

Muchammad Daniyal Kautsar
21/479 067 /TK/52 800

UTS TVM

Pakta Integritas

"Saya yang bertanda tangan dibawah ini, secara sadar dan sungguh-sungguh akan mengerjakan soal Ujian Tengah Semester Teori Vektor dan Matriks, tidak bertanya, berdiskusi, dan bekerja sama dengan teman/orang lain, tidak mencari pertolongan dengan cara, media dan bentuk apapun, dan tidak akan saling membagi jawaban selama masa ujian berlangsung. Bila saya melanggar, saya siap menerima konsekuensi berupa UTS saya tidak akan dinilai sama sekali dan dianggap bernilai Nol."

Sleman, 14 Desember 2021



Muchammad Daniyal Kautsar

0. Variable Initiation

$$NIV = 479\ 067$$

$$NIF = 52\ 800$$

$$\beta_1 = 1 + (7+5) \bmod 6 = 1 + 0 = 1$$

$$\beta_2 = 1 + (9+2) \bmod 6 = 1 + 5 = 6$$

$$\beta_3 = 1 + (0+8) \bmod 6 = 1 + 2 = 3$$

$$\beta_4 = 1 + (6+0) \bmod 6 = 1 + 0 = 1$$

$$\beta_5 = 1 + (7+0) \bmod 6 = 1 + 1 = 2$$

1.

$$\begin{aligned}\beta_3 x + 2\beta_5 y &= 5\beta_3 \\ 3\beta_5 x + 2\beta_5 y &= 11\beta_5\end{aligned}$$

$$\Rightarrow \begin{aligned}3x + 6y &= 15 \\ 6x + 4y &= 22\end{aligned}$$

a. Matrix Multiplication

$$3x + 6y = 15$$

$$6x + 4y = 22$$

\Rightarrow

$$\begin{bmatrix} 3 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 22 \end{bmatrix}$$

b. Find the solution!

$$\begin{bmatrix} 3 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 22 \end{bmatrix}$$

* Using elimination, ~~we~~ I will use the non matrix notation.

$$3x + 6y = 15$$

$$6x + 4y = 22 \quad \text{Eliminating both value of } x \text{ and } y. \text{ Thus,}$$

$$\begin{array}{r} * \quad 3x + 6y = 15 \\ \quad 3x + 2y = 11 \\ \hline \quad \quad 4y = 4 \end{array}$$

$$y = 1$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

So, the value of $x = 3$ and $y = 1$.

$$\begin{array}{r} * \quad 3x + 6y = 15 \\ \quad 3x + 6(1) = 15 \\ \quad 3x = 15 - 6 \end{array}$$

$$3x = 9$$

$$x = 3$$

c. Draw column picture

5. solving $Ax=b$

$$v_1 = \begin{bmatrix} 1\beta_1 \\ 2\beta_1 \\ 2\beta_2 \\ 2\beta_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \\ 12 \\ 12 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -2\beta_1 \\ 5\beta_1 \\ 1\beta_1 \\ 9\beta_1 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \\ 1 \\ 4 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 1\beta_3 \\ -3\beta_3 \\ -3\beta_3 \\ -2\beta_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \\ -9 \\ -6 \end{bmatrix}$$

$$v_4 = \begin{bmatrix} 2\beta_4 \\ -2\beta_4 \\ 3\beta_4 \\ 2\beta_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \\ 2 \end{bmatrix}$$

$$v_5 = \begin{bmatrix} 7\beta_5 \\ 3\beta_5 \\ -2\beta_5 \\ 4\beta_5 \end{bmatrix} = \begin{bmatrix} 14 \\ 3 \\ -6 \\ 24 \end{bmatrix}$$

$$A = [v_1, v_2, v_3, v_4]$$

$$A = \begin{bmatrix} 6 & -2 & 3 & 2 \\ 24 & 5 & -9 & -2 \\ 12 & 1 & -9 & 3 \\ 12 & 4 & -6 & 2 \end{bmatrix}$$

$$b = v_5$$

$$b = \begin{bmatrix} 14 \\ 3 \\ -6 \\ 24 \end{bmatrix}$$

a) Gauss elimination

Augmented matrix representation

$$\left[\begin{array}{cccc|c} 6 & -2 & 3 & 2 & 14 \\ 24 & 5 & -9 & -2 & 3 \\ 12 & 1 & -9 & 3 & -6 \\ 12 & 4 & -6 & 2 & 24 \end{array} \right]$$

$$\begin{array}{l} * \text{ } E_{41} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & \\ -2 & 0 & 0 & 1 & \end{array} \right] \left[\begin{array}{cccc|c} 6 & -2 & 3 & 2 & 14 \\ 24 & 5 & -9 & -2 & 3 \\ 12 & 1 & -9 & 3 & -6 \\ 12 & 4 & -6 & 2 & 24 \end{array} \right] \end{array}$$

$$= \left[\begin{array}{cccc|c} 6 & -2 & 3 & 2 & 14 \\ 24 & 5 & -9 & -2 & 3 \\ 12 & 1 & -9 & 3 & -6 \\ 0 & 8 & -12 & -2 & -4 \end{array} \right]$$

$$\begin{array}{l} * \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ -2 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{array} \right] \left[\begin{array}{cccc|c} 6 & -2 & 3 & 2 & 14 \\ 24 & 5 & -9 & -2 & 3 \\ 12 & 1 & -9 & 3 & -6 \\ 0 & 8 & -12 & -2 & -4 \end{array} \right] = \left[\begin{array}{cccc|c} 6 & -2 & 3 & 2 & 14 \\ 24 & 5 & -9 & -2 & 3 \\ 0 & 5 & -15 & -1 & -34 \\ 0 & 8 & -12 & -2 & -4 \end{array} \right] \end{array}$$

