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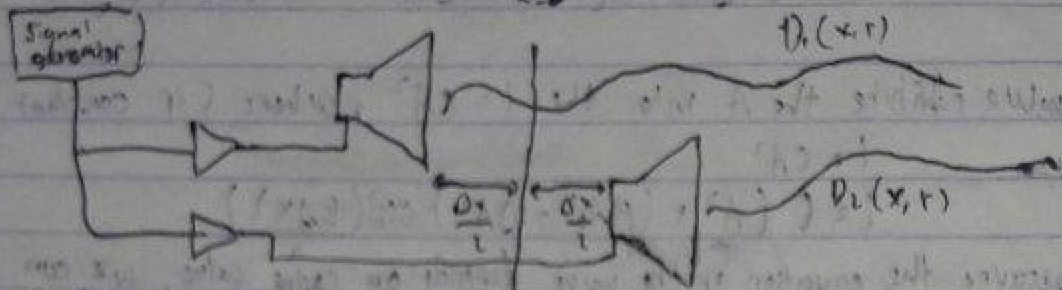
HW 01 Waves and acoustics

Problem 1



$$D_1(x,t) = A_1 \sin(k(x - \Delta x/2) - \omega t + \theta_1)$$
$$D_2(x,t) = A_2 \sin(k(x + \Delta x/2) - \omega t + \theta_2)$$

A. Suppose that $A_1 = A_0 - \Delta A/2$ and $A_2 = A_0 + \Delta A/2$. Determine Δx min and Δx max. Assume that $\theta_1 = \theta_2 = 0$!

Let $A_1 = A_0 - \Delta A/2$
$A_2 = A_0 + \Delta A/2$
$\theta_1 = \theta_2 = 0$

Adding two wave equation give

$$D(x,t) = D_1(x,t) + D_2(x,t)$$
$$= A_1 \sin(k(x - \Delta x/2) - \omega t + \theta_1) + A_2 \sin(k(x + \Delta x/2) - \omega t + \theta_2)$$
$$= (A_0 - \Delta A/2) \sin\left(kx - \frac{k\Delta x}{2} - \omega t\right) + (A_0 + \Delta A/2) \sin\left(kx + \frac{k\Delta x}{2} - \omega t\right)$$

Let $\frac{k\Delta x}{2} = m$ and $kx - \omega t = n$.

$$D(x,t) = (A_0 - \Delta A/2) \sin(n-m) + (A_0 + \Delta A/2) \sin(n+m)$$
$$= A_0(\sin(n-m) + \sin(n+m)) + (\Delta A/2(-\sin(n-m) + \sin(n+m)))$$
$$\cancel{= 2A_0 \sin}$$

we use trigonometri identity $\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$.
so, we get

$$D(x,t) = 2A_0(\sin(n) \cos(m)) + \Delta A(\cos(n) \sin(m))$$

we can let $2A_0 \cos(m) = A \cos\theta$
and $\Delta A \sin(m) = A \sin\theta$

now, we can calculate ~~the~~ combined A using $A = \sqrt{A^2\cos^2\theta + A^2\sin^2\theta}$

$$A = \sqrt{A^2\cos^2\theta + A^2\sin^2\theta}$$
$$A = \sqrt{4A_0^2\cos^2(m) + \Delta A^2\sin^2(m)}$$

Using trigonometry identity $\sin^2\theta + \cos^2\theta = 1$.

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we get $A = \sqrt{4A_0^2 \cos^2(m) + \Delta A^2 (1 - \cos^2(m))}$
$A = \sqrt{\Delta A^2 + (4A_0^2 - \Delta A^2)\cos^2(m)}$

we subtitute the m.
$$A = \sqrt{\Delta A^2 + (4A_0^2 - \Delta A^2)\cos^2\left(\frac{k\Delta x}{2}\right)}$$

now we subtitute the A into the $I = CA^2$, where C is constant variable.
$I = CA^2$
$I = C\left(A^2 + (4A_0^2 - \Delta A^2)\cos^2\left(\frac{k\Delta x}{2}\right)\right)$

