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Kalkulus Variabel Jamak B

<div align="center">UTS Kalkulus Variabel Jamak B</div>

1. p = tanggal lahir

(Tanggal, bulan, tahun lahir saya = 07 Juli 2002)

Sehingga $p = 7$.

Titik $A(1,1,-1)$, $B(-1,0,2)$

a. tentukan $\bar{u}$ dengan $\|\bar{u}\| = 7$, $v \perp \overline{AB}$

* $\overline{AB} = B - A = (-2,-1,3)$

karena $\bar{u} \perp \overline{AB}$ maka $\bar{u} \cdot \overline{AB} = 0$

Misalkan ~~titik~~ u yaitu $v(x,y,z)$.

Sehingga $\bar{u} \cdot \overline{AB} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} = -2x + -y + 3z$

$-2x - y + 3z = 0$

b. Tentukan persamaan garis l yang melalui A dgsearah vektor $v = 2i + -2j + pk$.

* $v = 2i - 2j + 7k$

$A(1,1,-1)$

sehingga, $a = 2$, $b = -2$, $c = 7$.

Maka persamaan garis menjadi $\begin{cases} x = 1 + 2t \\ y = 1 - 2t \\ z = -1 + 7t \end{cases}$

atau

$$\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+1}{7}$$

C. Jarak antara titik B ke garis l

• $B(-1, 0, 2)$

pers garis l : $\dfrac{x-1}{2} = \dfrac{y-1}{-2} = \dfrac{z+1}{7}$ dimana garis l melalui $A(1, 1, -1)$

Sehingga jarak B ke l adalah dan vektor sejajar l adalah $\vec{v} = 2i - 2j + 7k$

$$d = \left\| AB \times \frac{v}{\|v\|} \right\|$$

$$= \left\| (-2, -1, 1) \times \frac{(2, -2, 7)}{\sqrt{56}} \right\|$$

2. Sistem Persamaan

$$\begin{cases} 2x - 3y + z = -1 \\ x - 3y - 2z = 1 \\ -2x + y + az = 3 \end{cases}$$

a. Ubahlah menjadi $Ax = B$

$$\underbrace{\begin{bmatrix} 2 & -3 & 1 \\ 1 & -3 & -2 \\ -2 & 1 & a \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}}_{B}$$

b. Pilih nilai a agar $\det(A) \neq 0$ dan hitung invers A.

→ $\det(A) = (2 \cdot -3 \cdot a + -3 \cdot -2 \cdot -2 + 1 \cdot 1 \cdot 1) - (-2 \cdot -3 \cdot 1 + 1 \cdot -2 \cdot 2 + a \cdot 1 \cdot -3)$

$\quad = (-6a - 12 + 1) - (6 - 4 - 3a) = -6a - 11 - 2 + 3a = -3a - 13$

$\det(A) \neq 0$

$-3a - 13 \neq 0$

$-3a \neq 13$

$a \neq \dfrac{-13}{3}$ → Misalkan $a = 5$ dan $\det(A) = -3(-5) - 13$

$\qquad\qquad\qquad\qquad = 15 - 13 = 2$ (memenuhi

Sehingga $A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -7 & -2 \\ -2 & 1 & -5 \end{pmatrix}$.

* Invers dari A.

Menggunakan garis - jordan elimination. $(A|I) \to (I|A^{-1})$ dengan I adalah matriks identitas. Sehingga matriks A dirubah menjadi Augmented matrix menjadi

$$\begin{pmatrix} 2 & -3 & 1 \\ 1 & -3 & -2 \\ -2 & 1 & -5 \end{pmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \xrightarrow{R_3 + R_1} \begin{pmatrix} 2 & -3 & 1 \\ 1 & -3 & -2 \\ 0 & -2 & -4 \end{pmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \xrightarrow{R_2 - \frac{1}{2}R_1} \begin{pmatrix} 2 & -3 & 1 \\ 0 & -3/2 & -\frac{5}{2} \\ 0 & -2 & -4 \end{pmatrix} \begin{vmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$\xrightarrow{R_3 - \frac{4}{3}R_2} \begin{pmatrix} 2 & -3 & 1 \\ 0 & -5/2 & -5/2 \\ 0 & 0 & -2/3 \end{pmatrix} \begin{vmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{5}{3} & -\frac{4}{3} & 1 \end{vmatrix} \xrightarrow[R_3 \times \frac{-3}{2}]{R_2 \times -2} \begin{pmatrix} 2 & -3 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ \frac{-5}{2} & 2 & \frac{-3}{2} \end{vmatrix}$$

