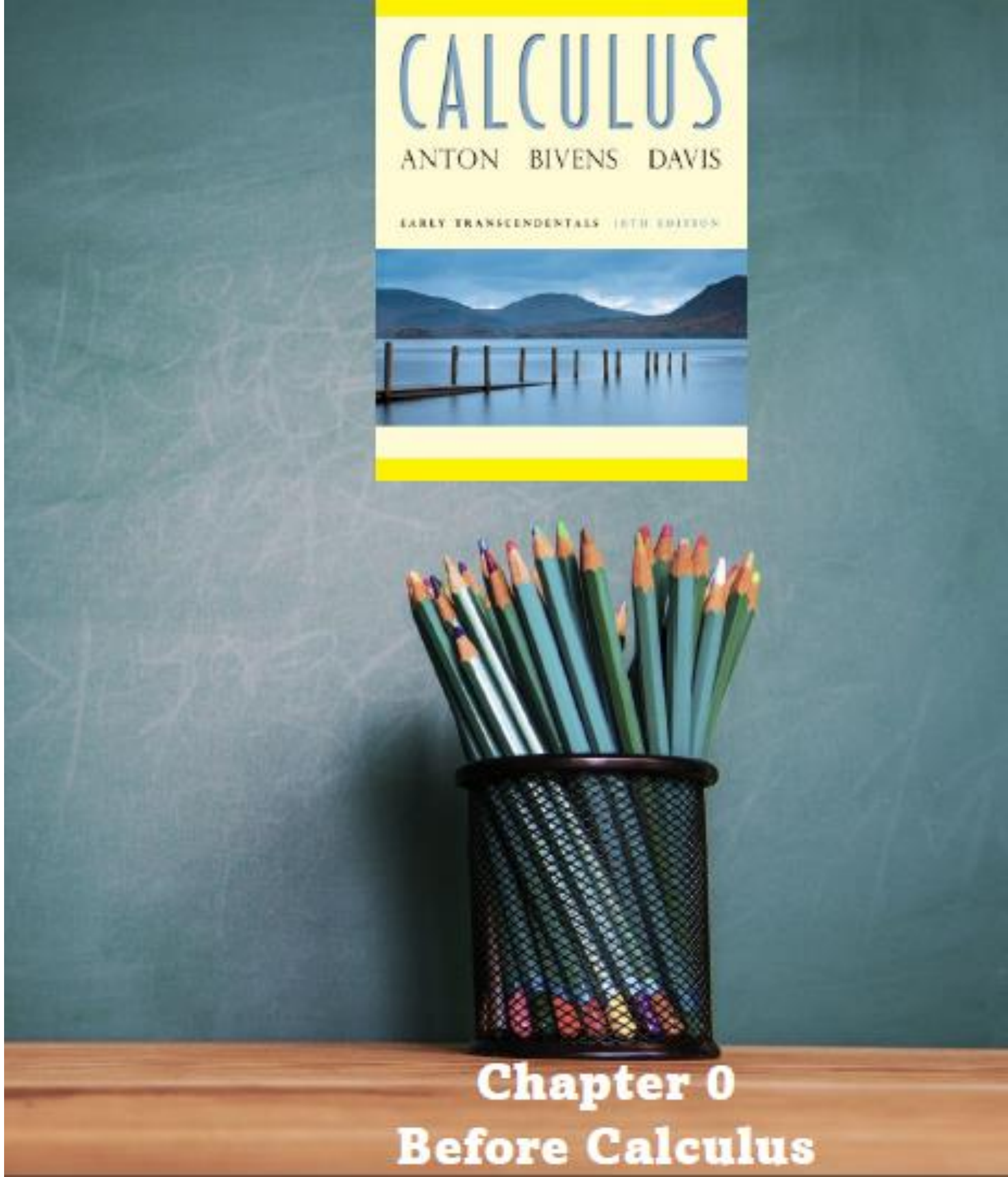
Function of Single Variable and Their Graphs



