Integration

Key Word

Area Problem

Derivative of a Function

DEFINITE INTEGRAL

If a function f is contunuous on an interval $[a,b]$, then f is integrable on $[a,b]$, and the net area A between the graph of f and the interval $[a,b]$ is

$$A = \int_a^b f(x)dx$$

THEOREM

If f and g are integrable on $[a,b]$ and if c is a constant, then cf, $f+g$ and $f-g$ are also integrable on $[a,b]$ and

(a) $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

(b) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

HOW TO PUT THE UPPER AND LOWER LIMIT AFTER INTEGRATION

OR

HOW TO GET DEFINITE VALUE

$$A = \int_a^b f(x)dx$$

$= [I(x)]_a^b$ [By considering the result of integration is $I(x)$]

$$= I(b) - I(a)$$

EXAMPLE

$$A = \int_1^2 xdx = \left[\frac{x^2}{2}\right]_1^2 = \frac{2^2}{2} - \frac{1^2}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

PROBLEMS

1. $\int_{-1}^{3}(x-5)\,dx$

2. $\int_{0}^{\pi} sin2xdx$

HOME-WORK

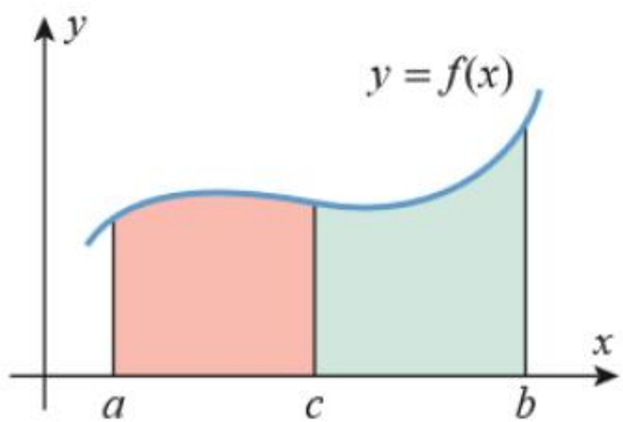
Evaluate the followings.

1. $\int_{0}^{1/2} e^{2x}dx$

2. $\int_{-\pi/2}^{\pi/2} cosxdx$

Property of Definite Integral

If f is continuous and nonnegative on the interval $[a, b]$, and if c is a point between a and b, then the area under $y = f(x)$ over the interval

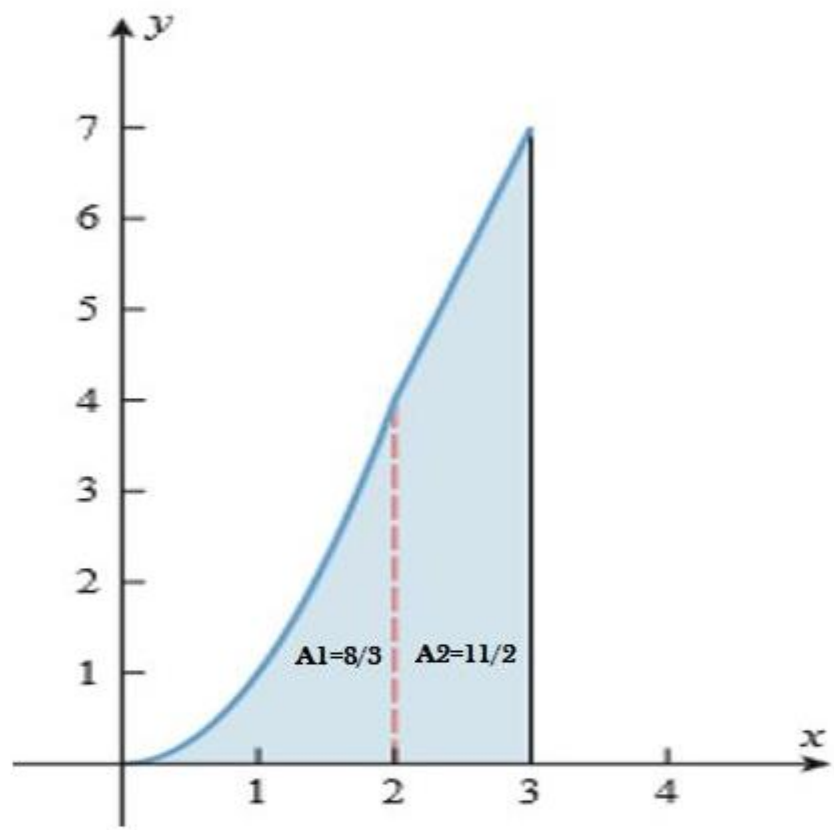


$[a, b]$ can be split into two parts and expressed as the area under the graph from a to c plus the area under the graph from c to b, that is,

