United International University

School of Science and Engineering

Practice Problems for MID Exam, Spring-2024

Course Code: Math 1151, Course Title: Fundamental Calculus

1. Determine whether the equation defines y as a function of x.

    (a) $2x - 5y = 7$

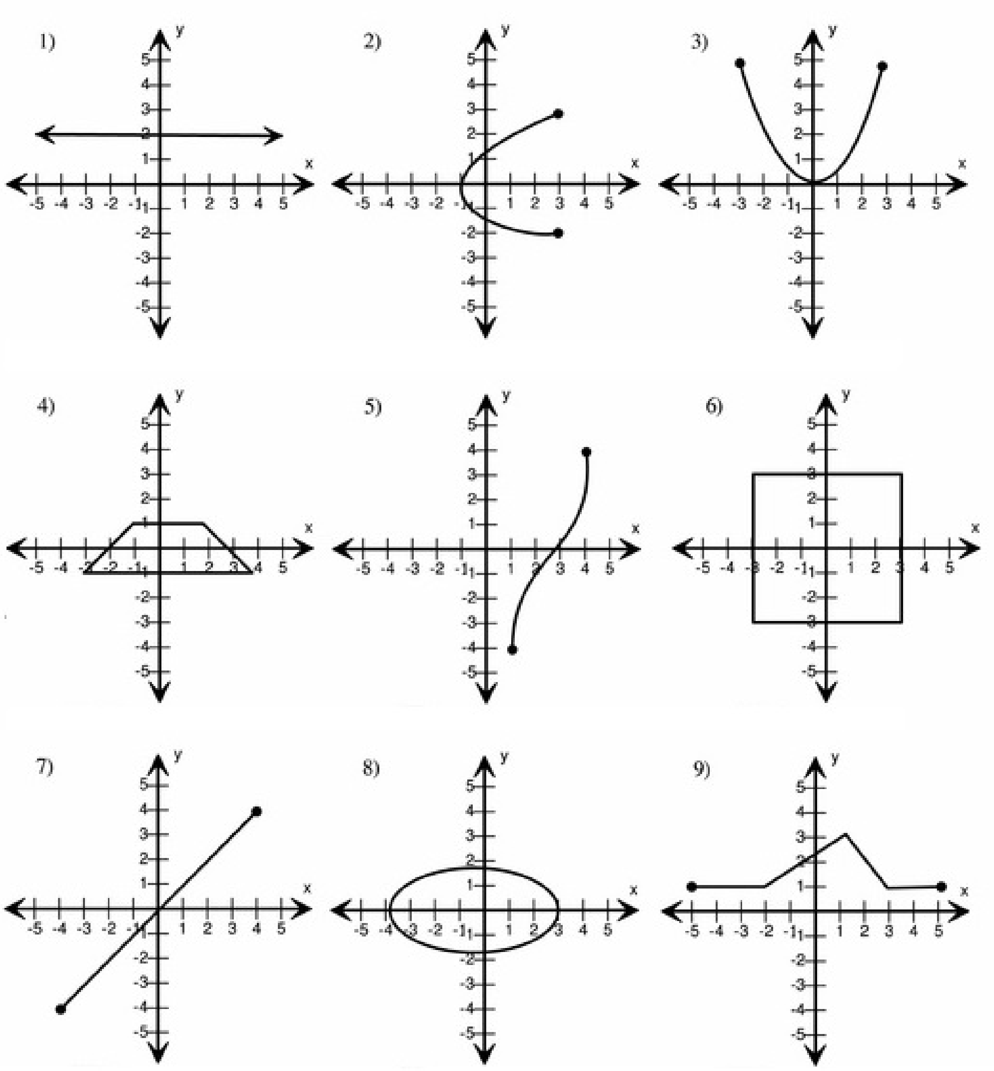
    (b) $(y+3)^3 + 1 = 2x$

    (c) $x^2 + (y-3)^2 = 5$

    (d) $2x - |y| = 0$

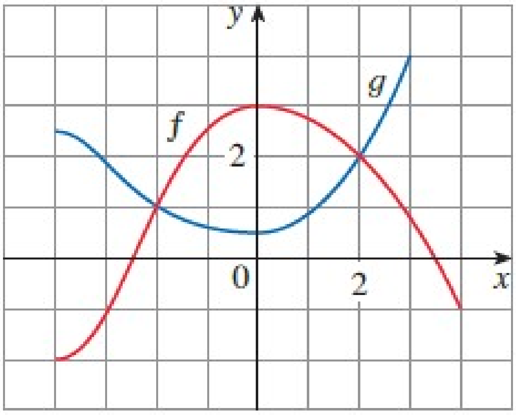
2. Determine whether the curve is the graph (Figure 1) of a function of x. If it is, sketch the domain and range of the function along with the corresponding function.

