Nama : Muchammad Daniyal Kautsar
NIM : 21/479067/TK/52800
NIU : 479067
Prodi : Teknologi Informasi

Tugas 3 Kalkulus Variabel Tunggal

- Find (a) the slope of the curve at the given point P and (b) an equation of the tangent line at P.

7. $y = x^2 - 3$, $P(2,1)$

$\rightarrow y = x^2 - 3$, $P(2,1)$

$Q(h+2, (h+2)^2 - 3)$

a. $\frac{\Delta y}{\Delta x} = \frac{(h+2)^2 - 3 - 1}{h+2-2} = \frac{h^2+4h+4-4}{h} = \frac{h^2+4h}{h} = h+4$

$h \to 0$

$\text{slope} = \frac{\Delta y}{\Delta x} = h+4 = 0+4 = 4$

b. $y - y_1 = m(x - x_1)$

$y - 1 = 4(x-2)$

$y - 1 = 4x - 8$

$y = 4x - 7$

8. $y = 5 - x^2$, $P(1,4)$

$\rightarrow y = 5 - x^2$, $P(1,4)$

$Q(h+1, 5-(h+1)^2)$

a. $\frac{\Delta y}{\Delta x} = \frac{5-(h+1)^2 - 4}{h+1-1} = \frac{1 - (h^2+2h+1)}{h} = \frac{1-h^2-2h-1}{h}$

$= \frac{-h^2-2h}{h}$

$= -h-2$

$h \to 0$

$\text{slope} = \frac{\Delta y}{\Delta x} = -h-2 = -2$

b. $y - y_1 = m(x - x_1)$

$y - 4 = -2(x-1)$

$y - 4 = -2x + 2$

$y = -2x + 6$

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9. $y = x^2 - 2x - 3,\quad P(2,-3)$

$\rightarrow y = x^2 - 2x - 3,\ P(2,-3)$

$Q(h+2, (h+2)^2 - 2h - 4 - 3)$

a. $\dfrac{\Delta y}{\Delta x} = \dfrac{(h+2)^2 - 2h - 4 - 3 + 3}{h+2-2} = \dfrac{h^2 + 4h + 4 - 4}{h} = \dfrac{h^2 + 4h}{h}$

$= h + 4$

$h \rightarrow 0$

slope $= \dfrac{\Delta y}{\Delta x} = h + 4 = 4$

b. $y - y_1 = m(x - x_1)$

$y + 3 = 4(x - 2)$

$y = 4x - 8 - 3$

$y = 4x - 11$

10. $y = x^2 - 4x,\ P(1,-3)$

$\rightarrow y = x^2 - 4x,\ P(1,-3)$

$Q(h+1, (h+1)^2 - 4h - 4)$

a. $\dfrac{\Delta y}{\Delta x} = \dfrac{(h+1)^2 - 4h - 4 + 3}{h+1-1} = \dfrac{h^2 + 2h + 1 - 4h - 1}{h} = \dfrac{h^2 - 2h}{h} = h - 2$

$h \rightarrow 0$

slope $= \dfrac{\Delta y}{\Delta x} = h - 2 = -2$

b. $y - y_1 = m(x - x_1)$

$y + 3 = -2(x - 1)$

$y = -2x + 2 - 3$

$y = -2x - 1$

20. let $f(t) = 1/t$ for $t \neq 0$

a. Find the average rate of change of f with respect to t over the intervals (i) from $t = 2$ to $t = 3$, and (ii) from $t = 2$ to $t = T$.

$\rightarrow$ i) $\dfrac{f(3) - f(2)}{3-2} = \dfrac{\frac{1}{3} - \frac{1}{2}}{1} = -\dfrac{1}{6}$

ii) $\dfrac{f(T) - f(2)}{T-2} = \dfrac{\frac{1}{T} - \frac{1}{2}}{T-2} = \dfrac{\frac{2-T}{2T}}{T-2} = \dfrac{-\cancel{2-T}}{2T\,(\cancel{T-t})} = \dfrac{-1}{2T},$

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b. Make a table of values of the average rate of change of f with respect to t over the intervals [2, T], for some values of T approaching 2, say T = 2.1, 2.01, 2.001, 2.0001, 2.00001, and 2.000001.

→

T	f (T)	(f (T) − f (2)) / (T − 2)
2.1	0,476190	−0,2381
2.01	0,497512	−0,2488
2.001	0,499750	−0,2500
2.0001	0,4999750	−0,2500
2.00001	0,499997	−0,2500
2.000001	0,499999	−0,2500

c. What does your table indicate is the rate of change of f with respect to t at t = 2?

