HW 1 No 1: Interference of Two Sound Waves

1) $D_1 = A_1 \sin[k(x - \Delta x/2) - \omega t + \phi_1]$

$D_2 = A_2 \sin[k(x + \Delta x/2) - \omega t + \phi_2]$

* Superposition principle

$D = D_1 + D_2$

general form : $D = A \sin[kx - \omega t + \varphi]$

$A \sin[kx - \omega t + \varphi] = D_1 + D_2$

$A \sin[kx - \omega t + \varphi] = A_1 \sin[k(x - \Delta x/2) - \omega t + \phi_1] + A_2 \sin[k(x + \Delta x/2) - \omega t + \phi_2]$

* Boundary condition

1) $kx - \omega t = 0$

$A \sin\varphi = A_1 \sin[-k\Delta x/2 + \phi_1] + A_2 \sin[k\Delta x/2 + \phi_2]$ ...... (1)

2) $kx - \omega t = \frac{\pi}{2}$

$A \sin(\frac{\pi}{2} + \varphi) = A_1 \sin[\frac{\pi}{2} - k\Delta x/2 + \phi_1] + A_2 \sin[\frac{\pi}{2} + k\Delta x/2 + \phi_2]$

$A \cos\varphi = A_1 \cos[-k\Delta x/2 + \phi_1] + A_2 \cos[k\Delta x/2 + \phi_2]$ ...... (2)

a) for $\phi_1 = \phi_2 = 0$ and $A_1 = A_0 +$

$A^2 \sin^2\varphi + A^2 \cos^2\varphi = (A_2 - A_1)^2 \sin^2(k\Delta x/2) + (A_2 + A_1)^2 \cos^2(k\Delta x/2)$

$A^2 = (A_2 - A_1)^2 \sin^2(k\Delta x/2) + (A_2 + A_1)^2 \cos^2(k\Delta x/2)$

$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(k\Delta x)$

$I \propto A^2$

• maxima intensity $\Rightarrow$ $\cos(k\Delta x) = 1$

$k\Delta x_{max} = 2m\pi$ with $m = 0, \pm 1, \pm 2, \ldots$

$\frac{2\pi}{\lambda} \Delta x_{max} = 2m\pi$

$\Delta x_{max} = m\lambda$ with $m = 0, \pm 1, \pm 2, \ldots$

•) minimum intensity $\Rightarrow \cos(k\Delta x) = -1$

$$k\Delta x_{min} = m\pi \quad \text{with } m = \pm 1, \pm 3, \pm 5, \dots$$

$$\frac{2\pi}{\lambda}\Delta x_{min} = m\pi$$

$$\Delta x_{min} = \frac{m}{2}\lambda \quad \text{with } m = \pm 1, \pm 3, \pm 5, \dots$$

b) for $\phi_1 = -\phi_2$ and $A_1 = A_2 = A_0$

(1)

$$A\sin\varphi = A_0 \sin\left(-k\Delta x/_2 - \phi_2\right) + A_0 \sin\left(k\Delta x/_2 + \phi_2\right) = 0$$

(2)

$$A\cos\varphi = A_0 \cos\left(-k\Delta x/_2 - \phi_2\right) + A_0 \cos\left(k\Delta x/_2 + \phi_2\right)$$

$$= 2A_0 \cos\left(k\Delta x/_2 + \phi_2\right)$$

$$D = A\sin\left[kx - \omega t + \varphi\right]$$

$$= A\sin(kx - \omega t)\cos\varphi + \underbrace{A\sin\varphi}_{0}\cos(kx - \omega t)$$

$$= 2A_0\cos\left(k\Delta x/_2 + \phi_2\right)\sin(kx - \omega t)$$

$$D = \underbrace{2A_0\cos\left(\frac{k\Delta x + \phi_0}{2}\right)}_{\text{Amplitudo}}\sin(kx - \omega t)$$

$$I \propto \text{Amplitude}^2 \Rightarrow I \propto 4A_0^2\cos^2\left(\frac{k\Delta x + \phi_0}{2}\right)$$

