14 feb. 2017

Probability

Random Enpariment :→ An Enpariment where all possible outcomes are know in advanced is called Random Enpariment.

In Random Enpriment Result of an Specific outcome can'nt be predicted.

Ex: tossing a Coin, Rolling a Dice.

Sample Space: the set of all possible outcomes of a Random Enpriment is called Sample Space.

Ex. E: tossing a Coin

$S = \{H, T\}$

Event: an event is the any Subset of the Sample Space.

Ex: $S = \{1, 2, 3, 4, 5, 6\}$

E: getting a Prime no.

$\{2, 3, 5\}$

types of Event: (i) **Simple event**: An Event is called Simple Event if it hass enactuly one possible outcome.

Ex: getting an even prime no. on Rolling a Dice.
→ $\{2\}$

(ii) **Compound Event**: Event which has more the one possible outcome is called Compound Eve.

Ex | Coming even no.)

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(iii) Mutually Exclusive Event :- two or more events are said to be mutualy Exclusive if they cannot occur simultaniously.

if A & B are two Events which are mutually Exclusive then $(A \cap B = \phi)$

Ex.: A = getting an even no.
B = getting an odd no.
$A \cap B = \phi$

(iv) Equally-Like Event :→ Events are said to be Equally likely if there is no reason for one Event to occur in preference to any other Event.

(v) Exaustive Event (परिपूर्ण घटनाएं) : if E_1, E_2, E_3 till E_n are n Events define on Sample Space "S" &

$$\boxed{E_1 \cup E_2 \cup E_3 \cdots \cup E_n = S}$$

then Events are said to be Exaustive Events.

Ex: $S = \{1, 2, 3, 4, 5, 6\}$
E_1 : getting an Even no. = $\{2, 4, 6\}$
E_2 : getting an odd no. = $\{1, 3, 5\}$
E_3 : getting an prime no = $\{2, 3, 5\}$

Classical defination of Probability :→ the Probability of an Event E is the ratio of no. of cases in it's favour to the total no. of cases.

(all cases must be Equally likely)

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{no. of cases in favour of } E}{\text{no. of elements in Sample Space}}.$$

$\boxed{0 \leq P(E) \leq 1}$ $\boxed{P(E') = 1 - P(E)}$

$\rightarrow P(E) = \frac{a}{a+b}$

then odds in favour of E is a/b

then odds against event E is b/a

Ex: $P(E) = 5/8$ $\rightarrow$ odds in favour of $E = 5/3$

= odds against of $E = 3/5$

Ex: odds in favour in of $E = 7/11$

$P(E) = \frac{7}{18}$

Q if three coin are tossed simultaneously the find the Probability of getting (i) Exactly 2H (ii) atleast 2H (iii) atmost 2H

Sol $S = \begin{Bmatrix} HHH, & THH \\ HHT, & THT \\ HTH, & TTH \\ HTT, & TTT \end{Bmatrix}$

(i) $P = 3/8$ (ii) $4/8$ (iii) $7/8$

Q if 3 Red & 5 Green are arranged in a row then find the Probability that Extrem balls are always red.

Sol $R + + + + + + R$ $\frac{\lfloor 6}{\lfloor 5} \cdot {}^{3}C_2$

total $= \frac{\lfloor 8}{\lfloor 5 \lfloor 3}$

$P(E) = \dfrac{\frac{\lfloor 6}{\lfloor 5}}{\frac{\lfloor 8}{\lfloor 5 \lfloor 3}} = \frac{\lfloor 6 \lfloor 3}{\lfloor 8} = \frac{3}{28}$

(ii) Let Balls are different

$P(E) = \frac{({}^{3}C_2 \lfloor 2)(\lfloor 6)}{\lfloor 8} = \frac{3 \cdot 2}{8 \cdot 7} = \frac{3}{28}$

Que two cards are selected from the pack of 52 cards find the probability find both the cards of spade.

Sol^n

$$P(E) = \frac{{}^{13}C_2}{{}^{52}C_2} = \frac{1}{17}$$

or

$$P(E) = \frac{13 \cdot 12}{52 \cdot 51} = \frac{1}{17}$$

Que: Pair of disc is rolled then find the probability.
(i) getting a sum of 10 (ii) — 8 (iii) 7 (iv) 5
(v) atlest 9

Sol^n $n(S) = 36$

(i) $\{(4,6), (5,5), (6,4)\}$ $P(E) = \frac{3}{36} = \frac{1}{12}$

(ii) $\{(2,6), (3,5), (4,4), (5,3), (6,2)\} = P(E) = \frac{5}{36}$

(iii) $\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\} = P(E) = \frac{6}{36} = \frac{1}{6}$

(iv) $\{(1,4), (2,3), (3,2), (4,1)\}$ $= P(E) = \frac{4}{36} = \frac{1}{9}$

(v) $P(E) = \frac{10}{36} = \frac{5}{18}$

or

$$\boxed{P(\text{getting Sum } n) = \begin{cases} \frac{n-1}{36}, & 2 \leq n \leq 7 \\ \frac{13-n}{36}, & 7 \leq n \leq 12 \end{cases}}$$

Q find the probability of getting a sum of 8 when three dise rolled simultaniousl.

Sol^n $(1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)$

$$\Rightarrow P(E) = \frac{1}{6}\left(\frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36}\right)$$

$$= \frac{1}{6}\left(\frac{21}{36}\right) = \frac{7}{72} = \frac{7}{72}$$

Ex: four dice are rolled simultaniously the find the Probabity of geti sum 7.

