Time complexity of Recursive Algorithms

```
int factorial(int n){
  if(n==1)
    return 1;
  else
    return n*factorial(n-1);
}
```

Required time = a

Total time = T(n)

Required time = T(n-1)

For multiplication, required Time = b

Recurrence Relation for Time

$$T(n) = \begin{cases} a & n = 1 \\ T(n-1) + b & n > 1 \end{cases}$$

Solution:

$$T(n) = T(n-1) + b \qquad [\because T(n-1) = T(n-2) + b]$$

$$= \underbrace{T(n-2) + b}_{} + b$$

$$= T(n-2) + 2b$$

$$\vdots$$

$$= T(n-k) + kb$$

Let $n - k = 1$

$\Rightarrow k = n - 1$

$$= T(1) + b(n-1)$$

$[T(n) = a$ when $n = 1$

$\Rightarrow T(1) = a]$

$$= a + b(n-1)$$

$$= bn + a - b$$

$$= O(n)$$

```
int Addition(int A[], int n){
    if (n==1)
        return A[n-1];          Required Time = a
    else
        return A[n-1] + Addition(A, n-1);
}
```

Total Time = $T(n)$

$\underbrace{\text{Addition}(A, n-1)}$ ⇓ Required Time = $T(n-1)$

$+$ → Required Time = b [for addition]

Recurrence Relation for Time

$$T(n) = \begin{cases} a & n = 1 \\ T(n-1) + b & n > 1 \end{cases}$$

Solution:

$$\begin{aligned} T(n) &= T(n-1) + b \\ &= \\ &= \\ &\vdots \\ &= O(n) \end{aligned}$$

~~Different~~

Growth of Functions

Different Notations

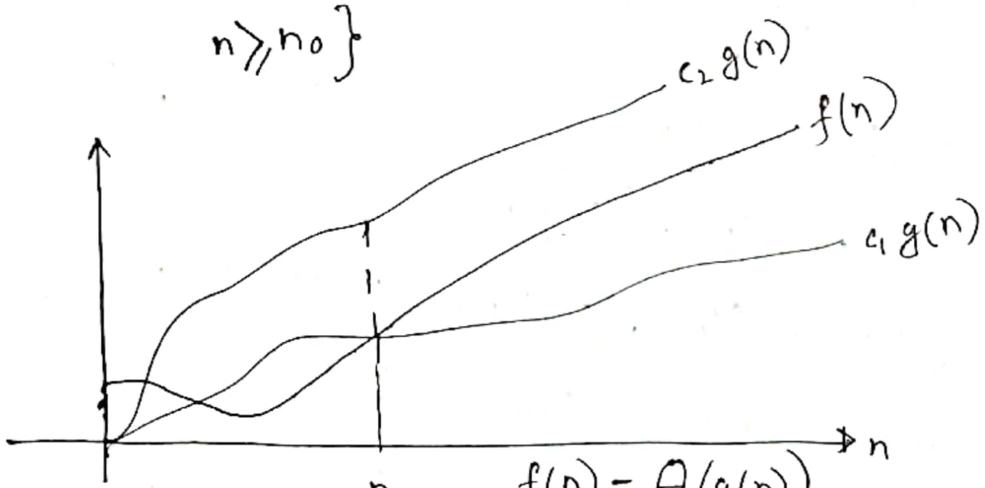
- Θ (Theta)
- O (Big O)
- Ω (Omega)
- o (Little o)

Θ- notation

$T(n) = \Theta(n^2)$

$\Rightarrow f(n) = \Theta(g(n))$ where $g(n) = n^2$

$= \{ f(n)$: there exists positive constants c_1, c_2 and n_0 such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0 \}$



$f(n) = \Theta(g(n))$

Prove that $f(n) = \frac{1}{2}n^2 - 3n = \Theta(n^2)$

From the definition Θ-notation

$c_1 g(n) \leq f(n) \leq c_2 g(n)$ Here, $g(n) = n^2$

$\Rightarrow c_1 n^2 \leq \frac{1}{2}n^2 - 3n \leq c_2 n^2$

$\Rightarrow c_1 \leq \frac{1}{2} - \frac{3}{n} \leq c_2$ [Divide by n^2]

Left side inequality

$c_1 \leq \frac{1}{2} - \frac{3}{n}$

$\therefore c_1 = \frac{1}{2} - \frac{3}{7}$ when $n \geq 7$

$= \frac{7-6}{14}$

$= \frac{1}{14}$

Positive

$n=1$	$\frac{1}{2} - 3$	→ Neg
$n=2$	$\frac{1}{2} - \frac{3}{2}$	
$n=3$	$\frac{1}{2} - \frac{3}{3}$	
$n=4$	$\frac{1}{2} - \frac{3}{4}$	
$n=5$	$\frac{1}{2} - \frac{3}{5}$	→ Neg
$n=6$	$\frac{1}{2} - \frac{3}{6}$	→ zero
$n=7$	$\frac{1}{2} - \frac{3}{7}$	→ positive

