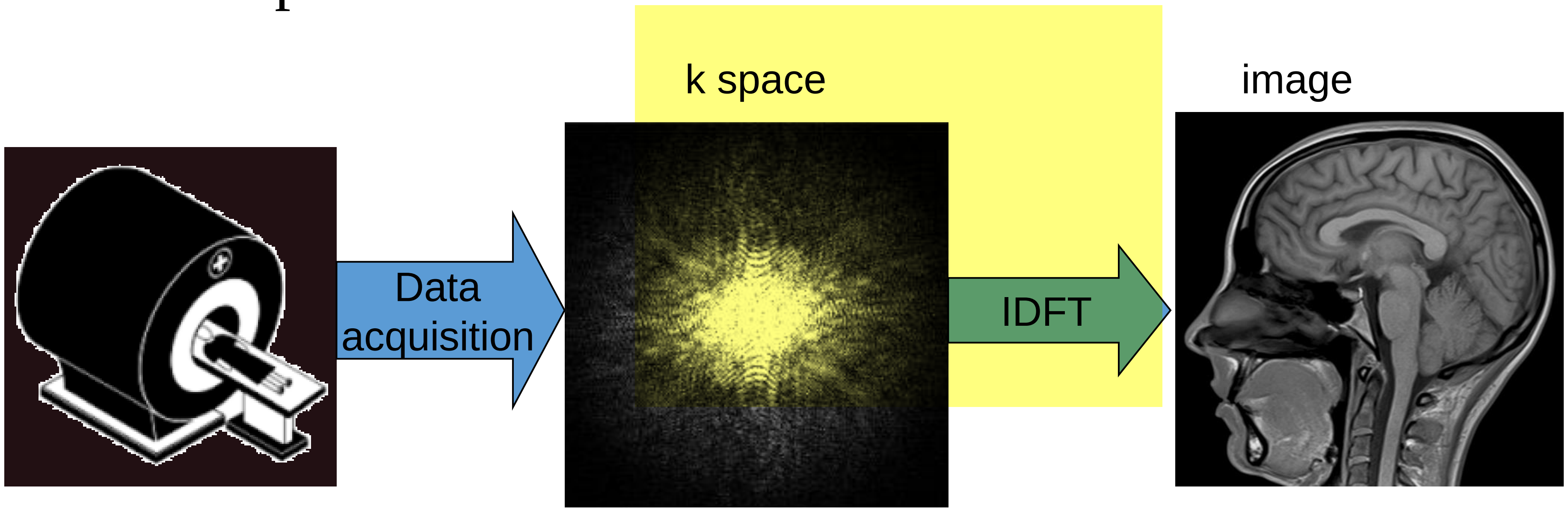
Compressed Sampling in MRI

- Compressed Sampling in MRI, while reducing acquisition time, enables high subsampling factors maintaining diagnosable image quality.

- This technique changes the goal based on three golden rules:
  1. Incoherent sub-sampling
  2. Transform sparsity
  3. Non-linear iterative reconstruction technique

