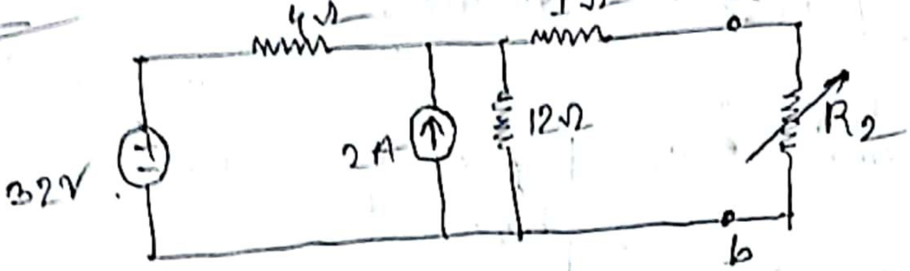
Date : 04-08-2025

Final

Thevenin Theorem

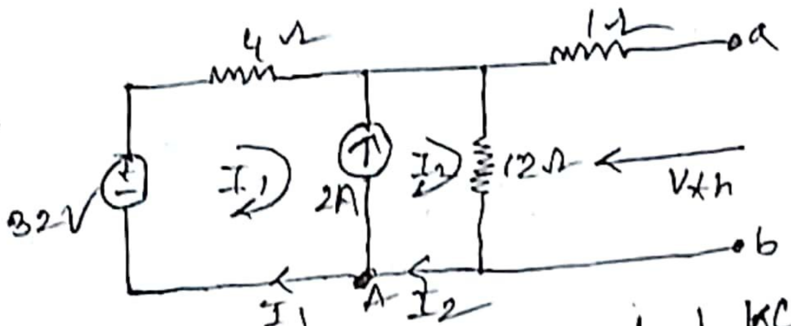
* $V_{th} \rightarrow$ Thevenin equivalant Voltage
* $R_{th} \rightarrow$ Thevenin " Resistance

1 Q1:



Draw the Thevenin equivalent circuit.

Solve



Applying KVL in supermesh,

$$-32 + 4I_1 + 12I_2 = 0$$

$$\Rightarrow 4I_1 + 12I_2 = 32$$

$$\Rightarrow I_1 + 3I_2 = 8 \quad \text{——} (1)$$

KCL in node-A,

$$I_2 = 2 + I_1 \quad \text{——} (11)$$

$\Rightarrow$ ~~$I_1 - I_2 =$~~

solving eq $\Rightarrow$

$$\therefore I_2 = 2.5A$$

$$\therefore V_{th} = I_2 \times 12$$

$$= 2.5 \times 12$$

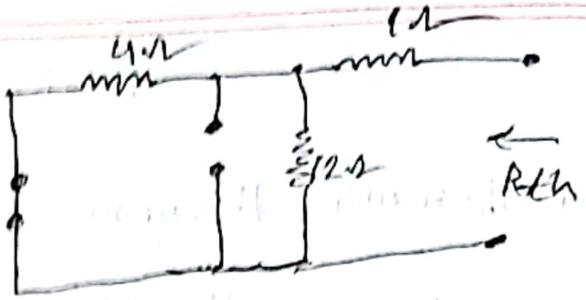
$$= 30V$$

$$R_{th} = (4 \| 12) + 1$$

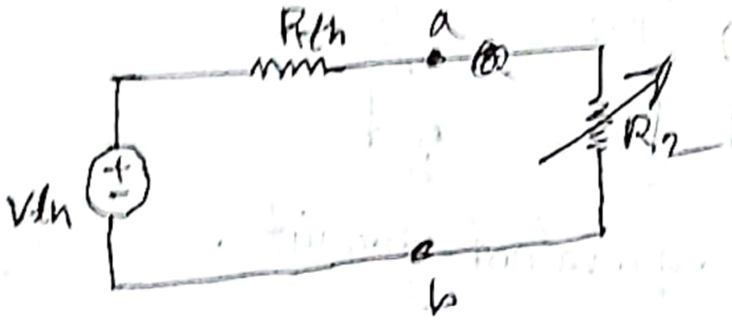
$$= \frac{4 \times 12}{4 + 12} + 1$$

$$= 3 + 1$$

$$= 4\,\Omega$$

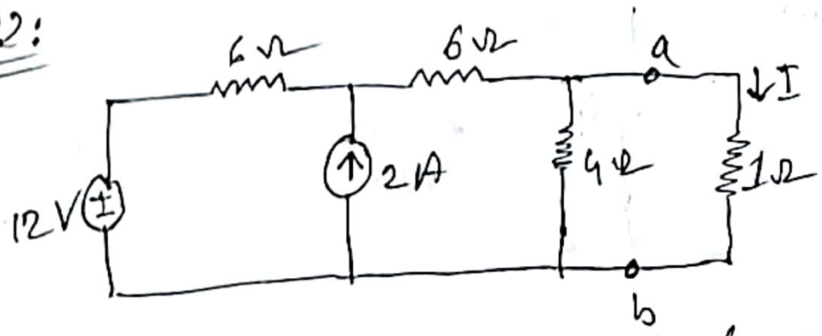


Thevenin equivalent circuit,



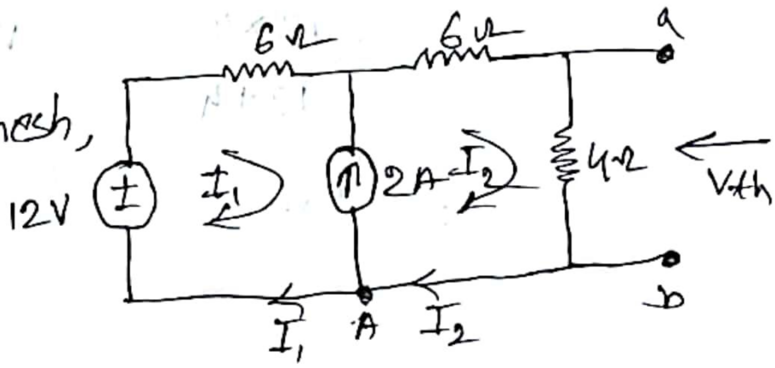
2

Q2:



Draw the thevenin equivalent circuit and Find I,

solve



Applying KVL in supermesh,

$-12+6I_1+6I_2+4I_2=0$

$\Rightarrow 6I_1+10I_2=12$

$\Rightarrow 3I_1+5I_2=6$ —— (i)

Applying KCL in node-A

$I_2=I_1-2$

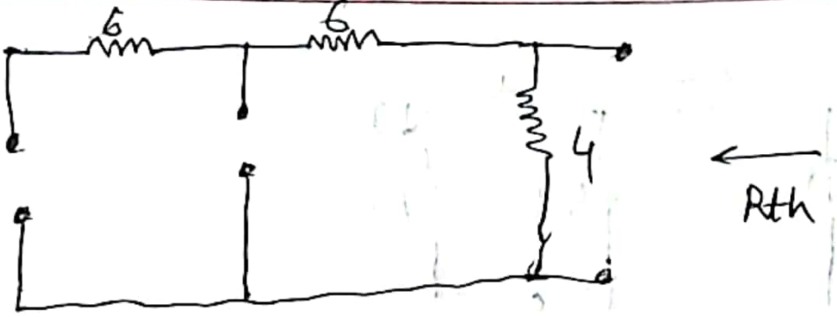
$\Rightarrow I_1-I_2=2$ —— (ii)

