Ch: 28
RHK

① $F = ma = (9.11\times10^{-31})(1.84\times10^{9}) = 1.68\times10^{-21}$

$E = \frac{F}{e} = \frac{1.68\times10^{-21}}{1.6\times10^{-19}} = 1.05\times10^{-2}$ N/C

Direction of E : westward

② (a) $F = Ee = (3.0\times10^{6})(1.6\times10^{-19}) = 4.8\times10^{-13}$ N

(b) $F = 4.8\times10^{-13}$ N

③ $qE = mg \Rightarrow E = \frac{mg}{q} = \frac{6.64\times10^{-27}\times9.8}{2\times1.6\times10^{-19}} = 2.03\times10^{-7}$ N/C

Direction of E : up.

④ (a) $q = -2.0\times10^{-9}$C , $F_e = 3.0\times10^{-6}$N (downward)

$E = \frac{F_e}{q} = \frac{-3.0\times10^{-6}}{-2.0\times10^{-9}} = 1.5$ kN/C

(b) $F_e = eE = (1.6\times10^{-19})(1500) = 2.4\times10^{-16}$ N (upward)

(c) $F_g = m_p g = (1.67\times10^{-27})\times9.8 = 1.64\times10^{-26}$ N

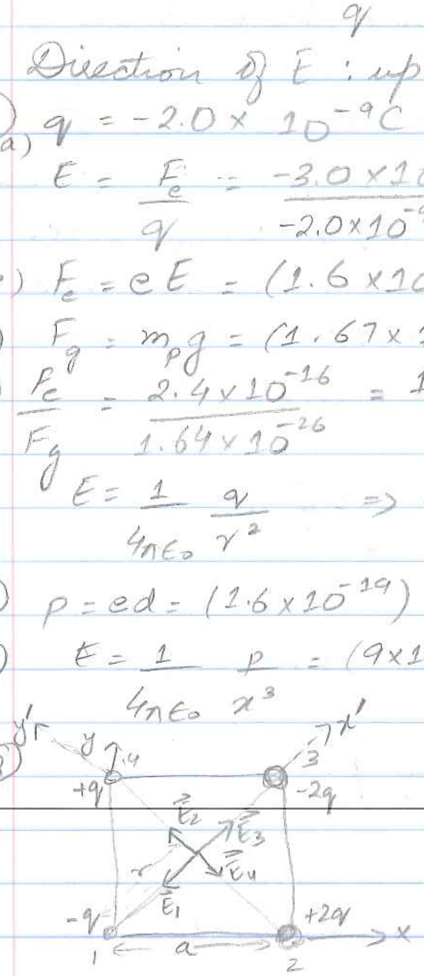
(d) $\frac{F_e}{F_g} = \frac{2.4\times10^{-16}}{1.64\times10^{-26}} = 1.46\times10^{10}$

⑤ $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \Rightarrow q = Er^2 4\pi\epsilon_0 = \frac{(2.30)(0.75)^2}{(9\times10^9)} = 1.44\times10^{-10}$C $= 144$ pC

⑥ $p = ed = (1.6\times10^{-19})(4.30\times10^{-9}) = 6.88\times10^{-28}$ Cm

⑦ $E = \frac{1}{4\pi\epsilon_0}\frac{p}{x^3} = (9\times10^{9})\frac{(3.56\times10^{-29})}{(25.4\times10^{-9})^3} = 1.96\times10^{4}$ N/C $= 19.6$ kN/C

⑧



Along x'-axis: $r = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

$E_1 = \frac{-q}{4\pi\epsilon_0(a/\sqrt{2})^2} = \frac{-q}{2\pi\epsilon_0 a^2}$

$E_3 = \frac{2q}{4\pi\epsilon_0(a/\sqrt{2})^2} = \frac{2q}{2\pi\epsilon_0 a^2}$

$E_{x'} = \frac{q}{2\pi\epsilon_0 a^2}$

Along y'-axis:

$E_2 = \frac{2q}{4\pi\epsilon_0(a/\sqrt{2})^2} = \frac{2q}{2\pi\epsilon_0 a^2}$, $E_4 = \frac{-q}{4\pi\epsilon_0(a/\sqrt{2})^2} = \frac{-q}{2\pi\epsilon_0 a^2}$

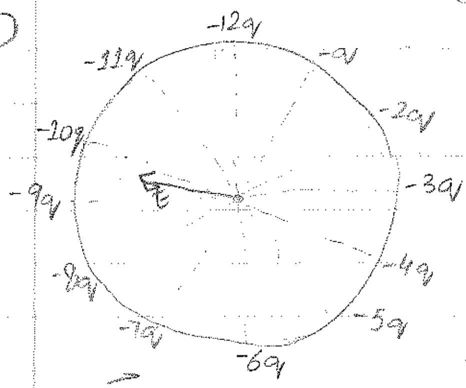
$E_{y'} = \frac{q}{2\pi\epsilon_0 a^2}$

$$E = \sqrt{E_x'^2 + E_y'^2} = \sqrt{2}\frac{q}{2\pi\epsilon_0 a^2} = \frac{q}{\sqrt{2}\pi\epsilon_0 a^2}$$

According to x'-y' plane, E is at 45° from its origin (at center of square). According to x-y plane, E is directed along y axis

$$\vec{E} = \frac{11.8\times10^{-9}}{\sqrt{2}\pi(8.85\times10^{-12})(0.052)^2}\hat{y} = 1.11\times10^{5}\ N/C\ \hat{y}$$

(9)



Consider electric fields of diametrically opposite charges:

For $-12q$ & $-6q$: $\vec{E}_1 = \frac{6q}{4\pi\epsilon_0 R^2}\hat{y}$

For $-9q$ & $-3q$: $\vec{E}_2 = \frac{-6q}{4\pi\epsilon_0 R^2}\hat{x}$

For $-q$ & $-7q$:

$$\vec{E}_3 = \frac{6q}{4\pi\epsilon_0 R^2}\left(\cos 240^\circ\hat{x} + \sin 240^\circ\hat{y}\right)$$

$$= \frac{6q}{4\pi\epsilon_0 R^2}\left(\frac{-1}{2}\hat{x} + \frac{-\sqrt{3}}{2}\hat{y}\right)$$

For $-2q$ & $-8q$:

$$\vec{E}_4 = \frac{6q}{4\pi\epsilon_0 R^2}\left(\cos 210^\circ\hat{x} + \sin 210^\circ\hat{y}\right)$$

$$= \frac{6q}{4\pi\epsilon_0 R^2}\left(\frac{-\sqrt{3}}{2}\hat{x} + \frac{-1}{2}\hat{y}\right)$$

For $-4q$ & $-10q$:

$$\vec{E}_5 = \frac{6q}{4\pi\epsilon_0 R^2}\left(\cos 150^\circ\hat{x} + \sin 150^\circ\hat{y}\right)$$

$$= \frac{6q}{4\pi\epsilon_0 R^2}\left(-\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}\right)$$

For $-5q$ & $-11q$:

$$\vec{E}_6 = \frac{6q}{4\pi\epsilon_0 R^2}\left(\cos 120^\circ\hat{x} + \sin 120^\circ\hat{y}\right)$$

$$= \frac{6q}{4\pi\epsilon_0 R^2}\left(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 + \vec{E}_6$$

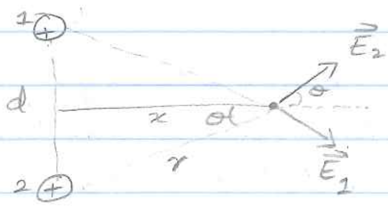
$$= \frac{6q}{4\pi\epsilon_0 R^2}\left[\hat{x}\left(-1 - \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - \frac{1}{2}\right) + \hat{y}\left(+1 - \frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{6q}{4\pi\epsilon_0 R^2}\left[\hat{x}(-2-\sqrt{3}) + \hat{y}\right]$$

$$\theta = -\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) + 180^\circ = 165^\circ$$

This corresponds to time 9:30.

