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Tugas 1 : Vektor dan Matriks

1. $A_{3\times 2} = \begin{pmatrix} 6 & -3 \\ 6 & -2 \\ 6 & -1 \end{pmatrix}$

$$A^T \times A = \begin{pmatrix} 6 & 6 & 6 \\ -3 & -2 & -1 \end{pmatrix} \times \begin{pmatrix} 6 & -3 \\ 6 & -2 \\ 6 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} 108 & -36 \\ -36 & 19 \end{pmatrix}$$

$$A \times A^T = \begin{pmatrix} 6 & -3 \\ 6 & -2 \\ 6 & -1 \end{pmatrix} \times \begin{pmatrix} 6 & 6 & 6 \\ -3 & -2 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} 45 & 42 & 39 \\ 42 & 40 & 38 \\ 39 & 38 & 37 \end{pmatrix}$$

$$|A \times A^T| = \begin{vmatrix} 45 & 42 & 39 \\ 42 & 40 & 38 \\ 39 & 38 & 37 \end{vmatrix}$$
$$= (45\cdot 40\cdot 37 + 42\cdot 38\cdot 39 + 39\cdot 38\cdot 42) - (39\cdot 40\cdot 39 + 38\cdot 38\cdot 45 + 37\cdot 42\cdot 42)$$
$$= (66600 + 62244 + 62244) - (60840 + 64980 + 65268)$$
$$= \cancel{790488}\ 191088 - 191088$$
$$= 0$$

2. $2x + 3y + 4z = 10$
$x - 2y + 3z = -2$
$x - y - z = 26$

$$\begin{pmatrix} 2 & 3 & 4 \\ 1 & -2 & 3 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \\ 26 \end{pmatrix}$$
$$A\,x = B$$

$$Kof(A) = \begin{pmatrix} 24 & 4 & 20 \\ -1 & -6 & 5 \\ 93 & -2 & -45 \end{pmatrix}$$

$$Adj\,A = \begin{pmatrix} 24 & -1 & 93 \\ 4 & -6 & -2 \\ 20 & 5 & -45 \end{pmatrix}$$

$$\det(A) = (42 + 9 + -4) - (-84 - 6 - 3)$$
$$= 140$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{140} \begin{pmatrix} 24 & -1 & 93 \\ 4 & -6 & -2 \\ 20 & 5 & -45 \end{pmatrix} \begin{pmatrix} 10 \\ -2 \\ 26 \end{pmatrix}$$
$$= \begin{pmatrix} 19 \\ 0 \\ -7 \end{pmatrix}$$

$x = 19$
$y = 0$
$z = -7$

Date

3. Bidang α // $\frac{2}{3} - x = \frac{y-7}{3} = 11 - z$ dan $x - 1 = \frac{y-2}{2} = \frac{z-3}{3}$

Tentukan pers bidang α.

* $T(x_0, y_0, z_0)$

$\frac{2-3x}{3} = \frac{y-7}{3} = 11 - z$

$3x = 2 + 3t \iff x = \frac{2}{3} + 3t$

$y = 7 + 3t$

$z = 11 + t$

$P(\frac{2}{3}, 7, 11)$

$N = (1, 3, 1) = (a, b, c)$

$PT = (x_0 - \frac{2}{3}, y_0 - 7, z_0 - 11)$

Pers. bidang

$n \cdot PT = 0$

~~$(1, 3, 1) \cdot (x_0 - \frac{2}{3}, y_0 - 7, z_0 - 11) = 0$~~

$\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x - x_0 + \frac{2}{3} \\ y - y_0 + 7 \\ z - z_0 + 11 \end{pmatrix} = 0$

$x - x_0 + \frac{2}{3} + 3y - 3y_0 + 21 + z - z_0 + 11 = 0$

$x + 3y + z - x_0 - 3y_0 - z_0 + \frac{98}{3} = 0$

* $T(x_0, y_0, z_0)$

$x - 1 = \frac{y-2}{2} = \frac{z-3}{3}$

$x = 1 + t$

$y = 2 + 2t$

$z = 3 + 3t$

$P(1, 2, 3)$

$N = (1, 2, 3) = (a, b, c)$

$PT = (x_0 - 1, y_0 - 2, z_0 - 3)$

$n \cdot PT = 0$

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} x - x_0 + 1 \\ y - y_0 + 2 \\ z - z_0 + 3 \end{pmatrix} = 0$

$x - x_0 + 1 + 2y - 2y_0 + 4 + 3z - 3z_0 + 9 = 0$

$x + 2y + 3z - x_0 - 2y_0 - 3z_0 + 14 = 0$

Date

9. Bidang α ~~A~~ melalui $(2,1,3)$, memuat garis g $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{5}$.
Jarak $(1,1,1)$ ke α = ?

Pers. parametrik

$$T(2,1,3)$$
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{5}$$

$$\left.\begin{aligned} x &= 1+2t \\ y &= 2+3t \\ z &= 4+5t \end{aligned}\right\} P(1,2,4)$$

$$\vec{N} = (2,3,5) = (a,b,c)$$
$$\cancel{PT = (-1,1,1)}$$
$$PT = (1,-1,-1)$$

Pers bidang

$$n \cdot PT = 0$$
$$\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y+1 \\ z+1 \end{pmatrix} = 0$$

$$2x - 2 + 3y + 3 + 5z + 5 = 0$$
$$2x + 3y + 5z + 6 = 0 \quad \rightarrow \text{(pers. bidang } \alpha\text{)}$$

jarak titik $(1,1,1)$ ke $2x + 3y + 5z + 6 = 0$

$$d = \frac{|2.1 + 3.1 + 5.1 + 6|}{\sqrt{2^2 + 3^2 + 5^2}} = \frac{|16|}{\sqrt{38}} = \frac{16}{\sqrt{38}} \approx 2{,}596$$