Tutorial KVJ

Pertemuan 4

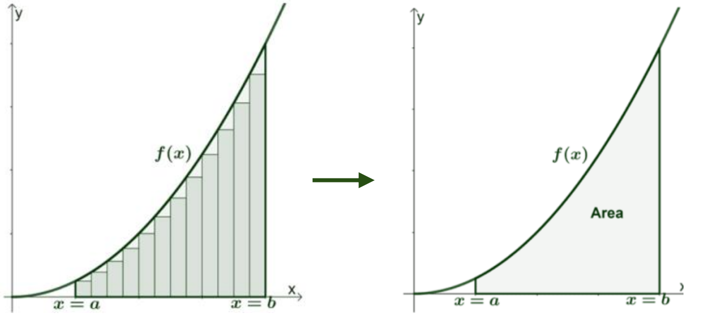
Pokok Pembahasan : *Double Integral*

1. Definisi
2. Koordinat Polar
3. Aplikasi
4. Metode Substitusi

Definisi *Double Integral*

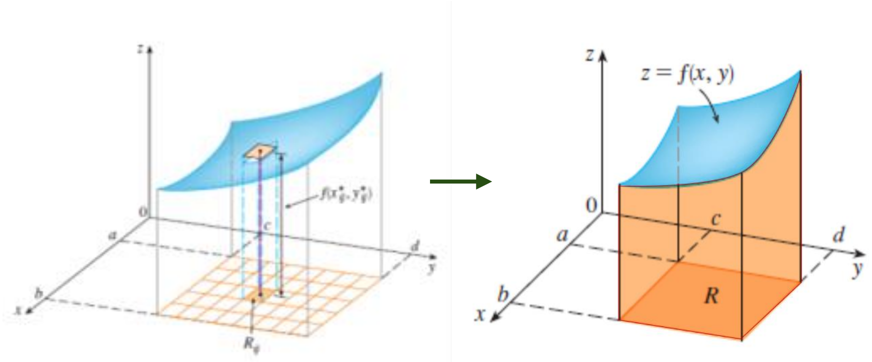
Let's see the pattern~

Variabel Tunggal



$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx = \mathit{area\ under\ the\ curve}$$

Variabel Jamak



$$\lim_{m,n\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A = \iint_R f(x,y)dA = \mathit{volume\ under\ the\ plane}$$

Integral Ganda

Definisi

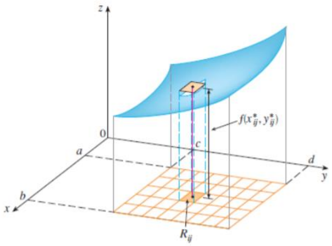
Integral ganda f atas **region** *R adalah*

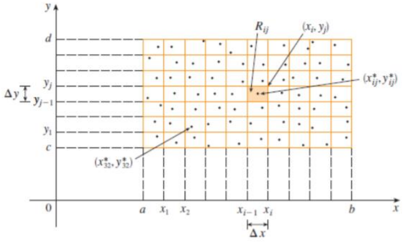
$$\iint_R f(x,y)\, dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$$

asalkan nilai limitnya ada.

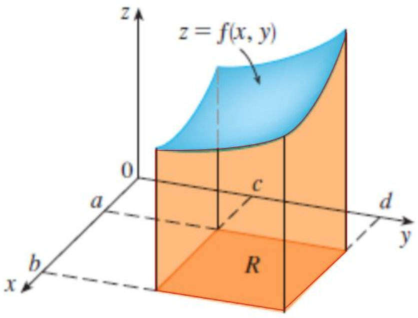
Akibatnya, jika $f(x,y) \geq 0$, maka **volume** V yaitu daaerah di bawah permukaan $z = f(x,y)$ dan diatas **region** R adalah

$$V = \iint_R f(x,y)\, dA.$$





$$V \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A, \qquad \Delta A = \Delta x \Delta y$$



Q : Gimana cara ngitung integralnya?

