Supplementary Material for: "Shifts in colour morph frequencies along an urbanisation gradient in the ground beetle *Pterostichus madidus*"

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S1 - Model description

For the present study, our response variable is $n_{i,j}/N_{i,j}$, the proportion of black-legged beetles captured in a given woodland i during a given sampling session j, with $N_{i,j}$ the corresponding total number of *P. madidus* beetles. Note that this naturally only includes woodland × session combinations with $N_{i,j} > 0$. We ran the below models for each of the 8 possible urbanisation metrics described in the main text.

We initially built binomial models as follows:

$$n_{i,j} \sim \text{Binomial}(p_{i,j}, N_{i,j}),$$
$$\text{logit}(p_{i,j}) = \beta_0 + (\beta_1 + \eta_j) \times x_i + \alpha_i + \gamma_j,$$

with x_i the (centered and scaled) urbanisation metric at site i, β_0 and β_1 the fixed-effects intercept and urbanisation slope, respectively, α_i the site-specific random intercept, and γ_j and η_j the session-specific random intercept and slope. Random effects are distributed as follows:

$$\alpha_i \sim \text{Normal}(0, \sigma_\alpha),$$
$$\begin{bmatrix}\gamma_j \\ \eta_j\end{bmatrix} \sim \text{MVNormal}\left(\begin{bmatrix}0 \\ 0\end{bmatrix}, \boldsymbol{\Omega}\right),$$

, where $\boldsymbol{\Omega}$ is the covariance matrix for the session-specific random effects, which can be decomposed into its constituent standard deviations and correlation matrix $\boldsymbol{R}$ in this way:

$$\boldsymbol{\Omega} = \begin{bmatrix}\sigma_\gamma & 0 \\ 0 & \sigma_\eta\end{bmatrix} \boldsymbol{R} \begin{bmatrix}\sigma_\gamma & 0 \\ 0 & \sigma_\eta\end{bmatrix}.$$

We used weakly informative priors inspired by @mcelreathStatisticalRethinkingBayesian2020. We used $\text{Normal}(0, 1)$ priors for the fixed effects β, and $\text{Half} - \text{Normal}(0, 1)$ priors for all standard deviations σ. We used a $\text{LKJ}(2)$ prior for the correlation matrix $\boldsymbol{R}$.

Evaluations of these models revealed slight but consistent evidence of overdispersion. We therefore fitted the equivalent beta-binomial models to account for that overdispersion:

$$n_{i,j} \sim \text{BetaBinomial}(p_{i,j}, N_{i,j}, \phi),$$

where ϕ is an added overdispersion parameter with prior $1/\phi \sim \text{HalfNormal}(0, 1)$. The remainder of the models is the same as in the binomial case.