5. Least Square and G-S. Process

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Sleman, 21 Desember 2021

Muchammad Daniyal Kautsar.

Initializing variable

NIU = 479067

NIF = 52800

$\beta_i = 1 + (NIU_{i+1} + NIF_i) \bmod 6$

$\beta_1 = 1 + (7+5) \bmod 6 = 1+2 = 3$

$\beta_2 = 1 + (9+2) \bmod 6 = 1+5 = 6$

$\beta_3 = 1 + (0+8) \bmod 6 = 1+2 = 3$

$\beta_4 = 1 + (6+0) \bmod 6 = 1+0 = 1$

$\beta_5 = 1 + (7+0) \bmod 6 = 1+1 = 2$

1. vector space and subspace.

a) Show that $N(A)$ is a subspace.

→ $N(A)$ is a subspace if $N(A)$ must satisfied the scalar multiplication and vector addition.

① vector addition.

Let v and w be the vector in $N(A)$.

Therefore,

$$Av = 0$$
$$Aw = 0$$

Thus,

$$A(v+w) = Av + Aw = 0+0 = 0$$

So, $N(A)$ is closed under addition.

②. scalar multiplication

Let v be the vector in $N(A)$ and $a \in \mathbb{R}$

therefore, $Av = 0$

Thus,

$$A(av) = aA(v)$$
$$= a \cdot 0 = 0$$

So, $N(A)$ is closed under scalar multiplication.

Since $N(A)$ satisfied both condition. therefore $N(A)$ is a subspace.

b). Determine α such that the set of $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^n, x_3 = x_1 + \alpha \right\}$ forms a subspace

* ① Vector addition

Let v and w be the sub set. where $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ and $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$.

Thus $v + w = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_3 + w_3 \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_1 + \alpha + w_1 + \alpha \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_1 + w_1 + 2\alpha \end{bmatrix}$.

Therefore, In order to be closed under addition, the α should be equal to 0, such that their addition will be meet the set criteria of $x_3 = x_1 + \alpha$.

③

② Scalar Multiplication

Let v be the subset and $c \in \mathbb{R}$ where $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

Thus, $cv = c\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} cv_1 \\ cv_2 \\ cv_3 \end{bmatrix} = \begin{bmatrix} cv_1 \\ cv_2 \\ cv_3 + c\alpha \end{bmatrix}$.

Therefore, to be closed under multiplication, the α should equal to 0. Since both condition need to have α with equal to 0. Therefore, $\underline{\alpha = 0.}$

c. Determine the set of polynomial

2. The complete solution to $Ax = b$

$$A = [V_1, V_2, V_3, V_4, V_5]$$

where

$$V_1 = \begin{bmatrix} 1\beta_5 \\ 3\beta_5 \\ 4\beta_5 \end{bmatrix} = \begin{bmatrix} 1\cdot 2 \\ 3\cdot 2 \\ 4\cdot 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 2\beta_4 \\ 6\beta_4 \\ 8\beta_4 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 \\ 6\cdot 1 \\ 8\cdot 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 3\beta_3 \\ 9\beta_3 \\ 12\beta_3 \end{bmatrix} = \begin{bmatrix} 3\cdot 3 \\ 9\cdot 3 \\ 12\cdot 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 27 \\ 36 \end{bmatrix}, \quad V_4 = \begin{bmatrix} 1\beta_2 \\ 2\beta_2 \\ 3\beta_2 \end{bmatrix} = \begin{bmatrix} 1\cdot 6 \\ 2\cdot 6 \\ 3\cdot 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$$

$$V_5 = \begin{bmatrix} 2\beta_1 \\ 5\beta_1 \\ 7\beta_1 \end{bmatrix} = \begin{bmatrix} 2\cdot 3 \\ 5\cdot 3 \\ 7\cdot 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 15 \\ 21 \end{bmatrix}$$

a. Determine the RREF of Matrix A.

$$R = rref(A)$$

* $$A = \begin{bmatrix} 2 & 2 & 9 & 6 & 6 \\ 6 & 6 & 27 & 12 & 15 \\ 8 & 8 & 36 & 18 & 21 \end{bmatrix} \sim \xrightarrow[R_3 - 4R_1]{R_2 - 3R_1} \begin{bmatrix} 2 & 2 & 9 & 6 & 6 \\ 0 & 0 & 0 & -6 & -3 \\ 0 & 0 & 0 & -6 & -3 \end{bmatrix} \xrightarrow[R_3 - R_2]{} \begin{bmatrix} 2 & 2 & 9 & 6 & 6 \\ 0 & 0 & 0 & -6 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 + R_2} \begin{bmatrix} 2 & 2 & 9 & 0 & 3 \\ 0 & 0 & 0 & -6 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_2 \times \frac{-1}{6}]{R_1 \times \frac{1}{2}} \begin{bmatrix} 1 & 1 & 9/2 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

* $R = rref(A)$

$$= \begin{bmatrix} 1 & 1 & 9/2 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

* Elimination matrix

$$E = E_{22} \cdot E_{11} \cdot E_{32} \cdot E_{12} \cdot E_{31} \cdot E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -9 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} -1 & 1/2 & 0 \\ 1/2 & -1/6 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

b. Determine the basis of C(A) and dimension of C(A)?