Summation of *slicing*! Misal kita *slice* pada sumbu yz:

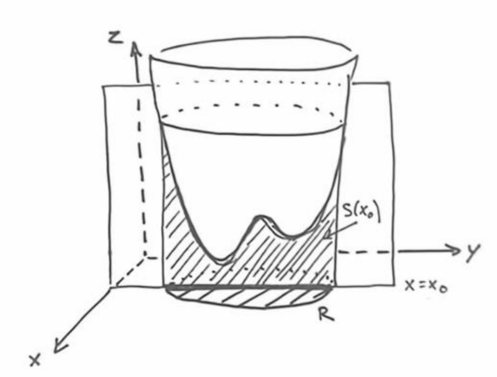
$S(x) \rightarrow Luas\ daerah\ satu\ slice\ plane\ yz\ pada\ titik\ x$

$$S(x) = \int_{y_{min}(x)}^{y_{max}(x)} f(x,y)dy$$

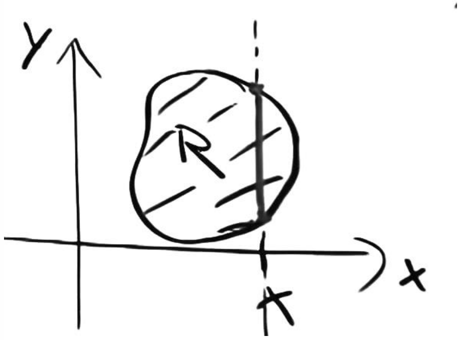
$$\iint_R f(x,y)dA = \int_{x_{min}}^{x_{max}} S(x)\ dx$$

$$\iint_R f(x,y)dA = \int_{x_{min}}^{x_{max}} \int_{y_{min}(x)}^{y_{max}(x)} f(x,y)\ dy\ dx$$

Tampak Samping

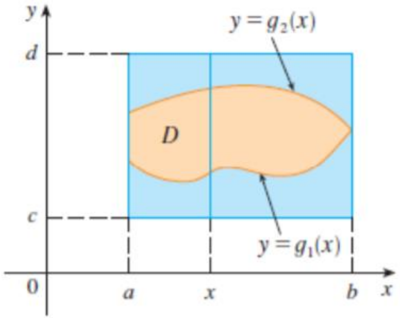


Tampak Atas



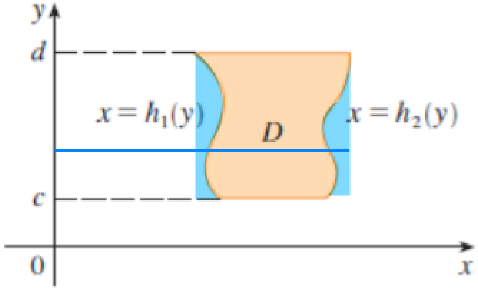
Generally speaking.. ada dua pendekatan penyelesaian:

Tipe 1 : Slice yz



$$\iint_D f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dydx.$$

Tipe 2 : Slice xz



$$\iint_D f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dxdy.$$

Contoh Soal

Rectangular Region

Tentukan volume di bawah bidang z dengan:

$$z = 1 - x^2 - y^2 \quad Region\ R:\ 0 \le x \le 1\,;0 \le y \le 2$$

Solusi : Misal kita pakai Tipe I

$$V = \int_0^1 \int_0^2 1 - x^2 - y^2\, dy\, dx$$

$$= \int_0^1 y - x^2 y - \frac{1}{3} y^3 |_0^2\, dx$$

$$= \int_0^1 -2x^2 - \frac{2}{3}\, dx$$

$$= -\frac{2}{3}x^3 - \frac{2}{3}x|_0^1$$

$$= -\frac{4}{3}$$

Trivia~
Untuk region berbentuk persegi panjang, urutan pengintegrasian bisa langsung dituker-tuker (kalo mau/butuh)!

Teorema (Fubini)

Jika f kontinu pada persegi panjang

$$R = \{(x, y) \mid a \le x \le b,\ c \le y \le d\}$$

maka,

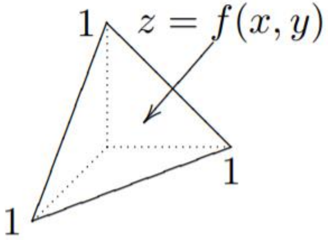
$$\iint_R f(x, y)dA = \int_a^b \int_c^d f(x, y)dydx = \int_c^d \int_a^b f(x, y)dxdy.$$

Mengapa bisa demikian?
Cermati pendekatan penyelesaian Tipe I dan Tipe II untuk region persegi panjang

Contoh Soal

Non-rectangular Region

Tentukan volume dari tetrahedron berikut!



