

2. I am going to describe the Turing Machine M which accepts the language $\{b(n)\#b(n+1) : n \geq 1\}$. I am going to assume that the input string is written on the tape with blank spaces extending indefinitely on both the right and left ends.

In essence M reading input s overwrites the portion of s before its first $\#$ with the value it would expect to occur after the first $\#$ and then checks if the string before the first hash is the same as the string after it (and then that the the string terminates).

- i. At its start state M checks that the first symbol is a one (if it is not a 1 it rejects the string). It then moves right until it finds a $\#$. If it finds the end of the string before finding a $\#$ it rejects the string. If M finds a $\#$ it moves left.
- ii. If the value directly to the left of the $\#$ is a 0, then M overwrites the 0 with a 1, returns to the beginning of the string, and uses the algorithm I wrote in problem 1 to check that a string is in the format $\{w\#w : w \in \{0,1\}^*\}$, and if that algorithm accepts them M accepts the string. (The case from problem one where $|w| = 0$ does not apply because M already checked for that.)
- iii. If the value directly to the left of the $\#$ is a 1, then M overwrites the 1 with a 0 and moves to the left. If the symbol to the left is a 1 then M overwrites the 1 with a 0 and moves to the left. M continues replacing 1 with 0 and moving left until it reads a 0 or a blank space (meaning that it has moved to the left of the beginning of the string).
- iv. If M reads a 0 it replaces it with a 1, returns to the beginning of the string, and uses the algorithm I wrote in problem 1 to check that a string is in the format $\{w\#w : w \in \{0,1\}^*\}$, and if that algorithm accepts them M accepts the string. (The case from problem one where $|w| = 0$ does not apply because M already checked for that.)
- v. But if M reads a blank space then it writes a 1. It is now at the beginning of the string so uses the algorithm I wrote in problem 1 to check that a string is in the format $\{w\#w : w \in \{0,1\}^*\}$, and if that algorithm accepts them M accepts the string. (The case from problem one where $|w| = 0$ does not apply because M already checked for that.)