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HW 5 Matematika diskrit

Strong Induction

3. Let P(n) be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The part of this exercise outline a strong induction proof that P(n) is true for all integers $n \geq 8$.

a) Show how that statement P(8), P(9), P(10) are true. completing the basis step of a proof by strong induction that P(n) is true for all integer $n \geq 8$.

→ P(8) = 3 + 5 = 8
P(9) = 3 + 3 + 3 = 9
P(10) = 5 + 5 = 10
∴ P(8), P(9), and P(10) are true.

b) What is the inductive hypothesis of a proof by strong induction that P(n) is true for all integer $n \geq 8$?

→ The inductive ~~...~~ hypothesis is that P(n) is true for $8 \leq n \leq k$ where $k \geq 10$

c) What do you need to prove in the inductive step of a proof by strong induction that P(n) is true for all integer $n \geq 8$?

→ We need to prove that P(k+1) is true in the inductive step.

d) Complete the inductive step for $k \geq 10$

→ $k \geq 10$
P(k+1) = P(k-2+3)
From induction hypothesis, $k-2 \geq 8$ and P(k-2) is true.
Then, P(k+1) is also true from adding one 3-cent stamps to P(k-2).
∴ P(k+1) is true.

e) Explain why these steps show that P(n) is true whenever $n \geq 8$.

→ Assume that for $8 \leq n \leq k$, where $k \geq 10$, n can be formed from 3-cent and 5-cent stamps. The n where $8 \leq n \leq k$ or $8 \leq n \leq 10$ for $k \geq 10$ will be true for n = 8, n = 9, n = 10. From that, we can use given n = k+1 in term of n = k-2+3. where k-2 will be in the range of $8 \leq n \leq k$ for $k \geq 10$. Also, k-2 is true, then for n = k+1 in term of n = k-2+3 will be true, because we just need to add a 3-cent stamp to k-2. From that, we can conclude that P(n) is true whenever $n \geq 8$.

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9. Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong inductive proof that $P(n)$ is true for all integers $n \geq 18$.

a) Show that the statement $P(18), P(19), P(20), P(21)$ are true, completing the basis step of a proof by strong induction that $P(n)$ is true for all integers $n \geq 18$.

→ $P(18) = 2.7 + 1.4 = 14 + 4 = 18$

$P(19) = 1.7 + 3.4 = 7 + 12 = 19$

$P(20) = 0.7 + 5.4 = 0 + 20 = 20$

$P(21) = 3.7 + 0.4 = 21 + 0 = 21$

$\therefore P(18), P(19), P(20), P(21)$ are true.

b). What is the inductive hypothesis of a proof by strong induction that $P(n)$ is true for all integers $n \geq 18$?

→ The inductive hypothesis is that $P(n)$ is true for all integer $18 \leq n \leq k$, where $k \geq 21$.

c). What do you need to prove in the inductive step of a proof that $P(n)$ is true for all integers $n \geq 18$?

→ We need to prove that $P(k+1)$ is true in the inductive step.

d). Complete the inductive step for $k \geq 21$.

→ $k \geq 21$

$P(k+1) = P(k-3+4)$

From induction hypothesis, $k-3 \geq 18$ and $P(k-3)$ is true, so, $P(k+1)$ is true by adding 4-cent stamp to $P(k-3)$.

$\therefore P(k+1)$ is true.

e). Explain why these steps show that $P(n)$ is true for all integers $n \geq 18$.

→ Assume for $18 \leq n \leq k$, where $k \geq 21$, n cents can be formed from 4-cent and 7-cent stamp. The n will be true for $n = 18, n = 19, n = 20, n = 21$. From that, we can use $n = k+1$ in term of $n = k-3+4$. $k-3$ is true for $18 \leq n \leq k, k \geq 21$. We just need to add 4 cent to $k-3$. So, we can conclude for $P(n)$ is true for integer $n \geq 18$.