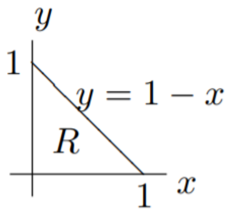
Tetrahedron

Solusi:

Step 1 - Cari f(x,y)

plane $ax + by + cz = d \quad \rightarrow \quad z = c_1x + c_2y + c_3$
$z = f(x, y) = c_1x + c_2y + c_3$
$f(0{,}0) = 1 \quad \rightarrow 1 = c_3$
$f(1, 0) = 0 \rightarrow c_1 + 1 = 0 \rightarrow c_1 = -1$
$f(1{,}0) = 0 \rightarrow c_2 + 1 = 0 \rightarrow c_2 = -1$
$f(x{,}y) = 1 - x - y$

Step 2 - Analisis bidang xy



Region R

Persamaan garis antara yang melewati titik $(1, 0)$ *dan* $(0, 1)$

$$m = \frac{\Delta y}{\Delta x} = -1$$

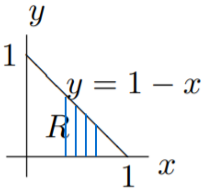
$$y - y_1 = m(x - x_1)$$

$$y = 1 - x \ldots (\textit{dipakai untuk Tipe } 1)$$

$$x = 1 - y \ldots (\textit{dipakai untuk Tipe } 2)$$

Step 3 - Kalkulasi Integral

Cara 1 - Tipe I (*slice* yz)



Region R

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx.$$

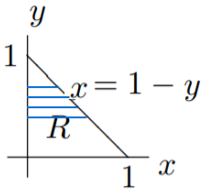
$$V = \int_0^1 \int_0^{1-x} 1 - x - y \, dy \, dx$$

$$= \int_0^1 y - xy - \frac{y^2}{2}\Big|_0^{1-x} dx$$

$$= \int_0^1 \frac{1}{2} - x + \frac{x^2}{2} dx$$

$$= \frac{1}{6}$$

Cara 2 - Tipe II (*slice* xz)



Region R

$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy.$$

$$V = \int_0^1 \int_0^{1-y} 1 - x - y \, dx \, dy$$

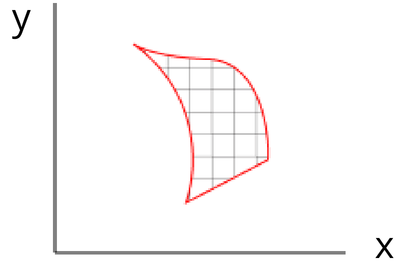
$$= \int_0^1 x - \frac{x^2}{2} - xy\Big|_0^{1-y} dy$$

$$= \int_0^1 \frac{1}{2} - y + \frac{y^2}{2} dy$$

$$= \frac{1}{6}$$

Double Integral dengan **Koordinat Polar**

Apa perbedaannya?

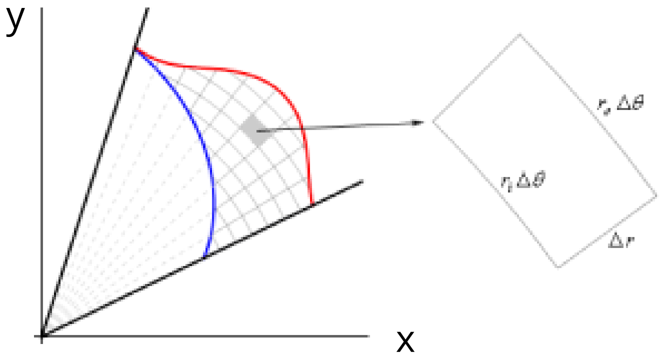


Cartesian Coordinate

$x, y\ as\ it\ is$

$dA = dxdy$

$$\iint_R f(x,y)\,dA = \iint_R f(x,y)\ dxdy$$



Polar Coordinate

$x = rcos\theta$

$y = rsin\theta$

$dA = rdrd\theta$

$$\boxed{\iint_R f(x,y)\,dA = \iint_R f(r\cos\theta,\, r\sin\theta)\, r\, dr\, d\theta.}$$

Contoh Soal

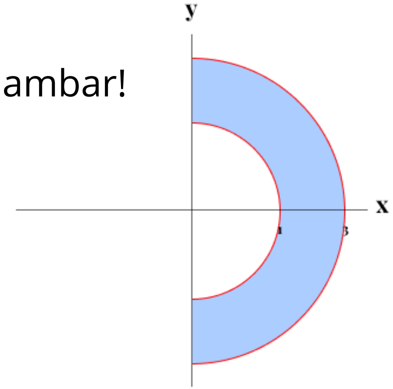
Hitung $\iint_R x\, dA.$

Dengan domain region

$$D = \{(r,\theta) | 1 \le r \le 2,\ -\tfrac{\pi}{2} \le \theta \le \tfrac{\pi}{2}\},$$

Solusi:

Step 1 : Gambar!