•) maximum Intensity $\Rightarrow \cos^2\left(\frac{k\Delta x + \phi_0}{2}\right) = 1$

$$\frac{k\Delta x + \phi_0}{2} = m\pi \quad \text{with } m = 0, \pm 1, \pm 2, \dots$$

$$\frac{2\pi}{\lambda}\Delta x + \phi_0 = 2m\pi$$

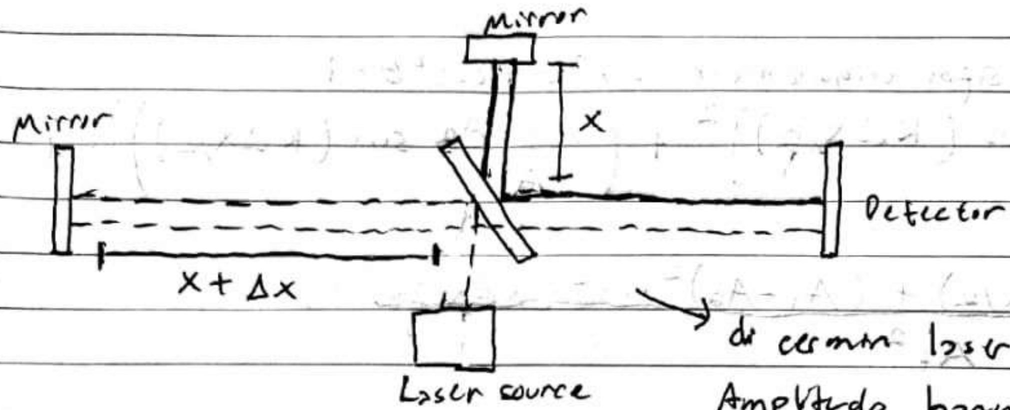
$$\Delta x_{max} = (2m\pi - \phi_0)\frac{\lambda}{2\pi} = \left(m - \frac{\phi_0}{2\pi}\right)\lambda \quad \text{with } m = 0, \pm 1, \pm 2, \dots$$

•) minimum Intensity $\Rightarrow \cos^2\left(\frac{k\Delta x + \phi_0}{2}\right) = 0$

$$\frac{k\Delta x + \phi_0}{2} = m\pi + \frac{\pi}{2} \quad \text{with } m = 0, \pm 1, \pm 2, \dots$$

$$\frac{2\pi}{\lambda}\Delta x + \phi_0 = 2m\pi + \pi$$

$$\Delta x_{min} = \left(m + \frac{1}{2} - \frac{\phi_0}{2\pi}\right)\lambda \quad \text{with } m = 0, \pm 1, \pm 2, \pm 3, \dots$$



Diketahui:
• panjang gelombang adalah λ

di cermin laser beam splitted sehingga Amplitudo hanya 50 %.

Karena berasal dari satu sinar yang sama maka amplitudo, fase, dan panjang gelombang kedua sinar tersebut sama. Namun, karena menempuh jarak yang berbeda maka persamaan kedua gelombang tersebut berbeda

D_1 = garis putus-putus $= \frac{A_0}{2} \sin(2k(x+\Delta x) - \omega t)$

D_2 = garis tanpa putus $= \frac{A_0}{2} \sin(2kx - \omega t)$

Saat berada di detektor, kedua gelombang akan berinteferensi sehingga persamaannya menjadi $D_t = \frac{A_0}{2}\left(\sin(2k(x+\Delta x) - \omega t) + \sin(2kx - \omega t)\right)$

Menurut identitas trigonometri $\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

sehingga $D_t = A_0 \cdot \sin\left(\frac{(2kx + 2k\Delta x - \omega t) + (2kx - \omega t)}{2}\right) \cos\left(\frac{(2kx + 2k\Delta x - \omega t) - (2kx - \omega t)}{2}\right)$

$= A_0 \sin\left(\frac{4kx + 2k\Delta x - 2\omega t}{2}\right) \cos\left(\frac{2k\Delta x}{2}\right)$

memenuhi pesamaan $D_t = A(\Delta x) \sin(kx - \omega t)$

$A(\Delta x) = A \cos(2k\Delta x / 2) = A \cos(k\Delta x)$

Intensitas gelombang pada detektor: $I = c(A(\Delta x))^2$

$I = A_0^2 \cos^2(k\Delta x)$. $c \rightarrow$ saat minimum $\theta = \cos^{-1}(\pm 0)$