(10)



$$E = E_1\cos\theta + E_2\cos\theta$$

$$= 2E\cos\theta \qquad (\because E_1 = E_2 = E)$$

$$= \frac{2q}{4\pi\epsilon_0 r^2}\cos\theta = \frac{2q}{4\pi\epsilon_0}\frac{x}{r^3}$$

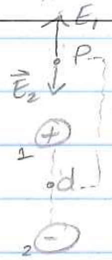
$$E = \frac{2q}{4\pi\epsilon_0}\frac{x}{(x^2+(d/2)^2)^{3/2}} = \frac{2q}{4\pi\epsilon_0}\frac{x}{x^3\left[1+\left(\frac{d}{2x}\right)^2\right]^{3/2}}$$

$$= \frac{2q}{4\pi\epsilon_0 x^2}\left[1+\left(\frac{d}{2x}\right)^2\right]^{-3/2}$$

$$= \frac{2q}{4\pi\epsilon_0 x^2}\left[1 + \left(-\frac{3}{2}\right)\left(\frac{d}{2x}\right)^2 + \cdots\right]$$

$$\simeq \frac{1}{4\pi\epsilon_0}\frac{2q}{x^2} \quad \text{when } x \gg d$$

(11)
(a)



$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{(z-d/2)^2}\hat{y}, \quad \vec{E}_2 = \frac{-1}{4\pi\epsilon_0}\frac{q}{(z+d/2)^2}\hat{y}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(z-d/2)^2} - \frac{1}{(z+d/2)^2}\right]\hat{y}$$

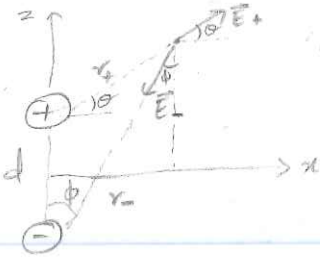
$$E = \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$

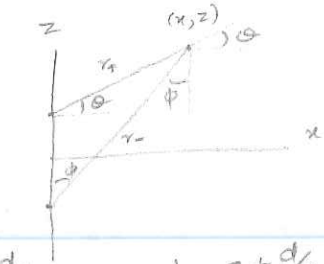
$$\simeq \frac{q}{4\pi\epsilon_0 z^2}\left[1 + (-2)\left(\frac{-d}{2z}\right) - 1 - (-2)\left(\frac{d}{2z}\right)\right]$$

$$= \frac{q}{4\pi\epsilon_0 z^2}\frac{2d}{z} = \frac{p}{2\pi\epsilon_0 z^3}$$ (along y axis for large values of z)

(b) Direction of $\vec{E}$: parallel to $\vec{p}$.

(12)





$$r_+ = \sqrt{x^2 + \left(z - \frac{d}{2}\right)^2}$$
$$r_- = \sqrt{\left(z + \frac{d}{2}\right)^2 + x^2}$$

$$\cos\theta = \frac{x}{r_+}, \quad \sin\theta = \frac{z - d/2}{r_+}, \quad \cos\phi = \frac{z + d/2}{r_-}, \quad \sin\phi = \frac{x}{r_-}$$

$$\vec{E}_+ = \frac{q}{4\pi\epsilon_0 r_+^2}\left[\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right]$$
$$= \frac{q}{4\pi\epsilon_0 r_+^3}\left[x\,\hat{x} + \left(z - \frac{d}{2}\right)\hat{y}\right]$$

$$\vec{E}_- = \frac{q}{4\pi\epsilon_0 r_-^2}\left[-\cos\phi\,\hat{y} - \sin\phi\,\hat{x}\right]$$
$$= \frac{q}{4\pi\epsilon_0 r_-^3}\left[-\left(z + \frac{d}{2}\right)\hat{y} - x\,\hat{x}\right]$$

$$E_x = \frac{q}{4\pi\epsilon_0}\left[\frac{x}{r_+^3} - \frac{x}{r_-^3}\right] = \frac{qx}{4\pi\epsilon_0}\left[\frac{1}{\left(x^2 + \left(z - \frac{d}{2}\right)^2\right)^{3/2}} - \frac{1}{\left(x^2 + \left(z + \frac{d}{2}\right)^2\right)^{3/2}}\right]$$
$$= \frac{qx}{4\pi\epsilon_0}\left[\left(x^2 + z^2 - dz + \frac{d^2}{4}\right)^{-3/2} - \left(x^2 + z^2 + dz + \frac{d^2}{4}\right)^{-3/2}\right]$$
$$= \frac{qx}{4\pi\epsilon_0}\left[(x^2+z^2)^{-3/2}\left(1 + \frac{\frac{d^2}{4} - dz}{x^2+z^2}\right)^{-3/2} - (x^2+z^2)^{-3/2}\left(1 + \frac{\frac{d^2}{4} + dz}{x^2+z^2}\right)^{-3/2}\right]$$
$$= \frac{qx}{4\pi\epsilon_0}(x^2+z^2)^{-3/2}\left[1 - \frac{3}{2(x^2+z^2)}\left(\frac{d^2}{4} - dz\right) - 1 + \frac{3}{2(x^2+z^2)}\left(\frac{d^2}{4} + dz\right)\right]$$
$$= \frac{qx}{4\pi\epsilon_0 (x^2+z^2)^{5/2}}\left[\frac{3}{2}dz - \frac{3}{2}\left(\frac{d^2}{4}\right) + \frac{3}{2}\left(\frac{d^2}{4}\right) + \frac{3}{2}dz\right]$$
$$= \frac{qx}{4\pi\epsilon_0 (x^2+z^2)^{5/2}}\left[3dz\right] = \frac{1}{4\pi\epsilon_0}\frac{3qdxz}{(x^2+z^2)^{5/2}} = \frac{1}{4\pi\epsilon_0}\frac{3pxz}{(x^2+z^2)^{5/2}}$$

$$E_y = \frac{q}{4\pi\epsilon_0}\left[\frac{(z - d/2)}{r_+^3} - \frac{(z + d/2)}{r_-^3}\right]$$
$$= \frac{q}{4\pi\epsilon_0}\left[z\left(\frac{1}{r_+^3} - \frac{1}{r_-^3}\right) - \frac{d}{2}\left(\frac{1}{r_+^3} + \frac{1}{r_-^3}\right)\right]$$
$$= \frac{q}{4\pi\epsilon_0}\left[z\left(\frac{3dz}{(x^2+z^2)^{5/2}}\right) - \frac{d}{2}(x^2+z^2)^{-3/2}\left(1 - \frac{3}{2(x^2+z^2)}\left(\frac{d^2}{4} - dz\right) + 1 - \frac{3}{2(x^2+z^2)}\left(\frac{d^2}{4} + dz\right)\right)\right]$$
$$= \frac{q}{4\pi\epsilon_0}\left[\frac{3dz^2}{(x^2+z^2)^{5/2}} - \frac{d}{2}(x^2+z^2)^{-5/2}\left(2(x^2+z^2) - \frac{3d^2}{4}\right)\right]$$
$$= \frac{q}{4\pi\epsilon_0}\left[\frac{3dz^2}{(x^2+z^2)^{5/2}} - \frac{d}{2(x^2+z^2)^{5/2}}\left(\frac{8x^2 + 8z^2 - 3d^2}{4}\right)\right]$$