Key words

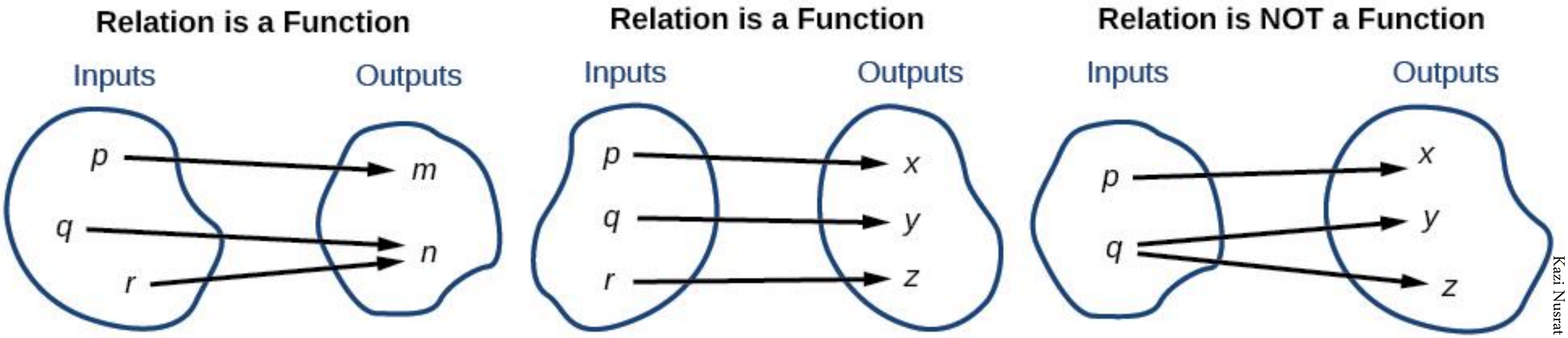
1. Variable
2. Relation
3. xy −plane

Variable, Relation and Function

A **variable** is a quantity that may be changed according to the mathematical problem. The generic letters which are used in many algebraic expressions and equations are x, y, z.

For a single input if there is any output (one or more) then it is known as a **relation**. In relation if for one input there is exactly one output, then it is known as a **function**.

