Practice Problem

1. Draw the piecewise functions.

a) $f(x) = \begin{cases} 3 - x & x \leq 3 \\ x^2 & x > 3 \end{cases}$

b) $f(x) = \begin{cases} -1 & x < -4 \\ x - 3 & -1 \leq x \leq 5 \\ 2x - 15 & x > 7 \end{cases}$

c) $f(x) = \begin{cases} -2x - 1 & x < 2 \\ x + 4 & x \geq 2 \end{cases}$

d) $f(x) = \begin{cases} 2 & x < -3 \\ -x - 1 & -3 \leq x < 2 \\ -\sqrt{4 - x^2} & x \geq 2 \end{cases}$

Practice Problem

2. Find the domain of the following functions.

a) $f(x) = x^4 - 2x + 4$ Ans: $(-\infty, \infty)$

b) $f(x) = \frac{3x+2}{x}$ Ans:$(-\infty, 0) \cup (0, \infty)$

c) $f(x) = \sqrt{x^2 - 6x + 8}$ Ans: $(-\infty, 2] \cup [4, \infty)$

d) $f(x) = \sqrt{2x^3 + 5x^2 - 3x}$ Ans:$(-\infty,3] \cup [3,0] \cup [\frac{1}{2}, \infty)$

e) $f(x) = \sqrt{-(x+2)(x+5)(x-1)}$ Ans:$(-\infty, -5] \cup [-2,1]$

Practice Problem

3. Find the domain and range of the following function

a) $f(x) = 3$

Ans: D=$(-\infty, \infty)$ or $\{x|x \in R\}, R = \{y|y = 3\}$

b) $f(x) = \sqrt{x^2 - 4x}$

Ans: D=$(-\infty, 0] \cup [4, \infty), R = [0, \infty)$

c) $f(x) = \sqrt{2 - x} + 1$

Ans: D= $(-\infty, 2], R = [1, \infty)$

d) $f(x) = \frac{5x-6}{x+3}$

Ans: D=$(-\infty, -3) \cup (-3, \infty)$,

$R = (-\infty, 5) \cup (5, \infty)$

Practice Problem

4. If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-1}$, then evaluate $(f \circ g)(x)$ and draw the function $y = (f \circ g)(x)$.
5. If $f(x) = x^2 - 9$ and $g(x) = x - 3$, then evaluate $(f \circ g)(x)$ and $(g \circ f)(x)$.

Practice Problem

6. Draw the followings and state whether it is a function or just a relation.

(a) $y = |x - 1|$

(b) $y = |4 - 2x|$

(c) $x = -4$

(d) $y = 5$

(e) $5x - 2y = 10$

(f) $3x + y = 6$

(g) $y^2 = x + 5$

(h) $x^2 = 2 - y$

(i) $(y - 1)^2 = x - 1$

(j) $x^2 + (y - 1)^2 = 4$

(k) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

(l) $y = 2 + \sin x$

Practice Problem

7. Identify the shape of the following curves. If the curves are (i) straight lines, then state whether they pass through the origin or not, (ii) in case of parabola state the vertex, and (iii) in case of circle, ellipse or hyperbola state the center.