Sol^n (1, 1, 5), (1, 2, 4), (2, 1, 4), (1, 3, 3)
(3, 1, 3)

(2, 2, 3), (2, 3, 2), (3, 2, 2)

$$\Rightarrow \frac{4}{36}\left(\frac{4}{36}+\frac{3}{36}+\frac{3}{36}+\frac{2}{36}+\frac{2}{36}+\frac{2}{36}+\frac{1}{36}+\frac{1}{36}\right)$$

$$= \frac{4}{36}\left(\frac{18}{36}\right) = \frac{20}{36\cdot 36}$$

art (2, 5), (3, 4), (4, 3), (2, 5)

$$= \frac{1}{36}\cdot\frac{4}{36}+\frac{2}{36}\cdot\frac{3}{36}+\frac{3}{36}\cdot\frac{2}{36},\ \frac{1}{36}*\frac{4}{36} = \frac{20}{36\times 36}$$

Q 52 cards distibuted omeny 4 place find the Probability that all 4 kings are held by one specific player.

Q $n(S) = \frac{\lfloor 52}{(\lfloor 13)^4 \cancel{\lfloor 4}} \cancel{\lfloor 4} = \frac{\lfloor 52}{(\lfloor 13)^4}$

favrable $\Rightarrow$ 48 $\leftarrow$ 13, 13, 13, 9 $= \left(\frac{\lfloor 48}{(\lfloor 13)^3 \lfloor 9 \lfloor 13}\right)\lfloor 13 = \frac{\lfloor 48}{(\lfloor 13)^3 \lfloor 9}$

$$P(E) = \frac{\frac{\lfloor 48}{(\lfloor 13)^3 \lfloor 9}}{\frac{\lfloor 52}{(\lfloor 13)^4}} = \frac{\lfloor 48}{\lfloor 52} \times \frac{\lfloor 13}{\lfloor 9}$$

$$= \frac{\cancel{13}\cdot\cancel{12}\cdot 11\cdot 10}{\cancel{52}\cdot\cancel{51}\cdot 50\cdot 49} = \frac{11}{4165}$$

Q $ax^2+bx+c=0$ each coefficient of Eq^n is determine by throughing a Disc. find the Proba. that Eq^n hass Equall Roots.

Sol^n $b^2 = 4ac$

	$D = b^2 - 4ac = 0$	no. of Case
$b=1$	$1-4ac=0$	0
$b=2$	$4-4ac=0$	1
$b=3$	$9-4ac=0$	0
$b=4$	$16-4ac=0$ $\begin{cases} a=1, b=4 \\ a=2, b=2 \\ a=4, b=1 \end{cases}$	3
$b=5$	$25-4ac=0$	0
$b=6$	$36-4ac=0$ $\{a=3, b=3$	1
		5

$$P = \frac{5}{216}$$

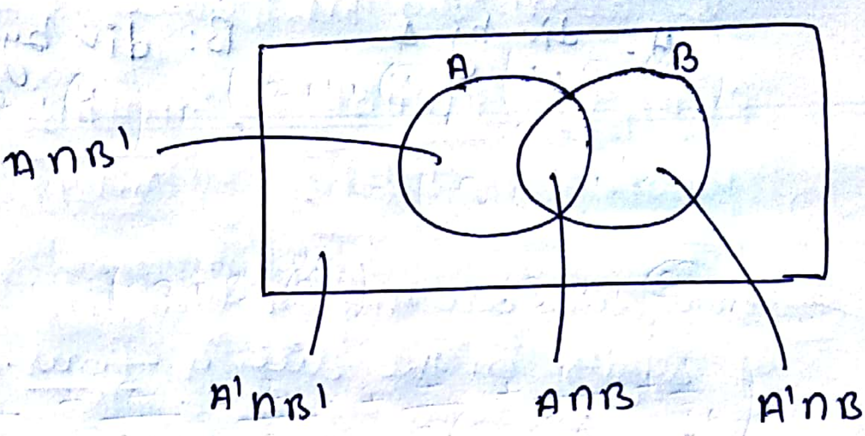
4.10: **for Real Root**

D	Case
$1-4ac>0$	0
$4-4ac>0$	0
$9-4ac>0$	3
$16-4ac>0$	5
$25-4ac>0$	14
$36-4ac>0$	16

$$P = \frac{38+5}{216}$$

$$P = \frac{43}{216}$$

Union & Intersection set of event



P(atlest on event occurs)

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cap B') = P(A) - P(A\cap B)$$

$$P(B\cap A') = P(B) - P(A\cap B)$$

Let A, B, C are define in Sample Space.

$$P(A\cup B\cup C) = \sum P(A) - \sum P(A\cap B) + P(A\cap B\cap C)$$

Que One of No. is selected from first 20 Natural no. Find the Probability it is Eighter div by 2 & 3.

Sol^n
$$S = \{1, 2, 3, \text{———} 20\}$$

A : no is div. by 2 , B: no is div by 3

$P(A\cup B) = ?$ $\quad P(A) = \frac{10}{20} = \frac{1}{2}$, $P(B) = \frac{6}{20} = \frac{3}{10}$

$$\left(P(A\cap B) = \frac{3}{20}\right)$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$\Rightarrow \frac{10}{20} + \frac{6}{20} - \frac{3}{20} = \frac{13}{20}$$

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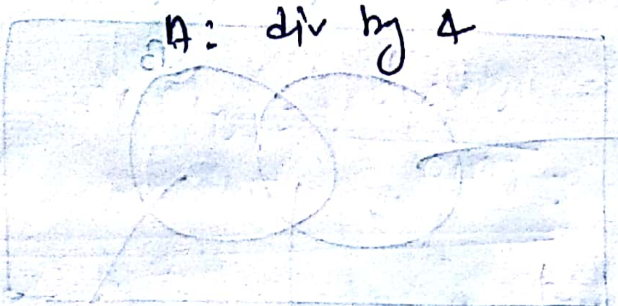
Ex: $S = \{1, 2, \text{———} 100\}$

A: div by 4   B: div by 6.



