PARTIAL
Differentiation

- *Derivative of a function*
- *Derivative partially*

Partial Derivative of a Function with Respect to an Independent Variable

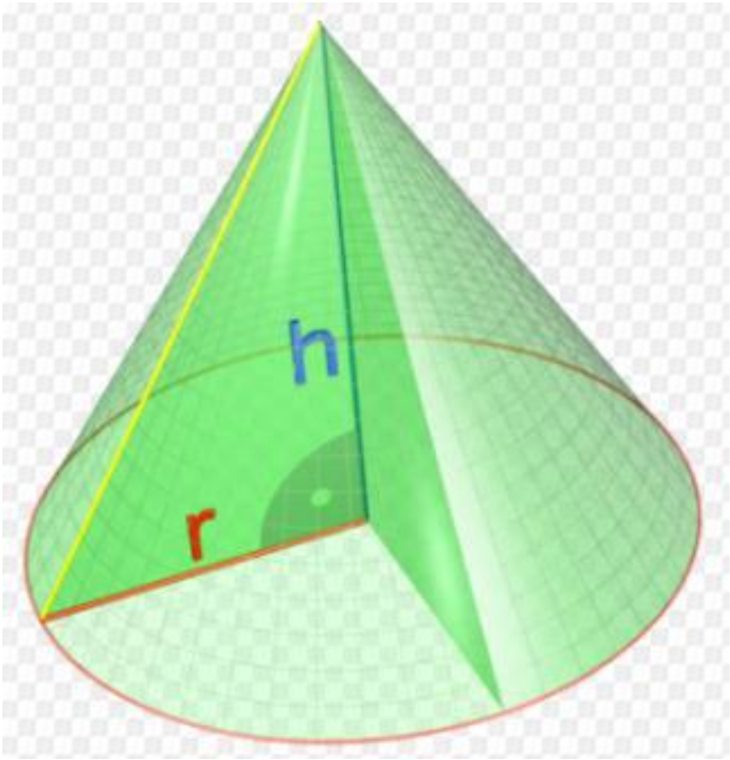
- Suppose that we have a real-valued function $z = f(x, y)$ of two real variables. Then, the derivative of f , with respect to x, holding y constant, is called the **Partial Derivative of f with respect to x,** and is denoted by any of $\frac{\partial z}{\partial x}$ $or\,\frac{\partial f}{\partial x}$ $or\ f_x\,(x, y)$.
- On the other hand, the derivative of f, with respect to y, holding x constant, is called the **partial derivative of *f* with respect to *y***, and is denoted by any of $\frac{\partial z}{\partial y}$ $or\,\frac{\partial f}{\partial y}$ $or\ f_y\,(x, y)$.

EXAMPLE

Example:

The volume of V of a cone depends on the cone's height h and its radius r according to the formula:

$$V(r,h) = \frac{1}{3}\pi r^2 h$$



Solution:

The partial derivative of V with respect to r is:

$$\frac{\partial V}{\partial r} = \frac{2}{3}\,\pi r h$$

which represents the rate with which a cone’s volume changes if its radius is varied and its height is kept constant.

The partial derivative with respect to h is:

$$\frac{\partial V}{\partial r} = \frac{1}{3}\,\pi r^2$$

which represents the rate with which the volume changes if its height is varied and its radius is kept constant.

EXAMPLE-1

If $z = 2x^2y - 3x + 2$ then evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Answer: $\frac{\partial z}{\partial x} = 4xy - 3; \frac{\partial z}{\partial y} = 2x^2$

EXAMPLE-2

If $f(\mathrm{x}, y) = \sqrt{xy} - \sin(x - y)$ then evaluate f_x and f_{yx}.

Answer: $f_x = \frac{y}{2\sqrt{xy}} - \cos(x - y)\,; f_{yx} = \frac{1}{4\sqrt{xy}} - \sin(x - y)$

MIXED PARTIAL DERIVATIVES

Suppose that we have a real-valued function $z = f(x, y)$ of two real variable. Then always $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

EXAMPLE-3

If z $= x - \frac{1}{2}y^2 - ln2x + e^{xy}$ then show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

EXAMPLE-4

If $f = 2x^3y^2 - 4x^2y^3 + 2xy$ then evaluate $f_x, f_y, f_{xx}, f_{yy},$ f_{xy} and f_{yx}

Practice Problem

1. If $f(x,y) = 3x^2y + 5xy^2 - 7x + 2y$ then find $f_x(x,y)$ and $f_y(x,y)$.

Ans: $f_x(x,y) = 6xy + 5y^2 - 7; f_y(x,y) = 3x^2 + 10xy + 2$

2. If $f(x,y) = x^3 - 4x^2y + 6xy^2 - y^3$ then find $f_x(x,y)$ and $f_y(x,y)$.

Ans: $f_x(x,y) = 3x^2 - 8xy + 6y^2; f_y(x,y) = -4x^2 + 12xy - 3y^2$

Practice Problem

3. If $f(x, y) = e^x + y^2 e^x - \ln(x + y)$ then find $f_x(x, y)$ and $f_y(x, y)$.

Ans: $f_x(x, y) = e^x + y^2 e^x - \frac{1}{x+y}$; $f_y(x, y) = 2ye^x - \frac{1}{x+y}$

4. If $f(x, y) = \frac{x^2+y^2}{x+y}$ then find $f_x(x, y)$ and $f_y(x, y)$.

Ans: $f_x(x, y) = \frac{x^2+2xy-y^2}{(x+y)^2}$; $f_y(x, y) = \frac{y^2+2xy-x^2}{(x+y)^2}$

Practice Problem

5. If $f(x,y) = \sin(xy) + x^2\cos(y)$ then find $f_x(x,y)$ and $f_y(x,y)$.

Ans: $f_x(x,y) = y\cos(xy) + 2x\cos(y)\,;\, f_y(x,y) = x\cos(xy) - x^2\sin(y)$

6. If $f(x,y) = \sqrt{64 - x^2 - y^2}$, then find $f_{xx}, f_{yy}, f_{xy}, f_{yx}$.

Ans: $f_{xx}(x,y) = -\dfrac{64-y^2}{(64-x^2-y^2)^{3/2}}\,;\, f_{xy}(x,y) = -\dfrac{xy}{(64-x^2-y^2)^{3/2}},$

$f_{yy}(x,y) = -\dfrac{64-x^2}{(64-x^2-y^2)^{3/2}},\; f_{yy}(x,y) = -\dfrac{xy}{(64-x^2-y^2)^{3/2}}$

Practice Problem

7. If $z(x,y) = e^{xy} + x^2y + \ln(x+y)$, find $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$, and verify they are equal.

8. If $z(x,y) = \sin(xy) + x^2\cos(y)$ then show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

9. If $z(x,y) = \frac{x^2+y^2}{x+y}$, find $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$, and verify they are equal.

10. If $z(x,y) = x^3y^2 + 4x^2y + 5$ then show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.