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HW 01 TVM

Problem 1 : Vector space and subspace

Proof whether the following sets of form subspace

a). The set of vectors in the following form $\{(x_1, x_2, x_3)\} \in \mathbb{R}^3 : x_1 = 2x_3$

Let $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 = 2x_3\}$. We need to show that

1). Let $x_1 = 0$, $x_2 = 0$, $x_3 = 0$. So $(0,0,0) \in W$. So, vector 0 is in the subset.

2). Let $w_1 = (x_{11}, x_{21}, x_{31})$ and $w_2 = (x_{12}, x_{22}, x_{32})$.
If we add w_1 and w_2. We get $w_1 + w_2 = (x_{11} + x_{12}, x_{21} + x_{22}, x_{31} + x_{32})$. We know that $x_1 = 2x_3$ and we can let $x_3 = x_{31} + x_{32}$, $x_2 = x_{21} + x_{22}$, and $x_1 = x_{11} + x_{12}$. So we get $w_1 + w_2 = (x_1, x_2, x_3)$ where $x_1 = 2x_3$, $w_1 + w_2 = (2x_3, x_2, x_3)$. So, $w_1 + w_2$ is closed under addition.

3). Suppose that we have $\alpha \in \mathbb{R}$ and let $w = (x_{11}, x_{21}, x_{31})$. If we multiplied α and w. we get $\alpha w = \alpha(x_{11}, x_{21}, x_{31}) = (\alpha x_{11}, \alpha x_{21}, \alpha x_{31})$.
We can let $x_1 = \alpha x_{11}$, $x_2 = \alpha x_{21}$, and $x_3 = \alpha x_{31}$. So, we get $\alpha w = (x_1, x_2, x_3)$. Since $x_1 = 2x_3$ and $\alpha w = (2x_3, x_2, x_3)$, thus condition implied that αw is closed under multiplication.

Since all of three condition satisfied, the set of vector in form of $\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 = 2x_3\}$ form subspace.

b). The set of vectors in the following form $\{(x_1, x_2, x_3)\} \in \mathbb{R}^3 : x_1 = 3x_2 + c$, $c \in \mathbb{R}$: any non zero real constant.

1). Let $x_2 = 0$, $x_1 = 0$, $x_3 = 0$. Since $x_1 = 3 \cdot x_2 + c$ and $0 \neq 3 \cdot 0 + c$, where c is constant. We can't have vector 0 on the subset.

2). Let $w_1 = (x_{11}, x_{21}, x_{31})$ and $w_2 = (x_{12}, x_{22}, x_{32})$. If we add w_1 and w_2, we get $w_1 + w_2 = (x_{11} + x_{12}, x_{21} + x_{22}, x_{31} + x_{32})$. Since $x_1 = 3x_2 + c$ and $x_{11} + x_{12} = 3(x_{21} + x_{22}) + 2c$, we know that those equation does not meet the given condition $(x_1 = 3x_2 + c)$. Thus, $w_1 + w_2$ is not closed under addition.

Since two of three condition are not satisfied, the set of vector does not form subspace.

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c). The set of all linear combination of rows of A.
Let A be the matrix $n \times m$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

We can denote the rows of A by $A_1, A_2, \ldots A_n$. Also, Let $\alpha_1, \alpha_2, \ldots \alpha_n$ to be the coeficient of linear combination.

1). Let every rows of A equal to $(\underbrace{0,0,\ldots 0}_{m})$. So, the linear combination will be $\alpha_1(\underbrace{0,0,\ldots 0}_{m}) + \alpha_2(\underbrace{0,0,\ldots 0}_{m}) + \ldots + \alpha_n(\underbrace{0,0,\ldots 0}_{n}) = (\underbrace{0,0,\ldots 0}_{m})$.

So, Vector 0 is in the subset.

2). Let v, w be the vector in A. Since A is the set of linear combination, Vector v and w are linear combination of $A_1, A_2, \ldots A_n$, and exist scalar $\alpha_1, \alpha_2, \ldots \alpha_n \in \mathbb{R}$ and $\beta_1, \beta_2, \ldots \beta_n \in \mathbb{R}$. We can write.

$$v = \alpha_1 A_1 + \alpha_2 A_2 + \cdots + \alpha_n A_n = \sum_{i=1}^{n} \alpha_i A_i$$

and

$$w = \beta_1 A_1 + \beta_2 A_2 + \cdots + \beta_n A_n = \sum_{i=1}^{n} \beta_i A_i$$

Then

$$v + w = \sum_{i=1}^{n} \alpha_i A_i + \sum_{i=1}^{n} \beta_i A_i = \sum_{i=1}^{n} (\alpha_i A_i + \beta_i A_i) = \sum_{i=1}^{n} (\alpha_i + \beta_i) A_i$$

So, the vector v+w is also linear combination of Vector $A_1, A_2, \ldots A_n$, So, v+w is closed under addition.

