Pembahasan Quiz 1 Tutorial KVJ

(Pertemuan 4)

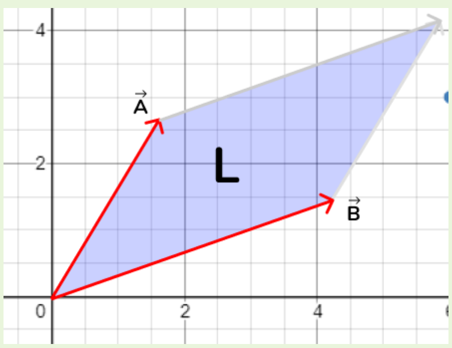
Nomor 1

Tentukan luas bidang R yang memiliki titik-titik sudut A(0, 0), B(2, 5), C(6, 3), D(4, -5), dan E(0, -4)!

Hint: Gunakan konsep luas dari interpretasi determinan yang sudah pernah dipelajari di kelas, ya!

Jawab:

Throwback konsep interpretasi determinan sebagai luasan:



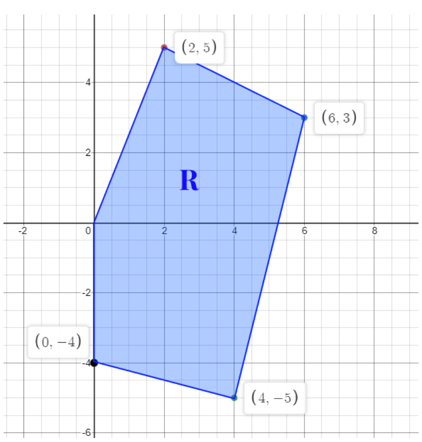
Misal kita mempunyai dua vektor:

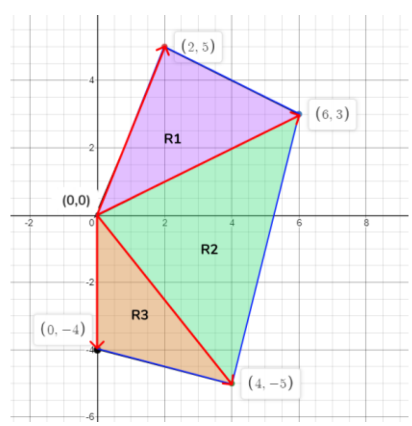
$$\vec{A} = (a, b) \quad ; \quad \vec{B} = (c, d)$$

Maka luasan jajargenjang L diberikan oleh:

$$L = \left\| \begin{matrix} a & b \\ c & d \end{matrix} \right\| = \left\| \begin{matrix} a & c \\ b & d \end{matrix} \right\|$$

Kita dapat menggunakan konsep itu untuk memecahkan soal ini.





Breakdown region R menjadi tiga: R1, R2, dan R3 dan definisikan vektor menuju titik-titik sudut:

$$\vec{P} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} ; \vec{Q} = \begin{bmatrix} 6 \\ 3 \end{bmatrix} ; \ \vec{R} = \begin{bmatrix} 4 \\ -5 \end{bmatrix} ; \vec{S} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

Perhatikan bahwa masing-masing luasan R1, R2, dan R3 merupakan bentuk **setengah** jajargenjang yang dibentuk dari operasi determinan!

$$L_{R1} = \frac{\left|\det\left(\left[\vec{P}\ \vec{Q}\right]\right)\right|}{2} = \frac{\left\|\begin{matrix} 2 & 6 \\ 5 & 3 \end{matrix}\right\|}{2} = \frac{|6-30|}{2} = 12$$

$$L_{R2} = \frac{\left|\det\left(\left[\vec{Q}\ \vec{R}\right]\right)\right|}{2} = \frac{\left\|\begin{matrix} 6 & 4 \\ 3 & -5 \end{matrix}\right\|}{2} = \frac{|-30-12|}{2} = 21$$

$$L_{R3} = \frac{\left|\det\left(\left[\vec{R}\ \vec{S}\right]\right)\right|}{2} = \frac{\left\|\begin{matrix} 4 & 0 \\ -5 & -4 \end{matrix}\right\|}{2} = \frac{|-16|}{2} = 8$$

Sehingga:

$$\boxed{L_R = L_{R1} + L_{R2} + L_{R3} = 12 + 21 + 8 = \mathbf{41}}$$

Nomor 2

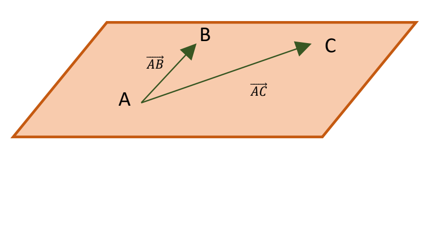
Tentukan persamaan bidang yang melewati titik A(2,0,2), B(0,4,4), dan C(1,1,0)!

