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Influence of Reflected Shock and Boundary-Layer Interaction on Shock-Tube Flows

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During the last decade the study of the interaction between normal shocks and boundary layers in the shock tube has been carried out by many research workers. This interest has been fostered by the requirement to understand the effects that this phenomenon may have on the flow properties in the region between the reflected shock and the end of the tube. This so-called reflected shock region is important in chemical kinetics studies and as a reservoir of gas for hypersonic shock tunnels. In this paper a review is given of the various analyses of the problem together with the results of experimental studies. Reflected and transmitted shock bifurcation, the rate of growth of the bifurcated foot, and the effects of transition to turbulence are discussed. The influence of bifurcation on the flow in the shock tube is assessed, especially as a mechanism for transporting cold driver gas to the end plate, causing early cooling of the gas on the reflected shock region.

I. INTRODUCTION

The first analytical approach to the problem of the interaction of the reflected shock with the boundary layer in a shock tube was made by Mark[1] who considered the boundary layer to be of unspecified thickness, stationary with respect to the wall, and to have a temperature equal to the initial test-gas temperature. Bifurcation was assumed to occur when the boundary layer could not negotiate the pressure rise across the reflected shock even when brought to rest relative to it, and was, therefore, trapped and carried along at the foot of the shock.

The shock angles in the bifurcated foot were calculated by Mark who assumed that the pressure behind the oblique shock OA in Fig. 1, was equal to the stagnation pressure in the boundary layer. Using this criterion he obtained fair agreement with experiment, as can be seen in Fig. 2.

An analysis of the interaction between the reflected shock and the laminar boundary layer has been carried out by Byron and Rott[2] using the more realistic model of the boundary layer given by Rott and Hartunian.[3] From this work, it was shown that under certain circumstances there exists a minimum stagnation pressure within the boundary layer, away from the wall. In these cases the critical ratio of boundary-layer stagnation pressure to pressure behind the normal reflected shock (p_{st}/p_5), below which bifurcation will occur, can be as high as 0.95, whereas Mark had suggested a value of 0.8.

Detailed experimental studies of bifurcation have been carried out[4-6] involving the use of schlieren photography, pressure measurements, heat transfer, and more recently by density measurements.[7] It was found that bifurcation for polyatomic gases is much more violent than for diatomic gases. It was thought that[1,2,8] the reflected shock should decelerate as a result of bifurcation, but experiments do not confirm this.[4,7] A dip in pressure at the end plate behind the reflected shock originally attributed to bifurcation,[9,10] has been shown to result from interface combustion.[11]

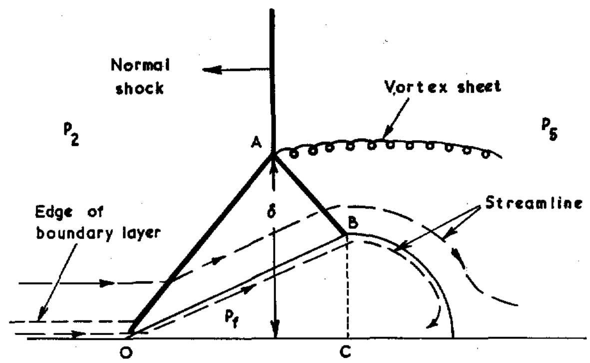

FIG. 1. Schematic diagram of bifurcated foot.

II. THE DURATION OF HIGH TEMPERATURE IN THE REFLECTED SHOCK REGION

The early cooling of the hot gas in the reflected

[1] H. Mark, NACA Technical Note 1418 (1958).
[2] S. Byron, and N. Rott, in *Proceedings of the 1961 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, California, 1961), p. 38.
[3] N. Rott and R. A. Hartunian, Cornell University Graduate School of Aeronautical Engineering Report (1955).
[4] R. A. Strehlow and A. Cohen, J. Chem. Phys. 30, 257 (1959).
[5] D. W. Holder, C. M. Stuart, and R. J. North, British Aeronautical Research Council Report No. 22 891 (1961).
[6] R. E. Center, Phys. Fluids 6, 307 (1963).
[7] H. B. Dyner, Phys. Fluids 10, 879 (1966).
[8] B. A. Woods, British Aeronautical Research Council Report No. 24 571 (1962).
[9] G. Rudinger, Phys. Fluids 4, 1463 (1961).
[10] D. W. Holder and D. L. Schultz, British Aeronautical Research Council Report Mem 3265 (1960).
[11] J. A. Copper, AIAA J. 2, 1669 (1964).

