Radial Basis Function Networks - Learning Algorithm for variable centres

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1 Notation

Weights (coefficients)	w	Centre	R
Target Output	t	Hidden Node/Centre Iterator	h
Actual Output	y	Number of Centres	m
Input Pattern	x	Output Iterator	o
Input Dimension Iterator	i	Number of Outputs	k
Input Dimension	d	Standard Deviation	σ
Number of Inputs	n	Learning Rates	η_R, η_σ, η_w
Input Pattern Iterator	p	Bias unit	b

2 Algorithm Pseudocode

1. Choose the number (m) and initial coordinates of the centres (R) of the RBF functions.
2. Choose the initial value of the spread parameter (σ) for each centre (R).
3. Initialise the weights/coefficients (w) to small random values [-1,1].
4. For each epoch (e)
5.   For each input vector/pattern ($x^{(p)}$)
6.   Calculate the output (y) of each output node (o) using eq. 1.
7.   Update the network parameters (w, R, σ) using eqs. 6, 7, 8, 9.
8.   end for (p = n)
9. end for (e = total epochs)

Note: Steps 1 and 2 can be performed using a clustering algorithm such as Kohonen SOMs.

3 Equations

1. The output of node o for pattern p is:

$$y_o^{(p)} = w_{bo} + \sum_{h=1}^{m} w_{ho}\phi(x^{(p)}, R_h, \sigma_h)$$

2. Gaussian function:

$$\phi(x^{(p)}, R_h, \sigma_h) = exp\left(-\frac{||x^{(p)} - R_h||^2}{2\sigma_h^2}\right)$$

3. Euclidean distance:

$$||x^{(p)} - R_h||^2 = \sum_{i=1}^{d}\left(x_i^{(p)} - R_{hi}\right)^2$$

4. Total Error:

$$E = \frac{1}{2}\sum_{p=1}^{n}\sum_{o=1}^{k}\left[\varepsilon_o^{(p)}\right]^2$$

5. Error of output neuron o for pattern p:

$$\varepsilon_o^{(p)} = t_o^{(p)} - y_o^{(p)}$$

6. Weight/coefficient update equation (from hidden unit/centre h to output node o):

$$w_{ho}^{'} = w_{ho} + \eta_w \varepsilon_o^{(p)}\phi(x^{(p)}, R_h, \sigma_h)$$

7. Weight/coefficient update equation (from bias unit b to output node o):

$$w_{bo}^{'} = w_{bo} + \eta_w \varepsilon_o^{(p)}$$

8. Centre update equation (i^{th}coordinate of centre h):

$$R_{hi}^{'} = R_{hi} + \eta_R \sum_{o=1}^{k}\varepsilon_o^{(p)} w_{ho}\phi(x^{(p)}, R_h, \sigma_h)\frac{x_i^{(p)} - R_{hi}}{\sigma_h^2}$$

9. Standard Deviation/width σ of Gaussian of centre h:

$$\sigma_h^{'} = \sigma_h + \eta_\sigma \sum_{o=1}^{k}\varepsilon_o^{(p)} w_{ho}\phi(x^{(p)}, R_h, \sigma_h)\frac{||x^{(p)} - R_h||^2}{\sigma_h^3}$$