Prelim 2 for BTRY6010/ILRST6100

November 3, 2016: 7:30pm-9:30pm

Name:________________________________

Lab: (circle one)

Lab 402: Tues 1:25PM - 2:40PM

Lab 403: Tues 2:55PM - 4:10PM

Lab 404: Wed 2:55PM - 4:10PM

Lab 405: Tues 7:30PM - 8:45PM

Score: ______ / 60

Instructions

1. Please **do not turn to next page** until instructed to do so.
2. You have 120 minutes to complete this exam.
3. The last page of this exam has some useful formulas.
4. No textbook, calculators, phone, computer, notes, etc. allowed (please keep your phones off or do not bring them to the exam).
5. Please answer questions in the spaces provided.
6. When asked to calculate a number, it is sufficient to write out the full expression in numbers without actually calculating the value. E.g., $\frac{1+3\times\frac{4}{7}}{3+0.7}$ is a valid answer.
7. Please read the following statement and sign before beginning the exam.

Academic Integrity

I, ____________________, certify that this work is entirely my own. I will not look at any of my peers' answers or communicate in any way with my peers. I will not use any resource other than a pen/pencil. I will behave honorably in all ways and in accordance with Cornell's Code of Academic Integrity.

Signature: ________________ **Date:**_________

For each problem in this section, please circle one of the following answers. No justification is required.

1. [2 points] Consider a 95% confidence interval. If you increase n, the interval will tend to get

(a) <u>narrower</u>

(b) wider

(c) stay the same

2. [2 points] If you change a 95% confidence interval to a 90% confidence interval, the interval will tend to get

(a) <u>narrower</u>

(b) wider

(c) stay the same

3. [2 points] Suppose you have a sample of size $n = 100$ of independent data, $X_1, \ldots, X_n$, that comes from a distribution that is not too skewed and has mean $\mu = 4$ and standard deviation $\sigma = 2$. We should expect the sample mean, $\bar{X}_n$, to be between 3.8 and 4.2 with probability about equal to

(a) <u>68%</u>

(b) 95%

(c) 99.7%

(d) cannot be determined from provided information

4. [2 points] Suppose you have a sample of size $n = 9$ of independent data, $X_1, \ldots, X_n$, that comes from a normal population with mean $\mu = 4$ and standard deviation $\sigma = 3$. We should expect the sample mean, $\bar{X}_n$, to be between 2.0 and 6.0 with probability about equal to

(a) 68%

(b) <u>95%</u>

(c) 99.7%

(d) cannot be determined from provided information

5. [2 points] Suppose you have a 95% confidence interval based on the central limit theorem. To make the interval half the width, you could

(a) double the sample size

(b) halve the sample size

(c) <u>none of the above</u>

6. [2 points] Suppose you have a 95% confidence interval based on the central limit theorem. To make the interval approximately half as wide, you could

(a) <u>lower the confidence level to 68%</u>

(b) increase the confidence level to 99.7%

(c) none of the above

True or False?

For each of the following questions, please answer either *True* or *False*. While justification is not required, it is encouraged and may allow in some cases for partial credit to be awarded.

7. [2 points] The sample mean $\bar{X}_n$ is approximately normal (or exactly normal) only in the case that the data, $X_1, \ldots, X_n$, is itself normal.

False.

8. [2 points] The significance level of a test depends on the distribution of the test statistic under the alternative hypothesis.

False.

9. [2 points] The power of a test is $1 - P(\text{Type I error})$.

False.

10. [2 points] The power of a test depends on the alternative hypothesis.

True.

11. [2 points] A p-value is the probability that the null hypothesis is true given the data.

False.

12. [2 points] Suppose $[4.2, 4.3]$ is a 99% confidence interval for μ. Then there is a 99% probability that μ is in the interval $[4.2, 4.3]$.

False.

13. [2 points] Suppose $[4.2, 4.3]$ is a 99% confidence interval for μ. Then if we could repeat the experiment over and over, we would expect that 99% of the time μ would be in the interval $[4.2, 4.3]$.

False.

14. [1 points] A probability mass function must everywhere be less than or equal to 1.

True.

15. [2 points] A probability density function must everywhere be less than or equal to 1.

False.

16. [2 points] Suppose $X_1, \dots, X_n$ are independent normal random variables with mean μ and standard deviation σ. If σ is **unknown**, then $\bar{X}_n$ is not normally distributed.

False.

Please answer the following questions. An answer without justification will not receive full credit.

17. [4 points] By a strange set of circumstances you end up at dinner with the CEO of a giant drug company. When he hears that you are taking a statistics class at Cornell, he says,

    "Tell me, something. My chief of research says that when testing new drugs we need to use tests with a low significance level and high power. As CEO, all I care about is how much money we make and lose. Do significance level and power have anything to do with this?"

Explain to him the practical monetary consequences of a high or low significance level and high or low power.

Setting a high significance level will lead to higher chance of type-I error and false discoveries, i.e., drugs which are not really effective will pass the test and will be recommended for sale on the market. This could result in large production cost and low sales, leading to loss of money. Setting a low significance level will help control the chances of this.

