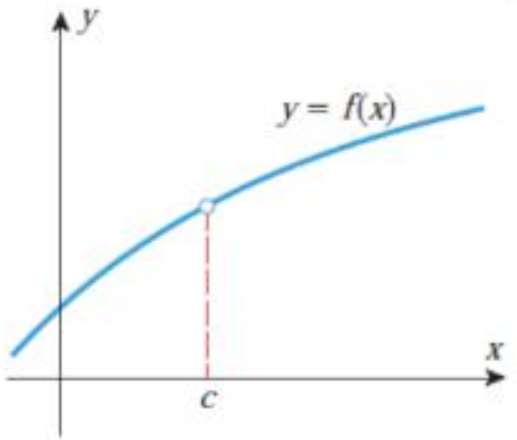
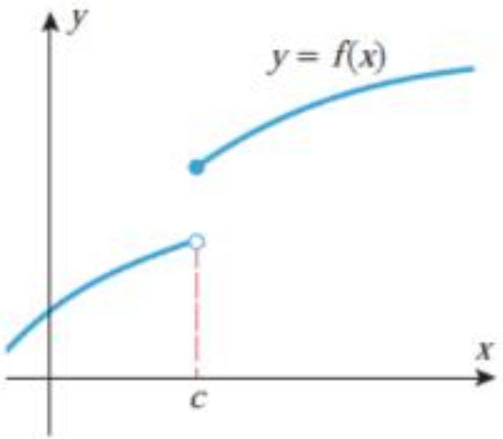
CONTINUITY

DEFINITION A function f is said to be ***continuous at $x = c$*** provided the following conditions are satisfied:

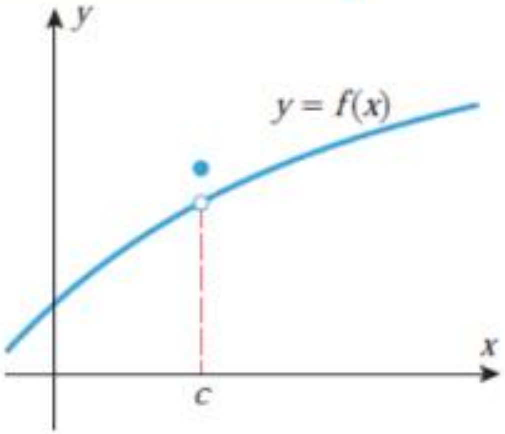
1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.



The function f is undefined at c



The limit of $f(x)$ does not exist as x approaches c



The value of the function and the value of the limit at c are different

CONTINUITY

SOME PROPERTIES

THEOREM *If the functions f and g are continuous at c, then*

(*a*) $f + g$ *is continuous at* c.

(*b*) $f - g$ *is continuous at* c.

(*c*) fg *is continuous at* c.

(*d*) f/g *is continuous at* c *if* $g(c) \neq 0$ *and has a discontinuity at* c *if* $g(c) = 0$.

CONTINUITY

CONTINUITY OF POLYNOMIALS AND RATIONAL FUNCTIONS

THEOREM

(a) *A polynomial is continuous everywhere.*

(b) *A rational function is continuous at every point where the denominator is nonzero, and has discontinuities at the points where the denominator is zero.*

CONTINUITY

Example 13:

A function $f(x)$ is defined as follows:

$$f(x) = x \quad \text{when } 0 < x < 1$$
$$= 2 - x \quad \text{when } 1 \le x \le 2$$
$$= x - \frac{x^2}{2} \quad \text{when } x > 2.$$

Prove that $f(x)$ is continuous at $x = 2$.

Solution: Given that

$$f(x) = x \quad \text{when } 0 < x < 1$$
$$= 2 - x \quad \text{when } 1 \le x \le 2$$
$$= x - \frac{x^2}{2} \quad \text{when } x > 2.$$

$$L.H.L. = \lim_{x \to 2^-} f(x)$$
$$= \lim_{x \to 2^-} (2 - x)$$
$$= 2 - 2$$
$$= 0$$

$$R.H.L. = \lim_{x \to 2^+} f(x)$$
$$= \lim_{x \to 2^+} \left(x - \frac{x^2}{2} \right)$$
$$= 2 - \frac{2^2}{2}$$
$$= 2 - \frac{4}{2}$$
$$= 2 - 2$$
$$= 0$$

When $x = 2$, $f(x) = 2 - x$

$$\therefore f(2) = 2 - 2$$
$$= 0$$

Since $L.H.L. = R.H.L. = f(2)$

Hence $f(x)$ is continuous at $x = 2$.

CONTINUITY

CONTINUITY OF POLYNOMIALS AND RATIONAL FUNCTIONS

Example 16 For what values of x is there a discontinuity in the graph of

$$y = \frac{x^2 - 9}{x^2 - 5x + 6}?$$

Solution. The function being graphed is a rational function, and hence is continuous at every number where the denominator is nonzero. Solving the equation

$$x^2 - 5x + 6 = 0$$

yields discontinuities at $x = 2$ and at $x = 3$

Practice Problem

4. (a) $f(x) = \begin{cases} \frac{2x-1}{x+4}, & x \neq -4 \\ 5, & x = -4 \end{cases}, a = -4$

(b) $f(x) = \begin{cases} \frac{x^2-3x+2}{x^2-2x}, & x \neq 2 \\ \frac{1}{2}, & x = 2 \end{cases}, a = 2$

(c) $f(x) = \begin{cases} \frac{x^2-9x-5}{x-5}, & x \neq 5 \\ 0, & x = 5 \end{cases}, a = 5$