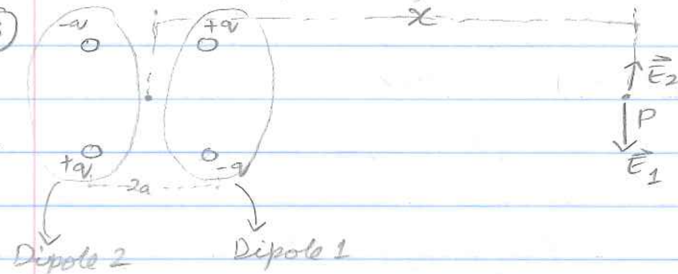
ignoring high powers of d

$$= \frac{q}{4\pi\epsilon_0 (x^2+z^2)^{5/2}} \left[3dz^2 - dx^2 - dz^2 + \frac{3}{8} d^3 \right]$$

$$= \frac{qd}{4\pi\epsilon_0 (x^2+z^2)^{5/2}} (2z^2 - x^2)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{p(2z^2-x^2)}{(x^2+z^2)^{5/2}}$$

(13)



Using

$$E_{\text{dipole}} = \frac{kdq}{[x^2 + (d/2)^2]^{3/2}}$$

where x = distance of point P from dipole center

d = length of dipole moment

$$E = E_1 - E_2$$

$$= \frac{kq\,2a}{[(x-a)^2 + a^2]^{3/2}} - \frac{kq\,2a}{[(x+a)^2 + a^2]^{3/2}}$$

$$= kq\,2a \left[(x^2 - 2xa + 2a^2)^{-3/2} - (x^2 + 2xa + 2a^2)^{-3/2} \right]$$

$$= kq\,2a \left[x^{-3} \left(1 - \frac{2xa - 2a^2}{x^2} \right)^{-3/2} - x^{-3} \left(1 + \frac{2xa + 2a^2}{x^2} \right)^{-3/2} \right]$$

$$\approx \frac{kq\,2a}{x^3} \left[1 + \frac{3}{2} \left(\frac{2xa - 2a^2}{x^2} \right) - 1 + \frac{3}{2} \left(\frac{2xa + 2a^2}{x^2} \right) \right]$$

$$= \frac{kq\,2a}{x^3} \left[\frac{6a}{x} \right] = \frac{1}{4\pi\epsilon_0} \frac{2qa^2}{x^3} \left(\frac{6}{x} \right) = \frac{3(2qa^2)}{2\pi\epsilon_0 x^4}$$

(14)

$$E = \frac{kq}{(z-d)^2} + \frac{kq}{(z+d)^2} - \frac{kq}{z^2} - \frac{kq}{z^2}$$

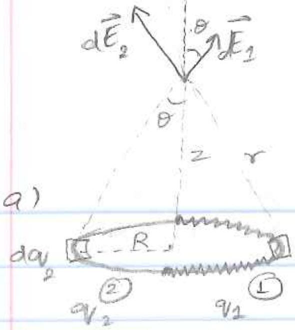
$$= kq \left[\frac{1}{(z-d)^2} + \frac{1}{(z+d)^2} - \frac{2}{z^2} \right]$$

$$= kq \left[\frac{1}{z^2} \left(1 - \frac{d}{z} \right)^{-2} + \frac{1}{z^2} \left(1 + \frac{d}{z} \right)^{-2} - \frac{2}{z^2} \right]$$

$$\approx \frac{kq}{z^2} \left[1 + \frac{2d}{z} + \frac{(-2)(-3)}{2!} \left(\frac{d^2}{z^2} \right) + 1 - \frac{2d}{z} + \frac{(-2)(-3)}{2!} \frac{d^2}{z^2} - 2 \right]$$

$$= \frac{kq}{z^2} \left[\frac{6d^2}{z^2} \right]$$

$$= \frac{3(2qd^2)}{4\pi\epsilon_0 z^4} = \frac{3Q}{4\pi\epsilon_0 z^4} \quad (\text{where } Q = 2qd^2)$$



$\lambda_1 = \frac{q_1}{\pi R}$, $\lambda_2 = \frac{q_2}{\pi R}$

$\cos\theta = \frac{z}{r}$, $\sin\theta = \frac{R}{r}$

15 (a) $E_z = \int dE_z = \int (dE_1 \cos\theta + dE_2 \cos\theta)$

$dE_1 = \frac{1}{4\pi\epsilon_0}\frac{dq_1}{r^2} = \frac{\lambda_1 ds_1}{4\pi\epsilon_0 r^2}$, $dE_2 = \frac{\lambda_2 ds_2}{4\pi\epsilon_0 r^2}$

$$E_z = \frac{\lambda_1}{4\pi\epsilon_0}\int \frac{z}{r^3}ds_1 + \frac{\lambda_2}{4\pi\epsilon_0}\int\frac{z}{r^3}ds_2$$

$$= \frac{\lambda_1 z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}\int ds_1 + \frac{\lambda_2 z}{4\pi\epsilon_0(z^2+R^2)^{3/2}}\int ds_2$$

$$= \frac{z}{4\pi\epsilon_0(z^2+R^2)^{3/2}}\frac{q_1}{\pi R}(\pi R) + \frac{z}{4\pi\epsilon_0(z^2+R^2)^{3/2}}\frac{q_2}{\pi R}(\pi R)$$

$$= \frac{z}{4\pi\epsilon_0(z^2+R^2)^{3/2}}(q_1+q_2) = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$$

(b) $$E_x = \frac{\lambda_1}{4\pi\epsilon_0}\int\frac{ds_1}{r^2}\sin\theta - \frac{\lambda_2}{4\pi\epsilon_0}\int\frac{ds_2}{r^2}\sin\theta$$

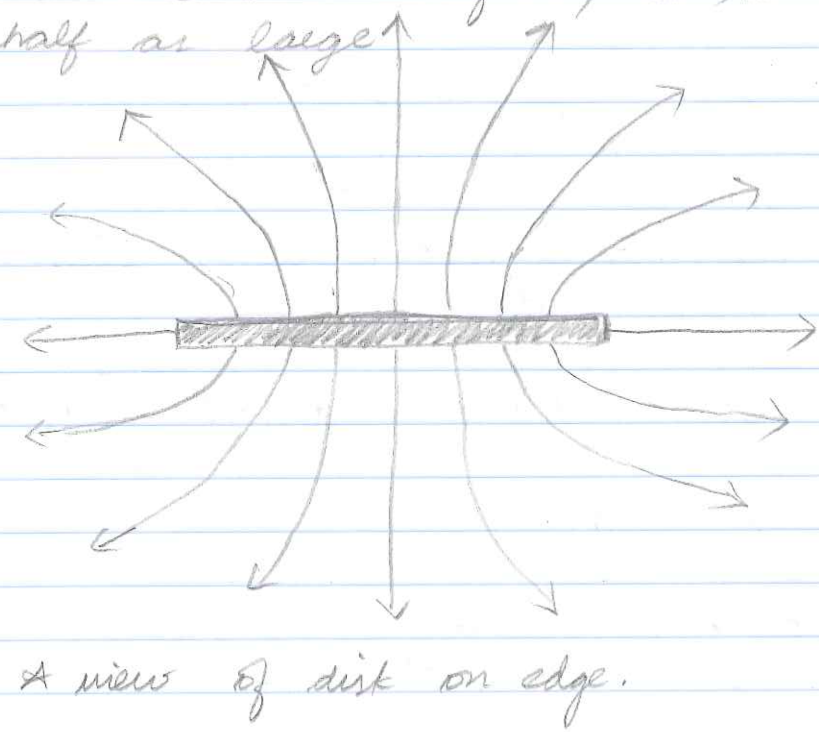
$$= \frac{\lambda_1 R}{4\pi\epsilon_0(z^2+R^2)^{3/2}}(\pi R) - \frac{\lambda_2 R}{4\pi\epsilon_0(z^2+R^2)^{3/2}}(\pi R)$$