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

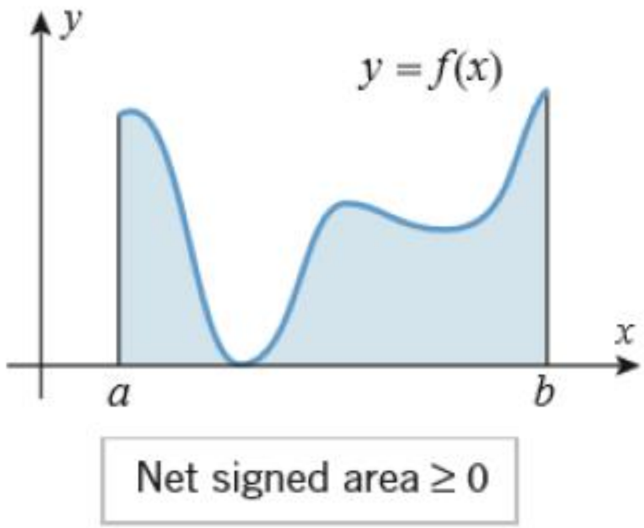
PROBLEM

Evaluate $\int_0^3 f(x)dx$ if $(x) = \begin{cases} x^2, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$.



THEOREM

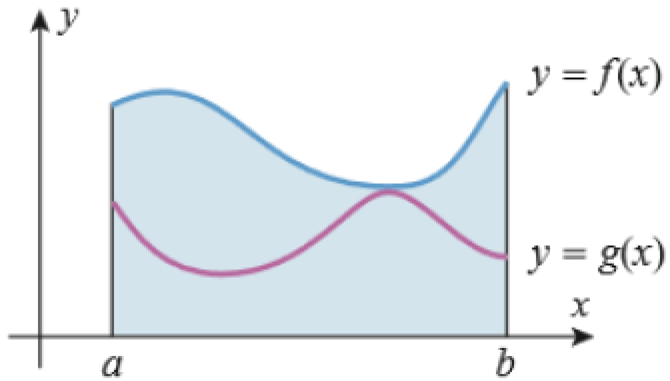
(a)



If f is integrable on $[a, b]$and $f(x) \geq 0$ for all x in $[a, b]$, then

$$\int_a^b f(x)dx \geq 0$$

(b)

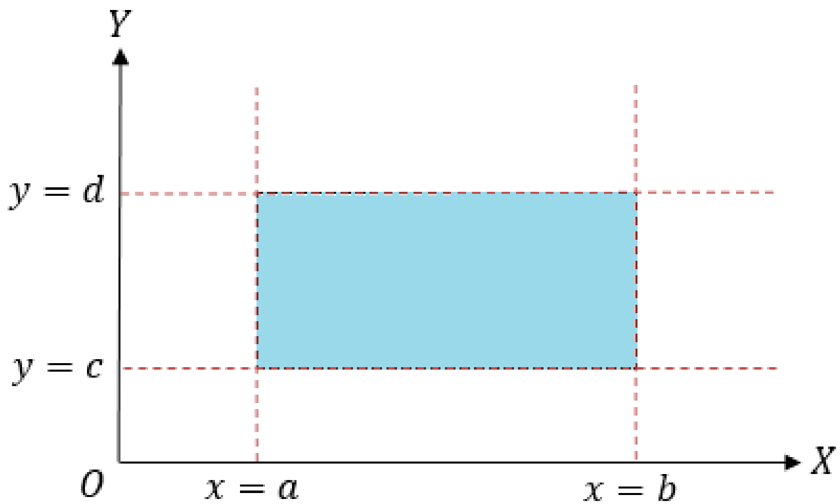


Area under $f \geq$ area under g

If f and g are integrable on $[a, b]$ and $f(x) \geq g(x)$ for all x in $[a, b]$, then

