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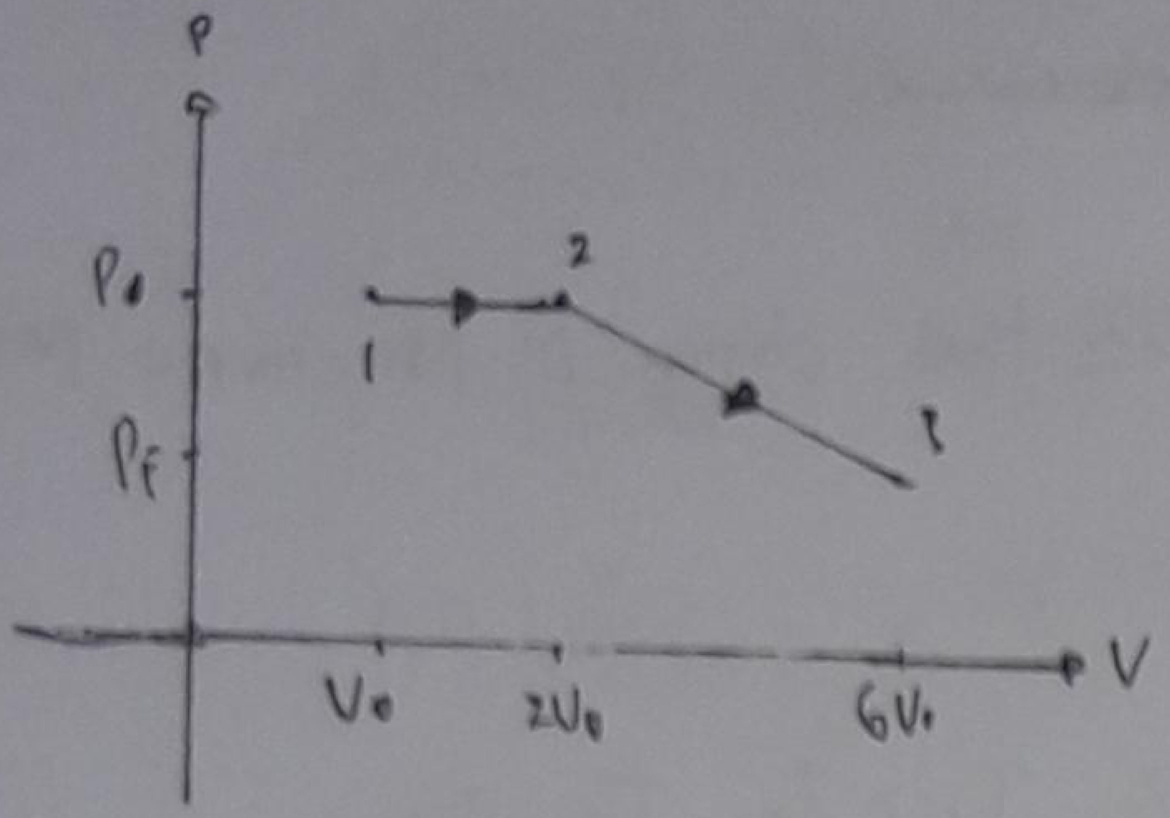
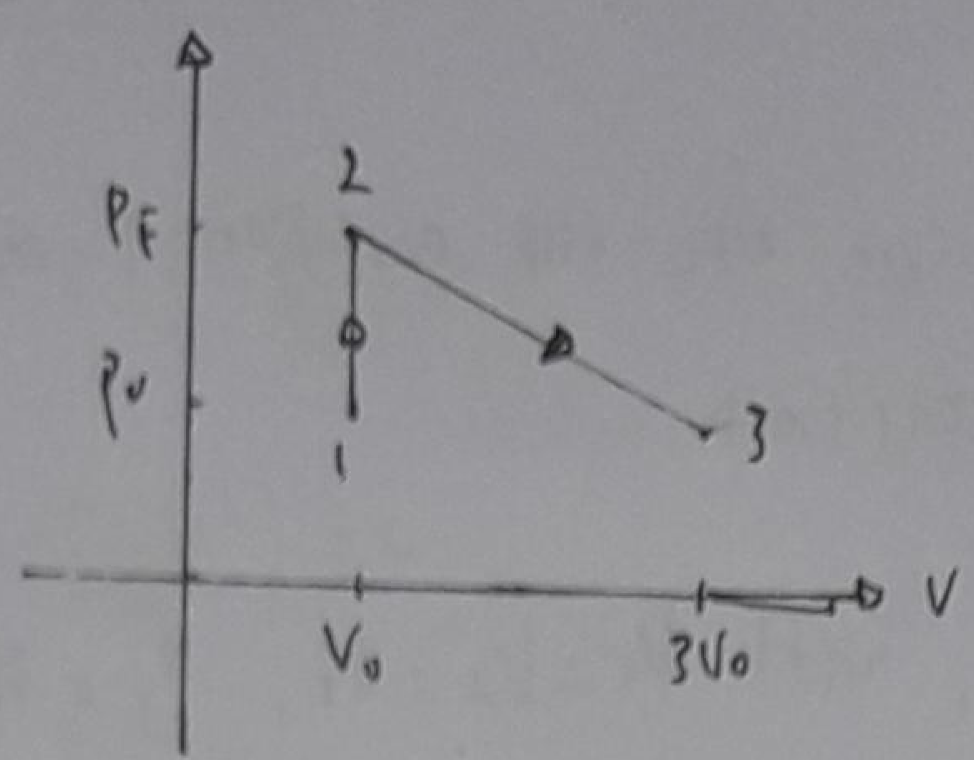
21/479067/TK/52800

