$x^2+y^2=16$

$y^2=16-x^2$

$r=4$

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UTS Kalkulus Variabel Tunggal

1. Tentukan

a. $\lim_{x\to 4} \frac{4-x}{5-\sqrt{x^2+9}} = \lim_{x\to 4} \frac{-1}{\frac{-1.2x}{2\sqrt{x^2+9}}}$ (l'hopital)

$= \lim_{x\to 4} \frac{\sqrt{x^2+9}}{x} = \frac{\sqrt{4^2+9}}{4} = \frac{5}{4} = 1{,}25$

b. $\lim_{x\to 0} \frac{x+x\cos x}{\sin x \cos x} = \lim_{x\to 0} \frac{x(1+\cos x)}{\sin x \cos x} = \lim_{x\to 0} \frac{1+\cos x}{\cos^2 x}$

$= \frac{1+\cos 0}{\cos 0} = \frac{1+1}{1} = 2$

2. Tentukan turunan berikut

a. $y = xe^{-x} + e^{3x}$

$y' = e^{-x} + (-1).e^{-x}.x + 3.e^{3x}$

$= e^{-x} - x.e^{-x} + 3.e^{3x}$

$= e^{-x}(1-x+3e^{4x})$

$= \frac{1-x+3e^{4x}}{e^x}$

b. $y = \frac{x\ln x}{1+\ln x}$

$y = \frac{u}{v}$

$u = x\ln x$

$u' = \ln x + \frac{x}{x} = \ln x + 1$

$v = 1+\ln x$

$v' = \frac{1}{x}$

$y' = \frac{u'v - v'u}{v^2}$

$= \frac{(\ln x+1)(\ln x+1) - \frac{x\ln x}{x}}{(1+\ln x)^2}$

$= \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2}$

$= 1 - \frac{\ln x}{(1+\ln x)^2}$

5. $y = (16 - x^2)^{-1/2}$ terhadap $(1, \sqrt{3})$

$y = \frac{1}{\sqrt{16 - x^2}}$ $\quad x^2 + y^2 = 16$

Date

3. Tentukan garis singgung dan garis normal dari $2xy + \pi \sin y = 2\pi$ di $(1, \pi/2)$.

$0 = \frac{d}{dx}\left(2xy + \pi \sin y - 2\pi\right)$

$0 = 2\left(x\frac{dy}{dx} + y\frac{d}{dx}\right) + \pi \cos y \frac{dy}{dx}$

$0 = 2x\frac{dy}{dx} + 2y + \pi \cos y \frac{dy}{dx}$

$-2y = (2x + \pi \cos y)\frac{dy}{dx}$

$\frac{-2y}{2x + \pi \cos y} = \frac{dy}{dx}$

$m = \left.\frac{dy}{dx}\right|_{(1, \pi/2)}$

$m = \frac{-2(\pi/2)}{2(1) + \pi \cos(\pi/2)} = \frac{-\pi}{2}$

* garis singgung

$y - y_1 = m(x - x_1)$

$y - (\pi/2) = -\frac{\pi}{2}(x - 1)$

$y - \pi/2 = -\pi x/2 + \pi/2$

$y = -\pi x/2 + 2\pi/2$

$2y = -\pi x + 2\pi$

$2y + \pi x - 2\pi = 0$

garis Normal

$y - y_1 = \frac{1}{m}(x - x_1)$

$y - (\pi/2) = \frac{-2}{\pi}(x - 1)$

$\pi y - (\pi^2/2) = -2x + 2$

$2\pi y - \pi^2 = -4x + 4$

$2\pi y + 4x - \pi^2 - 4 = 0$

4. $P = I^2 R$

a) Kaitkan antara dP/dt, dR/dt, dan dI/dt

* $dP/dt = \frac{d\, I^2 R}{dt}$

$= 2I \cdot R + I^2$

* $dR/dt = \frac{d\, P/I^2}{dt}$

$= \frac{I^2 - 2I \cdot P}{I^4}$

* $dI/dt = \frac{d\sqrt{P/R}}{dt} = \frac{1}{2} \cdot \frac{P - R \cdot \pi \cdot 1}{R^2 \sqrt{P/R}}$

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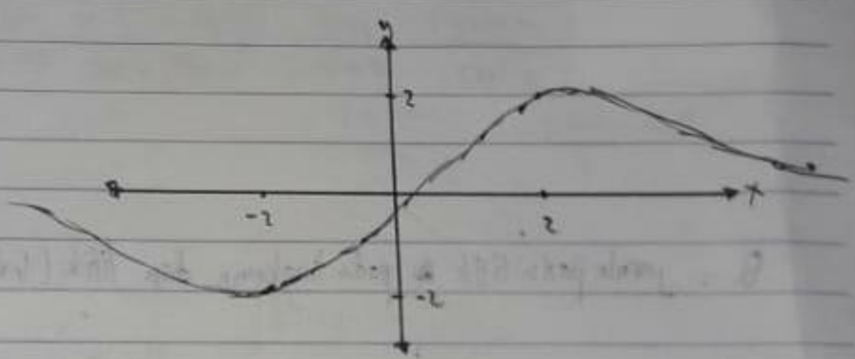
b. $dR/dt = \frac{d\ P/I^2}{dt}$

$= \frac{-P.2I}{I^4}$

$dI/dt = \frac{d\sqrt{P/R}}{dt}$

$= \frac{1}{2\sqrt{P/R}.R^2}$

5. $y = \frac{8x}{x^2+4}$



$0 = y'$

$0 = \frac{8(x^2+4) - 8x(2x)}{(x^2+4)^2}$

$0 = (x^2+4) - 2x^2$

$0 = -x^2+4$

$x^2 = 4$

$x = \pm 2$

$f(2) = \frac{8.2}{2^2+4} = \frac{16}{8} = 2$

$f(-2) = \frac{8.-2}{(-2)^2+4} = \frac{-16}{8} = -2$

Titik maksimum = (2,2)

Titik minimal = (-2,-2)

Interval cekung kebawah saat $x = [-\infty, 2]$

Interval cekung keatas saat $x = [-2, \infty]$

garis asimtot datar => $y = 0$

garis asimtot tegak => $x^2+4 = 0$

$x^2 = -4$

$x = \pm 2i$

$y = \frac{1}{\sqrt{16-x^2}}$

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6. $y = (16-x^2)^{1/2}$ terhadap $(1, \sqrt{3})$

→ $y^2 = 16 - x^2$

$x^2 + y^2 = 4^2$

$r = 4$

P : jarak terhadap pusat lingkaran dan titik $(1, \sqrt{3}) = \sqrt{(1-0)^2 + (\sqrt{3}-0)^2}$

$= \sqrt{1+3}$

$= \sqrt{4} = 2$

titik pada lingkaran $\Rightarrow * \sin\alpha = \frac{x_a}{4}$

$\frac{1}{2} = \frac{x_a}{4} \Leftrightarrow x_a = 2$

$\cos\alpha = \frac{y_a}{4}$

$\frac{\sqrt{3}}{2} = \frac{y_a}{4}$

$2\sqrt{3} = y_a$

Q : jarak pada titik * pada lingkaran dan titik $(1, \sqrt{3}) = \sqrt{(1-x_a)^2 + (\sqrt{3}-y_a)^2}$

$= \sqrt{(1-2)^2 + (\sqrt{3}-2\sqrt{3})^2}$

$= \sqrt{1^2 + \sqrt{3}^2}$

$= \sqrt{4} = 2$