(a) $x - 5y = 0$

(b) $3x - 6y = 9$

(c) $y = 6$

(d) $y^2 = x$

(e) $x^2 = 4 - y$

(f) $(y + 2)^2 = x - 3$

(g) $x^2 + y^2 = 4$

(h) $x^2 - \frac{y^2}{4} = 1$

(i) $\frac{x^2}{2} + \frac{y^2}{16} = 1$

Practice Problem

1. Find the limits.

(a) $\lim_{x \to -1} (-2x^5 + x^3 - x + 1)$

(b) $\lim_{x \to 0} (5x^2 + 3x - 1)^{11}$

(c) $\lim_{x \to 2} \sqrt[3]{x^5 - 2x^4 + 5x + 17}$

(d) $\lim_{x \to -5} \frac{2x+5}{x+4}$

(e) $\lim_{x \to -4} \frac{2x+8}{x+4}$

(f) $\lim_{x \to 2} \frac{x^2-x-2}{x^2-2x}$

(g) $\lim_{x \to -5} \frac{x^2+6x+9}{x^2+2x-3}$

(h) $\lim_{x \to \infty} (x^6 - 2x^4 + 6x^3 - x^2)$

(i) $\lim_{x \to -\infty} (-5x^4 + 3x^2 - 11)^{35}$

Practice Problem

2. If $f(x) = \begin{cases} 2x-1, & x < -5 \\ \frac{5}{2x+5}, & -5 \le x \le 0 \\ x^2 - 3x + 1, & 0 < x < 5 \\ \sqrt{x+4}, & x > 5 \end{cases}$, then find **(a)** $\lim_{x \to -5} f(x)$. **(b)** $\lim_{x \to -1} f(x)$. **(c)** $\lim_{x \to 0} f(x)$. **(d)** $\lim_{x \to 2} f(x)$. **(e)** $\lim_{x \to 5} f(x)$ and **(f)** $\lim_{x \to 10} f(x)$.

3. A function $f(x)$ as follows:

$$\begin{aligned} f(x) &= x^2 \quad \text{when } x < 1 \\ &= 2.5 \quad \text{when } x = 1 \\ &= x^2 + 2 \quad \text{when } x > 1. \end{aligned}$$

Does $\lim_{x \to 1} f(x)$ exist?

Practice Problem

4. (a) $f(x) = \begin{cases} \frac{2x-1}{x+4}, & x \neq -4 \\ 5, & x = -4 \end{cases}, a = -4$

(b) $f(x) = \begin{cases} \frac{x^2-3x+2}{x^2-2x}, & x \neq 2 \\ \frac{1}{2}, & x = 2 \end{cases}, a = 2$

(c) $f(x) = \begin{cases} \frac{x^2-9x-5}{x-5}, & x \neq 5 \\ 0, & x = 5 \end{cases}, a = 5$

Check continuity of the above functions.

Practice Problems

The Derivatives

1. Let $y = x^2 - 2x + 3$.

   (a) Find the average rate of change of y with respect to x over the interval $[1, 3]$.

   (b) Find the instantaneous rate of change of y with respect to x when $x = 2$.

2. Let $y = 5 + 3x - x^2$.

   (a) Find the average rate of change of y with respect to x over the interval $[-1,1]$.

   (b) Find the instantaneous rate of change of y with respect to x when $x = 0$.

Practice Problems

3. Find $\frac{dy}{dx}$.

(a) $y = \frac{1}{5}x^5 - 5 + \sqrt[3]{x}$

(b) $y = \frac{4x^3 - 12x + 1}{8}$

(c) $y = (2x - 3)(x^2 + 5)$

(d) $y = x\left(3 - \frac{2}{x}\right)$

(e) $y = \left(\frac{1}{x} + 2\right)(x - 3)$

(f) $y = \frac{x-3}{x+2}$

(g) $y = (2x^2 - 7x + 6)^8$

(h) $y = \frac{3-5x}{4+x^2}$

(i) $y = (x - 5)\sin x$

(j) $y = \frac{\cos x}{3x-4}$

(k) $y = 4\tan x + \ln(5x) + \sqrt[5]{x}$

(l) $y = e^{-x}\sin(3x - 2)$

4. Find $f'(x)$.

(a) $f(x) = (2x + 1)^{23}$

(b) $f(x) = x^{-4} + x^4 + 4$

Practice Problems

5. If $f(x) = x^2 - 7x + \frac{1}{x}$ then find the value of $f'''(x)$ at $x = -1$.

6. Use product rule to find $\frac{d^2y}{dx^2}$ at $x = 1$, when $y = x^3(3x - x^2)$.

7. Find $\frac{dy}{dx}$ at $x = -2$, if $y = \frac{2-x+3x^2+x^3-2x^4}{x^2}$.

8. Evaluate $f'''(x) = \ln x + 3x - e^{2x}$ at $x = 1$.