* Basis of C(A) is the pivot column from matrix A on the column 1, and 4

Basis of C(A) = $\left\{ \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}, \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix} \right\}$.

* Dimension of C(A) is the ~~number~~ sum of number basis in C(A).

Dimension of C(A) = 2. (since there are two basis in C(A))

c. Determine the N(A) and its basis and dimension.

*. From the rref (A) or R. we know that

$$\text{rref}(A) = R = \begin{bmatrix} 1 & 1 & 9/2 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

* $$\begin{bmatrix} 1 & 1 & 9/2 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

Therefore, we get.

$$x_1 + x_2 + 9/2\, x_3 + 3/2\, x_5 = 0 \qquad \rightarrow \quad x_1 = -x_2 - \frac{9}{2} x_3 - \frac{3}{2} x_5$$

$$x_4 + 1/2\, x_5 = 0 \qquad\qquad x_4 = -\frac{1}{2} x_5$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -x_2 - \frac{9}{2} x_3 - \frac{3}{2} x_5 \\ x_2 \\ x_3 \\ -\frac{1}{2} x_5 \\ x_5 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{9}{2} x_3 \\ 0 \\ x_3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} x_5 \\ 0 \\ 0 \\ -\frac{1}{2} x_5 \\ x_5 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -9/2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3/2 \\ 0 \\ 0 \\ -1/2 \\ 1 \end{bmatrix}$$

* So, $N(A) = a \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} -9/2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -3/2 \\ 0 \\ 0 \\ -1/2 \\ 1 \end{bmatrix}$, for $a, b, c \in \mathbb{R}$.

f. Which b_i leads to solvable $Ax = b_i$? why?

$$b_1 = \begin{bmatrix} 1\beta_1 \\ 4\beta_1 \\ 5\beta_1 \end{bmatrix} = \begin{bmatrix} 1\cdot 6 \\ 4\cdot 6 \\ 5\cdot 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \\ 30 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2\beta_1 \\ 4\beta_1 \\ 5\beta_1 \end{bmatrix} = \begin{bmatrix} 2\cdot 3 \\ 4\cdot 3 \\ 5\cdot 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 15 \end{bmatrix}$$

* to check the b_i, we can use the elimination from problem 2.a.

$$b_1 = \begin{bmatrix} 6 \\ 24 \\ 30 \end{bmatrix} \sim \xrightarrow[R_3-4R_1]{R_2-3R_1} \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} \xrightarrow[R_1+R_2]{R_3-R_2} \begin{bmatrix} 12 \\ 6 \\ 0 \end{bmatrix} \xrightarrow[R_2\times\frac{-1}{6}]{R_1\times 1/2} \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 6 \\ 12 \\ 15 \end{bmatrix} \sim \xrightarrow[R_3-4R_1]{R_2-3R_1} \begin{bmatrix} 6 \\ -6 \\ -9 \end{bmatrix} \xrightarrow[R_1+R_2]{R_3-R_1} \begin{bmatrix} 0 \\ -6 \\ -3 \end{bmatrix} \xrightarrow[R_2\times -1/6]{R_1\times 1/2} \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

From that, we know that the soluable one is b_1 ~~bec~~

* Why? because b_1 meet the ~~rref(A)~~ condition where the third row is all zero entries.
rref(A)

g. Determine the complete solution $x = x_p + x_n$.

$$Ax = b_1$$

*.

$$\begin{bmatrix} 2 & 2 & 9 & 6 & 6 \\ 6 & 6 & 27 & 12 & 15 \\ 8 & 8 & 36 & 18 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \\ 30 \end{bmatrix}.$$

Since, I already reduce it to rref in problem 2.a. and problem 2.f. we get

*

$$\begin{bmatrix} 1 & 1 & 9/2 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$$

Therefore, we get the $x_p = \begin{bmatrix} 6 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

* So, the complete solution $x = x_p + x_n$ will be

$$x = \begin{bmatrix} 6 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + a\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} -9/2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} -3/2 \\ 0 \\ 0 \\ -1/2 \\ 1 \end{bmatrix}, \text{ for } a,b,c \in \mathbb{R}$$

3. Least square and G-S. Process

a. show in $Y = Ax$ $y(t) = av^2(t) + by(t-1) + cr(t-1)$

$$Y = \begin{bmatrix} Y(2) \\ Y(1.5) \\ Y(1.0) \\ Y(0,0) \end{bmatrix} \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

* A will be

$$A = [v, y, r]$$

$$A = \begin{bmatrix} V(2.0)^2 & y(1.0) & r(1,0) \\ V(1,5)^2 & y(0.5) & r(0.5) \\ V(1,0)^2 & y(0.0) & r(0,0) \\ V(0,0)^2 & y(0.0) & r(0.0) \end{bmatrix}$$

So, the $Y = Ax$ will be

$$Y = Ax$$

$$\begin{bmatrix} y(2.0) \\ y(1.5) \\ y(1.0) \\ y(0.0) \end{bmatrix} = \begin{bmatrix} V(2.0)^2 & y(1,0) & r(1,0) \\ V(1,5)^2 & y(0,5) & r(0,5) \\ V(1,0)^2 & y(0,0) & r(0,0) \\ V(0,0)^2 & y(0,0) & r(0,0) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

b. Determine A using numerical data and calculate least square.