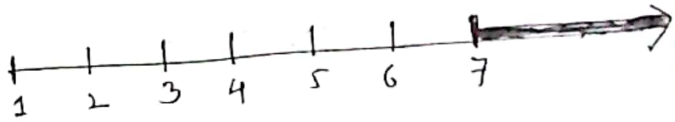
Right side inequality

$\frac{1}{2} - \frac{3}{n} \leq c_2$ → Positive

$\therefore c_2 = \frac{1}{2}$ when $n \geq 1$

$n=1$	$\frac{1}{2} - \frac{3}{1}$	→ True
$n=2$	$\frac{1}{2} - \frac{3}{2}$	→ True
$n=3$	$\frac{1}{2} - \frac{3}{3}$	→ True
⋮		
$n=\alpha$	$\frac{1}{2} - 0$	→ True

$\therefore c_1 = \frac{1}{14}$ when $n \geq 7$



$c_2 = \frac{1}{2}$ when $n \geq 1$

combindly $c_1 = \frac{1}{14}$, $c_2 = \frac{1}{2}$ when $n \geq 7$

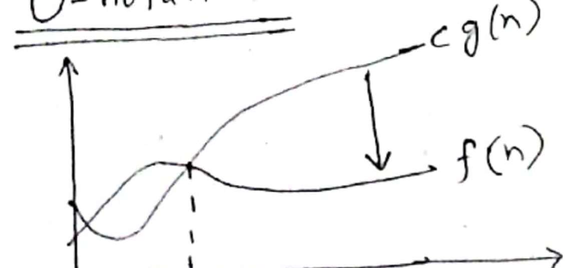
$\therefore c_1 g(n) \leq f(n) \leq c_2 g(n)$ when $n \geq 7$

$\Rightarrow \frac{1}{14} g(n) \leq f(n) \leq \frac{1}{2} g(n)$

$\therefore$ We can write, $f(n) = \theta(g(n))$

$= \theta(n^2)$ proved

O-notation

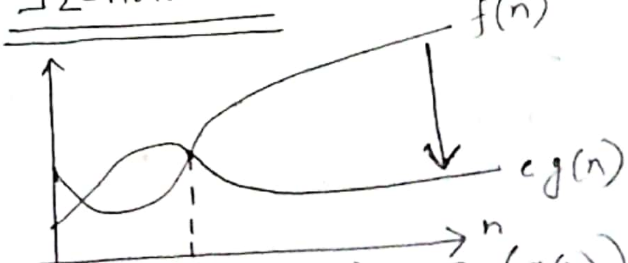


$f(n) = O(g(n))$

$f(n) \leq c\,g(n)$ when $n \geq n_0$

(Upper bound)

Ω-notation



$f(n) = \Omega(g(n))$

$c\,g(n) \leq f(n)$ when $n \geq n_0$

(Lower bound)

o-notation

$$f(n) = o(g(n)) \text{ when } \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

Prove that $f(n) = 6n^2 = o(n^3)$

Here, $f(n) = 6n^2$

$g(n) = n^3$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{6n^2}{n^3}$$

$$= \lim_{n \to \infty} \frac{6}{n}$$

$$= \frac{6}{\infty}$$

$$= 0$$

$\therefore f(n) = 6n^2 = o(n^3)$

Other Sorting Techniques

Bubble Sort:

10 5 4 7 ⟹ 5 10 4 7 ⟹ 5 4 10 7 ⟹ 5 4 7 | 10

5 4 7 | 10 ⟹ 4 5 7 | 10 ⟹ 4 5 | 7 10

4 5 | 7 10 ⟹ $\underbrace{4 \mid 5 \ 7 \ 10}_{\text{Sorted Order}}$

Selection Sort

10 5 4 7

7 5 4 | 10

4 5 | 7 10

4 | 5 7 10

Radix Sort

010 152 040 007

① ১ম বার

v_0: 040, 010 | v_1: | v_2: 152 | v_3: | v_4: | v_5: | v_6: | v_7: 007 | v_8: | v_9:

② 010 040 152 007

② ২য় বার

v_0: 007 | v_1: 010 | v_2: | v_3: | v_4: 040 | v_5: 152 | v_6: | ... | v_9:

007 010 040 152

③ ৩য় বার

v_0: 040, 010, 007 | v_1: 152 | v_2: | ... | v_9:

007 010 040 152

Bucket Sort

10 152 40 7 ⇒ 010 152 040 007

0 → 007 → 010 → 040 → NULL

1 → 152 → NULL

2

3

4

5

6

7

8

9

Sorted order

007 010 040 152