$k\Delta x = (m + \frac{1}{2})\pi$; $m = 0, \pm 1, \pm 2 \ldots$

$k = \frac{2\pi}{\lambda}$

$\Delta x_{min} = (m + \frac{1}{2}) \lambda/2$ //

saat maksimum $\theta = \cos^{-1}(\pm 1)$

$k \cdot \Delta x = m \cdot \pi$; $m = 0, \pm 1, \pm 2, \pm 3 \ldots$

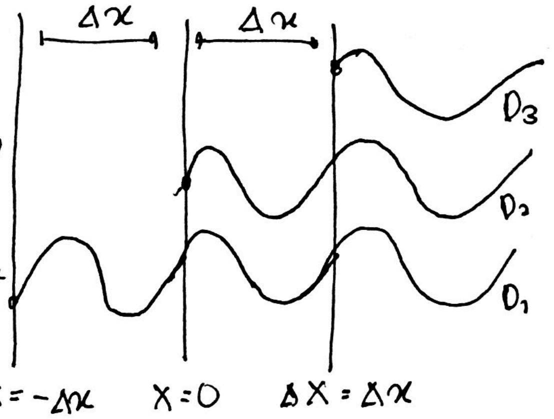
$k = \frac{2\pi}{\lambda}$

$\Delta x_{max} = m \cdot \lambda/2$ //

Jadi, Δx min adalah $(m + \frac{1}{2}) \lambda/2$ sedangkan $\Delta x_{max} = m \cdot \lambda/2$

HW Wave and Sound no. 3

3 gelombang !
Buktikan / Pastikan
$A(\Delta x)$ dan
$I(\Delta x) = |A(\Delta x)|^2$.



Rumus umum gelombang
$y = A \sin(kx \pm \omega t)$

karena gelombang semua bergerak / merambat ke kanan maka berlaku $-\omega t$.

D_1 (gelombang I) $= A \sin[k(x - \Delta x) - \omega t]$
D_2 (gelombang II) $= A \sin[kx - \omega t]$
D_3 (gelombang III) $= A \sin[k(x + \Delta x) - \omega t]$

D_1 dan D_3 bisa dijumlahkan.

dengan identitas trigonometri.

$$\sin\alpha + \sin\beta = 2\cos\frac{(\alpha - \beta)}{2}\sin\frac{(\alpha + \beta)}{2}$$

$$D_1 + D_3 = A\left(\sin\underbrace{[k(x - \Delta x) - \omega t]}_{\alpha} + \sin\underbrace{[k(x + \Delta x) - \omega t]}_{\beta}\right)$$

$$\sin\alpha + \sin\beta = 2\cos(-k\Delta x)\sin(kx - \omega t)$$

Ingat lagi bahwa $\cos(-\alpha) = \cos\alpha$. Ini akan memudahkan kalian di no 3b.

hingga $\sin\alpha + \sin\beta = 2\cos(k\Delta x)\sin(kx - \omega t)$

Total Interferensi konstruktif :
$D_1 + D_2 + D_3$.

$D_1 + D_3 = A\,2\cos(k\Delta x)\sin(kx - \omega t)$
$D_2 = A\sin(kx - \omega t)$
$D_1 + D_2 + D_3 = A(2\cos(k\Delta x)\sin(kx - \omega t) + \sin(kx - \omega t))$

keluarkan $\sin(kx-\omega t)$ menjadi

$$D = \underbrace{A\left(2\cos(k\Delta x)+1\right)}_{A(\Delta x)} \sin(kx-\omega t)$$

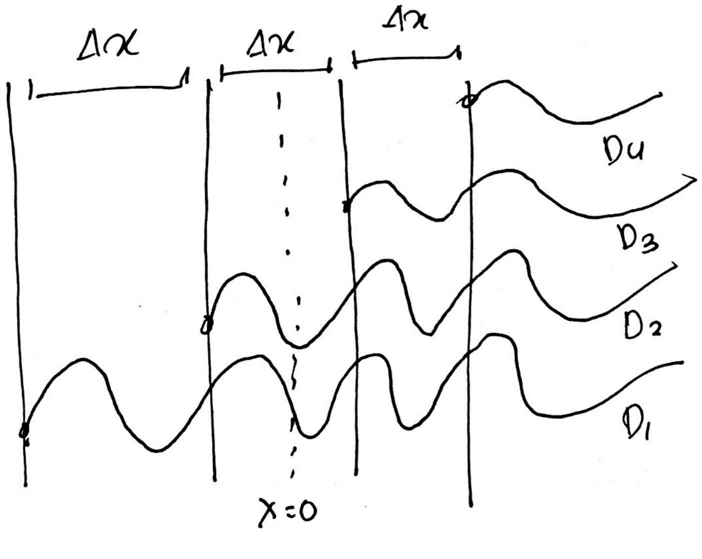
$$A(\Delta x) = A\left(2\cos(k\Delta x)+1\right)$$

