

Date

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Quiz Tutor KV

Materi 1-2

1. Luas R dg titik A(0,0), B(2,5), C(6,3), D(4,-5), E(0,-4)

$$R = \frac{1}{2} (|\Delta ABC| + |\Delta ACD| + |\Delta ADE|)$$

$$\Delta ABC = \begin{vmatrix} 0 & 0 & 1 \\ 2 & 5 & 1 \\ 6 & 3 & 1 \end{vmatrix} = 6 - (30 + 0 \cdot 0) = -24$$

$$\Delta ACD = \begin{vmatrix} 0 & 0 & 1 \\ 6 & 3 & 1 \\ 4 & -5 & 1 \end{vmatrix} = -30 - (12) = -42$$

$$\Delta ADE = \begin{vmatrix} 0 & 0 & 1 \\ 4 & -5 & 1 \end{vmatrix} = -4$$

$$R = \frac{1}{2} (|-24| + |-42| + |-4|) = \frac{1}{2} (82) = 41$$

2. Pers bidang yg melalui titik A(2,0,2), B(0,4,4), C(1,1,0)

$$\begin{aligned} \vec{N} &= \vec{P_1P_2} \times \vec{P_1P_3} \\ &= (-2, 4, 2) \times (-1, 1, -2) \\ &= \begin{vmatrix} i & j & k \\ -2 & 4 & 2 \\ -1 & 1 & -2 \end{vmatrix} = i(-10) + j(6) + k(2) \\ &= -10i + 6j + 2k \end{aligned}$$

Karena melalui A(2,0,2) dan tegak lurus \vec{N} maka pers bidang

$$-10x + 6y + 2z = -10(2) + 2(2) = -16$$

$$-10x + 6y + 2z = -16$$

Materi 3-4

1. Titik kritis dan klasifikasi dari

$$z = -x^2 + y^2$$

$$z_x = -2x = 0 \Rightarrow x = 0$$

$$z_y = 2y = 0 \Rightarrow y = 0$$

$$z_{xx} = -2 < 0$$

$$z_{yy} = 2 > 0$$

$$z_{xy} = 0$$

$$(0,0)$$

Saddle Point

Date

$$f\left(\frac{-18}{\sqrt{13}}, \frac{12}{\sqrt{13}}\right) = 6 \cdot \frac{-18}{\sqrt{13}} + 4 \cdot \frac{12}{\sqrt{13}} = \frac{-12}{\sqrt{13}} \quad (\text{Minimum})$$

$$f\left(\frac{18}{\sqrt{13}}, \frac{12}{\sqrt{13}}\right) = 6 \cdot \frac{18}{\sqrt{13}} + 4 \cdot \frac{12}{\sqrt{13}} = \frac{12}{\sqrt{13}} \quad (\text{Maximum})$$

Matr. 3

Moment inertia of half disk with $\delta(x,y) = \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{r}$

$$D = \{(r, \theta) \mid 1 \leq r \leq 2, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\}$$

a. about x axis

$$I_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_1^2 r^2 \sin^2 \theta \cdot \frac{1}{r} \cdot r \, dr \, d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{r^3}{3} \sin^2 \theta \right]_1^2 d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{7}{3} \sin^2 \theta \, d\theta = \frac{7}{6} \theta - \frac{7 \sin(2\theta)}{12} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{7\pi}{6}$$

b.

Date

2. Curvature from $r(t) = 5t\mathbf{i} + 3\sin(t)\mathbf{j} + 3\cos(t)\mathbf{k}$

$$r'(t) = (5, 3\cos(t), -3\sin(t))$$

$$\hat{T}(t) = \left(\frac{5}{\sqrt{43}}, \frac{3\cos(t)}{\sqrt{43}}, \frac{-3\sin(t)}{\sqrt{43}} \right)$$

$$\hat{T}'(t) = \left(0, \frac{-3\sin(t)}{\sqrt{43}}, \frac{-3\cos(t)}{\sqrt{43}} \right)$$

$$\|\hat{T}'(t)\| = \sqrt{43}$$

$$\|\hat{T}'(t)\| = \frac{9}{\sqrt{43}}$$

$$K = \frac{\|\hat{T}'(t)\|}{\|r'(t)\|} = \frac{9/\sqrt{43}}{\sqrt{43}} = \frac{9}{43}$$

Materi 5-6

1. $\phi = xy^2 + 6z^2x^2$. Carilah $\nabla\phi$ pada $(6, 2, 2)$

$$\nabla\phi = \frac{\partial}{\partial x}(xy^2 + 6z^2x^2)\mathbf{i} + \frac{\partial}{\partial y}(xy^2 + 6z^2x^2)\mathbf{j} + \frac{\partial}{\partial z}(xy^2 + 6z^2x^2)\mathbf{k}$$

$$= (y^2 + 12z^2x)\mathbf{i} + (2xy)\mathbf{j} + (12z^2x^2)\mathbf{k}$$

$$\nabla\phi(6, 2, 2) = (2^2 + 12 \cdot 2^2 \cdot 6)\mathbf{i} + (2 \cdot 6 \cdot 2)\mathbf{j} + (12 \cdot 2^2 \cdot 6^2)\mathbf{k}$$

$$= 580\mathbf{i} + 24\mathbf{j} + 2592\mathbf{k}$$

$$\|\nabla\phi(6, 2, 2)\| = \sqrt{580^2 + 24^2 + 2592^2} \approx 2656,21$$

2. Min & max dari $f(x, y) = 6x + 4y$ pada $x^2 + y^2 = 36$

$$F_x = 6 \Rightarrow x = 0$$

$$6x = 2x\lambda + 10 \Rightarrow 10 = 4x\lambda$$

$$F_y = 4 \Rightarrow y = 0$$

$$6 = 2x\lambda + 5 \Rightarrow 1 = 2x\lambda$$

$$2x\lambda - 6 = 0 \Leftrightarrow 2(x\lambda - 3) = 0 \Leftrightarrow x = \frac{3}{\lambda}$$

$$4 = 2y\lambda$$

$$2y\lambda - 4 = 0$$

$$2(y\lambda - 2) = 0 \Leftrightarrow y = \frac{2}{\lambda}$$

$$x^2 + y^2 = 36 \Rightarrow \left(\frac{3}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 = 36$$

$$\frac{9}{\lambda^2} + \frac{4}{\lambda^2} = 36 \Rightarrow \frac{13}{\lambda^2} = 36$$

$$\frac{13}{\lambda^2} - 36 = 0$$

$$13 - 36\lambda^2 = 0$$

$$\lambda = \pm \frac{\sqrt{13}}{6}$$

$$x = \frac{3}{\sqrt{13}/6} = \frac{18}{\sqrt{13}}$$

$$y = \frac{2}{\sqrt{13}/6} = \frac{12}{\sqrt{13}}$$

$$x = \frac{3}{-\sqrt{13}/6} = -\frac{18}{\sqrt{13}}$$

$$y = \frac{2}{-\sqrt{13}/6} = -\frac{12}{\sqrt{13}}$$

$$x = \frac{18}{\sqrt{13}}, y = \frac{12}{\sqrt{13}}$$

$$x = -\frac{18}{\sqrt{13}}, y = -\frac{12}{\sqrt{13}}$$