Introduction

- MRI Principle

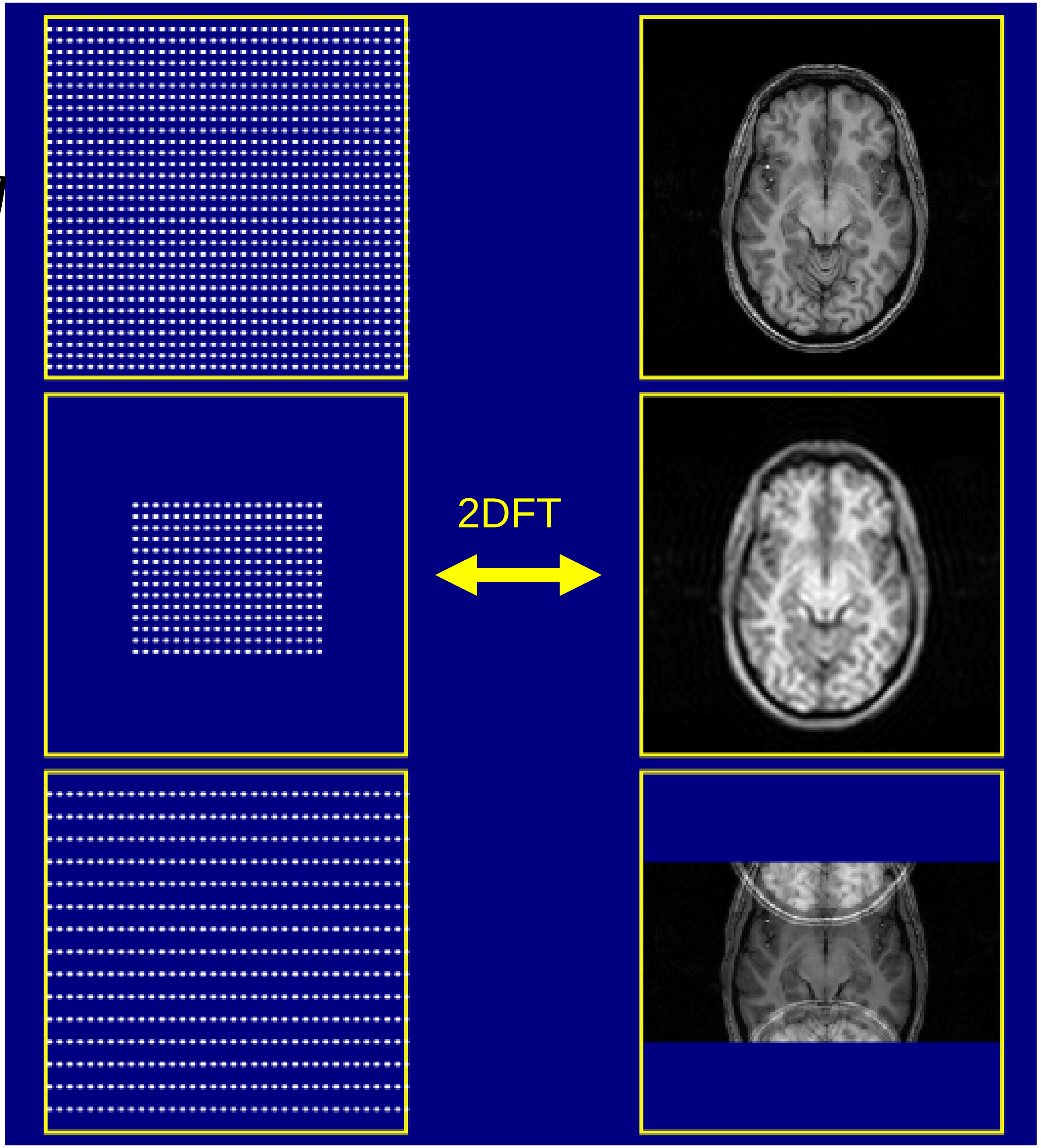


k-space | *Image*

Full sampling | *Full-FOV, high-res*

Reduce kmax | *Full-FOV, low-res: blurred*

Increase Δk | *Low-FOV, high-res: may be aliased*



Introduction

- Inherent slow data collection
  - Limits spatial resolution
  - Limits temporal resolution
  - Introduces artifacts in image
- Moreover, slow acquisition is uncomfortable for patients, especially-
  - Who are anxious
  - Who can not keep still or motionless
  - Who have limited breath-hold capacity, and
  - Who are uncooperative such as children

