

Counting Question

1. Fall 19:

- a) The vehicle registration numbers in Dhaka city are formed as follow: first, these registration numbers contain the words “Dhaka Metro”, followed by the vehicle class (represented by one of 31 Bangla letters), vehicle series (a 2-digit number from 11 to 99), and the vehicle number (represented by a 4-digit number). How many registration numbers can be created in this way?
- b) Among a set of 5 black balls and 3 red balls, how many selections of 5 balls can be made such that at least 3 of them are black balls.
- c) How many 4 digit numbers that are divisible by 10 can be formed from the numbers 3, 5, 7, 8, 9, 0 such that no number repeats?

2. Fall 20:

- a) A company stores its products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 80 aisles, 120 horizontal locations in each aisle, and 6 shelves throughout the warehouse. At least how many products must there be in the warehouse so that at least two products must be stored in the same bin?
- b) The Marvelous chocolate company makes 16 different flavors of chocolates, each of three different sizes – large, medium and small. The company makes gift boxes on special occasions which contain eight chocolates – all of different flavors. The boxes also contain chocolates of different sizes – three small chocolates, three medium ones, and two large ones. How many ways can the chocolate boxes made?

3. Spring 21:

- a) There are 64 districts in Bangladesh. At least how many students must be enrolled to United International University so that we can ensure that there are at least 20 students from the same district?
- b) Calculate how many ways the letters of the word “ARISTOTLE” can be arranged, if –
- (i) The T's are not together.
 - (ii) The vowels are together.
 - (iii) The consonants are together.
- c) You have to make a string of length 7, using lowercase letters only. Here are some conditions on the strings:
- (i) The first letter can be any lowercase letter.
 - (ii) A vowel must be followed by a consonant, and vice versa. (For example, if the first letter is a vowel, then the second one must be a consonant, and the third letter must be a vowel, and so on. Similarly evaluate the case of the first letter is a consonant.

(iii) A letter can be used as many times as possible. How many such strings are there?

4. Summer 21:

a) In a game of UNO, there are cards of 4 colors - red, green, blue and yellow. There are 25 cards for each color (there are some special cards, but we will not be considering them now). A player is dealt 7 cards in a round.

- i. Explain why there is no guarantee that a player will get at least 2 red cards.
- ii. How many cards should be picked to ensure that he gets at least 2 red cards?

b) A coin is tossed 6 times such that every time it can land either on heads or tails. How many possible outcomes contain an odd number of heads?

c) There are n_1 computer science courses and n_2 computer engineering courses available at a certain university. A student has to select r_1 courses from computer science courses and r_2 courses from computer engineering courses. If the order of the courses taken are important, then how many ways can a student complete the courses?

5. Fall 21:

a) Rahat has built up a mini library in his house. He has five shelves - one for Bangla novels, one for English novels, one for Maths and Science, one for current affairs, and one for comics. What is the minimum number of books Rahat must have if he has at least 150 books of the same category?

b) Grameenphone has recently been allowed to use the series 013 along with 017 in the mobile phone numbers it can provide. There are a total of 11 digits in its mobile numbers, where each of the remaining 8 digits are between 0 and 9. How many mobile phone numbers can Grameenphone provide?

c) Wordle is a new game where a player has to guess a five-letter word in six chances. In each chance, some letters are eliminated. Zinia was playing this game, and after five chances, she realized that the letters A, C, D, E, H, K, L, O, S, T, U and Y are eliminated. How many possible valid answers are remaining?

6. Spring 22:

a) If a librarian picks 5 Calculus books from 7 different Calculus books and 4 different Discrete Math books from 6 different Discrete Math books, how many ways he can arrange these 9 books on a shelf such that a Calculus book is at the beginning and a Discrete Math book is in the middle? [Note: each of the Calculus or Discrete Math books is written by different writers.]

b) From 6 CS faculty and 4 EE faculty, a committee of 6 is to be formed. In how many ways can this be done if the committee contains (i) exactly 2 EE faculty, or (ii) at least 2 EE faculty?

c) If a student writes random strings of length 3 using the vowels (A, E, I, O, U) only, how many times he is required to write such strings to be sure that he has written a string twice?