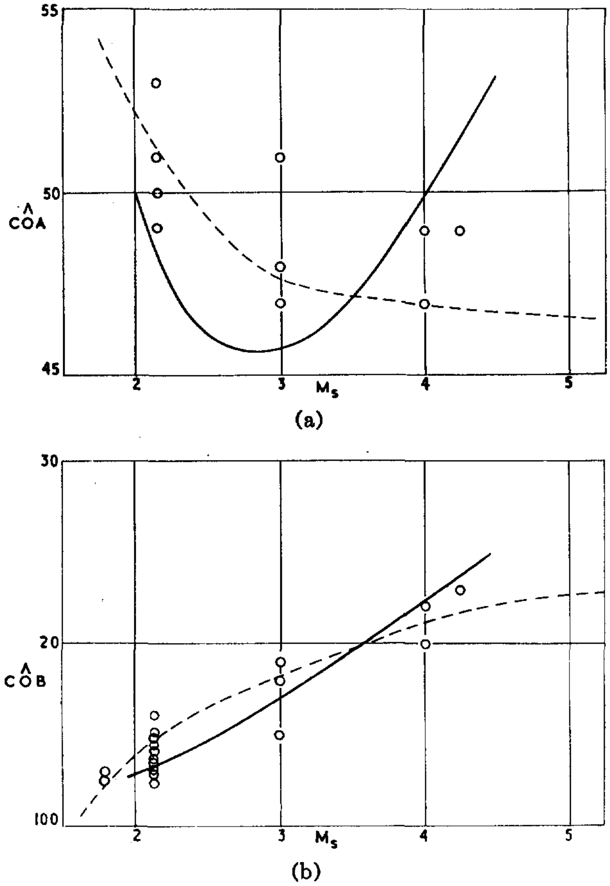

FIG. 2. Angles in bifurcated foot of reflected shock in nitrogen. Solid line—Mark's theory. Dashed line—present empirical theory. Circles—Mark's experimental points.

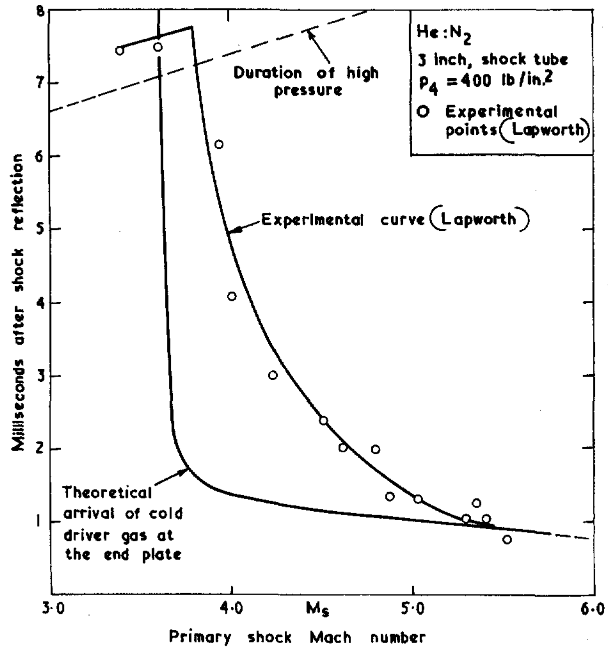

FIG. 3. Graph showing theoretical prediction of arrival of cold gas at end plate in National Physical Laboratory 3-in. shock tube compared with experimental high-temperature duration measurements.

shock region[12–14] is of great concern to shock tunnel users. It is found that the duration of high temperature becomes a decreasing fraction of high-pressure duration as primary shock Mach number increases, and this is shown in Fig. 3 (after Lapworth). One suggested flow mechanism[15] is based on the instability of the contact surface after interaction with the reflected shock, in a similar manner to the Rayleigh–Taylor instability of an accelerated interface. A small perturbation approach was adopted by Markstein,[15] and this has been successful in predicting certain features of the temperature duration and Mach number relationship. Copper[16] has shown that this model has only limited application.

