for $y \geqslant 0 \Rightarrow \quad f(y) - g(y) \geqslant f(0) - g(0) = 0 \Rightarrow f(y) \geqslant g(y)$

So $\quad 20(1 - e^{-4y}) \geqslant 16(1 - e^{-5y})$, for all $y \geqslant 0$.

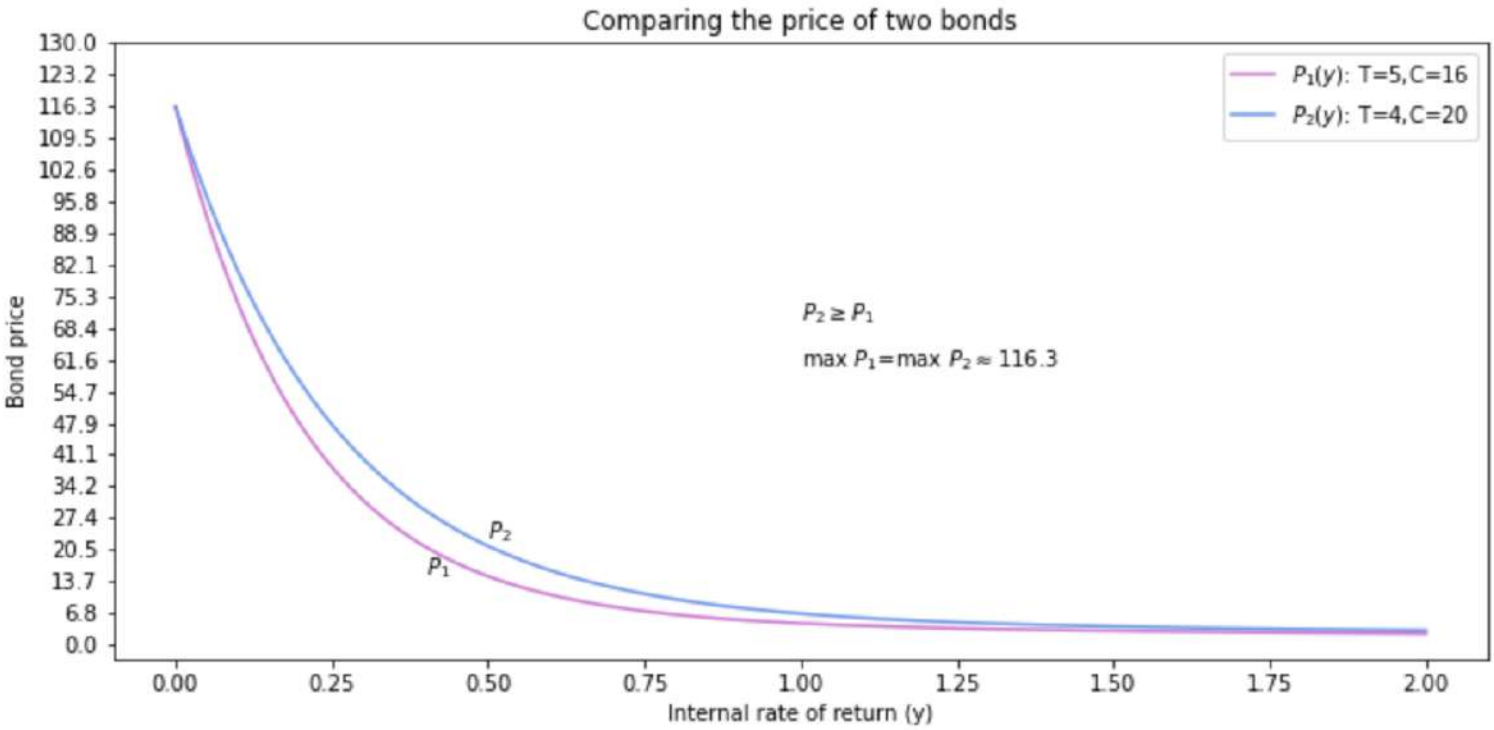
This proves that $P_2(y) \geqslant P_1(y)$, for $y > 0$.

If you don't want to use a mathematical proof, you can show this by any Programming language. For example in the next page you can see the plot for $P_1(y)$ and $P_2(y)$ and as you can see: $P_2(y) \geqslant P_1(y)$ for $y \in (0, \infty)$ and also it's clear that $\lim_{y \to 0} P_2(y) = \lim_{y \to 0} P_1(y)$.

This is where the maximum of two functions occurs and so you can conclude that both P_1 and P_2 have the same maximum which also answers the second part of the question, but we answer the second part separately.

