Soal 1:

Suatu ketika Prof. Gilbert Strang berkunjung ke UGM; beliau berkunjung ke warung siomay bersama 2 orang koleganya. Karena begitu mencintai Aljabar Linear, beliau berniat mengaplikasikan konsep Aljabar Linear untuk mengetahui harga masing - masing makanan (siomay, tahu, kentang). Oleh karena itu masing - masing kolega diminta untuk mengetahui harga total makanan yang mereka pesan masing - masing. Kemudian beliau bertanya langsung harga makanan yang dipesan kepada koleganya.

- Kolega pertama yang memesan 2 tahu dan 3 siomay menjawab harganya Rp 40.000,00.
- Kolega kedua yang memesan 1 tahu, 2 kentang, dan 2 siomay menjawab harganya Rp 40.000,00.
- Sementara itu, harga makanan yang dipesan Prof. Strang (1 tahu, 1 kentang, dan 2 siomay) adalah Rp 32.500,00.

Carilah harga masing-masing makanan dengan menggunakan metode (tuliskan tahapannya satu per satu):

a. Eliminasi Gauss
b. Eliminasi Gauss-Jordan
c. Faktorisasi LU

Pembahasan:

Dengan mengalokasikan variabel: x_1=siomay, x_2=kentang, x_3=tahu, dapat dibuat sekumpulan persamaan linear:

$$\begin{aligned} 3x_1 + 2x_3 &= 40 \cdot 10^3 \\ 2x_1 + 2x_2 + x_3 &= 40 \cdot 10^3 \\ 2x_1 + x_2 + x_3 &= 32{,}5 \cdot 10^3 \end{aligned}$$

Kemudian persamaan linear di atas dapat dijadikan bentuk umum $\boldsymbol{Ax} = \boldsymbol{b}$:

$$\begin{bmatrix} 3 & 0 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 40 \cdot 10^3 \\ 40 \cdot 10^3 \\ 32{,}5 \cdot 10^3 \end{bmatrix}$$

a. Eliminasi Gauss

Buat augmented matrix $[\boldsymbol{A}|\boldsymbol{b}]$

$$\left[\begin{array}{ccc|c} 3 & 0 & 2 & 40 \cdot 10^3 \\ 2 & 2 & 1 & 40 \cdot 10^3 \\ 2 & 1 & 1 & 32{,}5 \cdot 10^3 \end{array}\right]$$

Eliminasi $[\boldsymbol{A}|\boldsymbol{b}]$ hingga didapat matriks upper triangular $[\boldsymbol{U}|\boldsymbol{Eb}]$

$\underbrace{\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{M_{11}}} \left[\begin{array}{ccc\|c} 3 & 0 & 2 & 40 \cdot 10^3 \\ 2 & 2 & 1 & 40 \cdot 10^3 \\ 2 & 1 & 1 & 32.5 \cdot 10^3 \end{array}\right] = \left[\begin{array}{ccc\|c} 1 & 0 & \frac{2}{3} & \frac{40}{3} \cdot 10^3 \\ 2 & 2 & 1 & 40 \cdot 10^3 \\ 2 & 1 & 1 & 32.5 \cdot 10^3 \end{array}\right]$	$R_1 = \frac{1}{3} R_1$ Langkah menjadikan pivot=1 ini opsional tapi bisa memudahkan proses eliminasi.

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$$\begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\left[\begin{array}{ccc|c}\mathbf{1} & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ \mathbf{2} & 2 & 1 & 40\cdot 10^3\\ 2 & 1 & 1 & 32.5\cdot 10^3\end{array}\right] = \left[\begin{array}{ccc|c}1 & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 2 & -\frac{1}{3} & \frac{40}{3}\cdot 10^3\\ 2 & 1 & 1 & 32.5\cdot 10^3\end{array}\right]$$

$\boldsymbol{E_{21}}$

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Pivot = 1

$l_{21} = \frac{2}{1} = 2$

$R_2 = R_2 - 2\,R_1$

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$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}\left[\begin{array}{ccc|c}\mathbf{1} & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 2 & -\frac{1}{3} & \frac{40}{3}\cdot 10^3\\ \mathbf{2} & 1 & 1 & 32.5\cdot 10^3\end{array}\right] = \left[\begin{array}{ccc|c}1 & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 2 & -\frac{1}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 1 & -\frac{1}{3} & \frac{35}{6}\cdot 10^3\end{array}\right]$$

$\boldsymbol{E_{31}}$

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Pivot = 1

$l_{31} = \frac{2}{1} = 2$

$R_3 = R_3 - 2\,R_1$

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$$\begin{bmatrix}1 & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & 1\end{bmatrix}\left[\begin{array}{ccc|c}1 & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 2 & -\frac{1}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 1 & -\frac{1}{3} & \frac{35}{6}\cdot 10^3\end{array}\right] = \left[\begin{array}{ccc|c}1 & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 1 & -\frac{1}{6} & \frac{20}{3}\cdot 10^3\\ 0 & 1 & -\frac{1}{3} & \frac{35}{6}\cdot 10^3\end{array}\right]$$