HW 2 FFKG

Problem 1



Process A



Process B

An ideal gas (with number of moles n and molar specific heat Cv) inside a sealed container is expanded following two different processes shown in the figure above. Express only in terms of n, Cv, T0, V0, and R.

A. Determine the value of P_F and T_F in these processes!

* Process A.

We will use the $PV = nRT$ equation. Since n and R are constant, we can simplify and use the $\frac{P.V}{T}$ equation to compare it with another position.

→ For process A, compare the 1 and 2 position to find T_F.

$\frac{P_1 V_1}{T_1} = \frac{P_2 . V_2}{T_2}$, since $P_1 = P_0 = P_2$, $V_1 = V_0$, $V_2 = 2V_0$ and we can let $T_1 = T_0$, $T_2 = T_F$.

$\frac{\cancel{P_0} . \cancel{V_0}}{T_0} = \frac{\cancel{P_0} . 2\cancel{V_0}}{T_F} \Rightarrow \underline{T_F = 2T_0}$. So, T_F is equal to $2T_0$. Isobaric process occur in position 1 and 2.

→ To find P_F, we can compare the 2 and 3 position.

$\frac{P_2 . V_2}{T_2} = \frac{P_3 . V_3}{T_3}$, since ~~isothermal~~ adiabatic process occurs. $P_2 = P_0$, $P_3 = P_F$, $V_2 = 2V_0$, $V_3 = 6V_0$,

$T_2 = 2T_0$, $T_3 = \cancel{2T_0}\ T_0$

$\frac{P_0 . 2\cancel{V_0}}{2T_0} = \frac{P_F . 6\cancel{V_0}}{\cancel{2}T_0} \Rightarrow P_F = \frac{P_0}{6}$. So, P_F is equal to $\frac{P_0}{6}$. But, we can't express in term of P_0.

So, the P_F will be expressed by ~~$P_3.V_3$~~ $P_3 . V_3 = nRT_3$, with n and R constant

$P_F . 6V_0 = nR\cancel{2}T_0 \Rightarrow \underline{P_F = \frac{nRT_0}{6V_0}}$

~~$P_F = nRT_0/6V_0$~~

... the ...

Process B.

