3-D

Q. $A(4,8,10)$, $B(6,10,-8)$ find the ratio in which line AB divide By xy plane

Sol^n

$(4,8,10)$ λ : 1 $(6,10,-8)$

$$\left(\frac{6\lambda+4}{\lambda+1}, \frac{10\lambda+8}{\lambda+1}, \frac{-8\lambda+10}{\lambda+1}\right)$$

x-y-Plane $\quad \frac{-8\lambda+10}{\lambda+1} = 0 \qquad -8\lambda = -10$

ratio $(5:4)$ $\qquad (\lambda = 5/4)$

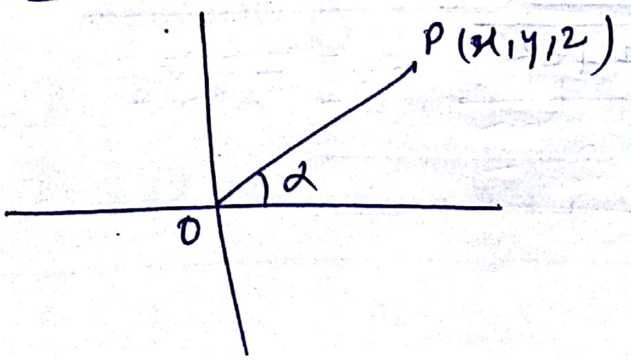
Direction co-sines & Direction ratio : (DC's)

Direction cosine : Direction cosine of a line are Difined as the cosines of the angles made by the line with Positive Direction of co-ordinate axis.

if α, β, γ are angles made by the line L with positive x y & z axis respectivily then Direction cosine of the line are $\cos\alpha, \cos\beta, \cos\gamma$

these are usally represent By l, m, n

$$\boxed{l = \cos\alpha, \ m = \cos\beta, \ n = \cos\gamma}$$



$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\cos\alpha = \frac{\vec{OP}\cdot\hat{i}}{|\vec{OP}|\,|\hat{i}|}$$

$\cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}}$; $\cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}}$; $\cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$$\boxed{l^2+m^2+n^2=1}$$

Note if l, m, n are Direction cosine of line then $-l, -m, -n$ are Direction cosine of Same line.

Direction Ratio's :→ (Dr's)

any three number a, b, c which are proportional to the Direction cosines of a line are called the Direction Ratios.

$$\boxed{\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \lambda}$$ $a = l\lambda$; $b = m\lambda$; $c = n\lambda$

$$a^2+b^2+c^2 = \lambda^2(l^2+m^2+n^2)$$

$$\lambda = \pm\sqrt{a^2+b^2+c^2}$$

$$l = \frac{a}{\lambda} = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \frac{b}{\lambda} = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \frac{c}{\lambda} = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

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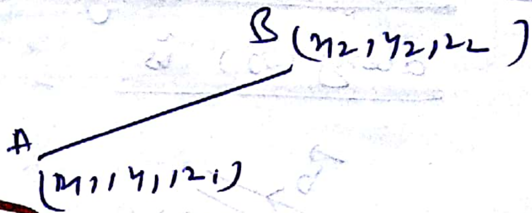
Que find the DC's of a line whose DR's are $(6, -3, 2)$ $\quad 36,\ 9,\ 4$

Soln $l = \dfrac{\pm 6}{7}$, $m = \dfrac{\pm(-3)}{7}$, $n = \dfrac{\pm 2}{7}$



~~Que~~
DR's of a line Joining two pts

$$\boxed{\text{dr's} = \left((x_2-x_1), (y_2-y_1), (z_2-z_1)\right)}$$

$$\text{DC's} = \frac{(x_2-x_1)}{|AB|}, \frac{(y_2-y_1)}{|AB|}, \frac{(z_2-z_1)}{|AB|}$$

Que find the DC's of a point passing through the pt $(1, 4, -2), (0, 2, 1)$

DR's $= (1, 2, -3)$ $\quad 9,\ 4,\ 1$

$l = \dfrac{1}{\sqrt{14}}$, $m = \dfrac{2}{\sqrt{14}}$, $n = \dfrac{-3}{\sqrt{14}}$

Q if a line makes 45 & 120 angle which positive x-axis & y-axis respectively then find the possible DC's

Q if a line makes α, β, γ angle with positive x, y, z axis respectively & find the value of

(a) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

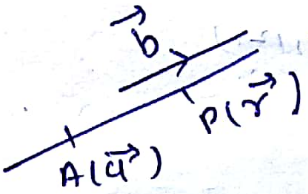
(b) find the $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

Line in a Space :→

- a Line in a space is uniquely determine if it passes threw a given Pt & has given direction.

1. Eqⁿ of line Passing threw a given Pt ($\vec{a}$) & hase given direction along ($\vec{b}$) is

$$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$$



$$\vec{AP} = \lambda\vec{b}$$

$$\vec{r} - \vec{a} = \lambda\vec{b}$$

$$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$$

Cartesion form

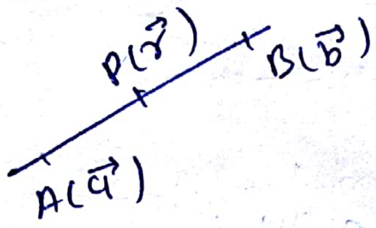
$$(x\hat{i} + y\hat{j} + z\hat{k}) = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a_1\hat{i} + b_1\hat{j} + c_1\hat{k})$$

$$x = x_1 + \lambda a$$

$$\boxed{\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} = \lambda}$$

2. Eqⁿ of line Passing threw two Points $A(\vec{a})$ & $b(\vec{b})$ is given by:

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$$



$$\vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$$

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$$

Ex: find the vector & cartesian Eqn of the line Passing through the pts $(4,2,-3)$ & $(-1,1,-1)$

Soln $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$\vec{r} = (4\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(-5\hat{i} - \hat{j} + 2\hat{k})$

$$\frac{x-4}{-1-4} = \frac{y-2}{1-2} = \frac{z+3}{-1+3} \Rightarrow \boxed{\frac{x-4}{-5} = \frac{y-2}{-1} = \frac{z+3}{2}}$$

Ex find the dc's of the vector Eqn of the line

$6x-2 = 3y+1 = 2z-2$

Soln $\frac{x-\cancel{1/3}}{\cancel{1}2} = \frac{y+\cancel{1/3}}{2} \Rightarrow \frac{x-1/3}{1/6} = \frac{y+1/3}{1/3} = \frac{z-1}{1/2}$

