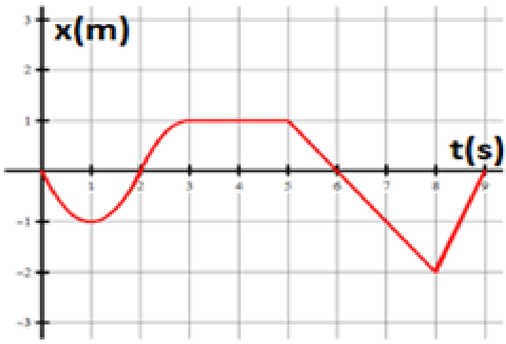
United International University
School of Science and Engineering
Practice Problems for Final Exam, Spring-2024
Course Code: Math 1151, Course Title: Fundamental Calculus

1. The following Figure 1 represents a position function of a particle at time t.

Figure 1:



   (a) Find the average velocities over the time intervals $[1, 3]$ and $[5, 8]$.

   (b) Find the value(s) of t at which the instantaneous velocity is zero.

   (c) On what interval(s), is the position graph increasing, decreasing, and stand-still?

   (d) Roughly sketch the velocity graph of the particle.

2. The graph in Figure 2 of the function $g(x)$ is given.

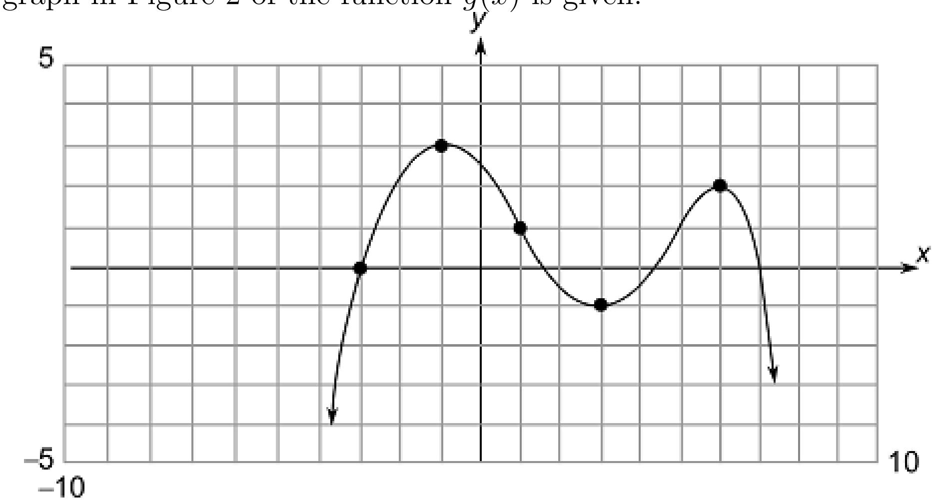


Figure 2:

   (a) Sate the sign of $g'(x)$ at the specified points.

   (b) Find the values of x at which $g'(x) = 0$. Explain the reason.

   (c) Roughly sketch the graph of $g'(x)$.

3. The position function of a moving particle is given by $s(t) = t^2 - 2$, where s is in meters and t is in seconds.

   (a) Find the average velocity of the particle over the time interval [2 3].

   (b) Find the particle's instantaneous velocity at time $t = 2$ s.

   (c) Draw the position function $s(t)$, and velocity function $v(t)$ in the same plot.

4. Consider the following functions:

   (a) $f(x) = x^3 + 3$ (b) $g(x) = 6x - x^2$

      i. Find the instantaneous rate of change of $f(x)$ and $g(x)$ with respect to x at an arbitrary value of x.

      ii. Use (i) to find the slope of the tangent lines for $x = 1$.

      iii. Find the equation of the tangent lines to the graphs at $x = 1$.

      iv. Find the equation of the secant lines in the interval $[-1, 1]$.

      v. Draw the graph of $f(x)$ and $g(x)$ together with the tangent lines and secant lines.

5. Consider the following functions:

   (a) $f(x) = -3x^2$ (b) $g(x) = x^3 - 4$

      i. Find the derivative of the functions with respect to x by using the formula $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

      ii. Find the equation of the tangent lines to the functions at $x = -1$.

      iii. Draw the functions and the tangents on the same plot.