$$\int_a^b f(x)dx \geq \int_a^b g(x)dx$$

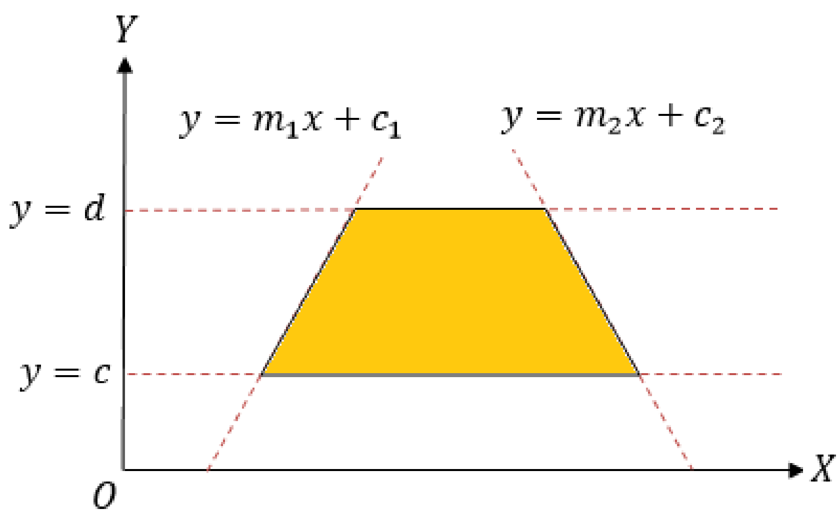
Area between Curves



Area = $\int_{x=a}^{x=b}(d-c)\,dx$

Or,

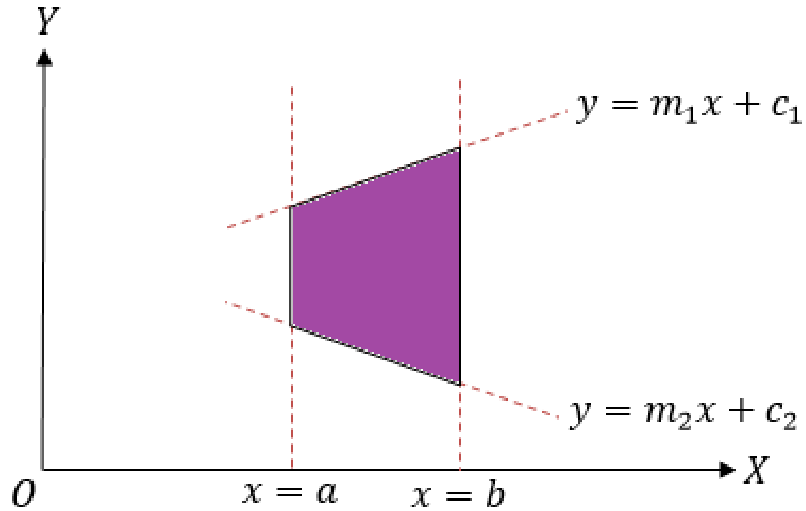
Area = $\int_{y=c}^{y=d}(b-a)\,dy$



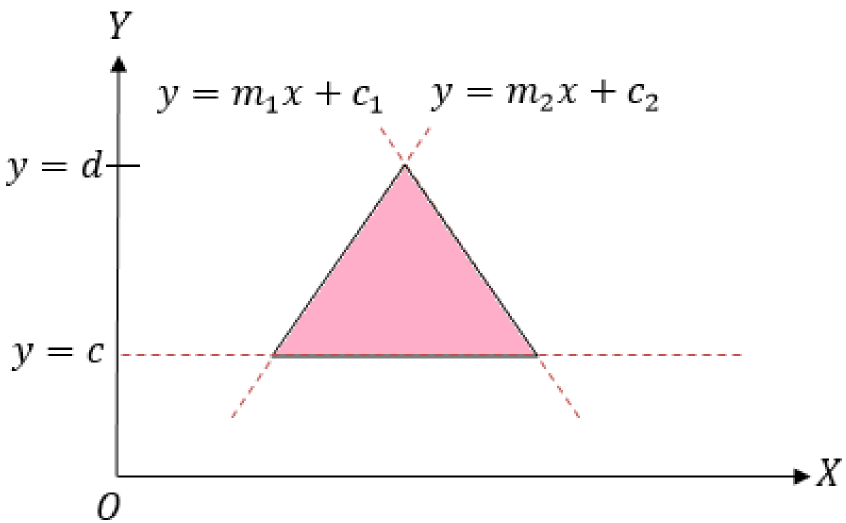
$y = m_1x + c_1 \quad \Rightarrow x = \frac{1}{m_1}(y - c_1)$

$y = m_2x + c_2 \quad \Rightarrow x = \frac{1}{m_2}(y - c_2)$

Area = $\int_{y=c}^{y=d}\left[\frac{1}{m_2}(y-c_2) - \frac{1}{m_1}(y-c_1)\right]dy$



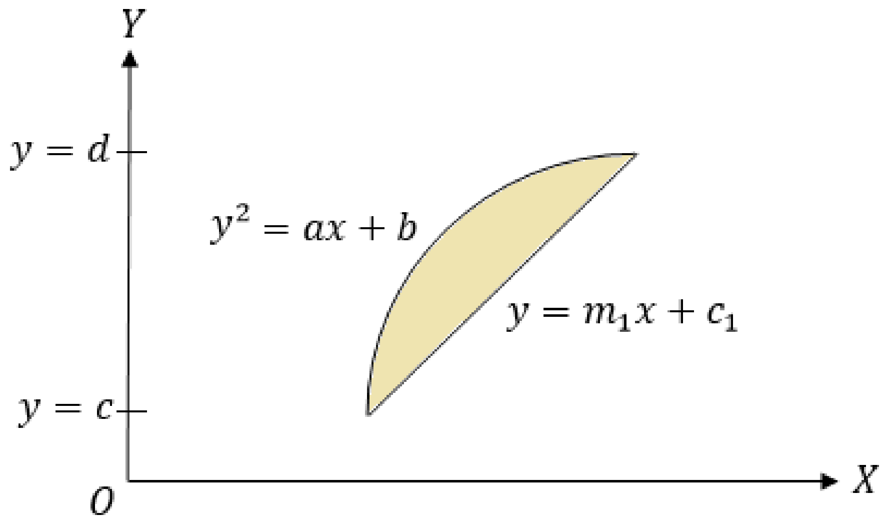
$$\textbf{Area} = \int_{x=a}^{x=b} [(m_1 x + c_1) - (m_2 x + c_2)]\,dx$$