Ex: three no. are chosen randomly without replacement from the set of first 10 natural number. Find the probability that minimum of the chosen no. is three or maximum is 7.

$S = \{1, 2, 3, \text{———} 10\}$

Sol^n A: min of chosen no. is 3,

B: max of chosen no is 7

$P(A) = \frac{7C_3}{10C_3}$ $P(B) = \frac{6C_2}{10C_3}$ $\frac{7\times6\times5}{3}$

$P(A\cap B) = \frac{3C_1}{10C_3}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{7C_2 + 6C_2 - 3C_1}{10C_3} = \frac{\cancel{70}\ 21+15-3}{120}$

$= \frac{11}{40}$

Ex: $P(A \cup B) = 0.65$ $P(A \cap B) = 0.15$

find $P(A') + P(B')$

$= 1 - P(A) + 1 - P(B)$

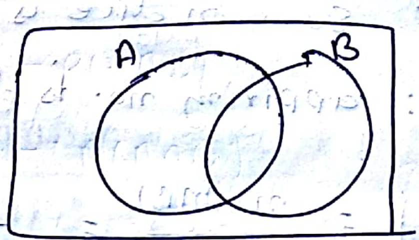
$= 2 - (P(A) + P(B))$

$= 2 - (P(A \cup B) + P(A \cap B))$

$= 2 - (0.65 + 0.15) = 2 - .80 =$

$= \underline{1.2}$

Conditional Probability :→



Concider two events A & B defined on Sample Space S, the Probility of occurence of Event A given that Even B has been already occurs is dinoted by.

$$P(A/B) = P(A \text{ given } B)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)}$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

Ex: two coins are tossed Simeultenously find the Probabilty that two heads appear if it is Known that atlest one head Commes.

Sol^n

$S \{ \underline{HH}, HT, TH \}$

$P = \frac{1}{3}$

OR: A: two Heads Come

B: atlest one head Come

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{n(A\cap B)}{n(B)} = \frac{1}{3}$$

Que two dice are thrown then find the Probability that Sum of digits on the dice is four. if it is known that no appairing on the dice are different

Sol^n A: Sum on dice is 4 ((1,3), (2,2) (3,1))

B: appiring no. is diff: $6\times5 = 30$

$$P(A/B) = \frac{n(A\cap B)}{n(B)} = \frac{2}{30} = \frac{1}{15}$$

Que three no. are chossen randomely without Replacement from the set of first 8 natural no. find the Probability that min of the choosen no. is 3, if it's know that their max is 6

A: min no. is three => (3, 4, 5, 6, 7, 8)

B: max no 6 ⟶ (1, 2, 3, 4, 5, 6)

$$P(A/B) = \frac{n(A\cap B)}{n(B)} = \frac{{}^{2}C_1}{{}^{5}C_2} = \frac{2}{8} = \frac{2}{10} = \frac{1}{5}$$

Que: if C is Subset of D & $P(D)\neq 0$ them

(A) $P(C/D) = \frac{P(D)}{P(C)}$ (B) $P(C/D) = P(C)$

(C) ✓ $P(C/D) \geq P(C)$ (D) $P(C/D) < P(C)$

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$$P\left(C/_D\right) = \frac{P(C\cap D)}{P(D)} = \frac{P(C)}{P(D)} \rightarrow \leq 1$$

$$\therefore P\left(C/_D\right) \geq P(C)$$

<u>Ex:</u> a no. is randomely selected from first 100 natural no. find the probability that it is div by 6. if it is know that no. is div. by 4

A: div by = 6

B: div by = 4

$$P\left(A/_B\right) = \frac{P(A\cap B)}{P(B)} = \frac{n(A\cap B)}{n(B)} = \frac{8}{25}$$

<u>Ex:</u> $P(A) > 0$, $P(B) \neq 1$ then

$P\left(A'/_{B'}\right) =$

(A) $1 - P\left(A/_B\right)$ (B) $1 - P\left(A'/_B\right)$ (C) $\frac{1 - P(A\cup B)}{P(B')}$ (D) $\frac{P(A')}{P(B')}$

<u>Sol^n</u> $P\left(A'/_{B'}\right) = \frac{P(A'\cap B')}{P(B')} = \frac{(P(A\cup B))'}{(P(B'))} = \frac{1 - P(A\cup B)}{P(B')}$

Multiplecation theorem in Probability

$$P\left(A/_B\right) = \frac{P(A\cap B)}{P(B)}$$

$$\boxed{P(A\cap B) = P(B)\cdot P\left(A/_B\right)}$$

OR

$$\boxed{P(A\cap B) = P(A)\cdot P\left(B/_A\right)}$$

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Let A, B, C are three Event defined on S.

$$P(A \cap B \cap C) = P(A) \cdot P\left(\frac{B}{A}\right) \cdot P\left(\frac{C}{B \cap A}\right)$$

Independent Event : if Events A & B are independent then occurrence or no-occurrence of one event doesn't affect the occurrence or non-occurrence of another event.

