Diberikan fungsi $f(x,y) = x^2 - y^2$ dan $p = (1,2)$. Tentukan

1. Vektor gradien $\nabla f(p)$
2. Vektor satuan u sehingga $D_u f(p)$
   a. Maksimum
   b. Minimum

Solusi :

1. $$\nabla f = (f_x, f_y) = (2x, -2y)$$
$$\nabla f(p) = \nabla f(1,2) = (2.1, -2.2) = \underline{(2,-4)}$$

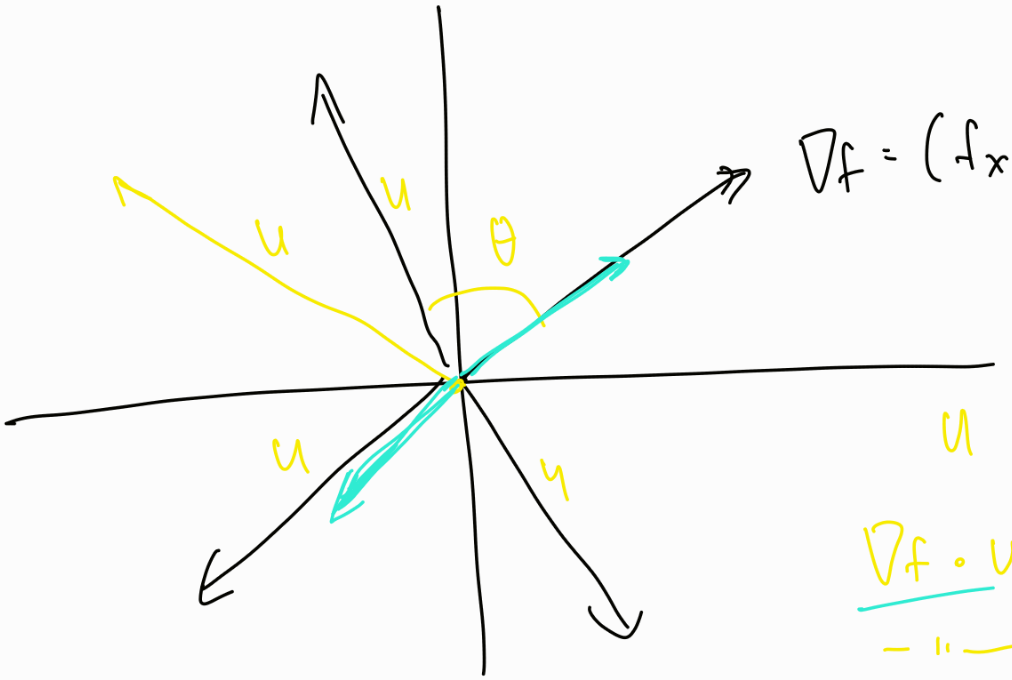
2. $$D_u f(P) = \nabla f(P) \bullet u$$
Misal $u = (a,b)$
$$\Rightarrow D_u f(P) = \nabla f(P) \bullet (a,b) = (2,-4) \bullet (a,b)$$

a.) $$u = \frac{\nabla f(p)}{\|\nabla f(p)\|} = \frac{(2,-4)}{\|(2,-4)\|}$$

$$\|\nabla f(p)\| = D_u f(p) = \frac{(2,-4) \bullet (2,-4)}{\|(2,-4)\|} = \frac{2^2 + (-4)^2}{\sqrt{2^2 + (-4)^2}} = \sqrt{2^2 + (-4)^2}$$
$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}.$$

b) $$u = \frac{-\nabla f(p)}{\|\nabla f(p)\|} \Rightarrow D_u f(p) = -\|\nabla f(p)\|$$
$$= -2\sqrt{5}.$$

(a, b)



$\nabla f = (f_x, f_y)$

u vektor satuan

$\nabla f \cdot u$ maksimum ?

— '' — minimum ?

