Latihan Soal 1 :

1) $\vec{m} = [5, -2, 1]$
$\vec{n} = [6, -2, 4]$

a) $\vec{p} = ?$

$\vec{p} = -2m + 0{,}5n$

$\vec{p} = -2[5, -2, 1] + 0{,}5[6, -2, 4]$

$\vec{p} = [-10, 4, -2] + [3, -1, 2]$

$\vec{p} = [-7, 3, 0]$

b) $\theta_{m,n} = ?$

$$\cos\theta = \frac{\vec{m} \cdot \vec{n}}{\|\vec{m}\| \cdot \|\vec{n}\|}$$

$$= \frac{[5, -2, 1] \begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix}}{\sqrt{5^2+(-2)^2+1^2} \cdot \sqrt{6^2+(-2)^2+4^2}}$$

$$= \frac{38}{\sqrt{30} \cdot \sqrt{56}}$$

$$\cos\theta \cong 0{,}927$$

$$\theta \cong \cos^{-1}(0{,}927)$$

$$\theta \cong \pm 22{,}01^\circ$$

c) Proyeksi vektor $\vec{m}$ terhadap vektor $\vec{n}$?

$$\text{Proj}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$\text{Proj}_n m = \frac{\vec{n} \cdot \vec{m}}{\|\vec{n}\|^2} \cdot \vec{n}$$

$$= \frac{38}{(\sqrt{56})^2} \cdot [6, -2, 4]$$

$\text{Proj}_n m = [4{,}07, -1{,}35, 2{,}71]$

2) Determinan matriks M

$$M = \begin{bmatrix} -5 & 3 & 1 \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{bmatrix} \Rightarrow \det M = ?$$

• mencari elemen matriks minor m_{11} sampai m_{13}

$$m_{11} = \det \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} = (1 \cdot 1) - (1 \cdot 3) = 1 - 3 = -2$$

$$m_{12} = \det \begin{bmatrix} 2 & 1 \\ -4 & 1 \end{bmatrix} = (2 \cdot 1) - (1 \cdot -4) = 2 + 4 = 6$$

$$m_{13} = \det \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} = (2 \cdot 3) - (1 \cdot -4) = 6 + 4 = 10$$

$$|M| = M_{11} \cdot C_{11} + M_{12} \cdot C_{12} + M_{13} \cdot C_{13}$$
$$= M_{11} \cdot (m_{11} \cdot -1^{(1+1)}) + M_{12} (m_{12} \cdot -1^{(1+2)}) + M_{13} (m_{13} \cdot -1^{(1+3)})$$
$$= -5 \cdot (-2 \cdot 1) + 3 (6 \cdot -1) + 1 (10 \cdot 1)$$
$$= 10 + (-18) + 10$$
$$|M| = 2$$

Latihan Soal 2 :

1) $\vec{u} = (1;0;0)$

$\vec{v} = (0;12;0)$

a) $\vec{u} \times \vec{v} = ?$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 12 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 12 & 0 \end{vmatrix} i + \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} -j + \begin{vmatrix} 1 & 0 \\ 0 & 12 \end{vmatrix} k$$

$$= 0i + 0j + 12k$$

$$= 12k$$

$$= (0, 0, 12) .$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{0^2 + 0^2 + 12^2}$$

$$= 12 .$$

Latihan Soal 3 :

Cari invers matriks B!

$$B = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{adj\,(B)}{|B|}$$

a) $adj\,(B) = \ldots ?$

$$adj\,(B) = [\text{cofactor}\,(B)]^T$$

$$adj\,(B) = \begin{bmatrix} m_{11} & -m_{12} & m_{13} \\ -m_{21} & m_{22} & -m_{23} \\ m_{31} & -m_{32} & m_{33} \end{bmatrix}^T$$

$$adj\,(B) = \begin{bmatrix} (-5) & -(-3) & (7) \\ -(-2) & 0 & -(1) \\ (4) & -(-3) & (-5) \end{bmatrix}^T$$

$$adj\,(B) = \begin{bmatrix} -5 & -3 & 7 \\ 2 & 0 & -1 \\ 4 & 3 & -5 \end{bmatrix}^T = \begin{bmatrix} -5 & 2 & 4 \\ -3 & 0 & 3 \\ 7 & -1 & -5 \end{bmatrix}$$

b) $|B| = ?$

$$|B| = B_{11} \cdot C_{11} + B_{12} \cdot C_{12} \cdot + B_{13} \cdot C_{13}$$

$$= 1\,(-5) + 2\,(-3) + 2\,(7)$$

$$= 3$$

o) $B^{-1} = \frac{adj(B)}{|B|}$

$$= \frac{\begin{bmatrix} -5 & 2 & 4 \\ -3 & 0 & 3 \\ 7 & -1 & -5 \end{bmatrix}}{3}$$

$$B^{-1} = \begin{bmatrix} -5/3 & 2/3 & 4/3 \\ -1 & 0 & 1 \\ 7/3 & -1/3 & -5/3 \end{bmatrix}$$