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Tugas 1 Kalkulus Variabel Tunggal

1.1

9. Express the area and Perimeter of an equilateral triangle as a function of the triangle's side length x

→ base $= x$ height $= h$, perimeter : $P(x)$, area : $a(x)$

$P(x) = x + x + x$
$= 3x$

$h = \sqrt{x^2 - \left(\frac{1}{2}x\right)^2}$
$= \sqrt{x^2 - \frac{1}{4}x^2}$
$= \sqrt{\frac{3}{2}}\, x$

$a(x) = \frac{x \cdot h}{2}$
$= \frac{x}{2} \cdot \sqrt{\frac{3}{2}}\, x$
$= \sqrt{\frac{3}{4}}\, x^2$

11. Express the edge length of a cube as a function of the cube's diagonal length d. Then express the surface area and volume of the cube as a function of the diagonal length.

→ diagonal $= d$, edge $= l$, area $= a(l)$, volume $= v(l)$

$d = \sqrt{l^2 + l^2 + l^2}$
$d = \sqrt{3l^2}$
$d = l\sqrt{3}$
$l = \frac{d}{\sqrt{3}}$

$a(l) = 6 \cdot l^2$
$= 6 \cdot \left(\frac{d}{\sqrt{3}}\right)^2$
$= \cancel{6}^{2} \cdot \frac{d^2}{\cancel{3}} = 2d^2$

$v(l) = l^3$
$= \left(\frac{d}{\sqrt{3}}\right)^3 = \frac{d^3}{3\sqrt{3}}$

59. The variable s is proportional to t, and $s = 25$ when $t = 75$. Determine t when $s = 60$

→ $s_1 = 25$; $t_1 = 75$
$s_2 = 60$

$s_1 t_2 = s_2 t_1$
$t_2 = \frac{t_1 s_2}{s_1}$
$= \frac{\cancel{75}^{3} \cdot 60}{\cancel{25}} = 180$

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62. Boyle's Law says that the volume V of a gas at constant temperature increases whenever the pressure P decreases, so that V and P are inversely proportional. If $P = 14.7\ lbs/in^2$ when $V = 1000\ in^3$, Then what is V when $P = 23.4\ lbs/in^2$?

$\rightarrow P_1 = 14,7\ lbs/in^2$

$V_1 = 1000\ in^3$

$P_2 = 23,4\ lbs/in^2$

Boyle Law $\Rightarrow P_1 \cancel{V_1} = P_2 V_2$

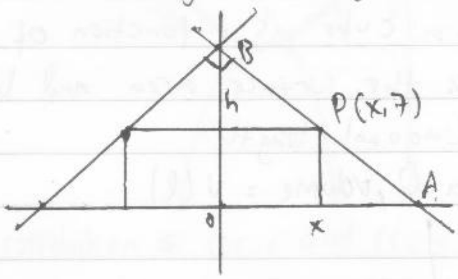
$$P_1 V_1 = P_2 V_2$$

$$V_2 = \frac{P_1 V_1}{P_2}$$

$$= \frac{14,7 \cdot 1000}{23,4}$$

$$V_2 = 628.205\ in^3$$

64. The accompanying figure shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.



a. Express the y-coordinate of P in terms of x.

$$2(AB)^2 = 2^2$$
$$2(AB)^2 = 4$$
$$AB^2 = 2$$
$$AB = \sqrt{2}$$

$$h = \sqrt{AB^2 - 1^2}$$
$$h = \sqrt{2-1}$$
$$h = 1$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{AB} = \frac{1-0}{0-1} = -1$$

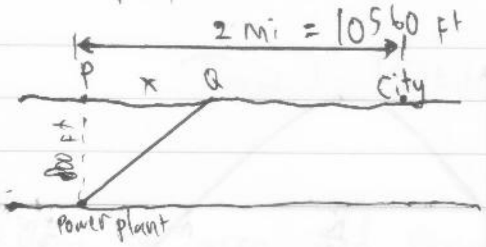
y coordinate of P $\rightarrow$ $y - y_1 = m_{AB}(x - x_1)$

$$y - 0 = -1(x - 1)$$
$$y = -x + 1$$

b. Express the area of rectangle in term of x

$$A(x) = 2xy$$
$$= 2x(-x+1)$$
$$= -2x^2 + 2x$$

72. Industrial costs. A power plant sits next to a river where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs \$180 per foot across the river and \$100 per foot along the land.



a. Suppose that the cable goes from the plant to a point in Q on the opposite side that is x ft from the point P directly opposite the plant. Write a function C(x) that gives the cost of laying the cable in terms of the distance x.