$\boldsymbol{M_{22}}$

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$R_2 = \frac{1}{2}R_2$

Langkah menjadikan pivot=1 ini opsional tapi bisa memudahkan proses eliminasi

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$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\left[\begin{array}{ccc|c}1 & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ 0 & \mathbf{1} & -\frac{1}{6} & \frac{20}{3}\cdot 10^3\\ 0 & \mathbf{1} & -\frac{1}{3} & \frac{35}{6}\cdot 10^3\end{array}\right] = \left[\begin{array}{ccc|c}1 & 0 & \frac{2}{3} & \frac{40}{3}\cdot 10^3\\ 0 & 1 & -\frac{1}{6} & \frac{20}{3}\cdot 10^3\\ 0 & 0 & -\frac{1}{6} & -\frac{5}{6}\cdot 10^3\end{array}\right]$$

$\boldsymbol{E_{32}}$

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Pivot = 1

$l_{32} = \frac{1}{1} = 1$

$R_3 = R_3 - R_2$

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$$\begin{bmatrix}1 & 0 & \frac{2}{3}\\ 0 & 1 & -\frac{1}{6}\\ 0 & 0 & -\frac{1}{6}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}\frac{40}{3}\cdot 10^3\\ \frac{20}{3}\cdot 10^3\\ -\frac{5}{6}\cdot 10^3\end{bmatrix}$$

$$x_1 + \frac{2}{3}x_3 = \frac{40}{3}\cdot 10^3$$

$$x_2 - \frac{1}{6}x_3 = \frac{20}{3}\cdot 10^3$$

$$-\frac{1}{6}x_3 = -\frac{5}{6}\cdot 10^3$$

Backward substitution

$$x_3 = \frac{-\frac{5}{6}\cdot 10^3}{-1/6} = 5\cdot 10^3$$

$$x_2 = \frac{20}{3}\cdot 10^3 + \frac{1}{6}x_3 = \frac{20}{3}\cdot 10^3 + \frac{1}{6}\cdot 5\cdot 10^3 = \frac{40+5}{6}\cdot 10^3 = 7{,}5\cdot 10^3$$

$$x_1 = \frac{40}{3}\cdot 10^3 - \frac{2}{3}x_3 = \frac{40}{3}\cdot 10^3 - \frac{2}{3}\cdot 5\cdot 10^3 = \frac{40-10}{3}\cdot 10^3 = 10\cdot 10^3$$

$$\therefore x_1 = 10 \cdot 10^3, \qquad x_2 = 7{,}5 \cdot 10^3, \qquad x_3 = 5 \cdot 10^3$$

Jadi, harga: 1 siomay Rp 10.000,00, 1 kentang Rp 7.500,00, dan 1 tahu Rp 5.000,00.

b. Eliminasi Gauss-Jordan

Eliminasi lagi $[\boldsymbol{U}|\boldsymbol{Eb}]$ hingga menjadi identitas $[\boldsymbol{I}|\boldsymbol{x}]$

<table>
<tr>
<td>$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -6 \end{bmatrix} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{40}{3} \cdot 10^3 \\ 0 & 1 & -\frac{1}{6} & \frac{20}{3} \cdot 10^3 \\ 0 & 0 & -\frac{1}{6} & -\frac{5}{6} \cdot 10^3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{40}{3} \cdot 10^3 \\ 0 & 1 & -\frac{1}{6} & \frac{20}{3} \cdot 10^3 \\ 0 & 0 & 1 & 5 \cdot 10^3 \end{array}\right]$$
$\boldsymbol{M_{33}}$</td>
<td>$R_3 = -6R_3$
Langkah menjadikan pivot=1 ini opsional tapi bisa memudahkan proses eliminasi</td>
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<td>$$\begin{bmatrix} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} 1 & 0 & \boldsymbol{\frac{2}{3}} & \frac{40}{3} \cdot 10^3 \\ 0 & 1 & -\frac{1}{6} & \frac{20}{3} \cdot 10^3 \\ 0 & 0 & \boldsymbol{1} & 5 \cdot 10^3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \cdot 10^3 \\ 0 & 1 & -\frac{1}{6} & \frac{20}{3} \cdot 10^3 \\ 0 & 0 & 1 & 5 \cdot 10^3 \end{array}\right]$$
$\boldsymbol{E_{13}}$</td>
<td>Pivot = 1
$l_{13} = \frac{2/3}{1} = \frac{2}{3}$
$R_1 = R_1 - \frac{2}{3} R_3$</td>
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<td>$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{6} \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \cdot 10^3 \\ 0 & 1 & -\boldsymbol{\frac{1}{6}} & \frac{20}{3} \cdot 10^3 \\ 0 & 0 & \boldsymbol{1} & 5 \cdot 10^3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \cdot 10^3 \\ 0 & 1 & 0 & 7{,}5 \cdot 10^3 \\ 0 & 0 & 1 & 5 \cdot 10^3 \end{array}\right]$$
$\boldsymbol{E_{23}}$ $[\boldsymbol{I}|\boldsymbol{x}]$</td>
<td>Pivot = 1
$l_{23} = \frac{-1/6}{1} = -\frac{1}{6}$
$R_2 = R_2 - \left(-\frac{1}{6}\right) R_3$
$= R_2 + \frac{1}{6} R_3$</td>
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</table>

