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Tugas 2 Derivatif Parsial

1. Tentukan kritis dan jenisnya dari $F(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$

* $\frac{\partial F}{\partial x} = 6xy - 6x$

$\frac{\partial F}{\partial x} = 0$

$6xy - 6x = 0$

$xy - x = 0$

$x(y-1) = 0$

$x = 0 \vee y = 1$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0 \vee y = 2$

$(0,0) \; (0,2)$

$\frac{\partial F}{\partial y} = 3x^2 + 3y^2 - 6y$

$\frac{\partial F}{\partial y} = 0$

$3x^2 + 3y^2 - 6y = 0$

$x^2 + y^2 - 2y = 0$

$x^2 - 1 = 0$

$(x-1)^2 = 0$

$x = 1$

$(1,1)$

* $D(x,y) = (6y-6)(6y-6) - [6x]^2$

$= 36y^2 + 36 - 72y - 36x^2 = 36(y^2 - 2y + 1 - x^2)$

* $D(0,0) = 36(0^2 - 2 \cdot 0 + 1 - 0^2)$

$= 36$

$f_{xx}(0,0) = (6 \cdot 0 - 6) = -6$

Relative maximum

* $D(0,2) = 36(2^2 - 2 \cdot 2 + 1 - 0^2)$

$= 36 \cdot 1 = 36$

$F_{xx}(0,2) = 6 \cdot 2 - 6 = 6$

Relative minimum

* $D(1,1) = 36(1^2 + 2 \cdot 1 + 1 - 1^2)$

$= 36 \cdot -1 = -36$

Saddle Point

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2. Titik pada $4x - 2y + z = 1$ yang memiliki jarak terdekat dengan $(-2, -1, 5)$.

* $f(x,y,z) = (x+2)^2 + (y+1)^2 + (z-5)^2$
  $g(x,y,z) = 4x - 2y + z - 1$
* $f_x = 2(x+2) = 2x+4$
  $f_y = 2(y+1) = 2y+2$
  $f_z = 2(z-5) = 2z-10$
* $g_x = 4$
  $g_y = -2$
  $g_z = 1$

* $2x+4 = 4\lambda$ | $2y+2 = -2\lambda$ | $2z-10 = \lambda$
  $x = 2\lambda - 2$ | $y = -\lambda - 1$ | $z = \frac{\lambda + 10}{2}$

$4x - 2y + z - 1 = 0$

$4(2\lambda - 2) - 2(-\lambda - 1) + \left(\frac{\lambda + 10}{2}\right) - 1 = 0$

$8\lambda - 8 + 2\lambda + 2 + \frac{\lambda}{2} + 5 - 1 = 0$

$10{,}5\lambda - 2 = 0$

$\lambda = \frac{2}{10{,}5} = \frac{4}{21}$

$x = 2\lambda - 2 = \frac{8}{21} - 2 = -\frac{34}{21}$ | $y = -\lambda - 1 = -\frac{4}{21} - 1 = -\frac{25}{21}$ | $z = \frac{\lambda}{2} + 5 = \frac{2}{21} + 5$

$z = \frac{107}{21}$

titik $= \left(-\frac{34}{21}, -\frac{25}{21}, \frac{107}{21}\right)$

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3. $\alpha : x + y + z = 1$ merupakan b. dang singgung $z = x^2 + ay^2$. Tentukan nilai a.

* bidang α bersinggungan dengan $z = x^2 + ay^2$ di titik (x_0, y_0, z_0).