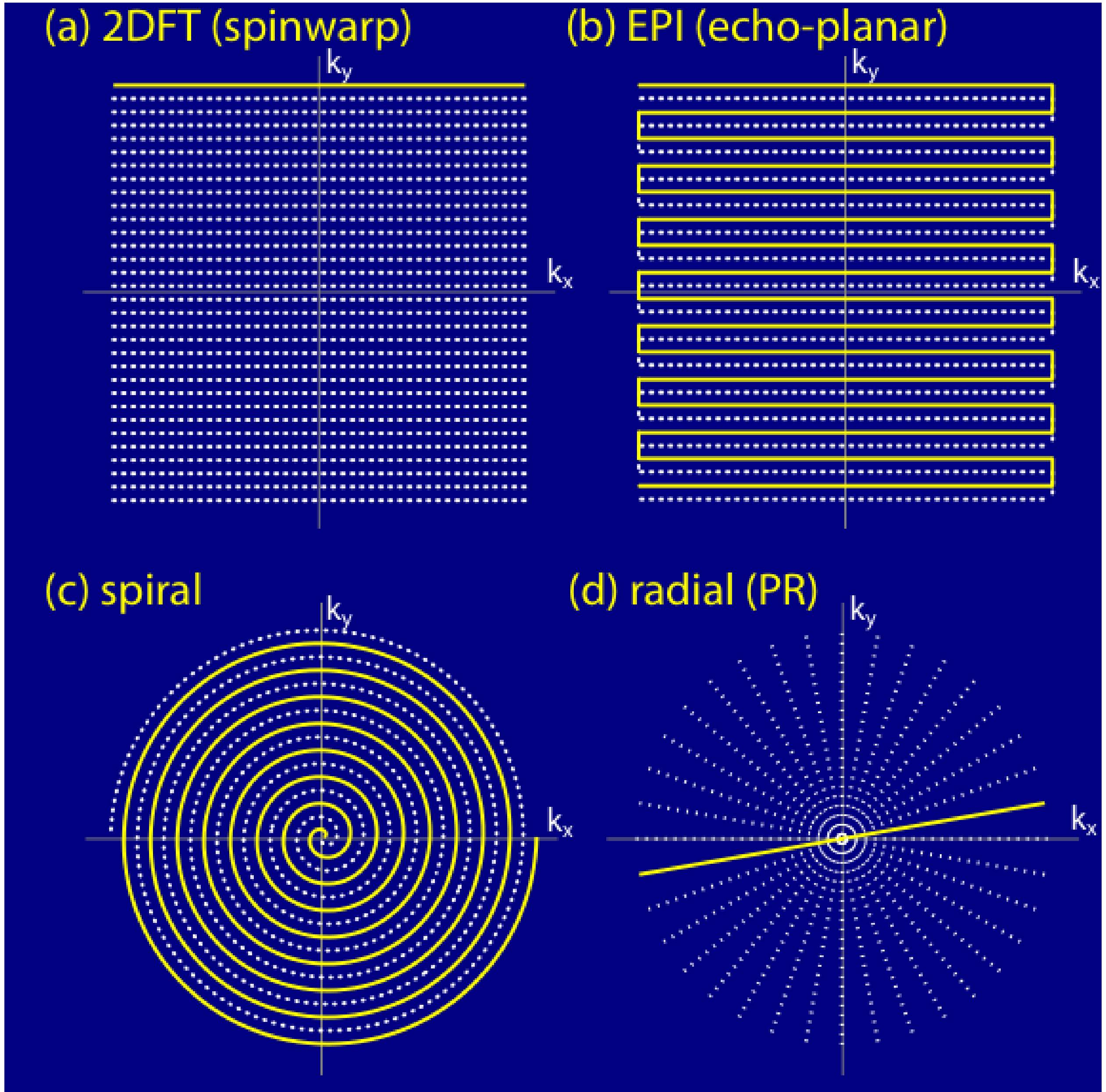
Introduction

- Acquisition of k-space data within reasonable time is a challenge

- Possible solution
  - Enables faster acquisition by reducing sampling data

- These challenges can be solved using compressed sampling

Many possible trajectories through *k*-space…



Compressed Sampling

	Nyquist's Sampling	**Compressed Sampling**
Sampling Frequency	$\geq 2f_b$	$< 2f_b$
Reconstruction	Low pass filter	Non-linear reconstruction

Non-linear Iterative Reconstruction

- Basic formulation of CS technique:

$$y = \Phi_c x + b$$

- Objective function:

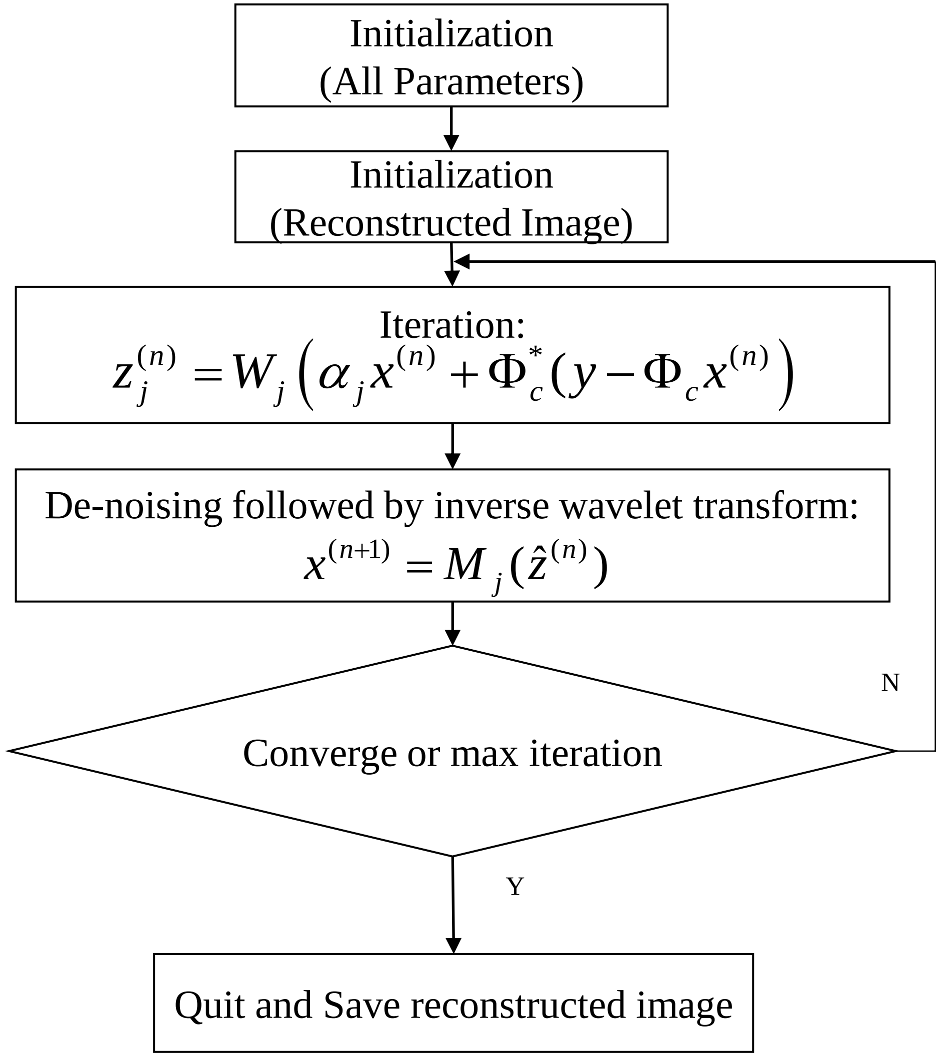
$$\mathrm{J(x)} = \frac{1}{2}\|\Phi_c \Psi x - y\|_2 + \tau\|\Psi x\|_1$$

  - $\Phi_c \Psi x$ generates low coherence
- Our goal is to achieve and optimal balance of data consistency and sparsity

Non-linear Iterative Reconstruction

- Total Variation (TV) based iterative algorithm
- Soft-thresholding based iterative algorithm
- Fast Composite Splitting Algorithm
- Many More….
- References
  - Beck A, Teboulle M. 2009.A fast iterative shrinkage-thresholding algorithm for linear inverse problems.
  - Guerquin-Kern M, Haberlin M, Pruessmann KP, Unser M. 2011. A fast wavelet-based reconstruction method for magnetic resonance imaging.
  - Huang J, Zhang S, Metaxas D. 2011. Efficient MR image reconstruction for compressed MR imaging.
  - R. Islam 2014, Improved regularisation constraints for compressed sensing of multi-slice MRI

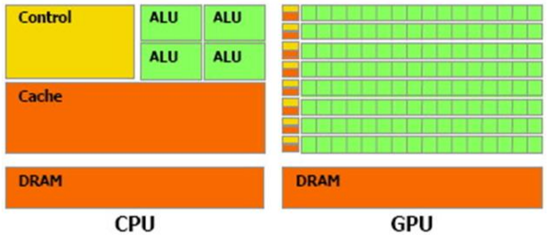
Proposed Methodology: Algorithm Flowchart



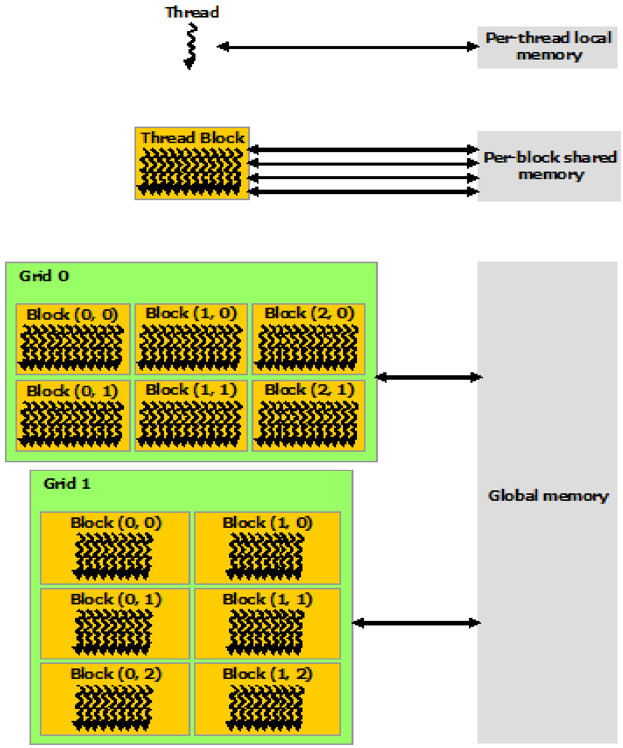
Implementation on GPU: Background

- GPU is coprocessor, which is controlled by CPU.
- It consists of hundreds of special purpose streaming multiprocessors (SMs)
- It is virtually partitioned into several grids
- Each grid is further partitioned into multiple blocks
- Each block comprises hundreds of threads,