Because the equation of the wave depends on cosine value, we can determine that when $\cos^2\left(\frac{k\Delta x}{2}\right) = 1$ is constructive interference and when $\cos^2\left(\frac{k\Delta x}{2}\right) = 0$ is destructive interference.
So,

$\cos^2\left(\frac{k\Delta x}{2}\right) = 1$
$\cos^2\left(\frac{k\Delta x}{2}\right) = \cos^2(\pi)$
$\frac{k\Delta x}{2} = \pi$
$\frac{2\pi}{2\lambda}\Delta x = \pi$
$\Delta x = \lambda$
$\Delta x_{max} = m\lambda$, where $m = \ldots, -1, 0, 1, 2, \ldots$
to calculate a period of λ distance

$\cos^2\left(\frac{k\Delta x}{2}\right) = 0$
$\cos^2\left(\frac{k\Delta x}{2}\right) = \cos^2\left(\frac{\pi}{2}\right)$
$\frac{k\Delta x}{2} = \frac{\pi}{2}$
$\frac{2\pi}{\lambda}\Delta x = \pi$
$\Delta x = \frac{\lambda}{2}$
$\Delta x_{min} = m\lambda + \frac{\lambda}{2}$
$= (m + \frac{1}{2})\lambda$

B Suppose that $\phi_1 = -\phi_0/2$ and $\phi_2 = \phi_0/2$. Assume that $A_1 = A_2 = A_0$. Determine the Δx_{min} and Δx_{max}!
* Let $\phi_1 = -\phi_0/2$
$\phi_2 = \phi_0/2$
$A_1 = A_2 = A_0$

We adding two wave equation
$D(x,t) = D_1(x,t) + D_2(x,t)$
$D(x,t) = A_1(\sin(k(x - \Delta x/2) - \omega t + \phi_1)) + A_2(\sin(k(x + \Delta x/2) - \omega t + \phi_2))$
$= A_0(\sin(kx - k\Delta x/2 - \omega t - \phi_0/2) + \sin(kx + k\Delta x/2 - \omega t + \phi_0/2))$

let $m = kx - \omega t$ and $n = k\Delta x/2 + \phi_0/2$
$D(x,t) = A_0(\sin(m-n) + \sin(m+n))$

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Using trigonometry identity, we can get

$\theta(x,t) = A_0 (\sin(m-n) + \sin(m+n))$

$= 2A_0 (\sin(m) \cdot \cos(n))$

We can consider that $A = 2A_0 \cos(n)$. So substituting A to the intensity equation, we get

$I = CA^2$

~~$I = C A A_0^2 \sin^2(m)$~~

$I = C 4 A_0^2 \cos^2(n)$.

we substitute the n

$I = C 4 A_0^2 \cos^2(n)$

$I = C 4 A_0^2 \cos^2(k\Delta x/2 + \phi_0/2)$

the interference is depend on the cosine value:

$\cos^2(k\Delta x/2 + \phi_0/2) = 1$

$\cos^2(k\Delta x/2 + \phi_0/2) = \cos^2(0)$

$k\Delta x/2 + \phi_0/2 = 0$

$k\Delta x = -\phi_0$

$\frac{2\pi}{\lambda} \Delta x = -\phi_0$

$\Delta x = \frac{-\phi_0}{2\pi} \lambda$

we can consider the constructive interference $\Delta x_{max} = m\lambda - \frac{\phi_0 \lambda}{2\pi}$

$\Delta x_{max} = \left(m - \frac{\phi_0}{2\pi}\right)\lambda$

$\cos^2(k\Delta x/2 + \phi_0/2) = 0$

$k\Delta x/2 + \phi_0/2 = \frac{\pi}{2}$

$k\Delta x = \pi - \phi_0$

$\Delta x = \frac{\pi - \phi_0}{2\pi} \lambda$

we can consider the destructive interference $\Delta x_{min} = m\lambda + \left(\frac{\pi - \phi_0}{2\pi}\right)\lambda$