Solusi $\boldsymbol{x}$ ada di ruas kanan matriks $[\boldsymbol{I}|\boldsymbol{x}]$

$$\therefore x_1 = 10 \cdot 10^3, \qquad x_2 = 7{,}5 \cdot 10^3, \qquad x_3 = 5 \cdot 10^3$$

Jadi, harga: 1 siomay Rp 10.000,00, 1 kentang Rp 7.500,00, dan 1 tahu Rp 5.000,00.

c. Faktorisasi LU

Dari langkah eliminasi Gauss, kita sudah mendapatkan matriks eliminasi:

$$\boldsymbol{E} = \boldsymbol{E_{32} M_{22} E_{31} E_{21} M_{11}}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Kemudian matriks lower triangular bisa didapat dari inverse matriks eliminasi:

$$\boldsymbol{L} = \boldsymbol{E^{-1}}$$
$$= (\boldsymbol{M_{11}})^{-1} (\boldsymbol{E_{21}})^{-1} (\boldsymbol{E_{31}})^{-1} (\boldsymbol{M_{22}})^{-1} (\boldsymbol{E_{32}})^{-1}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

Selesaikan $\boldsymbol{Lc} = \boldsymbol{b}$ untuk mendapatkan $\boldsymbol{c}$ dengan forward substitution

$$\boldsymbol{Lc} = \boldsymbol{b}$$
$$\begin{bmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 40 \cdot 10^3 \\ 40 \cdot 10^3 \\ 32{,}5 \cdot 10^3 \end{bmatrix}$$

$$3c_1 = 40 \cdot 10^3$$
$$2c_1 + 2c_2 = 40 \cdot 10^3$$
$$2c_1 + \mathrm{c}_2 + c_3 = 32{,}5 \cdot 10^3$$

$$c_1 = \frac{40}{3} \cdot 10^3$$
$$c_2 = \frac{40 \cdot 10^3 - 2c_1}{2} = \frac{40 \cdot 10^3 - 2 \cdot \frac{40}{3} \cdot 10^3}{2} = \frac{20}{3} \cdot 10^3$$
$$c_3 = 32{,}5 \cdot 10^3 - 2c_1 - c_2 = 32{,}5 \cdot 10^3 - 2 \cdot \frac{40}{3} \cdot 10^3 - \frac{20}{3} \cdot 10^3 = -\frac{5}{6} \cdot 10^3$$

Selesaikan $\boldsymbol{Ux} = \boldsymbol{c}$ untuk mendapatkan $\boldsymbol{x}$ dengan backward substitution

$$\boldsymbol{Ux} = \boldsymbol{c}$$
$$\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & -\frac{1}{6} \\ 0 & 0 & -\frac{1}{6} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{40}{3} \cdot 10^3 \\ \frac{20}{3} \cdot 10^3 \\ -\frac{5}{6} \cdot 10^3 \end{bmatrix}$$

$$x_1 + \frac{2}{3} x_3 = \frac{40}{3} \cdot 10^3$$
$$x_2 - \frac{1}{6} x_3 = \frac{20}{3} \cdot 10^3$$
$$-\frac{1}{6} x_3 = -\frac{5}{6} \cdot 10^3$$

$$x_3 = \frac{-\frac{5}{6} \cdot 10^3}{-1/6} = 5 \cdot 10^3$$

$$x_2 = \frac{20}{3} \cdot 10^3 + \frac{1}{6} x_3 = \frac{20}{3} \cdot 10^3 + \frac{1}{6} \cdot 5 \cdot 10^3 = \frac{40+5}{6} \cdot 10^3 = 7{,}5 \cdot 10^3$$

$$x_1 = \frac{40}{3} \cdot 10^3 - \frac{2}{3} x_3 = \frac{40}{3} \cdot 10^3 - \frac{2}{3} \cdot 5 \cdot 10^3 = \frac{40-10}{3} \cdot 10^3 = 10 \cdot 10^3$$

$$\therefore x_1 = 10 \cdot 10^3, \qquad x_2 = 7{,}5 \cdot 10^3, \qquad x_3 = 5 \cdot 10^3$$

Jadi, harga: 1 siomay Rp 10.000,00, 1 kentang Rp 7.500,00, dan 1 tahu Rp 5.000,00.