maka I adalah:

$$I(\Delta x) = \left|A\left(2\cos(k\Delta x)+1\right)\right|^2$$ atau dengan $k = \frac{2\pi}{\lambda}$

$$= \left|A\left(2\cos\left(\frac{2\pi\Delta x}{\lambda}\right)+1\right)\right|^2$$

4 gelombang



Jika diperhatikan jarak D_1 ke $x=0$ adalah $-1{,}5\,\Delta x$
D_2 ke $x=0$ adalah $-0{,}5\,\Delta x$
D_3 ke $x=0$ adalah $0{,}5\,\Delta x$
D_4 ke $x=0$ adalah $1{,}5\,\Delta x$.

$D_1 = A\sin\left[k\left(x-\frac{3}{2}\Delta x\right)-\omega t\right]$

$D_2 = A\sin\left[k\left(x-\frac{1}{2}\Delta x\right)-\omega t\right]$

$D_3 = A\sin\left[k\left(x+\frac{1}{2}\Delta x\right)-\omega t\right]$

$D_4 = A\sin\left[k\left(x+\frac{3}{2}\Delta x\right)-\omega t\right]$

D_1 dan D_4 bisa dijumlahkan:

$$D_1+D_4 = A\left(\sin\left[k\left(x-3/2\,\Delta x\right)-\omega t\right] + \sin\left[k\left(x+3/2\,\Delta x\right)-\omega t\right]\right)$$

dengan identitas trigonometri $\sin\alpha + \sin\beta$ maka

$$D_1+D_4 = A\left(2\cos\left(\frac{k3\Delta x}{2}\right)\sin(kx-\omega t)\right)$$

D_3 dan D_2 bisa dijumlahkan, dan dengan $\sin \alpha + \sin \beta$:

$$D_3 + D_2 = A \left[2 \cos \left(\frac{k \Delta x}{2} \right) \sin (kx - \omega t) \right]$$

total interferensi.

$$D = D_1 + D_2 + D_3 + D_4$$

$$= A \left(2 \cos \left(\frac{3k \Delta x}{2} \right) \sin (kx - \omega t) + 2 \cos \left(\frac{k \Delta x}{2} \right) \sin (kx - \omega t) \right)$$

keluarkan $\sin (kx - \omega t)$

$$= \underbrace{2A \left(\cos \left(\frac{3k \Delta x}{2} \right) + \cos \left(\frac{k \Delta x}{2} \right) \right)}_{A(\Delta x)} \sin (kx - \omega t)$$

$$A(\Delta x) = 2A \left(\cos \left(\frac{3k\Delta x}{2} \right) + \cos \left(\frac{k \Delta x}{2} \right) \right)$$

dengan Identitas trigonometri :

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$= 2 \cos \left(\frac{3k\Delta x + k \Delta x}{4} \right) \cos \left(\frac{3k \Delta x - k \Delta x}{4} \right)$$

$$= 2 \cos (k\Delta x) \cos \left(\frac{k \Delta x}{2} \right)$$

$$A(\Delta x) = 2A \left(2 \cos k \Delta x \cos \frac{k \Delta x}{2} \right)$$

$$= 4A \left(\cos \left(\frac{2\pi \Delta x}{\lambda} \right) \cos \left(\frac{2\pi \Delta x}{2\lambda} \right) \right) = 4A \left(\cos \left(\frac{2\pi \Delta x}{\lambda} \right) \cos (\pi \Delta x) \right)$$

$$I(\Delta x) = \left| 4A \left(\cos (k \Delta x) \cos \left(\frac{k \Delta x}{2} \right) \right) \right|^2$$

Bagaimana grafik $I(\Delta x)$ dengan $-3\lambda \leq \Delta x \leq 3\lambda$?

Que: - Pada umumnya grafik ini berpola.

