PARTIAL
Differentiation

- *Derivative of a function*
- *Derivative partially*

Homogeneous Polynomial/Function

A Polynomial is said to be homogeneous if degree of each term is **same**.

How do you calculate the degree of a term?

Term	Degree
x^2	2
$-x^3y^6$	3+6 = 9

How do you calculate the degree of a polynomial?

Polynomial	Degree of first term	Degree of second term	Degree of third term	Degree of the terms are same/different	Homogeneous/ Nonhomogeneous
$x^2y^3 - 2xy^4 + x^3y$	2+3 = 5	1+4 = 5	3+1 = 4	Different	Nonhomogeneous
$x^2 + \frac{1}{3}xy$	2	1+1 = 2	--	Same	Homogeneous of degree 2
$x^2yz + 6y^4 - yz^3$	2+1+1 = 4	4	1+3 = 4	Same	Homogeneous of degree 4

Homogeneous polynomials are also known as **homogeneous function.**

Euler's Theorem

If u is a homogeneous function of degree n, it must satisfy the condition:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

Practice Problems

Determine whether the following(s) is/are a homogeneous function. If yes, then prove Euler's Theorem for *u*.

$a)\ u = x^3 + y^3 + 3xy^2$

$b)\ u = x^5 - 2x^3y^2 + y^5$

$c)\ u = -y^6 + 5x^4y$

$d)\ u = 3x^4 - x^2y^2 + 2xy^3$

$e)\ u = sin^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{y}{x}\right)$

$f)\ u = \frac{x(x^3 - y^3)}{x^3 + y^3}$

$g)\ u = \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}$

Sample Answer

- **Solution of (a):**

Let us compute $u(tx, ty)$:

$$u(tx, ty) = (tx)^3 + (ty)^3 + 3(tx)(ty)^2$$

$$u(tx, ty) = t^3x^3 + t^3y^3 + 3t^3xy^2$$

$$u(tx, ty) = t^3(x^3 + y^3 + 3xy^2)$$

Thus, we see that:

$$u(tx, ty) = t^3u(x, y)$$

This shows that the function $u(x, y) = x^3 + y^3 + 3xy^2$ is homogeneous of degree 3.

Sample Answer

▪ Solution of (a):

Since $u(x, y)$ is homogeneous of degree 3, Euler's Theorem should hold, i.e., we should have:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u(x, y)$$

Now, let's compute the partial derivatives of $u(x, y)$:

- $\frac{\partial u}{\partial x} = 3x^2 + 3y^2$
- $\frac{\partial u}{\partial y} = 3y^2 + 6xy$

Now, compute $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$:

$$x\frac{\partial u}{\partial x} = x(3x^2 + 3y^2) = 3x^3 + 3xy^2$$

$$y\frac{\partial u}{\partial y} = y(3y^2 + 6xy) = 3y^3 + 6xy^2$$

Sample Answer

- **Solution of (a):**

Now, add them together:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = (3x^3 + 3xy^2) + (3y^3 + 6xy^2)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3x^3 + 3y^3 + 9xy^2$$

We observe that:

$$3x^3 + 3y^3 + 9xy^2 = 3(x^3 + y^3 + 3xy^2) = 3u(x, y)$$

Thus, we have verified that Euler's Theorem holds for $u(x, y)$.