UNIVERSITAS GADJAH MADA

FAKULTAS TEKNIK

Departemen Teknik Elektro dan Teknologi Informasi

Program Studi S1 Teknik Elektro

Soal Ujian Akhir Semester Genap 2015/2016

Aljabar Linear

Bersifat Terbuka terbatas (contekan HVS A4 1 lembar bolak balik), boleh menggunakan kalkulator, untuk dikerjakan selama 120 Menit

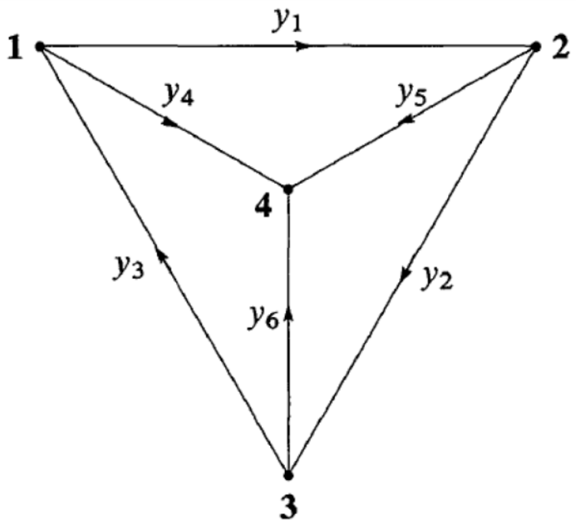
1. Which of the followingsubset of $\mathbb{R}^3$ are subspaces, state your reason!
   a. The plane of vectors (b1, b2, b3) with b1=b2
   b. The plane of vectors with b1=1
   c. The vectors with b1b2b3=0
   d. All linear combinations of v=(1,4,0) and w=(1,2,1)
   e. All vectors that satisfy b1+b2+b3=0
   f. All vectors with $b1 \leq b2 \leq b3$

2. For the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

   a. By inspection observe how many linear independent column vectors in A! what do you expect from your inspection?
   b. Reduce into echelon form R!
   c. What is the column space and the null space of A?
   d. Also what is the row space and the left null space of A? (see bonus contekan!)
   e. If Ax=b, what condition for b is solvable for A?
   f. For b=(16,19,1), find all solution for Ax=b!

3. (A bit challenge question) Kirchoff Law says that "the current in=the current out" at every node. For this network
   a. Obtain the equations for the four nodes and solve for the complete solutions.
   b. If additionally we know y1+y2+y3=0, 2y1=3y2, 4y1=4 find all solutions of the other ys!



Bonus contekan:
To find the left null space without explicitly compute from A^T, we can use the same elimination matrix E from [A I] → [R E], as E [A I]=[R E], the row space should be in the row of E that make row R=0.
Example [A I]=$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 2 & 4 & 6 & 0 & 1 \end{bmatrix} \dashrightarrow \begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 0 & 0 & 0 & 2 & -1 \end{bmatrix}$, then E=$\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$,
The row space is the second row of E=[2 -1], as [2 -1]A=0. The row space can be found using the same idea

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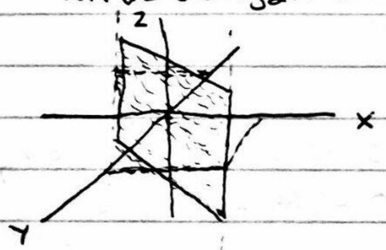
Kunci Aljabar Linear

1) Which are subspaces?

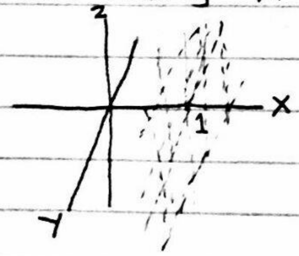
a) plane of vectors (b_1, b_2, b_3) with $b_1 = b_2$ ✓
b) plane of vectors (b_1, b_2, b_3) with $b_1 = 1$ ✗
c) the vectors with $b_1 b_2 b_3 = 0$ ✗
d) all linear combination of $v = (1,4,0)$ and $w = (1,2,1)$ ✓
e) all vectors that satisfy $b_1 + b_2 + b_3 = 0$ ✓
f) all vectors with $b_1 \leq b_2 \leq b_3$ ✗

Reason

a) set of vector (c_1, c_1, c_2). Titik2 vektor membentuk plane pada R^3 dengan: (1) memotong $(0,0,0)$, (2) pada bidang X, Y membuat diagonal dengan persamaan $Y = X$, (3) tegak lurus dengan bidang XY.



