Homework4

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1 Question 1

If one accepts s, the other one should accept 1000000-s, therefore the strategy is as follows:

if the first son says s_1, then the second son should say $1000000 - s_1$ and vise versa.

In this case we will have 1000001 Nash equilibrium for $s \in \{0, 1, ..., 1000000\}$

2 Question 2

2.1 part(a)

	ALLD	GRIM
ALLD	mP,mP	T+(m-1)P, S+(m-1)P
GRIM	S+(m-1)P, T+(m-1)P	mR,mR

2.2 part(b)

If GRIM is stable against the invasion of ALLD then for any small values of x, we must have

$$x(S + (m-1)P) + (1-x)mR > xmP + (1-x)(T + (m-1)P)$$

So by taking the limit as $x \to 0$ we must have

$$mR > T + (m-1)P \Rightarrow m > \frac{T-P}{R-P}$$

2.3 part(c)

	GRIM	GRIM*
GRIM	mR,mR	(m-1)R+S, (m-1)R+T
GRIM*	(m-1)R+T,(m-1)R+S	(m-1)R+P, (m-1)R+P

GRIM* dominates GRIM iff for any small values of x:

$$x((m-1)R+T)+(1-x)((m-1)R+P) > xmR+(1-x)((m-1)R+S))$$

and therefore in the limit

$$(m-1)R+P > (m-1)R+S \Rightarrow P > S$$

which is true and therefore GRIM* dominates GRIM

2.4 part(d)

	GRIM*	GRIM**
GRIM*	(m-1)R+P, (m-1)R+P	(m-2)R+S+P, (m-2)R+T+P
GRIM**	(m-2)R+T+P, (m-2)R+S+P	(m-2)R+2P,(m-2)R+2P

This strategy dominates GRIM* because again $P > S$.

2.5 part(e)

	$GRIM^{m-1}$	$GRIM^{m}$
$GRIM^{m-1}$	R+(m-1)P, R+(m-1)P	S+(m-1)P, T+(m-1)P
$GRIM^{m}$	T+(m-1)P, S+(m-1)P	mP,mP

This strategy dominates $GRIM^{m-1}$ because again $P > S$.

3 Question 3

3.1 part(a)

we have a geometric distribution for $m \in \{1, 2, 3, ...\}$ and probability $p = 1 - \delta$.
Therefore the mean is

$$\frac{1}{p} = \frac{1}{1-\delta} = 1 + \frac{\delta}{1-\delta}$$

3.2 part(b)

Expected payoff matrix:

	ALLD	GRIM
ALLD	$(1+\frac{\delta}{1-\delta})P$, $(1+\frac{\delta}{1-\delta})P$	$T+(\frac{\delta}{1-\delta})P$, $S+(\frac{\delta}{1-\delta})P$
GRIM	$S+(\frac{\delta}{1-\delta})P$, $T+(\frac{\delta}{1-\delta})P$	$(1+\frac{\delta}{1-\delta})R$, $(1+\frac{\delta}{1-\delta})$R

3.3 part(c)

GRIM is stable against ALLD iff

$$x(S+(\frac{\delta}{1-\delta})P)+(1-x)(1+\frac{\delta}{1-\delta})R > x(1+\frac{\delta}{1-\delta})P+(1-x)(T+(\frac{\delta}{1-\delta})P)$$

for any small value x. Therefore we must have

$$\frac{1}{1-\delta} > \frac{T-P}{R-P} \Rightarrow \delta > \frac{T-R}{T-P}$$