Successive Differentiation

KEYWORD:
DERIVATIVE OF A FUNCTION

Successive Differentiation

1. If $y = e^{ax}$, find y_n.

Solution: Given that $y = e^{ax}$

$$\therefore\ y_1 = ae^{ax}$$

$$y_2 = a^2 e^{ax}$$

$$y_3 = a^3 e^{ax}$$

$$\vdots$$

$$y_n = a^n e^{ax}$$

3. If $y = \frac{1}{x+a}$, find y_n.

Solution: Given that $y = \frac{1}{x+a}$

$$= (x+a)^{-1}$$

$$\therefore\ y_1 = -1.(x+a)^{-2}$$

$$y_2 = (-1)(-2)(x+a)$$

$$= (-1).(-1).1.\ 2\ (x+a)^{-3}$$

$$= (-1)^2.1.\ 2\ (x+a)^{-(2+1)}$$

Similarly, $y_3 = (-1)^3.1.2.3\ (x+a)^{-(3+1)}$

$$\vdots$$

$$y_n = (-1)^n.1.2.3.....n\ (x+a)^{-(n+1)}$$

$$= (-1)^n\, n!(x+a)^{-(n+1)}$$

$$= \frac{(-1)^n\, n!}{(x+a)^{n+1}}$$

Successive Differentiation

5. If $y = \sin(ax+b)$, find y_n.

Solution: Given that $y = \sin(ax+b)$

$$\therefore y_1 = a\cos(ax+b)$$

$$= a\sin\left(\frac{\pi}{2}+ax+b\right)$$

$$y_2 = a^2\cos\left(\frac{\pi}{2}+ax+b\right)$$

$$= a^2\sin\left(\frac{\pi}{2}+\frac{\pi}{2}+ax+b\right)$$

$$= a^2\sin\left(\frac{2\pi}{2}+ax+b\right)$$

$$y_3 = a^3\cos\left(\frac{2\pi}{2}+ax+b\right)$$

$$= a^3\sin\left(\frac{\pi}{2}+\frac{2\pi}{2}+ax+b\right)$$

$$= a^3\sin\left(\frac{3\pi}{2}+ax+b\right)$$

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$$y_n = a^n\sin\left(\frac{n\pi}{2}+ax+b\right)$$

Successive Differentiation

6. If $y = \cos(ax+b)$, find y_n.

Solution: Given that $y = \cos(ax+b)$

$$\therefore y_1 = -\sin(ax+b).(a.1+0)$$

$$= -a\sin(ax+b)$$

$$= a\cos\left(\frac{\pi}{2}+ax+b\right)$$

Similarly, $y_2 = -a^2\sin\left(\frac{\pi}{2}+ax+b\right)$

$$= a^2\cos\left(\frac{\pi}{2}+\frac{\pi}{2}+ax+b\right)$$

$$= a^2\cos\left(\frac{2\pi}{2}+ax+b\right)$$

$$y_3 = -a^3\sin\left(\frac{2\pi}{2}+ax+b\right)$$

$$= a^3\cos\left(\frac{\pi}{2}+\frac{2\pi}{2}+ax+b\right)$$

$$= a^3\cos\left(\frac{3\pi}{2}+ax+b\right)$$

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$$y_n = a^n\cos\left(\frac{n\pi}{2}+ax+b\right)$$

Successive Differentiation

❑ Practice Problems:

1. If $y = x^n$, find y_n.

2. If $y = \log(x + a)$, find y_n.

3. If $y = \sin 3x . \cos 2x$, find y_n.