Hence $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

if A & B are independent then

$$P(A/B) = P(A) = \frac{P(A \cap B)}{P(B)}$$

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

→ Events A, B, C are said to be Independent if

$P(A \cap B) = P(A) \cdot P(B)$
$P(B \cap C) = P(B) \cdot P(C)$
$P(A \cap C) = P(A) \cdot P(C)$
& $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$2^n - 1 - n$

→ if events A & B are independ

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)}$$

→ if Events A & B are independent then A' & B' are also independent.

$P(A' \cap B') = P(A) \cdot P(B)$

$P(A' \cap B') = P(A') \cdot P(B')$

$\Downarrow$

$P(A \cup B)' = 1 - P(A \cup B)$

$= \cancel{Br}\ 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - P(A) - P(B) + P(A) \cdot P(B)$

$= (1 - P(A)) - P(B)(1 - P(A))$

$= (1 - P(A))(1 - P(B))$

$$\boxed{P(A' \cap B') = P(A') P(B')}$$

→ Let $E_1 E_2$ ---- E_n are n <u>independent</u> events then

$P(E_1 \cup E_2 \cup$ ---- $\cup E_n) + P(E_1 \cup E_2$ ---- $\cup E_n)' = 1$

$\Rightarrow P(E_1 \cup E_2 \cup$ ---- $\cup E_n) = 1 - P(E_1' \cap E_2' \cap E_3'$ ---- $\cap E_n')$

$$\boxed{P(E_1 \cup E_2 \cup \text{——} \cup E_n) = 1 - (P(E_1') P(E_2') \text{——} P(E_n'))}$$

→ $\boxed{P(E_1/E_2) + P(E_1'/E_2) = 1}$

Que Identity whese Events are Independent or not.

$P(A \cup B) = 5/6 \qquad P(A \cap B) = 1/3, \ P(A) = 1/3$

$\frac{5}{6} = P(B) + \frac{1}{3} - \frac{1}{3} \qquad P(B) = 5/6$

$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{3} \cdot \frac{5}{6} = \frac{5}{18} \neq \frac{1}{3}$

Ex: Probability of solving a specific problem independently by student A & B are $\frac{1}{2}$ & $\frac{1}{3}$ respectively if both are try to solve the problem independently then find the probability.

(i) Problem solve. (ii) Exactly one of them solve the problem

Sol^n

(i) $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

or $P(A \cup B) = 1 - P(A')(B')$

$$= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{2}{3}$$

(ii) $P(A \cap B') + P(B \cap A') = \frac{1}{2} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2}$

$$= \frac{1}{2}$$

Ex: if E_1 —— E_8 are independent events such that $P(E_i) = \frac{1}{1+i}$ then probability that ~~none of the event~~ atleast one of the event occurs the.

Sol^n

$$P(E_1 \cup E_2 \cup E_3 —— E_8) = 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) —— \left(1 - \frac{1}{9}\right)$$

$$= 1 - \frac{1}{9} = \frac{8}{9}$$

Ex: from the set of first 20 natural no. three no. are selected randomely find the probability that

(i) these product is even.

(ii) these sum is even.

(iii) these form an A.P. (any order)

Sol^n (i) $\cancel{\frac{{}^{10}C_1 \cdot {}^{19}C_2}{{}^{20}C_3}} = \frac{10 \cdot \cancel{19} \cdot \cancel{18}/2}{\frac{20 \cdot \cancel{19} \cdot \cancel{18}}{\cancel{2} \cdot 3 \times 1}} =$

(i) $1 - \frac{{}^{10}C_3}{{}^{20}C_3} = 1 - \frac{\cancel{10} \cdot \cancel{9} \cdot \cancel{8}\,4}{\cancel{20} \cdot 19 \cdot \cancel{18}\,\cancel{2}}$

$=$

(ii) $P(\text{sum is Even}) = \frac{\overset{E}{{}^{10}C_3} + \overset{O}{{}^{10}C_2} \cdot \overset{E}{{}^{10}C_1} +}{{}^{20}C_3}$

(iii) $S = \{1, 2, 3, \longrightarrow 20\}$

(i) Let d is common diff of A.P.

$d=1 \quad \{(1,2,3), (2,3,4) \longrightarrow (18,19,20)\} = 18$

$d=2 \quad \{(1,3,5), (2,4,6), \quad (16,18,20)\} = 16$

$d=3 \quad \{(1,4,7), (2,5,8) \longrightarrow (14,17,20)\} = 14$

$\vdots$

2

$d=9 \quad \{(1,10,19), (2,11,19)\}$

$P(E) = \frac{2(1+2+3 \longrightarrow 9)}{{}^{20}C_3} = \frac{9 \cdot 10}{{}^{20}C_3} = \frac{90}{{}^{20}C_3}$

(ii)

$A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$P(E) = \frac{{}^{10}C_2 + {}^{10}C_2}{{}^{20}C_3} = \frac{10 \cdot 9}{{}^{20}C_3} = \frac{90}{{}^{20}C_3}$

Q three no. are selected randomely from first 100 natur. no find the probability that there (i) sum is div by three. (ii) sum is div by 4.

Sol:

$3n \quad A = \{3, 6, 8 —— 99\}$

$3n+1 \quad B: \{1, 4, 7 —— 100\}$

$3n+2 \quad C: \{2, 5, 8 —— 98\}$

$$P(ii) = \frac{n(E)}{n(S)} = \frac{{}^{33}C_3 + {}^{34}C_3 + {}^{33}C_3 + {}^{33}C_1 \cdot {}^{33}C_1 \cdot {}^{33}C_1}{{}^{100}C_3}$$

Que: the probability that an anti air craft gun can hit the target in the first, 2nd, 3rd attempt is 0.6, 0.7, & 0.1 respectively the probability that gun hit the target is.