Solving eq $\Rightarrow$

$I_1=3A$

$I_2=0A$

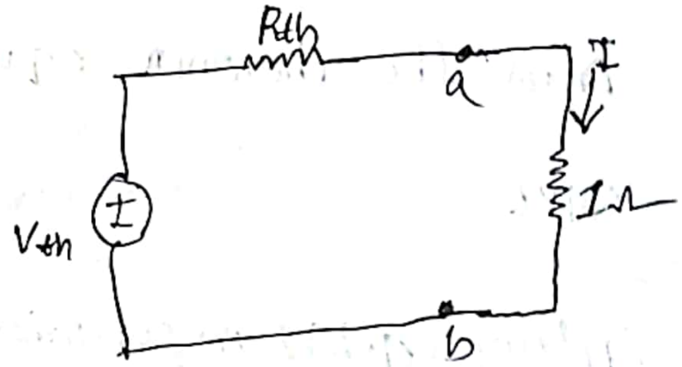
Vth =



$$R_{th} = \frac{(6+6) \parallel 4}{\cancel{6+6+4}}$$

$$= \frac{12 \times 4}{12+4}$$

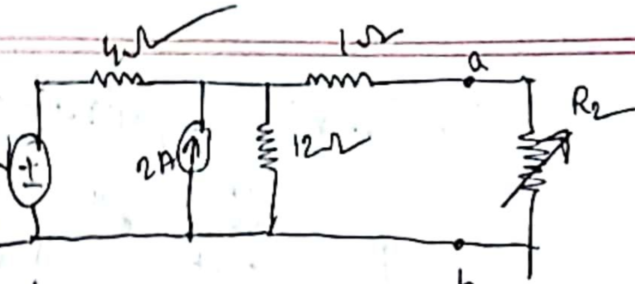
$$= 3\,\Omega$$

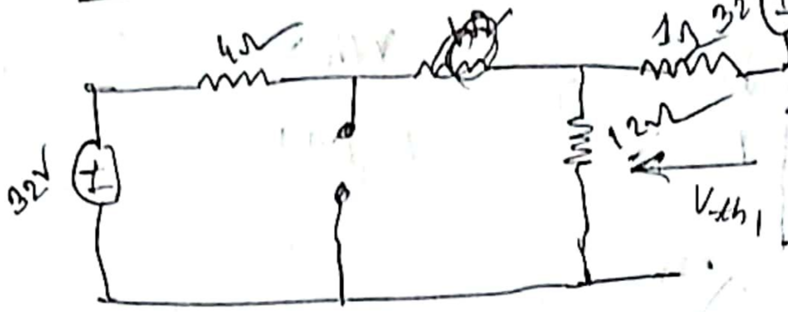


Applying CDR,

$$I = I_2 \times \frac{R_{th}}{R_{th}+1}$$

3 Superposition

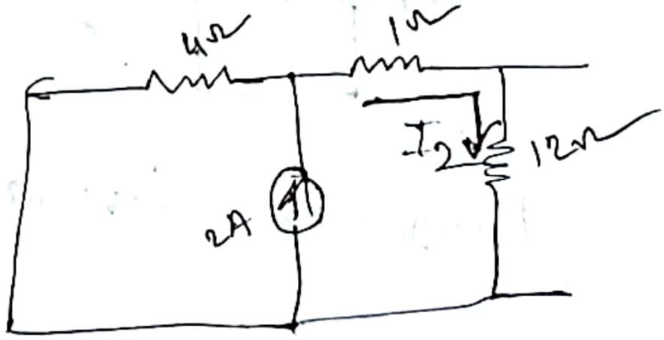




Using VDR,

$$V_{th_1} = \frac{12\times 4}{6+6+4} = 3V$$

Applying CDR,



$$I_2 = \frac{2\times 6}{6+(6+4)} = 3/4\ A$$

Using Ohm's Law,

$$V_{th_2} = I_2 \times 4$$

$$= \frac{3}{4}\times 4$$

$$= 3V$$

$$V_{th} = V_{th_1} + V_{th_2}$$

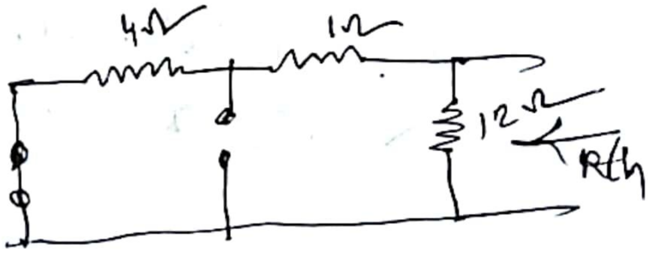
$$= 3+3$$

$$= 6V$$

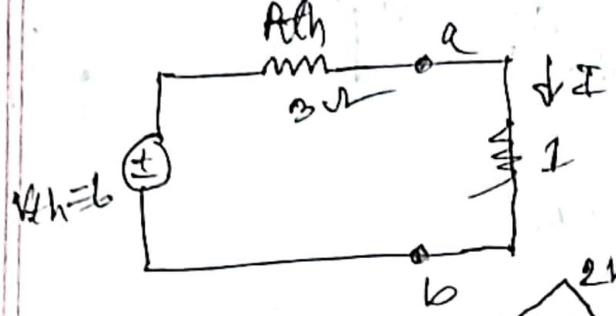
$$\therefore R_{th} = (6+6) \| 4$$

$$= \frac{12\times 4}{12+4}$$

$$= 3\Omega$$

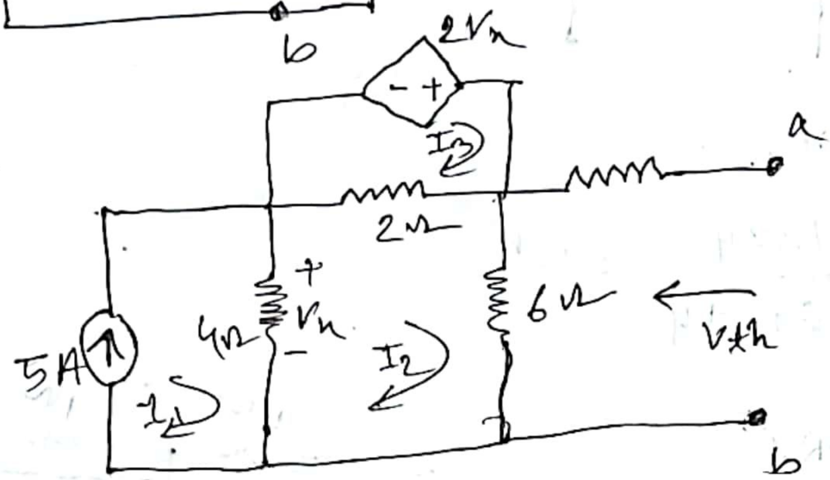


Thevenin equivalent circuit.



$$I = \frac{V_{th}}{R_{th}+1} = \frac{6}{3+1} = 1.5A$$

4 Q3 !.



Draw the thevenin equivalent circuit.

solve

From mesh-1,

$I_1 = 5\,A$

Applying KVL in mesh-2 →

$4(I_2 - I_1) + 2(I_2 - I_3) + 6I_2 = 0$

$\Rightarrow 4I_2 - 4I_1 + 2I_2 - 2I_3 + 6I_2 = 0$

$\Rightarrow -4I_1 + 12I_2 - 2I_3 = 0$

$\Rightarrow 2I_1 - 6I_2 + I_3 = 0$

$\Rightarrow 6I_2 - I_3 = 10$ —— (1)

$-4I_1 + 12I_2 - 2I_3 = 0$

$\Rightarrow -2I_1 + 6I_2 - I_3 = 0$

$\Rightarrow 6I_2 - I_3 = 10$ —— (1)

same

mesh-3

$$2(I_3 - I_2) - 2V_x = 0$$

$$\Rightarrow 2I_3 - 2I_2 - 2V_x = 0$$

$$\Rightarrow 2I_3 - 2I_2 - 2(4I_2 - 20) = 0$$

$$\Rightarrow 2I_3 - 2I_2 - 8I_2 + 40 = 0$$

$$\Rightarrow -10I_2 + 2I_3 = -40$$

$$\Rightarrow 10I_2 - 2I_3 = 40$$

$$\Rightarrow 5I_2 - I_3 = 20 \quad \text{—— (11)}$$

Using ohms law,
from mesh-2,

$$V_x = -4(I_2 - I_1)$$

$$= -4I_2 + 4I_1$$

$$= 4I_2 - 20$$

solving eq → $I_2 = -10\,A$

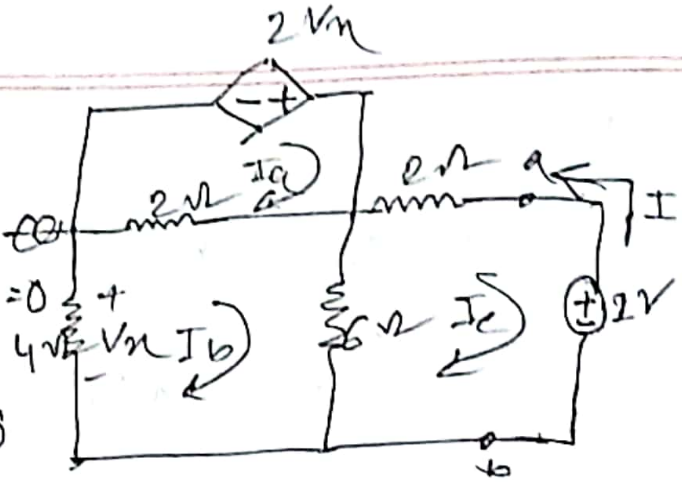
$$V_{th} = I_2 6$$

$$= -10 \times 6$$

$$= -60V$$

Mesh-2

$4I_b + 2(I_b - I_a) + 6(I_b - I_c) = 0$



$\Rightarrow 4I_b + 2I_b - 2I_a + 6I_b - 6I_c = 0$

$\Rightarrow -2I_a + 12I_b - 6I_c = 0$

$\Rightarrow I_a - 6I_b + 3I_c = 0$ —— (1)

Mesh-1

$V_x = -4I_b$

$2(I_a - I_b) - 2V_x = 0$

$\Rightarrow 2I_a - 2I_b - 2$

Mesh-3

$6(I_c - I_b) + 2I_c + 2 = 0$

$\Rightarrow 6I_c - 6I_b + 2I_c = -2$

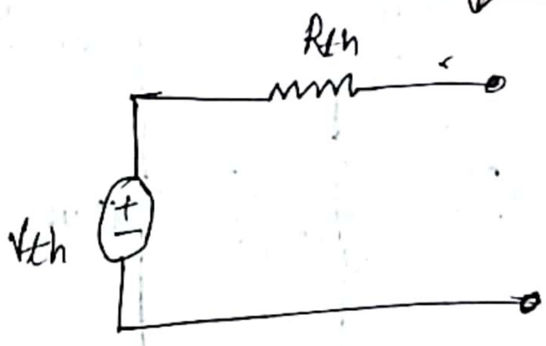
$\Rightarrow 6I_b - 8I_c = 2$

$\Rightarrow 3I_b - 4I_c = 1$ —— (III)

calculate, $I = -I_c$

Good

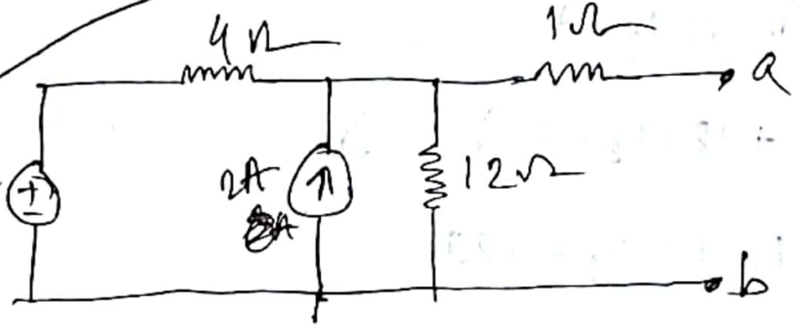
H.W



5

Norton Theorem

Q4:

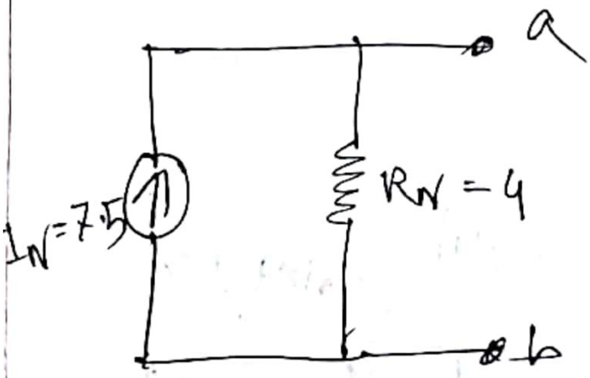


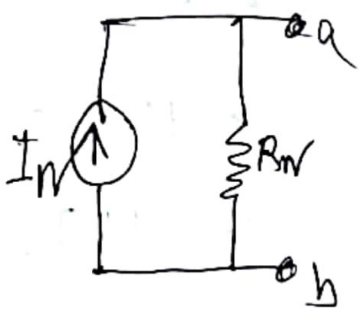
R_N = Norton eq Resistance

I_N = Norton eq current

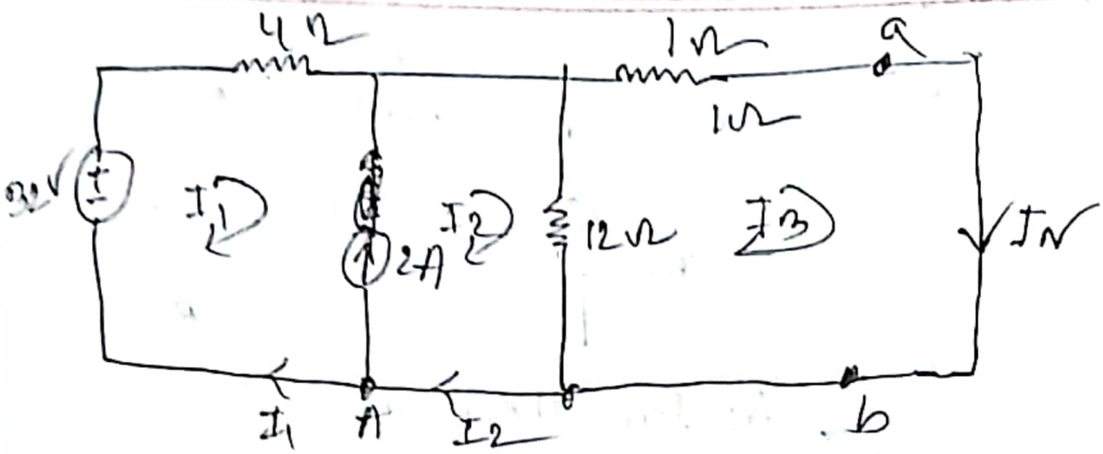
$$I_N = \frac{V_{th}}{R_{th}}$$

$$I_N = \frac{V_{th}}{R_{th}} = \frac{30}{4} = 7.5A$$





Type-2



From supermesh →

$-32 + 4I_1 + 12(I_2 - I_3) = 0$

$\Rightarrow -32 + 4I_1 + 12I_2 - 12I_3 = 0$

$\Rightarrow I_1 + 3I_2 - 3I_3 = 8$ —— (I)

from Mesh-3

$12(I_3 - I_2) + 1I_3 = 0$

$\Rightarrow 12I_3 - 12I_2 + I_3 = 0$

$\Rightarrow 12I_2 - 13I_3 = 0$ —— (II)