Step 2 : Kalkulasi Integral

$$\iint_R xdA = \iint_R rcos\theta\, r\, dr\, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_1^2 r^2 cos\theta\, drd\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{3} r^3 cos\theta \Big|_1^2\, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{7}{3} cos\theta\, d\theta$$

$$= \frac{7}{3} sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{14}{3}$$

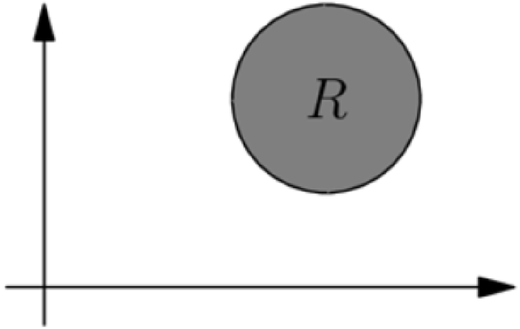
Aplikasi *Double Integral*

Mass and Average Value

Aplikasi double integral pada Mass and Average Value dapat kita pahami secara bertahap dengan kasus-kasus berikut.

1) Find area of region R

$$\text{Area}(R) = \iint_R dA$$



R=Region that you are integrating

dA= tiny chunk of area (can be dydx or dxdy)

Mass

Dengan konsep yang sama sekarang kita memberikan nilai/fungsi density pada tiap luasan integral.

Mass of a (flat) object with density $\delta = \dfrac{\text{mass}}{\text{unit area}}$

$$\Delta m = \delta \Delta A$$

$$\text{Mass} = \iint_R \delta \, dA$$

Average Value

2) Average value of f.

Average of $f = \bar{f} = \dfrac{1}{\text{Area}(R)} \iint\limits_R f\, dA$

Weighted average of f
with density δ.

$$\frac{1}{\text{Mass}(R)} \iint\limits_R f\delta\, dA$$

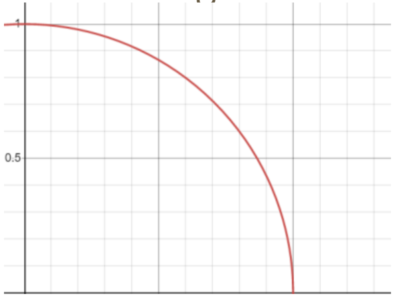
Contoh Kasus Penerapan Averague Value

TABLE 15.1 Mass and first moment formulas for thin plates covering a region R in the xy-plane

Mass:	$M = \iint_R \delta(x, y)\, dA$	$\delta(x, y)$ is the density at (x, y)
First moments:	$M_x = \iint_R y\delta(x, y)\, dA, \quad M_y = \iint_R x\delta(x, y)\, dA$	
Center of mass:	$\bar{x} = \frac{M_y}{M}, \quad \bar{y} = \frac{M_x}{M}$	

ContohSoal

- Let R be the quarter of the unit circle in the first quadrant with density δ(x, y) = y. (* Representasi parametrik lingkaran $x = r\cos\theta$ $y = r\sin\theta$)



-Analisis bidang xy

kita dapat menggunakan double integral pada koordinat polar. Sehingga batas batasnya menjadi $0 \le r \le 1$ and $0 \le \theta \le \pi/2$. Menggunakan representasi parametrik nilai $\delta = r\sin\theta$.

Kalkulasi Integral untuk mencari Mass

$$\begin{aligned} M &= \iint_R \delta \, dA \\ &= \int_0^{\pi/2} \int_0^1 (r \sin\theta)\, r \, dr \, d\theta \\ &= \int_0^{\pi/2} \int_0^1 r^2 \sin\theta \, dr \, d\theta. \end{aligned}$$

Inner: $\frac{1}{3} r^3 \sin\theta \Big|_0^1 = \frac{1}{3} \sin\theta.$

Outer: $-\frac{1}{3} \cos\theta \Big|_0^{\pi/2} = \frac{1}{3}.$

The region has mass $1/3$.

