Linear Algebra Error Mitigation in a GHZ state

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Abstract

The purpose of this project is to study the linear algebra error mitigation technique and to see if there is a relation between the fidelity and the noise probability by plotting the obtained data and observing the behavior of the results. This can be done by generating noise in a quantum circuit, computing the calibration matrix associated with the system, and getting the mitigated result. The fidelity obtained for a GHZ state with 10% of noise probability was 0.723 after applying the error mitigation technique, which corroborates that the algorithm successively mitigated the noise in the quantum circuit.

1 Introduction

New technologies are being built thanks to the basis of quantum computing, which has originated with the principles of quantum theory, with Planck and Einstein as protagonists. Problems that could take longer periods in a traditional computer can be done in seconds, using characteristics such as superposition, entanglement, storage, and sending information through qubits. [1]

Unfortunately, the biggest challenge for quantum computing is noise. Such noise can be due to imperfect control signals, interference from the environment, unwanted interactions between qubits, and quantum gates. These quantum gates perform operations on a quantum circuit, change the quantum state of a qubit, and then perform calculations to solve a specific problem.

In general, the effect of the noise that occurs throughout a calculation can be complex because it would be necessary to consider how each gate transforms the effect of each error. Error Mitigation is a series of protection techniques and algorithms to ensure the reduction of vulnerability and the attenuation of potential damage to the information transferred in a system.

For this project, it was used the linear algebra error mitigation technique. The noise was generated by a python function using Qiskit tools for noise models, and it was applied to a Bell state of three qubits, better known as the Greenberger–Horne– Zeilinger state (GHZ).

The GHZ state has its principles in a Bell state but is applied to three or more qubits. It was studied by Daniel Greenberger, Michael Horne, and Anton Zeilinger in 1989. [2] A bell state is a superposition of two qubits, and it represents the simplest and maximal quantum entanglement. [3]

The GHZ states are the maximally entangled states of N qubits, which have had many important applications in quantum information processing, such as quantum key distribution and quantum secret sharing. [4] This is beyond the scope of this project.

The motivation to choose this specific state was to understand the fundamentals of quantum mechanics and quantum error mitigation, doing the calculations by hand and knowing the final outcome in advance. The maximum entanglement of the GHZ state is very important to quantum computing, and it was an advantage for us to look forward to obtain a result of 50/50 probability of being in states $|000\rangle$ and $|111\rangle$.

2 Algorithm

This algorithm is based on a Qiskit code [5], and it intends to mitigate errors in a quantum system with linear algebra. There are other techniques, but we chose this one for its simplicity. In general:

1. Create a function that generates noise.
   (a) Include Pauli gates (we used X, Y and I gates) to cause errors related to such gates, with a respective probability.

   The X-Gate is represented by the Pauli-X matrix:

   $$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

   This gate switches the amplitudes of the states $|0\rangle$ and $|1\rangle$.

   The Y-Gate performs rotations by π around the y-axis of the Bloch sphere. It is represented by:

   $$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

   The I-Gate or the identity gate doesn't affect the circuit, [6] its matrix is the identity matrix:

   $$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   (b) Use the Qiskit tool `NoiseModel()`. This class is used to represent noise model for the `QasmSimulator` and it can be used to construct custom noise models for simulator.
2. Choose the probability of the noise function (we called it NP), in decimals.
3. Program the circuit. In this case, we chose the $|GHZ\rangle$ state, a Bell state with three qubits q_0, q_1 and q_2. To get this output:
   (a) Start with a $|00+\rangle$ state, applying a Hadamard gate to q_0.

   The Hadamard gate is a fundamental quantum gate. It allows us to create a superposition of $|0\rangle$ and $|1\rangle$. It has the matrix:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

The Hadamard gate performs the transformations: [6]

$$H|0\rangle = |+\rangle \text{ and } H|1\rangle = |-\rangle$$

The $|00+\rangle$ state can be represented mathematically as

$$\begin{aligned}|00+\rangle &= |0\rangle \otimes |0\rangle \otimes |+\rangle\\ &= \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\\ &= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \otimes \begin{bmatrix}1\\1\end{bmatrix}\end{aligned}$$

Then,

$$|00+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

This can be written as a sum of two states,

$$|00+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

And using $|0\rangle \otimes |0\rangle \otimes |0\rangle$ and $|0\rangle \otimes |0\rangle \otimes |1\rangle$:

$$|00+\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |001\rangle)$$

(b) Apply `CNOT` gates.