Identify Function



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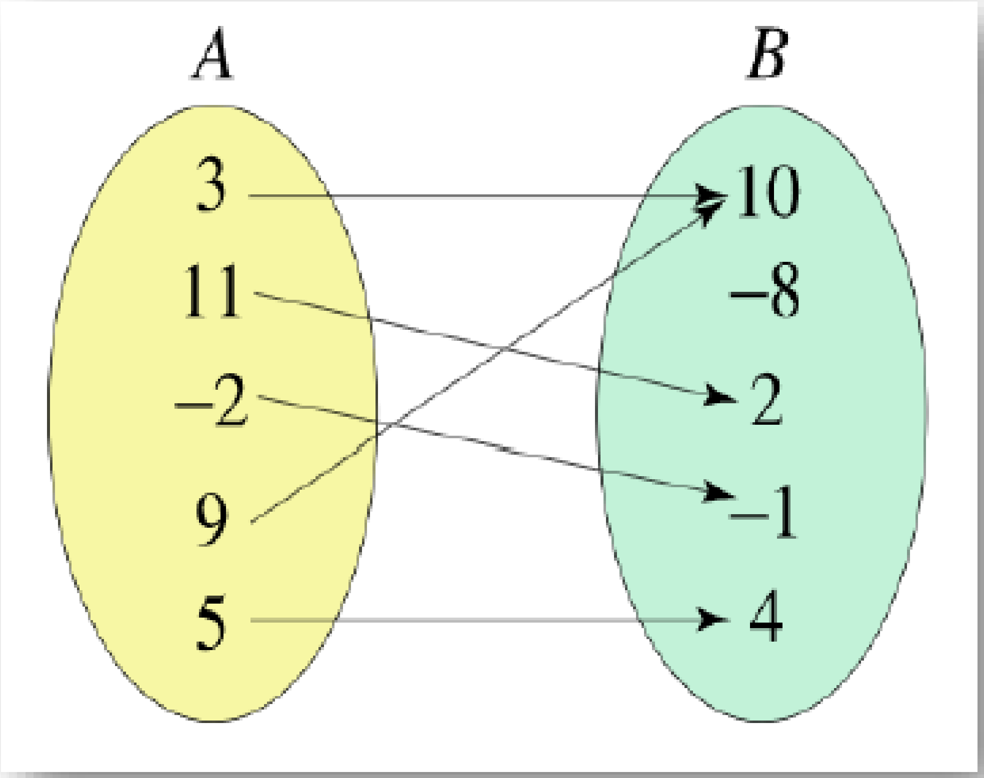
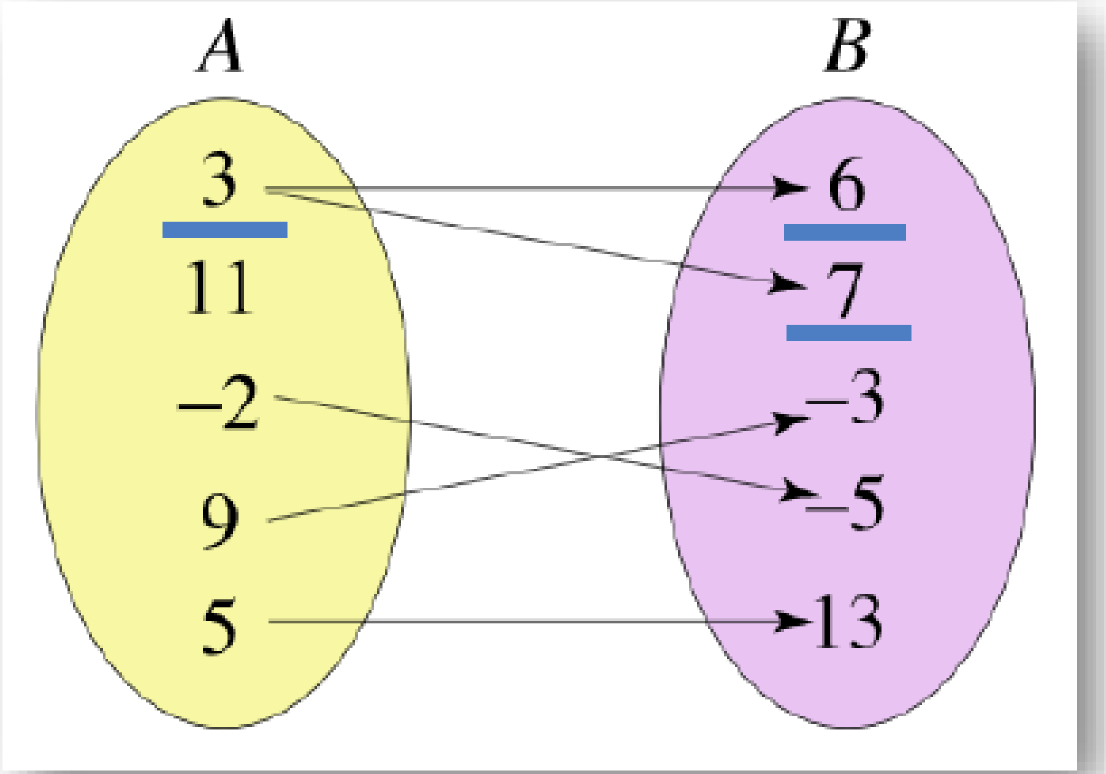
Identify Function



Function



Not a Function

Identify Function

Example

a. Is price a function of the item?

b. Is the item a function of the price?

Menu

Item	Price
Plain Donut	1.49
Jelly Donut	1.99
Chocolate Donut	1.99

Solution

a. Let's begin by considering the input as the items on the menu. The output values are then the prices.

