

# Quantum K-Nearest Neighbors

**Srijita Dutta**

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# What is k-nearest Neighbors (k-NN)?

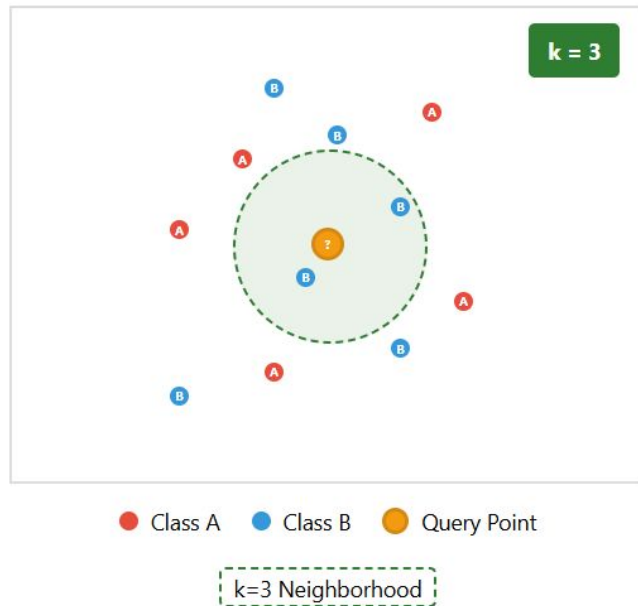
k-NN is a supervised machine learning algorithm that classifies data points based on the majority vote or average of their k nearest neighbors in the training dataset.

- k determines how many neighbors to consider
- Uses distance metrics to find nearest points
- Classification: majority vote among neighbors
- Regression: average of neighbor values
- Simple yet effective for many problems

## *Lazy Learner Algorithm*

k-NN doesn't learn during training. It stores the entire dataset and performs calculations only when making predictions, earning it the "lazy learner" designation.

k-NN Classification (k=3)



# How to choose the value of k for KNN Algorithm?

The choice of k significantly affects model performance. It balances between capturing local patterns and maintaining stability against noise and outliers.

**Small k** is more sensitive to noise and thus has a risk of **overfitting**, while a **Large k** is more stable but may miss patterns (**underfitting**)

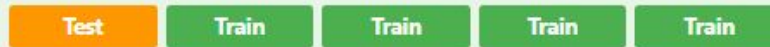
## Selection Methods

### Elbow Method Visualization



### Cross Validation Method

#### 5-Fold Cross-Validation Example



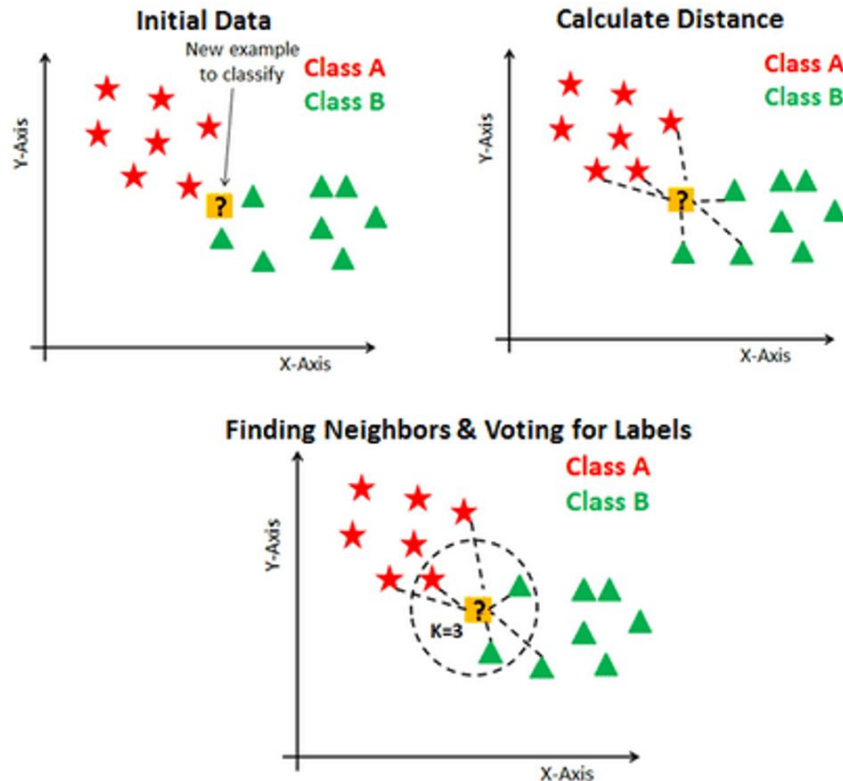
Each fold serves as test set once while others train the model.  
Average accuracy across all folds determines best k value.