KCL in node-A

$I_2 = I_1 + 2$

$\Rightarrow I_2 - I_1 = 2$ —— (III)

solving eqn

$I_1 = 6.12$

$I_2 = 8.12$

$I_3 = 7.5 = I_N$

$I_N = I_3 = 7.5A$

Rth
Vth/IN

$$\text{Power} = I_L^2 R_L$$

$$= \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \times R_L$$

Max Power Transfer Theorem:

$$\boxed{R_{th} = R_L}$$

$$= \frac{V_{th}^2}{(R_{th}+R_{th})^2} \times R_{th}$$

$$= \frac{V_{th}^2}{(2R_{th})^2} \times R_{th}$$

$$\therefore P = \frac{V_{th}^2}{4R_{th}}$$

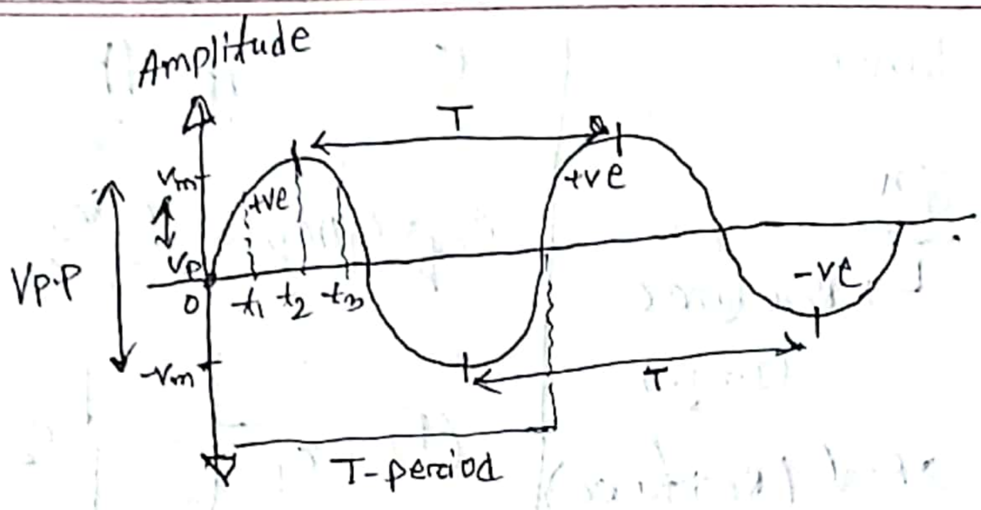
$$\therefore P_{max} = \frac{30^2}{4\times 4} \, W$$

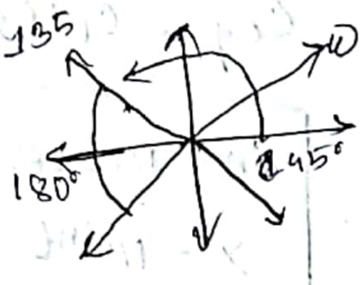
$$= 56.25 W$$

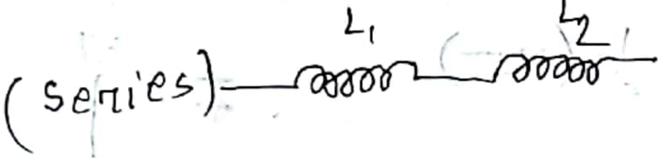
math 1

AC

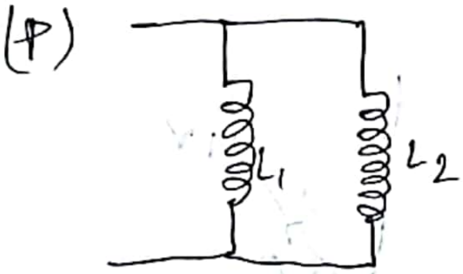
Date : 11-08-2025







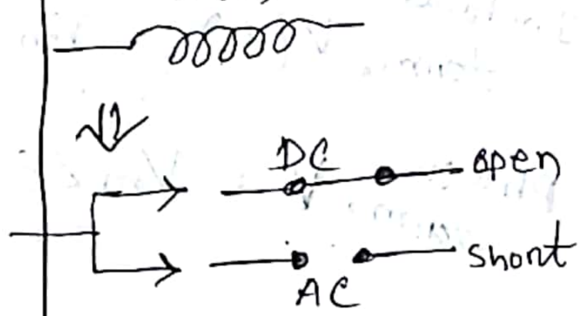
$$L_{eq} = L_1 + L_2$$

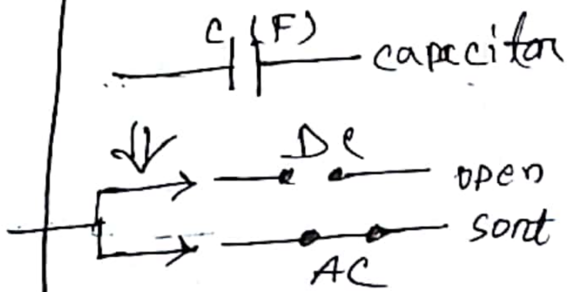


$$L_1 \| L_2 = \frac{L_1 L_2}{L_1 + L_2}$$

Inductor

L (H)



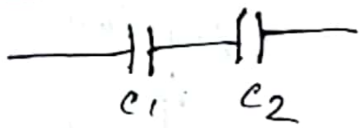


$\boxed{}$ $Z \Rightarrow$ Impedance

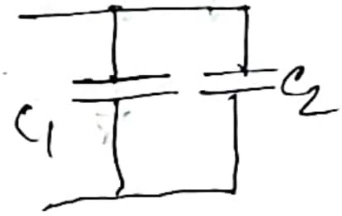
$Z = R + \underbrace{jx}$ Ω

$\hookrightarrow$ Reactance (Img.)

$\rightarrow$ Real (Resistance)

(S) 

$C_{eq} = C_1 \parallel C_2 = \dfrac{C_1 \times C_2}{C_1 + C_2}$

(P) 

$C_{eq} = C_1 + C_2$

Rec $\rightarrow x + jy = r(\cos\theta + j\sin\theta)$

Pol $\rightarrow r\angle\theta \rightarrow r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\dfrac{y}{x}$

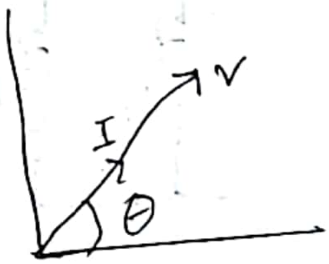
$x = r\cos\theta$

$y = r\sin\theta$

$\boxed{}$ Admittance, $y = \dfrac{1}{z}\,(\mho)$

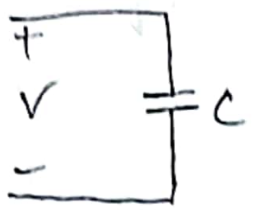
Instantaneous form, $V(t) = V_m \sin(\omega t + \theta)\,V$

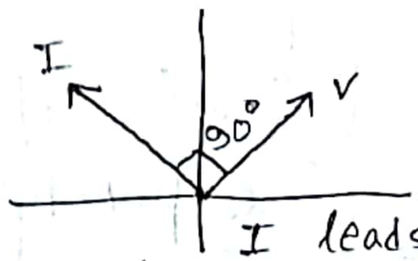
Phasor form, $V = V_m \angle\theta$ V



phase difence = 0

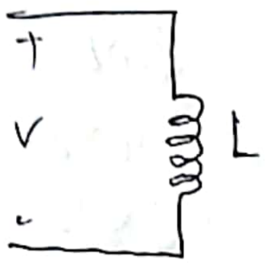
$I = V = \theta = 0$

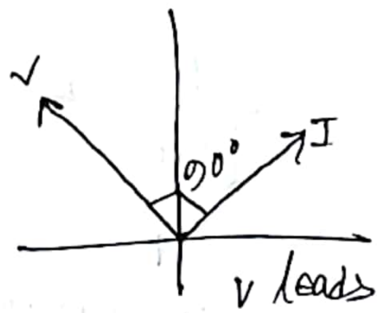




I leads V

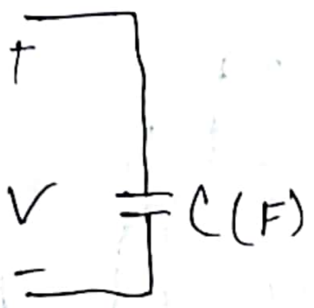
V lag I





V leads I

I lag V



F to Ω

$$F = \frac{1}{j\omega C}\ \Omega \quad \left[\frac{1}{j} = -j\right]$$

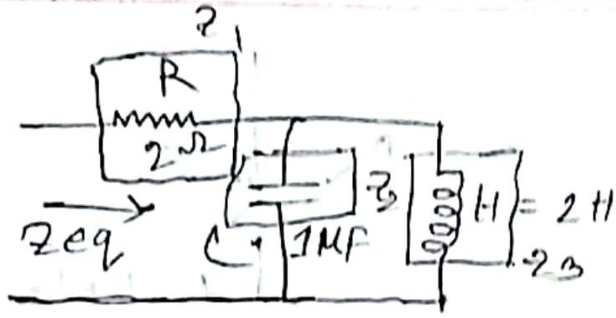
$$= -j\frac{1}{\omega C}$$

Hangry

H to Ω

$$H = j\omega L\ \Omega$$

1 Q1



* Find Z_{eq} when $\omega = 4$ rad/s ?

$Z = R + jx = 2+0$

$Z_1 = 2\Omega$

$\frac{1}{j} = -j$

$Z_2 = \frac{1}{j\omega C}\ \Omega$

$= \frac{1}{j \times 4 \times 1 \times 10^{-6}}\ \Omega$

$= -j(2.5 \times 10^5)\ \Omega$

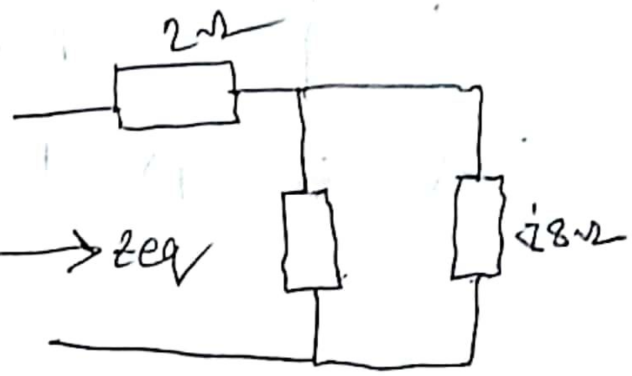
$Z_3 = j\omega L$

$= j4 \times 2$

$= j8\Omega$

$Z_{eq} = Z_1 + Z_2 \parallel Z_3$

$= 2 + \frac{-j(2.5\times10^5) \times j8}{-j(2.5\times10^5) + j8}$



rect → polar

$= (2 + j8)\ \Omega$

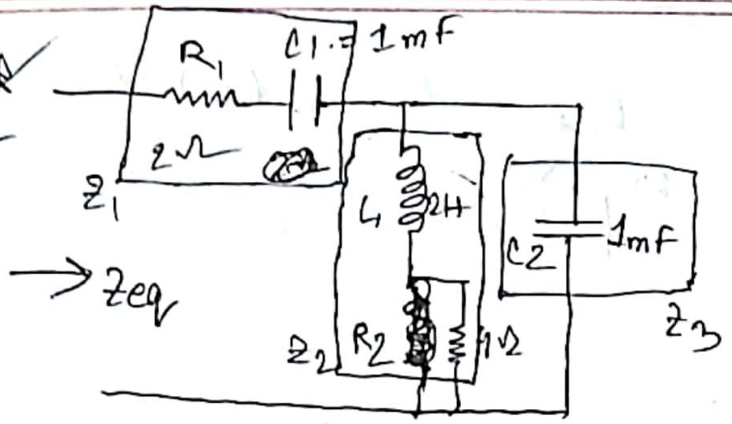
$= 2\sqrt{17} \angle 75.96$

[Using calculator

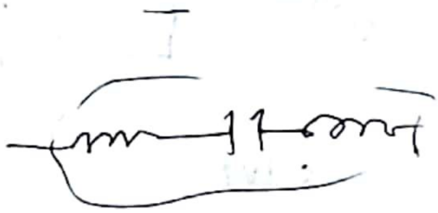
* mode → complex
* $\frac{-(2.5\times10^5)i \times 8i}{-(2.5\times10^5)i + 8i}$
* i = ENG (cal)

