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Tugas 2 Kalkulus Variabel Tunggal

1.5.

29. Population Growth. The population of Knoxville is 500.000 and increasing at the rate of 3,75% each year. Approximately when will the population reach 1 million?

→ Starting Pop ⇒ $P_0 = 500.000$
Rate ⇒ $r = 3,75\%$
$P(t) = 1.000.000$

$P(t) = P_0(1+r)^t$
$1.000.000 = 500.000\,(1+3,75\%)^t$
$2 = (103,75\%)^t$
$2 = 1,0375^t$
$t = \log_{1,0375} 2$

$t = 18.8237$
$t \approx 19$

So, the population will reach 1 million approximately in the next 19 years.

30. Population Growth. The population of Silver Run in 1890 was 6250. Assume the population increase at a rate of 2,75% per year.
a. Estimate the population in 1915 and 1940.
→ $t_1 = 1915$; $t_2 = 1940$; $P_0 = 6250$; $r = 2,75\%$.
$\Delta t_1 = 1915 - 1890 = 25$ years
$\Delta t_2 = 1940 - 1890 = 50$ years.
$P(\Delta t_1) = P_0(1+r)^{\Delta t_1}$
$= 6250\,(1+2,75\%)^{25}$
$= 6250\,(1,0275)^{25}$
$P(\Delta t_1) \approx 12.314,7551 \approx 12.315$
$P(\Delta t_2) = P_0(1+r)^{\Delta t_2}$
$= 6250\,(1+2,75\%)^{50}$
$= 6250\,(1,0275)^{50}$
$P(\Delta t_2) \approx 24.264,5111 \approx 24.265$

b. Approximately when did the population reach 50.000?

→ $P(t) = 50.000$

$P(t) = P_0 (1+r)^t$

$50.000 = 6250 (1 + 2,75\%)^t$

$8 = (1,0275)^t$

$t = \log_{1,0275} 8$

$t \approx 76,65708$

$t \approx 77$

So, approximately in next 77 years, the population reach 50000.

31. Radioactive decay The half-life of phosphorus-32 is about 14 days. There are 6,6 grams present initially.

a. Express the amount of phosphorus-32 remaining as a function of t.

→ $p(t) = p_0 (1-r)^t$

$p(t) = 6,6 (1-0,5)^{t/14}$

$p(t) = 6,6 (0,5)^{t/14}$

b. When will there be 1 gram remaining?

→ $p(t) = 1$ gram

$p(t) = 6,6 (0,5)^{t/14}$

$1 = 6,6 (0,5)^{t/14}$

$\frac{1}{6,6} = (0,5)^{t/14}$

$\frac{t}{14} = -\log_2 \left(\frac{1}{6,6}\right)$

$t = -14 \cdot \log_2 \left(\frac{1}{6,6}\right)$

$t \approx 38.11452$

32. If John invest \$ 2300 in a saving account with 6% interest rate compounded annually, how long does it take until John's account has a ballance of \$4150?

→ $b_0 = 2300$, $r = 6\%$, $b(t) = 4150$

$b(t) = b_0 \cdot (1+r)^t$

$4150 = 2300 \cdot (1 + 6\%)^t$

$\frac{415}{230} = (1,06)^t$

$t = \log_{1,06} \left(\frac{415}{230}\right)$

$t \approx 10,1289$

Date

33. Doubling your money. Determine how much time is required for an investment to double in value if interest is earned at the rate of 6,25% compounded annually.

$\rightarrow P(t) = 2P_0$; $r = 6{,}25\%$

$P(t) = P_0(1+r)^t$

$2P_0 = P_0(1+6{,}25\%)^t$

$2 = (1{,}0625)^t$

$t = \log_{1{,}0625} 2$

$t = 11{,}433$ years

34. Tripling your money. Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5,75% compounded continuously.

$\rightarrow P(t) = 3P_0$. $r = 5{,}75\%$

$P(t) = P_0 . e^{r.t}$

$3P_0 = P_0 . e^{0{,}0575.t}$

$3 = e^{0{,}0575.t}$

$0{,}0575 . t = \ln 3$

$t = \frac{100}{5{,}75} . \ln 3$

$t = 19.1063$ years.

35. Cholera bacteria. Suppose that a colony of bacteria starts with 1 bacterium and doubles in number every half hour. How many bacteria will the colony contain at the end of 24 hr?

$\rightarrow P_0 = 1$; $r = 2/0{,}5$ hour

$r = 4/$hour

$t = 24$ hours

$P(t) = P_0 . r^t$

$P(24) = 1 . 4^{24}$

$P(24) \approx 2{,}815 . 10^{14}$ bacteria

36. Suppose that in any given year the number of disease is reduced by 20%. If there are 10.000 cases today. how many years will it take

a. to reduce the number of cases to 1000?

$C_0 = 10.000$; $r = -20\%$, $C(t) = 1000$.

$C(t) = C_0 . (1+r)^t$

$\cancel{1000} = 10.\cancel{000} (1 - 0{,}2)^t$

$1 = 10 (0{,}8)^t$

$t = \log_{0,8} \frac{1}{10}$

$t \approx 10{,}3189$ years.

b. to eliminate the disease; that is, to reduce the number of cases to less than 1?

$C_0 = 10.000$; $r = -20\%$; $C(t) = 1$

$C(t) = C_0 . (1+r)^t$

$1 = 10.000 . (1-20\%)^t$

$1 = 10.000 . (0{,}8)^t$

$t = \log_{0,8} \frac{1}{10.000}$

$t \approx 41{,}2754$ years.

1.6.

19. Find a formula for f^{-1} in $f(x) = x^2 + 1, x \geq 0$

→ $f(x) = y$

$y = x^2 + 1$

$x^2 = y - 1$

$x = \sqrt{y-1}$

$f^{-1}(x) = \sqrt{x-1}$

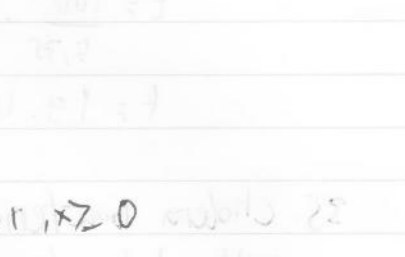
38. a. Find the inverse of $f(x) = -x + 1$. Graph the line $y = -x + 1$ together with the line $y = x$. At what angle do the line intersect?

→ $y = f(x) = -x + 1$

$y = -x + 1$

$x = -y + 1$

$f^{-1}(x) = -x + 1$



the line intersect at 90° angle crossing the line $y = x$ or perpendicular to the $y = x$

Date

38. b. Find the inverse of $f(x) = -x + b$ (b constant). What angle does the line $y = -x + b$ make with the line $y = x$?

→ $y = f(x) = -x + b$

$y = -x + b$

$x = -y + b$

$f^{-1}(x) = -x + b.$

the line intersect at $90°$ angle crossing the line $y = x$ or perpendicular to the $y = x$.

c. What can you conclude about the inverse of functions whose graphs are lines perpendicular to the line $y = x$?

→ When the inverse is perpendicular to $y = x$, Their slope will be -1 when being multiplied by slope of $y = x$.

43. Find the simpler expressions for

a. $e^{\ln 7.2}$

→ $e^{\ln 7.2} = 7.2$

b. $e^{-\ln x^2}$

→ $e^{-\ln x^2} = \frac{1}{x^2}$

b. $e^{\ln x - \ln y}$

→ $e^{\ln x - \ln y} = \frac{e^{\ln x}}{e^{\ln y}}$

$= \frac{x}{y}$

47. Solve for y in terms of t or x

$\ln y = 2t + 4$

→ $\ln y = 2t + 4$

$e^{\ln y} = e^{2t + 4}$

$y = e^{2t + 4}$

61. Simplify the expressions

a. $2^{\log_4 x}$

→ $2^{\log_4 x} = 2^{\frac{1}{2} \cdot \log_2 x}$

$= (2^{\log_2 x})^{\frac{1}{2}}$

$= x^{\frac{1}{2}} = \sqrt{x}$

b. $9^{\log_3 x}$

→ $9^{\log_3 x} = 3^{2 \cdot \log_3 x}$

$= (3^{\log_3 x})^2$

$= x^2$

61. c. $\log_2\left(e^{(\ln 2)(\sin x)}\right)$

$$\rightarrow \log_2\left(e^{(\ln 2)(\sin x)}\right) = \log_2\left(\left(e^{(\ln 2)}\right)^{\sin x}\right)$$

$$= \log_2\left(2^{\sin x}\right)$$

$$= \sin x \cdot \log_2 2$$

$$= \sin x$$

63. Express the ratios as ratios of natural logarithms and simplify.

a. $\dfrac{\log_2 x}{\log_3 x}$

$$\rightarrow \frac{\log_2 x}{\log_3 x} = \frac{\cancel{\ln x}}{\ln 2} \cdot \frac{\ln 3}{\cancel{\ln x}} = \frac{\ln 3}{\ln 2}$$

b. $\dfrac{\log_2 x}{\log_8 x}$

$$\rightarrow \frac{\log_2 x}{\log_8 x} = \frac{\cancel{\ln x}}{\ln 2} \cdot \frac{\ln 8}{\cancel{\ln x}}$$

$$= \frac{\ln 8}{\ln 2} = \frac{\ln 2^3}{\ln 2} = \frac{3 \cdot \ln 2}{\ln 2}$$

$$= 3$$

c. $\dfrac{\log_x a}{\log_{x^2} a}$

$$\rightarrow \frac{\log_x a}{\log_{x^2} a} = \frac{\cancel{\log a}}{\log x} \cdot \frac{\log x^2}{\cancel{\log a}}$$

$$= \frac{\log x^2}{\log x} = \frac{2 \cdot \cancel{\log x}}{\cancel{\log x}}$$

$$= 2$$