### Best Practices

- Start with  $k = \sqrt{n}$  (where n is training samples) and test multiple odd values around this estimate
- Consider dataset size and noise level
- Validate on separate test set

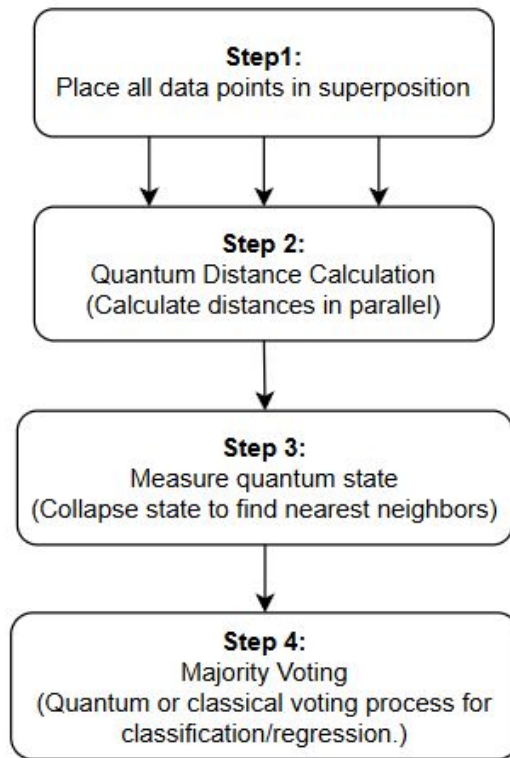
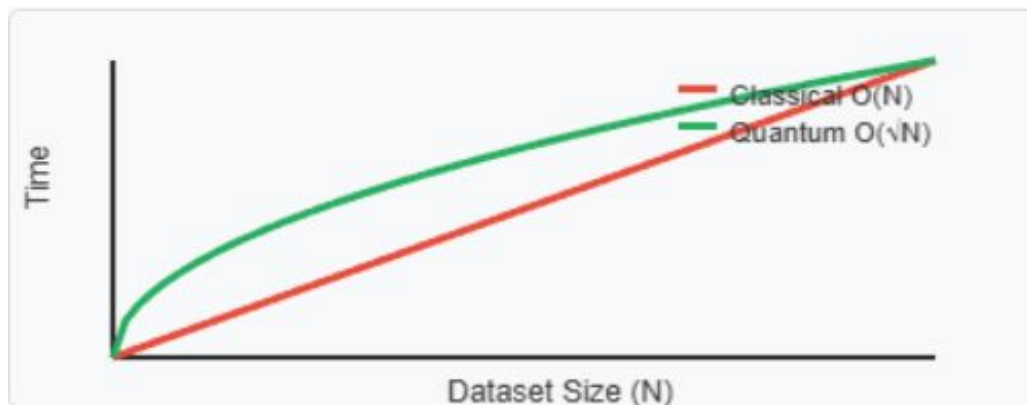
# Classical k-NN Process

- **Step 1:** Choose an optimal value of K.
- **Step 2:** Calculate the distance between the query point and each point in the dataset using traditional distance metrics like:
  - a. **Euclidean distance** =  $\sqrt{\sum (x_2 - x_1)^2}$
  - b. **Manhattan distance** =  $\sum |(x_2 - x_1)|$
- **Step 3:** Sort the distances and select the  $k(=3 \text{ in this case})$  nearest neighbors.
- **Step 4:** Assign a class label (or regression value) based on the nearest neighbors.

