Muchammad Daniyal Kautsar
21/479067/TK/52800

UTS TVM

0. Variable initiation

$NIU = 479067$
$NIF = 52800$

$\beta_1 = 1 + (7+5) \bmod 6 = 1 + 0 = 1$

$\beta_2 = 1 + (9+2) \bmod 6 = 1 + 5 = 6$

$\beta_3 = 1 + (0+8) \bmod 6 = 1 \times 2 = 3$

$\beta_4 = 1 + (6+0) \bmod 6 = 1 + 0 = 1$

$\beta_5 = 1 + (7+0) \bmod 6 = 1 + 1 = 2$

1.

$$\beta_3 x + 2\beta_3 y = 5\beta_3$$
$$3\beta_5 x + 2\beta_5 y = 11\beta_5$$
$$\Rightarrow \quad 3x + 6y = 15$$
$$6x + 4y = 22$$

a. Matrix Multiplication

$$3x + 6y = 15$$
$$6x + 4y = 22$$
$$\Rightarrow \quad \begin{bmatrix} 3 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 22 \end{bmatrix}$$

x^7

b. Find the solution!

$$\begin{bmatrix} 3 & 6 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 22 \end{bmatrix}$$

* Using elimination, ~~be to~~ I will use the non matrix notation.

$3x + 6y = 15$

$6x + 4y = 22$. Eliminating both value of x and y. Thus,

$$* \quad 3x + 6y = 15$$
$$\underline{ \quad 3x + 2y = 11}$$
$$4y = 4$$
$$\underline{\underline{y = 1}}$$

$$* \quad 3x + 6y = 15$$
$$3x + 6(1) = 15$$
$$3x = 15 - 6$$
$$3x = 9$$
$$\underline{\underline{x = 3}}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$. So, the value of $x = 3$ and $y = 1$.

C. Draw column Picture

5. solving $Ax = b$

$$V_1 = \begin{bmatrix} 1\beta_2 \\ 4\beta_2 \\ 2\beta_2 \\ 2\beta_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \\ 12 \\ 12 \end{bmatrix}, \quad V_2 = \begin{bmatrix} -2\beta_1 \\ 5\beta_1 \\ 1\beta_1 \\ 4\beta_1 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \\ 1 \\ 4 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 1\beta_3 \\ -3\beta_3 \\ -3\beta_3 \\ -2\beta_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \\ -9 \\ -6 \end{bmatrix}$$

$$V_4 = \begin{bmatrix} 2\beta_4 \\ -2\beta_4 \\ 3\beta_4 \\ 2\beta_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \\ 2 \end{bmatrix}, \quad V_5 = \begin{bmatrix} 7\beta_3 \\ 3\beta_4 \\ -2\beta_3 \\ 4\beta_2 \end{bmatrix} = \begin{bmatrix} 14 \\ 3 \\ -6 \\ 24 \end{bmatrix}$$

$$A = [V_1, V_2, V_3, V_4]$$

$$A = \begin{bmatrix} 6 & -2 & 3 & 2 \\ 24 & 5 & -9 & -2 \\ 12 & 1 & -9 & 3 \\ 12 & 4 & -6 & 2 \end{bmatrix}$$

$$b = V_5$$

$$b = \begin{bmatrix} 14 \\ 3 \\ -6 \\ 24 \end{bmatrix}$$

a.) Gaus elimination

Augmented matrix representation

$$\begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 24 & 5 & -9 & -2 & | & 3 \\ 12 & 1 & -9 & 3 & | & -6 \\ 12 & 4 & -6 & 2 & | & 24 \end{bmatrix} \quad * \quad E_{41} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 24 & 5 & -9 & -2 & | & 3 \\ 12 & 1 & -9 & 3 & | & -6 \\ 12 & 4 & -6 & 2 & | & 24 \end{bmatrix}}_{E_{41}}$$

$$= \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 24 & 5 & -9 & -2 & | & 3 \\ 12 & 1 & -9 & 3 & | & -6 \\ 0 & 8 & -12 & -2 & | & -4 \end{bmatrix}$$

$$* \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{E_{31}} \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 24 & 5 & -9 & -2 & | & 3 \\ 12 & 1 & -9 & 3 & | & -6 \\ 0 & 8 & -12 & -2 & | & -4 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 24 & 5 & -9 & -2 & | & 3 \\ 0 & 5 & -15 & -1 & | & -34 \\ 0 & 8 & -12 & -2 & | & -4 \end{bmatrix}$$

$$* \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{E_{21}} \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 24 & 5 & -9 & -2 & | & 3 \\ 0 & 5 & -15 & -1 & | & -34 \\ 0 & 8 & -12 & -2 & | & -4 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 0 & 13 & -21 & -10 & | & -53 \\ 0 & 5 & -15 & -1 & | & -34 \\ 0 & 8 & -12 & -2 & | & -4 \end{bmatrix}$$