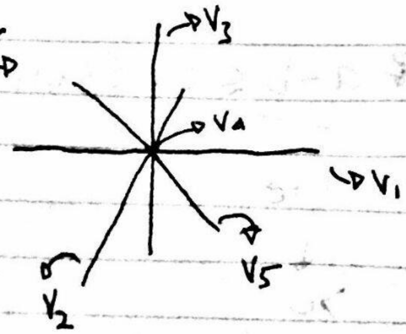
b) set of vector $(1, c_1, c_2)$. Plane parallel sumbu Y dan sumbu Z, memotong $X = 1$.



not satisf <u>origin</u>

c) Set of vector (c_1, c_2, c_3), with $c_1 \times c_2 \times c_3 = 0$. There will be

$$c_1 \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + c_5 \begin{bmatrix} p \\ g \\ 0 \end{bmatrix} + c_6 \begin{bmatrix} r \\ 0 \\ s \end{bmatrix} + c_7 \begin{bmatrix} 0 \\ t \\ u \end{bmatrix}$$



dll. Linear kombinasi V_1, V_2, V_3 menghasilkan

$c \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, not satisfy $c_1 \times c_2 \times c_3 = 0$ (keluar dari set).

independen

d) linear kombinasi 2 vektor akan membuat plane yang melewati (0,0,0). Karena linear kombinasi, hasil linear kombinasi titik² pada plane tersebut pasti tetap di dalam plane tersebut

e) $V = c_1 \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\quad W = c_2 \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

with $a+b+c=0$
$d+e+f=0$.

assume $X = V + W = [a+d \quad b+e \quad c+f]^T$
$Y = V - W = [a-d \quad b-e \quad c-f]^T$

$X \Rightarrow (a+d)+(b+e)+(c+f) = a+b+c+d+e+f = 0+0 = 0$ satisfy
$Y \Rightarrow (a-d)+(b-e)+(c-f) = a+b+c-d-e-f = 0-(d+e+f) = 0-0 = 0$

Syarat II terpenuhi
Syarat I : $V_1 = [a \; b \; c]^T$, bisa $0+0+0=0$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

f) $V = c_1 \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $W = c_2 \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

for $a=b=c=0 \Rightarrow 0 \leq 0 \leq 0$ terpenuhi syarat I.

assume : $Y = V - W = \begin{bmatrix} a-d \\ b-e \\ c-f \end{bmatrix}$

since $b \geq a$, there is possibility $b > a$, $a-b \leq 0$. then:

$Y \Rightarrow (a-d)-(b-e) = a-b-d+e = \underbrace{(a-b)}_{<0} + \underbrace{(-d+e)}_{>0}$.

So, $a-b+(-d+e)$ bisa lebih besar dari nol, keluar dari set. Not satisfy syarat II

$\boxed{2}$

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

a. By inspection,
Jumlah column vector yang linear independent adalah 2

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

→ Tidak L.I., karena kelipatan 3 dari kolom 3

→ Tidak L.I., karena kelipatan 2 dari kolom 3

→ pivot column

→ Tidak independent, karena merupakan kelipatan dari kolom 1

Pasti pivot colom

Sehingga hanya terdapat 2 pivot column atau kolom vector yang L.I. adalah 2. (Rank(A) = 2).

b. $$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \xrightarrow{E_{21}} A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\downarrow E_{32}$$

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{rref} A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Bentuk echelon}$$

c. Column Space → linear kombinasi dari kolom yang independent
Sehingga,

$$C(A) = \alpha_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Nullspace → $Ax = 0$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} x = 0 \xrightarrow{P_{23}} \underbrace{\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{P_{23}} x = 0$$

$$\begin{array}{c} \overbrace{\quad I \quad}^{} \quad F \\ \left[\begin{array}{cc|ccc} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right] x = 0 \end{array}$$

$$\left[\begin{array}{cc|ccc} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right] x = 0$$

$$\hat{N}(A) = \begin{bmatrix} -F \\ I \end{bmatrix}$$

$$= \left[\begin{array}{ccc} -2 & 0 & 0 \\ 0 & -2 & -3 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