Sol:

$$\cancel{P = \left(\frac{6}{10}\right)\left(1-\frac{7}{10}\right)\left(1-\frac{9}{10}\right) + \left(1-\frac{6}{10}\right)\left(\frac{7}{10}\right)\left(1-\frac{1}{10}\right)}$$

$$P = 1 - \left(1-\frac{6}{10}\right)\left(1-\frac{7}{10}\right)\left(1-\frac{1}{10}\right)$$

$$P = 1 - \left(\frac{4 \times 3 \times 9}{10^3}\right)$$

$$P = \frac{1000-108}{1000} = \frac{892}{1000} = .892$$

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Que three Person A, B & C discard a card in order from a pack of 52 plane card (with replacement) the first who discard a card of heart win the game. if process cep cogim on find the probability of win the match by A, B, C

Sol^n E_1: geting a card of heart

$P(E) = 1/4$, $P(E') = 3/4$

$$P(A) = P(E) + (P(E'))^3 P(E) + (P(E'))^6 P(E) + \_\_\_\_$$

$$P(A) = \frac{P(E)}{1-(P(E'))^3} = \frac{1/4}{1-(3/4)^3} = \frac{16}{37}$$

$$P(B) = P(\bar{E})P(E) + (P(\bar{E}))^4 \cdot P(E) + \_\_\_\_$$

$$= \frac{P(E)\ P(\bar{E})}{1-(P(E'))^3} = \frac{16}{37} \times \frac{3}{4} = \frac{12}{37}$$

$$P(C) = \frac{9}{37}$$

Binomial Enpriment :→ an Enpriment that can result in any two possible outcomes

is called Binomial Enpriment.

→ Usely on of the outcome is Success & nother is failer

Let Pro. of success is = p

& failure is = q

$$\boxed{p+q=1}$$

$$\boxed{P(\text{getting exactly } r \text{ Success}) = {}^nC_r\, p^r q^{n-r}}$$

$$P(\text{atlest } r \text{ Success}) = \sum_{r=r}^{n} {}^nC_r\, p^r q^{n-r}$$

$$P(\text{atmost } r \text{ Success}) = \sum_{r=0}^{r} {}^nC_r\, p^r q^{n-r}$$

mean, variance & standard daviation of binomial Distribution are np, npq, $\sqrt{npq}$ respectively.

Que: a coin is tossed 5 times then find the Probability.

(i) getting Exactly 3 head:

(ii) getting atlest 3 head

(iii) getting almost 3 head.

Sol^n $P(H) = \frac{1}{2}$, $P(F) = \frac{1}{2}$

(i) $^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3} = {}^5C_3\left(\frac{1}{2}\right)^5$

(ii) $^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5(10+5+1) = \frac{16}{32} = \frac{1}{2}$

(iii) $\left(\frac{1}{2}\right)^5\left({}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3\right)$

$\Rightarrow \frac{1+5+10+10}{32} = \frac{26}{32} = \frac{13}{16}$

Que: a coin is tossed 50 times then find the Pro. of getting

(i) Exactly $\cancel{3}$ head
(ii) atlest 2 head
(iii) atmost 2 head

S: getting a head
F: getting a tail
$p(S) = \frac{1}{2}$, $p(f) = \frac{1}{2}$

Soln (i) $P(S) = {}^{50}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{50-2} = {}^{50}C_2 \left(\frac{1}{2}\right)^{50}$

(ii) $P(A) = 1 - \left(\frac{1}{2}\right)^{50} \left({}^{50}C_0 + {}^{50}C_1\right) = 1 - \frac{51}{2^{50}}$

(iii) $P(A) = \left(\frac{1}{2}\right)^{50} \left({}^{50}C_0 + {}^{50}C_1 + {}^{50}C_2\right)$

Que: A Pair of dise is thrown 4 times then find the Probability of getting (i) Equal digits (doublets) Exactly two times.
(ii) Equall digits atlest two times.
(iii) Equall digits atmost two times.

Soln S: getting a doublet
F: not getting a doublet

$P(S) = \frac{6}{36} = \frac{1}{6}$, $P(F) = 5/6$

(i) $P(2\text{ success}) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{150}{1296}$

(ii) $P(\text{atlest 2 success}) = P(r=2) + P(r=3) + P(r=4)$

$= {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 + {}^4C_3 \left(\frac{1}{6}\right)^3 \left(5/6\right)^1 + {}^4C_4 \left(\frac{1}{6}\right)^4$

$= \frac{150 + 20 + 1}{1296} = \frac{171}{1296}$

(iii) $P(\text{atmost 2 success}) = 1 - \left(P(r=3) + P(r=4)\right)$

$= 1 - \frac{21}{1296} = \frac{1296 - 21}{1296}$

$= \frac{1275}{}$

Que: if a fair dice is thrown 20 times then the Probability that ~~4th, 6th~~ 4th six appear in the 10th thrown

Sol^n ~~p(atmost six in 10th time)~~ =

$\underline{\quad 6 \quad 6 \quad 6 \quad}(1)$

$$= \left[{}^{9}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^6\right]\left(\frac{1}{6}\right)$$

Que: Cards are delt one by one from a Pack of 52 cards. find the Probability that
(i) Exactly 5 cards are delt before first ace appear.
(ii) Exactly 10 cards are delt before second ace appear.

