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Quiz 1 KVJ

1. Volume S yg berada dibawah $z = 3 + 2\cos x^2$ dan dibatasi oleh segitiga pada bidang xy dengan titik sudut (0,0), (6,0) dan (6,2)

$\cancel{\int_0^2 \int_0^{3y} 3 + 2\cos x^2 \, dx\, dy}$

$\int_0^6 \int_0^{\frac{1}{3}x} 3 + 2\cos x^2 \, dy\, dx = \int_0^6 3y + 2y\cos x^2 \Big]_0^{\frac{1}{3}x} dx$

$= \int_0^6 x + \frac{2}{3}x \cos x^2 \, dx$

$= 17{,}669$

2. Volume S. $S = \{(x,y,z) \in \mathbb{R}^3 \mid z^2 \leq y \leq 2 - 2x^2 - z^2,\ x \geq 0\}$

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3. $\int_C f(x,y)\,ds$ $\quad C \to x^2+y^2=4,\ (0,0) \to (-\sqrt{3},1)$

$f(x,y) = x(y+1)+y$

$f(x,y) = xy+x+y$

* $x=t$, $-\sqrt{3} \le t < 0$

$y=\sqrt{4-t^2}$

$\times \left(\frac{dx}{dt}\right)^2 = 1$ $\qquad * \left(\frac{dy}{dt}\right)^2 = \left(\frac{d\sqrt{4-t^2}}{dt}\right)^2 = \left(\frac{-t}{\sqrt{4-t^2}}\right)^2 = \frac{t^2}{4-t^2}$

* $ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$= \sqrt{1+\frac{t^2}{4-t^2}}\,dt = \sqrt{\frac{4}{4-t^2}}\,dt = \frac{2}{\sqrt{4-t^2}}\,dt$

* $\int_C xy+x+y\,ds = \int_{-\sqrt{3}}^{0} \left(\sqrt{4-t^2}.t + t + \sqrt{4-t^2}\right) . \frac{2}{\sqrt{4-t^2}}\,dt$

$= \int_{-\sqrt{3}}^{0} \left(t+\sqrt{4-t^2}(t+1)\right) . \frac{2}{\sqrt{4-t^2}}\,dt$

$= \int_{\sqrt{3}}^{0} \frac{2t}{\sqrt{4-t^2}} + 2t+2\,dt = -2\sqrt{4-t^2}+t^2+2t\Big]_{-\sqrt{3}}^{0}$

$= -5+2\sqrt{3} \approx -1,5359.$

4. $F = (x+y)\,i + (1-y)\,j$ sepanjang $\frac{x^2}{9}+\frac{y^2}{4}=1$ CCW kuadran 4.

$\frac{3\pi}{2} < t \le 2\pi$

$x = 3\cos(t) \to dx/dt = -3\sin(t)$

$y = 2\sin(t) \to dy/dt = 2\cos(t)$

$W = \int_C F.dr = \int_C P(x,y)\,dx + q(x,y)\,dy$

$P(x,y) = x+y$

$q(x,y) = 1-y$

$W = \int_{\frac{3\pi}{2}}^{2\pi} x+y\ \cancel{dy}\ dx + 1-y\ dy$

$= \int_{\frac{3\pi}{2}}^{2\pi} (3\cos(t)+2\sin(t)).-3\sin(t).dt + (1-2\sin(t)).2\cos(t).dt$

$= \frac{17-3\pi}{2} \approx 3.788$

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3. $\int_C f(x,y)\, dr$ $\quad C \to x^2+y^2=4 \quad (0,0) \to (-\sqrt{3},1)$

$f(x,y) = (x(y+1)+y)$

(cara lain).

* $x = \sqrt{4-t^2}$

$y = t \qquad 0 \le t \le 1$

* $\left(\frac{dx}{dt}\right)^2 = \left(\frac{d\sqrt{4-t^2}}{dt}\right)^2 = \left(\frac{-t}{\sqrt{4-t^2}}\right)^2 = \frac{t}{4-t^2}$

* $\left(\frac{dy}{dt}\right)^2 = 1$

* $dr = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \sqrt{\frac{t}{4t^2}+1}\, dt = \sqrt{\frac{4}{4-t^2}}\, dt = \frac{2}{\sqrt{4-t^2}}\, dt$

* $\int_C xy+x+y\, dr = \int_0^1 \left(t\sqrt{4-t^2} + \sqrt{4-t^2} + t\right) \cdot \frac{2}{\sqrt{4-t^2}}\, dt$

$= \int_0^1 \left((t+1)\left(\sqrt{4-t^2}\right) + t\right) \frac{2}{\sqrt{4-t^2}}\, dt$

$= \int_0^1 2t+2 + \frac{2t}{\sqrt{4-t^2}}\, dt = t^2+2t - 2\sqrt{4-t^2} \Big|_0^1$

$= 1+2 - 2\sqrt{4-1}$

$= 3 - 2\sqrt{3} \approx -0,464$

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