6. The equation of motion of a particle is $s = t^4 - 2t^3 + t^2 - t$, where s is in meters and t is in seconds.

   (a) Find the velocity and acceleration as functions of t.

   (b) Find the acceleration after 1 s.

7. Consider the function $f(x) = 2 - |x|$.

   (a) Determine whether $f(x)$ is differentiable or, not at $x = 0$.

   (b) Find a formula for $f'(x)$.

8. The graph of $f(x)$ is given in Figure 3. State, with reasons, the numbers at which $f(x)$ is not differentiable.

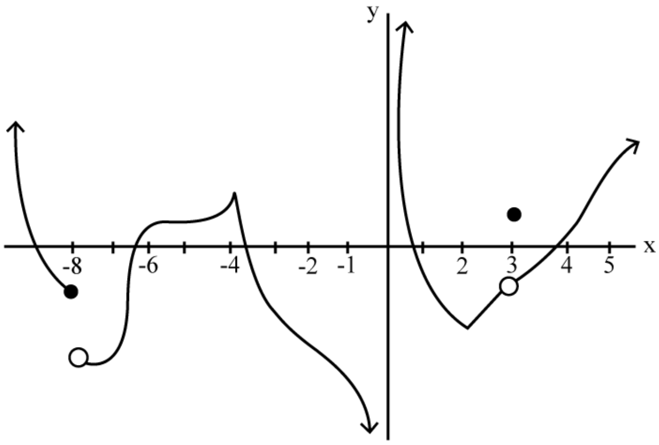
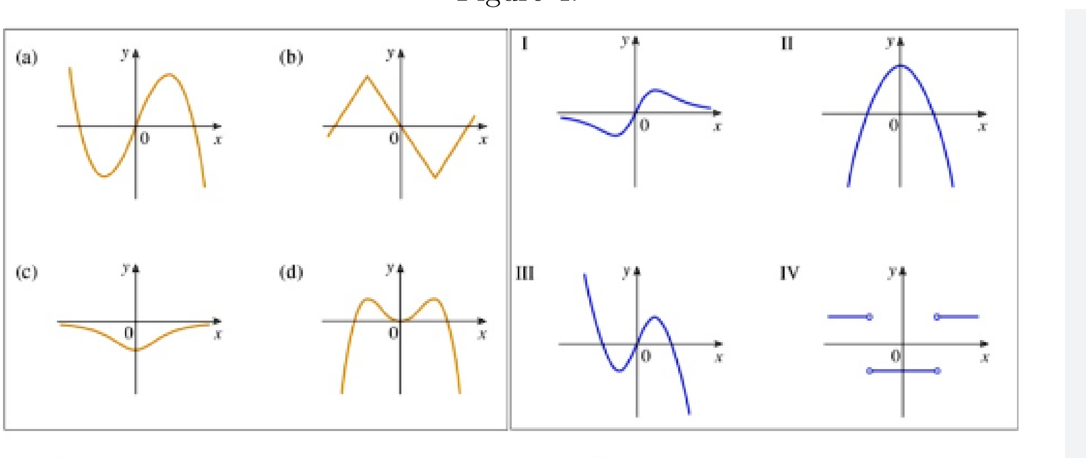


Figure 3:

9. Match the graphs of the functions shown in (a–d) (Figure 4) with the graphs of their derivatives in (I–IV) (Figure 4).

Figure 4:



10. Sketch the graph of the derivative of each function in Figure 5.

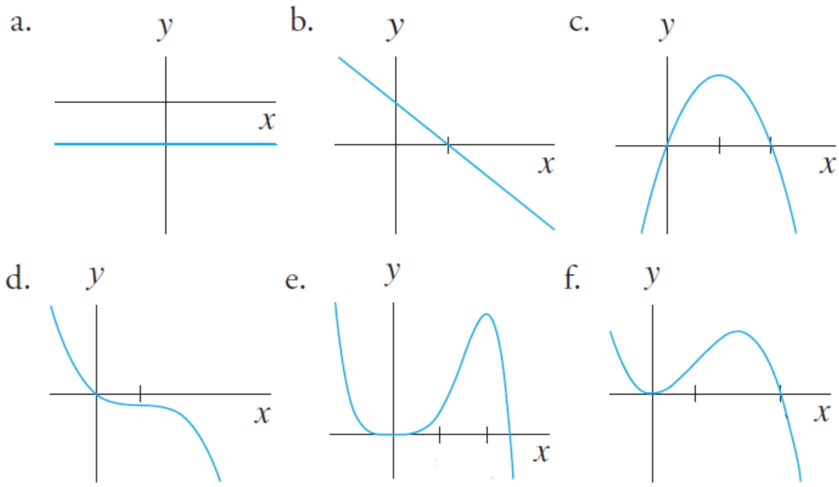


Figure 5:

11. Consider the function

$$f(x) = \begin{cases} 4 - x & \text{if } x < 0 \\ 4 - x^2 & \text{if } x \geq 0 \end{cases}$$

(a) Determine whether the function $f(x)$ is continuous and differentiable at $x = 0$.

(b) Sketch the graph of $f(x)$ and $f'(x)$.

12. (a) At what point(s) on the curve $y = x^3 - 3x^2 + 7$ is the tangent line parallel to the line $y = 9x - 1$?

(b) Find all points on the graph of the function $f(x) = x^3 + 3x^2 - 9x + 10$ at which the tangent line is horizontal.

(c) At what point on the curve $y = 2\sqrt{x}$ is the tangent line perpendicular to the line $y = 3 - 2x$?

13. Suppose $f(3) = 7$, $f'(3) = 6$, $g(3) = -5$, and $g'(3) = -2$. Find $h'(3)$ for the followings.

(a) $h(x) = 3f(x) - 2g(x) + f(x)g(x)$ (c) $h(x) = e^x g(x)$

(b) $h(x) = \frac{f(x)}{\ln x}$ (d) $h(x) = \frac{1+\sqrt{x}f(x)}{x^2+g(x)}$

14. (a) Consider $f(x) = \sec^3(x^5) - x^2 \cos 3x - 5$. find $f'(x)$.

(b) If $t = \ln(\sin(y^2 + 1))$ and $y = \cos(x^2 e^x)$, find $\frac{dt}{dx}$.

15. Write the composite function in the form of $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$]. Then, find $\frac{dy}{dx}$.

(a) $y = \ln(\cos x)$ (b) $y = \sqrt[3]{e^{2x} + 1}$ (c) $y = (x^5 + 3x^2 + 1)^{-10}$

16. Find an equation for the tangent line to the graph of $y = \cos(\cos x)$ at the point $x = \frac{\pi}{2}$.

17. Differentiate the following functions:

(a) $f(x) = \sec^3(\frac{x}{\sqrt{1-x^2}})$ (b) $g(x) = (x^7 - 3)^{-2} \tan(\frac{3}{x})$

18. The following table gives some information about $f(x), f'(x)$, $g(x)$ and $g'(x)$ at x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-3	4	7	-2	-1
0	7	-1	3	-2
2	-1	2	-3	5

If $u(x) = f(x)g(x)$, $v(x) = \frac{f(x)}{g(x)}$, $w(x) = f(g(x))$, and $z(x) = g(x^3 - 2x - 2)$, then use the Chain Rule to evaluate $u'(-3)$, $v'(0)$, $w'(2)$, and $z'(2)$.

19. Graph the function over the specified interval. Then use simple area formula from geometry to find the area function $A(x)$ that gives the area between the graph of the specified function $f(x)$ and the interval $[a, x]$. Confirm that $A'(x) = f(x)$. Also, verify your answer by direct integration.

    (a) $f(x) = 6 - x; [a, x] = [-2, x]$

    (b) $f(x) = 10; [a, x] = [2, x]$

    (c) $f(x) = 2x + 4; [a, x] = [\frac{1}{2}, x]$

20. Use the anti-derivative method to find the area under the graph of $y = x^3 - 1$ over the interval $[0, 2]$.

21. Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry, where needed:

    (a) $\int_0^2 |3x - 6|\, dx$

    (b) $\int_{-5}^0 |2x + 5|\, dx$

    (c) $\int_{-3}^3 \sqrt{9 - x^2}\, dx$

    (d) $\int_0^4 -\sqrt{16 - x^2}\, dx$

22. Evaluate $\int_0^3 (2x + \sqrt{9 - x^2})\, dx$

23. Consider the function:

$$f(x) = \begin{cases} |x + 4| & \text{if } x \leq 0 \\ -x + 4 & \text{if } x > 0 \end{cases}$$