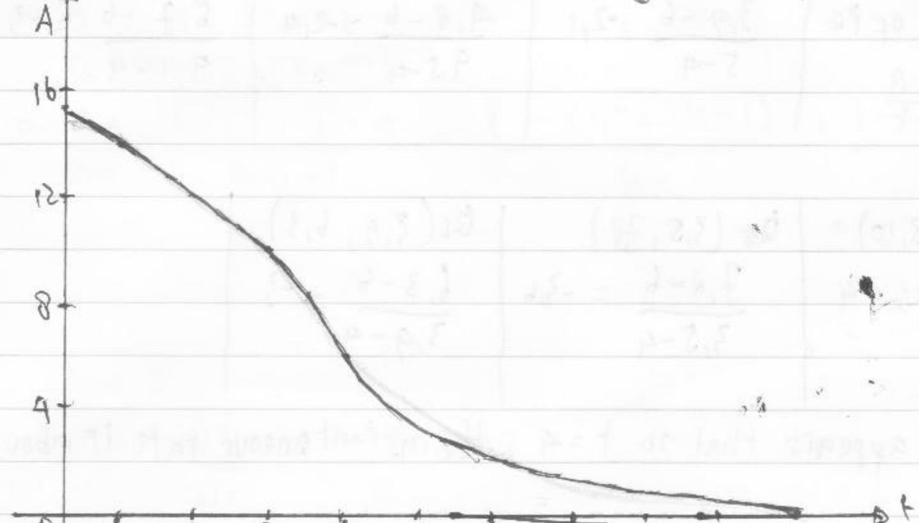
→ From the table, the rate of change indicate at −0,25 at t = 2.

d. Calculate the limit as T approaches 2 of the average rate of change of f with respect to t over the interval from 2 to T. You will have to do some algebra before you can substitute T = 2.

→ $\lim_{T \to 2} \frac{f(T) - f(2)}{T-2} = \lim_{T \to 2} \frac{\frac{1}{T} - \frac{1}{2}}{T-2} = \lim_{T-2} \frac{-(2-T)}{2T(T-2)} = \lim_{T-2} \frac{-1}{2T}$

$= -\frac{1}{4}$

22. The accompanying graph shows the total amount of gasoline A in the tank of an automobile after being driven for t days.



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a. Estimate the average rate of gasoline consumption over the time intervals [0,3], [0,5], and [7,10].

→ [0,3] : $\frac{\Delta A}{\Delta t} = \frac{10-15}{3-0} = \frac{-5}{3} \approx -1{,}67$ gal/day.

[0,5] : $\frac{\Delta A}{\Delta t} = \frac{3{,}9-15}{5-0} \approx \frac{-11{,}1}{5} \approx -2{,}2$ gal/day

[7,10] : $\frac{\Delta A}{\Delta t} = \frac{0-1{,}4}{10-7} = \frac{-1{,}4}{3} \approx -0{,}5$ gal/day

b. Estimate the instantaneous rate of gasoline consumption at the times $t=1$, $t=4$, and $t=8$.

→ $t=1$ at $P(1,14)$

Q	$Q_1(2, 12{,}2)$	$Q_2(1{,}5, 13{,}2)$	$Q_3(1{,}1, 13{,}85)$
slope of PQ $= \frac{\Delta A}{\Delta t}$	$\frac{12{,}2-14}{2-1} = -1{,}8$	$\frac{13{,}2-14}{1{,}5-1} = -1{,}6$	$\frac{13{,}85-14}{1{,}1-1} = -1{,}5$

$Q_4(0,15)$	$Q_5(0{,}5, 14{,}6)$	$Q_6(0{,}9, 14{,}06)$
$\frac{15-14}{0-1} = -1$	$\frac{14{,}6-14}{0{,}5-1} = -1{,}2$	$\frac{14{,}06-14}{0{,}9-1} = -1{,}4$

∴ It appears that in $t=1$, the instantaneous rate is about $-1{,}45$ gal/day

→ $t=4$ at $P(4,6)$

Q	$Q_1(5, 3{,}9)$	$Q_2(4{,}5, 4{,}8)$	$Q_3(4{,}1, 5{,}7)$
slope of PQ $= \frac{\Delta A}{\Delta t}$	$\frac{3{,}9-6}{5-4} = -2{,}1$	$\frac{4{,}8-6}{4{,}5-4} = -2{,}4$	$\frac{5{,}7-6}{4{,}1-4} = -3$

$Q_4(3,10)$	$Q_5(3{,}5, 7{,}8)$	$Q_6(3{,}9, 6{,}3)$
$\frac{10-6}{3-4} = 4$	$\frac{7{,}8-6}{3{,}5-4} = -3{,}6$	$\frac{6{,}3-6}{3{,}9-4} = -3$