$$= \frac{R}{4\pi\epsilon_0(z^2+R^2)^{3/2}}(q_1-q_2)$$

16 a) $F = eE = (1.6\times10^{-19})(40) = 6.4\times10^{-18}\,N$

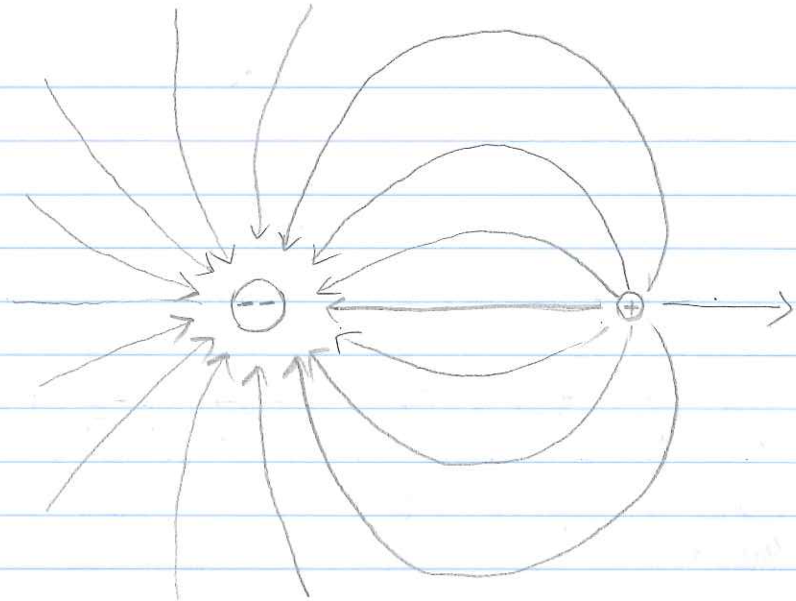
(b) Lines are twice as far apart, so the field is half as large

17



A view of disk on edge.

(18)



(19) To the right.

(20) The electric field is zero nearer to the smaller charge; since the charges have opposite signs it must be to the right of $+2q$ charge. Equating the magnitudes of the two fields:

$$\frac{5q}{4\pi\epsilon_0 (a+x)^2} = \frac{2q}{4\pi\epsilon_0 (x)^2}$$

$$\Rightarrow \quad 5x^2 = 2a^2 + 2x^2 + 4ax$$

$$\Rightarrow \quad 3x^2 - 4ax - 2a^2 = 0$$

$$x = \frac{4a \pm \sqrt{16a^2 + 24a^2}}{6} = \frac{4a \pm 2\sqrt{10}\,a}{6}$$

$$= \frac{2a \pm \sqrt{10}\,a}{3} = \frac{a}{3}\left(2 \pm \sqrt{10}\right) = 1.72a$$

(discarding "−" sign solution)

(21) For points between 0 and d:

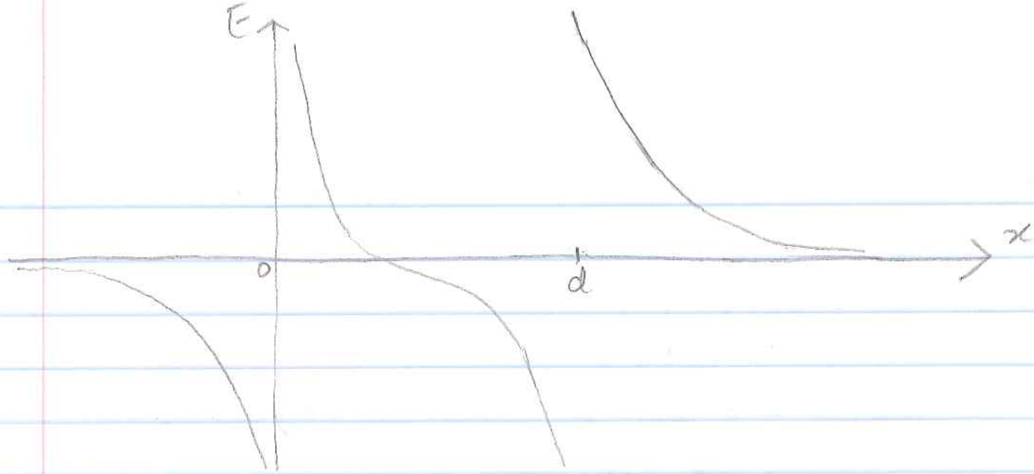
$$E = \frac{k q_1}{x^2} - \frac{k q_2}{(d-x)^2}$$

$$= \frac{(9\times10^9)(+1.0\times10^{-6})}{x^2} - \frac{(9\times10^9)(+3.0\times10^{-6})}{(0.1-x)^2}$$

$$= \frac{10^3}{x^2} - \frac{(2.7\times10^4)}{(0.1-x)^2}$$

For points beyond $x = d$.

$$E = \frac{k q_1}{x^2} + \frac{k q_2}{(x-d)^2} = \frac{10^3}{x^2} + \frac{(2.7\times10^4)}{(x-0.1)^2}$$



(22) $$E_A = \frac{-1}{4\pi\epsilon_0}\frac{q}{d^2} + \frac{1}{4\pi\epsilon_0}\frac{(2q)}{(2d)^2} = \frac{q}{4\pi\epsilon_0}\left(\frac{-1}{d^2} + \frac{1}{2d^2}\right)$$
$$= \frac{q}{4\pi\epsilon_0 d^2}\left(\frac{-1}{2}\right) = \frac{-q}{8\pi\epsilon_0 d^2}$$
$$E_B = \frac{+q}{4\pi\epsilon_0 (d/2)^2} + \frac{(2q)}{4\pi\epsilon_0 (d/2)^2} = \frac{3q}{\pi\epsilon_0 d^2}$$
$$E_C = \frac{-2q}{4\pi\epsilon_0 d^2} + \frac{q}{4\pi\epsilon_0 (2d)^2} = \frac{q}{4\pi\epsilon_0 d^2}\left(-2 + \frac{1}{4}\right) = \frac{-7q}{16\pi\epsilon_0 d^2}$$

(23) The number of field lines through a unit cross-section area is proportional to the electric field strength. If the exponent is n, then the electric field strength a distance r from a point charge is:
$$E = \frac{kq}{r^n}$$

and the total cross sectional area at a distance r is the area of a spherical shell, $4\pi r^2$. Then the number of field lines through the shell is proportional to:
$$EA = \frac{kq}{r^n} 4\pi r^2 = 4\pi k q r^{2-n}$$

The number of field lines varies with r if $n \neq 2$. This means that as we go farther from the point charge we need more and more field lines (or fewer and fewer). But the field lines can only start on charges, and we don't have any except for the point charge. We need $n = 2$, if we want workable field lines.