Q_2

2



$[\omega = 5 \text{ rad/s}]$



$$Z_1 = 2 + \frac{1}{j \times 5 \times 1 \times 10^{-3}}$$

$$= (2 - j(200))\,\Omega$$

$$Z_3 = \frac{1}{j\omega C}$$

$$= \frac{1}{j5 \times 1 \times 10^{-3}}$$

$$= (-j200\,\Omega)\,\Omega$$

$$Z_2 = j\omega L + 4$$

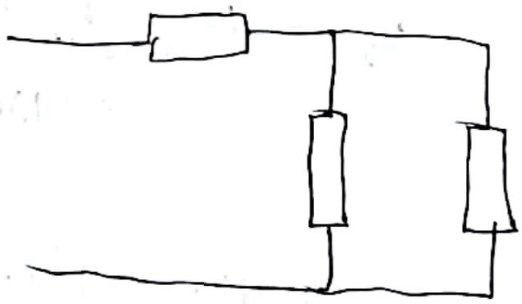
$$= j \times 5 \times 2 + 4$$

$$= (4 + j10)\,\Omega$$

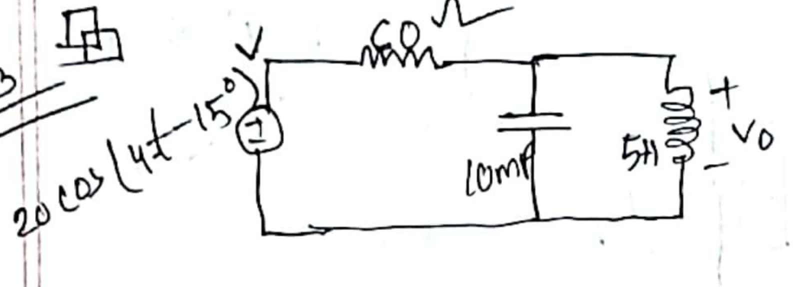
$$Z_{eq} = Z_1 + (Z_2 \| Z_3)$$

$$= (2 - j200) + \frac{(4+j10)(-j200)}{(4+j10) + (-j200)}$$

$$= 6.4 - j189.6\,\Omega$$



Q_3 3



* $V_0(t)$?

solve

$$20\cos(4t-15^\circ) \Rightarrow 20\angle -15^\circ \text{ V}$$

$$\omega = 4 \text{ rad/s}$$

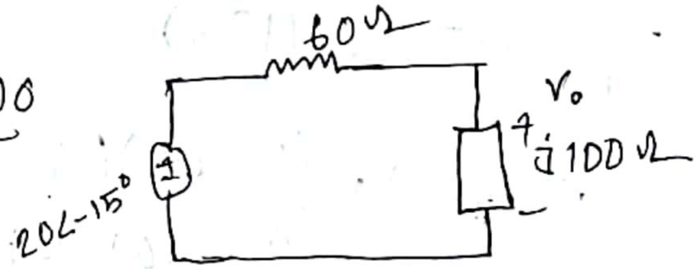
$$10\text{mF} \Rightarrow \frac{1}{j\omega C} = \frac{1}{j4\times 10\times 10^{-3}} = -j25\ \Omega$$

$$5\text{H} \Rightarrow j\omega L = j4\times 5 = j20\ \Omega$$

$$-j25 \parallel j20 = \frac{-j25\times j20}{-j25+j20} = j100\ \Omega$$

Using VDR,

$$V_0 = \frac{20\angle -15^\circ \times j100}{60+j100}$$

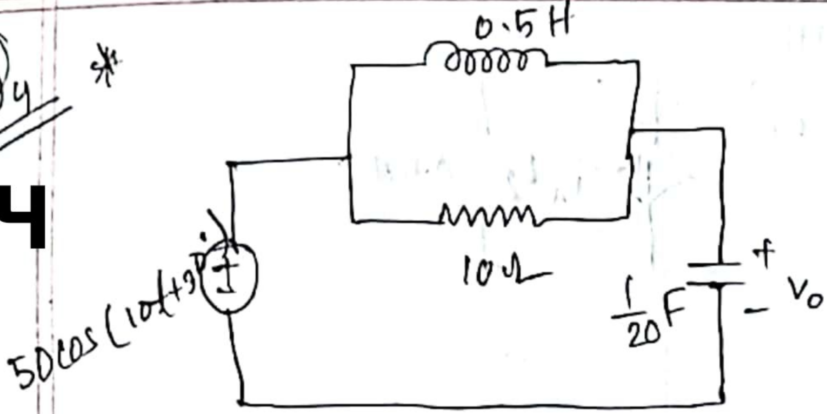


$$= 16.49 + j4.70$$

$$= 17.15\angle 15.96^\circ \text{ V}$$

$$\therefore V_0(t) = 17.15\cos(4t+15.96^\circ) \text{ Ans.}$$

Q4 * 4



* $V_o(t)$?

solve

$$50\cos(10t+30^\circ) \Rightarrow 50\angle 30^\circ \text{ V}$$

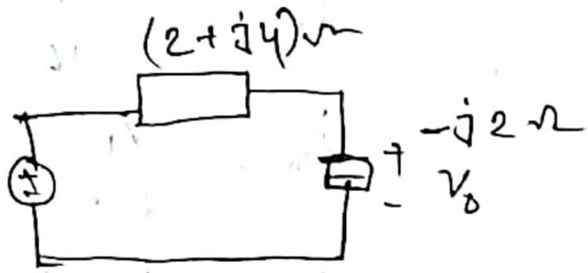
$$\omega = 10 \text{ rad/s}$$

$$0.5H \Rightarrow j\omega L = j10\times 0.5 = j5$$

$$\frac{1}{20}F \Rightarrow \frac{1}{j\omega C} = \frac{1}{j10\times 0.05} = -j2$$

$$j5 \parallel 10 = \frac{j5\times 10}{j5+10} = 2+j4$$

Using VDR,



$$V_o = \frac{50\angle 30 \times(-j2)}{-j2+(2+j4)}$$

$$= -9.15 - j34.15$$

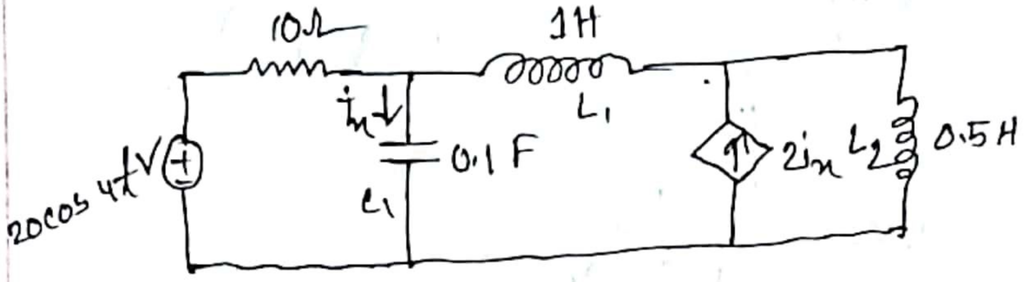
$$= 35.35\angle -104.9^\circ$$

$$V_o(t) = 35.35\cos(10t - 104.9^\circ) \text{ Ans}$$

Nodal Analysis

Date : 13-08-25

5



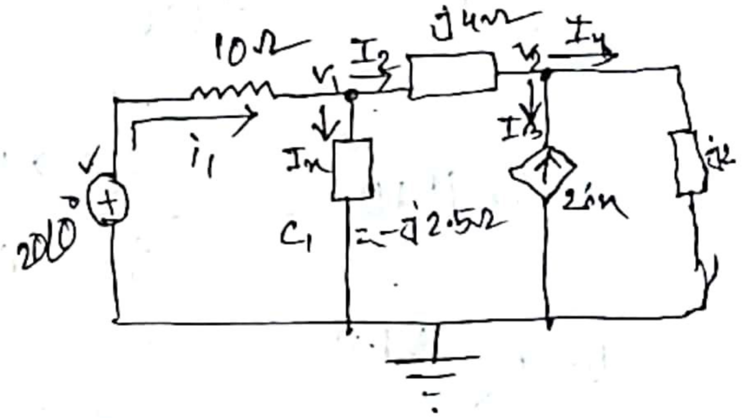
solve

$20\cos 4t\ V \Rightarrow 20\angle 0°\ V$, $\omega = 4\ rad/s$

$0.1F \Rightarrow \frac{1}{j\omega C_1} = \frac{1}{j4\times 0.1} = -j2.5\ \Omega$

$1H \Rightarrow j\omega L_1 = j4\times 1 = j4\ \Omega$

$0.5H \Rightarrow j\omega L_2 = j4\times 0.5 = j2\ \Omega$



Using ohm's Law,

$i_1 = \frac{20\angle 0° - V_1}{10}$

$i_x = \frac{V_1}{-j2.5}$

$i_2 = \frac{V_1 - V_2}{j4}$

$i_3 = 2i_x = \frac{2V_1}{-j2.5}$

$i_4 = \frac{V_2}{j2}$

Applying KCL in node $-V_1 \Rightarrow$

$$i_1 = i_x + I_2$$

$$\Rightarrow \frac{20\angle 0^\circ - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4}$$

$$\Rightarrow \frac{20\angle 0^\circ - V_1}{10} = \frac{j4V_1 - V_1 j2.5 + V_2 j2.5}{(-j2.5) \times j4}$$

$$\Rightarrow \frac{20\angle 0^\circ - V_1}{10} = \frac{j4V_1 - jV_1 2.5 + jV_2 2.5}{10}$$

$$\Rightarrow 20\angle 0^\circ - V_1 = j4V_1 - j2.5V_1 + j2.5V_2$$

$$\Rightarrow 20\angle 0^\circ - V_1 = j1.5V_1 + j2.5V_2$$

$$\Rightarrow V_1 + j1.5V_1 + j2.5V_2 = 20\angle 0^\circ$$

$$\Rightarrow V_1(1 + j1.5) + j2.5V_2 = 20\angle 0^\circ \quad \text{--- (1)}$$

Applying KCL in node $-V_2 \Rightarrow$

$$i_2 + i_3 = i_4$$

$$\Rightarrow \frac{V_1 - V_2}{j4} + 2i_x = \frac{V_2}{j2}$$

$$\Rightarrow \frac{V_1 - V_2}{j4} + 2 \times \frac{V_1}{-j2.5} = \frac{V_2}{j2}$$

$$\Rightarrow \frac{-j2.5V_1 + j2.5V_2 + (j4)(2V_1)}{j4 \times (-j2.5)} = \frac{V_2}{j2}$$

$$= \frac{(2V)(j1.5) + j2.5V_2}{10} = \frac{V_2}{j2}$$

$$\Rightarrow -11V_1 - 5V_2 = 10V_2 \Rightarrow$$

$$\Rightarrow -11V_1 - 5V_2 - 10V_2 = 0$$

$$\Rightarrow -11V_1 - 15V_2 = 0$$

$$\Rightarrow 11V_1 + 15V_2 = 0 \quad \text{---} (11)$$

$$\Rightarrow 5V_1 - 5V_2 - 16V_1 - 10V_2 = 0$$

$$\Rightarrow 5V_1 - 5V_2 - 16V_1 - 10V_2 = 0$$

$$\Rightarrow$$

Using cramer's rule,

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = (15+j22.5)-(j27.5)$$

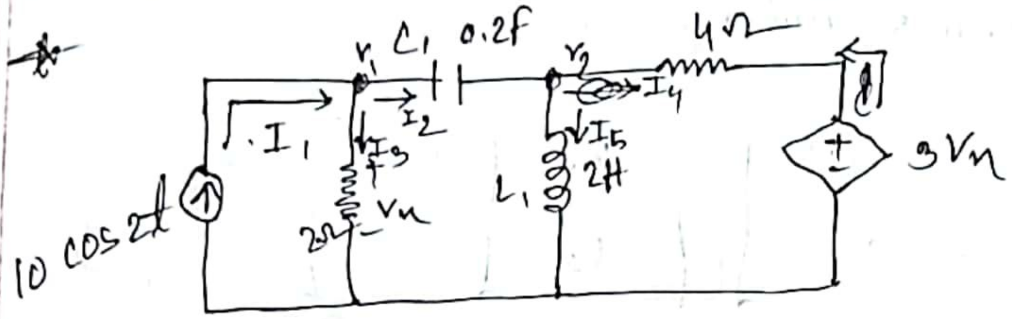
$$= 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20\angle 0^\circ & j2.5 \\ 0 & 15 \end{vmatrix} = 20\angle 0^\circ \times 15 = 300\angle 0^\circ$$