Dc's $\vec{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + 1\hat{k}\right) + \lambda\left(\frac{1}{6}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{2}\hat{k}\right)$

Ex: find the Dc's of the line

$\frac{x-2}{2} = \frac{2y-5}{-3}$; $z = -1$

Soln $\frac{x-2}{2} = \frac{y-5/2}{-3/2}$; $\frac{z+1}{0}$

fix Pt $(2, 5/2, -1)$ Dr's $= (2, -3/2, 0)$

Dc's $= \left(\frac{4}{5}, -3/5, 0\right)$

Angle b/w two lines :-

$\vec{r_1} = \vec{a_1} + \lambda\vec{b_1}$

$\vec{r_2} = \vec{a_2} + \mu\vec{b_2}$

Acute angle b/w the lines is θ

$$\boxed{\cos\theta = \left|\frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|}\right|}$$

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Q. find the angle b/w the lines $\vec{r} = -2\hat{i} + \hat{j} + 1\hat{k} + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$

(i) $\vec{r} = \hat{i} - 2\hat{j} + 1\hat{k} + \mu(3\hat{i} - 6\hat{j} + 2\hat{k})$

$$\cos\theta = \left| \frac{3 - 12 - 4}{\sqrt{1+4+4}\sqrt{9+36+4}} \right| = \frac{13}{21}$$

$$\theta = \cos^{-1}\left(\frac{13}{21}\right) \quad \text{or} \quad \pi - \cos^{-1}\left(\frac{13}{21}\right)$$

Que if the lines $\frac{x+3}{p+2} = \frac{y+3}{-2p} = \frac{z}{13}$ &

$\frac{x-1}{p-3} = \frac{y+2}{4} = \frac{z-3}{2}$ are perpendicular for. find P.

Sol $\vec{b}_1 \cdot \vec{b}_2 = 0$

$(p+2)(p-3) - 2p(4) + 26 = 0$

$p^2 - p - 8p - 6 + 26 = 0$

$p^2 - 9p + 20 = 0$

$p = 5, 4$

Q. find the angle b/w the lines whose DCs satisfy

$3l + m + 5n = 0$

$6nm - 2nl + 5lm = 0$

HW

Sol $m(6n + 5l) - 2nl = 0$

$(6n + 5l)(-3l - 5n) - 2nl = 0$

$[(5l + 6n)(3l + 5n) + 2nl] = 0$

$15l^2 + 45ln + 30n^2 = 0$

$5l^2 + 9ln + 6n^2 = 0$

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$3l^2 + 6ln + 3mn + 6n^2 = 0$

$3l(l+2n) + 3n(l+2n) = 0$

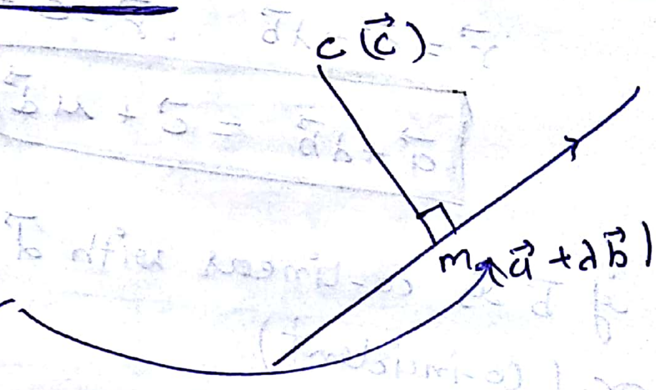
$(3l+3n)(l+2n) = 0$

$l+n=0$	$l=-2n$
$l=-n$	$m=n$
$m=-2n$	
$l \quad m \quad n$	$l \quad m \quad n$
$-n \quad -2n \quad n$	$-2n \quad n \quad n$
DRs $\langle -1, -2, 1\rangle$	DRs $(-2, 1, 1)$

$$\cos\theta = \frac{2-2+1}{\sqrt{6}\sqrt{6}} = \frac{1}{6}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{6}\right)}$$

Projection of a Point on the line :→



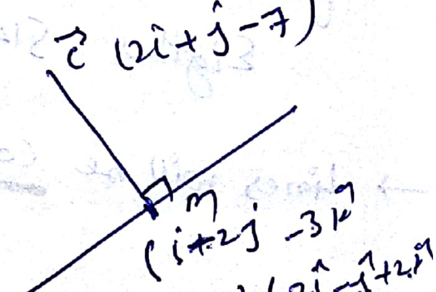
$\vec{CM}\cdot\vec{b} = 0$

$$\boxed{(\vec{a}+\lambda\vec{b}-\vec{c})\cdot\vec{b} = 0}$$

$\lambda = ?$

Ex find the Projection of a Pt $(2,1,-7)$ & line

$$\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$$



$\vec{CM}\cdot\vec{b} = 0$

$(-\hat{i} - 3\hat{i} + 4\hat{k} + \lambda(2\hat{i}-\hat{j}+2\hat{k}))\cdot(2\hat{i}-\hat{j}+2\hat{k}) = 0$

$(-2 + 2\lambda + 3 + \lambda + 8 + 4\lambda) = 0$

$9\lambda + 9 = 0$

$\boxed{\lambda = -1}$ $\quad M \Rightarrow (-\hat{i} - \hat{j} - 5\hat{k})$

Distance $\Rightarrow \sqrt{9+4+4} = \sqrt{17}$

Que n.10: find the co-ordinats of foot of Prependicular from the origin to the ling Passing threw $(-9,4,5)$ & $(11,0,-1)$.

Intersection of two lines :→

$$\vec{r} = \vec{a} + \lambda\vec{b} \quad , \vec{r} = \vec{c} + \mu\vec{d}$$

$$\boxed{\vec{a} + \lambda\vec{b} = \vec{c} + \mu\vec{d}}$$

→ if $\vec{b}$ is co-linear with $\vec{d}$ then lines are Parallel or (co-incident)

→ if $\vec{b}$ is not- co-linear with $\vec{d}$ then lins are Eigther Skew or intersecting.

→ Lines will be co-planer if $\boxed{[\vec{a}-\vec{c} \quad \vec{b} \quad \vec{d}] = 0}$

Castission form

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} = \lambda$$

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} = \mu$$

from intersection point.