(24) $$E_z = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$ (charged disk)

$$= \frac{Q}{2\epsilon_0(\pi R^2)}\left[1 - \left(1 + \frac{R^2}{z^2}\right)^{-1/2}\right]$$

$$= \frac{Q}{2\pi\epsilon_0 R^2}\left[1 - 1 + \frac{R^2}{2z^2}\right] \quad (\text{for } z >> R)$$

$$= \frac{Q}{4\pi\epsilon_0 z^2}$$

(25) Electric field at the surface of the disk at the centre:

$$E_s = \frac{\sigma}{2\epsilon_0} \quad (\text{for } z = 0 \text{ for a charged disk})$$

$$E_z = \frac{\sigma}{4\epsilon_0} = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$\Rightarrow \frac{1}{2} = 1 - \frac{z}{\sqrt{z^2+R^2}}$$

$$\Rightarrow \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \quad \Rightarrow 2z = \sqrt{z^2+R^2}$$

$$\Rightarrow 4z^2 = z^2 + R^2$$

$$\Rightarrow z = \frac{R}{\sqrt{3}}$$

(26) $$\frac{dE_z}{dz} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(z^2+R^2)^{3/2}} - \frac{3z}{2}\frac{(2z)}{(z^2+R^2)^{5/2}}\right] = 0$$

$$\Rightarrow \frac{1}{(z^2+R^2)^{3/2}} = \frac{3z^2}{(z^2+R^2)^{5/2}}$$

$$\frac{(z^2+R^2)^{5/2}}{(z^2+R^2)^{3/2}} = 3z^2$$

$$z^2 + R^2 = 3z^2$$

$$\Rightarrow 2z^2 = R^2$$

$$\Rightarrow z = \frac{R}{\sqrt{2}}$$

At $z = \frac{R}{\sqrt{2}}$ along axis of the charged ring, the axial electric field strength is maximum.

(27) (a) $E = 3\times10^{6}\ N/C$

$E = \dfrac{\sigma}{2\epsilon_0} \Rightarrow \sigma = E2\epsilon_0 \Rightarrow Q = AE2\epsilon_0$

$Q = \pi R^2 E 2\epsilon_0 = \pi(0.025)^2(3\times10^{6})(2\times8.85\times10^{-12})$

$= 1.04\times10^{-7}\ C$

(b) $N = \dfrac{\pi(0.025)^2}{0.015\times10^{-18}} = 1.31\times10^{17}$

(c) No. of charged surface atoms $= \dfrac{1.04\times10^{-7}}{1.6\times10^{-19}} = 6.5\times10^{11}$

Fraction : $\dfrac{6.5\times10^{11}}{1.31\times10^{17}} = 4.96\times10^{-6}$

(28) In carrying out the calculations, intermediate step is:

$$E_z = \frac{\sigma z}{2\epsilon_0}\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[\frac{z}{\sqrt{z^2}} - \frac{z}{\sqrt{z^2+R^2}}\right] \quad \text{(for both + \& } -z\text{)}$$

For $+z$:

$$E_{z_+} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{|z|}{\sqrt{z^2+R^2}}\right]$$

For $-z$:

$$E_{z_-} = \frac{\sigma}{2\epsilon_0}\left[-1 + \frac{|z|}{\sqrt{z^2+R^2}}\right] = \frac{-\sigma}{2\epsilon_0}\left[1 - \frac{|z|}{\sqrt{z^2+R^2}}\right]$$

$E_{z_+} = -E_{z_-}$

direction of $\vec{E}_z$ inverses (above & below the disk).

(29) For $z = 0$

$E = \dfrac{\sigma}{2\epsilon_0} \Rightarrow \sigma = 2\epsilon_0 E = 2(8.85\times10^{-12})(10^{7}\times2.043)$

$= 3.62\times10^{-4}\ C/m^2$

Now:

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \Rightarrow \frac{z}{\sqrt{z^2+R^2}} = 1 - \frac{2\epsilon_0 E}{\sigma}$$

$$\Rightarrow \frac{\sigma z}{\sqrt{z^2+R^2}} = \sigma - 2\epsilon_0 E \Rightarrow \frac{\sigma z}{\sigma - 2\epsilon_0 E} = \sqrt{z^2+R^2}$$

$$\Rightarrow R = \sqrt{\left(\frac{\sigma z}{\sigma - 2\epsilon_0 E}\right)^2 - z^2}$$

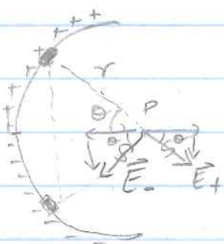
From given data; $z = 1cm = 10^{-2}m$, $E = 1.732\times10^{7}$ N/C

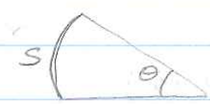
$$R = \sqrt{\left[\frac{3.62\times10^{-4}\times10^{-2}}{(3.62\times10^{-4})-(2\times8.85\times10^{-12}\times1.732\times10^{7})}\right]^2-(10^{-2})^2}$$

$= 0.065m = 6.5cm$

$Q = 6\times\pi R^2 = (3.62\times10^{-4})\times\pi\times(0.065)^2 = 4.8\times10^{-6}C = 4.8\mu C$

(30)





$s = \theta r$, $dq = \lambda ds$

$ds = r d\theta$

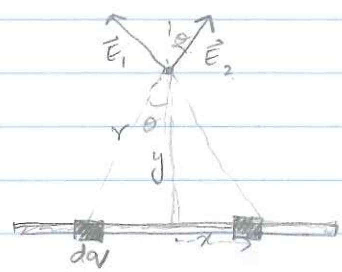
$$dE = 2\frac{dq}{4\pi\epsilon_0 r^2}\sin\theta = \frac{2\lambda ds}{4\pi\epsilon_0 r^2}\sin\theta = \frac{2\lambda r}{4\pi\epsilon_0 r^2}\sin\theta\, d\theta$$

B.C
∵ "2" factor added already

$$E = \int dE = \frac{2\lambda}{4\pi\epsilon_0 r}\int_0^{\pi/2}\sin\theta\, d\theta = \frac{-2\lambda}{4\pi\epsilon_0 r}\left[\cos\theta\right]_0^{\pi/2} = \frac{2\lambda}{4\pi\epsilon_0 r}$$

$$= \frac{2q}{4\pi\epsilon_0 r(\pi r/2)} = \frac{4q}{4\pi^2\epsilon_0 r^2} = \frac{q}{\epsilon_0\pi^2 r^2}$$

(31)



$dq = \lambda dx$, $\cos\theta = \frac{y}{r}$, $r = \sqrt{x^2+y^2}$

$$dE = 2\frac{dq}{4\pi\epsilon_0 r^2}\cos\theta = \frac{2\lambda dx}{4\pi\epsilon_0 r^3}y$$

$$E = \int dE = \frac{2\lambda y}{4\pi\epsilon_0}\int_0^{L/2}\frac{dx}{(x^2+y^2)^{3/2}}$$