More recently Davies[17] has suggested a cooling mechanism based on the bifurcation of the reflected shock. The physical situation envisaged is that part of the driver gas passes through the foot of the bifurcated transmitted shock, flows along the walls of the tube, and reaches the end wall this resulting in mixing, cooling and contamination of the test gas. Some experimental evidence of this phenomenon is found in photographs taken by Holder, Stuart, and North,[18] where a flow disturbance at the edge of the end plate is clearly seen (Ref. 5). This disturbance moves toward the center of the end plate. It is found that after the shock has interacted with the contact surface, a flow of driver gas of the type suggested above occurs.[18]

Since Mark's model for bifurcation was successful in predicting the shock angles which made up the foot over a range of Mach numbers, it was decided to use his analytical approach to obtain a qualitative description of the flow.

The angles of COA and COB (see Fig. 1) are obtained using the oblique shock relationships:

$$M_b^2 \sin^2(\mathrm{COA}) = \frac{(\gamma_1 + 1)p_{st}/p_2 + (\gamma_1 - 1)}{2\gamma_1},$$

[12] K. C. Lapworth, AGARDograph 68 (1964), p. 255.
[13] K. C. Lapworth and J. E. G. Townsend, British Aeronautical Research Council Report No. 26 110 (1964).
[14] K. D. Bird, J. F. Martin, and T. J. Bell, presented at the Third Hypervelocity Techniques Symposium, Denver (1964).
[15] G. H. Markstein, J. Aeron. Sci. 24, 238 (1957).
[16] J. A. Copper, presented at the Fourth Hypervelocity Techniques Symposium, Arnold Air Force Station (1965).
[17] L. Davies, British Aeronautical Research Council Report No. CP 880 (1966), Part 1.
[18] D. W. Holder, AGARDograph 68 (1964), p. 745.

$$\frac{\tan(\text{COA-COB})}{\tan(\text{COA})} = \frac{(\gamma_1 - 1)M_b^2 \sin^2(\text{COA}) + 2}{(\gamma_1 + 1)M_b^2 \sin^2(\text{COA})}.$$

M_b is the Mach number of the boundary layer in reflected-shock-fixed coordinates, and γ_1 is the specific heat ratio of the initial test gas. The ratio p_{st}/p_5 is obtained using the following equations depending on whether the boundary layer is supersonic or subsonic relative to the reflected shock:

$$M_b < 1, \frac{p_{st}}{p_2} = \left[1 + \frac{(\gamma_1 - 1)}{2} M_b^2\right]^{\gamma_1/(\gamma_1 - 1)},$$

$$M_b > 1, \frac{p_{st}}{p_2} = \left[\frac{(\gamma_1 + 1)}{2} M_b^2\right]^{\gamma_1/(\gamma_1 - 1)} \cdot \left[\frac{2\gamma_1}{(\gamma_1 + 1)} M_b^2 - \left(\frac{\gamma_1 - 1}{\gamma_1 + 1}\right)\right]^{1/(1-\gamma_1)}.$$

The third angle in the triangle OAB is determined in a similar manner by assuming that the pressure behind the limb AB is the static pressure behind the reflected shock. The angles obtained by this analysis for nitrogen as test gas, and for primary shock Mach numbers less than 4, agree closely with the experimental values (see Fig. 2). The velocity of the flow which emerges through the rear limb of the bifurcated foot is found to be higher than that of the flow which passes through the normal shock. Evidence of this is found in the photographs taken of bifurcated shocks where a vortex sheet is clearly seen originating at the triple point, suggesting the proximity of two flows of different velocity. Some measurements have been made by Seddon[19] where Pitot pressures were actually measured through the region behind the bifurcated foot. From this study Seddon concluded that a higher velocity flow emerged through the rear limb of the bifurcated foot than through the normal shock, and that this flow persisted for some considerable distance downstream. This flow will be "hot" test gas in the reflected shock case, and the cold driver gas flow starts after the shock has interacted with the contact surface. Heat-transfer gauges were used across the end plate of the National Physical Laboratory 6-in. × 3½-in.-shock tube to investigate the movement of this gas. The results of the tests clearly indicate that this movement begins immediately after reflection of the primary shock.[17] Similar results were obtained in the National Physical Laboratory 2-in. and 3-in. shock tunnels.

[19] J. Seddon, British Aeronautical Research Council Report No. 22 637 (1961).