$\Delta x_{min} = \left(m + \frac{1}{2} - \frac{\phi_0}{2\pi}\right)\lambda$

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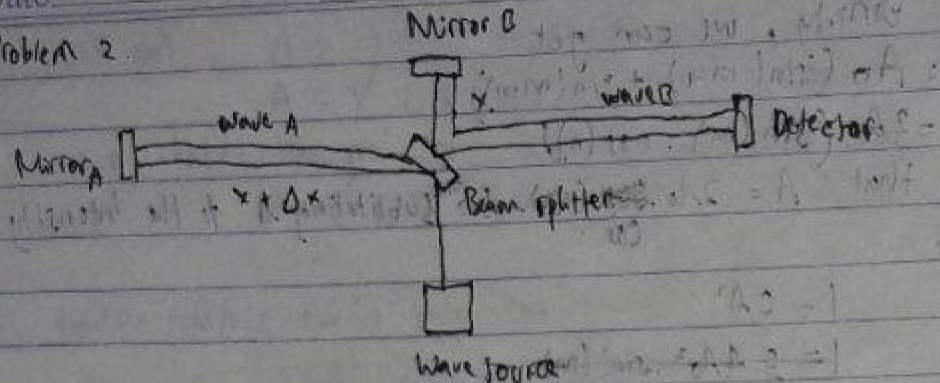
Problem 2



Base on the diagram, the wave A need to travel $2\Delta x$ extra compare to wave B. Therefore, the equation is

$$D_A(x,t) = A \sin(k(x + 2\Delta x) - \omega t)$$
$$D_B(x,t) = A \sin(kx - \omega t)$$

Adding two equation, we get

$$D(x,t) = D_A(x,t) + D_B(x,t)$$
$$= A\sin(kx + k2\Delta x - \omega t) + A\sin(kx - \omega t)$$
$$= A(\sin(kx + k2\Delta x - \omega t) + \sin(kx - \omega t))$$

Using the trigonometry identities we get

$$D(x,t) = A(\sin(kx + k2\Delta x - \omega t) + \sin(kx - \omega t))$$
$$= A(2 \sin(kx + k\Delta x - \omega t) \cos(k\Delta x))$$

Where $2A\cos(k\Delta x)$ is the A por intensity equation. So, subtituting A to the intensity equation we get

$$I = CA^2$$
$$I = C\,4A^2 \cos^2(k\Delta x)$$

The interference is depend on the cosine value. if the cosine square equal to 1, we get the contructive interference, when cosine square equal to 0, we get the destructive interference.

* $\cos^2(k\Delta x) = 1$ (Constructive interference).

$$\cos^2(k\Delta x) = \cos^2 n\pi$$
$$k\Delta x = n$$
$$\Delta x = \frac{n}{k} = \frac{n}{2\pi}\lambda = \frac{1}{2}\lambda$$
$$\Delta_{max} = \frac{m}{2}\lambda$$

* $\cos^2(k\Delta x) = 0$ (destructive interference).

$$\cos^2(k\Delta x) = \cos^2\left(\frac{\pi}{2}\right)$$
$$\frac{2\pi}{\lambda} \cdot \Delta x = \frac{\pi}{2}$$
$$\Delta x = \frac{\lambda}{4} \rightarrow \Delta x_{min} = \frac{m\lambda}{2} + \frac{\lambda}{4}$$

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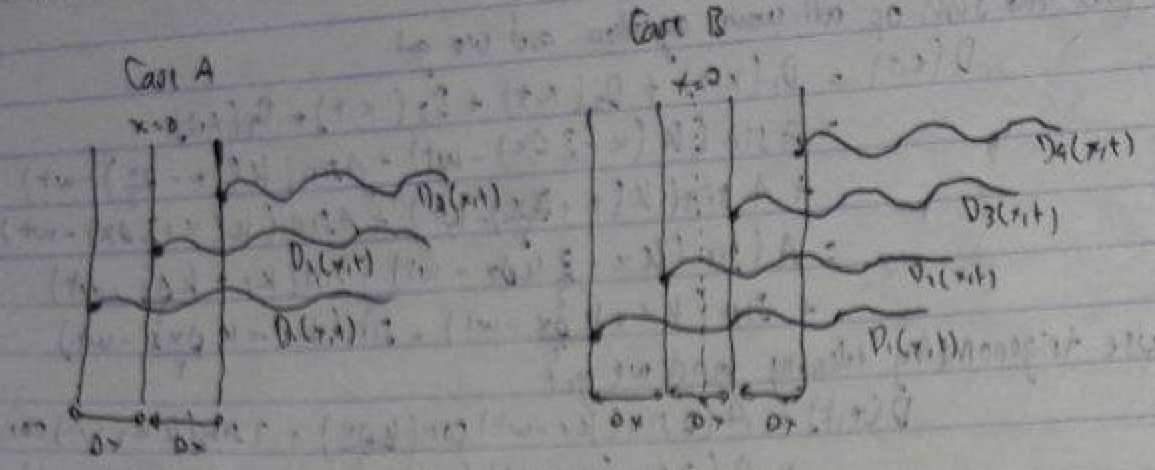
Problem 7.