Hint: Ada hubungannya dengan vektor normal! Gunakan cross product dan dot product.

Jawab:

Langkah 1: Cari dua vektor yang punya ekor yang sama, misal ($\overrightarrow{AB}$ & $\overrightarrow{AC}$) atau ($\overrightarrow{BA}$ & $\overrightarrow{BC}$) atau ($\overrightarrow{CA}$ & $\overrightarrow{CB}$). Misal kita ambil ekornya adalah titik A(2,0,2).

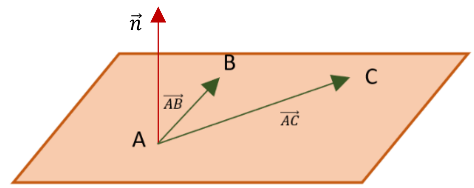
$$\overrightarrow{AB} = B - A = \begin{bmatrix} -2 \\ 4 \\ 2 \end{bmatrix}$$



$$\overrightarrow{AC} = C - A = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Langkah 2: Cari vektor normal dari kedua vektor di atas menggunakan *cross product*.

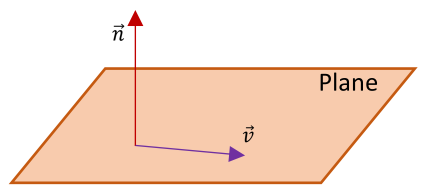
$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -2 & 4 & 2 \\ -1 & 1 & -2 \end{vmatrix} = \begin{bmatrix} -10 \\ -6 \\ 2 \end{bmatrix}$$



Langkah 3: Cari persamaan umum bidang menggunakan *dot product* antara $\vec{n}$ dan vektor umum dengan ekor sesuai Langkah 1

Untuk sebuah titik umum T(x, y, z):

$$\vec{v} = \overrightarrow{AT} = T - A = \begin{bmatrix} x - 2 \\ y \\ z - 2 \end{bmatrix}$$



$$Plane \equiv \vec{n} \,.\, \vec{v} = 0$$

$$Plane \equiv -10x + 20 - 6y + 2z - 4 = 0$$

$Plane \equiv \boldsymbol{-10x - 6y + 2z = -16}$, atau

$Plane \equiv \boldsymbol{10x + 6y - 2z = 16}$, atau

$Plane \equiv \boldsymbol{5x + 3y - z = 8}$

Kunci Jawaban Tutor 4

Quiz

Date

Materi 3 - 4

1. Find critical point and it's classification $z = -x^2 + y^2$

$f_x = -2x$ $\qquad f_y = 2y$

$f_{xx} = -2$ $\qquad f_{yy} = 2$ $\qquad f_{xy} = 0$

critical point

$-2x = 0$

$2y = 0$ $\rightarrow$ $(0,0)$ //

critical point classification

$D(x,y) = f_{xx} \cdot f_{yy} - (F_{xy})^2$

$= -2 \cdot 2 - 0$

$= -4$

untuk critical point $(0,0)$

$D(0,0) = -4 < 0 \rightarrow$ saddle point //

2. Find curvature

$r(t) = 5t\,i + 3\sin(t)\,j + 3\cos(t)\,k$

$K(t) = \dfrac{\| T'(t) \|}{\| r'(t) \|} = \dfrac{\| r'(t) \times r''(t) \|}{\| r'(t) \|^3}$

- $r'(t) = 5\bar{i} + 3\cos(t)\bar{j} - 3\sin(t)\,k = \langle 5, 3\cos(t), -3\sin(t) \rangle$
- $r''(t) = -3\sin(t)\bar{j} - 3\cos(t)\,k = \langle 0, -4\sin(t), -4\cos(t) \rangle$

$r'(t) \times r''(t) = \begin{vmatrix} \bar{i} & \bar{j} & k \\ 5 & 3\cos(t) & -3\sin(t) \\ 0 & -4\sin(t) & -4\cos(t) \end{vmatrix} = (-12\cos^2(t) - 12\sin^2(t))\bar{i} - (-20\cos(t))\bar{j} + (-20\sin(t))\,k$