III. BIFURCATION OF THE TRANSMITTED SHOCK

To obtain a model for the transmitted shock case, it is necessary to make certain simplifying assumptions concerning the contact surface and the state and character of the boundary layer.[17]

The contact surface is considered to be a discontinuity. The boundary layer is taken to be laminar and to consist only of test gas. This last point is based on arguments similar to those for the loss of test time in shock tubes.[20,21] The angles COA and COB in the foot are determined as before, but with velocities in transmitted-shock-fixed coordinates. The boundary-layer Mach number, when M_s is less than the tailored shock Mach number, is given by

$$M_b = A_{31}\left(\frac{P_{73} + \alpha_3}{\delta_3\gamma_3(\alpha_3 P_{73} + 1)^{\frac{1}{2}}} + \frac{2A_{53}}{(\gamma_1 - 1)}[1 - (P_{65})^{\beta_1}]\right)$$

and for M_s greater than the tailoring value by

$$M_b = A_{31}\left(\frac{P_{73} + \alpha_3}{\delta_3\gamma_3(\alpha_3 P_{73} + 1)^{\frac{1}{2}}} - \frac{A_{53}\delta_1(P_{65} - 1)}{(\alpha_1 P_{65} + 1)^{\frac{1}{2}}}\right),$$

where

$$\alpha_i = \frac{\gamma_i + 1}{\gamma_i - 1}, \qquad \beta_i = \frac{\gamma_i - 1}{2\gamma_i}$$

and

$$\delta_i = \left(\frac{2}{\gamma_i(\gamma_i - 1)}\right)^{\frac{1}{2}}.$$

P_{73} is the pressure ratio across the transmitted shock, P_{65} is the pressure ratio across the disturbance reflected back towards the end plate as a result of the interaction between the reflected shock and the boundary layer, γ_3 is the initial specific heat ratio of the driver gas, A_{53} is the ratio of expanded driver gas sound speed to reflected shock region sound speed, and A_{31} is the ratio of expanded driver gas sound speed to initial test-gas sound speed.

The angles calculated for a shock transmitted through a nitrogen to helium interface are presented in Fig. 4 together with experimental values. It is seen that the agreement obtained is fair.

In reality there exists a contact region, rather than a contact surface, and it is possible to use the simple model to predict the variation of COA and COB through this region. This has been done by Allan,[22] who has made a detailed study of the growth of the bifurcated foot under various conditions. In

[20] A. Roshko, Phys. Fluids **3**, 835 (1960).
[21] W. J. Hooker, Phys. Fluids **4**, 1451 (1961).
[22] W. Allan, M. Sc. thesis, University of Southampton, England (1967).

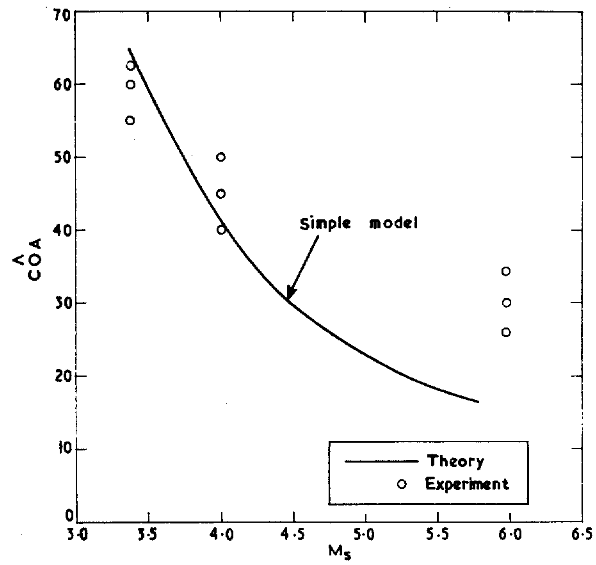

FIG. 4. Angle COA vs M_S for transmitted shock.

his experiments Allan found that the size of the bifurcated foot decreases as the shock entered the contact region, and the angle COA increased rapidly, see Fig. 5. After passing through the contact region the foot started to grow again, and the angle COA decreased to about the value predicted for the transmitted shock. By making an estimate of the gas composition in the contact region, it was possible to calculate the values of COA. These calculated values are shown in Fig. 5 as a function of distance from the end wall. The measurements were made in a 4-in. × 4-in. tube, with nitrogen as test gas and helium as driver gas, at a primary shock Mach number of 4.2. Although the position of the contact