Choosing a test with low power, on the other hand, will lead to higher chance of type-II error and missed opportunities, i.e., drugs which could have really been effective for a disease will not be recommended for sale, and the company will lose the chance to make more money.

18. [4 points] You form a 99% confidence interval for μ based on a sample of $n = 10$ independent data points where the data distribution is itself normal. The sample standard deviation is $s = 2$. The next day your colleague tells you that it is well known that $\sigma = 2$ (it turns out your estimate of the population standard deviation was perfect). Explain what effect if any this piece of information has on your confidence interval. Does the interval get wider, narrower, stay the same, or is there not enough information provided? Justify your answer as precisely as possible.

Since the population standard deviation σ is known, we will use $z_{.025}$ instead of $t_{9,.025}$ for margin of error calculation. Since the standard normal distribution has thinner tail than the t- distribution, $z_{.025}$ is smaller than $t_{9,.025}$, so our confidence interval will be narrower.

19. From years of experience, a local farmer knows his onions are dependably 3 inches wide on average. A Cornell researcher shows him a new technique that she claims will increase the size of his onions (making them sell for more). He grows 100 onions using the new technique and finds that the average size of an onion in this sample is 3.3 inches (with standard deviation 1 inch).

(a) [2 points] What is the null and alternative hypothesis?

$H_0 : \mu = 3$ and $H_A : \mu > 3$,

where μ is the population average size of an onion grown using the new technique.

(b) [2 points] What is the test statistic (expressed as a random variable) and what is its distribution under the null?

$Z = \frac{\bar{X}_n - \mu_0}{S_n/\sqrt{n}}$

where n is the sample size, μ_0 is the hypothesized value of population average onion size, $\bar{X}_n$ is the sample average and S_n is the sample standard deviation of onion sizes using the new technique. Under the null, its distribution is approximately N(0,1).

(c) [1 point] Calculate the realized value of the test statistic in this example.

$Z = \frac{3.3-3}{1/\sqrt{100}} = \frac{0.3}{0.1} = 3.$

(d) [2 points] What is the p-value of this test? (You should draw a picture as you answer this.)

p-value $= P(Z > 3) =$ `1-pnorm(3)` $\approx (1 - 0.997)/2 = 0.0015$.

(e) [4 points] Explain what you would need to know to answer whether this (i) a statistically significant effect and whether this (ii) a practically significant effect.

To know whether this is a statistically significant effect, we need to know whether the significance level is more than .0015.

To know whether this is a practically significant effect, we need to know whether an average increase of 0.3 inches in onion sizes is large enough to result in higher onion sales.

20. Consider performing a Monte Carlo simulation to estimate $P_{\text{Type I}}$, the probability that a test makes a Type I error. You have the computer generate a huge number ($n_{\text{simulation}} = 10,000$) of test statistics drawn from the null distribution. You have the computer record $x_i = 1$ if on the i th iteration the null is rejected and $x_i = 0$ if on the ith iteration we fail to reject the null.

(a) [1 point] How would you get an estimate of $P_{\text{Type I}}$ using $x_1, \ldots, x_{n_{\text{simulation}}}$? (You don't need to write code, just write out "in math" what you would do.)

$\hat{p} = \frac{x_1+x_2+\ldots+x_{n_{\text{simulation}}}}{n_{\text{simulation}}}$ can be used as an estimate of $P_{\text{Type I}}$.

(b) [4 points] Every time you run this simulation, you'd get a different set of numbers, $x_1, \ldots, x_{n_{\text{simulation}}}$ (assuming you haven't set a seed) and therefore a different estimate of $P_{\text{Type I}}$ (using the expression from part a). We'd like to report an interval for $P_{\text{Type I}}$ rather than just giving a single number. This interval should express in some way how sure we are of our Monte Carlo estimate. Write out a specific expression for this interval and explain how we should interpret it.

A 95% confidence interval for $P_{\text{Type I}}$ can be constructed as

$(\hat{p} - 1.96 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n_{\text{simulation}}}}, \hat{p} + 1.96 \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n_{\text{simulation}}}})$.

Interpretation: We are 95% confident that $P_{\text{Type I}}$ is somewhere between the upper and lower limits of the above confidence interval.

In other other words, if we run the simulation many times and create confidence intervals using the above formula, we expect about 95% of those intervals will contain the value of $P_{\text{Type I}}$.

21. Suppose you currently have two unrelated research questions. For each, you have a null and alternative hypothesis. For each, you gather data (completely independent of each other). For each, you perform a level $\alpha = 0.10$ test.

(a) [2 points] Suppose both null hypotheses are true. Under this assumption, what is the probability that you **do not reject either of the null hypotheses**?

$(1 - 0.10)(1 - .10) = 0.81$

(b) [3 points] Suppose you get to publish a paper if you are able to reject one or both of the null hypotheses. Interpret what your calculation in part (a) tells you about the probability of publishing a paper given that in fact both null hypotheses are true.

The calculation in part (a) tells us that there is a $(100 - 81)\% = 19\%$ chance of publishing a paper even when none of the alternative hypotheses is true.