Figure 1:



3. The graph (Figure 2) of function f and g are given.

Figure 2:



(a) State the values of $f(-4)$ and $g(3)$.

(b) Which is larger, $f(-3)$ or $g(-3)$?

(c) For what values of x is $f(x) = g(x)$?

(d) On what interval(s) is $f(x) \leq g(x)$?

(e) State the solution of the equation $f(x) = -1$.

(f) On what interval(s) is g decreasing?

(g) State the domain and range of f.

(h) Sketch the domain and the range of the function g.

(i) On what interval(s) is f increasing?

4. Find the domain of the following functions.

(a) $f(x) = \frac{x+4}{x^2-9}$

(b) $g(x) = \frac{x^2+1}{x^2+4x-21}$

(c) $h(x) = \sqrt{x^2 - 4x - 5}$

(d) $k(x) = \sqrt{3-x} - \sqrt{2+x}$

5. Evaluate $f(-4)$, $f(0)$, and $f(2)$ for the piecewise defined function. Then sketch the graph of the function.

(a) $f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

(b) $f(x) = \begin{cases} -1 & \text{if } x < -2 \\ \sqrt{4-x^2} & \text{if } -2 \leq x < 2 \\ x & \text{if } x \geq 2 \end{cases}$

6. Sketch the graph of the following functions and also find their domain and range. Don't use the graphing calculator or online tool.

(a) $f(x) = x + |x|$

(b) $f(x) = \frac{|x|}{x}$

(c) $f(x) = \frac{x^2-9}{x+3}$

(d) $y = 2(x-1)^2 + 3$

(e) $y = 4x - x^2$

(f) $y = 1 + 4x - x^2$

(g) $y = 2 - \sqrt{3-x}$ (k) $y = 1 + \sqrt[3]{2-x}$ (o) $y = \frac{1}{2}\sin(\frac{x}{2})$

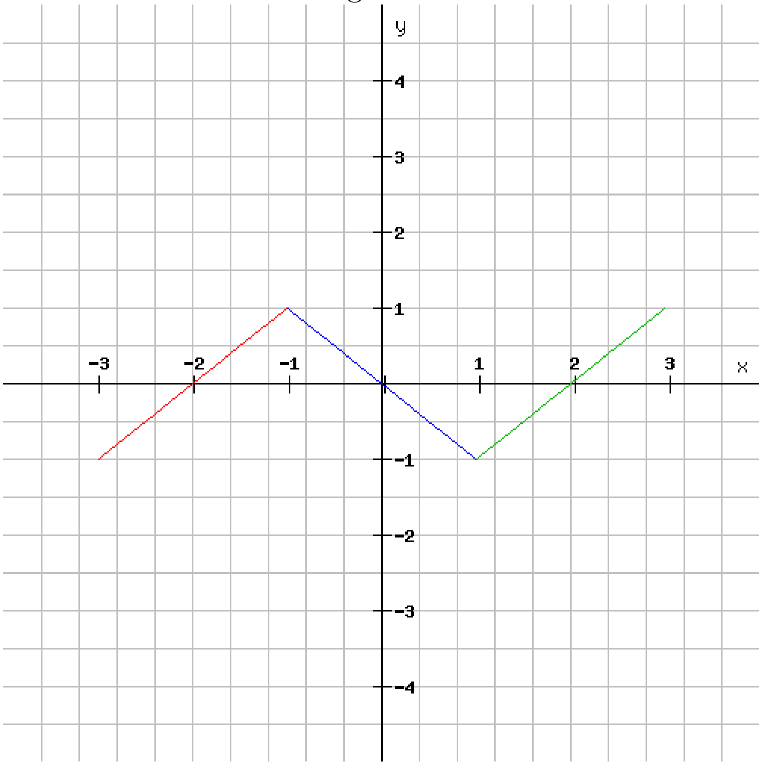
(h) $y = \frac{x}{x+1}$ (l) $y = 1 + \sin x$ (p) $y = 2 - 3^{-x}$ This thing need more attention..!

