Find the maximum and minimum of $f(x,y,z) = 6y - 2z$ subject to the constraints $x - 2y - 2z = 2$ and $x^2 + y^2 = 1$

•) Lagrangian formula

$\nabla f(x,y,z) = \nabla \lambda\, g_1(x,y,z) + \nabla \mu\, g_2(x,y,z)$

$\nabla f_x = \nabla \lambda\, g_{1x} + \nabla \mu\, g_{2x} \rightarrow 0 = \lambda + 2x\mu$ ①

$\nabla f_y = \nabla \lambda\, g_{1y} + \nabla \mu\, g_{2y} \rightarrow 6 = -2\lambda + 2y\mu$ ②

$\nabla f_z = \nabla \lambda\, g_{1z} + \nabla \mu\, g_{2z} \rightarrow -2 = -2\lambda$ ③

dari persamaan ③, didapatkan $\lambda = 1$, kemudian substitusi ke persamaan ① dan ②

① $\quad -1 = 2x\mu \rightarrow x = -\dfrac{1}{2\mu}$

② $\quad 6 = -2 + 2y\mu \rightarrow y = \dfrac{8}{2\mu}$

substitusi ke $g_2 = x^2 + y^2 = 1$

$\left(-\dfrac{1}{2\mu}\right)^2 + \left(\dfrac{8}{2\mu}\right)^2 = 1$

$\dfrac{1}{4\mu^2} + \dfrac{64}{4\mu^2} = 1$

$1 + 64 = 4\mu^2$

$\mu = \pm\sqrt{\dfrac{65}{4}} \rightarrow$ substitusi ke x dan y.

Maka didapatkan nilai x dan y yaitu.

$x = \pm\dfrac{1}{\sqrt{65}} \qquad y = \pm\dfrac{8}{\sqrt{65}}$

substitusi ke $g_1 = x - 2y - 2z = 2$.

didapatkan 4 nilai, yaitu:

$$\left(\frac{1}{\sqrt{65}}, \; 8/\sqrt{65}, \; \frac{-15-2\sqrt{65}}{2\sqrt{65}} \right)$$

$$\left(-1/\sqrt{65}, \; -8/\sqrt{65}, \; \frac{15-2\sqrt{65}}{2\sqrt{65}} \right)$$

$$\left(-1/\sqrt{65}, \; 8/\sqrt{65}, \; \frac{-17-2\sqrt{65}}{2\sqrt{65}} \right)$$

$$\left(1/\sqrt{65}, \; -8/\sqrt{65}, \; \frac{17-2\sqrt{65}}{2\sqrt{65}} \right)$$

didapatkan 4 titik, yaitu

$$\left(1/\sqrt{65}, \; 8/\sqrt{65}, \; \frac{-15-2\sqrt{65}}{2\sqrt{65}} \right) \;\to\; f(x,y,z) = 9,81$$

$$\left(-1/\sqrt{65}, \; -8/\sqrt{65}, \; \frac{+15-2\sqrt{65}}{2\sqrt{65}} \right) \;\to\; f(x,y,z) = -5,88$$

max
$$\left(-1/\sqrt{65}, \; 8/\sqrt{65}, \; \frac{-17-2\sqrt{65}}{2\sqrt{65}} \right) \;\to\; f(x,y,z) = 10,06$$

Min
$$\left(1/\sqrt{65}, \; -8/\sqrt{65}, \; \frac{17-2\sqrt{65}}{2\sqrt{65}} \right) \;\to\; f(x,y,z) = -6,06$$