Lanjutan Contoh Soal

-Find the Center of Mass

The center of mass (x_{cm}, y_{cm}) is described by

$$x_{cm} = \frac{1}{M}\iint_R x\delta\,dA \quad \text{and} \quad y_{cm} = \frac{1}{M}\iint_R y\delta\,dA.$$

$$\begin{aligned} x_{cm} &= \frac{1}{M}\iint_R x\delta\,dA \\ &= 3\int_0^{\pi/2}\int_0^1 (r\cos\theta)(r\sin\theta)r\,dr\,d\theta \\ &= \int_0^{\pi/2}\int_0^1 3r^3\cos\theta\sin\theta\,dr\,d\theta. \end{aligned}$$

Inner: $\frac{3}{4}r^4\cos\theta\sin\theta\Big|_0^1 = \frac{3}{4}\cos\theta\sin\theta.$

Outer: $\frac{3}{4}\frac{1}{2}(\sin\theta)^2\Big|_0^{\pi/2} = \frac{3}{8} = x_{cm}.$

$$\begin{aligned} y_{cm} &= \frac{1}{M}\iint_R y\delta\,dA \\ &= 3\int_0^{\pi/2}\int_0^1 (r\sin\theta)(r\sin\theta)r\,dr\,d\theta \\ &= \int_0^{\pi/2}\int_0^1 3r^3\sin^2\theta\,dr\,d\theta. \end{aligned}$$

Inner: $\frac{3}{4}r^4\sin^2\theta\Big|_0^1 = \frac{3}{4}\sin^2\theta.$

Outer: $\frac{3}{4}\left(\frac{\theta}{2} - \frac{1}{4}\sin(2\theta)\right)\Big|_0^{\pi/2} = \frac{3\pi}{16} = y_{cm}.$

TABLE 15.2 Second moment formulas for thin plates in the *xy*-plane

Moments of inertia (second moments):

About the x-axis: $$I_x = \iint y^2 \delta(x, y)\, dA$$

About the y-axis: $$I_y = \iint x^2 \delta(x, y)\, dA$$

About a line L: $$I_L = \iint r^2(x, y) \delta(x, y)\, dA,$$

where $r(x, y) =$ distance from (x, y) to L

About the origin (polar moment): $$I_0 = \iint (x^2 + y^2) \delta(x, y)\, dA = I_x + I_y$$

Radii of gyration:

About the x-axis: $R_x = \sqrt{I_x / M}$

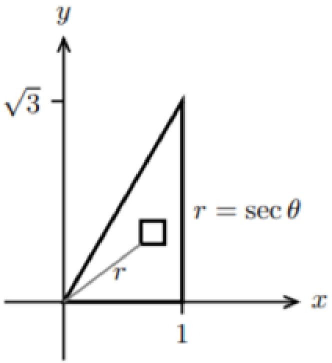
About the y-axis: $R_y = \sqrt{I_y / M}$

About the origin: $R_0 = \sqrt{I_0 / M}$

Contoh Soal

1. Let R be the triangle with vertices $(0,0)$, $(1,0)$, $(1,\sqrt{3})$ and density $\delta = 1$. Find the polar moment of inertia.

-Analisis Bidang xy



Batas Batas Integral yang didapat

$$x = 1 \;\Leftrightarrow\; r\cos\theta = 1 \;\Leftrightarrow\; r = \sec\theta.$$

Kalkulasi Integral

$$I = \iint_R r^2 \delta \, dA = \iint_R r^2 \, r \, dr \, d\theta = \iint_R r^3 \, dr \, d\theta.$$

$$I = \int_0^{\pi/3} \int_0^{\sec\theta} r^3 \, dr \, d\theta.$$

$$I = \int_0^{\pi/3} \int_0^{\sec\theta} r^3 \, dr \, d\theta.$$

Inner integral: $\dfrac{\sec^4\theta}{4}$.