$z = x^2 + ay^2 \rightarrow x^2 + ay^2 - z = 0$

$f(x,y,z) = x^2 + ay^2 - z$

$F_x = 2x$

$F_y = 2ay$

$F_z = -1$

- Pers bidang singgung di (x_0, y_0, z_0)

$F_x(x_0,y_0).(x - x_0) + F_y(x_0,y_0).(y - y_0) = (z - z_0)$

$2x_0.(x - x_0) + 2ay_0.(y - y_0) = z - z_0$

$2x_0.x - 2x_0^2 + 2ay_0.y - 2ay_0^2 + z_0 = z$

$-2x_0x - 2ay_0y + z = -2x_0^2 - 2ay_0^2 + z_0$

dimana pers. diatas adalah pers α. sehingga

$-2x_0x = x$ | $-2ay_0y = y$ | $z = z$ | $-2x_0^2 - 2ay_0^2 + z_0 = 1$

$x_0 = -\frac{1}{2}$ | $y_0 = -\frac{1}{2a}$ | | $-\frac{1}{2} - \frac{1}{2a} + z_0 = 1$

Sehingga, $z_0 = \frac{3}{2} + \frac{1}{2a}$, dimana $z_0 = x_0^2 + ay_0^2$

$z_0 = \frac{1}{4} + \frac{1}{4a}$

Sehingga, $\frac{3}{2} + \frac{1}{2a} = \frac{1}{4} + \frac{1}{4a}$

$6 + \frac{2}{a} = 1 + \frac{1}{a}$

$5 = \frac{1}{a} \Leftrightarrow a = \frac{1}{5}$

4. $F(x,y,z) = x^2 + y^2 + z^2$ dengan syarat $z = x + y$ dan $\frac{x^2}{9} + \frac{y^2}{5} + \frac{z^2}{25} = 1$

* $g_1(x,y,z) = x + y - z$

$\nabla g_1(x,y,z) = i + j - k$

* $g_2(x,y,z) = \frac{x^2}{9} + \frac{y^2}{5} + \frac{z^2}{25} - 1$

$\nabla g_2(x,y,z) = \frac{x}{2}i + \frac{2y}{5}j + \frac{2k}{25}z$

$\nabla F(x,y) = 2xi + 2yj + 2kz$

$2xi + 2yj + 2zk = \lambda i + \lambda j - \lambda k + N\frac{x}{2}i + N\frac{2y}{5}j + N\frac{2z}{25}k$

* $2x = \lambda + N\frac{x}{2}$ | $2y = \lambda + N\frac{2y}{5}$

$2z = -\lambda + N2z/25$

* $x + y - z = 0 \rightarrow$ ~~$2x + 2y - 2z = 0$~~

~~$\times 2N$~~

$\frac{2\lambda}{4-N} + \frac{5\lambda}{10-2N} + \frac{25\lambda}{50-2N} = 0$

$\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$

$\left(\frac{2\lambda}{4-N}\right)^2 \cdot \frac{1}{4} + \left(\frac{5\lambda}{10-2N}\right)^2 \cdot \frac{1}{5} + \left(\frac{-25\lambda}{50-2N}\right)^2 \frac{1}{25} = 1$

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dari pers. ⅔ diatas di dapat

$(x_1, y_1, z_1) = \left(\frac{-20\sqrt{646}}{323}, \frac{35\sqrt{646}}{646}, \frac{-5\sqrt{646}}{646}\right) \rightarrow F(x_1, y_1, z_1) = \frac{75}{17}$

$(x_2, y_2, z_2) = \left(\frac{20\sqrt{646}}{323}, \frac{-35\sqrt{646}}{646}, \frac{5\sqrt{646}}{646}\right) \rightarrow F(x_2, y_2, z_2) = \frac{75}{17}$

$(x_3, y_3, z_3) = \left(\frac{-2\sqrt{95}}{19}, \frac{-3\sqrt{95}}{19}, \frac{-5\sqrt{95}}{19}\right) \rightarrow F(x_3, y_3, z_3) = 10$

$(x_4, y_4, z_4) = \left(\frac{2\sqrt{95}}{19}, \frac{3\sqrt{95}}{19}, \frac{5\sqrt{95}}{19}\right) \rightarrow F(x_4, y_4, z_4) = 10$

Jadi nilai maksimum terdapat pada (x_3, y_3, z_3) dan (x_4, y_4, z_4).