$$V_1 = \frac{\Delta_1}{\Delta} = \cancel{\frac{15-j5}{300\angle 0^\circ}} = \frac{300\angle 0^\circ}{15-j5} = 18+j6$$

$$i_n = \frac{V_1}{-j2.5} = \frac{18+j6}{-j2.5} = 2.4 - \cancel{7.2}\, j7.2$$

6



solve

$$10\cos 2t \Rightarrow 10\angle 0^\circ \quad , \omega = 2\ rad/s$$

$$0.2F \Rightarrow \frac{1}{j\omega C_1} = \frac{1}{j2\times 0.2}$$

$$= -j2.5\,\Omega$$

$$I_1 = 10\angle 0^\circ$$

$$I_2 = \frac{V_1 - V_2}{-j2.5}$$

$$2H \Rightarrow j\omega L_1 = j2\times 2$$

$$= j4\,\Omega$$

$$I_3 = \frac{V_1}{2}$$

$$I_4 = \frac{3V_x - V_2}{4}$$

$$= \frac{3\times 2I_3 - V_2}{4}$$

$$= \frac{6I_3 - V_2}{4}$$

$$= \frac{6\cdot\frac{V_1}{2} - V_2}{4}$$

$$= \frac{3V_1 - V_2}{4}$$

$$I_5 = \frac{V_2}{j4}$$

$$V_x = 2I_3$$

Applying KCL in node - V_1,

$I_1 = I_2 + I_3$

$\Rightarrow 10\angle 0^\circ = \frac{V_1 - V_2}{-j2.5} + \frac{V_1}{2}$

$\Rightarrow 10\angle 0^\circ = \frac{2V_1 - 2V_2 - j2.5V_1}{-j2.5 \times 2}$

$\Rightarrow -j50 = 2V_1 - 2V_2 - j2.5V_1$

$\Rightarrow V_1(2 - j2.5) - 2V_2 + j50 = 0$

Applying KCL in node V_2,

$V_2 = 33.12\angle 57.12^\circ V$

$V_1 = 11.325\angle 60.01^\circ V$

$I_4 + I_2 = I_5$

$\Rightarrow \frac{3V_1 - V_2}{4} + \frac{V_1 - V_2}{-j2.5} = \frac{V_2}{j4}$

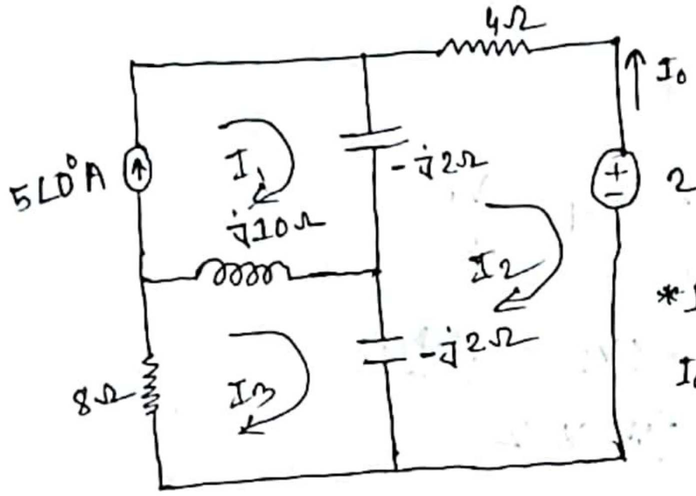
$\Rightarrow \frac{-j7.5V_1 + j2.5V_2 + 4V_1 - 4V_2}{4 \times (-j2.5)} = \frac{V_2}{j4}$

$\Rightarrow \frac{V_1(4 - j7.5) + V_2(j2.5 - 4)}{-j10} = \frac{V_2}{j4}$

$\Rightarrow V_1(30 + j16) + V_2(-10 - j16) = -j10V_2$

Date: 18-08-2025

Mesh Analysis



* Determine the current I_0 in the following circuit.

Solve:

From mesh-1,

$$I_1 = 5\angle 0^\circ \text{ A}$$

From mesh-3,

$$8I_3 + j10(I_3 - I_1) + (-j2)(I_3 - I_2) = 0$$

$$\Rightarrow 8I_3 + j10I_3 - j10I_1 - j2I_3 + j2I_2 = 0$$

$$\cancel{\Rightarrow 8I_3 + j10I_3} -$$

$$\Rightarrow -j10I_1 + j2I_2 + (8+j8)I_3 = 0$$

$$\Rightarrow -j10 \times 5\angle 0^\circ + j2I_2 + (8+j8)I_3 = 0$$

$$\Rightarrow -j50 + j2I_2 + (8+j8)I_3 = 0$$

$$\Rightarrow j2I_2 + (8+j8)I_3 = j50 \quad \text{——} \quad (1)$$

8

From mesh-2,

$$(-j2)(I_2-I_3)+(-j2)(I_2-I_1)+4I_2+20\angle 90^\circ=0$$

$$\Rightarrow -j2I_2+j2I_3-j2I_2+j2I_1+4I_2+20\angle 90^\circ=0$$

$$\Rightarrow j2I_1+(4-j4)I_2+j2I_3=-20\angle 90^\circ \text{ --- (11)}$$

$$\Rightarrow j2\times 5\angle 0^\circ+(4-j4)I_2+j2I_3=-20\angle 90^\circ$$

$$\Rightarrow \cancel{j10+}(4-j4)I_2+j2I_3=\overset{-j30}{\cancel{-20\angle 90^\circ}} \text{ --- (11)}$$

Using crammer's role,

$$\begin{bmatrix} j2 & 8+j8 \\ 4-j4 & j2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta=\begin{bmatrix} j2 & 8+j8 \\ 4-j4 & j2 \end{bmatrix}=-4-64=-68$$

$$\Delta_1=\begin{bmatrix} j50 & 8+j8 \\ -j30 & j2 \end{bmatrix}$$

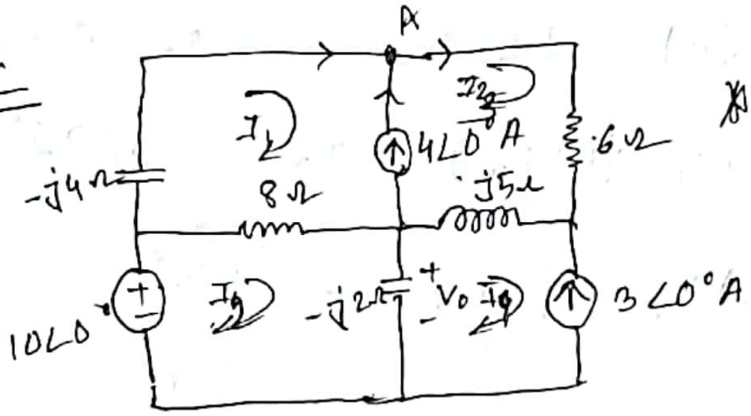
$$\Delta_2=\begin{bmatrix} j2 & j50 \\ 4-j4 & -j30 \end{bmatrix}=60-(200+200j)$$

$$=340-j240$$

$I_2 = \frac{\Delta_2}{\Delta} = \frac{340 - j240}{68} = 6.12\angle -35.21^\circ A$

$\therefore I_0 = I_2 = -6.12\angle -35.21^\circ A$

9 Q:



Find V_0 ?

From mesh-1,

$-j4 - 4\angle 0^\circ + 8(I_1 - I_2) = 0$

$\Rightarrow -j4 - 4\angle 0^\circ + 8I_1 - 8I_2 = 0$

$\Rightarrow$

From mesh-4,

$I_4 = -3\angle 0^\circ A$

~~From~~ KVL in supermesh,

$-j4I_1 + 6I_3 + j5(I_2 - I_4) + 8(I_1 - I_2) = 0$

$\Rightarrow -j4I_1 + 6I_3 + j5I_2 - j5I_4 + 8I_1 - 8I_2 = 0$

$\Rightarrow (8-j4)I_1 + (j5-8)I_2 + 6I_3 - j5I_4 = 0$

$$\Rightarrow 8I_1 + (j5-8)I_2 + 6I_3 - j0(-3\angle 0^\circ) = 0$$

KVL in supermesh.

$$-j4I_1 + 6I_3 + j5(I_2 - I_4) + 8(I_1 - I_2) = 0$$

$$\Rightarrow (8-j4)I_1 - 8I_2 + (6+j5)I_3 = -j15 \quad \text{— (I)}$$

<u>KVL in mesh - 2</u>

$$-10\angle 0^\circ + 8(I_2 - I_1) - j2(I_2 - I_4) = 0$$

$$\Rightarrow 8I_2 - 8I_1 - j2I_2 + j2I_4 = 10\angle 0^\circ$$

$$\Rightarrow -8I_1 + (8-j2)I_2 + j2I_4 = 10\angle 0^\circ$$

$$\Rightarrow -8I_1 + (8-j2)I_2 = 10\angle 0^\circ - j2(-3\angle 0^\circ)$$

$$= 11.66\angle 30.96^\circ \quad \text{— (II)}$$

Applying KCL in node A,

$$I_3 = I_1 + 4\angle 0^\circ$$

$$\Rightarrow I_3 - I_1 = 4\angle 0^\circ$$

Using cramer's rule,

$$\begin{bmatrix} 8-j4 & -8 & 6+j5 \\ -8 & 8-j2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -j15 \\ 11.66\angle 30.96^\circ \\ 4\angle 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 8-j4 & -8 & 6+j5 \\ -8 & 8-j2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= (8-j4)(8-j2)+8(-8)+(6+j5)(8-j2)$$

$$= 50-j20$$

$$\Delta_2 = \begin{bmatrix} 8-j4 & -j15 & 6+j5 \\ -8 & 11.66\angle 30.96 & 0 \\ -1 & 4\angle 0 & 1 \end{bmatrix}$$

$$= (8-j4)(11.66\angle 30.96)+(-8)(j15)$$
$$+(6+j5)(-32+11.66\angle 30.96)$$

$$= 58.02-j186.02$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-58.02 - j186.02}{50 - j20}$$