Outer integral: Use $\sec^4\theta = \sec^2\theta \sec^2\theta = (1 + \tan^2\theta)\, d(\tan\theta) \Rightarrow$ the outer integral is

$$\frac{1}{4}\left(\tan\theta + \frac{\tan^3\theta}{3}\right)\Big|_0^{\pi/3} = \frac{1}{4}\left(\sqrt{3} + \frac{(\sqrt{3})^3}{3}\right) = \frac{\sqrt{3}}{2}.$$

The polar moment of inertia is $\dfrac{\sqrt{3}}{2}$.

Subsitusi double integral

Subsitusi double integral

Jika region G pada bidang-uv ditransformasi ke region R pada bidang-xy yang memenuhi persamaan

$$x = g(u,v), \qquad y = h(u,v).$$

$$\iint_R f(x,y)\,dx\,dy = \iint_G f(g(u,v),h(u,v))|J(u,v)|\,du\,dv$$

Definisi (Jacobian)

Determinan Jacobian atau *Jacobian* dari transformasi koordinat $x = g(u,v)$, $y = h(u,v)$ adalah

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}.$$

Contoh soal

$$\iint_D e^{2x+3y} \cdot \cos(x - 3y)\, dx dy,$$

where D is the region bounded by the parallelogram with vertices $(0,0)$, $(1, \frac{1}{3})$, $(\frac{4}{3}, \frac{1}{9})$, and $(\frac{1}{3}, -\frac{2}{9})$.

$$\iint_D e^{2x+3y} \cdot \cos(x-3y)\,dx\,dy$$

dgn batas $(0,0)$, $(1, 1/3)$, $(4/3, \frac{1}{9})$, $(1/3, -\frac{2}{9})$

$$\iint_R f(x,y)\,dx\,dy = \iint_G f(g(u,v), h(u,v))\,|J(u,v)|\,du\,dv$$

- mencari jacobian

misal $u = 2x+3y$ dan $v = x-3y$

$$\begin{array}{l} u = 2x+3y \\ v = x-3y \end{array} \left| \begin{array}{l} \times 1 \\ \times 2 \end{array} \right| \quad \begin{array}{l} u = 2x+3y \\ 2v = 2x-6y \; - \\ \hline u-2v = 9y \end{array}$$

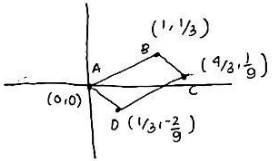
$$y = \frac{u-2v}{9} = \frac{1}{9}u - \frac{2}{9}v$$

$$\begin{array}{l} u = 2x+3y \\ v = x-3y \; + \\ \hline u+v = 3x \end{array}$$

$$x = 1/3\,u + 1/3\,v$$

$$J(u,v) = \begin{vmatrix} 1/3 & 1/3 \\ 1/9 & -\frac{2}{9} \end{vmatrix} = \frac{1}{3} \cdot \frac{-2}{9} - \frac{1}{3} \cdot \frac{1}{9} = \frac{-2}{27} - \frac{1}{27} = \frac{-3}{27} = \frac{-1}{9}$$

- transformasi titik



garis A-B

$$\frac{y-0}{1/3-0} = \frac{x-0}{1-0}$$

$$\frac{y}{1/3} = \frac{x}{1}$$

$$y = 1/3\,x$$

garis B-C

$$\frac{y-\frac{1}{3}}{\frac{1}{9}-\frac{1}{3}} = \frac{x-1}{\frac{4}{3}-1}$$

$$\frac{y-\frac{1}{3}}{-\frac{2}{9}} = \frac{x-1}{\frac{1}{3}}$$

$$2x+3y = 3$$

garis D-C

$$\frac{y-\frac{1}{9}}{-\frac{2}{9}-\frac{1}{9}} = \frac{x-4/3}{\frac{1}{3}-\frac{4}{3}}$$

$$x-3y = 1$$

garis A-D

$$\frac{y-0}{-\frac{2}{9}-0} = \frac{x-0}{\frac{1}{3}-0}$$

$$\frac{y}{-\frac{2}{9}} = \frac{x}{1/3}$$

$$2x+3y = 0$$

$$y = \frac{1}{3}x$$

$$\frac{1}{9}u - \frac{2}{9}v = \frac{1}{3}\left(\frac{1}{3}u + \frac{1}{3}v\right)$$

$$v = 0$$

$$x - 3y = 1$$

$$\left(\frac{1}{3}u + \frac{1}{3}v\right) - 3\left(\frac{1}{9}u - \frac{2}{9}v\right) = 1$$

