Nama : Muchammad Daniyal Kautsar

NIM : 21/479067/TK/52800

Homework 2

- Prove that $x = Ae^{-\frac{bt}{2m}} \cos(wt + \phi_0)$

$$w = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

is solution to differential equation : $\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$

* $\frac{dx}{dt} = \frac{d(Ae^{-bt/2m}\cos(wt+\phi_0))}{dt} = \cos(wt+\phi_0) - \frac{b}{2m}Ae^{-bt/2m} + (-w\sin(wt+\phi_0)Ae^{-bt/2m})$

$= -Ae^{-bt/2m}\left(\frac{b}{2m}\cos(wt+\phi_0) + w(\sin(wt+\phi_0))\right)$

* $\frac{d^2x}{dt^2} = \left(\frac{b}{2m}\cos(wt+\phi_0) + w\sin(wt+\phi_0)\right) - \frac{d}{dt}Ae^{-bt/2m}$ ~~$+(-Ae^{-bt/2m}$~~

$+ (-Ae^{-bt/2m})\frac{d}{dt}\left(\frac{b}{2m}\cos(wt+\phi_0) + w\sin(wt+\phi_0)\right)$

$= \frac{b}{2m}Ae^{-bt/2m}\left(\frac{b}{2m}\cos(wt+\phi_0) + w\sin(wt+\phi_0)\right)$

$- Ae^{-bt/2m}\left(\frac{b}{2m}\cdot w(-\sin(wt+\phi_0)) + w^2\cos(wt+\phi_0)\right)$

$= Ae^{-bt/2m}\left(\left(\frac{b}{2m}\right)\cos(wt+\phi_0) + \frac{b}{2m}\sin(wt+\phi_0) + \frac{bw}{2m}\sin(wt+\phi_0)\right.$

$\left. - w^2\cos(wt+\phi_0)\right)$

$= Ae^{-bt/2m}\left(\left(\frac{b^2}{4m^2} - w^2\right)\cos(wt+\phi_0) + \frac{bw}{m}\sin(wt+\phi_0)\right)$

* subtitute to the $\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$

$$Ae^{-bt/2m}\left(\left(\frac{b^2}{4m^2} - w^2\right)\cos(wt+\phi_0) + \frac{bw}{m}\sin(wt+\phi_0)\right) + \left(\frac{-b}{m}Ae^{-bt/2m}\left(\frac{b}{2m}\cos(wt+\phi_0)\right.\right.$$

$$\left.\left. + w\sin(wt+\phi_0)\right)\right) + \frac{k}{m}Ae^{-bt/2m}\cos(wt+\phi_0) = 0$$

$$\left(\frac{b^2}{4m^2} - \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right) + \frac{bw}{m}\sin(wt+\phi_0) - \frac{b^2}{2m^2} - \frac{bw}{m}\sin(wt+\phi_0) + \frac{k}{m} = 0$$

$$\frac{b^2}{4m^2} - \frac{k}{m} + \frac{b^2}{4m^2} - \frac{b^2}{2m^2} + \frac{k}{m} = 0$$

$$\frac{b^2}{2m^2} - \frac{b^2}{2m^2} = 0$$

$$0 = 0$$

Terbukti ▨