(ii) For part (2), we must have $\dfrac{c}{5P} + \dfrac{1}{T} \ln\left(\dfrac{100}{P}\right) \geqslant 0$

and so the value $P_{max} = \dfrac{100}{X}$ is the maximum where $\ln X = -\dfrac{cTX}{500}$. For both bonds we have $cT = 80$ and so they must have the same maximum (it's also clear from the plot)


Comparing the price of two bonds
$P_1(y)$: T=5,C=16
$P_2(y)$: T=4,C=20
$P_2 \geq P_1$
max P_1=max $P_2 \approx$ 116.3
P_2
P_1
Bond price
Internal rate of return (y)
130.0
123.2
116.3
109.5
102.6
95.8
88.9
82.1
75.3
68.4
61.6
54.7
47.9
41.1
34.2
27.4
20.5
13.7
6.8
0.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

and the maximum is obtained by solving $\ln x = -\frac{80x}{500}$ for x and then $P_{max} = \frac{100}{x}$. To slove $\ln x = -\frac{80x}{500}$

you can use Excel or any programming language and the solution is $\hat{x} \approx 0.87$ and hence $P_{max} = \frac{100}{0.87} = 114.94$ which is close to the exact P_{max} from the plot (≈ 116).

The results are reasonable, because the relation $\ln x = -\frac{CTx}{500}$ only depends on CT and CT is the same for both bonds although $P_2 \geqslant P_1$ for all $y \geqslant 0$, they have the same maximum and this can be seen from the plot.

$\boxtimes$

Question #4: solution

$c = 12$, $T = 4$, $P = 106$

1) current yield $= y_c = \dfrac{c}{5P} = \dfrac{12}{5(106)} = 0.02264 = 2.264\%$

2) capital gains $= y_{cg} = \dfrac{1}{T} \ln\left(\dfrac{100}{P}\right) = \dfrac{1}{4} \ln\left(\dfrac{100}{106}\right) = -0.0145$

3) $y_T = y_c + y_{cg} \approx 0.008 = 0.8\%$

4) To find IRR, we solve the equation

$$\frac{106}{100} = e^{-4y} + \frac{12}{1000(1-e^{y/2})} \left[1 - e^{-4y}\right]$$

for y and again this can be done by Excel, Mathematica

MATLAB, Python or any other PL and the solution is

$$\hat{y} \approx 0.009 = 0.9\%$$

$\boxtimes$

Question 5: solution :

half-year $\Rightarrow$ $t = \frac{1}{2}$

$r = 0.06$

$E_0 = 10000 \Rightarrow B_0 = 160(100) - 10000 = 6000$

(a) $V(\frac{1}{2}) = 164(100) = 16400 \Rightarrow E(\frac{1}{2}) = V(\frac{1}{2}) - B_0 e^{\frac{1}{2}r}$

$$= V(\tfrac{1}{2}) - 6000\, e^{0.03}$$

$$= 16400 - 6000\, e^{0.03}$$

$$= 10217.27$$

and return of equity after 6 months $(t = \frac{1}{2})$ is

$$r_E(t) = r_E(\tfrac{1}{2}) = \frac{1}{1/2} \ln\left(\frac{E(\frac{1}{2})}{E_0}\right) = 2 \ln\left(\frac{10217.27}{10000}\right)$$

$$= 0.043$$

(b) $V(\frac{1}{2}) = 17000 \Rightarrow E(\frac{1}{2}) = 17000 - 6000\, e^{0.03} = 10817.27$

and $r_E(\frac{1}{2}) = 2 \ln\left(\frac{10817.27}{10000}\right) = 0.157$

(c) $V(\frac{1}{2}) = 18000 \Rightarrow E(\frac{1}{2}) = 18000 - 6000\, e^{0.03} = 11817.27$

and $r_E(\frac{1}{2}) = 2 \ln\left(\frac{11817.27}{10000}\right) = 0.333$

Part (ii)

$E_0 = 12000 \Rightarrow B_0 = 16000 - 12000 = 4000$ and all other parameters are the same. The same calculations give us

(a) $r_E(t) = r_E(\frac{1}{2}) \simeq 0.0458$ ✓

$\boxed{\text{better return on equity}}$

(b) $r_E(\frac{1}{2}) \approx 0.141$ X

(c) $r_E(t) \simeq 0.29$ X

Comparing with the previous results, we see in part (a) we have a better return on equity. (For parts (b) and (c) $r_E(t)$ have decreased)