$$v = 1$$

$$2x + 3y = 3$$

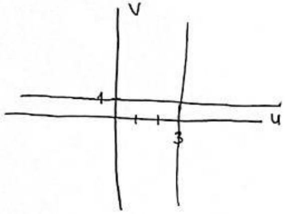
$$2\left(\frac{1}{3}u + \frac{1}{3}v\right) + 3\left(\frac{1}{9}u - \frac{2}{9}v\right) = 3$$

$$u = 3$$

$$2x + 3y = 0$$

$$2\left(1/3\,u + \frac{1}{3}v\right) + 3\left(\frac{1}{9}u - \frac{2}{9}v\right) = 0$$

$$u = 0$$



$$\iint e^u \cos(v)\,\left|-\frac{1}{9}\right|\,du\,dv$$

$$\frac{1}{9}\int_0^1\int_0^3 e^u \cos(v)\,du\,dv$$

$$\frac{1}{9}\int_0^1 \left[e^u \cos(v)\right]_0^3 dv$$

$$\frac{1}{9}\int_0^1 e^3\cos v - \cos v \quad dv$$

$$\frac{1}{9}\left[\int_0^1 e^3 \cos v\,dv - \int_0^1 \cos v\,dv\right]$$

$$\frac{1}{9}\left[\left[e^3 \sin v\right]_0^1 - \left[\sin v\right]_0^1\right]$$

$$\frac{1}{9}\left[e^3 \sin(1) - \sin(1)\right]$$

$$\frac{e^3 \sin(1) - \sin(1)}{9} = \frac{(e^3-1)\sin(1)}{9}$$

Contoh soal

$\iint_R \frac{x-2y}{3x-y}\,dA$, where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$, *and* $3x-y=8$

$\iint_R \frac{x-2y}{3x-y}\, dA$ dgn batas $x-2y=0$, $x-2y=4$, $3x-y=1$, dan $3x-y=8$

$$\iint_R f(x,y)\,dx\,dy = \iint_G f((g(u,v), h(u,v))\, |J(u,v)|\, du\, dv$$

• mencari jacobian

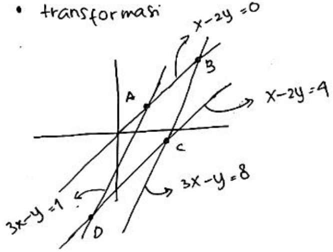
misal $u = x-2y$ dan $v = 3x-y$

$$\begin{array}{l} u = x-2y \\ v = 3x-y \end{array} \left| \begin{array}{l} \times 3 \\ \times 1 \end{array} \right| \quad \begin{array}{l} 3u = 3x-6y \\ v = 3x-y \quad - \\ \hline 3u - v = -5y \\ y = \frac{1}{5}v - \frac{3}{5}u \end{array}$$

$$\begin{array}{l} u = x-2y \\ v = 3x-y \end{array} \left| \begin{array}{l} \times 1 \\ \times 2 \end{array} \right| \quad \begin{array}{l} u = x-2y \\ 2v = 6x-2y \quad - \\ \hline u-2v = -5x \\ x = \frac{2}{5}v - \frac{1}{5}u \end{array}$$

$$J(u,v) = \begin{vmatrix} -\frac{1}{5} & \frac{2}{5} \\ -\frac{3}{5} & \frac{1}{5} \end{vmatrix} = -\frac{1}{25} - \left(-\frac{6}{25}\right) = \frac{5}{25} = \frac{1}{5}$$

• transformasi



$x-2y=0$

$\frac{2}{5}v - \frac{1}{5}u - 2\left(\frac{1}{5}v - \frac{3}{5}u\right) = 0$

$u = 0$

$x-2y=4$

$\frac{2}{5}v - \frac{1}{5}u - 2\left(\frac{1}{5}v - \frac{3}{5}u\right) = 4$

$u = 4$

$3x-y=1$

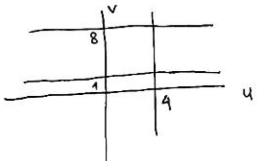
$3\left(\frac{2}{5}v - \frac{1}{5}u\right) - \left(\frac{1}{5}v - \frac{3}{5}u\right) = 1$

$v = 1$

$3x-y=8$

$3\left(\frac{2}{5}v - \frac{1}{5}u\right) - \left(\frac{1}{5}v - \frac{3}{5}u\right) = 8$