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Recursion

2. Find $f(1)$, $f(2)$, $f(3)$, $f(4)$, and $f(5)$ if $f(n)$ is defined recursively from $f(0) = 3$ and for $n = 1, 2, 3, ..$

a) $f(n+1) = -2f(n)$

$\rightarrow f(1) = -2f(0)$
$f(1) = -2 \cdot 3$
$f(1) = -6$

$\rightarrow f(2) = -2 \cdot f(1)$
$f(2) = -2 \cdot -6$
$f(2) = 12$

$\rightarrow f(3) = -2 \cdot f(2)$
$f(3) = -2 \cdot 12$
$f(3) = -24$

$\rightarrow f(4) = -2f(3)$
$f(4) = -2 \cdot -24$
$f(4) = 48$

$\rightarrow f(5) = -2f(4)$
$f(5) = -2 \cdot 48$
$f(5) = -96$

b). $f(n+1) = 3f(n) + 7$.

$\rightarrow f(1) = 3f(0) + 7$
$f(1) = 3 \cdot 3 + 7$
$f(1) = 9 + 7$
$f(1) = 16$

$\rightarrow f(2) = 3f(1) + 7$
$f(2) = 3 \cdot 16 + 7$
$f(2) = 48 + 7$
$f(2) = 55$

$\rightarrow f(3) = 3f(2) + 7$
$f(3) = 3 \cdot 55 + 7$
$f(3) = 165 + 7$
$f(3) = 172$

$\rightarrow f(4) = 3 \cdot f(3) + 7$
$f(4) = 3 \cdot 172 + 7$
$f(4) = 516 + 7$
$f(4) = 523$

$\rightarrow f(5) = 3 \cdot f(4) + 7$
$f(5) = 3 \cdot 523 + 7$
$f(5) = 1569 + 7$
$f(5) = 1576$

c) $f(n+1) = f(n)^2 - 2f(n) - 2$

$\rightarrow f(1) = f(0)^2 - 2f(0) - 2$
$f(1) = 3^2 - 2 \cdot 3 - 2$
$f(1) = 9 - 6 - 2$
$f(1) = 1$

$\rightarrow f(2) = f(1)^2 - 2f(1) - 2$
$f(2) = 1^2 - 2 \cdot 1 - 2$
$f(2) = -3$

$\rightarrow f(3) = f(2)^2 - 2f(2) - 2$
$f(3) = (-3)^2 - 2 \cdot -3 - 2$
$f(3) = 9 + 6 - 2$
$f(3) = 13$

$\rightarrow f(4) = f(3)^2 - 2f(3) - 2$
$f(4) = 13^2 - 2 \cdot 13 - 2$
$f(4) = 169 - 26 - 2$
$f(4) = 141$

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$\rightarrow F(5) = F(4)^2 - 2F(4) - 2$

$F(5) = 141^2 - 2 \cdot 141 - 2$

$F(5) = 19597$

d) $F(n+1) = 3^{F(n)/3}$

$\rightarrow F(1) = 3^{F(0)/3}$

$F(1) = 3^{3/3}$

$F(1) = 3$

$\rightarrow F(2) = 3^{F(1)/3}$

$F(2) = 3^{3/3}$

$F(2) = 3$

$\rightarrow F(3) = 3^{F(2)/3}$

$F(3) = 3^{3/3}$

$F(3) = 3$

$\rightarrow F(4) = 3^{F(3)/3}$

$F(4) = 3^{3/3}$

$F(4) = 3$

$\rightarrow F(5) = 3^{F(4)/3}$

$F(5) = 3^{3/3}$

$F(5) = 3$

6. Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If f is well defined, find a formula for $f(n)$ when n is a nonnegative integer and prove that your formula is valid.

a) $f(0) = 1$, $f(n) = -f(n-1)$ for $n \geq 1$

$\rightarrow f(0) = 1$

$f(n) = -f(n-1)$, $n \geq 1$

$n \rightarrow n = 1 \rightarrow f(1) = -f(0) = -1$

$n = 2 \rightarrow f(2) = -f(1) = -(-1) = 1$

$n = 3 \rightarrow f(3) = -f(2) = -1$

So, $F(n) = (-1)^n$

Basis step $\rightarrow F(1) = (-1)^1 = -1$

$f(1)$ is true

Inductive step $\rightarrow$ show that $F(k+1)$ is true

$F(k) = (-1)^k$

$F(k+1) = (-1)^{k+1} = (-1)^k \cdot (-1)$

$F(k+1) = F(k) \cdot (-1)$

$F(k+1) = -f(k)$

$\therefore f(k+1)$ is true and $F(n)$ is also true for $n \geq 1$

b) $F(0)=1,\ F(1)=0,\ F(2)=2,\ F(n)=2F(n-3)$ for $n \geq 3$.