- $\cos(-\alpha) = \cos \alpha$.

o) <u>3 gelombang</u>, anggap $\lambda = 2\pi$.

ambil sudut fundamental trigonometri: $\frac{1}{4}\pi$, $\frac{\pi}{2}$, $\frac{\pi}{3}$, π, 0, $2\pi/3$, $3\pi/4$ untuk cosinus.

θ =	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π	0
$\angle^\circ$ =	45°	90°	60°	120°	135°	180°	0
$\cos\alpha$ =	$\frac{1}{2}\sqrt{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	-1	1
$\angle^\circ$ =	225°	270°	240°	300°	315°	360	
$\cos -\alpha$ =	$-\frac{\sqrt{2}}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	1	

Oke, terlalu banyak matematika,...

$$I(\Delta x) = |A(2\cos(k\Delta x) + 1)|^2$$

$$= \left| A\left(2\cos\left(\frac{2\pi\Delta x}{\lambda}\right) + 1\right)\right|^2 \qquad -3\lambda \leq \Delta x \leq 3\lambda$$

o) <u>$\Delta x = 0$</u>

$I(0) = |A(2+1)|^2 = 9A^2$.

o) <u>$\Delta x = \frac{\lambda}{4}$</u>

$I(\lambda/4) = |A(1)|^2 = A^2$

o) $\Delta x = \lambda/3$

$I(\lambda/3) = |A(0)|^2 = 0$.

o) <u>$\Delta x = \lambda/2$</u>

$I(\lambda/2) = |A(-1)|^2 = A^2$

o) <u>$\Delta x = 2\lambda/3$</u>

$I(2\lambda/3) = |A(0)|^2 = 0$

o) $\Delta x = 3\lambda/4$

$I(3\lambda/4) = |A(1)|^2 = A^2$

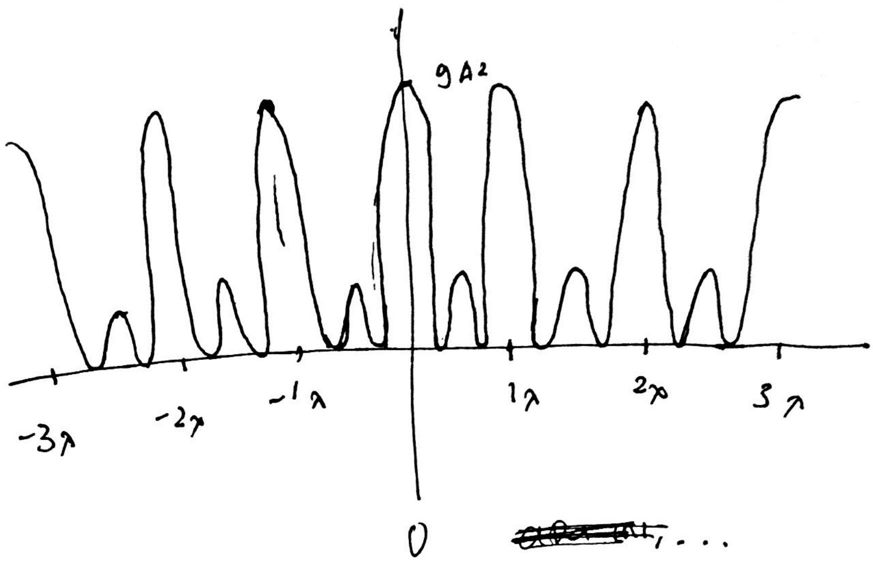
o) $\Delta x = 1$

$I(1) = |A(3)|^2 = 9A^2.$

Saat $\Delta x = 0$, $I(\Delta x) = 9A^2$

$\Delta x = 1$, $I(\Delta x) = 9A^2$.

Bilangan positif negatif antara $-3\lambda \leq \Delta x \leq 3\lambda$ bisa dianggap sama.