$$= 0.28 - j3.6$$

$\Delta = 50 - 20j$

$\Delta_1 = -182 - 184j$

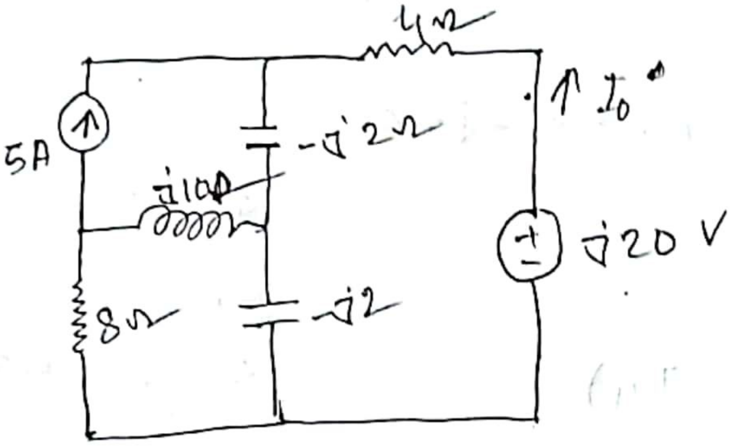
$\Delta_2 = -58 - 186j$

$$V_o = -j2 (I_2 - I_4)$$

$$= -j2 \{(0.28 - j3.6) + 3 \}$$

10

Superposition Theorem



fig

* Find I_o

Solve

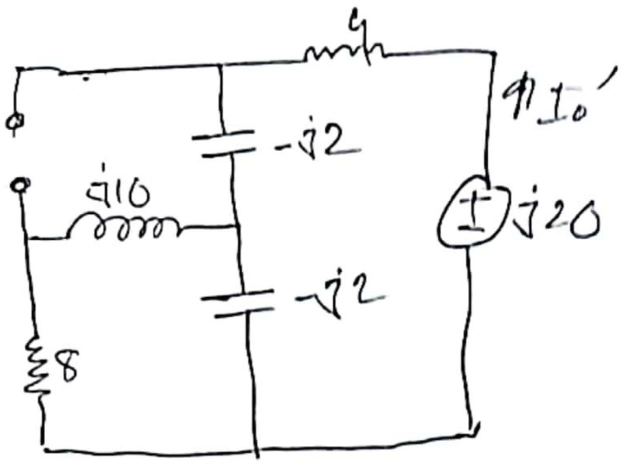


fig (1)

$$(8+j10) \parallel (-j2)$$

$$= \frac{(8+j10)(-j2)}{8+j10-j2}$$

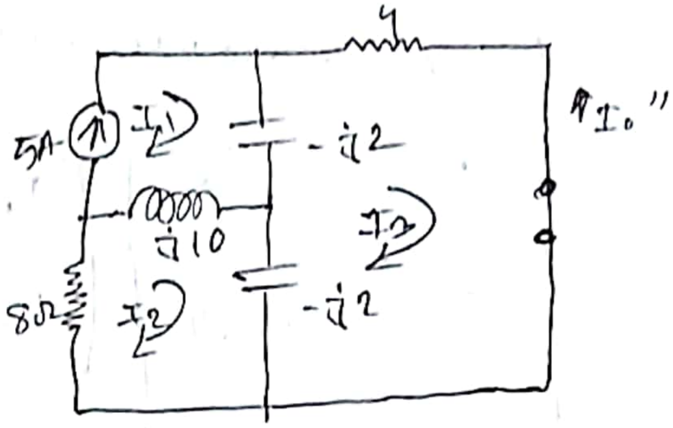
$$= (0.25 - j2.25)\ \Omega$$

$$I_o' = \frac{j20}{(0.25-j2.25)+(-j2)+4}$$

$$= 3.32 \angle 135^\circ$$

From mesh - 1,

$I_1 = 5A$



mesh - 2

$$8I_2 + j10(I_2 - I_1) - j2(I_2 - I_3) = 0$$

$$\Rightarrow 8I_2 + j10I_2 - j10I_1 - j2I_2 + j2I_3 = 0$$

$$\Rightarrow -j10I_1 + (8+j8)I_2 + j2I_3 = 0$$

$$\Rightarrow -j50 + (8+j8)I_2 + j2I_3 = 0 \quad \text{—— (I)}$$

mesh - 3

$$-j2(I_3 - I_2) - j2(I_3 - I_1) + 4I_3 = 0$$

$$\Rightarrow -j2I_3 + j2I_2 - j2I_3 + j2I_1 + 4I_3 = 0$$

$$\Rightarrow j2I_1 + j2I_2 + (4 - j4)I_3 = 0 \quad \text{—— (II)}$$

$$\Rightarrow j10 + j2I_2 + (4 - j4)I_3 = 0$$

Using cramer's rule,

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} j50 \\ -j10 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} = 68$$

$$\Delta_3 = \begin{bmatrix} 8+j8 & j50 \\ j2 & -j10 \end{bmatrix} = 180 - j80$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{180 - j80}{68} = 2.6 - j1.18$$

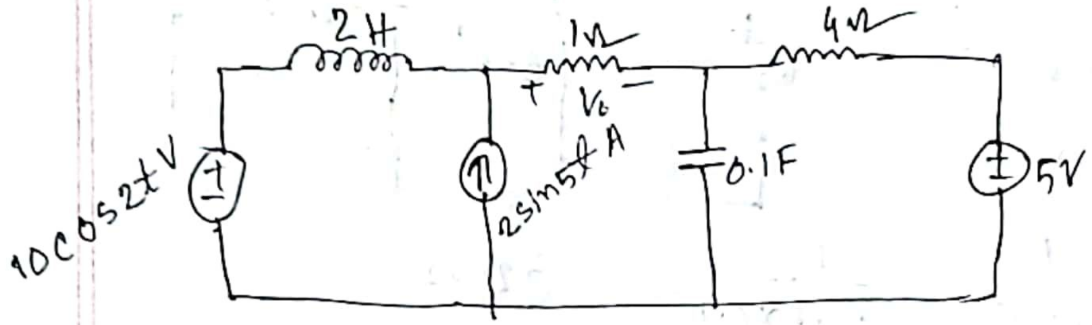
$$I_o'' = -I_3 = -2.6 - j1.18 = 2.9 \angle -23.96^\circ A$$

$$I_o = I_o' + I_o''$$

$$= (-5 + j3.5) A$$

* Inductor – sort
* capacitor – open

11 * Find V_0 ?



solve

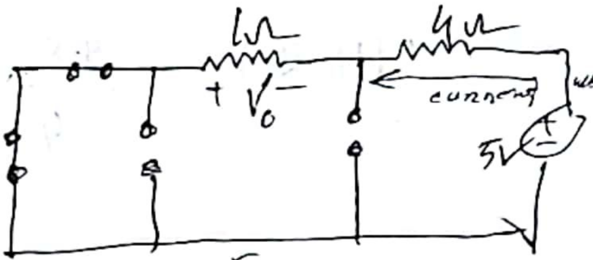


Fig-1

Using VDR,

$$V_0' = \frac{5\times 1}{1+4} = 1\,V$$

$10\cos 2t = 10\angle 0^\circ\ V$

$\omega = 2\ rad/s$

$2H = j\omega L = j4\ \Omega$

$$0.1F = \frac{1}{j\omega c} = \frac{1}{j2\times 0.1} = -j5$$

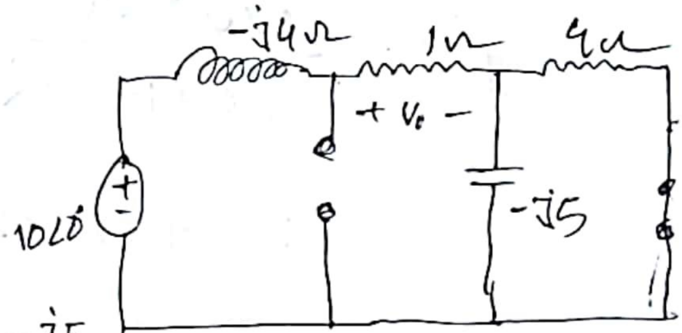


Fig-2

$$4 \parallel (-j5) = \frac{4\times(-j5)}{4-j5} = (2.43 - j1.95)\ \Omega$$

Using VDR,

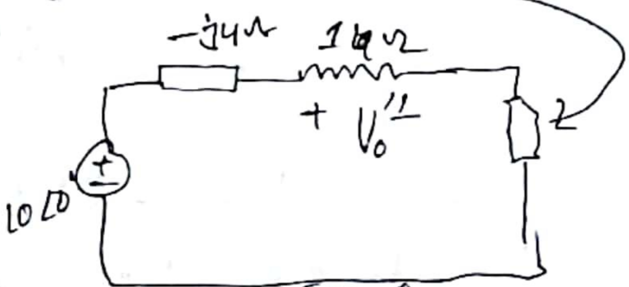


Fig-2

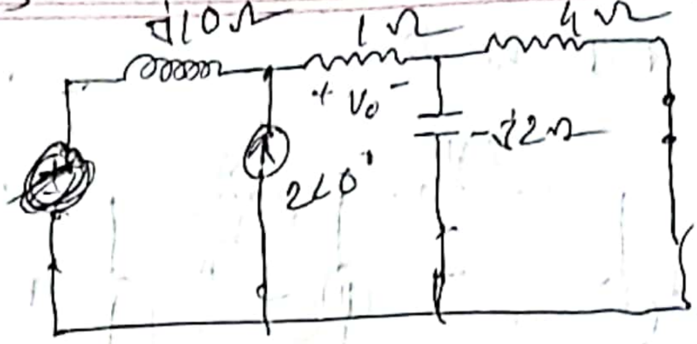
$$= \frac{10\angle 0^\circ \times 1}{-j4 + 1 + 2.43 - j1.95}$$

$= 2.14 - j1.27 = 2.49\angle -30.78^\circ\ V$

$v_0''(t) = 2.49\cos(2t - 30.78)\ V$

$2\sin 5t$ / $2\angle 0^\circ$ V

$\omega = 5$ rad/s



$2H = j\omega L$

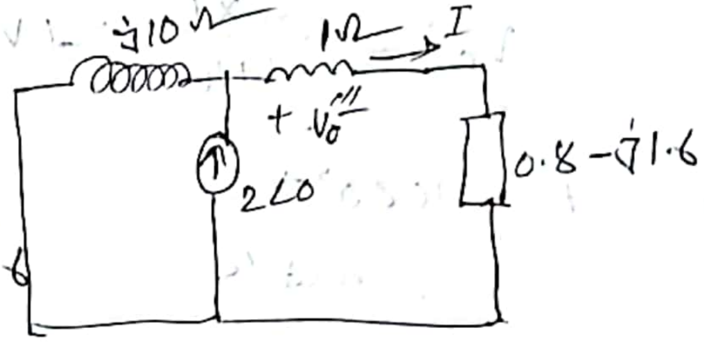
$= j \times 5 \times 2$

$= j10\,\Omega$

$$0.1F = \frac{1}{j\omega C} = \frac{1}{j \times 5 \times 0.1} = -j2\,\Omega$$

$$4 \| (-j2) = \frac{4 \times (-j2)}{4 - j2} = (0.8 - j1.6)\,\Omega$$

Using CDR,



$$I = \frac{2\angle 0 \times j10}{j10 + 1 + 0.8 - j1.6}$$

$= 23.48 \angle 12.09^\circ$ A

Using ohm's law,

$V_o''' = I \times 1$

$= 23.48 \angle 12.09^\circ$ V

$V_o'''(t) = 23.48 \sin(5t + 12.09^\circ)$ V

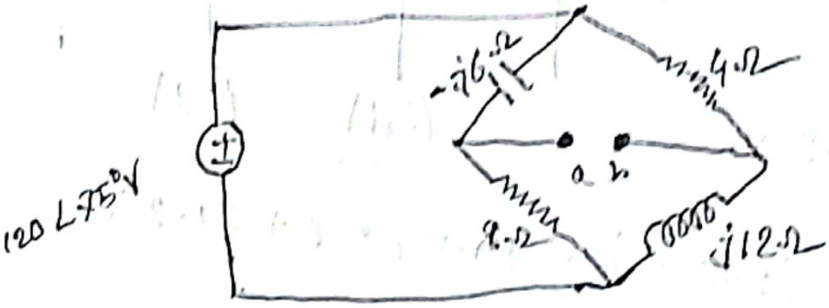
$V_o = V_o' + V_o'' + V_o'''$

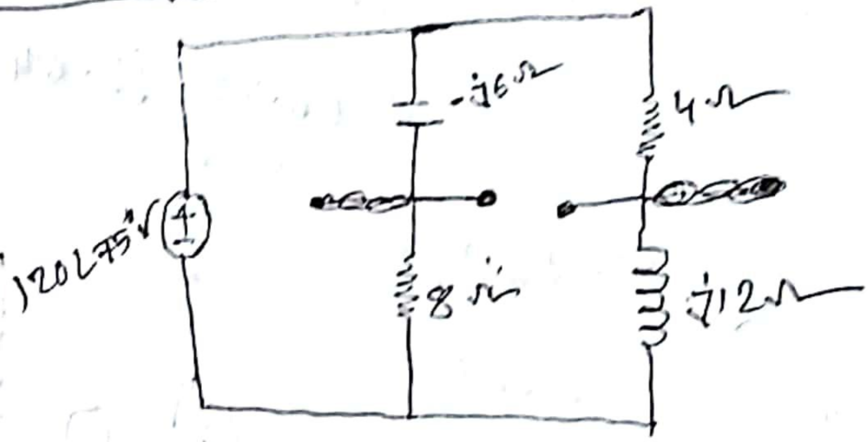
$= -1 + 2.49\cos(2t - 30.78^\circ) + 23.48\sin(5t + 12.09)$

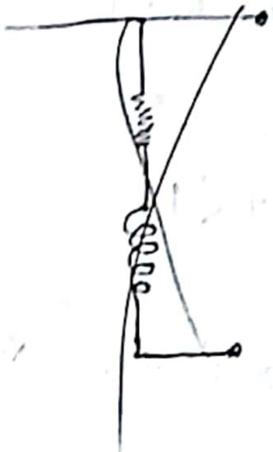
Date: 20-08-25

Thevenin's Theorem

12 #1 Obtain the Thevenin equivalent at terminal a-b.

