

$$* \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$* \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$* \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$* \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$* \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$* \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$* \cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$* \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$* \sin(A+B) \sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

$$* \cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

$$* \tan(A+B) \tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

$$* \cot(A+B) \cot(A-B) = \frac{\cot^2 A \cot^2 B - 1}{\cot^2 B - \cot^2 A}$$

$$* \sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$

$$* \sin(A+B) - \sin(A-B) = 2 \cos A \sin B$$

$$* \cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$* \cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$\text{or, } \cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

$$* \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$* \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$* \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$* \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$* \sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$* \cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$= \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$* \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$* \cot 2A = \frac{\cot^2 A - 1}{2 \cot A}$$

$$* \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$* \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$* \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$* \cot 3A = \frac{\cot^3 A - 3 \cot A}{3 \cot^2 A - 1}$$

$$\left[\begin{array}{l} 4 \cos^3 A = 3 \cos A + \cos 3A \\ 4 \sin^3 A = 3 \sin A - \sin 3A \end{array} \right] \begin{array}{l} 4 = 3 + 1 \\ 4 = 3 - 1 \end{array}$$

ত্রিভুজের গুণগুণিতিক:

* Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = \frac{a+b+c}{\sin A + \sin B + \sin C}$

* $\frac{a}{\sin A} = 2R$; $a = 2R \sin A$; $\sin A = \frac{a}{2R}$

* Cosine rule: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$; $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos A$

* $a = b \cos C + c \cos B$; $b = a \cos C + c \cos A$; $c = a \cos B + b \cos A$

* ত্রিভুজের ক্ষেত্রফল, $\Delta = \frac{1}{2} \times \text{দৈর্ঘ্য} \times \text{উচ্চতা}$

$\Delta = \frac{1}{2} ab \sin C$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

পারিধিমা, $2s = a+b+c$

অর্ধপারিধিমা, $s = \frac{a+b+c}{2}$

$\Delta = \frac{1}{2} ab \cdot \frac{c}{2R} = \frac{abc}{4R}$

* পরিবৃত্তের ব্যাসার্ধ, $R = \frac{abc}{4\Delta}$

* অর্ধপারিধিমা s ও অন্তঃব্যাসার্ধ r হলে $\Delta = rs$

* অন্তঃব্যাসার্ধ, $r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

* $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$; $\sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$; $\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

* $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$; $\cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$; $\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

* $\tan \frac{A}{2} = \frac{(s-b)(s-c)}{\Delta}$; $\tan \frac{B}{2} = \frac{(s-a)(s-c)}{\Delta}$; $\tan \frac{C}{2} = \frac{(s-a)(s-b)}{\Delta}$

* $\tan A = \frac{abc}{R} \cdot \frac{1}{b^2 + c^2 - a^2}$; $\tan B = \frac{abc}{R} \cdot \frac{1}{c^2 + a^2 - b^2}$; $\tan C = \frac{abc}{R} \cdot \frac{1}{a^2 + b^2 - c^2}$

$$* \text{ অক্ষবাহু ত্রিভুজের ক্ষেত্রফল} = \frac{\sqrt{3}}{4} \times \text{বাহু}^2$$

$$* \text{ সমদ্বিবাহু ত্রিভুজের ক্ষেত্রফল} = \frac{b}{4} \sqrt{4a^2 - b^2}$$

Shortcut

$$* \text{ কোনো সমান্তর বীজের যোগফল} = \frac{\text{পদসংখ্যা}}{2} \quad [\text{পদসংখ্যা জোড় হলে}]$$

$b = \text{দুটি}$
 $a = \text{সমান বাহুর দৈর্ঘ্য}$

যোগফল = জোড় পদের জন্য যোগফল + অবশিষ্ট

$$\text{পদসংখ্যা} = \frac{\text{সর্বোচ্চ মান} - \text{সর্বনিম্ন মান}}{\text{পার্থক্য}} + 1$$

