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Quiz 1 KVJ

1. Volume S yg berada dibawah $z = 3 + 2\cos x^2$ dan dibatasi oleh segitiga pada bidang xy dengan titik sudut $(0,0)$, $(6,0)$ dan $(6,2)$

$$\int_0^2 \int_0^6 (3 + 2\cos x^2) dx dy$$

$$\int_0^2 \int_0^6 (3 + 2\cos x^2) dy dx = \int_0^6 [3y + 2y\cos x^2]_0^2 dx = \int_0^6 (6 + 4\cos x^2) dx$$

$$= \int_0^6 (6 + 4\cos x^2) dx = 6x + 2\sin x^2 \Big|_0^6 = 36 + 2\sin 36$$

$$= 36 + 2(0.59) = 37.18$$

2. Volume S , $S = \{(x,y,z) \in \mathbb{R}^3 \mid z^2 \leq y \leq 2 - 2x^2 - z^2, x \geq 0\}$

$$\int_0^1 \int_0^{\sqrt{2-2x^2}} \int_0^{\sqrt{y}} dz dy dx$$

$$= \int_0^1 \int_0^{\sqrt{2-2x^2}} \frac{1}{2} y^{1/2} dy dx$$

$$= \int_0^1 \left[\frac{1}{2} \cdot \frac{2}{3} y^{3/2} \right]_0^{\sqrt{2-2x^2}} dx = \frac{1}{3} \int_0^1 (2-2x^2)^{3/2} dx$$

$$= \frac{1}{3} \int_0^1 2^{3/2} (1-x^2)^{3/2} dx = \frac{2\sqrt{2}}{3} \int_0^1 (1-x^2)^{3/2} dx$$

$$= \frac{2\sqrt{2}}{3} \left[\frac{x}{8} (1-x^2)^{3/2} + \frac{3}{8} \int (1-x^2)^{1/2} dx \right]_0^1$$

$$= \frac{2\sqrt{2}}{3} \left[\frac{1}{8} (1-1)^{3/2} + \frac{3}{8} \int_0^1 (1-x^2)^{1/2} dx \right]$$

$$= \frac{2\sqrt{2}}{3} \left[\frac{3}{8} \int_0^1 (1-x^2)^{1/2} dx \right]$$

$$= \frac{2\sqrt{2}}{3} \left[\frac{3}{8} \cdot \frac{\pi}{4} \right] = \frac{\pi\sqrt{2}}{4}$$

$$= \frac{\pi\sqrt{2}}{4} \approx 1.107$$

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$$3. \int_C F(x,y) ds \quad C \rightarrow x^2 + y^2 = 4, (0,0) \rightarrow (-\sqrt{3}, 1)$$

$$f(x,y) = x(y+1) + y$$

$$f(x,y) = xy + x + y$$

$$x = t, \quad -\sqrt{3} \leq t < 0$$

$$y = \sqrt{4-t^2}$$

$$x \left(\frac{dx}{dt} \right)^2 = 1 \quad * \left(\frac{dy}{dt} \right)^2 = \left(\frac{d\sqrt{4-t^2}}{dt} \right)^2 = \left(\frac{-t}{\sqrt{4-t^2}} \right)^2 = \frac{t^2}{4-t^2}$$

$$* ds = \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2} dt$$

$$= \sqrt{1 + \frac{t^2}{4-t^2}} dt = \sqrt{\frac{4}{4-t^2}} dt = \frac{2}{\sqrt{4-t^2}} dt$$

$$* \int_C xy + x + y ds = \int_{-\sqrt{3}}^0 (\sqrt{4-t^2} \cdot t + t + \sqrt{4-t^2}) \cdot \frac{2}{\sqrt{4-t^2}} dt$$

$$= \int_{-\sqrt{3}}^0 (t + \sqrt{4-t^2}(t+1)) \cdot \frac{2}{\sqrt{4-t^2}} dt$$

$$= \int_{-\sqrt{3}}^0 \frac{2t}{\sqrt{4-t^2}} + 2 + 2 dt = \left[-2\sqrt{4-t^2} + t^2 + 2t \right]_{-\sqrt{3}}^0$$

$$= -5 + 2\sqrt{3} \approx -1,5359$$

$$4. F = (x+y)i + (1-y)j \quad \text{sepanjang } \frac{x^2}{9} + \frac{y^2}{4} = 1 \quad \text{CCW kuadran 4.}$$

$$\frac{3\pi}{2} \leq t \leq 2\pi$$

$$x = 3\cos(t) \rightarrow dx/dt = -3\sin(t)$$

$$y = 2\sin(t) \rightarrow dy/dt = 2\cos(t)$$

$$W = \int_C F \cdot dr = \int_C p(x,y) dx + q(x,y) dy$$

$$p(x,y) = x+y$$

$$q(x,y) = 1-y$$

$$W = \int_{\frac{3\pi}{2}}^{2\pi} x+y dx + 1-y dy$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} (3\cos(t) + 2\sin(t)) \cdot (-3\sin(t)) dt + (1 - 2\sin(t)) \cdot 2\cos(t) \cdot dt$$

$$= \frac{17-3\pi}{2} \approx 3.788$$

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$$3. \int_C f(x,y) \, dr \quad C \rightarrow x^2 + y^2 = 4 \quad (0,0) \rightarrow (-\sqrt{3}, 1) \\ f(x,y) = (x(y+1) + y)$$

(cara lain).

$$* \quad x = \sqrt{4-t^2} \\ y = t \quad 0 \leq t \leq 1$$

$$* \quad \left(\frac{dx}{dt} \right)^2 = \left(\frac{d\sqrt{4-t^2}}{dt} \right)^2 = \left(\frac{-t}{\sqrt{4-t^2}} \right)^2 = \frac{t^2}{4-t^2}$$

$$* \quad \left(\frac{dy}{dt} \right)^2 = 1$$

$$* \quad dr = \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2} \, dt = \sqrt{\frac{t^2}{4-t^2} + 1} \, dt = \sqrt{\frac{4}{4-t^2}} \, dt = \frac{2}{\sqrt{4-t^2}} \, dt$$

$$\begin{aligned} * \quad \int_C xy + x + y \, dr &= \int_0^1 (t\sqrt{4-t^2} + \sqrt{4-t^2} + t) \cdot \frac{2}{\sqrt{4-t^2}} \, dt \\ &= \int_0^1 ((t+1)(\sqrt{4-t^2}) + t) \cdot \frac{2}{\sqrt{4-t^2}} \, dt \\ &= \int_0^1 2t + 2 + \frac{2t}{\sqrt{4-t^2}} \, dt = t^2 + 2t - 2\sqrt{4-t^2} \Big|_0^1 \\ &= 1 + 2 - 2\sqrt{4-1} \\ &= 3 - 2\sqrt{3} \approx 0,969 \end{aligned}$$