Soal 2 (Susulan):

2s - t + u + 5v = -4
3t + u - 2v - 5w = 7
4s - 5u + 10v = 5
2u + 2v + 5w = -1
5s - 3t - 4v - 8w =16

Cari solusi dari persamaan di atas menggunakan metode

a. Eliminasi Gauss-Jordan

b. Faktorisasi LU

c. Cari invers matriks **A** dari persamaan $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ persamaan di atas.

Pembahasan

Persamaan-persamaan di atas diubah terlebih dahulu menjadi ***Ax* = *b***

$$\begin{bmatrix} 2 & -1 & 1 & 5 & 0 \\ 0 & 3 & 1 & -2 & -5 \\ 4 & 0 & -5 & 10 & 0 \\ 0 & 0 & 2 & 2 & 5 \\ 5 & -3 & 0 & -4 & -8 \end{bmatrix} \begin{bmatrix} s \\ t \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \\ 5 \\ -1 \\ 16 \end{bmatrix}$$

Agar lebih ringkas, maka buat augmented matriks [***A*** | ***b***]

$$\left[\begin{array}{ccccc|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 4 & 0 & -5 & 10 & 0 & 5 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 5 & -3 & 0 & -4 & -8 & 16 \end{array}\right]$$

a. Eleminasi Gauss-Jordan

Eleminasi [$\boldsymbol{A}$ | $\boldsymbol{b}$] hingga membentuk [$\boldsymbol{U}$ | $\boldsymbol{Eb}$]

$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{E_{31}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 4 & 0 & -5 & 10 & 0 & 5 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 5 & -3 & 0 & -4 & -8 & 16 \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 2 & -7 & 0 & 0 & 13 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 5 & -3 & 0 & -4 & -8 & 16 \end{array}\right]$	$l_{31} = 2$ $R_3 = R_3 - 2R_1$
$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\frac{5}{2} & 0 & 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{E_{51}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 2 & -7 & 0 & 0 & 13 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 5 & -3 & 0 & -4 & -8 & 16 \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 2 & -7 & 0 & 0 & 13 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & -\frac{1}{2} & -\frac{5}{2} & -\frac{33}{2} & -8 & 26 \end{array}\right]$	$l_{51} = \frac{5}{2}$ $R_5 = R_5 - \frac{5}{2}R_1$
$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{E_{32}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 2 & -7 & 0 & 0 & 13 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & -\frac{1}{2} & -\frac{5}{2} & -\frac{33}{2} & -8 & 26 \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -\frac{23}{3} & \frac{4}{3} & \frac{10}{3} & \frac{25}{3} \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & -\frac{1}{2} & -\frac{5}{2} & -\frac{33}{2} & -8 & 26 \end{array}\right]$	$l_{32} = \frac{2}{3}$ $R_3 = R_3 - \frac{2}{3}R_2$
$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{E_{52}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -\frac{23}{3} & \frac{4}{3} & \frac{10}{3} & \frac{25}{3} \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & -\frac{1}{2} & -\frac{5}{2} & -\frac{33}{2} & -8 & 26 \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -\frac{23}{3} & \frac{4}{3} & \frac{10}{3} & \frac{25}{3} \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & 0 & -\frac{7}{3} & -\frac{101}{6} & -\frac{53}{6} & \frac{163}{6} \end{array}\right]$	$l_{52} = -\frac{1}{6}$ $R_5 = R_5 + \frac{1}{6}R_2$
$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{M_{33}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -\frac{23}{3} & \frac{4}{3} & \frac{10}{3} & \frac{25}{3} \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & 0 & -\frac{7}{3} & -\frac{101}{6} & -\frac{53}{6} & \frac{163}{6} \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & 0 & -\frac{7}{3} & -\frac{101}{6} & -\frac{53}{6} & \frac{163}{6} \end{array}\right]$	$R_3 = 3R_3$ (Opsional) Dapat langsung dilanjutkan tanpa perlu membagi baris ketiga dengan 3
$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{23} & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{E_{43}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 2 & 2 & 5 & -1 \\ 0 & 0 & -\frac{7}{3} & -\frac{101}{6} & -\frac{53}{6} & \frac{163}{6} \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 0 & \frac{54}{23} & \frac{135}{23} & \frac{27}{23} \\ 0 & 0 & -\frac{7}{3} & -\frac{101}{6} & -\frac{53}{6} & \frac{163}{6} \end{array}\right]$	$l_{43} = -\frac{2}{23}$ $R_4 = R_4 + \frac{2}{23}R_3$
$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{7}{69} & 0 & 1 \end{bmatrix}}_{\boldsymbol{E_{53}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 0 & \frac{54}{23} & \frac{135}{23} & \frac{27}{23} \\ 0 & 0 & -\frac{7}{3} & -\frac{101}{6} & -\frac{53}{6} & \frac{163}{6} \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 0 & \frac{54}{23} & \frac{135}{23} & \frac{27}{23} \\ 0 & 0 & 0 & -\frac{793}{46} & -\frac{453}{46} & \frac{1133}{46} \end{array}\right]$	$l_{53} = \frac{7}{69}$ $R_5 = R_5 - \frac{7}{69}R_3$
$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{793}{108} & 1 \end{bmatrix}}_{\boldsymbol{E_{54}}} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 0 & \frac{54}{23} & \frac{135}{23} & \frac{27}{23} \\ 0 & 0 & 0 & -\frac{793}{46} & -\frac{453}{46} & \frac{1133}{46} \end{array}\right] = \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 0 & \frac{54}{23} & \frac{135}{23} & \frac{27}{23} \\ 0 & 0 & 0 & 0 & \frac{133}{4} & \frac{133}{4} \end{array}\right]$	$l_{54} = -\frac{793}{108}$ $R_5 = R_5 + \frac{793}{108}R_4$