    Use $f(x)$ evaluate the following integrals:

    (a) $\int_{-8}^{-5} f(x)\, dx$

    (b) $\int_{-4}^4 f(x)\, dx$

    (c) $\int_4^6 f(x)\, dx$

24. Consider the function:

$$f(x) = \begin{cases} 2x + 3 & \text{if } x \leq 0 \\ 3 & \text{if } x > 0 \end{cases}$$

    Use $f(x)$ to evaluate: $\int_{-3/2}^2 f(x)\, dx$

25. The graph of f is shown in Figure 6. Evaluate the following integrals.

    (a) $\int_0^3 f(x)\, dx$

    (b) $\int_5^8 f(x)\, dx$

    (c) $\int_3^7 |f(x)|\, dx$

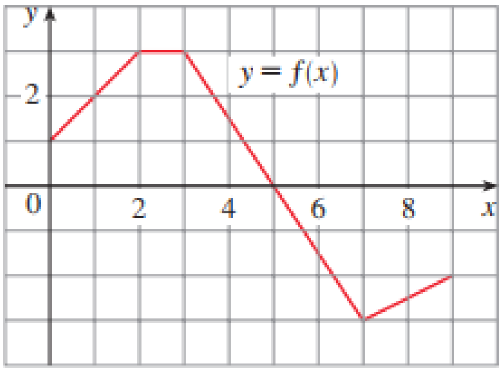


Figure 6:

26. If $\int_2^8 f(x)\,dx = 7.3$ and $\int_2^4 f(x)\,dx = 5.9$, find $\int_4^8 f(x)\,dx$.

27. Evaluate the following integrals:

(a) $\int x^{\frac{1}{3}}(1+x^2)\,dx$

(b) $\int_{-1}^{1} t(1-t)^2\,dt$

(c) $\int \frac{x^3+2x-5}{x^2}\,dx$

(d) $\int \frac{\sin x}{\cos^2 x}\,dx$

(e) $\int x^3\sqrt{3-2x^4}\,dx$

(f) $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$

(g) $\int \frac{e^x}{3+e^{2x}}\,dx$

(h) $\int \frac{1}{4+25x^2}\,dx$

(i) $\int \frac{x}{3+7x^2}\,dx$

(j) $\int \frac{x^4}{\sqrt{2+x^5}}\,dx$

(k) $\int \frac{\cos(\frac{5}{x})}{3x^2}\,dx$

(l) $\int \frac{e^{-\sqrt{2x}}}{2\sqrt{2x}}\,dx$

(m) $\int_{-\sqrt{2}}^{-2/\sqrt{3}} \frac{1}{x\sqrt{x^2-1}}\,dx$

(n) $\int \frac{x}{\sqrt{4-5x^2}}\,dx$

(o) $\int_0^4 3^s\,ds$

(p) $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$

(q) $\int \frac{\sin x}{\cos^2 x+1}\,dx$

(r) $\int \sin(\sin x)\cos x\,dx$

(s) $\int \csc x(\sin x+\cot x)\,dx$

(t) $\int \sin^3 x\cos x\,dx$

(u) $\int x\sqrt{x+1}\,dx$

28. Find the area of the following couple of curves by:

(a) integrating with respect to x

(b) integrating with respect to y

i. $y^2 = x$ and $y = 6 - x$

ii. $y = 2x^2$ and $y = 4 - 2x$

iii. $y = x^2 - 4x$ and $y = 2x$

iv. $y = 2x$, $y = -2$ and $x = 3$

29. Questions from 7.2 and 7.4

Evaluate the following integrals:

(a) $\int x^2\cos x\,dx$

(b) $\int x\tan^{-1}(3x)\,dx$

(c) $\int e^{-2x}\sin 3x\,dx$

(d) $\int \frac{xe^x}{(1+x)^2}\,dx$

(e) $\int_1^e x^2\ln x\,dx$

(f) $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

(g) $\int \frac{\sqrt{4+x^2}}{x}\,dx$

(h) $\int \frac{1}{(9-25x^2)^{\frac{3}{2}}}\,dx$

(i) $\int \frac{1}{\sqrt{x^2+4x+5}}\,dx$

(j) $\int \frac{x}{x^2+2x+2}\,dx$

(k) $\int_0^4 \sqrt{x(4-x)}\,dx$

(l) $\int_1^2 \frac{1}{\sqrt{x(4-x)}}\,dx$

30. Find the area of the shaded region in the following figures (Figure 7):

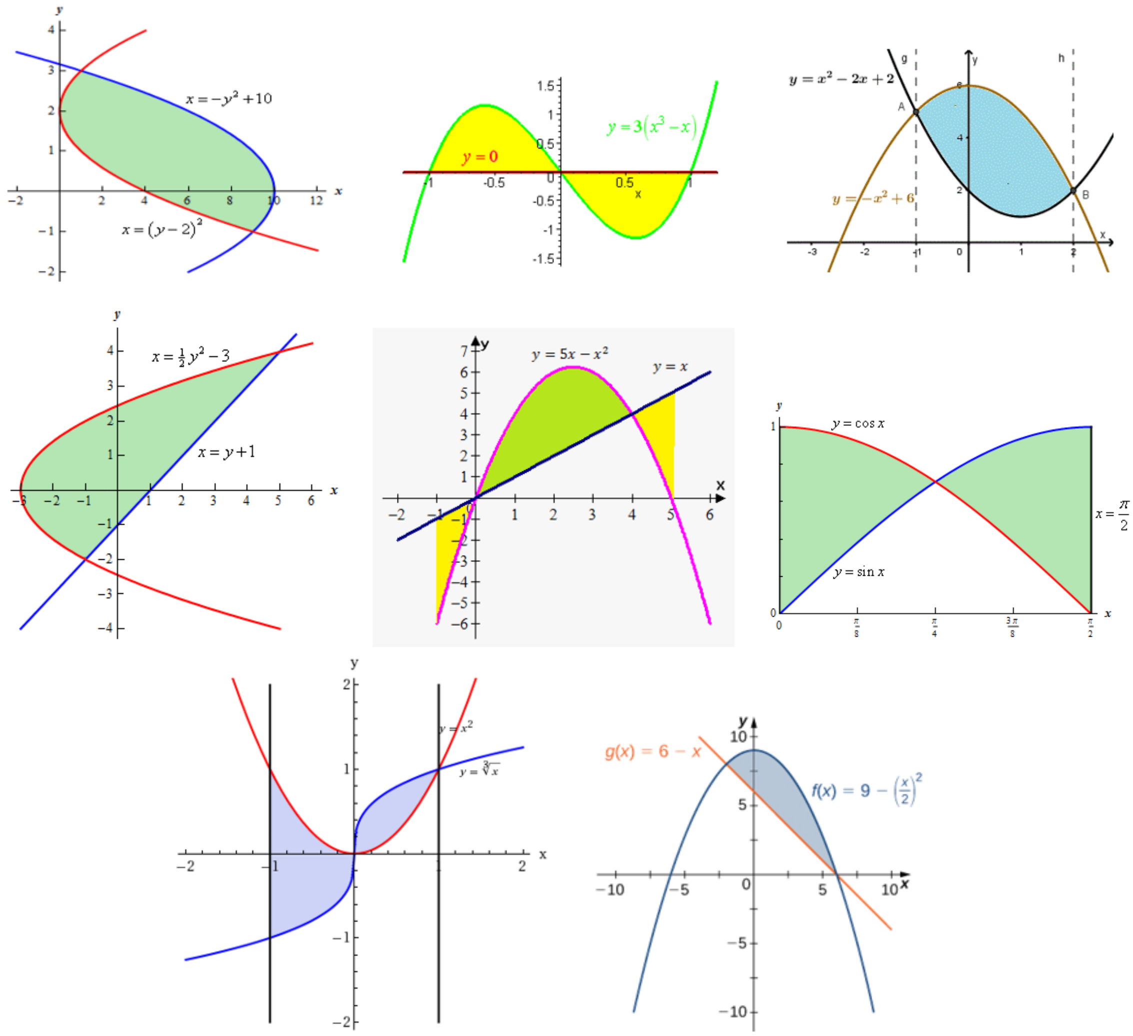


Figure 7: