PARALLEL ALGORITHM - MATRIX MULTIPLICATION

A matrix is a set of numerical and non-numerical data arranged in a fixed number of rows and column. Matrix multiplication is an important multiplication design in parallel computation. Here, we will discuss the implementation of matrix multiplication on various communication networks like mesh and hypercube. Mesh and hypercube have higher network connectivity, so they allow faster algorithm than other networks like ring network.

Mesh Network

A topology where a set of nodes form a p-dimensional grid is called a mesh topology. Here, all the edges are parallel to the grid axis and all the adjacent nodes can communicate among themselves.

$$\text{Total number of nodes} = numberofnodesinrow \times numberofnodesincolumn$$

A mesh network can be evaluated using the following factors −

- Diameter
- Bisection width

Diameter − In a mesh network, the longest **distance** between two nodes is its diameter. A p-dimensional mesh network having **kP** nodes has a diameter of $\mathbf{p}k-1$.

Bisection width − Bisection width is the minimum number of edges needed to be removed from a network to divide the mesh network into two halves.

Matrix Multiplication Using Mesh Network

We have considered a 2D mesh network SIMD model having wraparound connections. We will design an algorithm to multiply two n × n arrays using n^2 processors in a particular amount of time.

Matrices A and B have elements a_{ij} and b_{ij} respectively. Processing element PE_{ij} represents a_{ij} and b_{ij}. Arrange the matrices A and B in such a way that every processor has a pair of elements to multiply. The elements of matrix A will move in left direction and the elements of matrix B will move in upward direction. These changes in the position of the elements in matrix A and B present each processing element, PE, a new pair of values to multiply.

Steps in Algorithm

- Stagger two matrices.
- Calculate all products, $a_{ik} \times b_{kj}$
- Calculate sums when step 2 is complete.

Algorithm

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Procedure MatrixMulti

Begin
   for k = 1 to n-1

   for all Pij; where i and j ranges from 1 to n
      ifi is greater than k then
         rotate a in left direction
      end if

   if j is greater than k then
      rotate b in the upward direction
   end if
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   for all Pij ; where i and j lies between 1 and n
      compute the product of a and b and store it in c
   for k= 1 to n-1 step 1
   for all Pi;j where i and j ranges from 1 to n
      rotate a in left direction
      rotate b in the upward direction
      c=c+aXb
End
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Hypercube Network

A hypercube is an n-dimensional construct where edges are perpendicular among themselves and are of same length. An n-dimensional hypercube is also known as an n-cube or an n-dimensional cube.

Features of Hypercube with 2^k node

- Diameter = k
- Bisection width = 2^{k-1}
- Number of edges = k

Matrix Multiplication using Hypercube Network

General specification of Hypercube networks −

- Let $\mathbf{N = 2^m}$ be the total number of processors. Let the processors be $P_0, P_1 \ldots\ldots P_{N-1}$.
- Let i and i^b be two integers, $0 < i, i^b < N-1$ and its binary representation differ only in position b, $0 < b < k-1$.
- Let us consider two n × n matrices, matrix A and matrix B.
- **Step 1** − The elements of matrix A and matrix B are assigned to the n^3 processors such that the processor in position i, j, k will have a_{ji} and b_{ik}.
- **Step 2** − All the processor in position i, j, k computes the product

$$C_{i,j,k} = A_{i,j,k} \times B_{i,j,k}$$

- **Step 3** − The sum $C_{0,j,k} = \Sigma C_{i,j,k}$ for $0 \le i \le n-1$, where $0 \le j, k < n-1$.

Block Matrix

Block Matrix or partitioned matrix is a matrix where each element itself represents an individual matrix. These individual sections are known as a **block** or **sub-matrix**.

Example

$$\text{Matrix X} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Figure (a)

$$\text{Matrix A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Figure (b)

$$\text{Matrix B}=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$$

Figure (c)

$$\text{Matrix C}=\begin{pmatrix}4 & 5\\ 6 & 7\\ 8 & 9\end{pmatrix}$$

Figure (d)

$$\text{Matrix D}=\begin{pmatrix}5 & 0 & 5\\ 0 & 5 & 0\\ 5 & 0 & 5\end{pmatrix}$$

Figure (e)

$$\text{Matrix X}=\begin{pmatrix}1 & 2 & 1 & 1 & 1\\ 3 & 4 & 1 & 1 & 1\\ 4 & 5 & 5 & 0 & 5\\ 6 & 7 & 0 & 5 & 0\\ 8 & 9 & 5 & 0 & 5\end{pmatrix}$$

Figure (f)

In Figure a, X is a block matrix where A, B, C, D are matrix themselves. Figure f shows the total matrix.

Block Matrix Multiplication

When two block matrices are square matrices, then they are multiplied just the way we perform simple matrix multiplication. For example,

$$\begin{pmatrix}A1 & B1\\ C1 & D1\end{pmatrix} \text{X} \begin{pmatrix}A2 & B2\\ C2 & D2\end{pmatrix} =$$

$$\begin{pmatrix}A1A2 + B1C2 & A1B2+B1D2\\ C1A2 + D1C2 & C1B2+D1D2\end{pmatrix}$$