

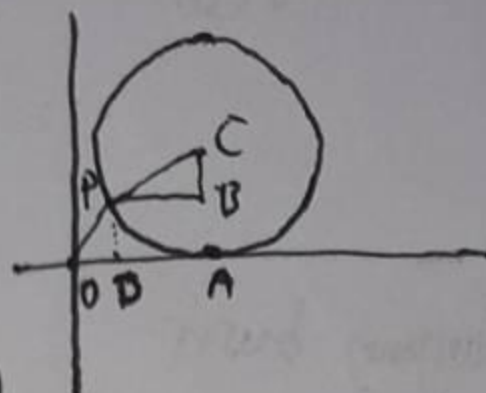
Latsol Tutor Kuj

$r=10$ satuan, bergerak cycloid.

Pada $P(x,y)$ dg sudut θ

a. Pers. parametrik

$$\begin{aligned} x &= OD = OA - PB = \cancel{PA} - \cancel{BB} \\ &= a\theta - a\sin\theta \\ &= a(\theta - \sin\theta) = 10(\theta - \sin\theta) \end{aligned}$$



$$\begin{aligned} y &= BP = AC - BC \\ &= a - a\cos\theta = a(1 - \cos\theta) \\ &= 10(1 - \cos\theta) \end{aligned}$$

b. Vektor & kecepatan

$$\begin{aligned} \vec{v}(\theta) &= \frac{d\vec{r}(\theta)}{d\theta} = \frac{d(10\theta - 10\sin\theta, 10 - 10\cos\theta)}{d\theta} \\ &= (10 - 10\cos\theta, 10\sin\theta) \end{aligned}$$

c. Kelajuan

$$\begin{aligned} \left\| \frac{d\vec{r}(\theta)}{d\theta} \right\| &= \left\| \vec{r}'(\theta) \right\| = \sqrt{(10 - 10\cos\theta)^2 + (10\sin\theta)^2} \\ &= \sqrt{100 - 200\cos\theta + 100\cos^2\theta + 100\sin^2\theta} \\ &= \sqrt{200 - 200\cos\theta} = \sqrt{200(1 - \cos\theta)} \\ &= 10\sqrt{2 - 2\cos\theta} \end{aligned}$$

d. Vektor singgung unit

$$\begin{aligned}\hat{T} &= \frac{\vec{v}(\theta)}{\|\vec{v}(\theta)\|} = \frac{10 - 10\cos\theta, 10\sin\theta}{10\sqrt{2-2\cos\theta}} \\ &= \frac{\sqrt{2+2\cos\theta}}{2}, \frac{\sin\theta}{\sqrt{2-2\cos\theta}}\end{aligned}$$

e. Panjang busur

$$\begin{aligned}s &= \int d\|\vec{v}(\theta)\| d\theta = \int 10\sqrt{2-2\cos\theta} d\theta \\ &= -20\sqrt{2-2\cos\theta} \cdot \cot\left(\frac{\theta}{2}\right) + C\end{aligned}$$

f. curvature

$$\begin{aligned}T'(\theta) &= \frac{dT}{d\theta} = \frac{d\sqrt{2+2\cos\theta}}{d\theta}, \frac{\sin\theta}{\sqrt{2-2\cos\theta}} \\ &= \frac{\sin\theta}{2\sqrt{2+2\cos\theta}}, \frac{\sqrt{2}\sin^2(\frac{\theta}{2})}{(1-\cos\theta)^{3/2}}\end{aligned}$$

$$\begin{aligned}K &= \frac{\|T'(\theta)\|}{\|\vec{r}'(\theta)\|} = \frac{\sqrt{\left(\frac{\sin\theta}{2\sqrt{2+2\cos\theta}}\right)^2 + \left(\frac{\sqrt{2}\sin^2(\theta/2)}{(1-\cos\theta)^{3/2}}\right)^2}}{10\sqrt{2-2\cos\theta}} \\ &= \frac{\sqrt{\sin^2(\frac{\theta}{2}) \sin^2(\theta)}}{(1-\cos^3\theta) \cdot (2\cos\theta + 2)} \\ &= \frac{1}{10\sqrt{2}} \frac{\sqrt{2-2\cos\theta}}{\sqrt{2-2\cos\theta}}\end{aligned}$$