$$\xrightarrow{R_2 - 5R_3} \begin{pmatrix} 2 & -3 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{vmatrix} 1 & 0 & 0 \\ \frac{23}{2} & -12 & \frac{15}{2} \\ \frac{-5}{2} & 2 & \frac{-3}{2} \end{vmatrix} \xrightarrow{R_1 + R_2} \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{vmatrix} 29/2 & -12 & 15/2 \\ 27/2 & -12 & 15/2 \\ -5/2 & 2 & -3/2 \end{vmatrix}$$

$$\xrightarrow{R_1 - R_3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{vmatrix} 17 & -14 & 9 \\ 27/2 & -12 & 15/2 \\ -5/2 & 2 & -3/2 \end{vmatrix} \xrightarrow[R_1 \times \frac{1}{2}]{R_2 \times \frac{1}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{vmatrix} 17/2 & -7 & 9/2 \\ 9/2 & -4 & 5/2 \\ -5/2 & 2 & -3/2 \end{vmatrix}.$$

Jadi, $A^{-1} = \begin{pmatrix} \frac{17}{2} & -7 & \frac{9}{2} \\ \frac{9}{2} & -4 & \frac{5}{2} \\ \frac{-5}{2} & 2 & \frac{-3}{2} \end{pmatrix}$

C. Selesaikan sistem persamaan tersebut

$$\begin{array}{ll} 2x - 3y + z = -1 \\ x - 3y - 2z = 1 \\ -2x + y - 5z = 3 \end{array} \longrightarrow$$

$$\begin{array}{l} *\ 2x - 3y + z = -1 \\ \underline{x - 3y - 2z = 1} \ + \\ 3x - z = 0 \\ 3x = z \end{array}$$

$$\begin{array}{l} *\ 2x - 3y + z = -1 \\ \underline{-2x + y - 5z = 3} \ + \\ -2y - 4z = 2 \\ y + 2z = -1 \end{array}$$

$*$ $\quad 2x - 3y + z = -1 \qquad \rightarrow \quad 2x - 3y + (3x) = -1 \quad \rightarrow \quad 5x - 3y + = -1$

$\qquad y + 2z = -1 \qquad\qquad\quad y + 2(3x) = -1 \qquad\qquad 6x + y = -1$

$$18x + 3y = -3$$
$$\underline{5x - 3y = -1}$$
$$23x = -4$$
$$x = \frac{-4}{23}$$

$\qquad\qquad\qquad\qquad\qquad 3x = z \qquad\qquad y + 2z = -1$

$\qquad\qquad\qquad\qquad\qquad 3 \cdot \frac{-4}{23} = z \qquad\quad y + 2 \cdot \frac{-12}{23} = -1$

$\qquad\qquad\qquad\qquad\qquad \frac{-12}{23} = z \qquad\qquad y - \frac{24}{23} = -1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y = -1 + \frac{24}{23}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y = \frac{1}{23}$

$\therefore$ jadi, $x = \frac{-4}{23}$, $y = \frac{1}{23}$, $z = \frac{-12}{23}$

3. a. Persamaan bidang yang melalui $(0,1,1)$ dan memuat garis

* Garis $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{2}$ atau $\begin{cases} x = -3 + 2t \\ y = 2 + 3t \\ z = -1 + 2t \end{cases}$

$$\frac{x+3}{2} = \frac{y-2}{3} = \frac{z+1}{2}$$

sehingga $\vec{v} = 2i + 3j + 2k$

dan terdapat suatu titik $(-3, 2, -1)$

* Misalkan $A(0,1,1)$ dan $B(-3,2,-1)$.