$\rightarrow$ $F(0)=1$

$F(1)=0$

$F(2)=2$

$F(n)=2F(n-3)$ for $n \geq 3$.

$n \rightarrow n=3 \rightarrow F(3)=2 \cdot F(0)=2 \cdot 1=2$

$n=4 \rightarrow F(4)=2 \cdot F(1)=2 \cdot 0=0$

$n=5 \rightarrow F(5)=2 \cdot F(2)=2 \cdot 2=4$

$n=6 \rightarrow F(6)=2 \cdot F(3)=4$

$n=7 \rightarrow F(7)=2 \cdot F(4)=0$

$n=8 \rightarrow F(8)=2 \cdot F(5)=8$

$\therefore F(n)=2^{n/3}$ if $n \bmod 3=0$, $F(n)=0$, if $n \bmod 3=1$, and

$F(n)=2^{(n+1)/3}$ if $n \bmod 3=2$.

show that $F(k+1)$ is true

$F(k)=2^{k/3}$ $(k \bmod 3=0)$

$F(k+3)=2^{(k+3)/3}$

$F(k+3)=2^{k/3} \cdot 2^{3/3}$

$F(k+3)=2F(k)$ (True)

$F(k)=0$ $(k \bmod 3=1)$

$F(k+1)=0$

$F(k+1)=2F(k)$ (True)

$F(k)=2^{(k+1)/3}$ $(k \bmod 3=2)$

$F(k+3)=2^{(k+4)/3}$

$F(k+3)=2^{(k+1)/3} \cdot 2^{3/3}$

$F(k+3)=2(k)$ (True)

$\therefore F(k+1)$ is true and $F(n)$ is also true for $n \geq 3$

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c) $f(0)=0$, $f(1)=1$, $f(n)=2f(n+1)$ for $n \geq 2$

→ $f(0)=0$

$f(1)=1$

$f(n)=2f(n+1)$, $n \geq 2$

$n \to n=2 \to f(2)=2.f(3)$

$f(3)$ is not defined yet!

∴ $f(n)$ is false.

d. $f(0)=0$, $f(1)=1$, $f(n)=2f(n-1)$ for $n \geq 1$

→ $f(0)=0$

$f(1)=1$

$f(n)=2.f(n-1)$, $n \geq 1$

$n=1 \to f(1)=2f(0)=0$

∴ is false because $f(1)=1$

$n=2 \to f(2)=2f(1)=2.1=2$

$n=3 \to f(3)=2f(2)=2.2=4$

$n=4 \to f(4)=2f(4)=2.4=8$

So, $f(n)=2^{n-1}$

Basic step → $f(1)=2^{1-1}=2^0=1$ (True)

Inductive → $f(k)=2^{k-1}=2^k/2$

$f(k+1)=2^{k+1-1}=2^k$

$f(k+1)=2.\frac{2^k}{2}$

$f(k+1)=2.f(k)$

∴ $f(k+1)$ is true and $f(n)$ is also true $n \geq 1$ ∎

e) $f(0)=2$, $f(n)=f(n-1)$ if n is odd and $n \geq 1$ and

$f(n)=2f(n-2)$ if $n \geq 2$

Odd	Even
$n=1 \to f(1)=f(0)=2$	$n=2 \to f(2)=2.f(0)=4$
$n=3 \to f(3)=f(2)=4$	$n=4 \to f(4)=2f(2)=8$
$n=5 \to f(5)=f(4)=8$	$n=6 \to f(6)=2f(4)=16$
So, $f(n)=2^{(n+1)/2}$ for n is odd	So, $f(n)=2^{(n+2)/2}$ for n is even
$f(1)=2^{(1+1)/2}=2^1=2$	$f(2)=2^{(2+2)/2}=2^2=4$
$f(k)=2^{(k+2)/2}$	$f(k)=2^{(k+2)/2}$
$f(k+1)=2^{(k+3)/2}$	$f(k+2)=2^{(k+4)/2}$
$f(k+1)=f(k)$	$=2^{(k+2)/2}.2^{2/2}$
	$=2.f(k)$ ∎

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$\therefore$ $F(k+1)$ is true for odd, $n \geq 1$ and $F(k+2)$ is true for even $n \geq 2$