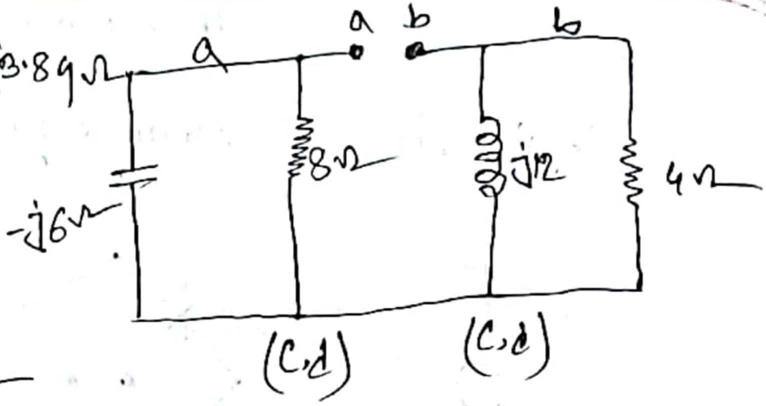






Z_{th}

$Z_1 = (-j6 \| 8) = \frac{-j6 \times 8}{-j6+8} = 2.88 - j3.84\ \Omega$



$Z_2 = (j12 \| 4) = \frac{j12 \times 4}{j12+4}$

$= 3.6 + j1.2\ \Omega$

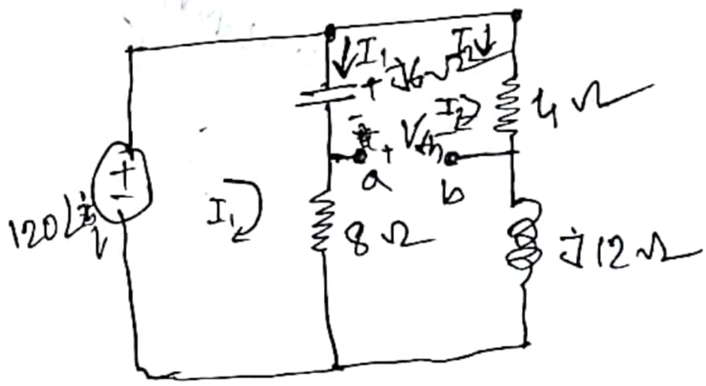
$Z_{th} = Z_1 + Z_2 = (2.88 - j3.84) + (3.6 + j1.2)$

$= (6.48 - j2.64)\ \Omega$ **Ans.**

Using Ohm's Law,

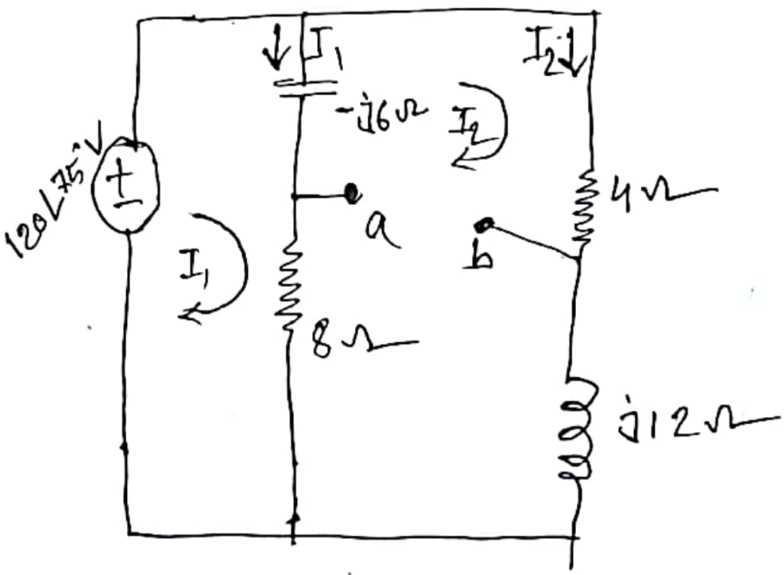
$I_1 = \frac{120\angle 75^\circ}{8 - j6}$

$= -4.47 + j11.13$



$I_2 = \frac{120\angle 75^\circ}{4 + j12}$

$= 9.47 + j0.56$



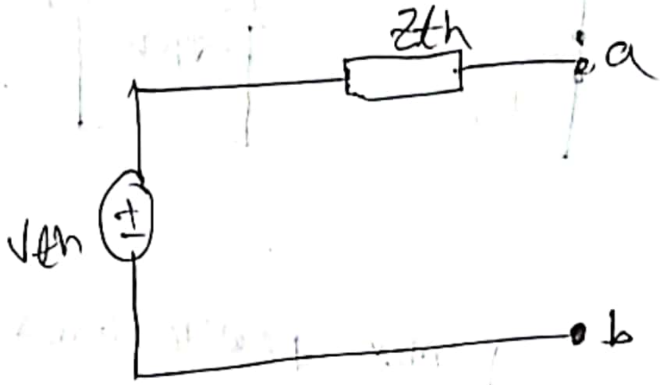
Applying KVL, in loop-2,

$$-(-j6)I_1 + I_2 4 = V_{th}$$

$$\Rightarrow V_{th} = j6 I_1 + I_2 4$$

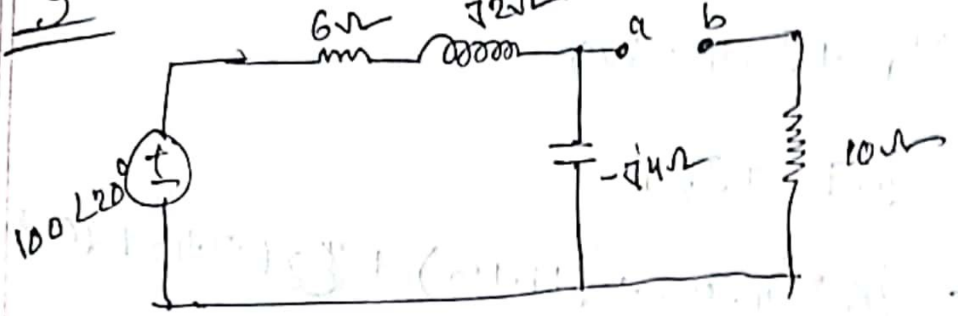
$$= j6(-4.47 + j11.13) + (0.47 + j0.56) \times 4$$

$$\therefore V_{th} =$$

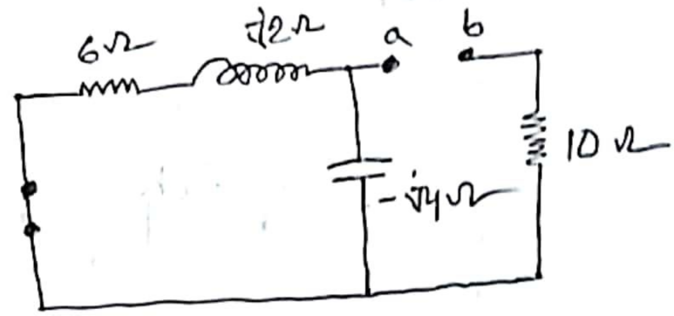


13

Q:



Solve



$((6+j2) \parallel -j4\,\Omega)$

$$z_1 = \frac{(6+j2)\times(-j4)}{6+j2-j4}$$

$$= 2.4 - j3.2$$

$$z_2 = 10\,\Omega$$

$$\therefore z_{th} = z_1 + z_2$$

$$= 2.4 - j3.2 + 10$$

$$= 12.4 - j3.2$$

Max power Transfer:

$$z_L = z_{th}^* = \underbrace{12.4}_{R_{th}} + j3.2$$

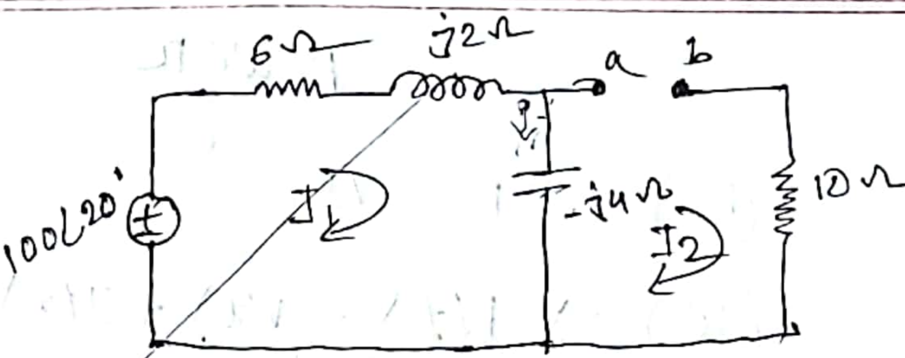
$$P_{max} = \frac{V_{th}^2}{8R_{th}}$$

$$= \frac{(63.25)^2}{8\times 12.4}$$

$$= 40.33\,W$$

Lab → 26 Aug

Quiz → 01 sept



KVL in mesh-1,

$$-100\angle 20 + 6I_1 + j2I_1 - j4(I_1 - I_2) = 0$$

$$\Rightarrow -100\angle 20 + 6I_1 + j2I_1 - j4I_1 + j4I_2 = 0$$

$$\Rightarrow -100\angle 20 + 6I_1 - j2I_1 + j4I_2 = 0$$

$$\Rightarrow -100\angle 20 + (6 - j2)I_1 + j4I_2 = 0$$

Using VDR,

$$V_{th} = \frac{100\angle 20^\circ \times (-j4)}{6 + j2 - j4}$$

$$= \underline{63.24} \angle -51.56^\circ \text{ V}$$

V_{th}

Power

(DC) → $P = VI$

(AC) → $p(t) = V(t)\,I(t)$

$$V(t) = V_m \cos(\omega_m t + \theta_v)$$

i(t

power

(DC) $\rightarrow P = VI$

(AC) $\rightarrow P(t) = v(t) + I(t)$

$$v(t) = V_m \cos(\omega_m t + \theta_v)$$
$$i(t) = I_m \cos(\omega_m t + \theta_i)$$

power : $p(t) = v(t) \cdot i(t)$

$$= V_m \cos(\omega_m t + \theta_v) \times I_m \cos(\omega_m t + \theta_i)$$

$$= V_m I_m \cos(\omega_m t + \theta_v) \cos(\omega_m t + \theta_i)$$

$$= \frac{V_m I_m}{2} \times 2 \cos(\omega_m t + \theta_v) \cos(\omega_m t + \theta_i)$$

$$= \frac{V_m I_m}{2} \times \left[\cos(\omega_m t + \theta_v + \omega_m t + \theta_i) + \cos(\omega_m t + \theta_v - \omega_m t - \theta_i)\right]$$

$$= \frac{1}{2} V_m I_m \left[\cos(2\omega_m t + \theta_v + \theta_i) + \overbrace{\cos(\theta_v - \theta_i)}^{\frac{1}{2} V_m I_m}\right]$$

Instantaneous power

$$\cancel{= \frac{1}{2} V_m I_m \times [\cos(2\omega_m t + 2\theta_v)}$$

$$\cancel{= V_m I_m \times \cos(\omega_m t + \theta_v)}$$

Avg power:

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

1

Date: 25-08-2025

Given,

$v(t) = 120\cos(377t + 45°)$ V and $i(t) = 10\cos(377t - 10°)$

Find the instantaneous power and the average power obsorbed by the passive linear network.

