Muchammad Daniyal Kautsar
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Latihan Soal

1. $f(x,y,z) = 5xyz$

$1 \le x \le 7$
$2 \le y \le 8$
$3 \le z \le 9$

$-2 \le x \le 4$
$-1 \le y \le 5$
$0 \le z \le 1$

$$\int_2^8 \int_1^7 \int_3^9 5xyz \, dz\,dx\,dy$$
$$= \int_2^8 \int_1^7 \frac{5xyz^2}{2}\Big|_3^9 \, dx\,dy$$
$$= \int_2^8 \int_1^7 180xy \, dx\,dy$$
$$= \int_2^8 90x^2y\Big|_1^7 \, dy$$
$$= \int_2^8 4320y \, dy$$
$$= \frac{2160y^2}{2}\Big|_2^8 = 129600$$

$$\int_{-1}^{5} \int_{-2}^{4} \int_{0}^{1} 5xyz \, dz\,dx\,dy$$
$$= \int_{-1}^{5} \int_{-2}^{4} \frac{5xyz^2}{2}\Big|_0^1 \, dx\,dy$$
$$= \int_{-1}^{5} \int_{-2}^{4} 90xy \, dx\,dy$$
$$= \int_{-1}^{5} 45x^2y\Big|_{-2}^{4} \, dy$$
$$= \int_{-1}^{5} 540y \, dy = 270y^2\Big|_{-1}^{5} = 6480$$

2. $\iiint_F 2x^2+2y^2 \, dz\,dy\,dx$. $F \to z = x^2+y^2$

$z = 4$

$z = x^2+y^2$
$z = r^2$

$0 \le z \le 4$
$0 \le \theta \le 2\pi$
$0 \le r \le 2$

$$\int_0^{2\pi} \int_0^2 \int_0^4 2r^2 \cdot r \, dz\,dr\,d\theta$$
$$= \int_0^{2\pi} \int_0^2 2r^3 \cdot 4 \, dr\,d\theta$$
$$= \int_0^{2\pi} 32 \, d\theta = 64\pi$$

3. $\iint_S 3xyz \, dS$. $S \to$ permukaan $3x+4y+2z = 12$

Proyeksi $S \to (0,3),(4,0)$

$z = f(x,y) = 6 - \frac{3}{2}x - 2y$

$F_x = -\frac{3}{2}$, $F_y = -2$

$3xyz = 3xy\left(6 - \frac{3}{2}x - 2y\right)$
$= 18xy - \frac{9}{2}x^2y - 6xy^2$

$dS = \sqrt{\frac{9}{4}+4+1} \, dA = \frac{\sqrt{29}}{2} dA$

$$\iint_S 3xyz \, dS = \iint_D \left(18xy - \frac{9}{2}x^2y - 6xy^2\right) \frac{\sqrt{29}}{2} dA$$
$$= \frac{\sqrt{29}}{2} \int_0^4 \int_0^{3-\frac{3}{4}x} 18xy - \frac{9}{2}x^2y - 6xy^2 \, dy\,dx$$
$$= \frac{\sqrt{29}}{2} \int_0^4 9xy^2 - \frac{9}{4}x^2y^2 - 2xy^3\Big|_0^{3-\frac{3}{4}x} \, dx$$
$$= \frac{\sqrt{29}}{2} \int_0^4 27x - \frac{81}{16}x^3 - \frac{81x^4 - 648x^3 + 1296x^2}{64} + \frac{27}{32}x^4 \, dx$$
$$= \frac{2214\sqrt{29}}{5} \approx 2384.55$$