- We can use Position 2 and 3 to find P_F. since that change is isothermal process, The temperature is constant.

$$\frac{P_2 \cdot V_2}{T_2} = \frac{P_3 \cdot V_3}{T_3}$$, with $P_2 = P_F$, $P_3 = P_0$, $V_2 = V_0$, $V_3 = 3V_0$, $T_2 = T_3 = T_F$.

$$\frac{P_F \cdot V_0}{T_F} = \frac{P_0 \cdot 3V_0}{T_F} \Rightarrow P_F = 3P_0.$$ ~~In other form, we can~~

- To find T_F. We can use the position 1 and 2. since that change is isochoric process, The volume is constant

$$\frac{P_1 \cdot V_1}{T_1} = \frac{P_2 V_2}{T_2}$$, with $V_1 = V_2 = V_0$, $P_1 = P_0$, $P_2 = P_F$, $T_1 = T_0$, $T_2 = T_F$

$$\frac{P_0 \cdot V_0}{T_0} = \frac{P_F \cdot V_0}{T_F} \Rightarrow \frac{P_0}{T_0} = \frac{P_F}{T_F}$$. From previous equation, we know that $P_F = 3P_0$.

$$\frac{P_0}{T_0} = \frac{3P_0}{T_F} \Rightarrow \underline{T_F = 3T_0.}$$

- Since P_F is expressed in term of P_0. We can express it in other form.

$$P_2 \cdot V_2 = nRT_2$$, with $P_2 = P_0$, $V_2 = V_0$, $T_2 = T_F = 3T_0$

$$P_F \cdot V_0 = nR3T_0$$

$$\underline{P_F = \frac{nR3T_0}{V_0}}$$

B. Determine the ammount of heat Q added into the system.

* Process A.

- Since the Position 2 and 3 is adiabatic process, there is no change in heat of the system or $\Delta Q = 0$.
- For the position 1 and 2. the process is isochoric process which means there is some amount of heat Q added. we can calculate by

$$Q = n(C_V + R) \cdot \Delta T$$

$$Q = n(C_V + R) \cdot (T_F - T_0)$$, since $T_F = 2T_0$, $T_F - T_0$ is equal to T_0

$$\underline{Q = n(C_V + R) \cdot T_0.}$$

So, the amount of heat Q added to the system is equal to $n(C_V + R) \cdot T_0$, as shown in the equation above.

* Process B

→ the position 1 and 2 is isochoric process. We know that there is some heat Q added by calculating

$Q_{12} = n . C_V . \Delta T$

$Q_{12} = n . C_V . (T_F - T_0)$, since $T_F = 3T_0$, $T_F - T_0 = 2T_0$.

$Q_{12} = n . C_V . 2T_0$

→ The position 2 and 3 is isothermal process. We can calculate the amount of heat added by

$Q_{23} = nRT_F \ln \frac{V_F}{V_0}$, since $V_F = 3V_0$, ~~and V_0~~ and $T_F = 3T_0$

$Q_{23} = 3nRT_0 \ln \frac{3V_0}{V_0}$

$Q_{23} = 3nRT_0 \ln 3$.

→ Sum up both Q_{12} and Q_{23}, we get the total amount of heat Q added.

$Q = Q_{12} + Q_{13} = nC_V . 2T_0 + 3nRT_0 \ln 3$

$\underline{Q = n . T_0 (2 . C_V + 3R \ln 3)}$

Problem 2.

M_A, C_A, T_A	M_B, C_B, T_B

Metal block with mass M_A and initial temperature T_A touches another metal block with mass M_B and initial temperature T_B. Suppose that the specific heat of two metal are C_A and C_B.

* A. Determine the final temperature T_F of the two metal blocks!

→ When two different metal blocks with their respective mass, initial temperature, and specific heat touches, there will be thermal equilibrium with given equation $Q_A + Q_B = 0$

⇒ $M_A . C_A . \Delta T_A + M_B . C_B . \Delta T_B = 0$, where ΔT is the difference between initial temperature and equilibrium or final temperature.

$M_A . C_A (T_F - T_A) + M_B . C_B (T_F - T_B) = 0$

$M_A . C_A . T_F - M_A . C_A . T_A + M_B . C_B . T_F - M_B . C_B . T_B = 0$

$T_F (M_A . C_A + M_B . C_B) = M_A . C_A . T_A + M_B . C_B . T_B$

$$T_F = \frac{M_A . C_A . T_A + M_B . C_B . T_B}{M_A . C_A + M_B . C_B}$$

So, the final temperature is $T_F = \dfrac{M_A . C_A . T_A + M_B . C_B . T_A}{M_A . C_A + M_B . C_B}$

B. Determine the ΔT_A and ΔT_B!

* $\Delta T_A = T_F - T_A$.