(i) $y = 2 + \frac{3}{1-x}$ (m) $y = \cos(\frac{x}{2} + \pi)$ (q) $g(x) = 2 + e^{x-1}$

(j) $y = 3 - |2x+1|$ (n) $y = -2\cos(2x)$ (r) $g(x) = -\ln(3-x)$

7. Use the given graph (Figure 3) of $y = f(x)$ to sketch the following functions:

Figure 3:



(a) $y = 1 + f(2-x)$ (c) $y = 3f(2x)$ (e) $y = -f(-x)$

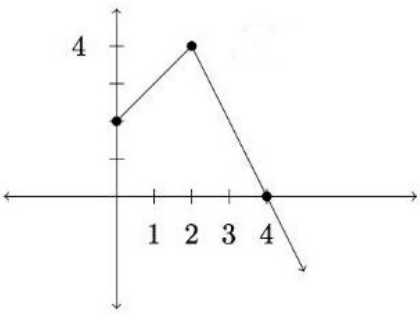
(b) $y = 2f(x+1)$ (d) $y = f(\frac{x}{2}) - 1$ (f) $y = 2 - |f(-x)|$

8. Find the functions $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$, and $(g \circ g)(x)$ and state their domains.

(a) $f(x) = \frac{1}{x}$, $g(x) = 2x + 1$ (b) $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

9. The given graph (Figure 4) of the function is defined for $x \geq 0$. Complete the graph for $x < 0$ to make it as (a) an odd function, and (b) an even function:

Figure 4:



10. Determine whether f is even, odd, or neither. You may use a graphing calculator to verify your answer:

(a) $y = \sqrt[3]{x} - 1$ (b) $y = \frac{x^3}{x^4-1}$ (c) $y = \frac{2}{x-3}$ (d) $y = 1 + e^{x^2}$

11. Sketch the graph of the following set of functions on a common screen. How are these graphs related?

(a) $y = 2e^x, y = 2e^{-x}$ (b) $y = 4^x, y = -4^x$ (c) $y = 8^x, y = -8^{-x}$

12. Complete the accompanying table so that the graph of $y = f(x)$ is symmetric (a) about y axis, and (b) about the origin.

x	-6	-3	-1	1	3	6
$f(x)$	7		2		-5	

13. (a) Find an equation for the family of lines whose members have slope 5.
    (b) Find an equation for the member of the family that passes through $(-2, 5)$.
    (c) Sketch some members of the family, and label them with their equations. Include the line in part (b).

14. (a) Find an equation for the family of lines with y-intercept 3.
    (b) Find an equation for the member of the family whose angle of inclination is $150°$.
    (c) Sketch some members of the family, and label them with their equations. Include the line in part (b).

15. Starting with the graph of $y = 3^x$, write the equation of the graph that results from

(a) shifting 2 units downward.
(b) shifting 2 units to the right.
(c) reflecting about the x-axis.
(d) reflecting about the y-axis.
(e) about the x-axis and then about the y-axis.

16. Determine whether the following functions are one-to-one, or many-to-one. Find the inverse of each function (if possible).