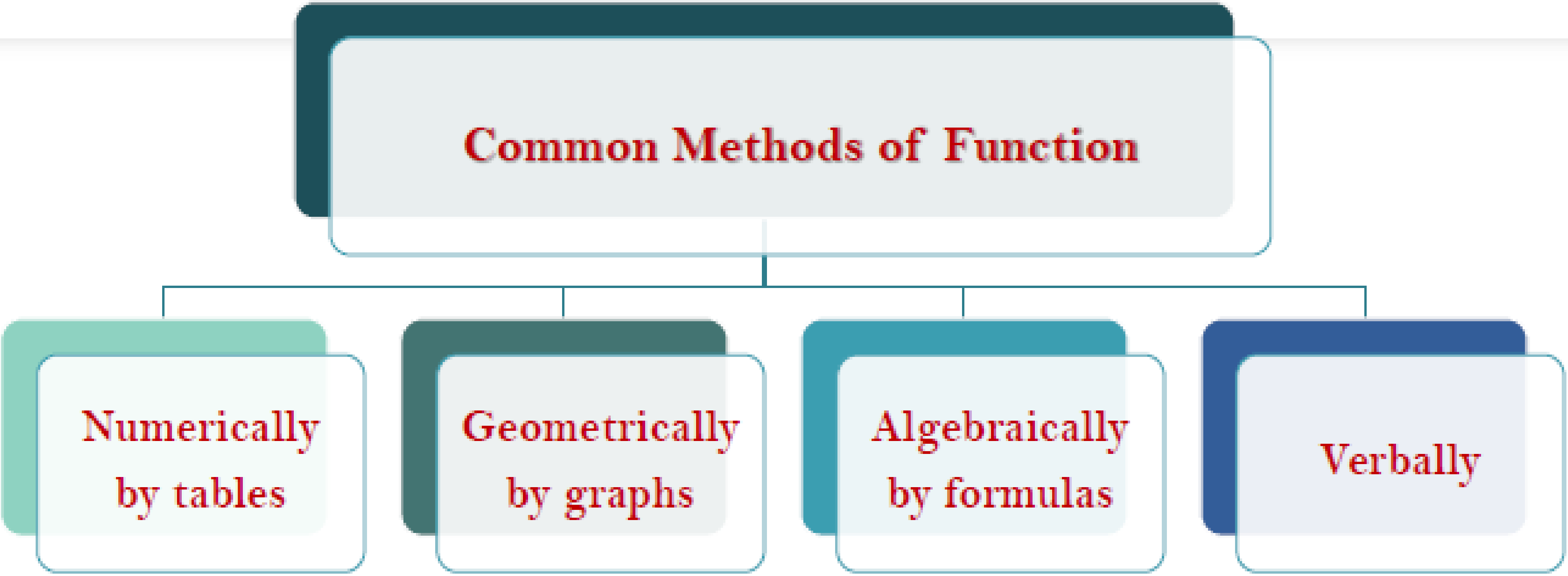
Each item on the menu has only one price, so the price is a function of the item.

b. Two items on the menu have the same price.

If we consider the prices to be the input values and the items to be the output, then the same input value could have more than one output associated with it.

Therefore, the item is a not a function of price.

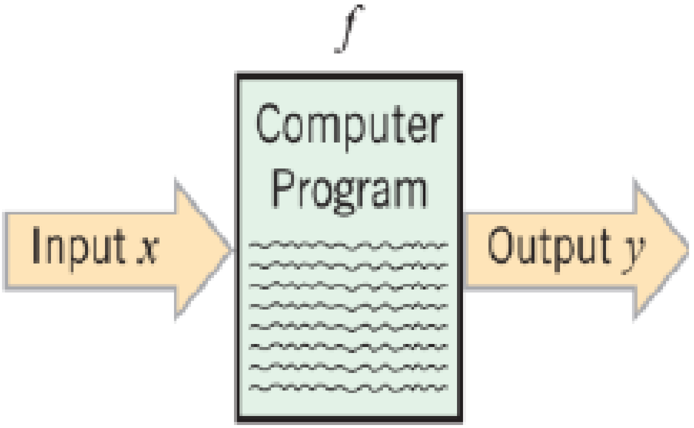
Common Methods for Representing Functions



Definition of Function Using Variables

If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that **y is a function of x**.

A function f is a rule that associates a unique output with each input. If the input is denoted by x, then the output is denoted by $f(x)$ (read "f of x").



Sometimes the output is denoted by a single letter, say y, and so $y = f(x)$.