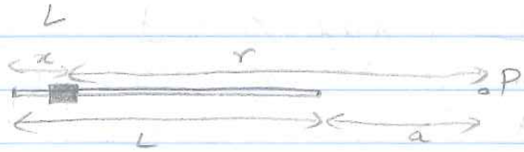
$$\Rightarrow E = \frac{2\lambda y}{4\pi\epsilon_0}\left[\frac{x}{y^2\sqrt{x^2+y^2}}\right]_0^{L/2}$$ (Using substitution: $x = y\tan\theta$)

$$= \frac{2\lambda y}{4\pi\epsilon_0}\left[\frac{L/2}{y^2\sqrt{\frac{L^2}{4}+y^2}}\right] = \frac{\lambda y L}{4\pi\epsilon_0 \frac{y^2}{2}\sqrt{L^2+4y^2}}$$

$$= \frac{q/L \cdot L}{2\pi\epsilon_0 y\sqrt{L^2+4y^2}} = \frac{q}{2\pi\epsilon_0 y}\frac{1}{\sqrt{L^2+4y^2}}$$

(32) a) $\lambda = \frac{-q}{L}$

(b)



$dq = \lambda dx$

$$E = \int \frac{dq}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dx}{(L+a-x)^2}$$

$$= \frac{q}{4\pi\epsilon_0 L}\int_0^L (L+a-x)^{-2}(-1)\, dx = \frac{q}{4\pi\epsilon_0 L}\left[\frac{(L+a-x)^{-1}}{(-1)}\right]_0^L$$

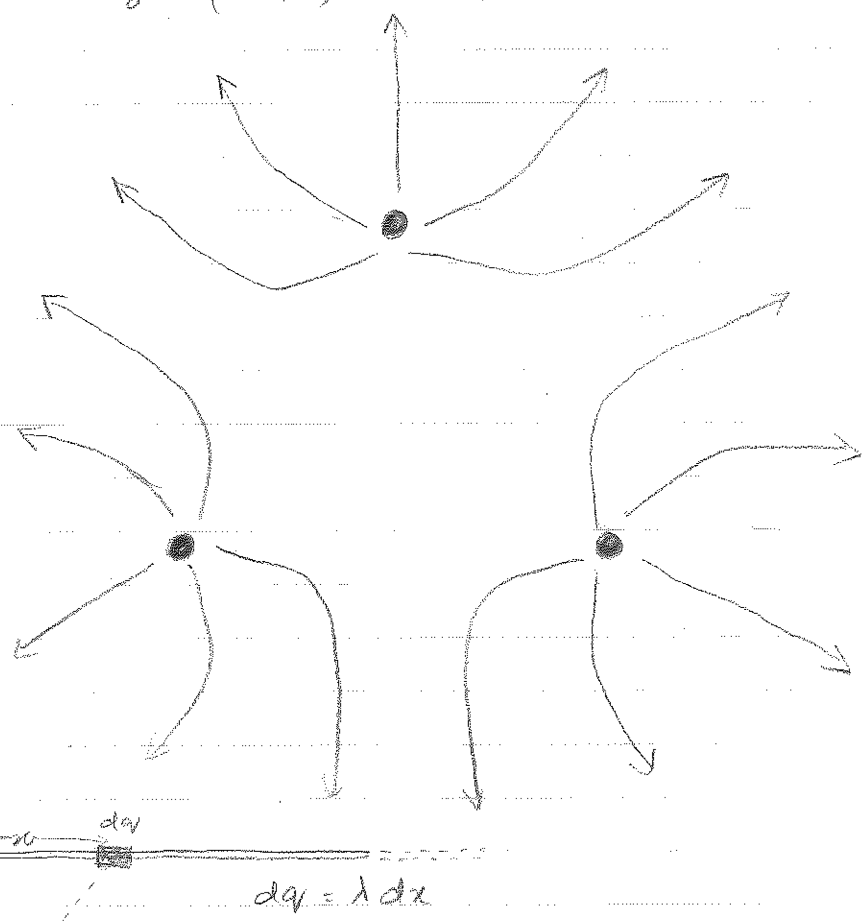
$$= \frac{-q}{4\pi\epsilon_0 L}\left[\frac{1}{a} - \frac{1}{L+a}\right] = \frac{-q}{4\pi\epsilon_0 L a(L+a)}\, L$$

$$= \frac{-q}{4\pi\epsilon_0 a(L+a)}$$

(c) For $a >> L$

$$E = \frac{-q}{4\pi\epsilon_0 a \cdot a(1+L/a)} = \frac{-q}{4\pi\epsilon_0 a^2} \quad \left(\text{ignoring } \frac{L}{a} \because \frac{L}{a} << 1\right)$$

33



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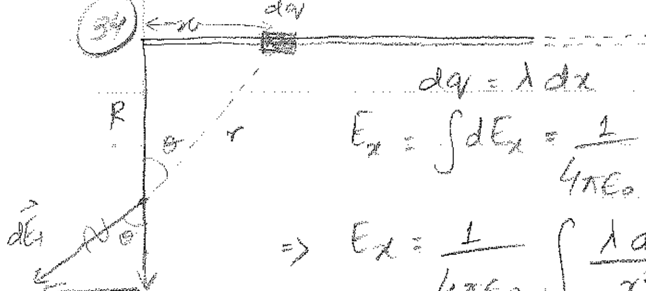


$$dq = \lambda\, dx$$

$$E_x = \int dE_x = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\sin\theta$$

$$\Rightarrow E_x = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dx}{r^2}\,\frac{x}{r} = \frac{\lambda}{4\pi\epsilon_0}\int_0^\infty \frac{x\, dx}{(x^2+R^2)^{3/2}}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{-1}{\sqrt{x^2+R^2}}\right]_0^\infty = \frac{\lambda}{4\pi\epsilon_0 R}$$

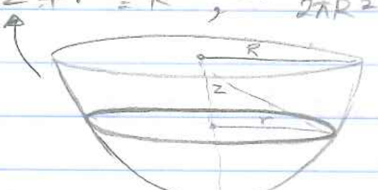
$$E_y = \int dE_y = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dx}{r^2}\frac{R}{r} = \frac{\lambda R}{4\pi\epsilon_0}\int_0^\infty \frac{dx}{(x^2+R^2)^{3/2}}$$

$$= \frac{\lambda R}{4\pi\epsilon_0}\left[\frac{x}{R^2\sqrt{x^2+R^2}}\right]_0^\infty = \frac{\lambda R}{4\pi\epsilon_0}\left[\frac{1}{R^2}\right] = \frac{\lambda}{4\pi\epsilon_0 R}$$

ϕ is the angle that the resultant electric field $\vec{E}$ makes with the rod.

$$\phi = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}(1) = 45°$$

(35) $z^2+r^2 = R^2$, $\sigma = \frac{q}{2\pi R^2}$ Consider the hemispherical cup as stacks of rings.



$$E = \int dE = \frac{1}{4\pi\epsilon_0}\int \frac{z\, dq}{(z^2+r^2)^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0}\int \frac{z\,(2\pi r\, dz)\sigma}{R^3} = \frac{\sigma}{2\epsilon_0 R^3}\int z\, r\, dz$$

$$= \frac{\sigma}{2\epsilon_0 R^3}\int_0^R z\sqrt{R^2-z^2}\, dz = \frac{\sigma}{2\epsilon_0 R^3}\left(\frac{-1}{2}\right)\left[\frac{(R^2-z^2)^{3/2}}{3/2}\right]_0^R$$

$$= \frac{\sigma}{2\epsilon_0 R^3}\left(-\frac{1}{3}\right)\left[-R^3\right] = \frac{\sigma}{6\epsilon_0} = \frac{q}{12\pi\epsilon_0 R^2}$$