$= -12(\cos^2(t) + \sin^2(t))\,\bar{i} + 20\cos(t)\,\bar{j} - 20\sin(t)\,k$

$= -12 + 20\cos(t)\,\bar{j} - 20\sin(t)\,\bar{j}$

$\| r'(t) \times r''(t) \| = \sqrt{144 + 400(\cos^2(t) + \sin^2(t))}$

$= \sqrt{544}$

$$\| r'(t) \| = \sqrt{25 + 9(\sin^2(t) + \cos^2(t))}$$

$$= \sqrt{34}$$

$$\| r'(t) \|^3 = 34\sqrt{34}$$

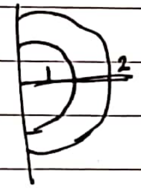
$$K(t) = \frac{\sqrt{544}}{34\sqrt{34}} = \frac{2}{17} \quad //$$

Materi 8

Find moment inerdia of halfdisk with $\delta = \frac{1}{\sqrt{(x^2+y^2)}}$

a. about x asis

b. about the origin



$D = \{(r,\theta) \mid 1 \le r \le 2,\ -\pi/2 \le \theta \le \pi/2\}$

note: $x = r\cos\theta$, $y = r\sin\theta$ $\quad x^2+y^2 = r^2$

a. about x asis $\quad I_x = \iint_R y^2 \,\delta \, dA$

$$\delta = \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{r}$$

$$I_x = \iint_R y^2 \,\delta\, dA = \int_{-\pi/2}^{\pi/2}\int_1^2 r^2 \sin^2\theta \cdot \frac{1}{\cancel{r}}\, \cancel{r}\, dr\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{r^3}{3}\sin^2\theta \Big]_1^2 = \int_{-\pi/2}^{\pi/2} \frac{7}{3}\sin^2\theta$$

$$= \frac{7}{3}\cdot\frac{1}{2}\left[\theta - \frac{1}{2}\sin(2\theta)\Big]_{-\pi/2}^{\pi/2}\right]$$

$$= \frac{7\pi}{6}$$

b. about the origin

$$I_0 = \iint_R (x^2+y^2)\,\delta\, dA = \int_{-\pi/2}^{\pi/2}\int_1^2 r^2 \cdot 1/\cancel{r}\,\cancel{r}\, dr\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_1^2 r^2\, dr\, d\theta = \int_{-\pi/2}^{\pi/2} \frac{r^3}{3}\Big]_1^2 d\theta = \int_{-\pi/2}^{\pi/2} 7\, d\theta$$

$$= 7\theta\Big]_{-\pi/2}^{\pi/2} = 7\pi \;//$$

5. $\nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$

$= \frac{\partial (xy^2 + 6z^3x^2)}{\partial x} i + \frac{\partial (xy^2 + 6z^3x^2)}{\partial y} j + \frac{\partial (xy^2 + 6z^3x^2)}{\partial z} k$

$= (y^2 + 12xz^3) i + 2xy\, j + 18\, x^2 z^2\, k$

$\nabla \phi (6,2,2) = 580 i + 24 j + 2592 k$

$|\nabla \phi| = \sqrt{580^2 + 24^2 + 2592^2} = 2656{,}20782$

6. $g(x,y) = x^2 + y^2 - 36$

$\nabla f = (6, 4)$

$\nabla g = (2x, 2y)$

$\nabla f = \lambda \nabla g$

$(6,4) = \lambda (2x, 2y)$

$6 = \lambda 2x \Rightarrow x = \frac{3}{\lambda}$

$4 = \lambda 2y \Rightarrow y = \frac{2}{\lambda}$

$x^2 + y^2 = 36$

$\left(\frac{3}{\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 = 36$

$\frac{9+4}{\lambda^2} = 36$

$\frac{13}{\lambda^2} = 36$

$\lambda^2 = \frac{13}{36}$

$\lambda = \pm \frac{\sqrt{13}}{6}$

$(x,y) = \frac{3 \pm 6}{\sqrt{13}}, \frac{2 \pm 6}{\sqrt{13}} = \left(\pm \frac{18}{\sqrt{13}}, \pm \frac{12}{\sqrt{13}}\right)$

$\max = 6 \cdot \left(\frac{18}{\sqrt{13}}\right) + 4\left(\frac{12}{\sqrt{13}}\right) = 12\sqrt{13}$

$\min = 6 \left(-\frac{18}{\sqrt{13}}\right) + 4\left(-\frac{12}{\sqrt{13}}\right) = -12\sqrt{13}$