$a_1\lambda + x_1 = a_2\mu + x_2$, $b_1\lambda + y_1 = b_2\mu + y_2$, $c_1\lambda + z_1 = c_2\mu + z_2$

$\lambda = 2$, $\mu = ?$

Lines will be co-planir if

$$\boxed{\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0}$$

Q find the pt of intersection of the lines

$\frac{x-3}{1} = \frac{y-4}{1} = \frac{z+5}{1}$ & $\frac{x-6}{1} = \frac{y-2}{-4} = \frac{z-4}{7}$

Sol^n

$\frac{x-3}{1} = \frac{y-4}{1} = \frac{z+5}{1} = \lambda$

$x = \lambda + 3$
$y = \lambda + 4$
$z = \lambda - 5$

$\frac{x-6}{1} = \frac{y-2}{-4} = \frac{z-4}{7} = \mu$

$x = \mu + 6$
$y = -4\mu + 2$
$z = 7\mu + 4$

$\lambda + 3 = \mu + 6$

$\lambda - \mu = 3$

$\lambda + 4 = -4\mu + 2$

$\lambda + 4\mu = -2$

$(\lambda = 2, \mu = -1)$

$(\lambda - 5) = (7\mu + 4)$

pt is $(5, 6, -3)$

HOW ① find the nature of lines if intersection find them

(i) $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-7}{5}$ & $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$

(ii) $\vec{r} = 2\hat{k} + \lambda(3\hat{i}+2\hat{j}+\hat{k})$ & $(3\hat{i}+2\hat{j}+3\hat{k}) + \mu(6\hat{i}+4\hat{j}+2\hat{k})$

② find value of k in the lines $\frac{x-3}{1} = \frac{y-4}{1} = \frac{z+5}{1}$ &

$\frac{x-6}{1} = \frac{y-k}{-4} = \frac{z-4}{7}$ are co-planer.

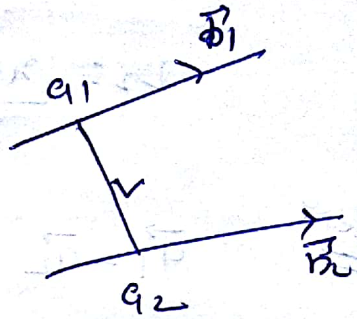
Shortest Distance b/w two Lines :→

(i) if two lines are intersecting the shortest distance is ze[ro]

(ii) Skew lines:

$\vec{r} = \vec{a_1} + \lambda \vec{b_1}$

$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$



direction of shortest Distance is $(\vec{b_1} \times \vec{b_2})$

S.D. is Projection $(\vec{a_2} - \vec{a_1})$ on $\vec{b_1} \times \vec{b_2}$

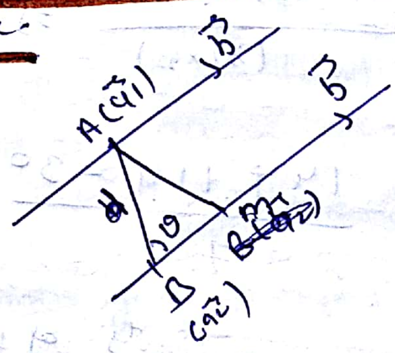
$$\boxed{\text{Shortest Distance} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|}$$

$$\boxed{\Rightarrow \left| \frac{[\vec{a_2} - \vec{a_1} \quad \vec{b_1} \quad \vec{b_2}]}{|\vec{b_1} \times \vec{b_2}|} \right|}$$

(iii) if lines are Parallel then shortest Distance b/w them is Prependicular Distance.

$\vec{r} = \vec{a_1} + \lambda \vec{b}$

$\vec{r} = \vec{a_2} + \mu \vec{b}$



$S.D = AB \sin\theta$

$= \dfrac{|\vec{a_1} - \vec{a_2}| \sin\theta \, |\vec{b}|}{|\vec{b}|}$

$$\boxed{S.D = \frac{|(\vec{a_1} - \vec{a_2}) \times \vec{b}|}{|\vec{b}|}}$$

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Que find the Shortes Distance b/w the lines

$$L_1 = \frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$$

$$L_2 : \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Sol^n

$$\vec{a_1} - \vec{a_2} = 7\hat{i} + 38\hat{j} - 5\hat{k}$$

$$\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}, \quad \vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} i & j & k \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = i(80-56) - j(-15-21) + k(24+48)$$

$$(\vec{b_1} \times \vec{b_2}) = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$$(\vec{b_1} \times \vec{b_2}) = 12(2\hat{i} + 3\hat{j} + 6\hat{k})$$

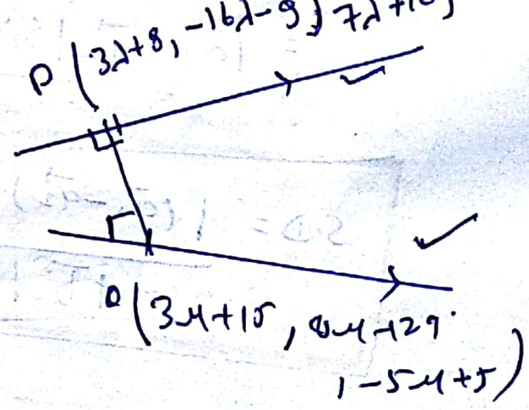
$$S.D: \frac{(\vec{a_1} - \vec{a_2})(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \Rightarrow \frac{(7\hat{i} + 38\hat{j} - 5\hat{k}) \times \cancel{(12)} \cdot (2\hat{i} + 3\hat{j} + 6\hat{k})}{\cancel{12}\sqrt{4+9+36}}$$

$$\Rightarrow \frac{14 + 114 - 30}{7} \Rightarrow \frac{98}{7} \Rightarrow \underline{\underline{14}}$$

13, 7, 128, 30

(ii) find the co-ordinats of the Pt on the line for which Shortest Distance occurs

$\mu, \lambda = ?$



Que find the Shortest Distance b/w the lines:

$$\frac{x-2}{-1} = \frac{y-4}{2} = \frac{z+1}{4} \quad \& \quad \frac{x+3}{1} = \frac{y-2}{-2} = \frac{z-2}{-4}$$