3 Tentukan maksimum dan minimum $f(x,y,z) = 6y - 2z$ dengan syarat

$$\underbrace{x - 2y - 2z = 2} \quad \text{dan} \quad \underbrace{x^2 + y^2 = 1}$$

$\nabla f = \lambda \nabla g$

-1

$-\frac{1}{2}$

Solusi :

$$f(x,y,z) = 6y - 2z$$

$$g_1(x,y,z) = x - 2y - 2z - 2 \checkmark$$

$$g_2(x,y,z) = x^2 + y^2 - 1 \checkmark$$

$$\nabla f = \mu \nabla g_1 + \lambda \nabla g_2$$

μ ? λ ?

$$\nabla f = (0, 6, -2) = 6\bar{j} - 2\bar{k}$$

$$\nabla g_1 = (1, -2, -2) = \bar{i} - 2\bar{j} - 2\bar{k}$$

$$\nabla g_2 = (2x, 2y, 0) = 2x\bar{i} + 2y\bar{j}$$

$$6\bar{j} - 2\bar{k} = \mu(\bar{i} - 2\bar{j} - 2\bar{k}) + \lambda(2x\bar{i} + 2y\bar{j})$$

$$6\bar{j} - 2\bar{k} = (\mu + 2\lambda x)\bar{i} + (-2\mu + \lambda 2y)\bar{j} + (-2\mu + 0)\bar{k}$$

$$\mu + 2\lambda x = 0 \Rightarrow 2\lambda x = -1$$
$$-2\mu + 2\lambda y = 6 \Rightarrow 2\lambda y = 8$$
$$-2\mu = -2 \quad \cdots\cdots \Rightarrow \boxed{\mu = 1}$$

$$\lambda \neq 0 \Rightarrow \begin{cases} x = -\dfrac{1}{2\lambda} \\ y = \dfrac{8}{2\lambda} = \dfrac{4}{\lambda} \end{cases}$$

$$(x, y) = \left(\underbrace{-\frac{1}{2\lambda}}, \underbrace{\frac{4}{\lambda}}\right)$$

$$g_1(x,y,z) = x - 2y - 2z - 2$$
$$g_2(x,y,z) = x^2 + y^2 - 1$$

$$\begin{cases} g_1 = 0 \\ g_2 = 0 \end{cases}$$

$$g_2(x,y,z) = 0 = x^2 + y^2 - 1$$

$$0 = \left(-\frac{1}{2\lambda}\right)^2 + \left(\frac{4}{\lambda}\right)^2 - 1 = \frac{1}{4\lambda^2} + \frac{16}{\lambda^2} - 1$$

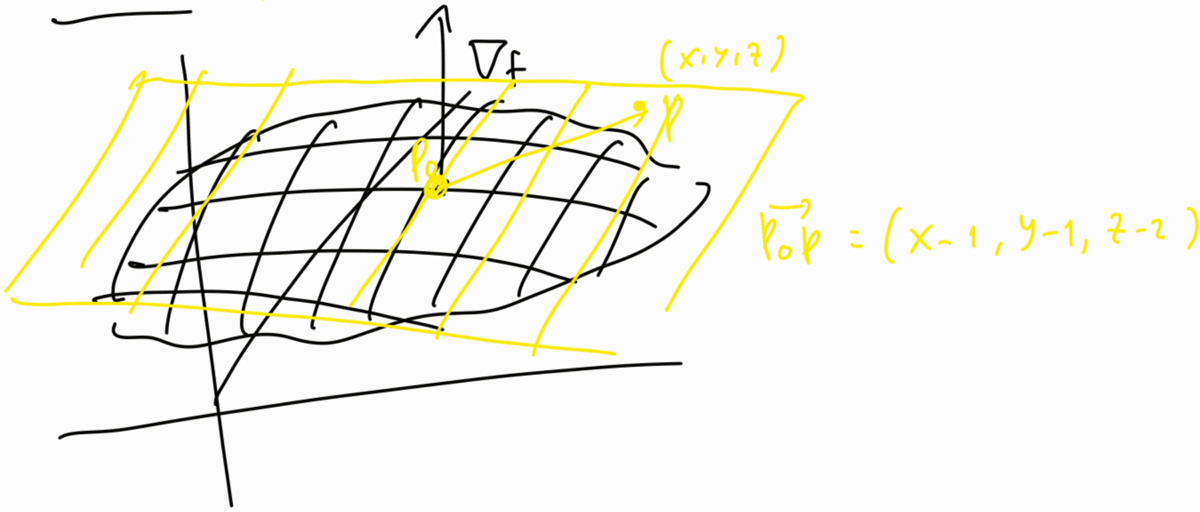
$$\Rightarrow \frac{1}{4\lambda^2} + \frac{16}{\lambda^2} = 1 \Rightarrow 1 + 64 = 4\lambda^2$$
$$\Rightarrow 4\lambda^2 = 65$$
$$\lambda^2 = \frac{65}{4} \Rightarrow \lambda = \pm\sqrt{\frac{65}{4}} = \pm\frac{1}{2}\sqrt{65}$$

