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# The History of Iterative Methods

Archeological evidence has revealed iterative calculations dating back further than 1500 BC[1]. Since then, iterative methods have advanced in both efficiency and accuracy. The following essay hopes to give an overview of these advancements and the great minds behind them. In the words of computer scientist Donald Knuth: “One of the ways to help make computer science respectable is to show that it is deeply rooted in history...”[2].

It begins, so far as we can tell, in ancient Babylon, where researchers have uncovered evidence of advanced numerical calculations in what is known as the Babylonian tablet[3], artifact 7289 of the Yale Babylonian Collection. This piece of stone shows the calculation of the square root of two using iterative strategies. On the stone is carved a square with two diagonal lines connecting the corners together and crossing at the center. The engravings show several numbers (written in base 60, different from our standard base 10 or binary base two) along the length of one diagonal. Researchers have interpreted the numbers to indicate additions to one by a series of ratios, achieving a closer approximation to root two with increasing iterations. Other ancient artifacts reveal that this stone is not merely a fluke. Ancient societies had discovered amazingly sophisticated mathematical techniques.[4]

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[1] https://en.wikipedia.org/wiki/YBC_7289
[2] Abstract, https://www.unige.ch/~gander/Preprints/LandmarksPaper.pdf
[3] Pages 2-3, https://www.unige.ch/~gander/Preprints/LandmarksPaper.pdf
[4] See part one, https://www.unige.ch/~gander/Preprints/LandmarksPaper.pdf

Moving forward in history, one sees iterative methods becoming very important for astrophysics, especially in the calculation of orbits. Kepler, who is famous for his law explaining the elliptical orbit of planets, utilized iterative techniques.

Then, in the 17th Century, a fresh wave of advancement hit the world of iterative methods. Great minds such as Francois Vieta, Isaac Newton, Joseph Raphson, and Thomas Simpson, each contributed to the development of strategies to find roots. Newton's method, which is often referred to as the Newton-Raphson method, boasted quadratic convergence for finding roots. Simpson ran with the momentum, formally introducing calculus into the equation (although Newton's "fluxion" was very similar in practice[5]). Further developments were pioneered by German mathematician Carl Friedrich Gauss, who discovered his own direct and indirect method of solving linear equations.[6]

Both Newton and Gauss had very interesting lives. Newton had a tough childhood, with his father dying before his birth and his mother and stepfather abandoning him in his early years. He was known to be unpleasant at times, an attribute presumably linked to his formative years.[7] A quote from Stephen Hawking says: "Isaac Newton was not a pleasant man. His relations with other academics were notorious, with most of his later life spent embroiled in heated disputes."[8] However, no one can doubt the influence of Isaac Newton, whose contributions to modern mathematics, Physics, Optics, and more are incredible. Newton spent two years of his time in University effectively quarantined because of the plague, and yet despite the apparent set back, he used this time to make leaps and bounds in science and mathematics, laying the groundwork for calculus and studying planetary circular motion.

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[5] Explanation on page 23. Simpson formally introduced Calculus, but also seemed to acknowledge that Newton and Raphson were using more than mere Algebra. Who can be credited with calculus in iterative methods seems to still be up for debate. https://www.unige.ch/~gander/Preprints/LandmarksPaper.pdf
[6] Page 1-2, https://www-users.cse.umn.edu/~saad/PDF/ys-2019-01.pdf
[7] https://www.britannica.com/biography/Isaac-Newton
[8] Page 18. https://www.unige.ch/~gander/Preprints/LandmarksPaper.pdf

Carl Friedrich Gauss grew up in a poor family, and was considered a calculating prodigy, able to perform difficult calculations extremely quickly in his head.[9] His remarkable talent earned him the funding needed to attend the University of Göttingen in Germany. His contributions to modern science span several fields, and particularly relevant have been his direct and indirect methods of elimination for solving linear equations. With Gaussian elimination, we see a distinct emphasis on efficiency. He has been quoted in a correspondence to a friend saying about his newly discovered indirect elimination: "I recommend this method to you for imitation. You will hardly ever again eliminate directly, at least not when you have more than 2 unknowns"[10]. Mathematicians after Gauss would continue his work with matrices, and would eventually discover increasingly efficient methods for solving linear equations using iterative methods, such as does the Gauss-Seidel method.

Thus, one can see that the use of iterative methods stretches as far back as ancient Babylon, and yet, as old as it is, recent centuries have shown breakthrough after breakthrough. The two motivations of accuracy and efficiency have continued and will continue to propel iterative methods and numerical analysis to new heights.

[9] https://www.britannica.com/biography/Carl-Friedrich-Gauss
[10] https://indico.math.cnrs.fr/event/9231/