Sol^n

$\vec{a}_1 - \vec{a}_2 = 5\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} - 4\hat{k}$

$$S.D = \frac{|(a_1 - a_2) \times (\vec{b})|}{|\vec{b}|} \Rightarrow$$

$$(a_1 - a_2) \times (\vec{b}) = \begin{vmatrix} i & j & k \\ 5 & 2 & -3 \\ 1 & -2 & -4 \end{vmatrix} \Rightarrow i(-8-6) - j(-20+3) + \hat{k}(-10-2)$$

$$\Rightarrow -14\hat{i} + 17\hat{j} - 12\hat{k}$$

$$S.D = \frac{\sqrt{196+144+289}}{\sqrt{1+4+16}} = \frac{\sqrt{629}}{\sqrt{21}} =$$

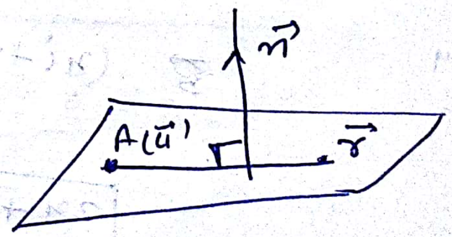
Plane :→ A Plane is a Surface such that a line Segment joining any two Pts on the Surface. Completely lying on it

a Plane is uniquely determin if it Passes threw a Point & Prependicular to a given direction

to find the Eqn of Plane if

① it Passes threw a Point $A(\vec{a})$ & Prependicular to $\vec{n}$:

$$\boxed{(\vec{r} - \vec{a}) \cdot \vec{n} = 0}$$



Cartesian :→

$A(x_1, y_1, z_1)$, $\vec{n}(a\hat{i}+b\hat{j}+c\hat{k})$

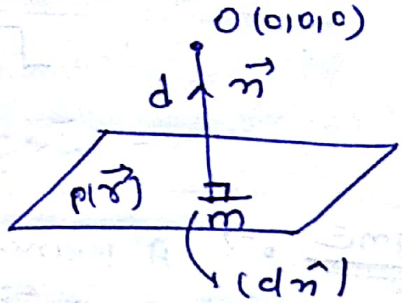
$$\boxed{(x-x_1)a+(y-y_1)b+(z-z_1)c=0}$$

Que find the vector & Cartesian Eqn of the Plane which passes through the Point $(-2,5,1)$ & Perpendicular to the line whose DRs are $(1,3,-2)$

Soln
$$(x+2)1+(y-5)3+(z-1)(-2)=0$$

$$\boxed{x+3y-2z=11}$$

(ii) Normal vector from origin to the Plane is along $\vec{n}$ & Perpendicular from origin is d



$$\vec{om} = d\hat{n}$$

$$\vec{mp}\cdot\hat{n} = 0$$

$$(\vec{r}-d\hat{n})\cdot\hat{n}=0$$

$$\boxed{\vec{r}\cdot\hat{n}=d}$$

unit vector

Que find the Eqn of the Plane whose Perpendicular distance from origin is 5 & normal from origin is $(2\hat{i}+6\hat{j}-3\hat{k})$

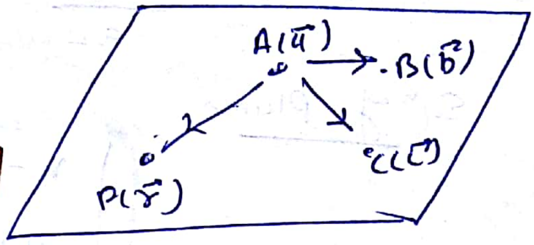
Soln
$$\vec{r}\quad (x\hat{i}+y\hat{j}+z\hat{k})\left(\frac{2\hat{i}+6\hat{j}-3\hat{k}}{7}\right)=5$$

$$\boxed{2x+6y-3z=35}$$

(iii) it passes through three non co-linear points :

A.t.a $A(x_1,y_1,z_1)$ $B(x_2,y_2,z_2)$ & $C(x_3,y_3,z_3)$

$$\boxed{[\vec{AP}\ \ \vec{AB}\ \ \vec{AC}]=0}$$



$$\boxed{[\vec r-\vec a\ \ \ \vec b-\vec a\ \ \ \vec c-\vec a]=0}$$

Cartesian

$$\boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0}$$

Ex: $(2,2,-1)$, $(3,4,2)$, $(7,0,6)$ — plane ka eqn

Soln

$$\begin{vmatrix} x-7 & y & z-6 \\ -5 & 2 & -7 \\ -4 & 4 & -4 \end{vmatrix}=0 \rightarrow (x-7)(-8+28)$$
$$-y(20-28)$$
$$+(z-6)(-20+8)=0$$

$\Rightarrow 20(x-7)+8(y)-12(z-6)=0$

$\Rightarrow 5(x-7)+2y-3(z-6)=0$

$=\boxed{5x+2y-3z=17}$

(iv) it contains a line L_1 & parallel to the line L_2.

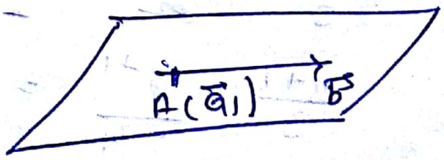
$r=\vec{a_1}+\lambda\vec b$

$r=\vec{a_2}+\mu\vec c$

normal vector is along $\vec b\times\vec c$

Eqn of Plane

$$\boxed{(\vec r-\vec{a_1})\cdot(\vec b\times\vec c)=0}$$



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Cartesian $L_1: \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

$L_2 = \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

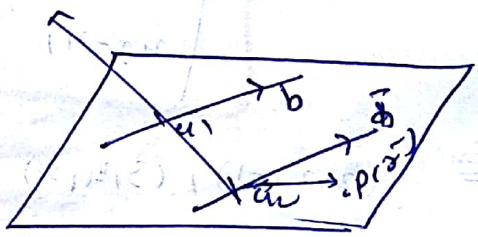
Eqn of Plane

$$\boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0}$$

(v) it contains two Parallel lines :-

L_1: $\vec{r} = \vec{a_1} + \lambda \vec{b}$

$\vec{r} = \vec{a_2} + \mu \vec{b}$



normal vector to the Plane

$(\vec{a_1} - \vec{a_2}) \times \vec{b}$.

