Standard Integrals (2)

PX: Integrate $\int \frac{7x-9}{x^2-2x+35}\,dx$ $\left[\frac{\text{Type - 2}}{\int \frac{px+q}{ax^2+bx+c}\,dx} \quad a \neq 0,\ p \neq 0 \right]$

Solⁿ: Let, $I = \int \frac{7x-9}{x^2-2x+35}\,dx$

$$= \int \frac{\frac{7}{2}(2x-2)-2}{x^2-2x+35}\,dx$$

$$= \frac{7}{2}\int \frac{2x-2}{x^2-2x+35}\,dx - 2\int \frac{dx}{x^2-2x+35}$$

$$= \frac{7}{2}\ln|x^2-2x+35| - 2\int \frac{dx}{(x-1)^2+34}$$

$$= \frac{7}{2}\ln|x^2-2x+35| - 2\int \frac{dx}{(x-1)^2+(\sqrt{34})^2}$$

$$= \frac{7}{2}\ln|x^2-2x+35| - 2\cdot\frac{1}{\sqrt{34}}\tan^{-1}\left(\frac{x-1}{\sqrt{34}}\right)+C$$

$$= \frac{7}{2}\ln|x^2-2x+35| - \frac{2}{\sqrt{34}}\tan^{-1}\left(\frac{x-1}{\sqrt{34}}\right)+C$$

Ans:

Ans (10)

$I = \int \frac{x\,dx}{x^2+2x+1}$, $I = \int \frac{x+1}{3+2x-x^2}\,dx$

UAP

$I = \int \frac{x\,dx}{2-x-x^2}$

(40)

P4: Find $\int \frac{e^x\,dx}{e^{2x}+2e^x+5}$

Sol^n: Let $e^x = z$ ① $\therefore e^x\,dx = dz$

Now $\int \frac{e^x\,dx}{e^{2x}+2e^x+5} = \int \frac{dz}{t^2+2t+5}$

$$= \int \frac{dt}{(t+1)^2+(2)^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{t+1}{2}\right) + c$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{e^x+1}{2}\right) \text{ [by (1)]} + C.$$

Ans.

* Type-2

P5: $I = \int \frac{(4x+3)\,dx}{3x^2+3x+1}$

$$= \int \frac{\frac{2}{3}(6x+3)+1}{3x^2+3x+1}\,dx$$

$$= \frac{2}{3}\int \frac{6x+3}{3x^2+3x+1}\,dx + \int \frac{1}{3x^2+3x+1}\,dx$$

$$I = \frac{2}{3}\ln\left|3x^2+3x+1\right| + \int \frac{1}{3\left(x^2+x+\frac{1}{3}\right)}\,dx$$

$$= \frac{2}{3}\ln\left|3x^2+2x+1\right| + \frac{1}{3}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{1}{12}}\,dx$$

$$= \frac{2}{3}\ln\left|3x^2+2x+1\right| + \frac{1}{3}\cdot\frac{1}{1/2\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{1}{2\sqrt{3}}}\right) + C$$

$$= \frac{2}{3}\ln\left|3x^2+2x+1\right| + \frac{2\sqrt{3}}{3}\tan^{-1}\left(\sqrt{3}(2x+1)\right) + C$$

$$= \frac{2}{3}\ln\left|3x^2+2x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}(2x+1)\right) + C$$ Ans:

(Type-5, $\int \frac{dx}{(ax+b)\sqrt{cx+d}}$, $a\neq0, c\neq0$)

P6: $I = \int \frac{dx}{(2x+1)\sqrt{4x+3}}$, | put $4x+3 = z^2$ —(1)

$\Rightarrow 4\,dx = 2z\,dz$

$\Rightarrow dx = \frac{z}{2}\,dz$

$$= \frac{1}{2}\int \frac{z\,dz}{\left\{2\cdot\left(\frac{z^2-3}{4}\right)+1\right\}z}$$

$$= \frac{1}{2}\int \frac{dz}{\frac{z^2-3}{2}+1} = \frac{1}{2}\int \frac{2\,dz}{z^2-1}$$

$$= \int \frac{dz}{z^2-1} = \frac{1}{2}\ln\left|\frac{z-1}{z+1}\right| + C$$

[by (1)]

$$= \frac{1}{2}\ln\left|\frac{\sqrt{4x+3}-1}{\sqrt{4x+3}+1}\right| + C$$ Ans:

Prove that: P = 60 (DAS)

1. $\int \sqrt{a^2-x^2}\, dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$ [Let $x = a\sin\theta$]

2. $\int \sqrt{a^2+x^2}\, dx = \frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + c$ [$x = a\sinh\theta$]

3. $\int \sqrt{x^2-a^2}\, dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a} + c$ [$x = a\cosh\theta$]

$= \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right) + c$

Assignment (Das)

Integrate the following standard integrals:

1. $I = \int \frac{dx}{x^2(2+3x)^2}$, [Hints, put $2+3x = zx$].

2. $I = \int \frac{x\,dx}{x^4+2x^2+2}$

3. $I = \int \frac{x\,dx}{2-6x-x^2}$

4. $I = \int \frac{(x+1)}{\sqrt{4+8x-5x^2}}\,dx$

5. $I = \int \frac{dx}{(2x-1)\sqrt{(x+1)}}$

6. $I = \int \frac{x+1}{3+2x-x^2}\,dx$

7. $I = \int \frac{dx}{(x+1)\sqrt{1+2x-x^2}}$

8. $I = \int \frac{dx}{\sqrt{6+11x-10x^2}}$

23, P=700

9) $\int \sqrt{(4-3x-2x^2)}\,dx$

10) $\int \frac{dx}{(x^2+1)\sqrt{x^2+4}}$, P=716

$\to$ 2013

11) $\int \frac{3x+2}{\sqrt{x^2+4x+2}}\,dx$

12) $\int \sqrt{(x-\alpha)(\beta-x)}\,dx$

Put $x = \alpha\cos^2\theta + \beta\sin^2\theta$.

Soln:

1. $I = -\frac{1}{8}\left[\frac{2+3x}{x} - \frac{9x}{2+3x} - 6\ln\left|\frac{2+3x}{x}\right|\right] + C$

2. $I = \frac{1}{2}\tan^{-1}(x^2+1) + C$

3. $I = \frac{-3}{2\sqrt{11}} \ln\left|\frac{\sqrt{11}+3+x}{\sqrt{11}-3-x}\right| - \frac{1}{2}\ln|2-6x-x^2| + C$

4. $I = -\frac{1}{5}\sqrt{4+8x-5x^2} + \frac{9}{5\sqrt{5}} \sin^{-1}\left(\frac{5x-4}{6}\right) + C$

5. $I = 2\tan^{-1}(\sqrt{x+1}) + C$

6. $I = -\ln(x-3) + C$

7. $I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x\sqrt{2}}{1+x}\right) + C$

8. $I = \sqrt{\frac{2}{5}} \sin^{-1}\sqrt{\frac{10x+4}{9}} + C$

9. $I = \frac{4x+3}{8}\sqrt{4-3x-2x^2} + \frac{41\sqrt{2}}{32} \sin^{-1}\frac{4x+3}{\sqrt{41}} + C$

10. $I = \frac{-1}{\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} \sqrt{(x^2+4)/x^2} + C$

$= \frac{-1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{x^2+4}}{x\sqrt{3}} + C$