Sol^n
(i) $P(E) = \dfrac{{}^{48}C_5 \cdot {}^{4}C_1}{{}^{52}C_6}$

5 cards, ace

relt

43

(ii)

5, 1st ace, ~~10 cards~~ 7, 8, 9, 10, 2nd ace

$$P(E) = \boxed{\frac{{}^{48}C_5 \cdot {}^{4}C_1}{{}^{52}C_6} \times \frac{{}^{43}C_4 \cdot {}^{3}C_1}{{}^{46}C_5}}$$

Ex: if 4 cards are drawn from a Pack of 52 Plane cards then find the Probility that ~~all 4 cards~~ Cards are from different suit & denomination.

Sol^n

$$P(E) = \frac{{}^{13}C_1 \cdot {}^{12}C_1 \cdot {}^{11}C_1 \cdot {}^{10}C_1}{{}^{52}C_4}$$

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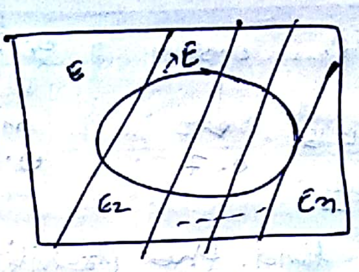
Total Probability theorem :→



Let $E_1, E_2, \ldots E_n$ are n mutually Exclusive and Exhaustive events defined on Sample Space S.

Let E be any event in S overlapping ~~over~~ each of the event then.

$$E = (E \cap E_1) \cup (E \cap E_2) \cup \text{———} (E \cap E_n)$$

$$P(E) = P(E \cap E_1) + P(E \cap E_2) + \text{———} + P(E \cap E_n)$$

$$P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + \text{——} P(E_n) \cdot P(E/E_n)$$

$$\boxed{P(E) = \sum_{i=1}^{n} P(E_i) \cdot P(E/E_i)}$$

Ex: a bag contains four copper & three silver coins & another bag contains 6 copper & 2 silver coins. A coin is drawn randomely from one of the bag. the find the probability that Drawn coin is of copper.

Sol^n

E_1 : coin is drawn is drawn form first Bag

E_2 : coin is ——— 2nd Bag

E : coin is copper.

$$P(E) = \sum_{i=1}^{2} P(E_i) \cdot P(E/E_i)$$

$$= P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$$

$$\Rightarrow \frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{6}{9} = \frac{2}{7} + \frac{3}{9}$$

Que

	A	B	C
win p.	0.5	0.3	0.2
Product is introduce:	0.7	0.6	0.5

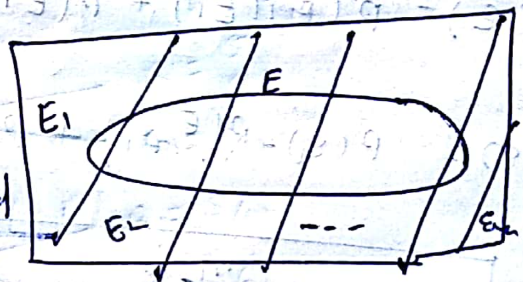
find the Probility that new Product is Introduce.

$$P(E) = 0.5\times0.7 + 0.3\times0.6 + 0.2\times0.5$$

$$= 0.35 + 0.18 + 0.10 = \underline{\underline{0.63}}$$

Bayers theorem :→

Let $E_1, E_2 \dots E_n$ are n mutually Enclusive of Enchaustive event defined on Sample Space.



$$P(E_i/E) = \frac{P(E\cap E_i)}{P(E)}$$

$$\boxed{P(E_i/E) = \frac{P(E_i)\,P(E/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(E/E_i)}}$$

Que a Bag Contains A (2W 3R)   Bag B (4W 5R)

Ball Drawn is found to be red - find the Prob. that Ball Came out form Bag A

$$P(E) \Rightarrow \frac{\frac{1}{2}\times\frac{3}{5}}{\frac{1}{2}\times\frac{3}{5} + \frac{5}{9}\times\frac{1}{2}} \Rightarrow \frac{3/10}{\frac{3}{10} + 5/18}$$

$$\Rightarrow \frac{3\times18}{\cancel{56}\,54+50} = \frac{\cancel{8}\,54}{104} = \frac{27}{52}$$

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Que: 3 machine Produce bots: A → 25% → Def. 5%
B → 35% → 4%
C → 40% → 2%

A bots is drawn randomly fom Defective & find the Prob. that it was Produced By machine A.

Sol^n

$$P(E) = \frac{0.25 \times 0.05}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02} = \frac{125}{125 + 140 + 80}$$

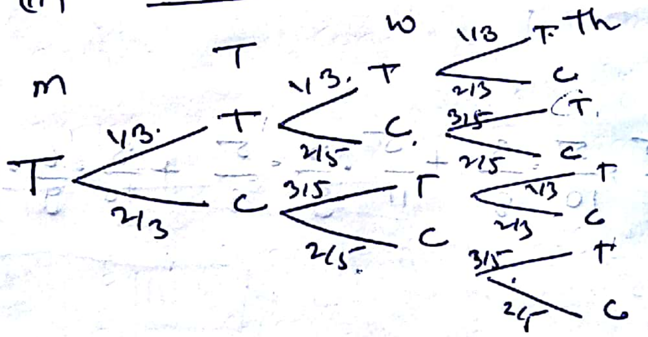
$$= \frac{125}{345} = \frac{25}{69}$$

Tree diagram :

Q: if mister A takes coeefee on monday then Probabilty of taking tea is 3/5 & his coeefee on next day is 2/5 & tea is 3/5 & he takes tea on mondy then Probility of taking tea one next monday. 1/3 & coeef = 2/3

(i) find the Probalitily that he takes coeefee on weeleneday

(ii) ______ tea on thrusday



(i) $P(E) = \frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{2}{5} = \frac{2}{9} + \frac{4}{15}$