Eqn of Plane

$$\boxed{(\vec{r} - \vec{a_1})((\vec{a_1} - \vec{a_2}) \times \vec{b}) = 0}$$

$[\vec{r} - \vec{a_1} \quad \vec{a_1} - \vec{a_2} \quad \vec{b}] = 0$

Cartesian

$$\boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a & b & c \end{vmatrix} = 0}$$

Q. find the Eqn of plane Containing the line

$\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2}$ & Parallel to the line

$\frac{x+1}{3} = \frac{y+1}{-4} = \frac{z}{1}$

Sol^n

$$\begin{vmatrix} x-1 & y-4 & z-4 \\ 3 & 2 & -2 \\ & & 1 \end{vmatrix} = 0$$

$(x-1)(2-8)+(y-4)(3+6)+(z-4)(-12-6)=0$

$(x-1)(-6)+(y-4)(9)+(z-4)(-18)=0$

$2(x-1)+(-3)(y-4)+6(z-4)=0$

$2x-3y+6z-2+12-24=0$

$$\boxed{2x-3y+6z=14}$$

Q. Containing two Parallel Lines

$$\frac{x+2}{-1}=\frac{y-3}{-2}=\frac{z-1}{2} \ \& \ \frac{x-5}{1}=\frac{y+1}{2}=\frac{z+3}{-2}$$

Sol^n

$$\begin{vmatrix} x+2 & y-3 & z-1 \\ -7 & 4 & 4 \\ 1 & 2 & -2 \end{vmatrix}=0$$

$(x+2)(-8-8)-(y-3)(14-4)+(z-1)(-14-4)=0$

$(-16)(x+2)-10(y-3)+(z-1)(-18)=0$

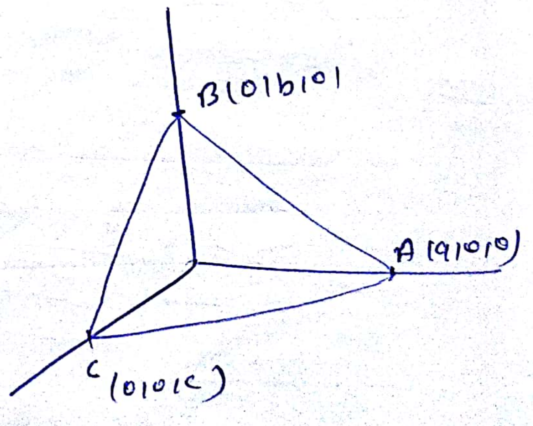
$8(x+2)+5(y-3)+9(z-1)=0$

$8x+5y+9z+16-15-9=0$

$$\boxed{8x+5y+9z=8}$$

(vi) Eq^n of Plane whose x, y & z-intercepts are a, b, & c respectively.

is $\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$



~~Ans~~

Area $\Delta ABC=\frac{1}{2}|\vec{AB}\times\vec{BC}|$

$=\frac{1}{2}\left|(-a\hat{i}+b\hat{j})\times(-a\hat{i}+c\hat{k})\right|$

$=\frac{1}{2}|$

$$= \frac{1}{2} \left| ac\hat{j} + ab\hat{k} + bc\hat{i} \right|$$

$$\boxed{\text{area } \Delta ABC = \frac{1}{2} \sqrt{(ab)^2 + (bc)^2 + (ac)^2}}$$

<u>volume of tetrahedron</u> $= \frac{1}{6} [\vec{OA} \ \ \vec{OB} \ \ \vec{OC}]$

$$= \frac{1}{6} [a\hat{i} \ \ b\hat{j} \ \ c\hat{k}]$$

$$\boxed{\text{volume of tetrahedron} = \frac{abc}{6}}$$

<u>Que</u>

$2x + 3y + 6z = 12$

Plane intercept x, y, z axes at A, B, C respectively

Find Area of Δ ABC, & volume of tetrahedron.

<u>Soln</u>

$$\frac{x}{6} + \frac{y}{4} + \frac{z}{2} = 1$$

Area of ABC $= \frac{1}{2} \sqrt{(24)^2 + (8)^2 + (12)^2} = \frac{1}{2} \sqrt{576 + 64 + 144}$

$\Rightarrow \frac{1}{2} \sqrt{784} \Rightarrow \sqrt{196} = 14$

625
49

16

576
64
144

784

volume of ABC $= \frac{abc}{6} = \frac{(6)(4)(2)}{6} = 8$

angle b/w two Plans :→

$$\vec{r}\cdot\vec{n_1} = d_1 \quad , \quad \vec{r}\cdot\vec{n_2} = d_2$$

Let θ is acute angle b/w Planes.

$$\boxed{\cos\theta = \left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|}$$

$\pi_1 : A_1x + B_1y + C_1z + d_1 = 0$

$\pi_2 : A_2x + B_2y + C_2z + D_2 = 0$

$$\boxed{\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|}$$

Q find angle b/w the Planes $2x+y-2z=6$ & $-3x+2y+6z=0$

Sol^n

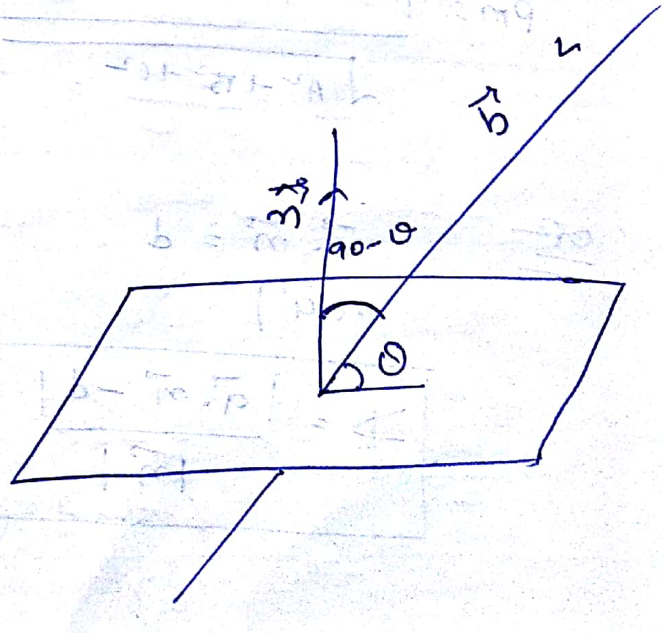
$$\cos\theta = \left|\frac{-6+2-12}{\sqrt{4+1+4}\sqrt{9+36+4}}\right| = \left|\frac{-16}{3\times 7}\right|$$

acute angle $\theta = \cos^{-1}\left(\frac{16}{21}\right)$ optuse ang. $\phi = \pi - \theta$

Angle b/w Plane & a line.