$$\begin{array}{l} * \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ -4 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{array} \right] \left[\begin{array}{cccc|c} 6 & -2 & 3 & 2 & 14 \\ 24 & 5 & -9 & -2 & 3 \\ 0 & 5 & -15 & -1 & -34 \\ 0 & 8 & -12 & -2 & -4 \end{array} \right] = \left[\begin{array}{cccc|c} 6 & -2 & 3 & 2 & 14 \\ 0 & 13 & -21 & -10 & -53 \\ 0 & 5 & -15 & -1 & -34 \\ 0 & 8 & -12 & -2 & -4 \end{array} \right] \end{array}$$

$$\star \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{8}{13} & 0 & 1 \end{bmatrix}}_{E_{41}} \begin{bmatrix} 6 & -2 & 3 & 2 & 14 \\ 0 & 13 & -21 & -10 & -53 \\ 0 & 5 & -15 & -1 & -34 \\ 0 & 0 & -12 & -2 & -4 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & 14 \\ 0 & 13 & -21 & -10 & -53 \\ 0 & 5 & -15 & -1 & -34 \\ 0 & 0 & 12/13 & 54/13 & 372/13 \end{bmatrix}$$

$$\star \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5/13 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{E_{32}} \begin{bmatrix} 6 & -2 & 3 & 2 & 14 \\ 0 & 13 & -21 & -10 & -53 \\ 0 & 5 & -15 & -1 & -34 \\ 0 & 0 & 12/13 & 54/13 & 372/13 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & 14 \\ 0 & 13 & -21 & -10 & -53 \\ 0 & 0 & -90/13 & 37/13 & -173/13 \\ 0 & 0 & 12/13 & 54/13 & 372/13 \end{bmatrix}$$

$$\star \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{15} & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 & 3 & 2 & 14 \\ 0 & 13 & -21 & -10 & -53 \\ 0 & 0 & -90/13 & 37/13 & -173/13 \\ 0 & 0 & 12/13 & 54/13 & 372/13 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 3 & 2 & 14 \\ 0 & 13 & -21 & -10 & -53 \\ 0 & 0 & -90/13 & 37/13 & -173/13 \\ 0 & 0 & 0 & 68/15 & 134/5 \end{bmatrix}$$

$$\star \begin{aligned} 6x_1 - 2x_2 + 3x_3 + 2x_4 &= 14 \\ 13x_2 - 21x_3 - 10x_4 &= -53 \\ -90/13 x_3 + 37/13 x_4 &= -173/13 \\ 68/15 x_4 &= 134/5 \end{aligned}$$

$$\star x_4 = \frac{134}{5} \cdot \frac{15}{68} = \frac{201}{34}$$

$$\star \frac{-90}{13} x_3 + \frac{37}{13} \cdot \frac{201}{34} = \frac{-173}{13}$$

$$x_3 = \frac{299}{68}$$

$$\star 13x_2 - 21 \cdot \frac{299}{68} - 10 \cdot \frac{201}{34} = -53$$

$$x_2 = \frac{515}{68}$$

$$\star 6x_1 - 2 \cdot \frac{515}{68} + 3 \cdot \frac{299}{68} + 2 \cdot \frac{201}{34} = 14$$

$$x_1 = \frac{281}{408}$$

Therefore, $x_1 = \frac{281}{408}$, $x_2 = \frac{515}{68}$, $x_3 = \frac{299}{68}$, $x_4 = \frac{201}{34}$

$$E_{41} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix}, \quad E_{41}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_{31}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{42} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{8}{13} & 0 & 1 \end{bmatrix}, \quad E_{42}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{8}{13} & 0 & 1 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5/13 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 5/13 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{15} & 1 \end{bmatrix}, \quad E_{43}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{2}{15} & 1 \end{bmatrix}$$

b. Solve using $A = LU$

Since $A = LU$

$A = E^{-1}U$, where E^{-1} is already calculate from question A.

$$E^{-1} = E_{21}^{-1} E_{31}^{-1} E_{41}^{-1} E_{32}^{-1} E_{42}^{-1} E_{43}^{-1}$$

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ -2 & -5/13 & 1 & 0 \\ -2 & -8/13 & 2/15 & 1 \end{bmatrix} = L$$

$$\text{and } U = \begin{bmatrix} 6 & -2 & 3 & 2 \\ 0 & 13 & -21 & -10 \\ 0 & 0 & -90/13 & 37/13 \\ 0 & 0 & 0 & 68/15 \end{bmatrix}$$

$$* Lc = b$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -9 & 1 & 0 & 0 \\ -2 & -5/13 & 1 & 0 \\ -2 & -8/13 & 2/15 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 19 \\ 13 \\ -6 \\ 24 \end{bmatrix}$$

$$* C_1 = 19$$

$$* -4C_1 + C_2 = 13 \rightarrow -4 \cdot 19 + C_2 = 13 \rightarrow C_2 = 69$$

$$* -2C_1 - \frac{5}{13}C_2 + C_3 = -6 \rightarrow -2 \cdot 19 - \frac{5}{13} \cdot 69 + C_3 = -6 \rightarrow C_3 = \frac{631}{13}$$

$$* -2C_1 - \frac{8}{13}C_2 + \frac{2}{15}C_3 + C_4 = 24 \rightarrow C_4 = \frac{16183}{195}$$

$$* Ux = C$$

$$\begin{bmatrix} 6 & -2 & 3 & 2 \\ 0 & 13 & -21 & -10 \\ 0 & 0 & -90/13 & 32/13 \\ 0 & 0 & 0 & 68/15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 19 \\ 13 \\ -6 \\ 24 \end{bmatrix}$$