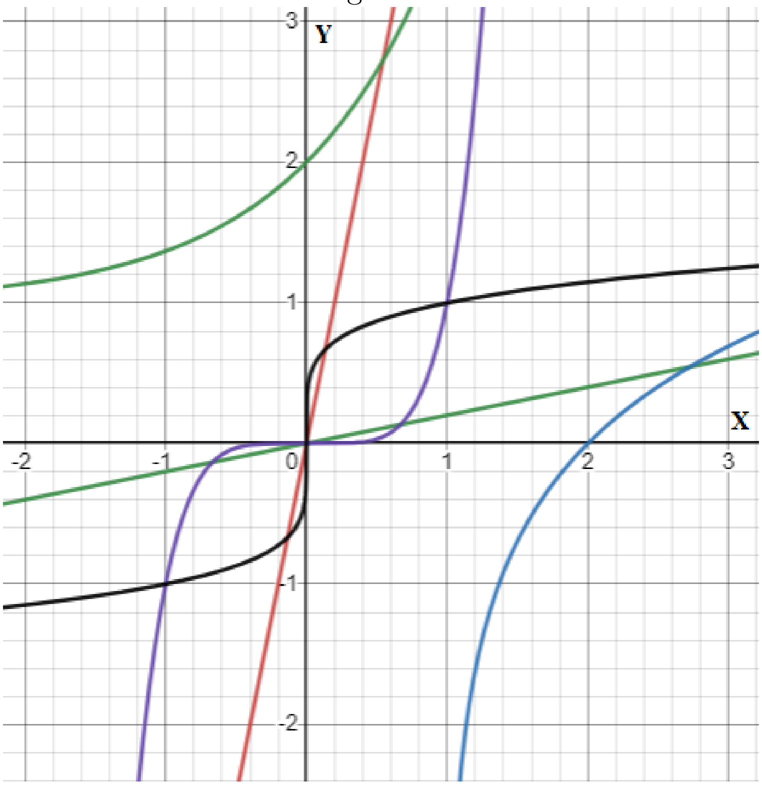
(a) $f(x) = 1 + 2^{-x}$ (b) $f(x) = (x-3)^2, x \geq 3$

17. Find the inverse of the following functions, draw the graph of the function and its inverse in the same diagram. Also, state the domain and range of the inverse.

(a) $f(x) = (x-2)^3 + 1$ (b) $f(x) = 2 + e^{-x}$ (c) $f(x) = \ln(3 - x)$

18. The following graphs in (Figure 5) are functions of $y = 5x, y = \frac{x}{5}, y = x^5, y = \sqrt[5]{x}, y = 1+e^x$ and $y = \ln(x-1)$. Identify the graph of the functions and determine which graph is inverse of the other graph.

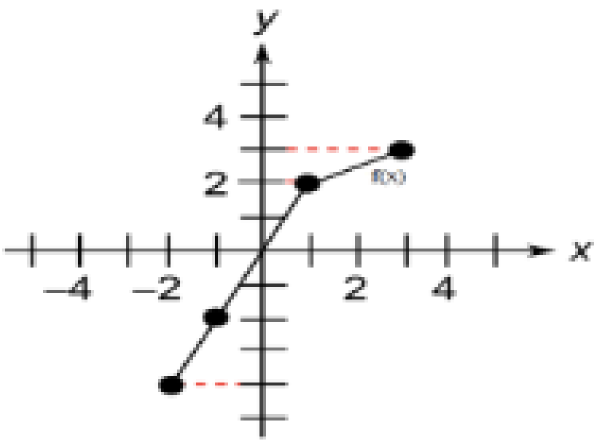
Figure 5:



19. Use the following graph (Figure 6) of the function $f(x)$ to complete the table and sketch the graph of $f^{-1}(x)$.

x	-4	-2	2	3
$f^{-1}(x)$				

Figure 6:

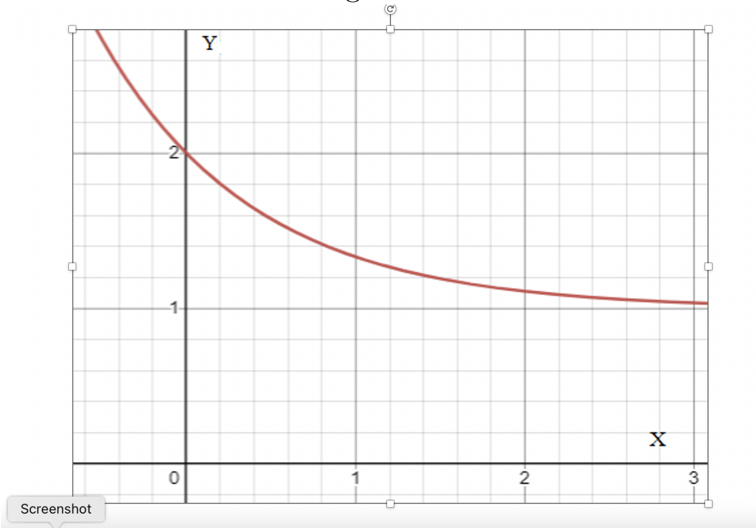


20. Determine whether the following set of functions are inverse, or not.

(a) $f(x) = \sqrt[5]{x+2}$, $g(x) = x^5 - 2$ (b) $f(x) = x^4$, $g(x) = \sqrt[4]{x}$