$$\begin{cases} (x,y) = \left(-\dfrac{1}{\sqrt{65}}, \dfrac{8}{\sqrt{65}}\right) \cdots (1) \\ (x,y) = \left(\dfrac{1}{\sqrt{65}}, \dfrac{-8}{\sqrt{65}}\right) \cdots (2) \end{cases}$$

$$g_1(x,y,z) = 0 \Rightarrow x - 2y - 2z = 2$$
$$\Rightarrow z = \frac{x}{2} - y - 1$$

④ Cari persamaan bidang singgung terhadap $z = x^2 + y^2$ di titik $\underline{(1,1,2)} = P_0$

$f(x,y,z) = x^2 + y^2 - z$



$$\nabla f = (2x, 2y, -1) \Rightarrow \nabla f \Big|_{(1,1,2)} = (2, 2, -1)$$

$$\nabla f \Big|_{(1,1,2)} \cdot \overrightarrow{P_0P} = 0$$

$$(2, 2, -1) \cdot (x-1, y-1, z-2) = 0$$

$$2(x-1) + 2(y-1) - (z-2) = 0 \Rightarrow \underline{2x + 2y - z = 2}$$

⑤ Tentukan titik pada bidang $\alpha : x + 2y + 3z = 12$ yang paling dekat dengan titik $(1,0,1)$.

$$f(x,y,z) = (x-1)^2 + (y)^2 + (z-1)^2$$ minimalkan.

$$g(x,y,z) = x + 2y + 3z - 12 = 0$$

$$\nabla f = \lambda \nabla g \quad , \quad g(x,y,z) = 0$$

$$\nabla f = (2(x-1), 2y, 2(z-1)) = \lambda (1, 2, 3)$$
$$= (\lambda, 2\lambda, 3\lambda)$$

$$2x - 2 = \lambda \quad , \quad 2y = 2\lambda \quad , \quad 2z - 2 = 3\lambda$$

$$\left. x = \frac{\lambda + 2}{2} \quad , \quad y = \lambda \quad , \quad z = \frac{3\lambda + 2}{2} \right\}$$

$$x + 2y + 3z - 12 = 0$$

$$\frac{\lambda+2}{2} + 2\lambda + 3\left(\frac{3\lambda+2}{2}\right) - 12 = 0$$

$$\lambda + 2 + 4\lambda + 9\lambda + 6 - 24 = 0$$

$$14\lambda = 24 - 2 - 6 = 16$$

$$\lambda = \frac{16}{14} = \frac{8}{7}$$

$$x = 1 + \frac{4}{7} = \frac{11}{7}$$

$$y = \frac{8}{7}$$

$$z = 1 + \frac{12}{7} = \frac{19}{7}$$

$$(x, y, z) = \left(\frac{11}{7}, \frac{8}{7}, \frac{19}{7}\right)$$

$$d = \sqrt{(x-1)^2 + y^2 + (z-1)^2} \quad //$$

$$1^2 \qquad x^2 \qquad 2^2 \qquad \sqrt{x^2} \qquad \sqrt{2^2} \qquad \sqrt{1^2}$$

$$[1, 2]$$

$x = 1$ $\qquad$ $x = 2$