The CNOT gate is a conditional gate that performs an X-gate on the target qubit if the state of the control qubit is $|1\rangle$. In Qiskit, you can apply a CNOT gate on two qubits using: `circuit.cx(control, target)`. [7]

i. First, apply a CNOT gate, with q_0 as the control qubit and q_1 as the target qubit. This will flip the state of the target (applying an X gate) if the state of the control qubit is equal to 1.

ii. Apply another CNOT gate, but choose q_0 as the control qubit and q_2 as the target qubit.

The output of the circuit is:

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

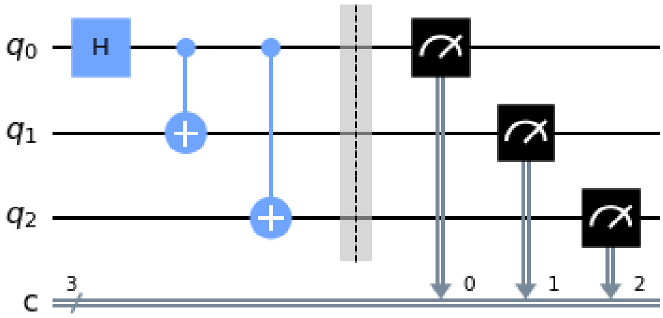

Figure 1: GHZ state used.

4. Run the circuit in the simulator `aer_sim` with the noise model. Use 10000 shots.
5. Get the results and the counts for every possible quantum state of the circuit. This will give an output of the counts for 8 different states in a python dictionary.
6. Take the counts dictionary for each basis states, and normalise it.
7. Create the calibration matrix M using the data obtained in the last step.
8. Invert the calibration matrix M, because the noisy result is:

$$C_{\text{noisy}} = MC_{\text{mitigated}}$$

   Where C refers to the result, the probability of measuring the states.

9. Apply the inverse of the calibration matrix M to the noisy results C_{noisy}, to get $C_{\text{mitigated}}$:

$$C_{\text{mitigated}} = M^{-1}C_{\text{noisy}}$$

3 Results and Analysis

We applied noise to the circuit shown in Fig. (1), and we ran it in a simulator. First, with the noise generated by the pauli error function, and then applying the noise calibration, with 10% of noise probability in both cases. Afterward, we plotted a histogram with the results as Fig. (2) shows,

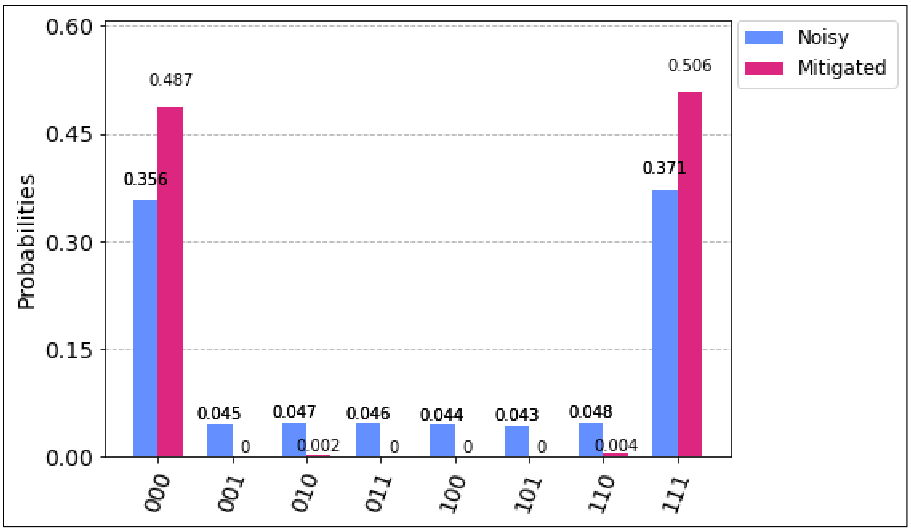

Figure 2: Noisy and Mitigated probabilities at 10% noise prob.

It is visible that without noise mitigation (in blue) and 10% of noise probability the states are not clearly defined for the GHZ states. With the mitigated noise (in pink) we can appreciate partial and total reductions for the unwanted states of the system, and an overall increase on both 000 and 111 states to a probability of 0.5 according to the scale, those being the desired states.

Subsequently we calculated the average fidelity for the measurement, resulting in 0.723 for the given noise reduction method applied to the GHZ state, where the ideal average fidelity is 1.