(36) (a) $F = Ee = ma \Rightarrow a = \frac{Ee}{m} = \frac{(2.16\times10^4)(1.6\times10^{-19})}{(1.67\times10^{-27})} = 2.07\times10^{12}\ m/s^2$

(b) $v_f = \sqrt{2aS} = \sqrt{2\times2.07\times10^{12}\times1.22\times10^{-2}} = 2.2\times10^5\ m/s$

(37) (a) $W = \Delta K \Rightarrow -F.S = -\frac{1}{2}mv_i^2 \Rightarrow S = \frac{1}{2}\frac{mv_i^2}{eE}$

$$S = \frac{1}{2}\frac{(9.11\times10^{-31})(4.86\times10^6)^2}{(1.6\times10^{-19})(1030)} = 6.53\,cm$$

(b) $a = \frac{Ee}{m} = \frac{(1030)(1.6\times10^{-11})}{(9.11\times10^{-31})} = 1.81\times10^{14}\ m/s^2$

$$t = \frac{v_f - v_i}{a} = \frac{-(4.86\times10^6)}{-(1.81\times10^{18})} = 2.69\times10^{-8}\,s = 26.9\,ns$$

(c) K.E lost = Work done by $E = -eEs = -(1.6\times10^{-19})(1030)(7.88\times10^{-3})$
$= -1.3\times10^{-18}\,J$

$$\frac{K.E\ lost}{K.E_i} = \frac{1.3\times10^{-18}}{\frac{1}{2}(9.11\times10^{-31})(4.86\times10^6)^2} = 0.121$$

(38)(a) $S = \frac{1}{2}at^2 \Rightarrow a = \frac{2s}{t^2} = \frac{2\times1.95\times10^{-2}}{(14.7\times10^{-9})^2} = 1.80\times10^{14}\ m/s^2$

$v_f = v_i + at = (1.80\times10^{14})(14.7\times10^{-9}) = 2.65\times10^{6}\ m/s$

(b) $E = \frac{F}{e} = \frac{ma}{e} = \frac{(9.11\times10^{-31})(1.80\times10^{14})}{(1.6\times10^{-19})} = 1025\ N/C$

(39)(a) E (directed towards "−" charge) $= \frac{2q}{4\pi\epsilon_0 r^2} = \frac{2(9\times10^{9})(1.88\times10^{-7})}{(15.2\times0.5\times10^{-2})^2} = 5.9\times10^{5}\ N/C$

(b) F (directed towards "+" charge) $= eE = (1.6\times10^{-19})(5.9\times10^{5}) = 9.44\times10^{-14}\ N$

(40)(a) $E_1 = \frac{q_1}{4\pi\epsilon_0 r^2} = \frac{(2.26\times10^{-6})(9\times10^{9})}{(0.117)^2} = 1.42\times10^{6}\ N/C$

$E_2 = \frac{q_2}{4\pi\epsilon_0 r^2} = \frac{(85.3\times10^{-9})(9\times10^{9})}{(0.117)^2} = 5.61\times10^{4}\ N/C$

(b) $F_1 = q_1E_2 = (2.16\times10^{-6})(5.61\times10^{4}) = 1.21\times10^{-1}\ N$

$F_2 = q_2E_1 = (85.3\times10^{-9})(1.42\times10^{6}) = 1.21\times10^{-1}\ N$

(Also by Coulomb's law $\frac{q_1q_2}{4\pi\epsilon_0 r^2}$)

(41) $m = V\rho = \frac{4}{3}\pi r^3\rho = \frac{4}{3}\pi(1.64\times10^{-6})^3\left(\frac{0.851\times10^{-3}}{10^{-6}}\right) = 1.57\times10^{-14}\ kg$

$\Sigma F = 0 \Rightarrow mg = qE \Rightarrow q = \frac{mg}{E} = \frac{1.57\times10^{-14}\times9.8}{1.92\times10^{5}} = 8.01\times10^{-19}\ C$

$\frac{q}{e} = \frac{8.01\times10}{1.6\times10^{-19}} = 5$ so $q = 5e$

(42) $F_e = eE$, $F_p = eE$

$a_e = \frac{eE}{m_e}$, $a_p = \frac{eE}{m_p}$

$x = \frac{1}{2}a_pt^2 \Rightarrow t^2 = \frac{2x}{a_p}$, $d-x = \frac{1}{2}a_et^2 \Rightarrow t^2 = \frac{2(d-x)}{a_e}$

so: $\frac{2x}{a_p} = \frac{2(d-x)}{a_e} \Rightarrow xa_e = (d-x)a_p$

$\Rightarrow x\frac{eE}{m_e} = (d-x)\frac{eE}{m_p} \Rightarrow xm_p = m_e(d-x)$

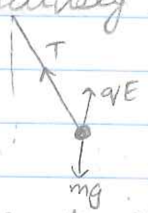
$\Rightarrow x = \frac{m_ed}{(m_p+m_e)} = \frac{(9.11\times10^{-31})(5.00\times10^{-2})}{(1.67\times10^{-27} + 9.11\times10^{-31})} = 2.72\times10^{-5}\ m$

(43) If each value of q measured by Milikan was a multiple of e, then the difference between any two values of q must also be a multiple of q. The smallest difference would be the smallest multiple, and this multiple might be unity. The differences are 1.641, 1.63, 1.60, 1.63, 3.30, 3.35, 3.18 and 3.24, all times $10^{-19}C$. This is a pretty clear indication that the fundamental charge is on the order of $1.6\times10^{-19}C$. If so, the likely number of fundamental charges on each of the drops is shown below:

4	8	12
5	10	14
7	11	16

The total no. of charges is 87, while the total charge is $142.69\times10^{-19}C$, so the average charge per quanta is $1.64\times10^{-19}C$.

(44) For positively charged lower plate:



$T=(mg-qE)\cos\theta$, $F_r=-(mg-qE)\sin\theta$

For negatively charged lower plate

$T=(mg+qE)\cos\theta$, $F_r=-(mg+qE)\sin\theta$

Because of the electric field, the acceleration toward the ground of a charged particle is not g, but $g\pm qE/m$, where the sign depends on the direction of the electric field.

