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1. Describe geometrically (line, plane, or all of \mathbb{R}^3)

a. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 6 \\ 9 \end{bmatrix} \rightarrow$ line in \mathbb{R}^3

b. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow$ plane in \mathbb{R}^3

c. $\begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \rightarrow$ all of \mathbb{R}^3

5. compute $u+v+w$ and $2u+2v+w$. How do you know u, v, w lie in a plane?

$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, w = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$

$u+v+w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

u, v, w in a plane because $u+v+w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
and also $u = (-v) + (-w)$

$$\begin{aligned} 2u+2v+w &= 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + \begin{bmatrix} -6 \\ 2 \\ -4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \end{aligned}$$

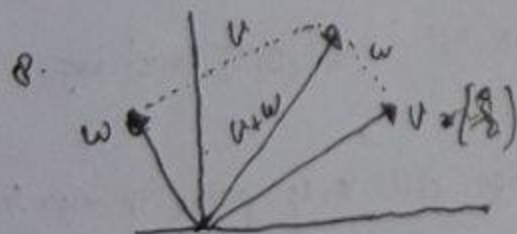
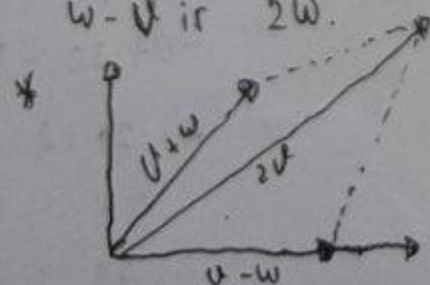


Fig. 1.1

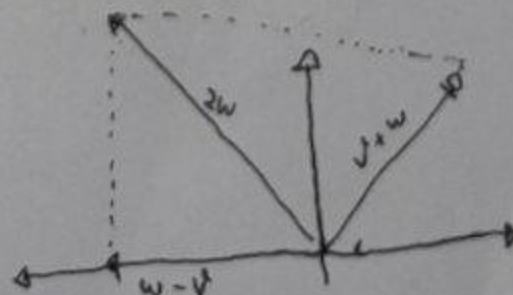
The parallelogram in figure 1.1 has diagonal $u+v$. What is its other diagonal? What is the sum of the ~~other~~ two diagonals? Draw the vector sum.

* Also stated in fig. 1.1, the other diagonal is $u-w$ or it can be $w-v$.

* the sum of $u+w$ and $u-w$ is $2u$ or sum of $u+w$ and $w-v$ is $2w$.



or

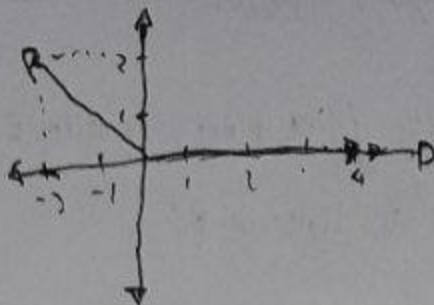


9. three corner of a parallelogram are $(1,1)$, $(4,2)$, $(1,3)$ what are all three of the fourth possible corners? Draw two of them

$$* (1,1) + (4,2) - (1,3) = (4,0)$$

$$* (1,1) + (1,3) - (4,2) = (-2,2)$$

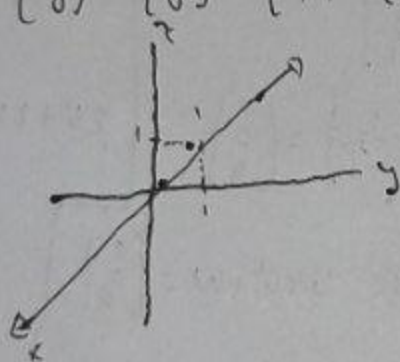
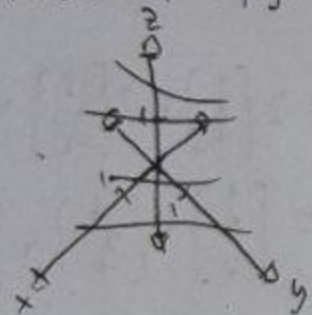
$$+ (4,2) + (1,3) - (1,1) = (4,4)$$



10. Which point of the cube is $i+j$? Which point is the vector sum of $i = (1,0,0)$ and $j = (0,1,0)$, and $k = (0,0,1)$? Describe all points (x,y,z) in the cube.

$$i+j = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{Vector sum of } i, j, k = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$



$$\text{where } \begin{aligned} 0 &\leq x \leq 1 \\ 0 &\leq y \leq 1 \\ 0 &\leq z \leq 1 \end{aligned}$$

11.

27. In xyz space, where is the plane of all linear combination of $i = (1,0,0)$ and $i+j = (1,1,0)$?

When i and $i+j$ isn't perpendicular, their combination fill x,y plane in xyz space.

31. $cu + dv + ew = b$.

$$c \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + d \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + e \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{aligned} 2c - d &= 1 \\ -c + 2d - e &= 0 \\ -d + 2e &= 0 \end{aligned}$$

$$\begin{aligned} + 2e &= d \\ 2d - c &= e \\ 4e - c &= e \\ c &= 3e \end{aligned}$$

$$\begin{aligned} 2c - d &= 1 \\ 2d - c &= e \\ \cancel{2e} &= d \end{aligned}$$

$$\begin{aligned} * 2c - d &= 1 \\ 2e - 2e &= 1 \\ 6e - 2e &= 1 \end{aligned}$$

$$4e = 1$$

$$e = \frac{1}{4}$$

$$c = \frac{3}{4}$$

$$d = \frac{1}{2}$$