We can subtitute T_F from previous equation.

$$\Delta T_A = \frac{M_A . C_A . T_A + M_B . C_B . T_B}{M_A . C_A + M_B . C_B} - T_A$$

$$= \frac{M_A . C_A . T_A + M_B . C_B . T_B - M_A . C_A . T_A - M_B . C_B . T_A}{M_A . C_A + M_B . C_B}$$

$$= \frac{M_B . C_B . T_B - M_B . C_B . T_A}{M_A . C_A + M_B . C_B} = \frac{M_B . C_B . (T_B - T_A)}{M_A . C_A + M_B . C_B}$$

So, the temperature change of metal A is $\Delta T_A = \dfrac{M_B . C_B . (T_B - T_A)}{M_A . C_A + M_B . C_B}$.

* $\Delta T_B = T_F - T_B$

Also, we can subtitute T_F from previous equation.

$$\Delta T_B = \frac{M_A . C_A . T_A + M_B . C_B . T_B}{M_A . C_A + M_B . C_B} - T_B$$

$$= \frac{M_A . C_A . T_A + M_B . C_B . T_B - M_A . C_A . T_B - M_B . C_B . T_B}{M_A . C_A + M_B . C_B}$$

$$= \frac{M_A . C_A . T_A - M_A . C_A . T_B}{M_A . C_A + M_B . C_B} = \frac{M_A . C_A . (T_A - T_B)}{M_A . C_A + M_B . C_B}$$

So, the temperature change of metal B is $\Delta T_B = \dfrac{M_A . C_A \, (T_A - T_B)}{M_A . C_A + M_B . C_B}$.

c. Let assume that $M_A = M_B$. and $|\Delta T_A| > |\Delta T_B|$. Which metal block has higher specific heat?

* Since both metal have same mass, we can just compare the temperature change and specific heat.

~~Q...~~ $C_A . |\Delta T_A| = C_B . |\Delta T_B|$

$$\frac{C_B}{C_A} = \frac{|\Delta T_A|}{|\Delta T_B|}$$

Since $|\Delta T_A| > |\Delta T_B|$ as the ~~condi~~ given condition, we know that from that comparison that $\underline{C_B > C_A}$. So, that the Metal B has higher specific heat. Proved by the ~~condition~~ above condition.

Problem 3.

Inside the sealed container with temperature T, there exist N_0 atoms / moleculer of gas with mass m. The speed distribution of the gas atoms / moleculer follow a functions describe as follows.

$$N_v(v) = 4\pi N_0 . \left(\frac{m}{2\pi K_B T}\right)^{3/2} v^2 e^{-mv^2/2K_BT}$$

Which called as a Maxwell - Boltzmann distribution. The distribution have speed between v and $v + dv$ is given by

$$dN = N_v(v)\, dv$$

So that when we want to determine the total atom / moleculer that have speed between V_1 and V_2, we need to perform the equation

$$N = \int_{V_1}^{V_2} N_v(v)\, dv$$

Hint: $\int_0^\infty x^3 e^{-ax^2} = \frac{1}{2a^2}$, $\int_0^\infty x^4 e^{-ax^2} = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$

Prove the following statement.

a. RMS of speed

$$V_{RMS} = \sqrt{\frac{3K_BT}{m}}$$

* $V_{RMS} = \sqrt{\bar{v}^2}$

$$\bar{v}^2 = \frac{\int_0^\infty v^2 N_v(v)\, dv}{N_0}$$, subtituting from the equation above. we get

$$\bar{v}^2 = \frac{\int_0^\infty v^2 . 4\pi N_0 . \left(\frac{m}{2\pi K_B T}\right)^{3/2} v^2 e^{-mv^2/2K_BT}}{N_0} dv$$

$$\bar{v}^2 = \frac{4\pi \cancel{N_0} \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_0^\infty v^4 e^{-mv^2/2k_BT} dv}{\cancel{N_0}} \text{, since } \int_0^\infty v^4 e^{-av^2} = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