3). Let v be the vector in A. and $c \in \mathbb{R}$. Since A is the set of linear combination and v is linear combination of $A_1, A_2, \ldots A_n$. Then

$$v = \sum_{i=1}^{n} \alpha_i A_i$$

We multiplie v and c,

$$cv = c \sum_{i=1}^{n} \alpha_i A_i$$

$$= \sum_{i=1}^{n} c(\alpha_i A_i) = \sum_{i=1}^{n} (c\alpha_i) A_i$$

So, vector cv is also linear combination of $A_1, A_2, \ldots A_n$. So, cv is closed under multiplication.

$\therefore$ Since all of three conditions satisfied. The set form subspace. $\boxed{}$

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d) The set of ~~solution~~ All solutions to $Ax = b$

1). Let $x = 0$. We get $A(0) = b$. Since b can be equal to 0 but not for All solution. We can assume that $b \neq 0$. So, the condition for $x = 0$ isn't satisfied.

2). Let x_1 and x_2 be the solutions. We get

$$A(x_1 + x_2) = Ax_1 + Ax_2 = b + b = 2b.$$

Since $2b \neq b$, the condition isn't satisfied. So $x_1 + x_2$ is not closed under addition.

3). Let x be a solution and c a constant where $c \in \mathbb{R}$. We get $A(cx) = cAx = cb$. Since $cb \neq b$, the condition isn't satisfied and is not closed under multiplication.

Since all of three condition is not satisfied. The set of all solutions to $Ax = b$ does not form ~~a~~ subspace.

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e). The set of differentiable function

Let $W = \{f : f'(x) \text{ is exist}\}$. ~~Let f and g~~

1). Let f and $g \in W$. Therefore:

$$\frac{d}{dx}(f + g) = \frac{df}{dx} + \frac{dg}{dx}$$

Which meant that there condition ~~there~~ is closed under addition.

2). Let $f \in W$ and $c \in \mathbb{R}$ is a constant. Therfore:

$$\frac{d(cf)}{dx} = c \cdot \frac{df}{dx}$$

which means that there condition is closed under multiplication.

Since both condition satisfied. The set of differentiable functions form subspace.

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Problem 2: Null space and complete solutions to $Ax = b$.

Consider the set of linear equations $Ax = b$ with:

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

a. Find the $N(A)$:

We can convert to augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 2 & 0 \end{array}\right]$$

Row-reduce to RREF

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 2 & 0 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & -3 & -2 & -2 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 * -1]{R_2 * -1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 3 & 2 & 2 & 0 \end{array}\right] \xrightarrow[R_3 - 3R_2]{R_1 - 2R_2} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 * 1/2} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 0 \end{array}\right] \Rightarrow \begin{matrix} x_1 + x_3 = 0 \\ x_2 + x_4 = 0 \\ x_3 - 1/2\, x_4 = 0 \end{matrix}$$

$$\Rightarrow \begin{matrix} x_1 = -x_3 \\ x_2 = -x_4 \\ x_3 = 1/2\, x_4 \end{matrix} \quad \Rightarrow \begin{matrix} x_1 = -1/2\, x_4 \\ x_2 = -x_4 \\ x_3 = 1/2\, x_4 \end{matrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1/2\, x_4 \\ -x_4 \\ 1/2\, x_4 \\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} -1/2 \\ -1 \\ 1/2 \\ 1 \end{bmatrix}}_{\text{Basis}} x_4$$

$$\Rightarrow x_n = \begin{bmatrix} -1/2 \\ -1 \\ 1/2 \\ 1 \end{bmatrix} x_4$$

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b. Find the particular solution to $Ax = b$.
We can convert to augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 2 & 3 \end{array}\right]$$

Row Reduce to RREF

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 2 & 3 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & -3 & -2 & -2 & -3 \end{array}\right]$$

$$\xrightarrow[R_3 \times -1]{R_2 \times -1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 3 & 2 & 2 & 3 \end{array}\right] \xrightarrow[R_3 - 3R_2]{R_1 - 2R_2} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \times 1/2} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1/2 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 + x_3 = 1 \\ x_2 + x_4 = 1 \\ x_3 - 1/2\, x_4 = 0 \end{array}$$

$$\Rightarrow \begin{array}{l} x_1 = 1 - x_3 = 1 - 1/2\, x_4 \\ x_2 = 1 - x_4 \\ x_3 = 1/2\, x_4 \end{array}$$

Let x_4 (free variable) equal to 0

$$x_p = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 - 1/2\, x_4 \\ 1 - x_4 \\ 1/2\, x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

c. Find the complete solution using a) and b).

$$x = x_p + x_n = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -1/2 \\ -1 \\ 1/2 \\ 1 \end{bmatrix} \quad \text{where } c \in \mathbb{R}.$$

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