Integration, the inverse process of differentiation:

If $f(x)$ be a given function of x and if another function $F(x)$ be obtained such that its differential "Coefficient with respect to x is equal to $f(x)$, then $F(x)$ is defined as an integral, or more Properly an indefinite integral of $f(x)$ with respect to x.

The process of ~~findi~~ finding an integral (antiderivative) of a function of x is called Integration and the operation is indicated by writing the integral sign $\int$ before the given function and dx after the given function, the symbol dx indicating that x is the variable of integration.

The function $f(x)$ to be integrated, is called the Integrand.

Symbolically, if $\frac{d}{dx}F(x) = f(x)$

then $\int f(x)\,dx = F(x) + C$, where C is called constant of integration.

where $\int f(x)dx$ is called an indefinite integral (অনির্দিষ্ট) of $f(x)$ with respect to x.

Thus, considered as symbols of operation, $\frac{d}{dx}(\,)$ and $\int(\,)dx$ are inverse to each other.

Standard Method of Integration:

There are two ~~principle~~ principal processes:

1) The method of substitution, i.e. change of the independent variable.

2) Integration by parts.

Method of Substitution

P1: Integrate $\int (a+bx)^n dx$

Sol^n: Let $(a+bx) = z$ ——— (1)

$\therefore b\,dx = dz$

$\Rightarrow dx = \frac{dz}{b}$

Now $\int (a+bx)^n dx = \int z^n \cdot \frac{dz}{b}$

$= \frac{1}{b} \int z^n dz$

$= \frac{1}{b} \frac{z^{n+1}}{n+1} + c$ | where c is an integra constant.

$= \frac{1}{b} \frac{(a+bx)^{n+1}}{n+1} + c$ [by (1)]

Ans

P2: Integrate $\int \frac{dx}{x^3 (a+bx)^2}$

Sol^n: Let $a+bx = zx \Rightarrow \frac{a}{x} + b = z$ —— (1)

$$\Rightarrow -\frac{a}{x^2} dx = dz \Rightarrow dx = -\frac{x^2}{a} dz$$

Now $\int \frac{dx}{x^3 (a+bx)^2} = -\frac{1}{a} \int \frac{dz}{x z^2 x^2}$

$$= -\frac{1}{a} \int \frac{dz}{z^2} \left(\frac{z-b}{a}\right)^3$$

Since $x = \frac{a}{z-b}$

$$= -\frac{1}{a^4} \int \frac{(z-b)^3}{z^2} dz$$

$$= -\frac{1}{a^4} \int \left(z - 3b + \frac{3b^2}{z} - \frac{b^3}{z^2}\right) dz$$

$$= -\frac{1}{a^4} \left[\frac{z^2}{2} - 3bz + 3b^2 \ln z + \frac{b^3}{z}\right] + C$$

$$= -\frac{1}{a^4} \left[\frac{1}{2}\left(\frac{a+bx}{x}\right)^2 - 3b\left(\frac{a+bx}{x}\right) + 3b^2 \ln \frac{a+bx}{x} + b^3 \left(\frac{x}{a+bx}\right)\right] + C$$

[by (1)]

Ans:

Ass (GW)

P3: Integrate $\int \sqrt{\frac{x}{a-x}}\,dx$

Sol^n: Let $x = a\sin^2\theta$ —— (1)

$\therefore dx = 2a\sin\theta\cos\theta\,d\theta$

Now,

$$\int \sqrt{\frac{x}{a-x}}\,dx = \int \sqrt{\frac{a\sin^2\theta}{a(1-\sin^2\theta)}}\;2a\sin\theta\cdot\cos\theta\,d\theta$$

$$= a\int \frac{\sin\theta}{\cos\theta}\cdot 2\sin\theta\cos\theta\,d\theta$$

$$= a\int 2\sin^2\theta\,d\theta$$

$$= a\int (1-\cos 2\theta)\,d\theta$$

$$= a\left(\theta - \frac{\sin 2\theta}{2}\right) + c$$

$$= a\left(\theta - \sin\theta\cdot\cos\theta\right) + c$$

$$= a\sin^{-1}\sqrt{\frac{x}{a}} - \sqrt{x(a-x)} + c \quad \text{[by (1)]}$$

Ans:

P4: $$I = \int \frac{dx}{\{(x-3)+(x-4)\}\sqrt{(x-3)(x+4)}}$$

Soln:

$$I = \int \frac{dx}{(2x-7)\sqrt{x^2-7x+12}}$$

$$= \int \frac{2\,dx}{(2x-7)\sqrt{4x^2-28x+48}}$$

$$= \int \frac{2\,dx}{(2x-7)\sqrt{(2x-7)^2-1}}$$

$$= \int \frac{dt}{t\sqrt{t^2-1}}$$, Let $2x-7 = t$ — (1)

$\Rightarrow 2\,dx = dt$

$dx = \frac{dt}{2}$.

$$= \sec^{-1}(t) + C$$

$$= \sec^{-1}(2x-7) + C.$$ [by (1)]

Ans.