Examples of Function

Example 1

Table 1

x	0	1	2	3
y	3	4	-1	6

Table 1 describes a functional relationship $y = f(x)$ for which

$f(0) = 3$ — f associates $y = 3$ with $x = 0$.

$f(1) = 4$ — f associates $y = 4$ with $x = 1$.

$f(2) = -1$ — f associates $y = -1$ with $x = 2$.

$f(3) = 6$ — f associates $y = 6$ with $x = 3$.

Example 2

The equation $y = 3x^2 - 4x + 2$ has the form $y = f(x)$ in which the function f is given by the formula $f(x) = 3x^2 - 4x + 2$.

$$f(0) = 3(0)^2 - 4(0) + 2 = 2$$

$$f(-1.7) = 3(-1.7)^2 - 4(-1.7) + 2 = 17.47$$

$$f(\sqrt{2}) = 3(\sqrt{2})^2 - 4\sqrt{2} + 2 = 8 - 4\sqrt{2}$$

For each input x, the corresponding output y is obtained by substituting x in this formula.

Independent and Dependent Variables and Real-Valued Function

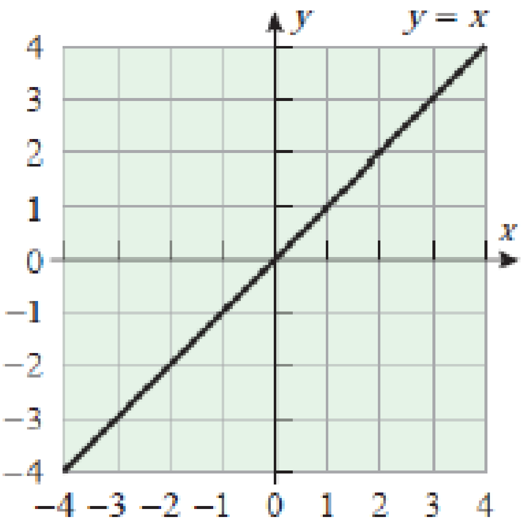
In the equation $y = f(x)$ expresses y as a function of x; the variable **x is called the independent variable** (or argument) of f, and the variable **y is called the dependent variable** of f.

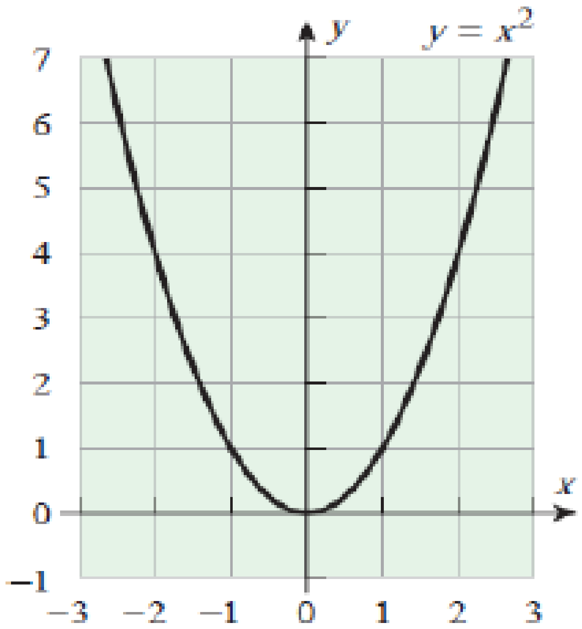
In a function f if both the independent and dependent variables are real numbers, then the function **f is a real-valued function of a real variable**.

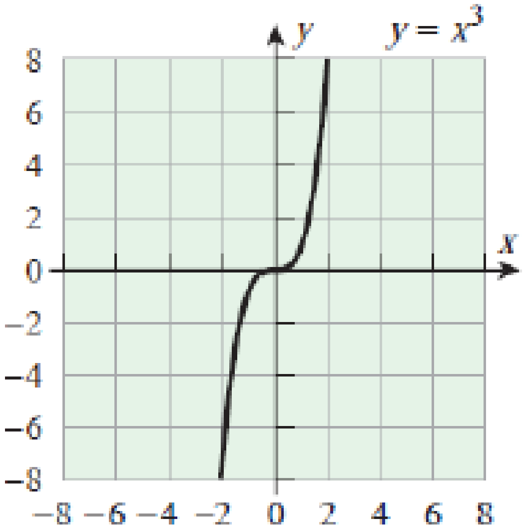
Graph of Functions
and
Vertical Line Test

If f is a real-valued function of a real variable, then the ***graph*** of f in the xy-plane is defined to be the graph of the equation $y = f(x)$.