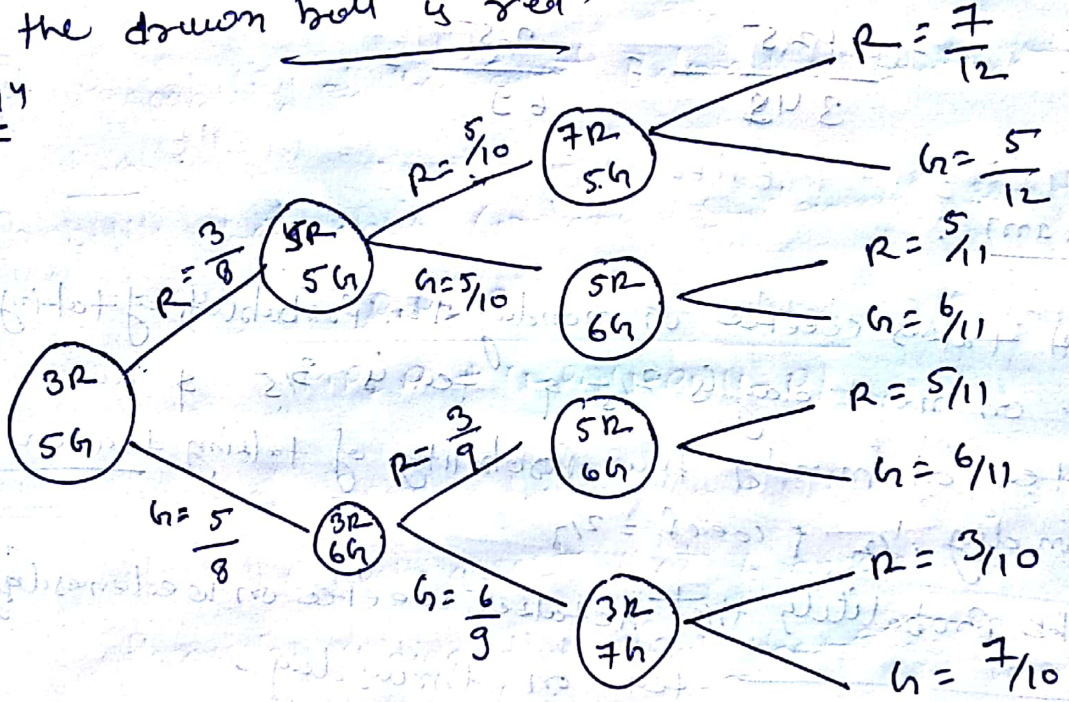
(ii) $P(E) = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{3}{5} + \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{2}{5} \cdot \frac{3}{5}$

$$= \frac{1}{9} + \frac{6}{45} + \frac{6}{45} + \frac{12}{75}$$

$$= \frac{1}{9} + \frac{12}{45} + \frac{4}{25} \Rightarrow \frac{1}{9} +$$

Que: a Beg Contains 3 Red & 5 Green Balls a Ball is drawn randomely & if it is found to be red then two more red balls are added in the beg & if it is found to be Green then one more Green Ball added in the Beg. if Enpriment is repeated two time & a ball is randomely from the beg then find the Probability that the drawn ball is red.

Soln



P(drawn ball is red)

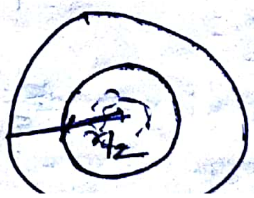
$$\Rightarrow \frac{7}{12}\cdot\frac{5}{10}\cdot\frac{3}{8} + \frac{5}{11}\cdot\frac{5}{10}\cdot\frac{3}{8} + \frac{5}{11}\cdot\frac{3}{9}\cdot\frac{5}{8} + \frac{3}{10}\cdot\frac{6}{9}\cdot\frac{5}{8}$$

$\Rightarrow$

Geometrical Probability : (less Important)

Q a Point is Rendomely Selected inside the circle find the Probability that Point is closses to the Centre of the circle than to its circumference.

Soln $P(E) = \dfrac{\pi (r/2)^2}{\pi r^2} = \dfrac{1}{4}$



Que: a wire of length l is cut into three Pieces then find the Probability these three Pieces formed a triangle:

Sol^n

$x, y, l-(x+y)$

$x>0, y>0, (x+y)<l$

$0 < x, y < l$

$x+y > l-(x+y)$

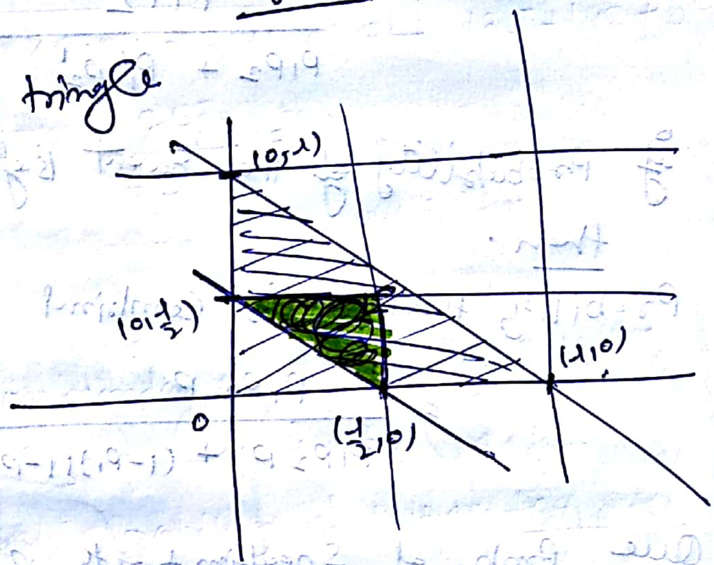
$x+y > l/2$

$(l-x-y)+y > x$

$x < l/2$

$y < l/2$

for triangle



$$p = \frac{\frac{1}{2}(\;)^2 + \frac{1}{2}\left(\frac{l}{2}\right)^2}{\frac{1}{2}l^2}$$