Simpan ***U*** dan seluruh matriks eleminasi untuk digunakan pada Faktorisasi LU. Lanjutkan eleminasi hingga membentuk [***I*** | ***x***]

$\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{23} & 0 & 0 \\ 0 & 0 & 0 & \frac{23}{54} & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{133} \end{bmatrix} \left[\begin{array}{ccccc\|c} 2 & -1 & 1 & 5 & 0 & -4 \\ 0 & 3 & 1 & -2 & -5 & 7 \\ 0 & 0 & -23 & 4 & 10 & 25 \\ 0 & 0 & 0 & \frac{54}{23} & \frac{135}{23} & \frac{27}{23} \\ 0 & 0 & 0 & 0 & \frac{133}{4} & \frac{133}{4} \end{array}\right] = \left[\begin{array}{ccccc\|c} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} & 0 & -2 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$ ***D***	$R_1 = \frac{1}{2}R_1 \| R_4 = \frac{23}{54}R_4$ $R_2 = \frac{1}{3}R_1 \| R_5 = \frac{4}{133}R_5$ $R_3 = -\frac{1}{23}R_3$ Mempermudah proses eleminasi. Matriks ***D*** merupakan matriks diagonal yang menyimpan nilai diagonal matriks ***U***
$\begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccccc\|c} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} & 0 & -2 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{ccccc\|c} 1 & 0 & \frac{2}{3} & \frac{13}{6} & -\frac{5}{6} & -\frac{5}{6} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$ ***E12***	$l_{12} = -\frac{1}{2}$ $R_1 = R_1 + \frac{1}{2}R_2$
$\begin{bmatrix} 1 & 0 & -\frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccccc\|c} 1 & 0 & \frac{2}{3} & \frac{13}{6} & -\frac{5}{6} & -\frac{5}{6} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{ccccc\|c} 1 & 0 & 0 & \frac{105}{46} & -\frac{25}{46} & -\frac{5}{46} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$ ***E13***	$l_{13} = \frac{2}{3}$ $R_1 = R_1 - \frac{2}{3}R_3$
$\begin{bmatrix} 1 & 0 & 0 & -\frac{105}{46} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccccc\|c} 1 & 0 & 0 & \frac{105}{46} & -\frac{25}{46} & -\frac{5}{46} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{ccccc\|c} 1 & 0 & 0 & 0 & -\frac{25}{4} & -\frac{5}{4} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$ ***E14***	$l_{14} = \frac{105}{46}$ $R_1 = R_1 - \frac{105}{46}R_4$
$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{25}{4} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccccc\|c} 1 & 0 & 0 & 0 & -\frac{25}{4} & -\frac{5}{4} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{ccccc\|c} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$ ***E15***	$l_{15} = -\frac{25}{4}$ $R_1 = R_1 + \frac{25}{4}R_5$
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccccc\|c} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & -\frac{5}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{ccccc\|c} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & -\frac{14}{23} & -\frac{35}{23} & \frac{62}{23} \\ 0 & 0 & 1 & -\frac{4}{23} & -\frac{10}{23} & -\frac{25}{23} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$ ***E23***	$l_{23} = \frac{1}{3}$ $R_2 = R_2 - \frac{1}{3}R_3$