Assume that all waves have the same amplitude and frequency.

A. Determine the Amplitude $(A(\Delta x))$ and intensity $(I(\Delta x) = |A(\Delta x)|^2)$ of the resulting wave.

Case A.

We find the sum of all three wave equations. We get

$$D(x,t) = D_1(x,t) + D_2(x,t) + D_3(x,t)$$
$$= A\sin(k(x-\Delta x) - \omega t) + A\sin(kx - \omega t) + A\sin(k(x+\Delta x) - \omega t)$$
$$= A(\sin(k(x-\Delta x) - \omega t) + \sin(kx - \omega t) + \sin(k(x+\Delta x) - \omega t))$$

Using trigonometry identities we get

$$D(x,t) = A(\sin(kx - k\Delta x - \omega t) + \sin(kx - \omega t) + \sin(kx + k\Delta x - \omega t))$$
$$= A(2\sin(kx - \omega t)\cos(k\Delta x) + \sin(kx - \omega t))$$
$$= A(\sin(kx - \omega t))(2\cos(k\Delta x) + 1)$$

we can simply find the $D(x,t)$:

$$D(x,t) = A(\sin(kx - \omega t))(2\cos(k\Delta x) + 1)$$
$$= A(\Delta x) \cdot \sin(kx - \omega t).$$

From that, we get the $A(\Delta x)$ that equal to

$$A(\Delta x) = A(2\cos(k\Delta x) + 1)$$

where $k = \frac{2\pi}{\lambda}$.

To find the intensity, we subtitute $A(\Delta x)$ to the $I(\Delta x) = |A(\Delta x)^2|$.

we get →
$$I(\Delta x) = |((A(2\cos(k\Delta x) + 1))^2|$$
$$I(\Delta x) = A^2(2\cos(k\Delta x) + 1)^2$$

where k is also $2\pi/\lambda$.

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Case B

We find the sum of all wave equation and we get

$$D(x,t) = D_1(x,t) + D_2(x,t) + D_3(x,t) + D_4(x,t)$$
$$= A\sin(k(x-\tfrac{3}{2}\Delta x)-\omega t) + A\sin(k(x-\tfrac{\Delta x}{2})-\omega t)$$
$$+ A\sin(k(x+\tfrac{\Delta x}{2})-\omega t) + A\sin(k(x+\tfrac{3}{2}\Delta x)-\omega t)$$
$$= A(\sin(kx-\tfrac{3}{2}k\Delta x-\omega t) + \sin(kx-\tfrac{k\Delta x}{2}-\omega t)$$
$$+ \sin(kx+\tfrac{k\Delta x}{2}-\omega t) + \sin(kx+\tfrac{k\Delta x 3}{2}-\omega t))$$

We use trigonometry identity and we get

$$D(x,t) = A(2\sin(kx-\omega t)\cos(\tfrac{k\Delta x}{2}) + 2\sin(kx-\omega t)\cos(\tfrac{3}{2}k\Delta x))$$
$$= A(2\sin(kx-\omega t)(\cos(k\Delta x/2) + \cos(3k\Delta x/2))$$
$$= A(2\sin(kx-\omega t)(2\cos(k\Delta x)\cdot\cos(k\Delta x/2))$$

We can simplified the equation and we can get

$$D(x,t) = A(\Delta x)\cdot \sin(kx-\omega t)$$

And, we get $A(\Delta x)$ that equal to

$$A(\Delta x) = 4A\cos(k\Delta x)\cdot\cos(k\Delta x/2)$$

where $k = \frac{2\pi}{\lambda}$

Also, we can subtitute $A(\Delta x)$ to the Intensity equation $I(\Delta x) = |A(\Delta x)^2|$

