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Lapol TUM 7

Soal 2

a. Show $\left\{ \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\2\\1\end{bmatrix} \right\}$ is basis for R^3

* RREF

$$\begin{bmatrix}1&-1&1\\1&1&2\\1&0&1\end{bmatrix} \xrightarrow[R_3-R_1]{R_2-R_1} \begin{bmatrix}1&-1&1\\0&2&1\\0&1&0\end{bmatrix} \xrightarrow{R_2 * 1/2} \begin{bmatrix}1&-1&0\\0&1&1/2\\0&1&0\end{bmatrix} \xrightarrow{R_3-R_2} \begin{bmatrix}1&-1&0\\1&\cancel{1}&1/2\\0&0&-1/2\end{bmatrix}$$

$$\xrightarrow{R_3*(-2)} \begin{bmatrix}1&-1&1\\0&1&1/2\\0&0&1\end{bmatrix} \xrightarrow[R_2-\frac{1}{2}R_3]{R_1-R_3} \begin{bmatrix}1&-1&0\\0&1&0\\0&0&1\end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Since rref is identity matrix, the set of original vector is basis of R^3

b. Why this is not ~~ortogonal~~ ortonormal basis for R^3.

We calculate the dot product of every vector to check the ortogonal.

$V_1 . V_2 = \begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}-1\\1\\0\end{bmatrix} = 0$ | $V_1 . V_3 = \begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}1\\2\\1\end{bmatrix} = 4$ | $V_2 . V_3 = \begin{bmatrix}-1\\1\\0\end{bmatrix} \cdot \begin{bmatrix}1\\2\\1\end{bmatrix} = 1$

Since not all of the vector are ortogonal so that is not ortonormal basis.
if we check the magnitude, we conclude that all of the vector magnitude isn't equal to 1

$\|V_1\| = \sqrt{1+1+1} = \sqrt{3}$, $\|V_2\| = \sqrt{1+1+0} = \sqrt{2}$, $\|V_3\| = \sqrt{1+4+1} = \sqrt{5}$

c. Using Gram-schmidt process, transform the basis to an orthonormal basis for R^3

$\vec{w_1} = \begin{bmatrix}1\\1\\1\end{bmatrix}$; $\vec{w_2} = \vec{v_2} - \frac{\vec{w_1} . \vec{v_2}}{\vec{w_1} . \vec{w_1}} \vec{v} = \begin{bmatrix}-1\\1\\0\end{bmatrix} - \frac{0}{3}\vec{v} = \begin{bmatrix}-1\\1\\0\end{bmatrix}$; $\vec{w_3} = \vec{v_3} - \frac{\vec{w_1} . \vec{v_3}}{\vec{w_1} . \vec{w_1}} \vec{w_1} - \frac{\vec{w_2} . \vec{v_3}}{\vec{w_2} . \vec{w_2}} . \vec{w_2}$

$= \begin{bmatrix}1\\2\\1\end{bmatrix} - \frac{4}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} - \frac{1}{2}\begin{bmatrix}-1\\1\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{6}\\\frac{1}{6}\\-\frac{1}{3}\end{bmatrix}$

The ortogonal basis $\rightarrow \left\{ \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}, \begin{bmatrix}1/6\\1/6\\-1/3\end{bmatrix} \right\}$

Normalizing and we find ortonormal basis $\rightarrow \left\{ \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\\0\end{bmatrix}, \sqrt{6}\begin{bmatrix}1/6\\1/6\\-1/3\end{bmatrix} \right\}$

Soal 1

$$Ax = b \rightarrow \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} x = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

a. Is there a solution?

* If we eliminate A and B. we get

$$Ax = b$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$$

Since the ~~row~~ third row of A equal to 0, the third row of vector B should be 0. In that particular condition, the equation didn't have a solution.

b. Find $\hat{b} \in C(A)$, that minimize $\|b - \hat{b}\|$.

* $\hat{b} = A\hat{x} = A(A^TA)^{-1}A^Tb$

$$A^TA = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 8 & 14 \end{pmatrix}$$

$$(A^TA)^{-1} = \frac{1}{20} \begin{pmatrix} 14 & -8 \\ -8 & 6 \end{pmatrix}$$

$$\hat{b} = A(A^TA)^{-1}A^Tb = A\,\frac{1}{20} \begin{pmatrix} 14 & -8 \\ -8 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0{,}8 \\ 1{,}6 \\ 4{,}0 \end{pmatrix}$$

c. $\varepsilon = b - \hat{b}$. Explain the relation beetween ε and $C(A)$.

$$\varepsilon = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 0{,}8 \\ 1{,}6 \\ 4{,}0 \end{pmatrix} = \begin{pmatrix} 1{,}2 \\ -0{,}6 \\ 0 \end{pmatrix}$$

Is the left null space matrix A. $\varepsilon = (N(A^T))$. $\varepsilon \perp C(A)$