∴ It appears that in $t=4$, the instantaneous rate is about -3 gal/day

→ $t = 8$ at $p(8,1)$

Q	$Q_1(9, 0{,}5)$	$Q_2(8{,}5, 0{,}7)$	$Q_3(8{,}1, 0{,}95)$
slope of PQ $= \frac{\Delta A}{\Delta t}$	$\frac{0{,}5-1}{9-8} = -0{,}5$	$\frac{0{,}7-1}{8{,}5-8} = -0{,}6$	$\frac{0{,}95-1}{8{,}1-8} = -0{,}5$

$Q_4(7, 1{,}4)$	$Q_5(7{,}5, 1{,}3)$	$Q_6(7{,}9, 1{,}04)$
$\frac{1{,}4-1}{7-8} = -0{,}4$	$\frac{1{,}3-1}{7{,}5-8} = -0{,}6$	$\frac{1{,}04-1}{7{,}9-8} = -0{,}4$

∴ It appears that in $t = 8$, the instantaneous rate is about $-0{,}55$ gal/day

c. From the graph, the curve decreasing fastest at $t = 3{,}5$. So, on the $P(3{,}5, 7{,}8)$

→ $P(3{,}5, 7{,}8)$

Q	$Q_1(4{,}5, 4{,}8)$	$Q_2(4, 6)$	$Q_3(3{,}6, 7{,}4)$
slope of PQ $= \frac{\Delta A}{\Delta t}$	$\frac{4{,}8-7{,}8}{4{,}5-3{,}5} = -3$	$\frac{6-7{,}8}{4-3{,}5} = -3{,}6$	$\frac{7{,}4-7{,}8}{3{,}6-3{,}5} = -4$

$Q_4(2{,}5, 11{,}2)$	$Q_5(3, 10)$	$Q_6(3{,}4, 8{,}2)$
$\frac{11{,}2-7{,}8}{2{,}5-3{,}5} = -3{,}4$	$\frac{10-7{,}8}{3-3{,}5} = -4{,}4$	$\frac{8{,}2-7{,}8}{3{,}4-3{,}5} = -4$

∴ it appears that in $t = 3{,}5$, the rate is about -4 gal/day

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15. $\lim_{x \to 2} \frac{x+3}{x+6} = \frac{2+3}{2+6} = \frac{5}{8}$

27. $\lim_{t \to 1} \frac{t^2+t-2}{t^2-1} = \lim_{t \to 1} \frac{(t+2)\cancel{(t-1)}}{(t+1)\cancel{(t-1)}} = \lim_{t \to 1} \frac{t+2}{t+1} = \frac{1+2}{1+1} = \frac{3}{2}$

47. $\lim_{x \to 0} \frac{1+x+\sin x}{3\cos x} = \frac{1+0+\sin 0}{3\cos 0} = \frac{1+0+0}{3.1} = \frac{1}{3}$

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55. Before contracting to grind engine cylinders to a cross-sectional area of $9\,in^2$, you need to know how much deviation from the ideal cylinder diameter of $x_0 = 3.385$ in. you can allow and still have the area come within $0{,}01\,in^2$ of the required $9\,in^2$. To find out, you let $A = \pi(x/2)^2$ and look for the interval in which you must hold x to make $[A-9] \leq 0.01$. What interval do you find?

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$\rightarrow |A-9| \leq 0{,}01$

$|\pi(x/2)^2 - 9| \leq 0{,}01$

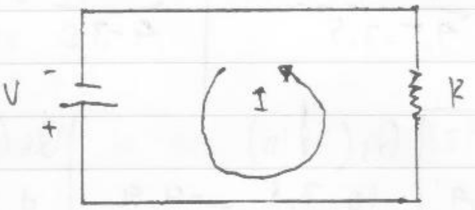
$-0{,}01 \leq \pi(x/2)^2 - 9 \leq 0{,}01$

$8{,}99 \leq \dfrac{\pi x^2}{4} \leq 9{,}01$

$4 \cdot \dfrac{8{,}99}{\pi} \leq x^2 \leq 4 \cdot \dfrac{9{,}01}{\pi}$

$2\sqrt{\dfrac{8{,}99}{\pi}} \leq x \leq 2\sqrt{\dfrac{9{,}01}{\pi}}$

56. Ohm's law for electrical circuits like the one shown in the accompanying figure states that $V = RI$. In this equation, V is a constant Voltage, I is the current in amperes and R is the resistance in ohms. Your firm has been asked to supply the resistors for a circuit in which V will be 120 Volts and I will be $5 \pm 0{,}1$ amp. In what interval does R have to lie for I to be within $0{,}1$ amp of the value $I_0 = 5$?