→ C(x) = (distance in land × cost on land) + (distance across water × cost on water)

$C(x) = ((10560 - x) \cdot \$100) + (\sqrt{800^2 + x^2} \cdot \$180)$

b. Generate a table of values to determine if the least expensive location for point Q is less than 2000 ft or greater than 2000 ft from P.

→ $C(x) = ((10560 - x) \cdot \$100) + (\sqrt{800^2 + x^2} \cdot \$180)$

$C(0) = \$ 1.200.000$

$C(500) \approx \$ 1.175.812$

$C(1000) \approx \$ 1.186.512$

$C(1500) \approx \$ 1.212.000$

$C(2000) \approx \$ 1.243.732$

$C(2500) \approx \$ 1.278.479$

So, the least expensive is less than 2000 ft from point P.

1.3

12. $\sin x = -\frac{1}{2}$, $x \in \{\pi, \frac{3\pi}{2}\}$. $\cos x = ?$, $\tan x = ?$

→ $\sin x = -\frac{1}{2}$, $x \in \langle \pi, \frac{3\pi}{2} \rangle$

$x = \frac{7}{6}\pi$

· $\cos \frac{7}{6}\pi = -\frac{1}{2}\sqrt{3}$

· $\tan \frac{7}{6}\pi = \frac{1}{3}\sqrt{3}$

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19- Graph the functions and what is the period of $\cos\left(x - \frac{\pi}{2}\right)$.

$\rightarrow \cos\left(x - \frac{\pi}{2}\right) = \sin(x)$.

let $x \in \{0, 2\pi\}$

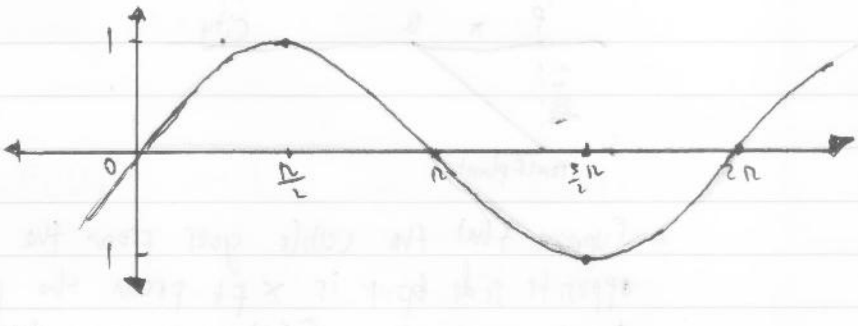
$\sin(0) = 0$

$\sin\left(\frac{\pi}{2}\right) = 1$

$\sin(\pi) = 0$

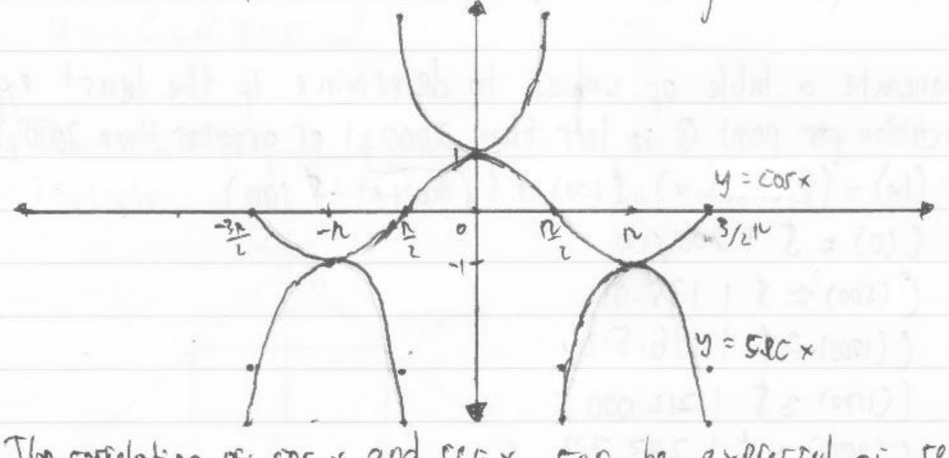
$\sin\left(\frac{3}{2}\pi\right) = -1$

$\sin(2\pi) = 0$



period $= 2\pi$

27 a. Graph $y = \cos x$ and $y = \sec x$ together for $-3\pi/2 \leq x \leq 3\pi/2$. Comment on the behavior of $\sec x$ in relation to the sign and values of $\cos x$.