* অবশ্যই \sin বা \cos এর দ্বিগুণবিশিষ্ট হতে হবে।

$$* \frac{\cos \theta \pm \sin \theta}{\cos \theta \mp \sin \theta} = \tan(45^\circ \pm \theta) \quad [\theta < 45^\circ]$$

$$* \frac{\sin \theta_{\text{ক}} \pm \sin \theta_{\text{খ}}}{\sin \theta_{\text{ক}} \mp \sin \theta_{\text{খ}}} = \tan(45^\circ \pm \theta_{\text{খ}}) \quad [\theta < 45^\circ]$$

$$* \frac{\cos \theta_{\text{ক}} \pm \cos \theta_{\text{খ}}}{\cos \theta_{\text{ক}} \mp \cos \theta_{\text{খ}}} = \tan(45^\circ \pm \theta_{\text{ক}}) \quad [\theta > 45^\circ]$$

$$\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$$

$$\sin 36^\circ = \cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

$$\sin 72^\circ = \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\sin 75^\circ = \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan 15^\circ = 2 - \sqrt{3}$$

$$\tan(-15^\circ) = \sqrt{3} - 2$$

$$\tan 75^\circ = 2 + \sqrt{3}$$

$$\tan 7\left(\frac{1}{2}\right) = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$$

কিছু important objective:

$$\Rightarrow * 2 \cos \frac{\pi}{2^n} = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots (n-1) \text{ সংখ্যক}}}}$$

$$\Rightarrow * 2 \sin \frac{\pi}{2^n} = \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots (n-2) \text{ সংখ্যক}}}}$$

$$\Rightarrow * 2 \cos \frac{\pi}{3 \cdot 2^n} = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots (n-1) \text{ সংখ্যক} + \sqrt{3}}}}$$

$$* 2 \sin \frac{\pi}{3 \cdot 2^n} = \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots (n-2) \text{ সংখ্যক} + \sqrt{3}}}}$$

Example

$$* 2 \cos \frac{\pi}{2^4} = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$$

$$2 \cos \frac{\pi}{16}$$

$$2 \sin \frac{\pi}{3 \cdot 2^4} = \sqrt{2 - \sqrt{2 + \sqrt{3}}}$$

$$2 \sin \frac{\pi}{48}$$

[$\theta > 0$]

$$(\theta \pm \pi) \text{ not} = \frac{\sin \theta \pm \sin \pi}{\sin \theta \pm \sin \pi}$$

* a, b, c ত্রাকোণের কোণমানগুলো হলে, $a + c = 2b$

কিছিত কোণমানগুলো হলে, $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

[$\theta < 0$]

$$(\theta \pm \pi) \text{ not} = \frac{\sin \theta \pm \sin \pi}{\sin \theta \pm \sin \pi}$$

$$\sin \theta - \sin \phi = 2 \cos \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2}$$

$$\sin \theta + \sin \phi = 2 \sin \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2}$$

$$\cos \theta - \cos \phi = -2 \sin \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2}$$

$$\cos \theta + \cos \phi = 2 \cos \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2}$$

$$\frac{\sin \theta - \sin \phi}{p}$$

$$= \frac{1 - \cos \theta}{2 \sin \frac{\theta}{2}} = \sin \frac{\theta}{2} = \sin \theta$$

$$\frac{1 - \cos \phi}{p} = \sin \frac{\phi}{2} = \sin \phi$$

$$\frac{\sin \theta - \sin \phi}{p} = \sin \frac{\theta}{2} - \sin \frac{\phi}{2}$$

$$\frac{1 - \cos \theta}{p} = \sin \frac{\theta}{2} = \sin \theta$$

$$\frac{\sin \theta + \sin \phi}{p} = \sin \frac{\theta}{2} + \sin \frac{\phi}{2}$$

$$\frac{\sin \theta + \sin \phi}{p}$$

$$= \frac{1 + \cos \theta}{2 \cos \frac{\theta}{2}} = \cos \frac{\theta}{2} = \cos \theta$$