Dekker-Brent Algorithm

Numerical Analysis Final Presentation

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Overview

1. Other Root Finding Algorithms
2. Dekker
3. Brent

Remember?

- The **Bisection Method**: halving and keeping opposite signs
- The **Secant Method**: uses a succession of roots of secant lines to better approximate
- The **Inverse Quadratic Interpolation**

Inverse Quadratic Interpolation

$$x_{n+1} = \frac{f(x_{n-1})f(x_n)}{(f(x_{n-2}) - f(x_{n-1}))(f(x_{n-2}) - f(x_n))}x_{n-2}$$
$$+\frac{f(x_{n-2})f(x_n)}{(f(x_{n-1}) - f(x_{n-2}))(f(x_{n-1}) - f(x_n))}x_{n-1}$$
$$+\frac{f(x_{n-2})f(x_{n-1})}{(f(x_n) - f(x_{n-2}))(f(x_n) - f(x_{n-1}))}x_n$$

We use the three preceding iterates, x_{n2}, x_{n1} and x_n, with their function values, $f(x_{n2})$, $f(x_{n1})$ and $f(x_n)$. Applying the Lagrange interpolation formula to do quadratic interpolation on the inverse of f.

Dekker's Method

Secant method: $s = \begin{cases} b_k - \frac{b_k - b_{k-1}}{f(b_k) - f(b_{k-1})} f(b_k), & \text{if } f(b_k) \neq f(b_{k-1}) \\ m & \text{if } f(b_k) = f(b_{k-1}) \end{cases}$

Bisection method: $m = \frac{a_k + b_k}{2}$

if $|b_k| < |s| < |m|$ then $b_{k+1} = s$, otherwise $b_{k+1} = m$

Brent's Method

An additional test must be satisfied. The following inequalities for a given δ:

If the previous step used bisection: $|\delta| < |b_k - b_{k-1}|$ and $|s - b_k| < \frac{1}{2}|b_k - b_{k-1}|$

If the previous step used interpolation: $|\delta| < |b_{k-1} - b_{k-2}|$ and $|s - b_k| < \frac{1}{2}|b_{k-1} - b_{k-2}|$

Why?

This modification ensures that at the k^{th} iteration, a bisection step will be performed in at most $2\log_2(|b_{k-1} - b_{k-2}|/\delta)$ additional iterations, because the above conditions force consecutive interpolation step sizes to halve every two iterations, and after at most $2\log_2(|b_{k-1} - b_{k-2}|/\delta)$ iterations, the step size will be smaller than δ, which invokes a bisection step. Brent proved that his method requires at most N^2 iterations, where "N" denotes the number of iterations for the bisection method.

If the function "f" is well-behaved, then Brent's method will usually proceed by either inverse quadratic or linear interpolation, in which case it will converge [[rate of convergence—superlinearly]].

Code

`https://gitlab.com/gnu-octave/octave/-/blob/default/scripts/optimization/fzero.m?ref_type=heads`

References

https://en.wikipedia.org/wiki/Brent%27s_method

The End