$\underbrace{\begin{bmatrix}1&0&0&0&0\\0&1&0&\frac{14}{23}&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}}_{E_{24}}\left[\begin{array}{ccccc\|c}1&0&0&0&0&5\\0&1&0&-\frac{14}{23}&-\frac{35}{23}&\frac{62}{23}\\0&0&1&-\frac{4}{23}&-\frac{10}{23}&-\frac{25}{23}\\0&0&0&1&\frac{5}{2}&\frac{1}{2}\\0&0&0&0&1&1\end{array}\right]=\left[\begin{array}{ccccc\|c}1&0&0&0&0&5\\0&1&0&0&0&3\\0&0&1&-\frac{4}{23}&-\frac{10}{23}&-\frac{25}{23}\\0&0&0&1&\frac{5}{2}&\frac{1}{2}\\0&0&0&0&1&1\end{array}\right]$	$l_{24} = -\frac{14}{23}$ $R_2 = R_2 + \frac{14}{23}R_4$
$\underbrace{\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&\frac{4}{23}&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}}_{E_{34}}\left[\begin{array}{ccccc\|c}1&0&0&0&0&5\\0&1&0&0&0&3\\0&0&1&-\frac{4}{23}&-\frac{10}{23}&-\frac{25}{23}\\0&0&0&1&\frac{5}{2}&\frac{1}{2}\\0&0&0&0&1&1\end{array}\right]=\left[\begin{array}{ccccc\|c}1&0&0&0&0&5\\0&1&0&0&0&3\\0&0&1&0&0&-1\\0&0&0&1&\frac{5}{2}&\frac{1}{2}\\0&0&0&0&1&1\end{array}\right]$	$l_{34} = -\frac{4}{23}$ $R_3 = R_3 + \frac{4}{23}R_4$
$\underbrace{\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&-\frac{5}{2}\\0&0&0&0&1\end{bmatrix}}_{E_{45}}\left[\begin{array}{ccccc\|c}1&0&0&0&0&5\\0&1&0&0&0&3\\0&0&1&0&0&-1\\0&0&0&1&\frac{5}{2}&\frac{1}{2}\\0&0&0&0&1&1\end{array}\right]=\underbrace{\left[\begin{array}{ccccc\|c}1&0&0&0&0&5\\0&1&0&0&0&3\\0&0&1&0&0&-1\\0&0&0&1&0&-2\\0&0&0&0&1&1\end{array}\right]}_{[\boldsymbol{I}\mid\boldsymbol{x}]}$	$l_{45} = \frac{5}{2}$ $R_4 = R_4 - \frac{5}{2}R_5$

Solusi $\boldsymbol{x}$ ada di ruas kanan matriks $[\boldsymbol{I}|\boldsymbol{x}]$

$$\therefore \mathrm{s} = 5, \quad \mathrm{t} = 3, \quad \mathrm{u} = -1, \quad v = -2, \quad w = 1$$

b. Fakorisasi LU

Dari langkah eliminasi Gauss, kita sudah mendapatkan matriks eliminasi:

$$\boldsymbol{E} = \boldsymbol{E_{54}E_{53}E_{43}M_{33}E_{52}E_{32}E_{51}E_{31}}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&\frac{793}{108}&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&-\frac{7}{69}&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&\frac{2}{23}&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&3&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&\frac{1}{6}&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&-\frac{2}{3}&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\-\frac{5}{2}&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\-2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$$

Kemudian matriks lower triangular bisa didapat dari inverse matriks eliminasi:

$$\boldsymbol{L} = \boldsymbol{E^{-1}}$$

$$= \boldsymbol{(E_{31})^{-1}(E_{51})^{-1}(E_{32})^{-1}(E_{52})^{-1}(M_{33})^{-1}(E_{43})^{-1}(E_{53})^{-1}(E_{54})^{-1}}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\\frac{5}{2}&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&\frac{2}{3}&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&-\frac{1}{6}&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&\frac{1}{3}&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&-\frac{2}{23}&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&\frac{7}{69}&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&-\frac{793}{108}&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\\frac{5}{2}&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&\frac{2}{3}&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&-\frac{1}{6}&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&\frac{1}{3}&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&-\frac{2}{23}&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\\frac{5}{2}&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&\frac{2}{3}&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&-\frac{1}{6}&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&\frac{1}{3}&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&-\frac{2}{23}&1&0\\0&0&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\\frac{5}{2}&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&\frac{2}{3}&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&-\frac{1}{6}&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&\frac{1}{3}&0&0\\0&0&-\frac{2}{23}&1&0\\0&0&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\\frac{5}{2}&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&\frac{2}{3}&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&\frac{1}{3}&0&0\\0&0&-\frac{2}{23}&1&0\\0&-\frac{1}{6}&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\\frac{5}{2}&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&\frac{2}{3}&\frac{1}{3}&0&0\\0&0&-\frac{2}{23}&1&0\\0&-\frac{1}{6}&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&\frac{2}{3}&\frac{1}{3}&0&0\\0&0&-\frac{2}{23}&1&0\\\frac{5}{2}&-\frac{1}{6}&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&\frac{2}{3}&\frac{1}{3}&0&0\\0&0&-\frac{2}{23}&1&0\\\frac{5}{2}&-\frac{1}{6}&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}$$

