**CS 537: Artificial Intelligence**

**HW#3 Sudoku Report**

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This report is on the HW#3: Sudoku and highlights the 5 different algorithms used to solve the Sudoku board. It also shows the performance of the 5 algorithms and also gives an insight on which of the 5 implementation methods should be used when. The 5 implementation methods are:

1. Backtracking
2. Backtracking + MRV heuristic
3. Backtracking + MRV + Forward Checking
4. Backtracking + MRV + Constraint Propagation
5. Min-conflicts Heuristic

**1. Backtracking**

This algorithm chooses the first empty cell in the board, finds all the valid numbers possible for that cell, iterates on that list and keeps setting numbers in a Depth First manner. Recursion is used to implement the DFS nature. The algorithm unsets the values that do not result in a valid Sudoku board.

**2. Backtracking + MRV heuristic**

This algorithm implements the same backtracking algorithm along with Minimum Remaining Values and Least Constraining Values heuristics. This heuristic helps in solving the complex boards faster.

**3. Backtracking + MRV + Forward Checking**

This algorithm implements the Backtracking + MRV heuristic algorithm along with Forward Checking Heuristic. In this heuristic we check if there are no valid values remaining for the neighboring cells. This heuristic helps in pruning the tree and lessen the execution time.

**4. Backtracking + MRV + Constraint Propagation**

This algorithm implements the Backtracking + MRV heuristic algorithm along with 3-ARC constraint propagation. This also helps in pruning the tree and lessen the execution time.

**5. Min-conflicts Heuristic**

This algorithm requires a completely filled board. If the board is not completely filled, then this algorithm fills the empty cells randomly. The following steps are done in an iteration which runs for a counter = 10000. Next it finds the set of conflicting cells, randomly chooses a cell from the set, finds a number that that minimizes the conflict for that cell, sets that number to the cell and proceeds with the iteration. The number that minimizes the conflict is calculated by finding the frequency of numbers present in the domain of the cell and choosing a number that has the least frequency. There is a lot of randomization in this heuristic and thus this algorithm sometimes fails to find a solution for a board.

**Performance of the 5 implementation methods**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **BT** | **BT+MRV** | **BT+MRV+FC** | **BT+MRV+CP** | **Min Conflicts** |
| **Empty 4\*4** | 0.000237  16 | 0.001836  16 | 0.002091  16 | 0.007693  16 | 0.00347  50 |
| **Empty 9\*9** | 0.007189  407 | 0.055347  81 | 0.052324  81 | 0.321908  81 | No  Solution |
| **Empty 12\*12** | 0.030915  1040 | 0.199009  144 | 0.198532  144 | 1.385617  144 | No  Solution |
| **Wrongly filled 12\*12** | No  Solution | No  Solution | No  Solution | No  Solution | 0.320524  262 |
| **Board 1** | 0.004765  184 | 0.041713  76 | 0.038924  76 | 0.048022  76 | No  Solution |
| **Board 2** | 3.06689  130315 | 2.011561  2746 | 1.947563  2746 | 1.972546  2746 | No  Solution |
| **Board 3** | 11.671281  416165 | 0.212882  247 | 0.206673  247 | 0.233329  247 | No  Solution |

The table above shows the execution time and number of consistency checks for each of the 5 implementation methods. The boards used for testing are listed at the end of this report.

**Conclusion**

* We see that for simpler boards, the Backtracking algorithm takes lesser execution time than the algorithms which use heuristics to prune the valid numbers. This is because for uncomplicated boards, the heuristics are not required and BT does a fair job in finding the solution board by DFS method.
* For larger complex boards such as Board 2 and Board 3, the Backtracking algorithm takes a lot of time to build the large DFS tree. The heuristics perform way better as they prune the tree and the numbers which may lead to an invalid solution board are not used from the start itself. The performance of BT+MRV+FC and BT+MRV+CP is better than the performance of BT+MRV as the former two heuristics do a 1 step forward check of numbers and 3-ARC consistency check respectively that may lead to invalid boards and prune the tree accordingly.
* Min Conflict heuristic involves a lot of randomization and thus produces results for smaller boards and larger boards which are partially filled. We cannot let the Min Conflict algorithm run for long for obvious performance constraints.

**The boards used for testing mentioned in the performance table are as follows:**

**Wrongly filled 12\*12:**

12,3,4  
1,1,3,4,5,6,7,8,9,10,12,12;  
5,5,7,8,9,10,11,12,1,2,3,4;  
9,10,10,12,1,2,3,4,8,5,6,7;  
2,11,4,1,1,9,8,3,10,7,12,5;  
8,7,5,6,2,1,1,10,11,9,4,3;  
10,9,12,3,11,5,4,7,2,1,8,6;  
3,4,6,5,12,8,10,2,7,11,1,9;  
8,8,1,9,7,4,6,5,3,12,10,2;  
7,7,10,2,3,11,1,9,3,6,5,8;  
12,1,2,11,8,3,9,6,5,4,7,10;  
4,4,9,7,10,12,5,1,6,8,2,11;  
6,6,8,10,4,7,2,11,12,3,1,1;

**Board 1:**

12,3,4;  
1,9,-,-,11,8,-,-,-,10,-,-;  
5,2,10,-,-,12,1,-,-,8,-,-;  
-,8,3,4,-,7,-,9,-,1,6,5;  
-,-,9,-,4,-,-,5,12,6,3,-;  
11,12,8,-,3,1,-,-,-,9,7,-;  
3,5,4,-,9,11,-,-,-,-,10,-;  
9,-,2,12,-,-,-,-,-,-,-,7;  
-,3,-,8,-,5,-,7,10,-,-,2;  
-,-,-,-,1,6,-,10,3,-,9,-;  
10,1,-,2,-,-,11,8,9,-,-,-;  
-,-,11,-,-,9,12,6,-,-,2,-;  
-,-,5,-,10,-,-,-,4,-,-,12;

**Board 2:**

12,3,4;  
-,2,12,-,-,-,1,4,-,8,-,-;  
-,9,-,-,-,2,-,-,-,4,1,10;  
10,1,-,-,5,-,6,9,-,-,-,12;  
-,-,-,-,1,7,-,-,-,-,-,-;  
9,-,6,-,2,-,-,12,5,7,-,-;  
4,-,3,-,-,-,-,-,11,-,2,-;  
-,5,-,12,-,-,-,-,-,1,-,4;  
-,-,1,2,11,-,-,5,-,10,-,6;  
-,-,-,-,-,-,3,7,-,-,-,-;  
12,-,-,-,10,3,-,8,-,-,6,11;  
1,10,5,-,-,-,9,-,-,-,12,-;  
-,-,7,-,12,5,-,-,-,9,10,-;

**Board 3:**

12,3,4;  
-,3,-,-,11,-,-,-,10,-,-,12;  
-,11,-,-,-,-,-,-,-,-,7,-;  
7,-,-,2,-,-,-,9,-,-,6,-;  
-,4,-,5,-,-,-,-,-,6,-,-;  
-,-,7,-,4,10,9,12,-,5,-,-;  
-,-,11,-,-,-,-,-,4,-,10,-;  
-,8,-,6,-,-,-,-,-,11,-,-;  
-,-,3,-,10,6,12,4,-,7,-,-;  
-,-,5,-,-,-,-,-,12,-,9,-;  
-,5,-,-,9,-,-,-,8,-,-,11;  
-,9,-,-,-,-,-,-,-,-,2,-;  
3,-,-,8,-,-,-,7,-,-,5,-;