Question #3: solution

For the first bond with $T=5$, $c=16$ we have

$$\frac{P_1(y)\,e^{yT}}{100} = e^{-5y} + \frac{16}{1000(1-e^{-y/2})}(1-e^{-5y})$$

and for the second bond with $T=4$, $c=20$ we have

$$\frac{P_2(y)\,e^{yT}}{100} = e^{-4y} + \frac{20}{1000(1-e^{-y/2})}(1-e^{-4y})$$

we can show that for $y \geq 0$, we have always

$P_2 \geq P_1 \rightarrow P_2$ is likely to cost more

and the reason is as follows:

First, we can see for $y \geq 0$: $e^{-4y} \geq e^{-5y}$. Moreover we can show $20(1-e^{-4y}) \geq 16(1-e^{-5y})$ for all $y \geq 0$. because let $f(y) = 20(1-e^{-4y})$ and $g(y) = 16(1-e^{-5y})$ so $f'(y) = \frac{\partial f}{\partial y} = 80e^{-4y}$ and $g'(y) = 80e^{-5y}$ and since $e^{-4y} \geq e^{-5y} \Rightarrow f'(y) \geq g'(y)$. This means $(f(y)-g(y))' \geq 0$ or $f(y)-g(y)$ is an increasing function of y. Therefore