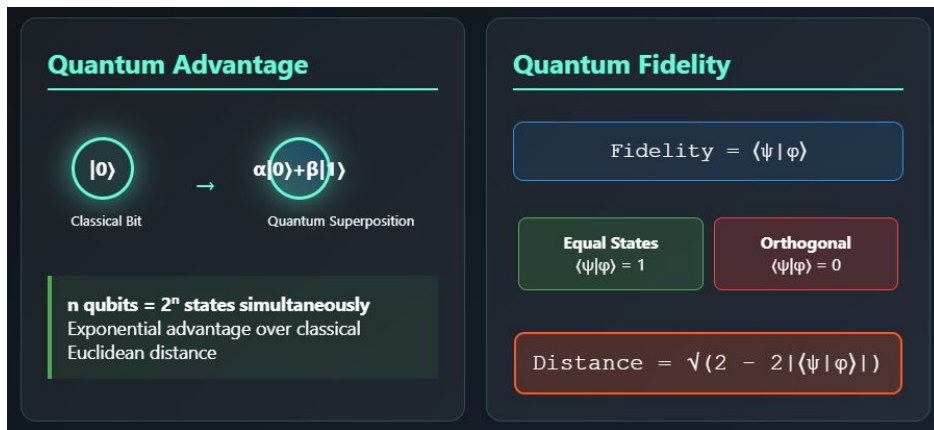


# Quantum k-NN Process

- **High-Dimensional Advantage:** Quantum k-NN excels in high-dimensional spaces
- **Quantum Minimization Algorithm:** Uses Grover's search to find nearest neighbors efficiently
- **Speedup Achievement:** Reduces classical  $O(n)$  to quantum  $O(\sqrt{n})$  time complexity



# Distance calculation in Quantum k-nn

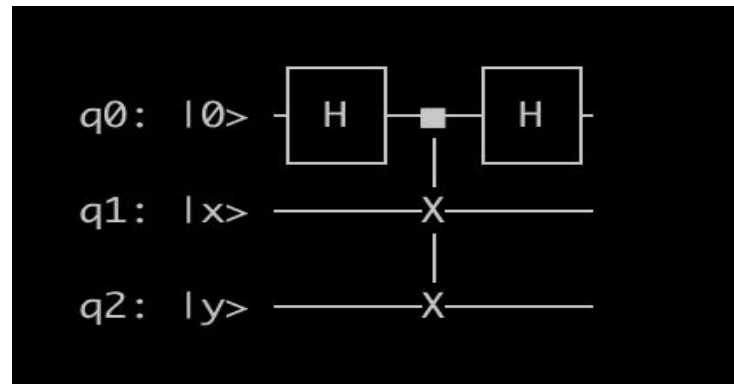


Using this method to compare two quantum states distance can be described as  $\sqrt{2-2|\langle\psi|\varphi\rangle|}$ . This algorithm uses an auxiliary qubit set to  $|0\rangle$  that gets passed through a **Hadamard gate**. Then the two vectors  $|x\rangle$  and  $|y\rangle$  are switched using the **Controlled-SWAP gate** otherwise known as the **Fredkin gate**.

The **circuit for SWAP test** looks something like this:

In quantum, distance is measured using **fidelity**. The fidelity of two quantum states  $|\psi\rangle$  and  $|\varphi\rangle$  is a measure of their similarity. Fidelity is the same as the inner product of two states and is represented by  $F = |\langle\psi|\varphi\rangle|^2$

- If the two quantum states are equal then  $F$  evaluates to 1.
- If the two states are orthogonal then  $F$  evaluates to 0.



# Why Quantum k-NN?

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Quantum k-NN can achieve a **quadratic** speedup using algorithms like Quantum Approximate Nearest Neighbor (QANN), hence, it can give faster solutions to certain problems, particularly those that involve large datasets or complex data structures.

## Key Advantages:

- **Speed:** Faster processing using quantum algorithms.
- **Efficiency:** Quantum features help reduce computational complexity.
- **Better with large data:** Quantum systems can handle high-dimensional data better than classical systems.

## Applications of Quantum k-NN:

Quantum k-NN can work better with high-dimensional data, which is often found in fields like:

- Image recognition
- Natural language processing
- Bioinformatics (protein folding, gene classification)

# References and resources

- [1] R. S. Chitambar and G. Gour, “Quantum Principal Component Analysis Using Hybrid Classical-Quantum Methods,” *arXiv preprint arXiv:2505.06441*, May 2025. [Online]. Available: <https://arxiv.org/abs/2505.06441>
- [2] M. Schuld and N. Killoran, “Quantum Machine Learning in Feature Hilbert Spaces,” *arXiv preprint arXiv:2003.09187*, Mar. 2020. [Online]. Available: <https://arxiv.org/abs/2003.09187>
- [3] A. Y. Guerrero-Estrada, L. F. Quezada, and G. H. Sun, “Benchmarking quantum versions of the kNN algorithm with a metric based on amplitude-encoded features,” *Scientific Reports*, vol. 14, no. 1, p. 16697, 2024. [Online]. Available: <https://doi.org/10.1038/s41598-024-67392-0>
- [4] J. Biamonte, P. Wittek, N. Pancotti *et al.*, “Quantum machine learning,” *Nature*, vol. 549, pp. 195–202, 2017. [Online]. Available: <https://doi.org/10.1038/nature23474>
- [5] <https://github.com/nagarx/Quantum-KNN-Classfier-using-Qiskit/tree/main>