~~and a = m/2kBT~~

$$\bar{v}^2 = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \frac{3}{8_2} \sqrt{\frac{\pi}{\left(\frac{m}{2\pi k_B T}\right)^5}}$$

$$\cancel{\bar{v}^2 = \frac{3\pi}{2} \sqrt{\left(\frac{m}{2\pi k_B T}\right)^3} \cdot \sqrt{\frac{\pi}{\left(\frac{m}{2\pi k_B T}\right)^2}}}$$

$$\cancel{= \frac{3}{2}\pi \cdot 1}$$

$$\Rightarrow \bar{v}^2 = \frac{3\pi}{2} \cdot \sqrt{\left(\frac{m}{2\pi k_B T}\right)} \cdot \frac{m}{2\pi k_B T} \cdot \frac{4 k_B^2 T^2 \sqrt{2\pi k_B T}}{m^2 \sqrt{m}}$$

$$= \frac{3 k_B T}{m}$$

Since $V_{RMS} = \sqrt{\bar{v}^2}$

$$V_{RMS} = \sqrt{3 k_B T / m}$$ ☒

So, the equation is proven.

B. $V_{avg} = \sqrt{\frac{8 k_B T}{\pi m}}$

* $V_{avg} = \frac{\int_0^\infty v N(v)\, dv}{N_0}$, subtituting from the Maxwell Boltzmann function, we get

$$V_{avg} = \frac{\int_0^\infty v \cdot 4\pi \cdot N_0 \cdot \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT} dv}{N_0}$$

$$V_{avg} = \frac{4\pi \cancel{N_0} \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_0^\infty v^3 e^{-mv^2/2k_BT} dv}{\cancel{N_0}} \text{, since } \int_0^\infty x^3 e^{-ax^2} = \frac{1}{2a^2}$$

$$V_{avg} = \overset{2}{\cancel{4}}\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \frac{1}{\cancel{2}(m/2k_BT)^2} = 2\pi \left(\frac{m}{2\pi k_B T}\right) \sqrt{\frac{m}{2\pi k_B T}} \cdot \frac{4 k_B^2 T^2}{m^2}$$

$$V_{avg} = \sqrt{\frac{8 k_B T}{\pi m}}$$ ☒

~~Since~~ So, the equation is proven.

c. Most probable speed of the gas atoms / moleculer is given by

$$V_{mp} = \sqrt{\frac{2K_BT}{m}}$$

* since most probable speed or V_{mp} is the maximum value of distribution ($N_v(v)$). we can calculate when the derivative is equal to zero.

$$\frac{dN_v(v)}{dv} = 0$$

$$\frac{d\left(4\pi N_0 \left(\frac{m}{2\pi K_BT}\right)^{3/2} v^2 e^{-mv^2/2K_BT}\right)}{dv} = 0$$

$$4\pi \left(\frac{m}{2\pi K_BT}\right)^{\frac{3}{2}} \left(2ve^{-mv^2/2K_BT} + v^2\left(-\frac{mv}{K_BT}\right)e^{-mv^2/2K_BT}\right) = 0$$

$$4\pi \left(\frac{m}{2\pi K_BT}\right)^{\frac{3}{2}} e^{-mv^2/2K_BT} \, v\left(2 - \frac{m}{K_BT}v^2\right) = 0$$

Thus, we have two possible solution

$$V = 0 \quad \text{or} \quad 2 - \frac{m}{K_BT}V^2 = 0$$

since $V = 0$ is the minimum value. we use the $2 - \frac{m}{K_BT}V^2 = 0$.

$$2 - \frac{m}{K_BT}V^2 = 0$$

$$\frac{m}{K_BT}V^2 = 2$$

$$V^2 = \frac{2K_BT}{m}$$

$$V = \sqrt{\frac{2K_BT}{m}}$$

since we calculate the most probable speed, $V = V_{mp}$, thus, we know that the condition satisfied and the formula ~~given~~ is proven. ■