MATH 333 HOMEWORK

Please complete the problem below before our next class. If you chose to collaborate with a friend, please include the names of those you worked with. Lecture slides and code may be found in the course repository on Github.

(1) Consider computing a numerical solution to the IVP

$$\begin{cases} y'(x) = f(x,y), \ \ x \text{ in } [a,b] \\ y(a) = y_0 \end{cases} \tag{1}$$

(a) Following the procedure used in class to derive RK2 from the Trapezoid rule, show that we can obtain the so-called **modified Euler's method**

$$u_{n+1} = u_n + hf\left(x_n + \frac{h}{2}, u_n + \frac{hf_n}{2}\right). \tag{2}$$

using the **midpoint rule**.

(b) Using the exponential model problem from class, verify numerically that modified Euler is order 2.

(c) Compare the performance of modified Euler's method with Forward Euler, RK2, and RK4 to solve the following IVP. Which method performs the best? Does this agree with the theory?

$$\begin{cases} y'(x) = \cos(x)y(x), \ \ x \text{ in } [0,6] \\ y(0) = 1 \end{cases} \tag{3}$$