The correlation of $\cos x$ and $\sec x$ can be expressed as $\sec x = \frac{1}{\cos x}$

Proof: let $x = \frac{\pi}{2}$; $\sec x = \frac{1}{\cos x}$

$\sec\frac{\pi}{2} = \frac{1}{\cos\frac{\pi}{2}} = \frac{1}{0}$ = undefined.

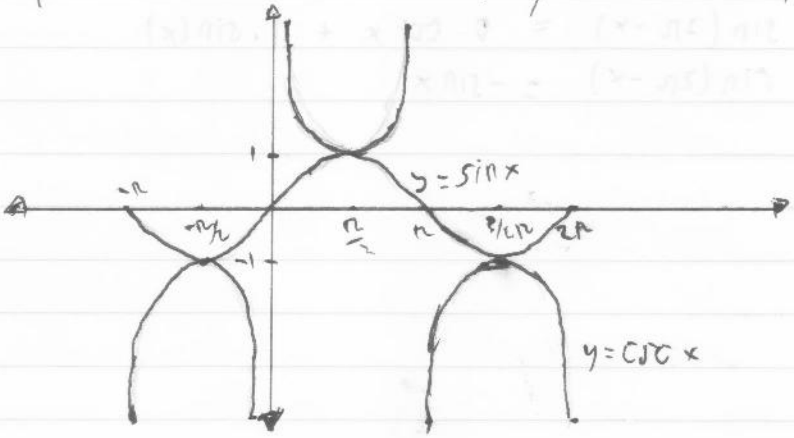
let $x = 0$; $\sec x = \frac{1}{\cos x}$

$\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1$.

Also, $\cos x$ and $\sec x$ are positive for x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$: and negative in the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

Range of $\sec x$ is $\{-\infty, -1] \cup [1, \infty\}$ and the range of cos is $(-1, 1)$.

27. b. Graph $y = \sin x$ and $y = \csc x$ together for $-\pi \leq x \leq 2\pi$. Comment on the behavior of $\csc x$ in relation to the signs and values of $\sin x$



The correlation of $\sin x$ and $\csc x$ can be expressed as $\csc = \frac{1}{\sin x}$.

Proofing : let $x = 0$; $\csc(0) = \frac{1}{\sin(0)}$

$\csc(0) = \frac{1}{0}$ = undefined.

let $x = \frac{\pi}{2}$; $\csc\left(\frac{\pi}{2}\right) = \frac{1}{\sin\left(\frac{\pi}{2}\right)}$

$\csc\left(\frac{\pi}{2}\right) = \frac{1}{1} = 1$

Also, $\csc x$ and $\sin x$ will be positive for x in the intervals $(-\frac{3}{2}\pi, -\pi)$ and $(0, \pi)$; and will be negative for x in the interval $(-\pi, 0)$ and $(\pi, \frac{3}{2}\pi)$. The range of $\csc x$ is $\langle -\infty, -1) \cup (1, \infty\rangle$ and $\sin x$ is $(-1, 1)$.

38. What happen if you take $B = 2\pi$ in the addition formulas? Do the results agree with something you already know?

→ if $B = 2\pi$.

$\cos(A + B) = \cos(A + 2\pi)$

$\cos(A + 2\pi) = \cos A \cdot \cos 2\pi - \sin A \cdot \sin 2\pi$

$= \cos A$.

$\sin(A + B) = \sin(A + 2\pi)$

$\sin(A + 2\pi) = \sin A \cdot \cos 2\pi + \cos A \cdot \sin 2\pi$

$= \sin A$.

The results agree with the fact that cos and sine functions have period of 2π.

40. $\sin(2\pi - x)$

→ $\sin(2\pi - x) = \sin 2\pi \cdot \cos x + \cos 2\pi \cdot \sin(-x)$

$= 0 \cdot \cos x + 1 \cdot \sin(-x)$

$= \sin(-x) = -\sin x$