Date

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Integral garis

1. $\int_C (2+x^2y)\,ds$ dimana C adalah separuh lingkaran $x^2+y^2=1$, $y \geq 0$

* $x = \sqrt{1-t^2}$
  $y = t$ $\quad 0 \leq t \leq 1$

* $\frac{dx}{dt} = \frac{d\sqrt{1-t^2}}{dt} = \frac{-t}{\sqrt{1-t^2}}$
  $\left(\frac{dx}{dt}\right)^2 = \left(\frac{-t}{\sqrt{1-t^2}}\right)^2 = \frac{t^2}{1-t^2}$

  $\frac{dy}{dt} = 1$
  $\left(\frac{dy}{dt}\right)^2 = 1$

* $ds = \sqrt{\left(\frac{t^2}{1-t^2}\right)+(1)}\,dt = \sqrt{\frac{t^2+1-t^2}{1-t^2}}\,dt$
  $= \sqrt{\frac{1}{1-t^2}}\,dt = \frac{1}{\sqrt{1-t^2}}\,dt$

* $\int_C (2+x^2y)\,ds = \int_0^1 (2+t-t^3)\cdot\frac{1}{\sqrt{1-t^2}}\,dt$
  $= \int_0^1 \frac{2}{\sqrt{1-t^2}} + t\sqrt{1-t^2}\,dt = 2\sin^{-1}(t) - \left[\frac{(1-t^2)\sqrt{1-t^2}}{3}\right]_0^1$
  $= 2\pi - \frac{1}{3}$

2. $\int_C F\cdot dr = ?$, $F(x,y,z) = z\,i + xy\,j - y^2 k$, $C: r(t) = t^2 i + t\,j + \sqrt{t}\,k$, $0 \leq t \leq 1$

$\frac{dr}{dt} = 2t\,i + 1\,j + \frac{1}{2\sqrt{t}}\,k$ $\quad F(r) = \sqrt{t}\,i + t^2 j - t^2 k$

$\int_C F\cdot dr = \int_0^1 F\cdot dr = \int_C F(r)\cdot dr = \int_0^1 2t\sqrt{t} + t^2(-t^2)\frac{1}{2\sqrt{t}}\,dt$

$= \frac{4t^2\sqrt{t}}{5} + \frac{t^4}{4} - \frac{t^2\sqrt{t}}{5}\Big|_0^1$

$= \frac{4}{5} + \frac{1}{4} - \frac{1}{5} = 0{,}85$

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Aplikasi teorema green

1. $F(x,y) = (-3y, 3x)$

Daerah D dibatasi kurva C berupa lingkaran arah positif.

Hitunglah sirkulasi F sekeliling C dan Fluks F meninggalkan D melalui C.

$x^2 + y^2 = r^2$ → $y = r\sin(x)$

$0 \le x \le \pi$

sirkulasi

$$\int_C F \cdot T \, ds = \iint_D \left(\frac{dN}{dx} - \frac{dM}{dy}\right) dA$$

$M = -3$

$N = 3$

$$\int_C F \cdot T \, ds = \int_{-r}^{r} \int_{0}^{\sqrt{r^2-x^2}} (3+3) \, dy \, dx$$

$$= \int_{-r}^{r} 6y \Big|_0^{\sqrt{r^2-x^2}} dx = \int_{-r}^{r} 6\sqrt{r^2-x^2} \, dx$$

$$= \int_{-r}^{r} 6r \, dx = 6rx \Big|_{-r}^{r} = 6$$

$$\int_C F \cdot T \, ds = \int_0^{\pi} \int_0^{r\sin(x)} (3+3) \, dy \, dx = \int_0^{\pi} 6y \Big|_0^{r\sin(x)} dx$$

$$= \int_0^{\pi} 6r \cdot \sin(x) \, dx = -6r\cos(x) \Big|_0^{\pi} = 6r$$