$$\bullet f(x, y, z) = x^2 + y^2 + 5xy + z$$

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \text{ at point } (-3, 2, 0)$$

$$\frac{\partial f}{\partial x} = \frac{\partial x^2 + y^2 + 5xy + z}{\partial x} = 2x + 5y$$

$$(-3, 2, 0) \rightarrow 2(-3) + 5(2) \\ = -6 + 10 = 4$$

$$\frac{\partial f}{\partial y} = \frac{\partial x^2 + y^2 + 5xy + z}{\partial y} = 5x + 2y$$

$$(-3, 2, 0) \rightarrow 5(-3) + 2(2) \\ = -15 + 4 = -11$$

$$\frac{\partial f}{\partial z} = \frac{\partial x^2 + y^2 + 5xy + z}{\partial z} = 1$$

$$(-3, 2, 0) \rightarrow 1$$

$$f(x, y, z) = x \sin(5y + 10z) \cos(2x + 12z)$$

$$\frac{\partial F}{\partial z} = \frac{\partial}{\partial z} x \sin(5y + 10z) \cos(2x + 12z)$$

$$= 10 \cos(5y + 10z) \cdot \cos(2x + 12z)$$

$$+ 12 \cdot (\sin(2x + 12z)) \cdot \sin(5y + 10z)$$

$$= 10 \cos(5y + 10z) \cdot \cos(2x + 12z)$$

$$- 12 \sin(2x + 12z) \cdot \sin(5y + 10z)$$

$$f(x, y) = 7x - 8y + 2xy - x^2 + y^3$$

$$\frac{\partial F}{\partial x} = 7 + 2y - 2x$$

$$\frac{\partial F}{\partial y} = -8 + 2x + 3y^2$$

$$\frac{\partial^2 F}{\partial^2 x} = -2$$

$$\frac{\partial^2 F}{\partial^2 y} = 6y$$

$$\frac{\partial F}{\partial x} = 0$$

$$7 + 2y - 2x = 0$$

$$2x - 2y = 7$$

$$2x = 2y + 7$$

$$2y + 7 + 3y^2 = 8$$

$$3y^2 + 2y - 1 = 0$$

$$y_1 = -1 \quad y_2 = \frac{1}{3}$$

$$\frac{\partial F}{\partial y} = 0$$

$$-8 + 2x + 3y^2 = 0$$

$$2x + 3y^2 = 8$$

$$y = x - \frac{7}{2}$$

$$2x + 3\left(x - \frac{7}{2}\right)^2 = 8$$

$$0 = 2x + 3x^2 - 21x + 147$$

$$x = \frac{7 \pm 2\sqrt{2}}{2}$$

$$y_1 = -1$$

$$2x = 2y + 7$$

$$2x = -2 + 7$$

$$x = \frac{5}{2}$$

$$\left(\frac{5}{2}, -1\right)$$

$$y_2 = \frac{1}{3}$$

$$2x = 2y + 7$$

$$2x = \frac{2}{3} + 7$$

$$x = \frac{1}{3} + \frac{7}{2}$$

$$x = 3\frac{5}{6} = \frac{23}{6}$$

$$\left(3\frac{5}{6}, \frac{1}{3}\right)$$

$$d \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$D(x, y) = (7 + 2y - 2x)(-8 + 2x + 3y^2) - 4$$

$$D\left(\frac{5}{2}, -1\right) = (7 - 2 - 5)(-8 + 5 + 3) - 4$$

$$= -4 \quad (-4 < 0) \text{ saddle point}$$

$$D\left(\frac{23}{6}, \frac{1}{3}\right) = \left(7 + \frac{2}{3} - \frac{23}{3}\right)\left(-8 + \frac{23}{3} + \frac{1}{3}\right) - 4$$

$$= -9 \quad (-9 < 0) \text{ saddle point}$$