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Tugas 4 Matematika Diskrit

4. Let P(n) be the statement that $1^3 + 2^3 + \dots + n^3 = (n(n+1)/2)^2$ for the positive integer n.

a) What is the statement P(1)?

→ $P(1) : 1^3 = (1(1+1)/2)^2$

b) Show that P(1) is true, completing the basic step of the proof of P(n) for all positive integer n.

→ $P(1) : 1^3 = (1((1+1)/2)^2$

$1^3 = (1(2)/2)^2$

$1^3 = (1.1)^2$

$1^3 = 1^2$

$1 = 1$

∴ P(1) is true

c) What is the inductive hypothesis of a proof that P(n) is true for all positive integers n?

→ The inductive hypothesis is $P(n) : 1^3 + 2^3 + \dots + n^3 = (n(n+1)/2)^2$

d) What do you need to prove in the inductive step of a proof that P(n) is true for all positive integers n?

→ Need to proof P(n+1) so

$1^3 + 2^3 + \dots + n^3 + (n+1)^3 = (n+1((n+1+1)/2)^2$

e) Complete the inductive step of a proof that P(n) is true for all positive integer n, identifying where you use the inductive hypothesis.

→ $P(n) : 1^3 + 2^3 + \dots + n^3 = (n(n+1)/2)^2$

$P(n+1) : 1^3 + 2^3 + \dots + n^3 + (n+1)^3 = (n+1(n+1+1)/2)^2$

$$(n(n+1)/2)^2 + (n+1)^3 = (n+1(n+2)/2)^2$$

$$\frac{n^2(n+1)^2}{4} + (n+1).(n+1)^2 = (n+1)^2 \frac{(n+2)^2}{4}$$

$$(n+1)^2\left(\frac{n^2}{4} + \frac{4(n+1)}{4}\right) = (n+1)^2 \frac{(n+2)^2}{4}$$

$$(n+1)^2\left(\frac{n^2+4n+4}{4}\right) = (n+1)^2\left(\frac{n^2+4n+4}{4}\right)$$

f.) Explain why these steps show that this formula is true whenever n is a positive integer.

→ So, if $P(n)$ is true, then $P(n+1)$ is true.

∴ $P(n)$ is true for all positive integer n.

11. a). Find a formula for

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n}$$

by examining the values of this expression for small values of n.

→ $P(n) = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n}$

$n=1$ → $P(1) = \frac{1}{2^1} = \frac{1}{2}$

$n=2$ → $P(2) = \frac{1}{2^1}+\frac{1}{2^2} = \frac{1}{2}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4} = \frac{3}{4} = \frac{2^2-1}{2^2}$

$n=3$ → $P(3) = \frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \frac{4+2+1}{8} = \frac{7}{8} = \frac{2^3-1}{2^3}$

∴ $P(n) = \frac{2^n-1}{2^n}$

b). Prove the formula you conjectured in part (a)

→ $P(n) : \frac{1}{2}+\frac{1}{4}+\dots+\frac{1}{2^n} = \frac{2^n-1}{2^n}$

$P(n+1) : \frac{1}{2}+\frac{1}{4}+\dots+\frac{1}{2^n}+\frac{1}{2^{(n+1)}} = \frac{2^{(n+1)}-1}{2^{(n+1)}}$

$$\frac{2^n-1}{2^n}+\frac{1}{2^{(n+1)}} = \frac{2^{(n+1)}-1}{2^{(n+1)}}$$

$$\frac{2^n-1}{2^n}+\frac{1}{2^n.2} = \frac{2^{(n+1)}-1}{2^{(n+1)}}$$

$$\frac{2(2^n-1)}{2^n.2}+\frac{1}{2^n.2} = \frac{2^{(n+1)}-1}{2^{(n+1)}}$$

$$\frac{2^n.2-2+1}{2^n.2} = \frac{2^{(n+1)}-1}{2^{(n+1)}}$$

$$\frac{2^{(n+1)}-1}{2^{(n+1)}} = \frac{2^{(n+1)}-1}{2^{(n+1)}}$$ $\blacksquare$

∴ $P(n) : \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n} = \frac{2^n-1}{2^n}$ is true for all integer

27. Prove that for every positive integer n,

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots+\frac{1}{\sqrt{n}} > 2\left(\sqrt{n+1}-1\right)$$

→ Let $n=1$

$P(1)$: $1 > 2\left(\sqrt{1+1}-1\right)$

$1 > 2\left(\sqrt{2}-1\right)$

$1 > 2\sqrt{2}-2$

$1 > 2{,}8-2$

$1 > 0{,}8$ ■

→ $P(n): 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots+\frac{1}{\sqrt{n}} > 2\left(\sqrt{n+1}-1\right)$

$p(n+1): 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots+\frac{1}{\sqrt{n}}+\frac{1}{\sqrt{n+1}} > 2\left(\sqrt{n+2}-1\right)$

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots+\frac{1}{\sqrt{n}}+\frac{1}{\sqrt{n+1}} > 2\left(\sqrt{n+1}-1\right)+\frac{1}{\sqrt{n+1}}$$

$$2\left(\sqrt{n+1}-1\right)+\frac{1}{\sqrt{n+1}} > 2\left(\sqrt{n+2}-1\right)$$

$$2\sqrt{n+1}-2+\frac{1}{\sqrt{n+1}} > 2\sqrt{n+2}-2$$

$$\frac{1}{\sqrt{n+1}} > 2\sqrt{n+2}-2\sqrt{n+1}$$

$$\frac{1}{\sqrt{n+1}} > 2\left(\sqrt{n+2}-\sqrt{n+1}\right)$$

$$\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+1}} > 2\left(\sqrt{n+2}-\sqrt{n+1}\right)\left(\sqrt{n+2}+\sqrt{n+1}\right)$$

$$\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+1}} > 2\left(n+2-n-1\right)$$

$$\frac{\sqrt{n+2}+\sqrt{n+1}}{\cancel{\sqrt{n+1}}} > \frac{2\sqrt{n+1}}{\cancel{\sqrt{n+1}}}$$

$$\sqrt{n+2} > \sqrt{n+1}$$

$$\left(\sqrt{n+2}\right)^2 > \left(\sqrt{n+1}\right)^2$$

$$\cancel{n}+2 > \cancel{n}+1$$

$$2 > 1$$

■

∴ The formula is true for every positive integer of n.