THE VERTICAL LINE TEST *A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.*





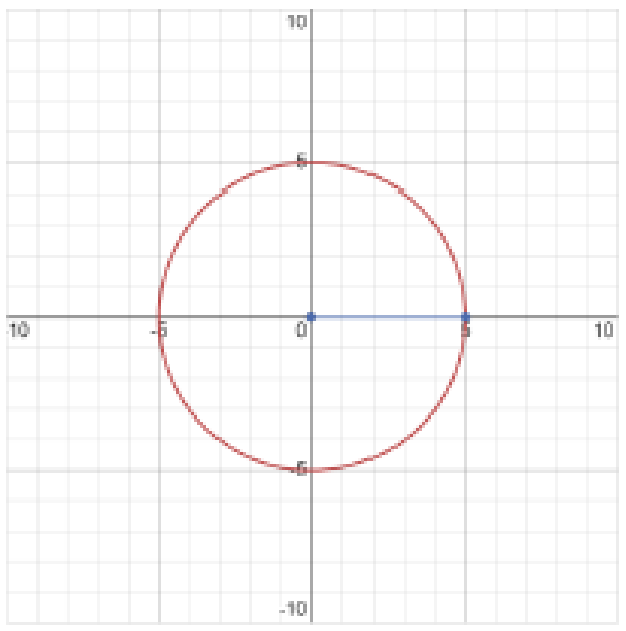


Graph of Functions
and
Vertical Line Test

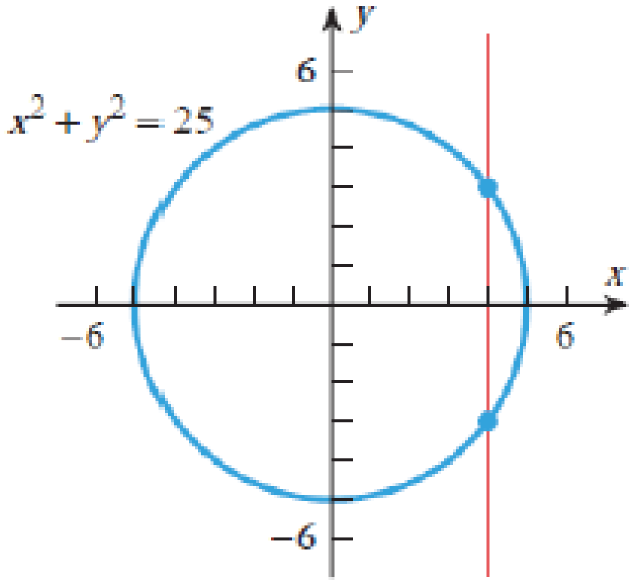
The graph of the equation

$$x^2 + y^2 = 25$$

is a circle of radius 5 centered at the origin and hence there are vertical lines that cut the graph more than once. Thus this equation does not define y as a function of x.