Afterward, we increased the noise percentage in 50 steps and calculated the fidelity for each one of the given values, starting with a noise percentage of 5% up to 50%. This data was plotted in Fig 3, one can notice that the fidelity tends to decrease exponentially with respect to the increase in the noise percentage. Applying an exponential fitting $y = a \cdot e^{b \cdot x}$ to the curve using Python and the Scipy library, we found the decay rate for the system with value $b = -0.039$.

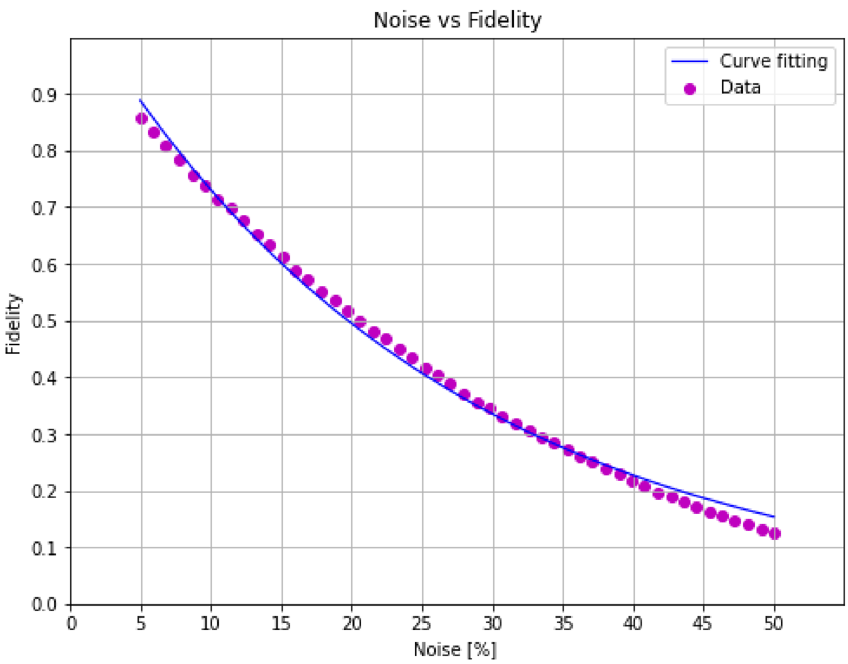

Figure 3: Increase in noise with mitigated data

4 Conclusions

The linear algebra error mitigation technique was able to obtain an average fidelity of 0.723 with a 10% noise probability, which is close to the ideal value of 1. When comparing the mitigated results and the results with noise, it is observed that the applied filter fulfilled its objective of attenuating errors and it gets closer to the expected values. Likewise, it was found that the fidelity and the percentage of the probability of noise are related, and in this case it behaves as a decreasing exponential curve. That is, the higher the percentage of noise, the lower the fidelity.

Although this technique worked correctly in the chosen circuit, the biggest disadvantage is that the calibration matrix is specific to the system, that means if we try to apply it to another circuit, it won't work. Nonetheless, this technique works on the specific system where it is applied. The level of mitigation is high, approaching the results near an ideal level.

For qubits is quite easy to lost information due to decoherence and that is the main reason behind the error mitigation models, preventing this from happening. This particular study case is open to further development, even to develop an optimization for the models improving the average fidelity, not limited to this particular case but with more qubits in the system.

References

[1] Reyes Alvarez, M. F., "Evolución de la computación cuántica y los retos para la seguridad de la información," *Cuaderno activa*, , No. 8, 2016, pp. 49–63.

[2] Greenberger, D. M., Horne, M. A., and Zeilinger, A., "Going Beyond Bell's Theorem," 2007.

[3] Inspire, Q., "Bell states," `https://www.quantum-inspire.com/kbase/bell-states/`, Accessed: April 29, 2022.

[4] Fan, X.-Y., Zhou, J., Meng, H.-X., Wu, C., Pati, A. K., and Chen, J.-L., "Greenberger–Horne–Zeilinger states: Their identifications and robust violations," *Modern Physics Letters A*, Vol. 36, No. 31, oct 2021.

[5] Qiskit, "Measurement Error Mitigation," `https://qiskit.org/textbook/ch-quantum-hardware/measurement-error-mitigation.html`, Accessed: April 28, 2022.

[6] Qiskit, "Single Qubit Gates," `https://qiskit.org/textbook/ch-states/single-qubit-gates.html`, Accessed: April 28, 2022.

[7] Qiskit, "Multiple Qubits and Entangled States," `https://qiskit.org/textbook/ch-gates/multiple-qubits-entangled-states.html#3.1-The-CNOT-Gate-`, Accessed: April 28, 2022.