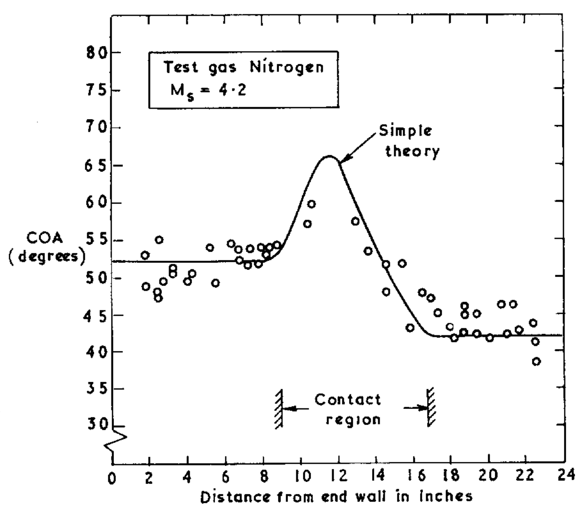

FIG. 5. Angle COA versus distance from the end wall (after Allan, Ref. 22).

region and the gas composition were assumed, the trends are clearly in agreement.

According to the calculations carried out using the simple model, for helium as driver gas and nitrogen as driven gas, no bifurcation of the transmitted shock should occur for $M_s < 3.5$. The reflected shock will be bifurcated, however, and so the observed result should be a rapid decrease in bifurcation after interaction with the contact region. For $M_s > 3.5$ the ratio of p_{st}/p_5 for the transmitted shock decreases rapidly as M_s increases, and so a considerable increase in bifurcation growth rate should occur. Both effects are illustrated in Ref. 17.

From the available experimental evidence it appears that the simple model gives a fairly representative description of the bifurcation phenomenon for the transmitted shock case.

IV. TIME OF ARRIVAL OF COLD GAS AT THE END PLATE

The time of arrival of cold gas at the end plate is obtained from the velocity of the flow which emerges through the bifurcated foot, and the distance from the end plate of the point of interaction of the reflected shock and the contact surface.

An expression for this time is given by Davies,[17] as

$$t = \frac{LU_r}{(U_r + u_2)}\left(1 - \frac{u_2}{U_1}\right)\left(\frac{1}{U_r} + \frac{1}{(U_T^* - U_T)}\right).$$

Here U_r is the reflected shock velocity, u_2 is the flow velocity behind the primary shock, U_1 is the primary shock velocity, U_T is the transmitted shock velocity, U_T^* is the velocity of the flow which has passed through the bifurcated foot of the transmitted shock, and L is the length of the channel.

The validity of this equation has been demonstrated by Edwards and Bull.[23] In their experiments the driver gas contained a small percentage of CO_2, and the infrared absorption of this gas was monitored at the end plate. It was found that when a realistic value of the distance between the primary shock and the contact surface was used, the predicted time of arrival of driver gas at the end plate agreed almost exactly with the experimental values. This is shown in Fig. 6.

In these calculations no estimate was made of the time taken for the mixing and cooling processes to occur. Therefore, no absolute comparison can be made with measurements of the high-temperature duration at the end plate. Even so it is found that the predicted trend is in fair agreement with Lapworth's experiments. See Fig. 3.

[23] D. H. Edwards and D. Bull, AIAA J. 6, 1620 (1968).

In order to evaluate the relative importance of contact-surface instability and bifurcation, it is necessary to use a monatomic test gas, since these mechanisms occur almost coincidentally for diatomic gases. One possible gas is argon, where the transmitted shock does not bifurcate, according to simple theory, at the primary shock Mach numbers for which contact-surface instability causes cooling. In argon, of course, no significant bifurcation of the reflected shock occurs. Bifurcation of the transmitted shock, when using helium as driver and argon as test gas, has been shown to occur by Davies and Bridgman.[24] The best combination for the study suggested above would be to use hydrogen as driver gas, because the Mach number for which the two forms of cooling occur are then more widely separated. Since the theories are based on very simple models of the flow it will be necessary to fix clearly, by experiment, the threshold Mach number for bifurcation of the transmitted shock.

V. MIRELS' LAMINAR AND TURBULENT BOUNDARY LAYERS

In Ref. 25 bifurcation is studied using the realistic boundary-layer profiles of Mirels.[26] It is shown how the stagnation pressure varies through the turbulent boundary layer as a function of Mach number in the same way that Byron and Rott[2] had calculated it for an empirical laminar boundary-layer profile.

In the present paper we use the Mirels' boundary-layer models to calculate the growth rate of the bifurcated foot.