- Karena A dikalikan dengan P_{23}, maka $\hat{N}(A)$ juga demikian

$$\boxed{A P_{23} x = 0} \rightarrow \boxed{P_{23} \hat{N}(A)}$$

$$N(A) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{P_{23}} \underbrace{\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & -3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\hat{N}(A)}$$

$$N(A) = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -2 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{matriks nullspace}$$

$$x_N = \alpha_1 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

nullspace of A atau nullspace solution

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d. Row space → linear kombinasi dari baris yang independent

$$R(A) = d_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + d_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}^T$$

Left Nullspace → $y^T A = 0$

$A^T y = 0$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 2 & 3 & 1 \\ 4 & 6 & 2 \\ 6 & 9 & 3 \end{bmatrix} y = 0 \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{bmatrix} y = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} y = 0 \xleftarrow{\text{rref}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} y = 0$$

↓ menukar baris 2 ke 3 dengan P_{23}

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{P_{23}} \cdot \underbrace{\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]}_{A^T} y = 0$$

$$N(A) = \begin{bmatrix} -F \\ I \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$ → matriks left nullspace

$$y_N = d_1 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$ → left nullspace

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e. $Ax=b$ solvable if b merupakan dan atau berada di column space A

f. $Ax=b \rightarrow b=(16,19,1)$

$$\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} x = \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} \rightarrow \left[\begin{array}{ccccc|c} 1 & 2 & 2 & 4 & 6 & 16 \\ 1 & 2 & 3 & 6 & 9 & 19 \\ 0 & 0 & 1 & 2 & 3 & 1 \end{array}\right]$$

$$\xrightarrow{E_{21}} \begin{bmatrix} 1 & 2 & 2 & 4 & 6 & 16 \\ 0 & 0 & 1 & 2 & 3 & 3 \\ 0 & 0 & 1 & 2 & 3 & 1 \end{bmatrix} \xrightarrow{E_{32}} \left[\begin{array}{ccccc|c} 1 & 2 & 2 & 4 & 6 & 16 \\ 0 & 0 & 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

- X_p = no solution, karena $\boxed{0x=-2}$
- $X_N \rightarrow Ax=0$

Sama seperti poin c

$$X_N = d_1 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + d_2 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + d_3 \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

All solution $\rightarrow X_c$

$X_c = X_p + N_N$

$$X_c = d_1 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + d_2 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + d_3 \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

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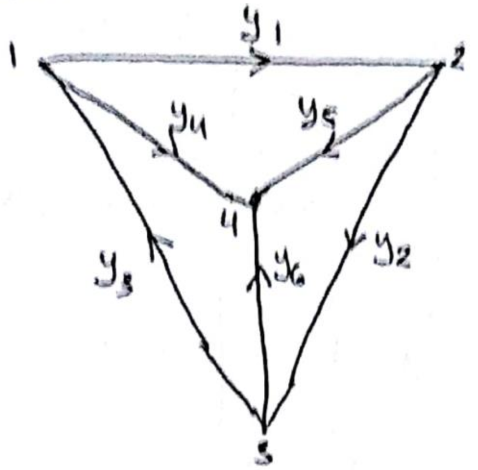
3. A bit Challenge Question

→ Using KCL



a. Obtain the equations for the four nodes and solve for the complete solutions

- node 1 → $y_3 = y_1 + y_4$

$y_1 - y_3 + y_4 = 0 \quad \ldots 1)$

- node 2 → $y_1 = y_2 + y_5$

$y_1 - y_2 - y_5 = 0 \quad \ldots 2)$

- node 3 → $y_2 = y_3 + y_6$

$y_2 - y_3 - y_6 = 0 \quad \ldots 3)$

- node 4 → $y_4 + y_5 + y_6 = 0 \quad \ldots 4)$

Dari keempat persamaan diatas, dapat dibuat menjadi matriks $Ay = 0$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = 0$$