4 gelombang

$$I(\Delta x) = \left| 4A \left(\cos\left(\frac{k\Delta x}{2}\right) \cos(k\Delta x) \right) \right|^2$$

$$= \left| 4A \left(\cos\left(\frac{\pi \Delta x}{\lambda}\right) \cos\left(\frac{2\pi \Delta x}{\lambda}\right) \right) \right|^2$$

o) $\Delta x = \lambda/4$

$I(\lambda/4) = \left|4A\left(\frac{\sqrt{2}}{2} \cdot 0\right)\right|^2$

$= 0.$

o) $\Delta x = \lambda/3$

$I(\lambda/3) = \left|4A\left(\frac{1}{2} \cdot -\frac{1}{2}\right)\right|^2$

$= A^2$

o) $\Delta x = \lambda/2$

$I(\lambda/2) = |4A(0 \cdot -1)|^2$

$= 0.$

$\Delta x = 2\lambda/3$

$I(2\lambda/3) = \left|4A\left(-\frac{1}{2} \cdot -\frac{1}{2}\right)\right|^2$

$= A^2$

o) $\Delta x = 3\lambda/4$

$I(3\lambda/4) = \left|4A\left(-\frac{\sqrt{2}}{2} \cdot 0\right)\right|^2$

$= 0.$

o) $\Delta x = 0$

$I(0) = |4A(1)|^2$

$= 16A^2$

o) $\Delta x = 1$

$I(1) = |4A(1 \cdot -1)|^2$

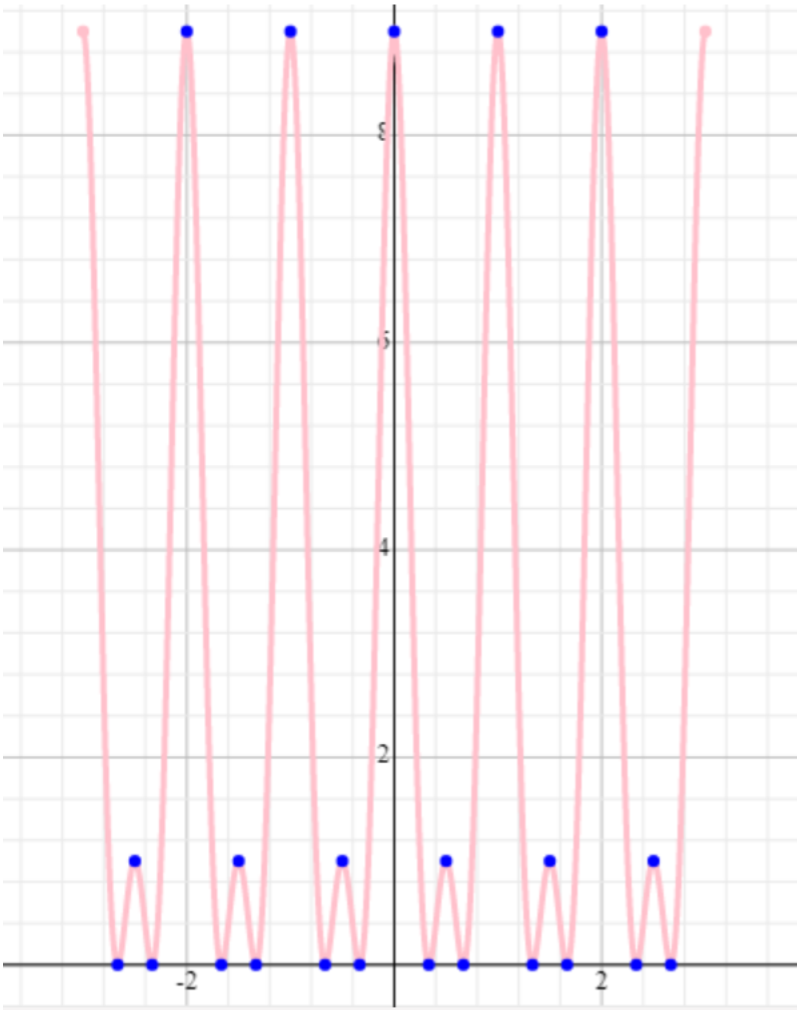
$= 16A^2.$

ketika $\Delta x = 0$, $I(\Delta x) = 16A^2$

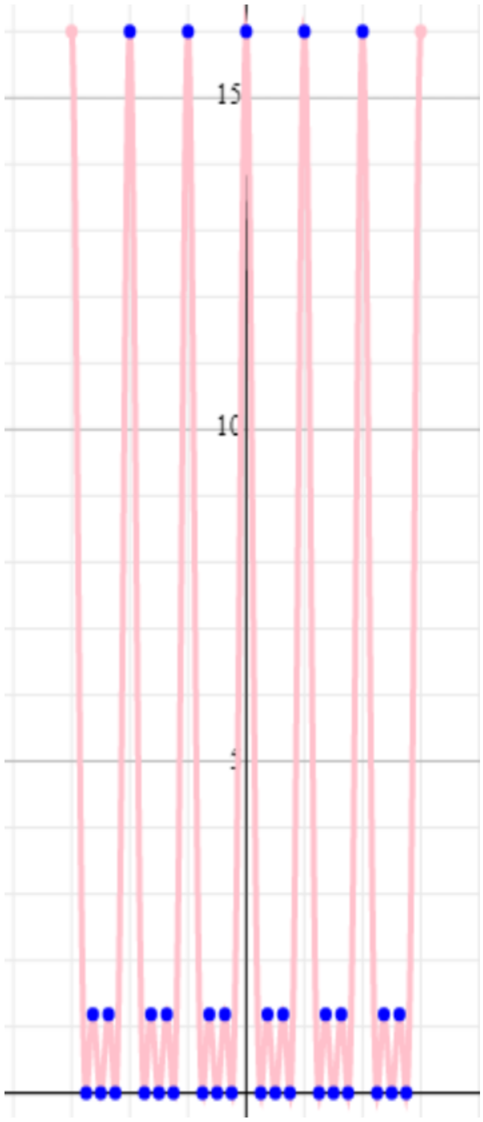
$\Delta x = 1$, $I(\Delta x) = 16A^2$.

berpola

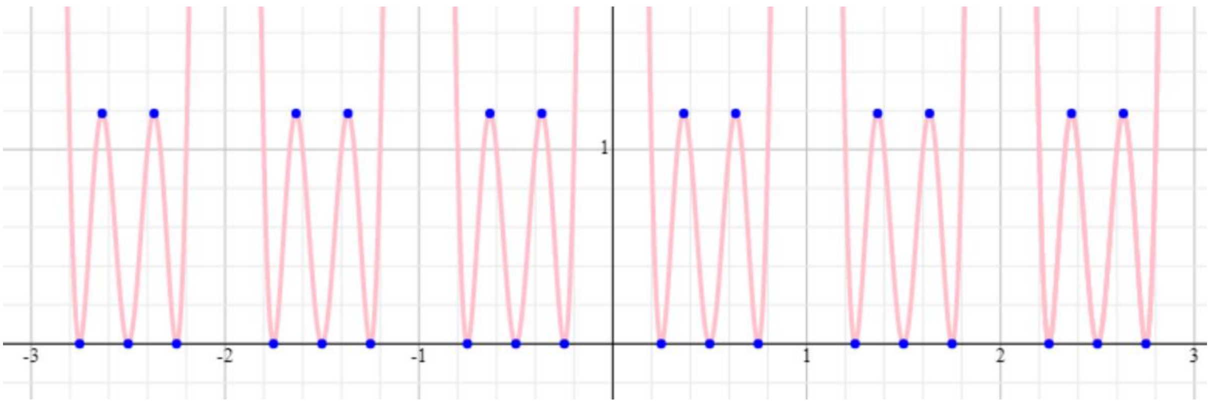