7. Summer 22:

- a) How many strings (considering only lowercase alphabets) are there of length five or less that begin and end with the alphabet "a"? (Do not consider empty string).
- b) Suppose that a software company has 7 front-end developers and 9 back-end developers. A standard team is formed by having at least one front-end developer and at least one back-end developer. How many ways are there to form a standard team of seven members so that it must have more front-end developers than back-end developers?
- c) In a game me of UNO, there are cards of 4 colors- red, green, blue and yellow. There are 25 cards for each color (there are some special cards, but we will not be considering them now). A player is dealt 7 cards in a round. Now using the pigeonhole or generic pigeonhole principle do the followings:
- Explain why there is no guarantee that a player will get at least 2 blue cards.
 - Minimum how many cards should be picked to ensure that he gets at least 3 cards of the same color?

8. Fall 22:

- a) How many cards must be selected from a deck of 52 standard playing cards to guarantee that at least a King, a Queen and a Red card is picked? Explain with proper reasoning.
- b) In how many ways can the word EXAMINATION be arranged such that the word starts with an N, and the letters EX always stays together in any order?
- c) How many ways can UIU make a committee of 7 faculties from 7 CS Faculties and 7 Math Faculties so that the number of CS Faculties always remains more than the number of Math Faculties. The committee must contain at least one faculty from both departments.
- d) Assume that Vowel Strings are the strings having vowels (A EIOU) only. How many vowel strings of length 4 or less can you build that begin and end with the same vowel? (Do not consider empty strings)

9. Spring 23:

- a) The students of your class are arranging a raffle draw and forty five tickets, numbered 1, 2, 3,...,45 are made for each student to draw one ticket from. Four different prizes are awarded in this event, including a grand prize (a trip to Maldives). How many ways are there to award the prizes if students holding ticket number 10 and 17 are winners but students holding tickets 27,37 and 45 do not win prizes?
- b) A factory stores their manufactured products in a warehouse. Storage bins in this warehouse are specified by their row, location in the row, and shelf. There are 50 rows, 85 horizontal locations in each row, and 5 shelves throughout the warehouse. What is the least number of products the factory can produce so that at least one bin contains four products?

c) Find out the number of four letter words that can be created using letters from the word 'EXAMINATION'.

10. Summer 23:

a) It has been a long day of classes at UIU. You just finished your DM theory class and brainstorming about counting problems made you really hungry! So, you decided to grab a quick lunch from the cafeteria.

However, on your way, you want to go for a quick counselling to your course teacher's room at the 6th floor before going to the café. So, your path is: Classroom -> 6th floor Café.

Suddenly, you were curious to know how many ways you can make this journey. For each part of your journey (i.e., Classroom to 6th Floor, and then 6th Floor to Café), you have two options: taking one of the 7 available lifts or choosing one of the 3 staircases. Calculate the numbers of ways you can complete this "Classroom -> 6th floor -> Café" journey with the necessary steps.

b) Suppose that, BRTC is introducing a new type of numberplates for electric cars in Bangladesh that is in the format W-XXXX-NNNN, where X is any English letter (case sensitive) and N is a digit. W is a special character that can be either 'F' or 'B'. Now, calculate how many different license plates are available if:

i. Letters (X) and Numbers (N) cannot be repeated.

ii. Letters (X) can be repeated, but numbers (N) cannot be repeated.

c) You are arranging a party where you have kept three boxes of chocolates containing KitKat, Dairy Milk and Snickers. You will let your friends take only one chocolate. What is the minimum number of friends you have to invite to ensure that 10 of them end up with the same type of chocolate?

d) 15 students from CSE and 12 students from BBA are appearing in term final exams in the same room. Some answers for the CSE question can easily be copied. However, BBA students cannot copy since all questions are descriptive.

Can you calculate how many ways may we seat all students so that no two students from CSE seat next to each other? For simplicity, consider that the all students are seated in a single continuous line.

11. Fall 23:

a) Seven (7) women and nine (9) men are on the faculty in the Mathematics department at a school. How many ways are there to select a committee of six (6) members of the department if at least two (2) women and at least one (1) man must be on the committee?

b) How many 4-digit numbers are possible that are divisible by 3 can be formed using the digits 3, 5, 6, 1 and 7 such that no digit is repeated? (Hint: A number is divisible by 3 if the sum of the digits is divisible by 3.)

c) How many cards must be chosen from a standard deck of 52 cards to guarantee that there are at least two (2) cards of each of two (2) different suits? (Note: There are a total of 4 suits of cards)