$\rightarrow V = R \cdot I \iff I = \dfrac{V}{R}$

$I = 5 \pm 0{,}1\ A \quad ; \quad V = 120$

$\left| \dfrac{V}{R} - 5 \right| \leq 0{,}1$

$-0{,}1 \leq \dfrac{V}{R} - 5 \leq 0{,}1$

$4{,}9 \leq \dfrac{120}{R} \leq 5{,}1$

$\dfrac{10}{49} \geqslant \dfrac{R}{120} \geqslant \dfrac{10}{51}$

$\dfrac{1200}{49} \geqslant R \geqslant \dfrac{1200}{51}$

$24{,}49 \geqslant R \geqslant 23{,}53$

$\rightarrow 23{,}53 \leq R \leq 24{,}49$

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44. $\lim_{x \to \infty} \frac{1}{x^2 - 7x + 1} = \lim_{x \to \infty} \frac{\frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{7x}{x^2} + \frac{1}{x^2}} = \lim_{x \to \infty} \frac{\frac{1}{x^2}}{1 - \frac{7}{x} + \frac{1}{x^2}}$

$= \frac{0}{1 - 0 + 0} = 0$

46. $\lim_{x \to \infty} \frac{x^4 + x^3}{12x^3 + 128} = \lim_{x \to \infty} \frac{\frac{x^4}{x^3} + \frac{x^3}{x^3}}{\frac{12x^3}{x^3} + \frac{128}{x^3}} = \lim_{x \to \infty} \frac{x + 1}{12 + \frac{128}{x^3}} = \infty$

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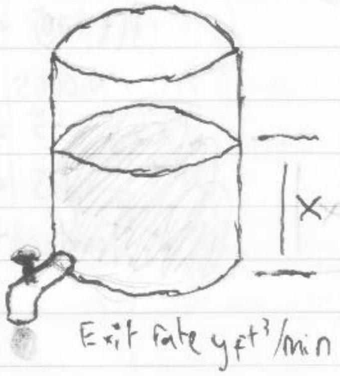
4. Torricelli's law says that if you drain a tank like the one in the figure shown, the rate y at which water runs out is a constant times the square root of the water's depth x. the constant depends on the size and shape of exit valve.

Suppose that $y = \sqrt{x}/2$ for a certain tank. You are trying to maintain a fairly constant exit rate by adding water to the tank with a hose from the time to time. How deep must you keep the water if you want to maintain the exit rate



a. within 0,2 ft³/min of the rate $y_0 = 1$ ft³/min

$\rightarrow y = \sqrt{x}/2$

$|y - y_0| <$ rate

$|\sqrt{x}/2 - 1| < 0{,}2$

$-0{,}2 < \sqrt{x}/2 - 1 < 0{,}2$

$0{,}8 < \sqrt{x}/2 < 1{,}2$

$1{,}6 < \sqrt{x} < 2{,}4$

$2{,}56 < x < 5{,}76$

b. within 0,1 ft³/min of the rate $y_0 = 1$ ft³/min

$\rightarrow |y - y_0| <$ rate

$|\sqrt{x}/2 - 1| < 0{,}1$

$-0{,}1 < \sqrt{x}/2 - 1 < 0{,}1$

$0{,}9 < \sqrt{x}/2 < 1{,}1$

$1{,}8 < \sqrt{x} < 2{,}2$

$3{,}24 < x < 4{,}84$

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5. As you may know, most metals expand when heated and contract when cooled. The dimensions of a piece of laboratory equipment are sometimes so critical that the shop where the equipment is made must be held at the same temperature as the laboratory where the equipment is to be used. A typical aluminium bar that is 10 cm wide at 70°F will be

$$y = 10 + (t-70) \times 10^{-4}$$

centimeter wide at a nearby temp. t. Suppose that you are using a bar like this in a gravity wave detector, where its width must stay within 0,0005 cm of the ideal 10 cm. How close to $t_0 = 70°F$ must you maintain the temperature to ensure that this tolerance is not exceeded?

$\rightarrow y_0 = 10$; $\Delta y = \pm 0{,}0005$

$|y - y_0| < \Delta y$

$|10 + (t-70) \times 10^{-4} - 10| < 0{,}0005$

$|(t-70) \times 10^{-4}| < 0{,}0005$

$-0{,}0005 < (t-70) \cdot 10^{-4} < 0{,}0005$

$-5 < t-70 < 5$

$65 < t < 75$

$\therefore$ Temperature must be around $65°$ to $75°F$.