$4 \times 6 \qquad 6 \times 1$

$\downarrow E_{21} = E_2 - E_1$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} y = 0 \xrightarrow[E_{32} = E_3 + E_2]{} \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$\downarrow E_{43} = E_4 + E_3$

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$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} y = 0$$

$\downarrow$ Diubah menjadi bentuk echelon form

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} y = 0$$

$$\begin{array}{cccccc} & & \downarrow & & \downarrow & \downarrow \\ & & y_3 & & y_5 & y_6 \end{array}$$

free column

- $y_p \rightarrow$ particular solution
  Set $y_3 = y_5 = y_6 = 0$, cek masing-masing persamaan dari persamaan 3 hingga 1
  - Pers 3
    $y_4 + 0 \cdot y_5 + 0 \cdot y_6 = 0$
    $y_4 = 0$
  - Pers 2
    $y_2 + (-1) y_3 + y_4 + y_5 + 0 \cdot y_6 = 0$
    $y_2 = 0$
  - Pers 1
    $y_1 + 0 + -1\, y_3 + y_4 = 0$
    $y_1 = 0$

  $y_p = 0$

- $y_N \rightarrow$ Nullspace solution
  Set $y_3 = 1$, $y_5 = y_6 = 0$ && $y_3 = y_6 = 0$, $y_5 = 1$ && $y_3 = y_5 = 0$, $y_6 = 1$
  Cek masing-masing persamaan dari pers 3 hingga 1

$$d_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + d_2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + d_3 \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

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- Complete Solution

$$y_c = y_p + y_N$$

$$= \alpha_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

b. If additionally we know

$$y_1 + y_2 + y_3 = 0 \quad \ldots 5)$$

$$2y_1 = 3y_2$$

$$2y_1 - 3y_2 = 0 \quad \ldots 6)$$

$$4y_1 = 4 \quad \ldots 7)$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & -3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$$\Big\downarrow E_{51}\ E_{61}\ E_{71}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -3 & 2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 4 & -4 & 0 & 0 & 4 \end{array}\right]$$

$$\Big\downarrow E_{52}\ E_{62}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 3 & 0 & 0 \\ 0 & 0 & 4 & -4 & 0 & 0 & 4 \end{array}\right] \Big\downarrow E_{65}\ E_{75}$$

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$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2/3 & 11/3 & 0 & 0 \\ 0 & 0 & 0 & -4/3 & 4/3 & 0 & 4 \end{array}\right]$$

$$\downarrow E_{63}, E_{73}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 8/3 & 4/3 & 4 \end{array}\right]$$

$$\downarrow E_{76}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 124/117 & 4 \end{array}\right]$$

dibuat echelon form

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{117}{31} \end{array}\right]$$

→ Karena tidak ada free column, maka hanya menggunakan back substitution

$$y_6 = \frac{117}{31} \quad \ldots 6)$$

$$y_5 + \frac{2}{3} y_6 = 0$$
$$y_5 = -\frac{78}{31} \quad \ldots 5)$$

$$y_4 + y_5 + y_6 = 0$$
$$y_4 = \frac{-117}{31} + \frac{78}{31}$$
$$= \frac{-39}{31} \quad \ldots 4)$$

$$y_3 - \frac{2}{3} y_4 - \frac{1}{3} y_5 = 0$$
$$y_3 = \frac{2}{3}\left(-\frac{39}{31}\right) + \frac{1}{3}\left(-\frac{78}{31}\right)$$
$$= -\frac{52}{31} \quad \ldots 3)$$

$$y_2 - y_3 + y_4 + y_5 = 0$$
$$y_2 = -\frac{52}{31} + \frac{39}{31} + \frac{78}{31}$$
$$= \frac{65}{31} \quad \ldots 2)$$

$$y_1 - y_3 + y_4 = 0$$
$$y_1 = -\frac{52}{31} + \frac{39}{31}$$
$$= -\frac{13}{31} \quad \ldots 1)$$

$$y = \begin{bmatrix} -13/31 \\ 65/31 \\ -52/31 \\ -39/31 \\ -78/31 \\ 117/31 \end{bmatrix}$$

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