Not a Function

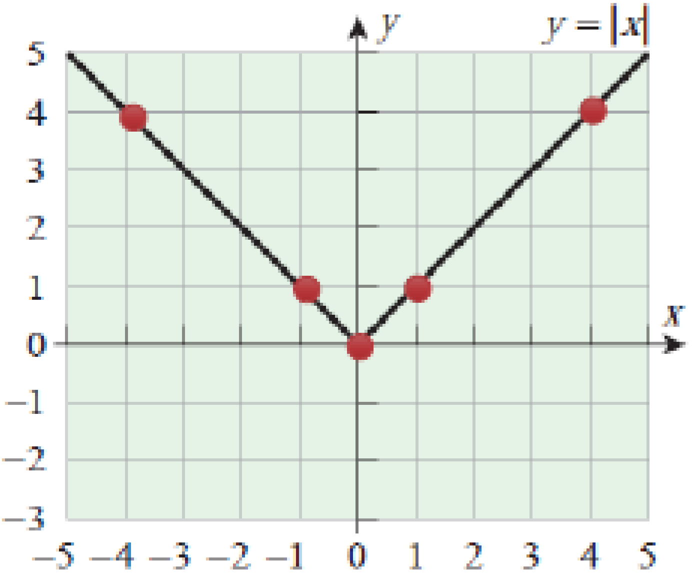


Absolute Value Function

The *absolute value* or *magnitude* of a real number x is defined by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

x	-4	-1	0	1	4
$y = \lvert x \rvert$	4	1	0	1	4



The effect of taking the absolute value of a number is to strip away the minus sign if the number is negative and to leave the number unchanged if it is nonnegative. Thus,

$$|5| = 5, \quad \left|-\tfrac{4}{7}\right| = \tfrac{4}{7}, \quad |0| = 0$$

The graph of the function $f(x) = |x|$ can be obtained by graphing the two parts of the equation

$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

separately. Combining the two parts produces the V-shaped graph in Figure.

Properties Absolute Value

If a and b are real numbers, then

(a)	$\|-a\| = \|a\|$	A number and its negative have the same absolute value.
(b)	$\|ab\| = \|a\|\,\|b\|$	The absolute value of a product is the product of the absolute values.
(c)	$\|a/b\| = \|a\|/\|b\|,\ b \neq 0$	The absolute value of a ratio is the ratio of the absolute values.
(d)	$\|a+b\| \leq \|a\| + \|b\|$	The ***triangle inequality***

Piecewise-Defined Function

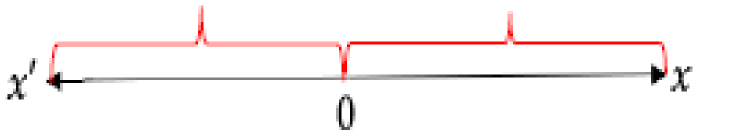
A **piecewise-defined** function is one which is defined not by a single equation, but by two or more. Each equation is valid for some interval .

The absolute value function $f(x) = |x|$ is an example of a function that is defined ***piecewise*** in the sense that the formula for f changes, depending on the value of x.

$$y = \; f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

breakpoint $\quad x = 0$

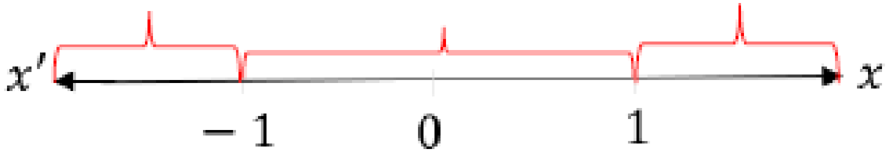
intervals $\quad -\infty < x < 0$ and $0 \leq x < \infty$



Example Sketch the graph of the function defined piecewise by the formula

$$f(x) = \begin{cases} 0, & x \leq -1 \\ \sqrt{1-x^2}, & -1 < x < 1 \\ x, & x \geq 1 \end{cases}$$