$v = 8$



$$\iint \frac{u}{v} \left|\frac{1}{5}\right| du\, dv$$

$$\frac{1}{5}\int_0^4\int_1^8 \frac{u}{v}\, dv\, du$$

$$\frac{1}{5}\int_0^4 \Big[u \ln(v)\Big]_1^8 du$$

$$\frac{1}{5}\int_0^4 u\ln(8) - u\ln(1)^{\to 0}\, du$$

$$\frac{1}{5}\int_0^4 u\ln(8)\, du$$

$$\frac{\ln(8)}{5}\int_0^4 u\, du$$

$$\frac{\ln(8)}{5}\left[\frac{u^2}{2}\right]_0^4 = \frac{\ln(8)}{5}\left[8-0\right]$$

$$= \frac{8\ln(8)}{5}$$

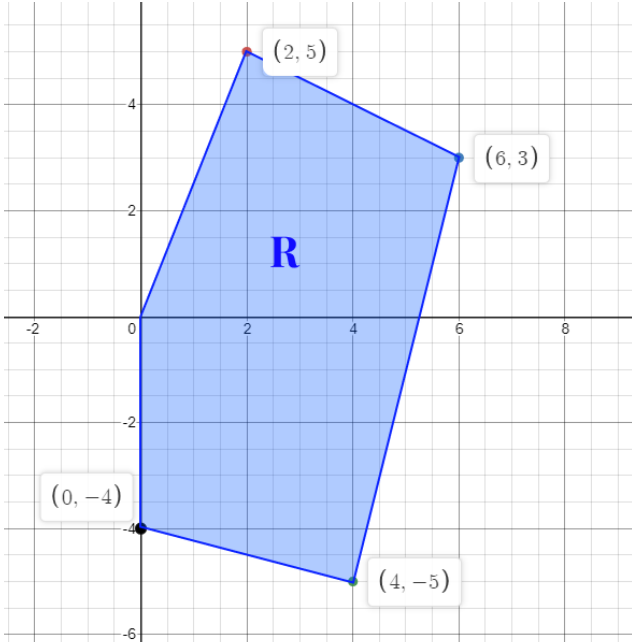
Quiz Materi 1-2

1. Tentukan luas bidang R yang memiliki titik-titik sudut A(0, 0), B(2, 5), C(6, 3), D(4, -5), dan E(0, -4)!

   Hint: Gunakan konsep luas dari interpretasi determinan yang sudah pernah dipelajari di kelas, ya!

1. Tentukan persamaan bidang yang melewati titik A(2,0,2), B(0,4,4), dan C(1,1,0)!

   Hint: Ada hubungannya dengan vektor normal! Gunakan cross product dan dot product.



Gambar untuk Nomor 1

Quiz materi 5-6

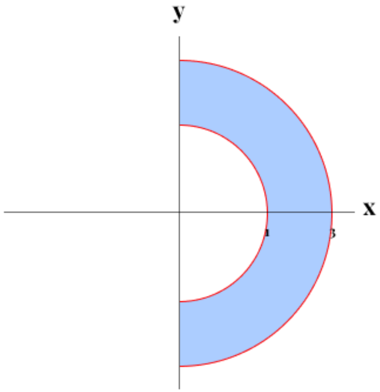
Jika $\Phi = xy^2 + 6 z^3 x^2$, carilah $\nabla\Phi$ dan $|\nabla\Phi|$ pada titik (6,2,2)

Carilah nilai maksimum dan minimum dari $f(x,y) : 6x+4y$ pada lingkaran $x^2+y^2 = 36$

Quiz Materi 8

Find moment inertia of the halfdisk with mass density $\delta(x, y) = \dfrac{1}{\sqrt{x^2+y^2}}$

a. About the x axis
b. About the origin



$$D = \{(r, \theta) | 1 \leq r \leq 2, -\tfrac{\pi}{2} \leq \theta \leq \tfrac{\pi}{2}\},$$

Pengerjaan Quiz

- Dikerjakan seperti latihan soal biasa
- Dikumpulkan di form pengumpulan tugas https://tinyurl.com/PengumpulanTugasKVJ (diisi pertemuan 4)
- Waktunya sampai jam 10.30 + toleransi pengumpulan 15 menit
- Format nama file NIU_Nama_Quiz1