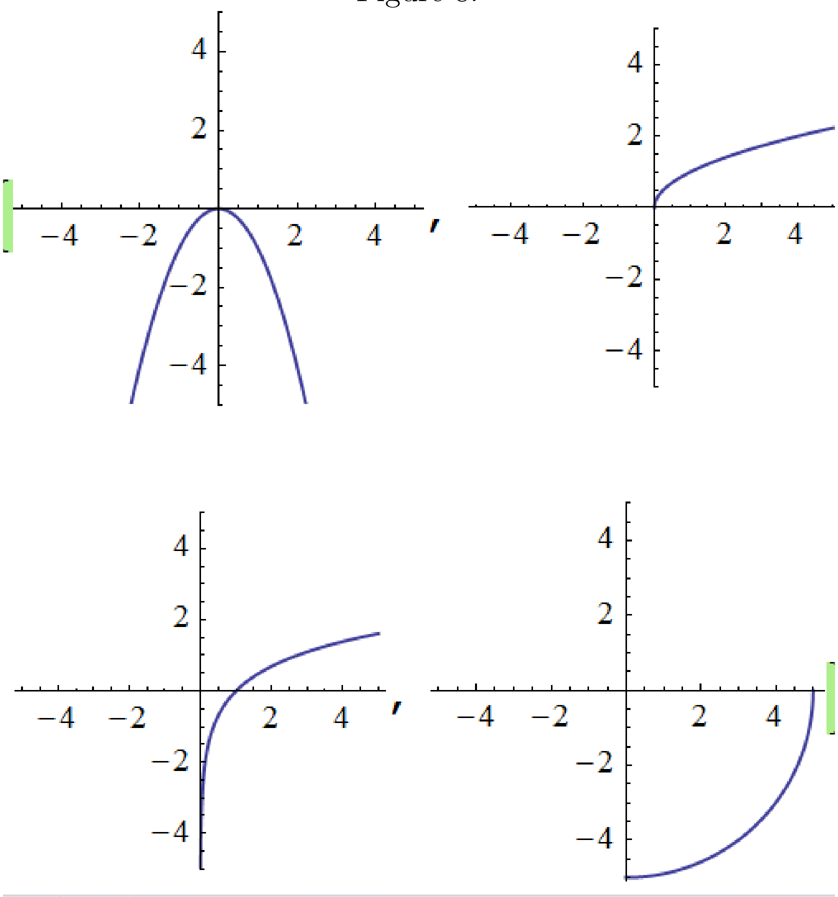
21. Determine whether the following graph in (Figure 7) has inverse or not. If so, write down the equation of the graph. Sketch the graph of its inverse in the same diagram. Also, write down the equation of the inverse graph.

Figure 7:



22. Consider the figures given in (Figure 8). Determine which function is one-to-one and which is not. Draw the inverse function along with the main function (if exists).

Figure 8:



23. Consider the following graph in (Figure 9) of a function $y = f(x)$. From the figure (Figure 9) write the answer of the following questions:

(a) $\lim_{x\to -1^-} f(x)$

(b) $\lim_{x\to 0^+} f(x)$

(c) $\lim_{x\to -1} f(x)$

(d) $\lim_{x\to 1} f(x)$

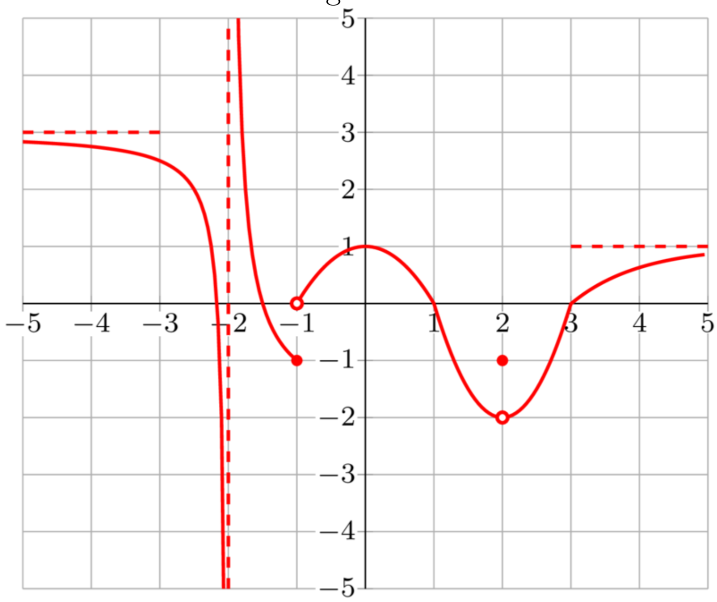
(e) $f(-2), f(-1), f(2), f(3)$

(f) check the continuity at $x = -2, 0, 2$.

(g) Find the vertical and horizontal asymptotes of $f(x)$.

(h) Find $\lim_{x\to -\infty} f(x)$ and $\lim_{x\to +\infty} f(x)$. What do you understand by those limits?

Figure 9:



24. Show that, $f(x) = 2 - |x - 1|$ is continuous at $x = 1$.

25. Check the continuity at $x = 2$ for the function $f(x) = \frac{1}{x-5} - 2$

26. Draw the graphs and determine whether the following functions are continuous at $x = 3$.

(a) $f(x) = \frac{x^2-3}{x-3}$

(b) $f(x) = \begin{cases} \frac{x^2-3}{x-3} & \text{if } x \neq 3 \\ 5 & \text{if } x = 3 \end{cases}$

(c) $f(x) = \begin{cases} \frac{x^2-3}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

27. Find the numbers at which $f(x)$ is discontinuous. At which of these numbers is $f(x)$ continuous from the right, from the left, or neither? Sketch the graph of $f(x)$.

(a) $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ \frac{1}{x} & \text{if } x \geq 1 \end{cases}$

(b) $f(x) = \begin{cases} 2^x & \text{if } x \leq 1 \\ 3 - x & \text{if } 1 < x \leq 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$

28. Consider a function

$$f(x) = \begin{cases} \frac{1}{x+2} & x \le -2 \\ x^2 - 5 & -2 < x \le 3 \\ \sqrt{x+13} & x > 3 \end{cases}$$

Find

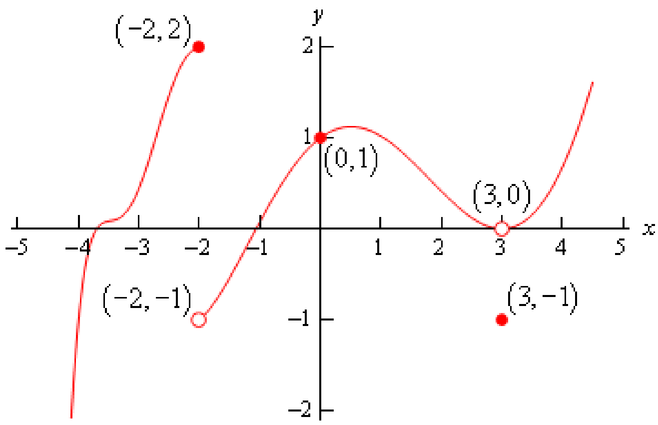
(a) $\lim_{x \to -2} f(x)$ (b) $\lim_{x \to 3} f(x)$

29. Find values of the constants k and m, if possible, that will make the function $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x+1) + k, & -1 < x \le 2 \\ 2x^3 + x + 7, & x \le -1 \end{cases}$$

30. For the following figure (Figure 10) find the point of discontinuities over the interval $[-4, 4]$ showing the appropriate reason. Which one is removable and/or which one is permanent?

Figure 10:



31. Consider the function $f(x) = \frac{x^2-1}{x^3+x^2}$

(a) Find the point of discontinuities of $f(x)$.

(b) Identify the removable and permanent discontinuities. Which one is the vertical asymptotes?

(c) Find the horizontal asymptotes of $f(x)$.

(d) Find $f(-1)$.

32. Find the vertical and horizontal asymptotes of the following functions.

(a) $y = x + 7$

(b) $y = \frac{2}{x-3}$

(c) $y = \frac{2x+1}{x+5}$

(d) $y = \frac{x^3+2x-1}{x^2-3x-10}$