$\pi : \vec{r}\cdot\vec{n} = d$

$L : \vec{r} = \vec{a} + \lambda\vec{b}$



$$\cos(90-\theta) = \frac{\vec{b}\cdot\vec{n}}{|\vec{n}||\vec{b}|}$$

$$\boxed{\sin\theta = \frac{\vec{b}\cdot\vec{m}}{|\vec{b}|\,|\vec{m}|}}$$

2

Q Find the angle b/w the line $\frac{x-2}{1} = \frac{y+2}{-2} = \frac{z+3}{2}$ & the plane $10x - 2y + 11z = 20$

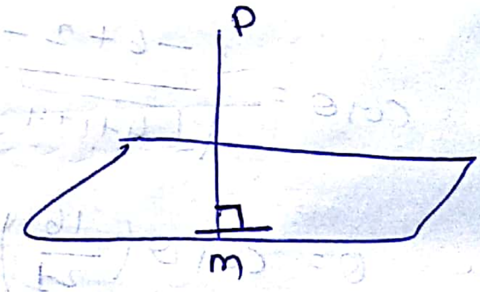
Sol^n $\sin\theta = \left| \frac{10 + 4 + 22}{\sqrt{121+100+4}\,\sqrt{1+4+4}} \right| = \left| \frac{\cancel{36}}{\cancel{3\times 15}} \right|$

$\sin\theta = 4/5$ $\quad \underline{\theta = \sin^{-1}(4/5)}$ acute angle

Distance of a Point from a Plane

$\pi : Ax + By + cz + D = 0$

$P(x_1, y_1, z_1)$



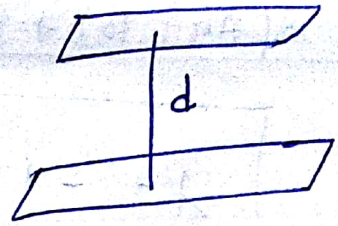
$$Pm = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

OR $\vec{r}\cdot\vec{n} = d$

$P(\vec{a})$

$$\boxed{D = \frac{|\vec{a}\cdot\vec{n} - d|}{|\vec{n}|}}$$

Distance B/w two Parallel Planes



$\pi_1 : Ax + By + Cz + D_1 = 0$

$\pi_2 = Ax + By + Cz + D_2 = 0$

$$\boxed{d = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}}$$

Que (i) find the Distance of the pt $(-2, 4, 3)$ from the plane

$10x + 2y - 11z = 3$

Sol^n

$$d = \frac{|-20 + 8 - 33|}{\sqrt{100 + 4 + 121}} \quad -12 \quad -3 \quad = \frac{48}{15} = 8\ 16/5$$

(ii) find the Distance b/w the planes

$\vec{r}.(2\hat{i} + \hat{j} - 2\hat{k}) = 4$ $ \vec{r}.\ \vec{r}.$

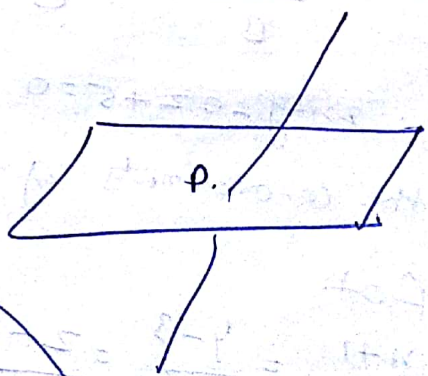
$\vec{r}(4\hat{i} + 2\hat{j} - 4\hat{k}) = 15$

$$d = \frac{|15 - 8|}{\sqrt{16 + 4 + 16}} = \frac{7}{6}$$

Intersection of line & a Plane :

$\pi : Ax + By + Cz + D = 0$

$L : \frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$



any pt on line is

$(a\lambda + x_1, b\lambda + y_1, c\lambda + z_1)$

lies on Plane

$A(a\lambda + x_1) + B(b\lambda + y_1) + C(c\lambda + z_1) + D = 0$

$\lambda = ?$

$$\lambda(Aa+Bb+Cc) = -(Ax_1+By_1+Cz_1+D)$$

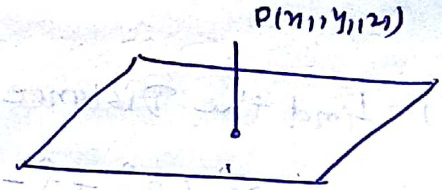
$$\boxed{\lambda = -\left(\frac{Ax_1+By_1+Cz_1+D}{Aa+Bb+Cc}\right)}$$

→ if $Aa+Bb+Cc=0$ then line is parallel to the plane

→ if $Aa+Bb+Cc=0$ & (x_1,y_1,z_1) lies on the plane then plane contains the line.

co-ordinates of foot of perpendicular & image of a pt :-

$\pi: Ax+By+Cz+D=0$



foot of perpendicular

$$\frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{C} = -1\left(\frac{Ax_1+By_1+Cz_1+D}{A^2+B^2+C^2}\right)$$

co-ordinates of image of pt P

$$\frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{C} = -2\left(\frac{Ax_1+By_1+Cz_1+D}{A^2+B^2+C^2}\right)$$

$\frac{6}{9}=\frac{2}{3}$

Q $\pi: 2x+y-2z+5=0$ $\quad P(-1,3,0)$

find the co-ordinates of foot of perpendicular & image of pt P

Sol^n foot

$$\frac{x+1}{2} = \frac{y-3}{1} = \frac{z-0}{-2} = -1\left(\frac{-2+3+5}{4+1+4}\right) = -\left(\frac{2}{3}\right)$$

foot $\left(-7/3, 7/3, 4/3\right)$

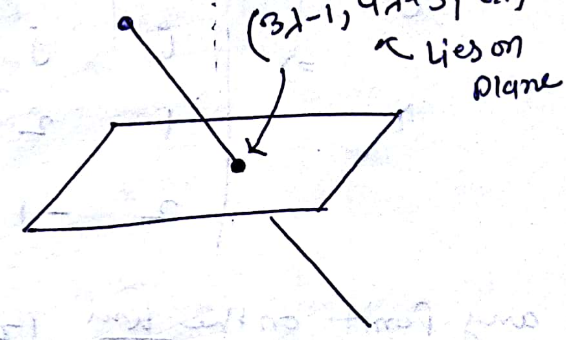
Image $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z}{-2} = -2\left(\frac{2}{3}\right)$

Image $\left(-11/3, 5/3, 8/3\right)$

Q $\pi : 2x + y - 2z + 8 = 0$, $L: \frac{x+1}{3} = \frac{y-3}{4} = \frac{z-0}{6} = \lambda$ (sai)

(i) intersection Point (ii) Eqn of Image line (iii) Eqn of Projection line

(iv) find the angle b/w line & plane.