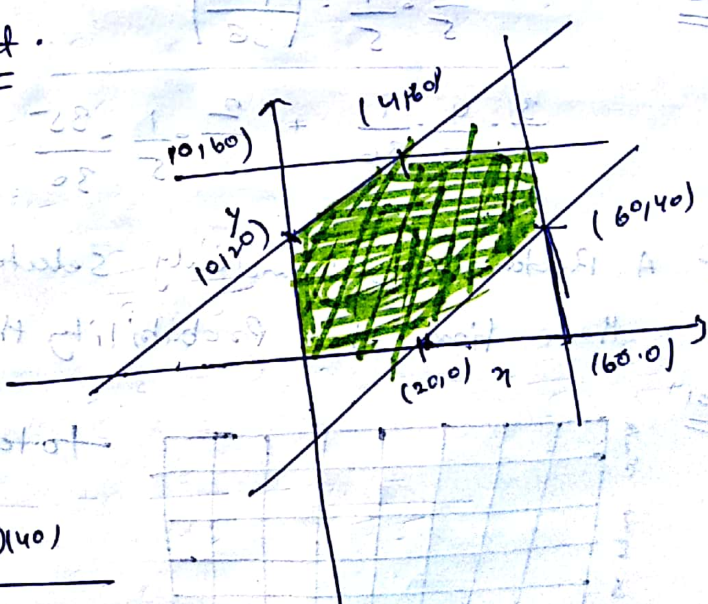
$$p = \frac{1}{4}$$

Que: A •   •   • B   then 11—12 Am

wait time $= 20$ min.

A & B are meet.

Sol^n $\boxed{11;x}$; $\boxed{11:y}$



$|x-y| < 20$

$x-y < 20$

$x-y > -20$

$$\text{prob.} = \frac{3600 - 2\frac{1}{2}(40)(40)}{3600}$$

$$= \frac{3600-1600}{3600} = \frac{20}{36} = \frac{5}{9}$$

Coincidence ~~testimony~~ :-

if p_1 & p_2 are the Probability of speaking the truth of two independent witness A & B then Probability that their Combined Statement is true.

$$\frac{p_1 p_2}{p_1 p_2 + p_1' p_2'}$$

$\cancel{1225}$
41
$\cancel{1296}$

if Probability of the Event Before their Statement is p then.

Probility that their Combined Statement is true

$$= \frac{p_1 p_2 p}{p_1 p_2 p + (1-p_1)(1-p_1)(1-p)}$$

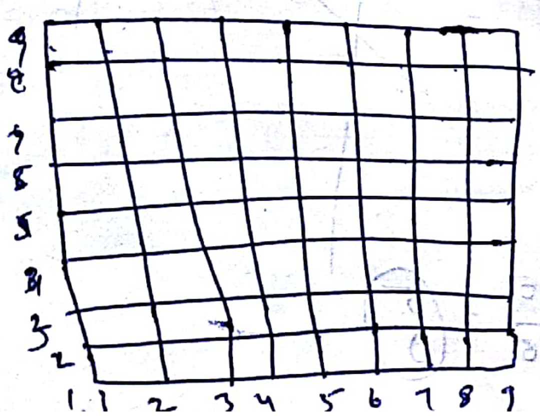
Que Prob. of Speaking trouth By A & B is $3/5$ & $4/5$ resp. if A & B both assert that 6 appere two times on throwing a dice then find the Probability that their Combined Statement is true.

Sol^n

$$\frac{\frac{3}{5}\cdot\frac{4}{5}\cdot\left(\frac{1}{36}\right)}{\frac{3}{5}\cdot\frac{4}{5}\cdot\frac{1}{36} + \frac{2}{5}\cdot\frac{1}{5}\cdot\frac{35}{36}} = \frac{12}{12+70} = \frac{12}{82} = \frac{6}{41}$$

Que A Rectangle is randomly selected on the chess bord then find the Probability that it is Squre.

Sol^n



total no of recttangle

$= {}^9C_2 \cdot {}^9C_2 = 36 \cdot 36$
$= 1296$

Square => $\overset{(8\times8)}{64} + (\cancel{7\times7}) + (6\times6) + (5\times5) + (4)(4) + (3\times3) \times (2\times2) + 1$

$\Rightarrow 1^2 + 2^2 + 3^2 \longrightarrow 82$

$$= \frac{4 - 8 \cancel{(9)}^{3}(17)}{\cancel{6}} = 12 \times 17 = \underline{204}$$

$$P = \frac{204}{1296} = \frac{51}{324} = \frac{17}{108}$$

<u>Que:</u> find the Probability of Sunday in leap year

<u>Sol^n</u> $52 \times 7 = 364$

{SM, MT, TW, WT, TF, FS), S}

$$P = \frac{2}{7}$$

<u>Ex:</u> A = {1801, 1802, ...1900}

$$P = \frac{24}{100} \times \frac{2}{7} + \frac{76}{100} \times \frac{1}{7}$$

for <u>leap year</u>
centuary Div By 400
& others Div by 4