breakpoints $\quad x = -1$ and $x = 1$



First interval: $-\infty < x \leq -1$

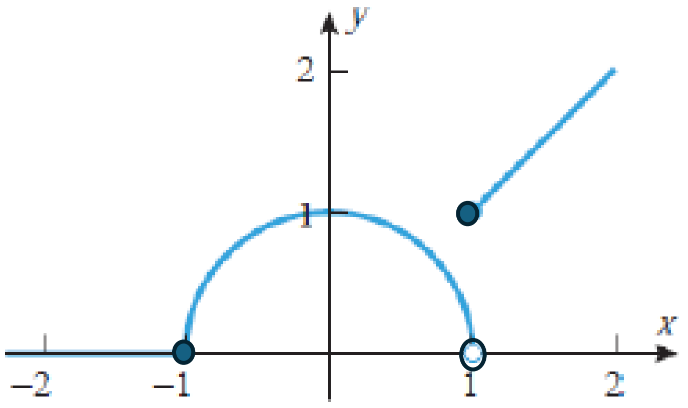
x	-2	-1
$y = 0$	0	0

Second interval: $-1 < x < 1$

x	-1	0	1
$y = \sqrt{1-x^2}$	0	1	0

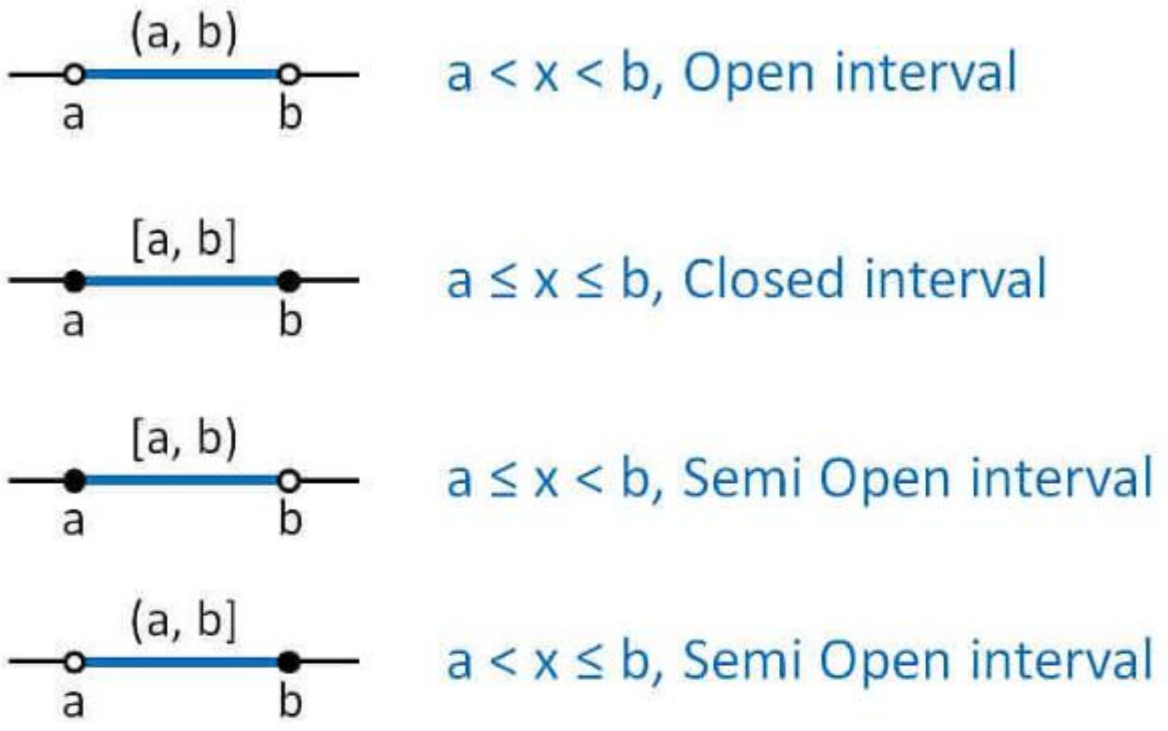
Third interval: $1 \leq x < \infty$

x	1	2
$y = x$	1	2



Interval Notation

A closed interval, [a, b], is an interval that includes all of its endpoints, and an open interval, (a, b), is an interval that does not contain its endpoints.



Practice Problem

1. Draw the piecewise functions.

a) $f(x) = \begin{cases} 3 - x & x \le 3 \\ x^2 & x > 3 \end{cases}$

b) $f(x) = \begin{cases} -1 & x < -4 \\ x - 3 & -1 \le x \le 5 \\ 2x - 15 & x > 7 \end{cases}$

c) $f(x) = \begin{cases} -2x - 1 & x < 2 \\ x + 4 & x \ge 2 \end{cases}$

d) $f(x) = \begin{cases} 2 & x < -3 \\ -x - 1 & -3 \le x < 2 \\ -\sqrt{4 - x^2} & x \ge 2 \end{cases}$