$\vec{AB} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$ dan $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$

sehingga $\vec{n} = \vec{AB} \times \vec{v} = \begin{vmatrix} i & j & k \\ -3 & 1 & -2 \\ 2 & 3 & 2 \end{vmatrix} = 8i + 2j - 11k$

$\vec{n} = 8i + 2j - 11k$

* Persamaan bidang yang melalui $(0,1,1)$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$, dimana $a = 8, b = 2, c = -11$, didapat dari $\vec{n}$

$8(x - 0) + 2(y - 1) + -11(z - 1) = 0$

$8x + 2y - 11z = 2 - 22$

$\boxed{8x + 2y - 11z = -20}$ → adi pers. bidang.

b. Persamaan garis yang merupakan perpotongan bidang $3x + y - 4z = 0$ dan $2x + 3y + 4z = 0$

* misalkan $\alpha : 3x + y - 4z = 0$, $\beta : 2x + 3y + 4z = 0$

$\alpha \times \beta = \begin{vmatrix} i & j & k \\ 3 & 1 & -4 \\ 2 & 3 & 4 \end{vmatrix} = -8i - 20j + 7k = \begin{bmatrix} -8 \\ -20 \\ 7 \end{bmatrix}$

misalkan $x = 0$ → $y - 4z = 0$

$\underline{3y + 4z = 0}$

$4y = 0$

$y = 0$

$z = 0$

$(0,0,0) = (x, y, z)$

Misalkan $z = 0 \rightarrow \begin{array}{l} 3x + y = 0 \\ 2x + 3y = 0 \end{array}$ $\rightarrow \begin{array}{l} 9x + 3y = 0 \\ 2x + 3y = 0 \\ \hline 7x = 0 \\ x = 0 \\ y = 0 \end{array}$ $(x, y, z) = (0, 0, 0)$

Sehingga persamaan garis menjadi $\begin{cases} x = -8t \\ y = -20t \\ z = 7t \end{cases}$ atau $\dfrac{x}{-8} = \dfrac{y}{-20} = \dfrac{z}{7}$

4. q adalah bulan lahir. $q = 7$ (karena bulan juli)

$$F(x, y) = 3x^2 y + y \ln x^2 + \cos(2yx)$$

a. $f_{xx}(2, q)$, $f_{xy}(2, 0)$, $f_{yy}(0, q)$

$\times$ $f_{xx}(2, q)$

$\rightarrow f_{xx}(7, 7)$

$\cdot$ $F_{xx}(x, y) = \dfrac{\partial}{\partial x}\left(6xy + \dfrac{2y}{x} \cdot -2y\sin(2yx)\right)$

$\qquad = 6y - \dfrac{2y}{x^2} - 4y^2 \cos(2xy)$

$F_{xx}(7, 7) = 6 \cdot 7 - \dfrac{2 \cdot 7}{7 \cdot 7} - 4 \cdot 7^2 \cos(2 \cdot 7 \cdot 7)$

$\qquad = 42 - \dfrac{2}{7} - 196 \cos(98) \approx 20$

$\qquad \approx 41,714 - 196 \cos(98) \approx 202,295$

b. $\times$ $F_{xy}(2, 0)$

$\rightarrow F_{xy}(7, 0)$

$\times$ $F_{xy}(x, y) = \dfrac{\partial}{\partial y}\left(6xy + \dfrac{2y}{x} - 2y\sin(2xy)\right)$

$\qquad = 6x + \dfrac{2}{x} - 2\sin(2xy) - 4xy\cos(2xy)$

$F_{xy}(7, 0) = 6 \cdot 7 + \dfrac{2}{7} - 2\sin(0) - 4 \overset{7 \cdot 0}{xy} \cos(0)$

$\qquad = 42 + \dfrac{2}{7} - 0 - 0 = \dfrac{296}{7} \approx 42,286$

* $F_{yy}(0,2)$

→ $F_{yy}(0,7)$

* $F_{yy}(x,y) = \frac{\partial}{\partial y}\left(3x^2 + 2\ln(x) + -2x\sin(2xy)\right)$

$= -4x^2\cos(2xy)$

$F_{yy}(0,7) = -4 \cdot 0 \cos(2 \cdot 0 \cdot 7) = 0$

a. b jika