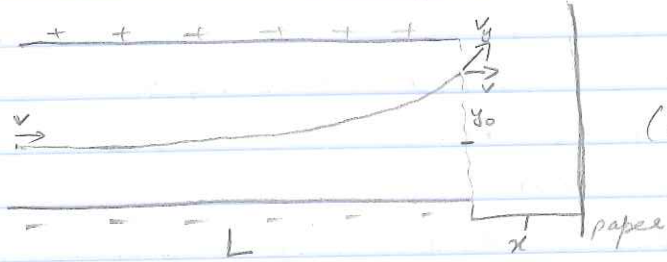
(a) If the lower plate is positively charged then $a=g-qE/m$. Replacing g in the pendulum period expression by this, and then:

$$T=2\pi\sqrt{\frac{L}{g-Eq/m}}$$

(b) If the lower plate is negatively charged then $a = g + Eq/m$. Replacing g in the pendulum expression by this, and then:

$$T = 2\pi \sqrt{\frac{L}{g + Eq/m}}$$

(45)



$x = vt \Rightarrow t = \frac{x}{v}$

(a from sample problem) $a = \frac{qE}{m}$

$$v_y = at_0 = \frac{aL}{v} = \frac{qEL}{mv}$$

$$y = v_y t = \frac{qEL}{mv}\left(\frac{x}{v}\right)$$

$$= \frac{qELx}{mv^2}$$

$$= \frac{(1.5\times10^{-13})(1.4\times10^{6})(0.016)(6.8\times10^{-3})}{(1.3\times10^{-10})(18)^2}$$

$$= 5.4\times10^{-4}\,m$$

$$Y = y + y_0 = (5.4\times10^{-4}) + (6.4\times10^{-4}) = 1.18\times10^{-3} = 1.18\,mm$$

(46) $F = -eE = -e\left[\frac{qz}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}\right]$

For $z \ll R$:

$$\frac{1}{(z^2+R^2)^{3/2}} = \frac{1}{R^3}\left(1 + \frac{z^2}{R^2}\right)^{-3/2} = \frac{1}{R^3}\left(1 - \frac{3}{2}\frac{z^2}{R^2} + \dots\right) \approx \frac{1}{R^3}$$

$$F = -e\left[\frac{qz}{4\pi\epsilon_0 R^3}\right] = -\left(\frac{eq}{4\pi\epsilon_0 R^3}\right)z = -kz$$

where $k = \frac{eq}{4\pi\epsilon_0 R^3}$. So, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}}$

(47) $x = v_0 \cos\theta\, t$

$y = v_0 \sin\theta\, t + \frac{1}{2}at^2$, $F_e = -eE$

$$ma = -mg - eE \Rightarrow a = -g - \frac{eE}{m} = -3.28\times10^{14}\,m/s^2$$

Solving for $y = 1.97\,cm$. If we get real + value of t, it means that electron strikes the upper plate.

$$t^2\left(\frac{a}{2}\right) + t(v_0 \sin\theta) - y = 0$$

$$t^2(-1.64\times10^{14}) + t(3.67\times10^{6}) + (-1.97\times10^{-2}) = 0$$

$$t = \frac{-(3.67\times10^6) \pm \sqrt{(3.67\times10^6)^2 - 4(-1.64\times10^{14})(-1.97\times10^{-2})}}{2(-1.64\times10^{14})}$$

$$= 8.94\times10^{-9}\,s$$

when electron strikes the upper plate, x is:

$$x = (5.83\times10^6)(\cos 39^\circ)(8.94\times10^{-9})$$

$$= 4.05\times10^{-2}\,m$$

(48) (a) $p = qd = (1.48\times10^{-9})(6.23\times10^{-6}) = 9.22\times10^{-15}\,Cm$

(b) $U_{parall} = -\vec{p}\cdot\vec{E} = -pE$

$U_{antiparallel} = -\vec{p}\cdot\vec{E} = pE$

$\Delta U = U_a - U_p = pE - (-pE) = 2pE = 2(9.22\times10^{-15})(1100) = 2.03\times10^{-11}\,J$

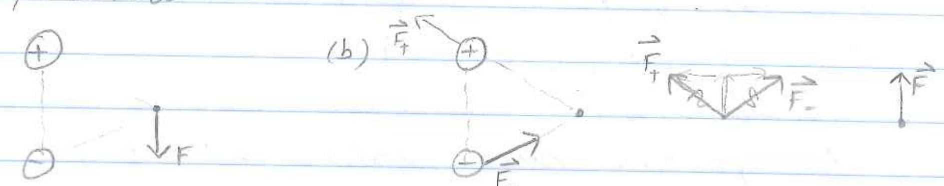
(49) $p = qd = (2\times1.6\times10^{-19})(0.78\times10^{-9}) = 2.496\times10^{-28}\,Cm$

(a) $\tau = pE\sin 0^\circ = 0$

(b) $\tau = pE\sin 90^\circ = pE = (2.496\times10^{-28})(3.4\times10^6) = 8.49\times10^{-22}\,Nm$

(c) $\tau = pE\sin 180^\circ = 0$

(50) (a) (b)



(c) Force on charge (by dipole) = $-$ Force on dipole (by charge)

So magnitude of the force on dipole $= 5.22\times10^{-16}\,N$

(d) $E = \dfrac{p}{4\pi\epsilon_0 x^3} \Rightarrow qE = \dfrac{qp}{4\pi\epsilon_0 x^3} \Rightarrow \dfrac{F(4\pi\epsilon_0 x^3)}{q} = p$

$\Rightarrow p = \dfrac{5.22\times10^{-16}\times(28.5\times10^{-2})^3}{(9\times10^9)(3.16\times10^{-6})} = 4.25\times10^{-22}\,C.m.$

(51)

$\theta_f = \pi - \theta_0$, $U = -\vec{p}\cdot\vec{E}$

$W = -\Delta U = -(U_f - U_i) = U_i - U_f$

$W = pE\cos\theta_0 - pE\cos(\pi - \theta_0)$

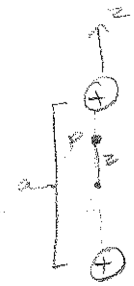
$= 2pE\cos\theta_0$

(52) $|\tau| = K\theta$, $|\tau| = pE\sin\theta \simeq pE\theta$ (for small θ in radians)

$\Rightarrow K = pE$ $\quad f = \dfrac{1}{2\pi}\sqrt{\dfrac{K}{I}} = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}}$

(53)

(a)



$$E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\left(\frac{a}{2}-z\right)^2} - \frac{1}{\left(\frac{a}{2}+z\right)^2}\right]$$

$$= \frac{q}{\pi\epsilon_0}\left[\frac{1}{(a-2z)^2} - \frac{1}{(a+2z)^2}\right]$$

$$\frac{dE}{dz} = \frac{q}{\pi\epsilon_0}\left[-2(a-2z)^{-3}(-2) - (-2)(a+2z)^{-3}(2)\right]$$

$$= \frac{q}{\pi\epsilon_0}\left[\frac{4}{(a-2z)^3} + \frac{4}{(a+2z)^3}\right]$$

$$\left.\frac{dE}{dz}\right|_{z=0} = \frac{q}{\pi\epsilon_0}\left[\frac{4}{a^3} + \frac{4}{a^3}\right] = \frac{8q}{\pi\epsilon_0 a^3}$$

(b) The electrostatic force on a dipole is the difference in the magnitudes of the electrostatic forces on the two charges that make up the dipole. Near the center of the given charge arrangement:

$$E \approx E(0) + \left.\frac{dE}{dz}\right|_{z=0} z + \text{higher order terms.}$$

The net force on dipole is:

$F = F_+ - F_- = q(E_+ - E_-)$

where "+" & "−" subscripts refer to the locations of the positive and negative charges.

$$F = q\left(E(0) + \left.\frac{dE}{dz}\right|_{z=0} z_+ - E(0) - \left.\frac{dE_z}{dz}\right|_{z=0} z_-\right)$$

$$= q\left.\frac{dE}{dz}\right|_{z=0}(z_+ - z_-) = qd\left.\frac{dE}{dz}\right|_{z=0}$$

Thus: $F = qd\left(\frac{dE}{dz}\right) = p\left(\frac{dE}{dz}\right)$