Selesaikan $\boldsymbol{Lc} = \boldsymbol{b}$ untuk mendapatkan $\boldsymbol{c}$ dengan forward substitution

$$\boldsymbol{Lc} = \boldsymbol{b}$$

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\2&\frac{2}{3}&\frac{1}{3}&0&0\\0&0&-\frac{2}{23}&1&0\\\frac{5}{2}&-\frac{1}{6}&\frac{7}{69}&-\frac{793}{108}&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\\c_4\\c_5\end{bmatrix}=\begin{bmatrix}-4\\7\\5\\-1\\16\end{bmatrix}$$

$$c_1 = -4$$
$$c_2 = 7$$
$$2c_1 + \frac{2}{3}c_2 + \frac{1}{3}c_3 = 5$$
$$-\frac{2}{23}c_3 + c_4 = -1$$

$$\frac{5}{2}c_1 - \frac{1}{6}c_2 + \frac{7}{69}c_3 - \frac{793}{108}c_4 + c_5 = 16$$

$c_1 = -4$

$c_2 = 7$

$c_3 = 3(5 - 2c_1 - \frac{2}{3}c_2) = 3[5 - 2(-4) - \frac{2}{3}(7)] = 25$

$c_4 = -1 + \frac{2}{23}c_3 = -1 + \frac{2}{23}(25) = \frac{27}{23}$

$c_5 = 16 - \frac{5}{2}c_1 + \frac{1}{6}c_2 - \frac{7}{69}c_3 + \frac{793}{108}c_4 = 16 - \frac{5}{2}(-4) + \frac{1}{6}(7) - \frac{7}{69}(25) + \frac{793}{108}(\frac{27}{23}) = \frac{133}{4}$

Selesaikan $\boldsymbol{Ux} = \boldsymbol{c}$ untuk mendapatkan $\boldsymbol{x}$ dengan backward substitution

$$\boldsymbol{Ux} = \boldsymbol{c}$$

$$\begin{bmatrix} 2 & -1 & 1 & 5 & 0 \\ 0 & 3 & 1 & -2 & -5 \\ 0 & 0 & -23 & 4 & 10 \\ 0 & 0 & 0 & \frac{54}{23} & \frac{135}{23} \\ 0 & 0 & 0 & 0 & \frac{133}{4} \end{bmatrix} \begin{bmatrix} s \\ t \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \\ 25 \\ \frac{27}{23} \\ \frac{133}{4} \end{bmatrix}$$

$$2s - t + u + 5v = -4$$

$$3t + u - 2v - 5w = 7$$

$$-23u + 4v + 10w = 25$$

$$\frac{54}{23}v + \frac{135}{23}w = \frac{27}{23}$$

$$\frac{133}{4}w = \frac{133}{4}$$

$w = \frac{133}{4} \cdot \frac{4}{133} = 1$

$v = \left[\frac{27}{23} - \frac{135}{23}w\right] \cdot \frac{23}{54} = \left[\frac{27}{23} - \frac{135}{23}(1)\right] \cdot \frac{23}{54} = -2$

$u = \frac{25 - 4v - 10w}{-23} = \frac{25 - 4(-2) - 10(1)}{-23} = -1$

$t = \frac{7 - u + 2v + 5w}{3} = \frac{7 - (-1) + 2(-2) + 5(1)}{3} = 3$

$s = \frac{-4 + t - u - 5v}{2} = \frac{-4 + (3) - (-1) - 5(-2)}{2} = 5$

$$\therefore \mathrm{s} = 5, \quad \mathrm{t} = 3, \quad \mathrm{u} = -1, \quad v = -2, \quad w = 1$$

c. Invers $Ax = b$.

$$\mathbf{E} = \mathbf{E}_{45}\mathbf{E}_{34}\mathbf{E}_{24}\mathbf{E}_{23}\mathbf{E}_{15}\mathbf{E}_{14}\mathbf{E}_{13}\mathbf{E}_{12}\mathbf{D}\mathbf{E}_{54}\mathbf{E}_{53}\mathbf{E}_{43}\mathbf{M}_{33}\mathbf{E}_{52}\mathbf{E}_{32}\mathbf{E}_{51}\mathbf{E}_{31}$$

$$\mathbf{E}[\mathbf{A} \quad | \quad \mathbf{I}] = [\mathbf{EA} \quad | \quad \mathbf{EI}]$$

$$= [\mathbf{I} \quad | \quad \mathbf{E}]$$

Kita tahu bahwa, $\mathbf{A^{-1}A = I}$. Jika $\mathbf{EA = A^{-1}A = I}$, maka $\mathbf{E = A^{-1}}$. Jika menggunakan MATLAB, kita akan mendapatkan nilai E sebagai berikut:

$$\mathbf{E} = \mathbf{A}^{-1} = \begin{bmatrix} -\frac{290}{1197} & \frac{55}{513} & \frac{163}{1197} & \frac{461}{1130} & \frac{25}{133} \\ -\frac{2}{9} & \frac{7}{27} & \frac{1}{9} & \frac{7}{27} & 0 \\ \frac{2}{9} & \frac{2}{27} & -\frac{1}{9} & \frac{2}{27} & 0 \\ \frac{83}{399} & -\frac{1}{171} & -\frac{4}{399} & -\frac{151}{1197} & -\frac{10}{133} \\ -\frac{206}{1197} & -\frac{14}{513} & \frac{58}{1197} & \frac{282}{1277} & \frac{4}{133} \end{bmatrix}$$

$$\mathbf{E} = \mathbf{A}^{-1} = \begin{bmatrix} -0.2423 & 0.1072 & 0.1362 & 0.4080 & 0.1880 \\ -0.2222 & 0.2593 & 0.1111 & 0.2593 & 0 \\ 0.2222 & 0.0741 & -0.1111 & 0.0741 & 0 \\ 0.2080 & -0.0058 & -0.0100 & -0.1261 & -0.0752 \\ -0.1721 & -0.0273 & 0.0485 & 0.2208 & 0.0301 \end{bmatrix}$$

MATLAB CODE:

```
clear all;close all;clc
E31 = [ 1  0  0  0  0;
        0  1  0  0  0;
       -2  0  1  0  0;
        0  0  0  1  0;
        0  0  0  0  1];

E51 = [ 1     0  0  0  0;
        0     1  0  0  0;
        0     0  1  0  0;
        0     0  0  1  0;
       -5/2   0  0  0  1];

E32 = [1   0    0   0   0;
       0   1    0   0   0;
       0  -2/3  1   0   0;
       0   0    0   1   0;
       0   0    0   0   1];

E52 = [1   0    0  0  0;
       0   1    0  0  0;
       0   0    1  0  0;
       0   0    0  1  0;
```

```
       0 1/6  0 0 1];

M33 = [1  0  0  0  0;
       0  1  0  0  0;
       0  0  3  0  0;
       0  0  0  1  0;
       0  0  0  0  1];

E43 = [1  0  0     0  0;
       0  1  0     0  0;
       0  0  1     0  0;
       0  0  2/23  1  0;
       0  0  0     0  1];

E53 = [1  0  0    0  0;
       0  1  0    0  0;
       0  0  1    0  0;
       0  0  0    1  0;
       0  0 -7/69 0  1];

E54 = [1  0  0  0       0;
       0  1  0  0       0;
       0  0  1  0       0;
       0  0  0  1       0;
       0  0  0 793/108 1];

D   = [1/2  0    0     0     0;
       0   1/3   0     0     0;
       0    0  -1/23   0     0;
       0    0    0   23/54   0;
       0    0    0     0   4/133];

E12 = [1  1/2  0  0  0;
       0   1   0  0  0;
       0   0   1  0  0;
       0   0   0  1  0;
       0   0   0  0  1];

E13 = [1  0  -2/3  0  0;
       0  1    0   0  0;
       0  0    1   0  0;
       0  0    0   1  0;
       0  0    0   0  1];

E14 = [1  0  0  -105/46  0;
       0  1  0      0    0;
       0  0  1      0    0;
       0  0  0      1    0;
       0  0  0      0    1];

E15 = [1  0  0  0  25/4;
       0  1  0  0    0;
       0  0  1  0    0;
       0  0  0  1    0;
       0  0  0  0    1];

E23 = [1  0   0   0  0;
       0  1 -1/3  0  0;
       0  0   1   0  0;
       0  0   0   1  0;
       0  0   0   0  1];

E24 = [1  0  0    0   0;
       0  1  0  14/23 0;
```

```
       0  0  1    0   0;
       0  0  0    1   0;
       0  0  0    0   1];

E34 = [1  0  0   0   0;
       0  1  0   0   0;
       0  0  1  4/23 0;
       0  0  0   1   0;
       0  0  0   0   1];

E45 = [1  0  0   0   0;
       0  1  0   0   0;
       0  0  1   0   0;
       0  0  0   1 -5/2;
       0  0  0   0   1];

E = E45*E34*E24*E23*E15*E14*E13*E12*D*E54*E53*E43*M33*E52*E32*E51*E31;

b = [-4;7;5;-1;16];

A = [2 -1  1  5 0;
     0  3  1 -2 -5;
     4  0 -5  10 0;
     0  0  2  2  5;
     5 -3  0 -4 -
```