The laminar boundary-layer development is calculated from the Blasius equation

$$f''' + f''f = 0$$

in coordinates:

$$\eta = \left(\frac{u_e}{2x\nu_w}\right)^{\frac{1}{2}} \int_0^y \frac{T_w}{T}\, dy,$$

$$\psi = (2u_e x\nu_w)^{\frac{1}{2}} f(\eta).$$

Since the equations are in shock-fixed coordinates, the boundary conditions include a moving wall.

The velocity profile is then obtained from

$$u/u_e = f'.$$

The turbulent velocity profile is represented by an empirical power-law relationship. Within the boundary layer we have

[24] L. Davies and K. Bridgman, British Aeronautical Research Council Report No. CP 879 (1966).
[25] L. Davies and D. Schofield, British Aeronautical Research Council Report No. ARC 28 112 (1966).
[26] H. Mirels, NACA Technical Note 3712 (1956).

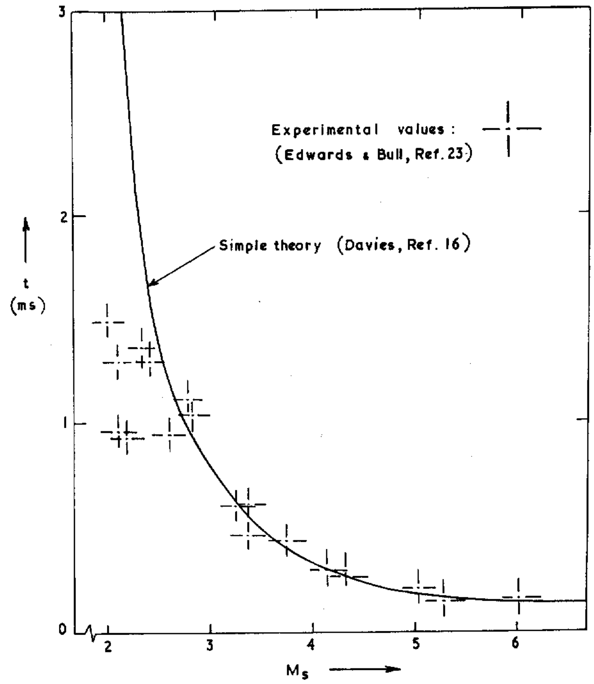

FIG. 6. First arrival time of cold driver gas at end plate (Edward and Bull).

$$\left|\frac{u - u_w}{u_e - u_w}\right| = \left(\frac{y}{\bar{\delta}}\right)^{1/7}.$$

The growth rate, which is based on an incompressible flow empirical relationship, is

$$\bar{\delta} = 0.0574\left(\phi\,\frac{1 - U_w/U_e}{\theta/\bar{\delta}}\right)^{4/5}\left|1 - \frac{U_w}{U_e}\right|^{3/5}\left(\frac{\nu_e}{U_e x}\right)^{1/5} x,$$

where θ is the momentum thickness and ϕ depends on the variation of viscosity with temperature.

Both the laminar and turbulent boundary-layer formulas have been shown to represent the profiles and growth rates found in practice in shock tubes[27,28] quite well.

VI. THE BIFURCATED FOOT MODEL

The angles of the two shocks in a bifurcated flow are uniquely determined in a given flow by the pressure under the foot p_f, assuming no pressure gradients under the foot normal to the wall. In previous papers this pressure has been assumed to depend on the boundary layer which feeds the stagnation region. Mark assumed that it was the stagnation pressure of the boundary layer at the wall, while Byron and Rott took it to be the minimum value of the stagnation pressure in the boundary layer. It would appear, however, that the pres-

[27] E. J. Gion, Phys. Fluids 8, 546 (1965).
[28] W. A. Martin, J. Aerospace Sci. 25, 644 (1958).

sure of the feed gas should have little influence on the foot pressure p_f. It is suggested that this is determined by the geometry and mechanics of the gas at the rear of the stagnation region.