Soln (i) $2(3\lambda-1)+(4\lambda+3)-2(6\lambda)+8=0$

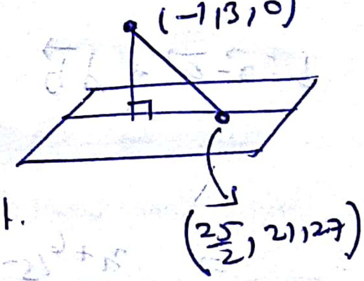
$6\lambda - 2 + 4\lambda + 3 - 12\lambda + 8 = 0$

$-2\lambda = -9$, $\lambda = 9/2$

Pt of intersection $= \left(\frac{25}{2}, 21, 27\right)$

(iv) Angle b/w line & plane $\sin\theta = \frac{(3i+4j+6k)(2i+j-2k)}{\sqrt{9+16+36}\,\sqrt{4+1+4}}$

$\sin\theta = \frac{|6+4-12|}{\sqrt{61}\,(3)} = \frac{2}{3\sqrt{61}}$



(iii) Foot of perpendicular

$\frac{x+1}{2} = \frac{y-3}{1} = \frac{z-0}{-2} = -1\left(\frac{-2+3+8}{4+1+4}\right) = -1.$

Foot ~~x=3~~ ~~(−3,~~ $(-3, 2, 2)$

$\boxed{\vec{r} = (\vec{a}) + \lambda(\vec{b} - \vec{a})}$

$\vec{a}$ $(-1,2,2)$ —— $\vec{b}$ $\left(\frac{25}{2}, 21, 27\right)$

(ii) for Image

pt $(-5, 1, 4)$

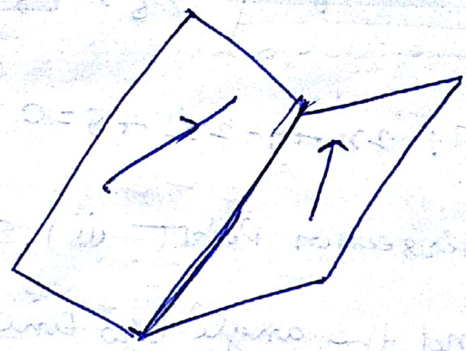
$\vec{c}$ $(-3,1,4)$ —— $\vec{d}$ $\left(\frac{25}{2}, 21, 27\right)$

Eqn $\boxed{\vec{r} = (\vec{a}) + \lambda(\vec{d} - \vec{c})}$

Intersection of two Planes :

$\pi_1 = x+2y-z-4=0$

$\pi_2 = 2x-y+2z+5=0$



d r's of line = $\vec{n}_1 \times \vec{n}_2$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 2 \end{vmatrix} = 3\hat{i} - 4\hat{j} - 5\hat{k}$$

any Point on the line $(z=0)$

$x+2y=4$
$2x-y=-5$ $\Rightarrow$ $5x=-6$

$\left(x=-6/5,\ 13/5,\ 0\right)$

$L: \vec{r} = \vec{a} + \lambda \vec{b}$

$$\Rightarrow \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\Rightarrow \frac{x+6/5}{3} = \frac{y-13/5}{-4} = \frac{z-0}{-5}$$

~~#~~ family of Plane :→

Eqn of Plane Passing threw intersection of Planes π_1 & π_2

is $\boxed{\pi_1 + \lambda \pi_2 = 0}$,

$\pi_1 = A_1x + B_1y + C_1z + D_1 = 0$

$\pi_2 = A_2x + B_2y + C_2z + D_2 = 0$

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Q. Find the Eqⁿ of Plane Passing through intersection of the Plane $x+2y-3z-5=0$ & $2x+y+z-7=0$ &

(i) which Passes through origin

$\pi = \pi_1 + \lambda \pi_2 = 0$

$\pi = (1+2\lambda)x + (2+\lambda)y + (-3+\lambda)z - 5 - 7\lambda = 0$

(0, 0, 0)

$-5-7\lambda=0$

$\lambda = -5/7$

(ii) which is Perpendicular to $2x+y+z-7=0$

$\vec{\pi} \cdot \vec{\pi_2} = 0 \quad \{\cos\theta = 0\}$

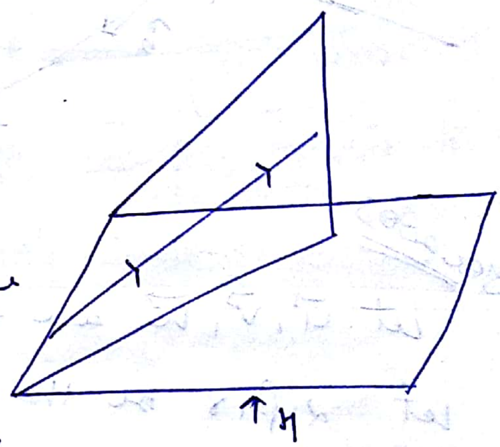
$2(1+2\lambda) + 1(2+\lambda) + 1(-3+\lambda) = 0$

$2 + 4\lambda + 2 + \lambda - 3 + \lambda = 0$

$6\lambda = -1$

$\boxed{\lambda = -1/6}$

Line of greatest slope :→



Que assume the plane $4x-3y+7z=0$ as horizontal Plane & $2x+y-5z=0$ as inclined Plane. Find the Eqⁿ of line of greatest slope. Passing through (2, 1, 1)

