KUNCI JAWABAN

LATSOL 9 : INTEGRAL GARIS

Referensi:

James Stewart - Multivariable Calculus-Brooks (2012)

1. Page 1088

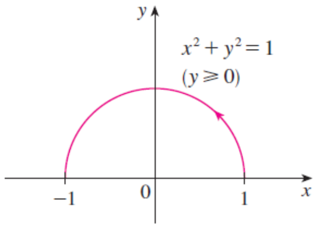

FIGURE 3

EXAMPLE 1 Evaluate $\int_C (2 + x^2y)\, ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$.

SOLUTION In order to use Formula 3, we first need parametric equations to represent C. Recall that the unit circle can be parametrized by means of the equations

$$x = \cos t \qquad y = \sin t$$

and the upper half of the circle is described by the parameter interval $0 \leqslant t \leqslant \pi$. (See Figure 3.) Therefore Formula 3 gives

$$\begin{aligned}
\int_C (2 + x^2y)\, ds &= \int_0^\pi (2 + \cos^2 t \sin t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \\
&= \int_0^\pi (2 + \cos^2 t \sin t)\sqrt{\sin^2 t + \cos^2 t}\, dt \\
&= \int_0^\pi (2 + \cos^2 t \sin t)\, dt = \left[2t - \frac{\cos^3 t}{3}\right]_0^\pi \\
&= 2\pi + \tfrac{2}{3}
\end{aligned}$$

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Figure 12 shows the force field and the curve in Example 7. The work done is negative because the field impedes movement along the curve.

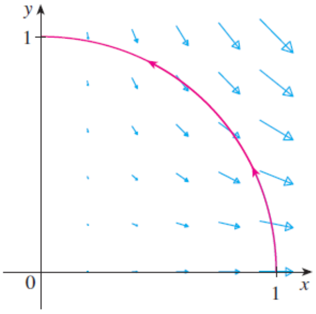

FIGURE 12

EXAMPLE 7 Find the work done by the force field $\mathbf{F}(x, y) = x^2\,\mathbf{i} - xy\,\mathbf{j}$ in moving a particle along the quarter-circle $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j}$, $0 \leqslant t \leqslant \pi/2$.

SOLUTION Since $x = \cos t$ and $y = \sin t$, we have

$$\mathbf{F}(\mathbf{r}(t)) = \cos^2 t\,\mathbf{i} - \cos t \sin t\,\mathbf{j}$$

and

$$\mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j}$$

Therefore the work done is

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{\pi/2} \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\,dt = \int_0^{\pi/2} (-2\cos^2 t \sin t)\,dt$$

$$= 2\,\frac{\cos^3 t}{3}\bigg]_0^{\pi/2} = -\frac{2}{3}$$

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LATSOL 10 : TEOREMA GREEN

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EXAMPLE 1 Evaluate $\int_C x^4\,dx + xy\,dy$, where C is the triangular curve consisting of the line segments from (0, 0) to (1, 0), from (1, 0) to (0, 1), and from (0, 1) to (0, 0).

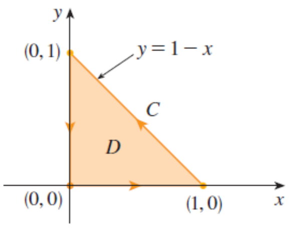

FIGURE 4

SOLUTION Although the given line integral could be evaluated as usual by the methods of Section 16.2, that would involve setting up three separate integrals along the three sides of the triangle, so let's use Green's Theorem instead. Notice that the region D enclosed by C is simple and C has positive orientation (see Figure 4). If we let $P(x, y) = x^4$ and $Q(x, y) = xy$, then we have

$$\int_C x^4\,dx + xy\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_0^1 \int_0^{1-x} (y - 0)\,dy\,dx$$

$$= \int_0^1 \left[\tfrac{1}{2}y^2\right]_{y=0}^{y=1-x} dx = \tfrac{1}{2}\int_0^1 (1 - x)^2\,dx$$

$$= -\tfrac{1}{6}(1 - x)^3\Big]_0^1 = \tfrac{1}{6}$$

LATIHAN SOAL APLIKASI THEOREMA GREEN

$F(x,y) = (-3y, 3x)$

Daerah D dibatasi kurva sederhana c berupa lingkaran

Sirkulasi

$$F(x,y) = (\overset{M}{-3y}, \overset{N}{3x})$$

$M_y = -3$

$N_x = 3$

$$\int_C F \cdot T \, ds = \iint_D (N_x - M_y)\, dA$$

$$= \int_0^{2\pi}\int_0^1 [3-(-3)]\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 6r\,dr\,d\theta$$

$$= \int_0^{2\pi} 3r^2 \Big|_0^1 \, d\theta$$

$$= \int_0^{2\pi} [3-0]\, d\theta$$

$$= \int_0^{2\pi} 3\,d\theta = [3\theta]_0^{2\pi} = 6\pi - 0 = \underline{\underline{6\pi}}$$

Fluks

$M_x = 0$

$N_y = 0$

$$\int_C F \cdot n \, ds = \iint_D (M_x + N_y)\, dA$$

$$= \int_0^{2\pi}\int_0^1 (0+0)\, r\,dr\,d\theta$$

$$= \underline{\underline{0}}$$