The pressure under the foot is found from measured shock angles to be approximately $0.55p_5$. This is less than the stagnation pressure of most of the boundary layer (see the tabulated values in Ref. 25, for example). Thus, almost all the boundary layer is turned through the angle COB. At the rear of the foot it is unable to enter the p_5 region and it, therefore, turns back towards the wall, as in Fig. 1. The free-stream gas, having passed through the bifurcated region, also turns down towards the side wall and the flow divides at a stagnation point. The high curvature of the flow in this region gives rise to centrifugal forces and, therefore, pressure gradients. It is suggested that it is this which enables the pressure p_f to be maintained at a lower value than p_5. The centrifugal forces which could occur are more than enough to account for the observed pressure differences and so p_f is probably determined by the detailed geometry of the flow. Pressure measurements in the side wall of a shock tube by Davies[17] show that there is a rapid rise of pressure at the front of the stagnation region to the pressure p_f, followed by a more gradual rise to p_5 at the rear of the region.

From the measured shock angles it is found that, except at low primary shock Mach numbers, the flow emerges from the bifurcated region at an angle of about 10° to the wall. In the present study, since we are assuming that p_f does not depend upon the particular boundary layer, we have assumed that this fact determines the pressure p_f of the foot. At Mach numbers less than about 2.5, the measured shock angles are inconsistent with the pressure ratio p_2/p_5, and we conclude that a higher pressure p_5 exists just behind the second shock and that a further adiabatic compression of the flow to p_5 takes place. In this Mach number region the shock angles agree with the pressure, being given by

$$p_f/p_5 = (p_2/p_5)^{0.4}.$$

It can be seen from Fig. 2 that these empirical relationships, while not differing from Mark's theory at low Mach numbers, do not predict a rise in the angle COA at high Mach numbers.

VII. GROWTH RATE OF THE BIFURCATED FOOT

Using Mirels' boundary layers, the foot height to the apex of the two shock waves has been computed

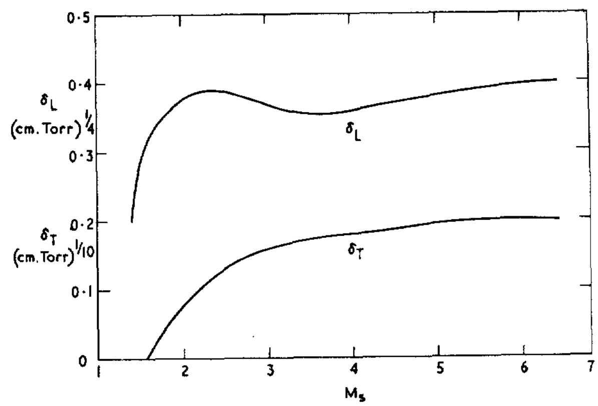

FIG. 7. Height of the bifurcated foot.

for nitrogen, as a function of x, the distance of the reflected shock from the end wall.

The stagnation region beneath the shock is assumed to be wedge shaped with a quadrant of a circle attached downstream. The angle COB is calculated from the assumed value of p_f. The numerical value of p_f has very little effect on these calculations, since if the pressure were greater, the volume would be smaller but the angles COA and COB would be increased leaving the foot-height approximately unchanged.

The rate of mass input to the stagnation region is determined by integrating the mass flow in the boundary layer up to a certain value of p_{st}. In the past, several assumptions have been made as to how much gas is trapped. The present value of $p_{st\,max} = 0.9p_5$ is empirically determined by comparison with experiment. The factor 0.9 allows for the gas which is not trapped due to mixing at the rear of the foot, for trapped gas which escapes from the stagnation region, possibly as a new boundary layer on the wall, and for the possibly conservative estimate of the size of the stagnation region at its rear.

The temperature in the stagnation region was found by averaging the stagnation temperature weighted with the mass flow through the trapped part of the boundary layer and then allowing for the fact that the gas is at the pressure p_f. The Prandtl number is assumed to be constant, and γ is taken to be constant and equal to its value behind the incident shock. The flow conditions as a function of Mach number are taken from Bernstein.[29]

The volume of the trapped gas is then calculated and the height of the triple point found by geometry.

[29] L. Bernstein, British Aeronautical Research Council Report No. CP 626 (1963).