Grafik I untuk 3 sumber gelombang



Grafik I untuk 4 sumber gelombang



Detailnya



4. Siswa tinggal di tempat yang berjarak 2,5 km dari rel kereta. Ia bisa mendengar suara kereta pada malam hari namun tidak pada siang hari.

A) . Penjelasan dari fenomena tersebut yaitu dengan konsep pembiasan. Bunyi akan merambat lebih cepat pada suhu udara yang lebih hangat. Saat siang hari, suhu udara di sekitar permukaan bumi lebih hangat daripada suhu udara di sekitar atmosfer, sehingga bunyi yang merambat dibiaskan menuju atmosfer. Sebaliknya, saat malam hari, suhu udara di sekitar permukaan bumi lebih dingin daripada suhu udara di sekitar atmosfer, sehingga bunyi yang merambat dibiaskan menuju permukaan bumi.
B) Fenomena fatamorgana mirip dengan fenomena ini, karena sama-sama disebabkan karena pembiasan. Suhu yang dingin memiliki kerapatan udara yang lebih tinggi daripada suhu panas. Contohnya pada padang pasir. Ketika siang hari suhu panas, sinar matahari mengenai permukaan padang pasir dan menyebabkan terjadinya perbedaan indeks bias antara padang pasir dengan udara sekitarnya dan akan muncul ilusi optik seperti genangan air.