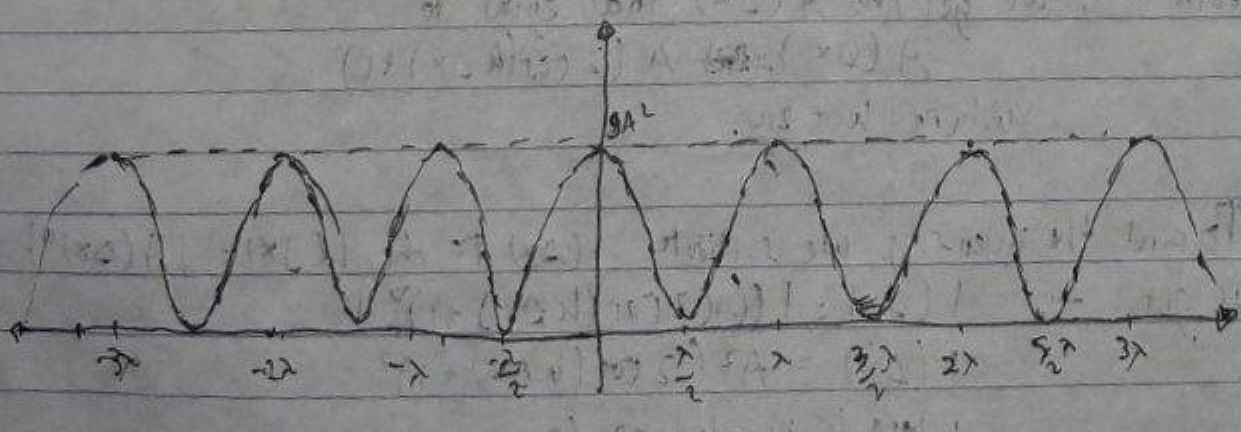
$$I(\Delta x) = |A(\Delta x)^2|$$
$$= (4A\cdot\cos(k\Delta x)\cdot\cos(k\Delta x/2))^2$$
$$= 16A^2\cdot\cos^2(k\Delta x)\cdot\cos^2(k\Delta x/2)$$

where k is also $k = \frac{2\pi}{\lambda}$

B. Plot a graph of $I(\Delta x)$ for $-3\lambda \le \Delta x \le 3\lambda$!

Case A : $I(\Delta x) = A^2(2\cos(k\Delta x)+1)^2$

$I(-3\lambda) = 9A^2$	$I(\lambda) = 9A^2$	$I(-1{,}5\lambda) = 0$	$I(2{,}5\lambda) = 0$
$I(-2\lambda) = 9A^2$	$I(2\lambda) = 9A^2$	$I(-0{,}5\lambda) = 0$	
$I(-\lambda) = 9A^2$	$I(3\lambda) = 9A^2$	$I(0{,}5\lambda) = 0$	
$I(0) = 9A^2$	$I(-2{,}5\lambda) = 0$	$I(1{,}5\lambda) = 0$	



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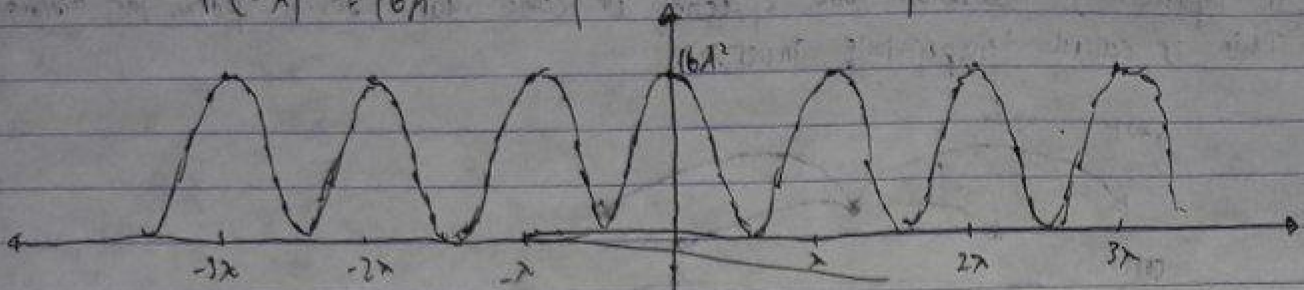
Case B: $I(\Delta x) = 16A^2 \cos^2(k\Delta x) \cdot \cos^2(k\Delta x/2)$