Solⁿ let $\vec{n_1} = 4\hat{i} - 3\hat{j} + 7\hat{k}$

$\vec{n_2} = 2\hat{i} + \hat{j} - 5\hat{k}$

dr's of req. line $(\vec{n_1} \times \vec{n_2}) \times \vec{n_2} = (\vec{n_1} \cdot \vec{n_2}) \cdot \vec{n_2} - (|\vec{n_2}|^2) \cdot \vec{n_1}$

$= -30(\vec{n_2}) - 30(\vec{n_1})$

$= -30(\vec{n_1} + \vec{n_2})$

$= -30(6\hat{i} - 2\hat{j} + 2\hat{k})$

Eqⁿ of line

$$\frac{x-2}{6} = \frac{y-1}{-2} = \frac{z-1}{2}$$

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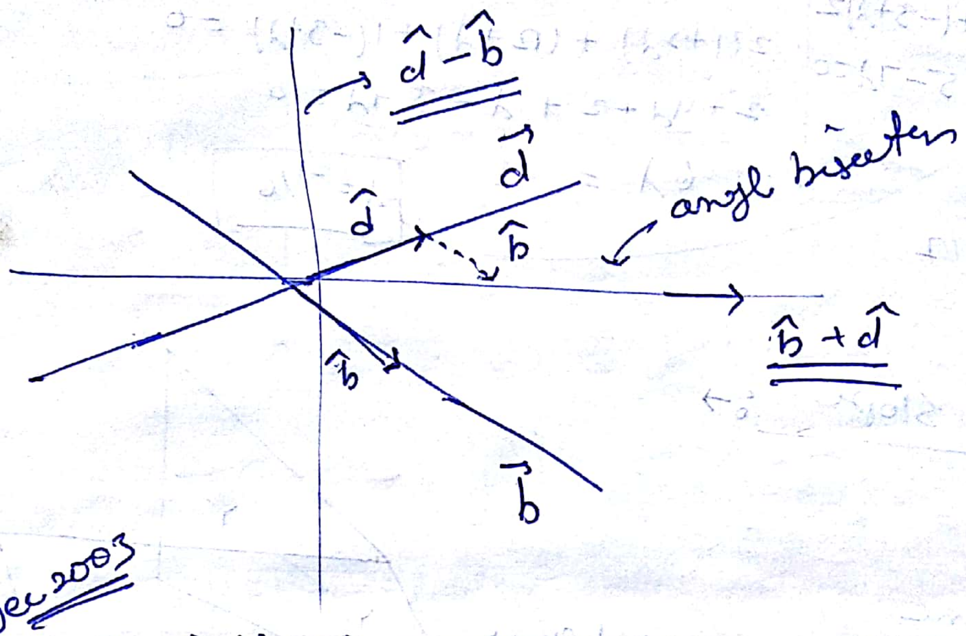
Angle bisectors of Plane

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\left(\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$



Jee 2003

Q Let $\vec{u}, \vec{v}, \vec{w}$ are three non-co-planes unit vector Let α, β, γ be the angles b/w $\vec{u}$ & $\vec{v}$, $\vec{v}$ & $\vec{w}$ & $\vec{w}$ & $\vec{u}$ respectively let $\vec{a}, \vec{b}$ & $\vec{c}$ be unit vectors along the bisectors of the angles α, β, γ respectively. P.T.

$$[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}] = \frac{1}{16}[\vec{u}\,\vec{v}\,\vec{w}]^2 \sec^2\alpha/_2 \sec^2\beta/_2 \sec^2\gamma/_2$$

Sol^n $\vec{a} = \dfrac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}$ $\qquad |\vec{u}+\vec{v}| = \sqrt{1+1+2\cos\alpha}$

$|\vec{u}+\vec{v}| = 2\cos\alpha/_2$

$$\vec{a} = \frac{\vec{u}+\vec{v}}{2\cos\alpha/_2}, \quad \vec{b} = \frac{\vec{v}+\vec{w}}{2\cos\beta/_2}, \quad \vec{c} = \frac{\vec{u}+\vec{w}}{2\cos\gamma/_2}$$

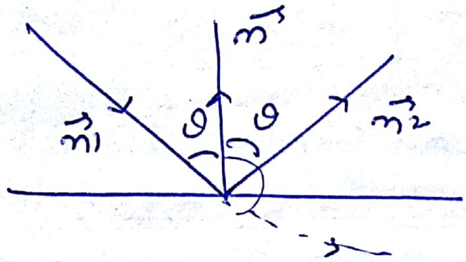
$$[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}] = [abc]^2$$

$$\Rightarrow \left[\frac{\vec{u}+\vec{v}}{2\cos\alpha/_2} \quad \frac{\vec{v}+\vec{w}}{2\cos\beta/_2} \quad \frac{\vec{w}+\vec{u}}{2\cos\gamma/_2}\right]^2$$

$$\Rightarrow \left(\frac{1}{8}\sec\alpha/_2 \cos\beta/_2 \sec\gamma/_2\right)^2 \underbrace{[\vec{u}+\vec{v} \quad \vec{v}+\vec{w} \quad \vec{w}+\vec{u}]^2}_{2[abc]}$$

$$\Rightarrow \frac{1}{16}(\sec^2\alpha/_2 \sec^2\beta/_2 \sec^2\gamma/_2)\, \cancel{[4u]}\, [\vec{u}\,\vec{v}\,\vec{w}]^2$$

Que



$$\boxed{\vec{n_2} = \vec{n_1} - 2(n\cdot n_1)\hat{n}}$$

$$\hat{n}_2 = x\hat{n}_1 + y\hat{n}$$

dot with $\hat{n}_1$

$$\hat{n}_2\cdot\hat{n}_1 = x(1) + y(\hat{n}\cdot\hat{n}_2)$$

$$-\cos 2\theta = x + y\cos\theta \longrightarrow ①$$

dot with $\hat{n}_2$

$$1 = -x\cos 2\theta + y\cos\theta \longrightarrow ②$$

$$x(1+\cos 2\theta) = \cos 2\theta + 1 \qquad \boxed{x = +1}$$

$$\boxed{y = 2\cos\theta}$$

$$\cos\theta = -\hat{n}_1 - \hat{n}$$

$$\hat{n}_2 = x\hat{n}_1 + y\hat{n}$$

$$\hat{n}_2 = \hat{n}_1 + (2\cos\theta)\hat{n}$$

$$\boxed{\hat{n}_2 = \hat{n}_1 - 2(\hat{n}_1\cdot\hat{n})\hat{n}}$$