$$y = m_1 x + c_1 \qquad \Rightarrow x = \frac{1}{m_1}(y - c_1)$$

$$y = m_2 x + c_2 \qquad \Rightarrow x = \frac{1}{m_2}(y - c_2)$$

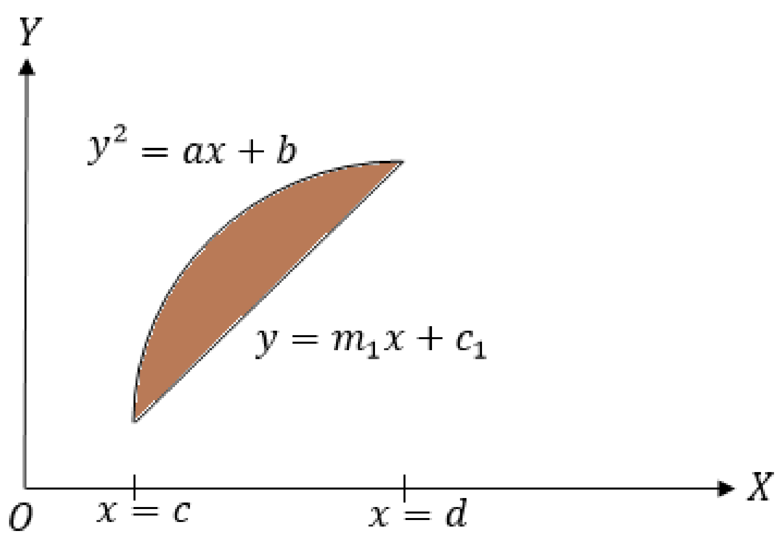
$$\textbf{Area} = \int_{y=c}^{y=d} \left[\frac{1}{m_1}(y - c_1) - \frac{1}{m_2}(y - c_2)\right] dy$$



$$y^2 = ax + b \qquad \Rightarrow x = \frac{1}{a}(y^2 - b)$$

$$y = m_1x + c_1 \qquad \Rightarrow x = \frac{1}{m_1}(y - c_1)$$

Area = $\displaystyle\int_{y=c}^{y=d} \left[\frac{1}{m_1}(y - c_1) - \frac{1}{a}(y^2 - b)\right] dy$



$$y^2 = ax + b \qquad \Rightarrow y = \sqrt{ax + b}$$

Area = $\displaystyle\int_{x=c}^{x=d} \left[\sqrt{ax + b} - (m_1x + c_1)\right] dx$

Problems

Find the area of the region bounded by the following curves:

a) $x = -3, x = 2, y = 0$ and $y = 4$ Result: 20

b) $x = 5, y = 0, y = 4$ and $y = 2x$ Result: 16

c) $x = 0, y = 1, y = 4$ and $x + y = 5$ Result: 15/2

d) $x = 0, x - 3y = 0$ and $2x + 3y = 9$ Result: 9/2

e) $y = x + 1$ and $y = x^2 - 1$

f) $x^2 + y^2 = 4, x = -1$ and $x = 1$

g) $y^2 = x$ and $x^2 = y$

Home-Works

Find the area of the region bounded by the following curves:

1. $x = \frac{1}{2},\ x = 4,\ y = -1$ and $y = 1$ Result: 7

2. $x = 0,\ x = 1,\ y = x$ and $y = 2$ Result: $\frac{3}{2}$

3. $x = 0,\ y = x,$ and $y = 2$ Result: 2

4. $x = -3, x = 0, y = 0$ and $y = 3x - 1$ Result: $\frac{33}{2}$

5. $x = -2, x = 4, \mathrm{y} = x - 2$ and $x^2 = 4y$ Result: 12

6. $y = x$ and $\mathrm{y} = x^2 - 2x$ Result: $\frac{9}{2}$

7. $x^2 + y^2 = 1,\ x = -1$ and $x = 0$ Result: $\frac{\pi}{2}$

8. $x = -\sqrt{1 - y^2}$ and $x = 0$ Result: $\frac{\pi}{2}$