$I(-3\lambda) = 16A^2$ | $I(\lambda) = 16A^2$ | $I(3\lambda) = 16A^2$
$I(-2\lambda) = 16A^2$ | $I(2\lambda) = 16A^2$
$I(-\lambda) = 16A^2$

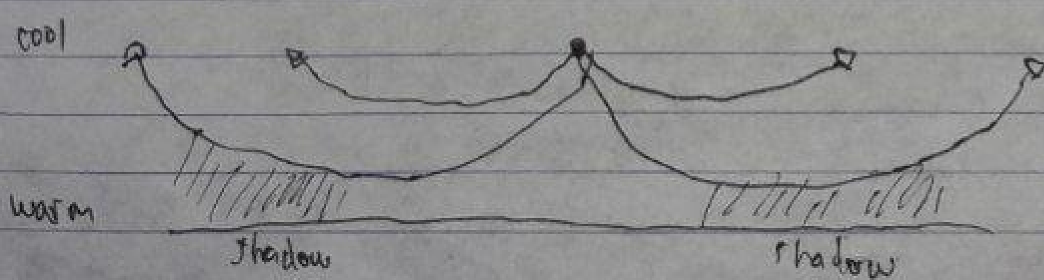


Problem 4.

A student lives around 2.5 km from the closest railroad track. Even though the track is quite far, he can still hear the sound of a train in the evening. He is wondering why this kind of phenomenon happens because he never hear the train sound from his house in the morning or afternoon.

A. Please explain to him the reason why can this ~~hap~~ kind of phenomenon happen?

* This phenomenon is due to the refraction of the sound wave. Wave speed changes gradually over a given distance. The speed of a sound wave in air depends on the temperature ($c = 331 + 0.6T$) where T is temp in °C. Often, it resulting refraction. For example, during the day, the air is warmest right next to the ground and grows cooler above the ground. This is called temperature lapse. Since the temperature decrease with height, the speed of sound is also decrease. This mean when the sound is traveling close to the ground, the part of the wave close to the ground is traveling the fastest, and the part furtest above the ground is slowest. As the result the wave change direction and bend upwards. This create a "shadow zone" which zone that the sound wave cannot penetrate. A person standing in shadow zone cannot hear the sound even the source is visible.



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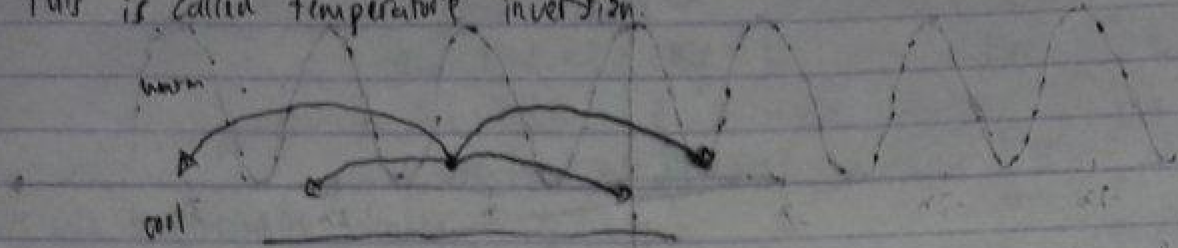
Another example is at night when the temperature coolest right next to the ground and warmer as increasing the height. The speed is increase with height. Thus, create an oposite effect that happen in day time. The sound is refracting downward and effecting us could hear a sound from far distance. This is called temperature inversion.



B. Can the same explanation be applied to explain an optical phenomenon called Fata Morgana? Please explain your reason!

→ Yes, the refraction of sound and light happen in same principle. When the light travel, light depends on the temperature. Higher the temperature create less dense their optic and viceversa. The density of the optic effecting the ~~[illegible]~~ refraction index of light.