Problem:

Suppose that we the following vectors:

$$\boldsymbol{v_1} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}; \boldsymbol{v_2} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}; \boldsymbol{v_3} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}; \boldsymbol{v_4} = \begin{bmatrix} 2 \\ 1 \\ 1.5 \end{bmatrix}$$

a) Determine the length of vector $\boldsymbol{v_1}$.
b) Determine the length of vector $(\boldsymbol{v_2} + \boldsymbol{v_3})$.
c) Determine the angle between vectors $(\boldsymbol{v_2} + \boldsymbol{v_3})$ and $\boldsymbol{v_4}$.

Solution:

a) Length of vector $\boldsymbol{v_1}$

$$|\boldsymbol{v_1}|$$
$$= \sqrt{3^2 + 0^2 + (4)^2}$$
$$= \sqrt{9 + 0 + 16}$$
$$= \sqrt{25}$$
$$= \mathbf{5}$$

b) Length of vector $(\boldsymbol{v_2} + \boldsymbol{v_3})$.

$$(\boldsymbol{v_2} + \boldsymbol{v_3})$$
$$= \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$
$$= \begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}$$

$$|\boldsymbol{v_2} + \boldsymbol{v_3}|$$
$$= \sqrt{3^2 + 0^2 + (-4)^2}$$
$$= \sqrt{9 + 0 + 16}$$
$$= \sqrt{25}$$
$$= \mathbf{5}$$

c) Angle between vectors $(\boldsymbol{v_2} + \boldsymbol{v_3})$ and $\boldsymbol{v_4}$

$$\cos\theta$$
$$= \frac{(\boldsymbol{v_2} + \boldsymbol{v_3}) \cdot \boldsymbol{v_4}}{\|\boldsymbol{v_2} + \boldsymbol{v_3}\|. \|\boldsymbol{v_4}\|}$$
$$= \frac{\begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 1.5 \end{bmatrix}}{(5)(2.693)}$$
$$= \frac{6 + 0 + (-6)}{13.463}$$
$$= 0$$

So,

$$\boldsymbol{\theta} = \frac{\boldsymbol{\pi}}{\mathbf{2}}$$

~ Lathif Ma'arif