Nama : Muchammad Daniyal Kautsar
NIU 479067
Kalkulus Variable Jamak B

UAS

~~L = p = 7 (bulan Juli)~~

2. $p = 7$ (bulan Juli)

$F(x,y,z) = (3xy + z^3, x^2, 3xz^3)$

a. Buktikan bahwa integral garis $p\int_A^B F \cdot dr$, $A = (-2,1,1)$, $B = (1,3,4)$ bebas lintasan.

→ Kita cek $F(x,y,z)$ dengan $x_y = y_x$, $x_z = z_x$, $y_z = z_y$ untuk mengetahui apakah bebas lintasan

$x_z = z_x$

$$\frac{d(3xy+z^3)}{dz} = \frac{(3xz^3)}{dx}$$

$$3z^2 \neq 3z^3$$

$x_y = y_x$

$$\frac{d(3xy+z^3)}{dy} = \frac{d(x^2)}{dx}$$

$$3x \neq 2x$$

$y_z = z_y$

$$\frac{d(x^2)}{dz} = \frac{d(3xz^3)}{dy}$$

$$0 = 0$$

Karena $x_y \neq y_x$ dan $x_z \neq z_x$, maka $F(x,y,z)$ tidak bebas lintasan.

b. Karena $F(x,y,z)$ tidak bebas lintasan, maka $F(x,y,z)$ tidak konservatif

c.

1. $P = 7$

$z = 2 + x^2 + (y-2)^2$

$z = 1 \quad , x = 2 , x = 0 , y = 7 , y = -1$

$f(x,y) = 2 + x^2 + (y-2)^2 - 1$

$= 1 + x^2 + y^2 + 4 - 4y$

$$\int_0^2 \int_{-1}^7 (5 + x^2 + y^2 + -4y)\, dy\, dx = \int_0^2 \left(5y + yx^2 + \frac{y^3}{3} - 2y^2 \Big|_{-1}^{7} \right) dx$$

$$= \int_0^2 \frac{176}{3} + 8x^2\, dx \quad = \frac{176}{3} x + \frac{8}{3} x^3 \Big|_0^2$$

$$= \frac{352}{3} + \frac{69}{3} = \frac{416}{3}$$

6

3. Dengan teorema green hitunglah $\iint_D (x^2 - y^2)\, dx\, dy$

a. Jika D adalah persegi panjang dg batas $x = 0,\ y = 0\ \ x = 2,\ y = 1$

* $\iint_D (x^2 - y^2)\, dx\, dy = \int_0^1\int_0^2 (x^2 - y^2)\, dx\, dy = \int_0^1 \left(\frac{x^3}{3} - xy^2 \Big|_0^2\right) dy$

$= \int_0^1 \left(\frac{8}{3} - 2y^2\right) dy = \left(\frac{8}{3}y - \frac{2}{3}y^3 \Big|_0^1\right)$

$= \frac{8}{3} - \frac{2}{3} = \frac{5}{3}$

b. Jika D daerah yang dibatasi oleh lingkaran $(x+1)^2 + y^2 = 4$

* $r^2 = 4$ $\quad$ $x+1 = 2\cos\theta$

$r = 2$ $\quad$ $x = 2\cos\theta - 1$ $\quad$ $y = 2\sin\theta$

$0 \le r \le 2$ $\quad$ $0 \le \theta \le 2\pi$

$\iint_D (x^2 - y^2)\, dx\, dy = \iint_D (4\cos^2\theta + 1 - 4\cos\theta - 4\sin^2\theta)\, r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^2 (4r\cos^2\theta + r - 4r\cos\theta - 4r\sin^2\theta)\, dr\, d\theta$

$= \int_0^{2\pi} \left(4r^2\cos^2\theta - 2r^2 + \frac{r^2}{2} - 2r^2\cos\theta \Big|_0^2\right) d\theta$

$= \int_0^{2\pi} (16\cos^2\theta - 6 - 8\cos\theta)\, d\theta = \left(2\theta + 8\sin\theta\cos\theta - 8\sin\theta \Big|_0^{2\pi}\right)$

$= 4\pi$

4.a. $\iiint_E 2xy\, dV$ dg E berada dibawah bidang $z = 1 + x + y$ dan diatas region di bidang xy yang dibatasi oleh kurva $y = \sqrt{x}$, $y = 0$, dan $x = 1$

* Batas-batas:

$0 \le x \le 1$

$0 \le y \le \sqrt{x}$ $\quad | \quad 0 \le z \le 1 + x + y$

* $\int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} (2xy)\, dz\, dy\, dx = \int_0^1\int_0^{\sqrt{x}} \left(2xyz \Big|_0^{1+x+y}\right) dy\, dx = \int_0^1\int_0^{\sqrt{x}} (2xy(1+x+y))\, dy\, dx$

$= \int_0^1\int_0^{\sqrt{x}} (2xy + 2x^2y + 2xy^2)\, dy\, dx = \int_0^1 \left(xy^2 + x^2y^2 + \frac{2}{3}xy^3 \Big|_0^{\sqrt{x}}\right) dx$

$= \int_0^1 \left(x^2 + x^3 + \frac{2x^{5/2}}{3}\right) dx = \left(\frac{x^3}{3} + \frac{x^4}{4} + \frac{2}{3} \cdot \frac{2}{7} \cdot x^{7/2} \Big|_0^1\right) = \frac{1}{3} + \frac{1}{4} + \frac{4}{21} = \frac{65}{84}$