$*$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{8}{13} & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 0 & 13 & -21 & -10 & | & -53 \\ 0 & 5 & -15 & -1 & | & -74 \\ 0 & 8 & -12 & -2 & | & -4 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 0 & 13 & -21 & -10 & | & -53 \\ 0 & 5 & -15 & -1 & | & -34 \\ 0 & 0 & 12/13 & 54/13 & | & 372/13 \end{bmatrix}$$

$\underbrace{\qquad\qquad}$
E_{42}

$*$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5/13 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 0 & 13 & -21 & -10 & | & -53 \\ 0 & 5 & -15 & -1 & | & -34 \\ 0 & 0 & 12/13 & 54/13 & | & 372/13 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 0 & 13 & -21 & -10 & | & -53 \\ 0 & 0 & -90/13 & 37/13 & | & -173/13 \\ 0 & 0 & 12/13 & 54/13 & | & 372/13 \end{bmatrix}$$

$\underbrace{\qquad\qquad}$
E_{32}

$*$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{15} & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 0 & 13 & -21 & -10 & | & -53 \\ 0 & 0 & -90/13 & 37/13 & | & -173/13 \\ 0 & 0 & 12/13 & 54/13 & | & 372/13 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & | & 14 \\ 0 & 13 & -21 & -10 & | & -53 \\ 0 & 0 & -90/13 & 37/13 & | & -173/13 \\ 0 & 0 & 0 & 68/15 & | & 134/5 \end{bmatrix}$$

$*$ $6x_1 + -2x_2 + 3x_3 + 2x_4 = 14$

$13x_2 - 21x_3 - 10x_4 = -53$

$-90/13 x_3 + 37/13 x_4 = -173/13$

$68/15 \, x_4 = 134/5$

$*$ $x_4 = \dfrac{134}{5} \cdot \dfrac{15}{68} = \dfrac{201}{34}$

$*$ $\dfrac{-90}{13} x_3 + \dfrac{37}{13} \cdot \dfrac{201}{34} = \dfrac{-173}{13}$

$x_3 = \dfrac{299}{68}$

$*$ $13 x_2 - 21 \cdot \dfrac{299}{68} - 10 \cdot \dfrac{201}{34} = -53$

$x_2 = \dfrac{515}{68}$

$*$ $6x_1 - 2 \cdot \dfrac{515}{68} + 3 \cdot \dfrac{299}{68} + 2 \cdot \dfrac{201}{34} = 14$

$x_1 = \dfrac{281}{408}$

Therefore, $x_1 = \dfrac{281}{408}$, $x_2 = \dfrac{515}{68}$, $x_3 = \dfrac{299}{68}$, $x_4 = \dfrac{201}{34}$

$$E_{41} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix} \quad , \quad E_{41}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad E_{31}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{42} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{8}{13} & 0 & 1 \end{bmatrix} \quad , \quad E_{42}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{8}{13} & 0 & 1 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5/13 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 5/13 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{15} & 1 \end{bmatrix} \quad , \quad E_{43}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{15} & 1 \end{bmatrix}$$

b Solve using $A = LU$

Since $A = LU$

$A = \overset{-1}{E} U$, where E^{-1} is already calculate from question A.

$E^{-1} = E_{21}^{-1} E_{31}^{-1} E_{41}^{-1} E_{32}^{-1} E_{42}^{-1} E_{43}^{-1}$

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ -2 & -5/13 & 1 & 0 \\ -2 & -8/13 & 2/15 & 1 \end{bmatrix} = L$$

and
$$U = \begin{bmatrix} 6 & -2 & 3 & 2 \\ 0 & 13 & -21 & -10 \\ 0 & 0 & -90/13 & 37/13 \\ 0 & 0 & 0 & 68/15 \end{bmatrix}$$

* $Lc = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ -2 & -5/13 & 1 & 0 \\ -2 & -8/13 & 2/15 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 19 \\ 3 \\ -6 \\ 24 \end{bmatrix}$$

* $c_1 = 19$
* $-4c_1 + c_2 = 13 \quad \to -4 \cdot 14 + c_2 = 13 \to c_2 = 69$
* $-2c_1 - \dfrac{5}{13} c_2 + c_3 = -6 \to -2 \cdot 14 - \dfrac{5}{13} \cdot 69 + c_3 = -6 \to c_3 = \dfrac{631}{13}$
* $-2c_1 - \dfrac{8}{13} c_2 + \dfrac{2}{15} c_3 + c_4 = 24 \to \quad c_4 = \dfrac{16183}{195}$

* $Ux = c$

$$\begin{bmatrix} 6 & -2 & 3 & 2 \\ 0 & 13 & -21 & -10 \\ 0 & 0 & -90/13 & 37/13 \\ 0 & 0 & 0 & 68/15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$