  which run concurrently on SMs

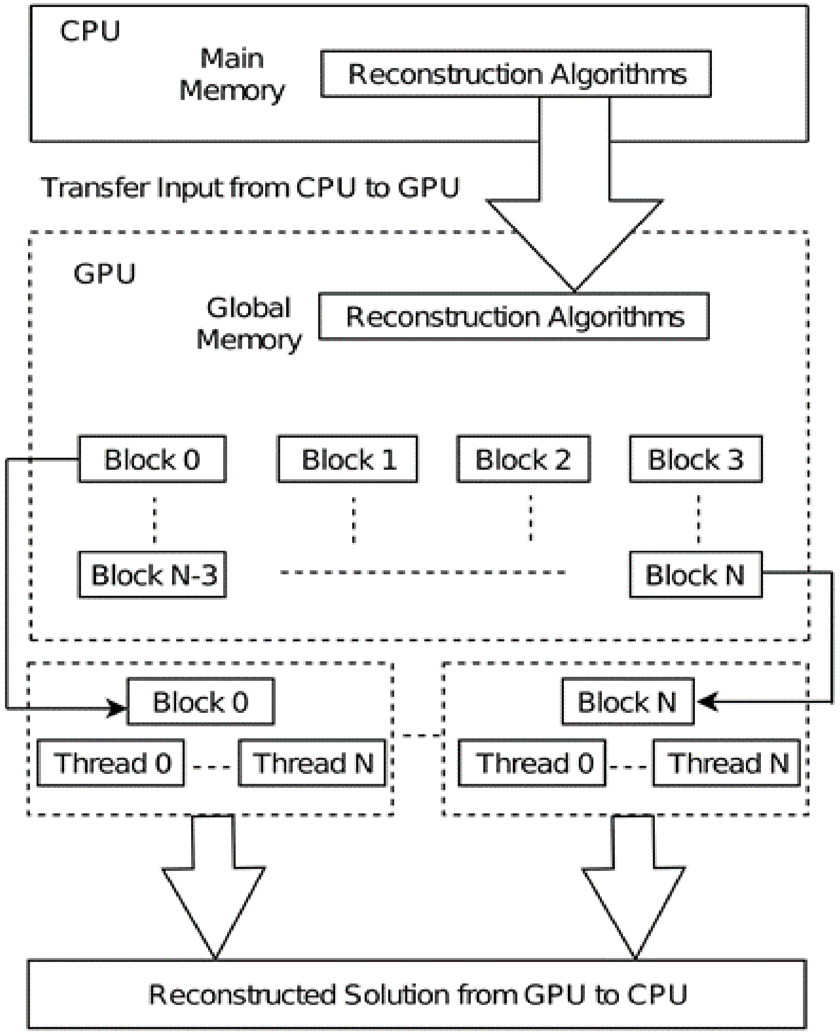


CPU-GPU comparison



GPU

Implementation on GPU: Mapping

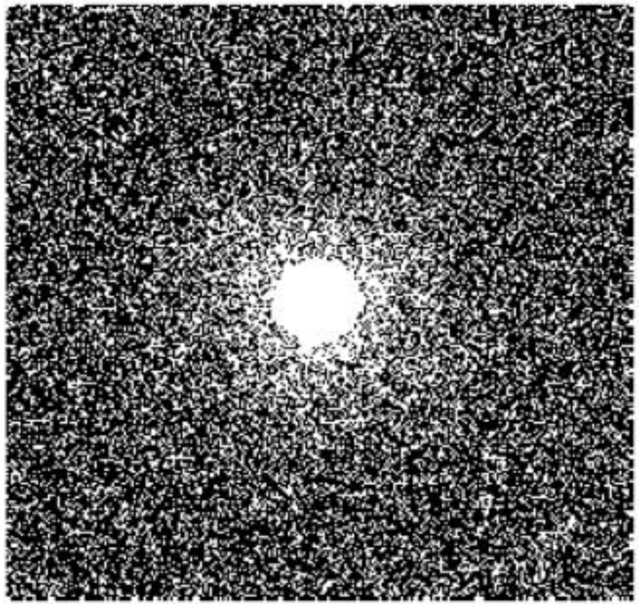


Experimental Setup: System Configuration

Property	CPU	Property	GPU
Processor	Core i7	**Brand**	NVIDIA Geforce 730
Clock Speed	3.40 GHz	**Graphics Speed**	602 MHz
No. of Cores	2	**CUDA Cores**	384
No. of Threads	4	**Max Threads**	1024
RAM	8 GB	**Graphics**	2024 MB
Cache	3 M	**Bandwidth**	14.40 GB/s

Experimental Setup : Input and sampling mask

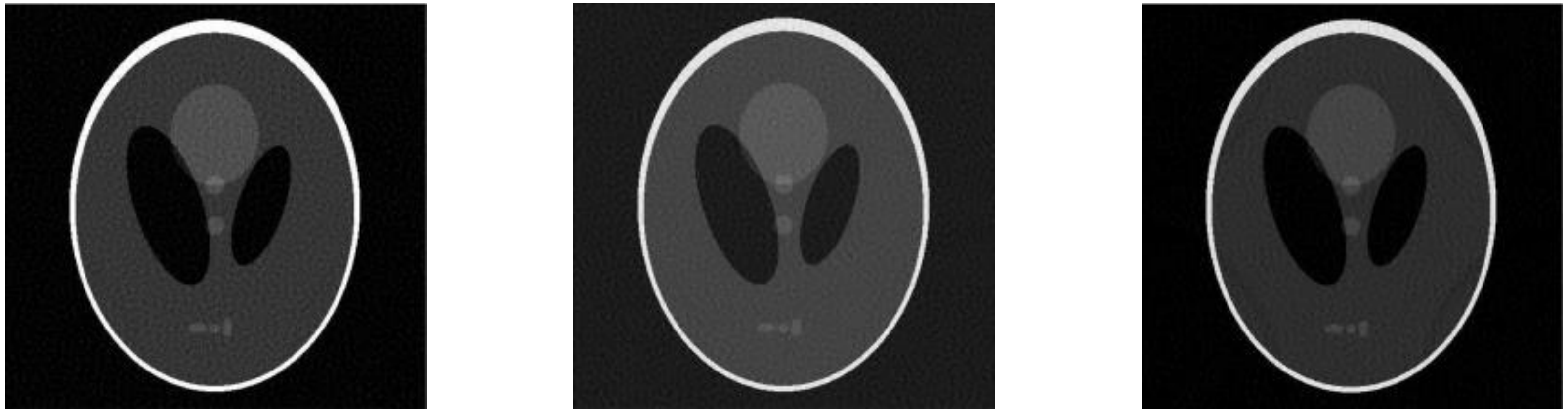
(a) Original Image



(a) Undersampled Mask

Fig: Visual display of (a) Original Image (b) Random Undersampled Mask

Experimental Result: Visualization



(a) TV (b) ST (c) Proposed method

Fig. Visual display of the output of the algorithms using (a) TV (b) ST and (c) proposed prior.

Experimental Result: PSNR

Image Size	Sampling measurements	Algorithm Name		
		TV	ST	Proposed
256×256	50%	27.52	30.02	31.95
	66.7%	23.34	24.32	28.40
	75%	21.16	22.54	26.86
512×512	50%	31.73	33.27	35.61
	66.7%	25.87	26.72	29.97
	75%	24.70	25.13	28.73

TABLE. PSNR COMPARISON OF DIFFERENT ALGORITHMS

Experimental Result: Execution Time

Processor	Algorithm Name	Execution time (256×256)	Execution time (512×512)
CPU	TV	294	2135
	ST	228	1829
	Proposed	274	2062
GPU	TV	14	80
	ST	07	71
	Proposed	09	78
Speed-up	TV	21	26
	ST	32	25
	Proposed	30	26

TABLE. EXECUTION TIME, IN SECONDS

Conclusion

- The proposed prior-based method exhibits improved performance in terms of quality of the reconstructed image
- The proposed GPU implementation exceedingly outperforms the CPU implementation, yielding a speedup of 26x, 32x, and 30x over the CPU-based methods, for TV, ST and proposed implementation respectively
- The proposed GPU implementation exhibits massive parallelism that can enable real-time reconstruction of CS MRI

Thank you

Any Questions?

[GPU based Real Time Reconstruction of Compressed Sampling MRI]