<u>Quick Sort Algorithm</u>

A[f] ≥ A[i] i →
A[f] < A[j] ← j

Mechanism:

27 8 9 32 29 18 15 26 55 14
f=1 ~~i=2~~ ~~i=3~~ i=4 j, l=10

27 8 9 14 29 18 15 26 55 32
f=1 i=5 j=9 l=10

27 8 9 14 29 18 15 26 55 32
f=1 i=5 j=8 l=10

27 8 9 14 26 18 15 29 55 32
f=1 ~~i=6~~ j=7 ~~i=7~~ i=8 l=10

15 8 9 14 26 18 27 29 55 32
f=1 j-1 j=7 j+1 l=10
Qsort(A, f, j-1) Qsort(A, j+1, l)

First partitioning element

Qsort Algorithm

```
void Qsort(int A[], int f, int l)
{
  if(l>f){
    i=f+1
    j=l
    while(i<=j){
      while((i<=l)&&(A[f]>=A[i]))
      { i=i+1
      }
      while((j>=f)&&(A[f]<A[j]))
      { j=j-1
      }
      if(i>j) break;
      A[i] <-> A[j]
      i=i+1
      j=j-1
    }
    A[f] <-> A[j]
    Qsort(A, f, j-1)
    Qsort(A, j+1, l)
  }
}
```

Time Complexity Analysis

$c(n)$ = element Comparison over n elements (Average)

$$c(n) = \frac{1}{n}\left[\sum_{j=1}^{n}\{(n+1) + c(j-1) + c(n-j)\}\right]$$

$j-1-1+1 = j-1$

$f=1$ —— $j-1$

$$= \frac{1}{n}\left[\sum_{j=1}^{n}(n+1) + \sum_{j=1}^{n}\{c(j-1) + c(n-j)\}\right]$$

$n-(j+1)+1 = n-j$

$j+1$ —— $l=n$

$$= \frac{1}{n}\left[n(n+1) + 2\{c(0) + c(1) + \cdots + c(n-1)\}\right]$$

1, 2, 3, 4 : $4-1+1 = 4$

$$\Rightarrow c(n) = (n+1) + \frac{2}{n}\{c(0) + c(1) + \cdots + c(n-1)\}$$ [Multiply by n]

$$n\,c(n) = n(n+1) + 2\{c(0) + c(1) + \cdots + c(n-1)\} \cdots\cdots \text{eq } (1)$$

Replace n by $(n-1)$

$$(n-1)c(n-1) = (n-1)n + 2\{c(0) + c(1) + \cdots + c(n-2)\} \cdots\cdots \text{eq } (2)$$

eq (1) − eq (2) $\Rightarrow$

$$n\,c(n) - (n-1)c(n-1) = \underline{\underline{n(n+1) - (n-1)n}} + 2c(n-1)$$

$$\Rightarrow n\,c(n) - (n-1)c(n-1) = 2n + 2c(n-1)$$

$$\Rightarrow n\,c(n) = 2n + (n-1)c(n-1) + 2c(n-1)$$

$$\Rightarrow n\,c(n) = 2n + \{(n-1) + 2\}c(n-1)$$

$$\Rightarrow n\,c(n) = 2n + (n+1)c(n-1)$$

$$\Rightarrow \frac{c(n)}{n+1} = \frac{2}{n+1} + \frac{c(n-1)}{n}$$ [Divide by $n(n+1)$]

[Recurrence Relation]

By Solving the Recurrence

$$\frac{c(n)}{n+1} = \frac{2}{n+1} + \frac{c(n-1)}{n}$$

Replace n by n−1

$$\because \frac{c(n-1)}{n} = \frac{2}{n} + \frac{c(n-2)}{n-1}$$

$$= \frac{2}{n+1} + \left[\frac{2}{n} + \frac{c(n-2)}{n-1}\right]$$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{c(n-2)}{n-1}$$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \frac{c(n-3)}{n-2}$$

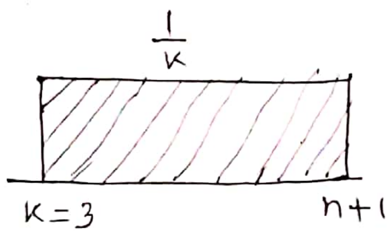
$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \cdots\cdots + \frac{2}{3} + \frac{c(1)}{2}$$

$\because c(1) = 0$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \cdots + \frac{2}{3} + \frac{0}{2}$$

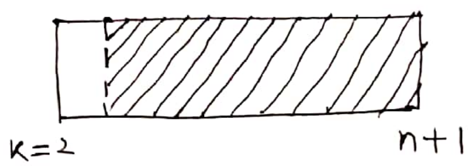
$$= 2\left(\frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n+1}\right)$$

$$= 2 \sum_{k=3}^{n+1} \frac{1}{k}$$



$$= 2 \int_{3}^{n+1} \frac{1}{k} \cdot dk$$

$$\Rightarrow \frac{c(n)}{n+1} \leq 2 \int_{2}^{n+1} \frac{1}{k} \cdot dk$$



$$= 2 \left[\log_e k\right]_2^{n+1}$$

$$= 2\left[\log_e (n+1) - \log_e 2\right]$$

$$\therefore c(n) \leq 2(n+1)\left[\log_e (n+1) - \log_e 2\right]$$

$$c(n) \leq O(n \log_e n)$$