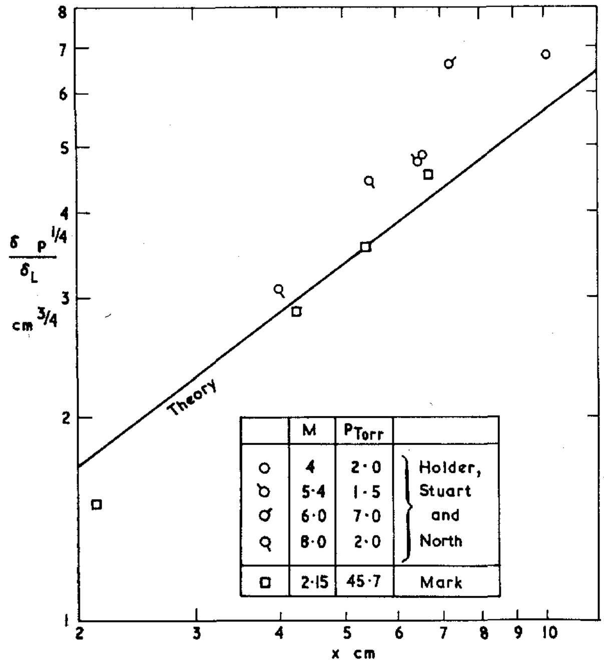

FIG. 8. Laminar boundary layer. Comparison of experimental foot height with theory.

The results can be expressed in the form

Laminar flow: $\delta = \delta_L(M_s)x^{3/4}/p_1^{1/4}$,

Turbulent flow: $\delta = \delta_T(M_s)x^{9/10}/p_1^{1/10}$,

where x and δ are in centimeters, and p_1 is in Torr. The dimensional factors δ_L and δ_T are plotted in Fig. 7 as a function of Mach number.

Experimental determinations of the foot size have been made by Mark, Holder, Stuart, and North, and by Allan.[1,5,22] These results are plotted using the above relationships in Fig. 8 for the laminar cases and Fig. 9 for the turbulent flow. Two facts should be noted. While the measurements from laminar boundary layers are spread about the theoretical line, and the dependence on pressure appears to be correct, a linear function of x would give a better fit than the $x^{3/4}$ dependence of the theory. In the turbulent boundary-layer case the fit of $x^{9/10}$ is excellent but the weak pressure dependence is untested since unfortunately all the available measurements have been made at one pressure.

VIII. CONCLUSIONS

The following conclusions are drawn:

(1) Although the Mark model of the shock–boundary-layer interaction is crude, the general features of the foot geometry as a function of Mach number are well described for the Mach number range $M_s < 3.6$. Above this limit a more realistic

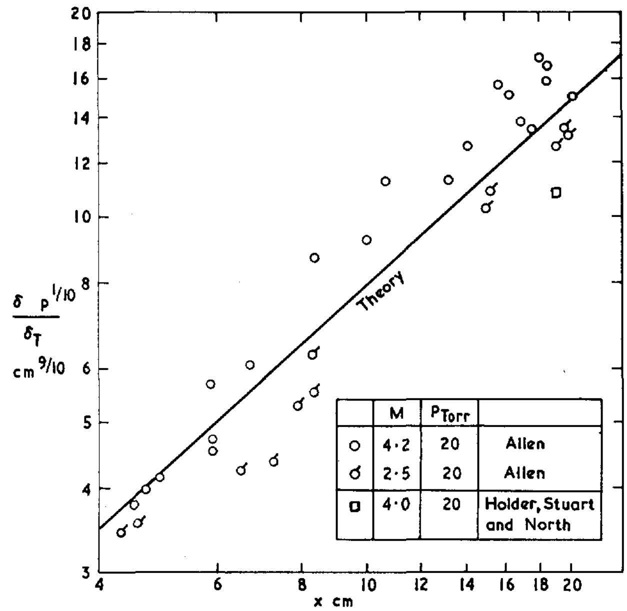

FIG. 9. Turbulent boundary layer. Comparison of experimental foot height with theory.

model of the boundary layer must be used, as has been done by Byron and Rott.

(2) The early cooling of the test gas at the end plate is due at least in part to bifurcation. The flow of gas which passes through the bifurcated foot has a higher velocity than the flow which has passed through the normal shock. In the case of the transmitted shock, this provides a means of transporting cold driver gas to the end plate before the arrival of the contact surface.

(3) Mark's simplified model is most useful in outlining the general features of the flow discussed in conclusion (2). In particular the bifurcation of the transmitted shock is reasonably well represented.

(4) In a turbulent boundary layer, the bifurcated foot is predicted to grow in a manner roughly proportional to x, the distance from the end wall. This is observed experimentally. In a laminar boundary layer a growth proportional to $x^{3/4}$ is expected but experimentally it is again proportional to x. The growth rate in both cases is only weakly dependent on pressure and Mach number.

ACKNOWLEDGMENTS

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