Solve

The instantaneous power,

$$p(t) = v(t)\, I(t)$$

$$= 120\cos(377t + 45)^{\circ} \cdot 10\cos(377t - 10°)$$

$$= \frac{120 \times 10}{2} \times 2\cos(377t + 45°)\cos(377t - 10°)$$

$$= \frac{120 \times 10}{2} \times [\cos(377t + 45° + 377t - 10°) + \cos(377t + 45° - 377t + 10°)]$$

$$= 600[\cos(754t + 35°) + \cos(55°)]$$

$$\therefore p(t) = 600\cos(754t + 35°) + 600 \times \cos(55°)$$

$$= 600\cos(754t + 35°) + 344.14 \text{ W}$$

The average power :

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 120 \times 10 \times \cos(45 - (-10))$$

$$= 344.2 W.$$

2 Ex Calculate the average power absorbed by an impedance $Z = 30 - j70\,\Omega$, when a voltage $V = 120\angle 0^\circ V$ is applied across it.

Solve

Given, $Z = 30 - j70\,\Omega$; $V = 120\angle 0^\circ V$

$$I = \frac{120}{30 - j70} = 1.57\angle 66.8^\circ$$

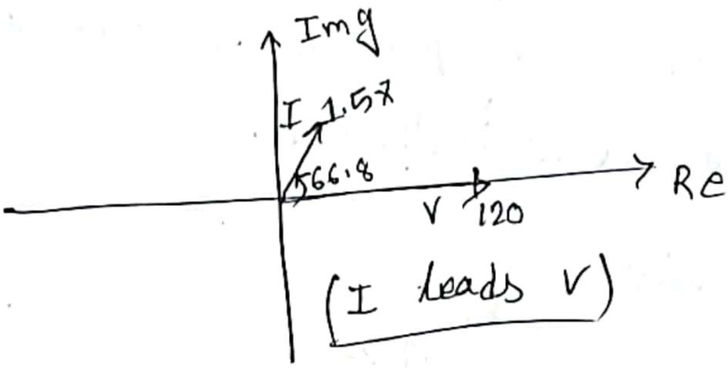
Average power,

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

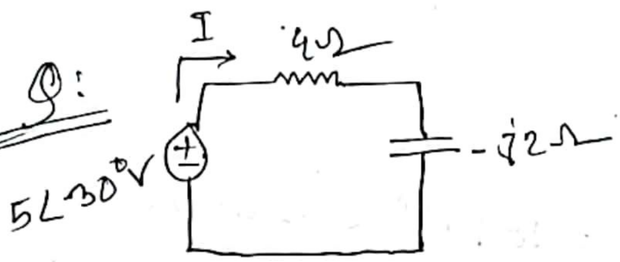
$$= \frac{1}{2} \times 120 \times 1.57 \cos(0^\circ - 66.8^\circ)$$

$$= 37.1 W$$

Phasor diagram of v & I :



3 Q:



Find the avg power supplied by the source and the avg power absorbed by the resistor.

solve

$$I = \frac{5\angle 30^\circ}{4-j2} = 0.6 + j0.9 = 1.11\angle 56.56^\circ$$

$V_1 = 4.44\angle 56.56^\circ$

$V_2 = 2.22\angle -33.44$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 5 \times 1.11 \times \cos(30^\circ - 56.56^\circ)$$

$$= 2.48\ W$$

$$P_{avg_1} = \frac{1}{2} \times 4.44 \times 1.11 \times \cos(56.56 - 56.56)$$

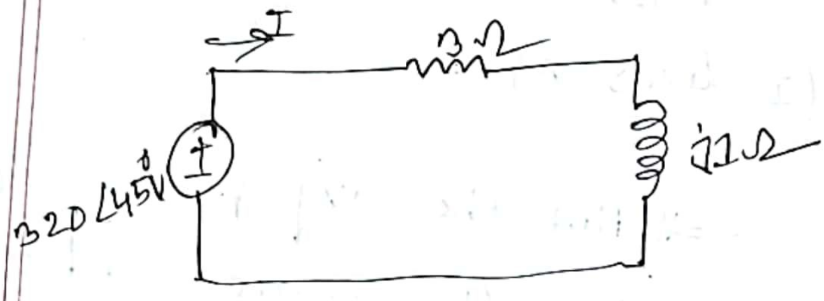
$$= 3.19\ W$$

$$P_{avg_2} = \frac{1}{2} \times 2.22 \times 1.11 \times \cos(-33.44 - 56.56^\circ)$$

$$= 0.62\ W$$

4 Q:

Find the average power supplied by the source and the avg power absorbed by the Resistor and the inductor.



Solve

$$I = \frac{320\angle 45^\circ}{3 + j1} = 101.19\angle 26.56^\circ$$

$$P_{avg} = \frac{1}{2} \times 320 \times 101.19 \times \cos(45 - 26.56)$$

$$= \cancel{1546.69\ W}\ 15359.11\ W = 15.35\ kW$$

$$V_R = 303.57\angle 26.56^\circ$$

$$V_L = 101.19\angle 116.56^\circ$$

$$P_{R\,avg} = \frac{1}{2} \times 303.57 \times 101.19 \times \cos(26.56^\circ - 26.56^\circ)$$

$$= \cancel{1546}\ 15359.11\ W = 15.35\ kW.$$

$$P_{L\,avg} = \frac{1}{2} \times 101.19 \times 101.19 \times (11.56 - 26.56)$$

$$= 0\ W$$

$Z = R + jX$ — R → Real power, X → Reactive power.

Complex power: $S = P + jQ = V_{r.m.s} I^*_{r.m.s}$

$= |V_{r.m.s}||I_{r.m.s}| \angle \theta_v - \theta_i$

Apparent power:

$$S_A = |V_{r.m.s}||I_{r.m.s}|$$

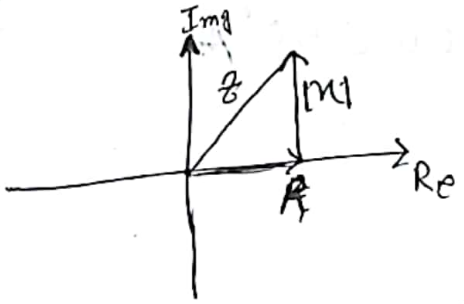
$$= \sqrt{P^2 + Q^2}$$

Real power: $P = Re(S) = S_A \cos(\theta_v - \theta_i)$

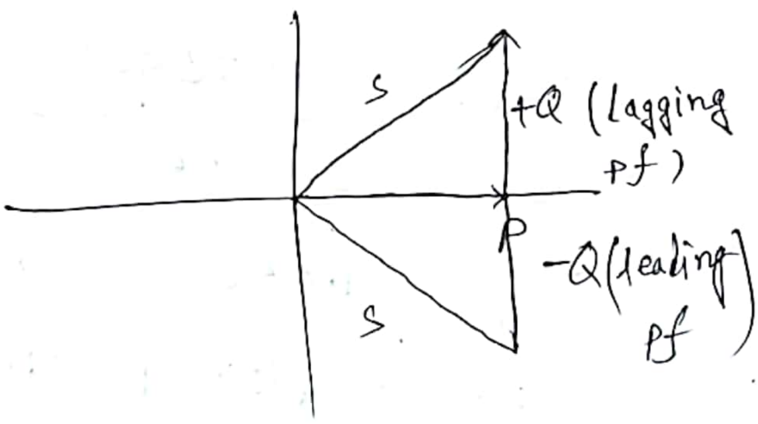
Img / Reactive power: $Q = Im(S) = S_A \sin(\theta_v - \theta_i)$

power factor : $Pf = \frac{P}{S_A} = \cos(\theta_v - \theta_i)$

Ex $Z = R + jx$



power



5 Q: The voltage across a load is $v(t) = 60\cos(\omega t - 10°)$ and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50°)$ A

Find. a) The complex and apparent power.

b) The real and the reactive power and the power factor and the load impedance.

solve $v(t) = 60\cos(\omega t - 10°) = 60\angle -10°$ V

$I(t) = 1.5\cos(\omega t + 50°) = 1.5\angle 50°$ A

$$V_{r.m.s} = \frac{60\angle -10°}{\sqrt{2}} = 42.42\angle -10 \text{ V}$$

$$I_{r.m.s} = \frac{1.5\angle 50°}{\sqrt{2}} = 1.06\angle 50° \text{ A}$$

$$I^{*}_{rm.s} = 1.06\angle -50° \text{ A}$$

The complex power:

$$S = P + jQ = V_{r.m.s}\, I^{*}_{r.m.s}$$

$$= 42.42\angle -10 \times 1.06\angle -50°$$

$$= 44.96\angle -60° \text{ VA} = (22.48 - j38.92) \text{ VA}$$

CamScanner

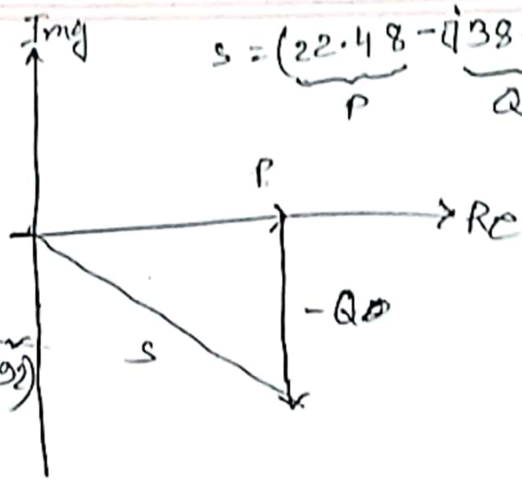
apperant power.

$S = (\underbrace{22.48}_{P} - \underbrace{j38.92}_{Q})$

$S_A = |V_{r.m.s}|\,|I_{r.m.s}|$

$= 44.96$ $\cancel{\sqrt{P^2+Q^2}}$

$\cancel{= \sqrt{(22.48)^2+(j38.92)^2}}$



b) Real power:

$P = Re(S) = S_A \cos(\theta_v - \theta_i)$

$= 44.96 \times \cos(-10 - 50)$

$= 22.48\ W$

$Q = Im(S) = S_A \sin(\theta_v - \theta_i)$

$= 44.96 \times \sin(-10^\circ - 50^\circ)$

$= -38.92\ VAR$

power factor:

$Pf = \cos(\theta_v - \theta_i)$

$= \cos(-10 - 50)$

$= 0.5$ leading $(-Q)$

Impedance

$Z = \dfrac{V}{I}$

$= \dfrac{60\angle -10^\circ}{1.5\angle 50^\circ}$

$= 20 - j34.64$

$= 40\angle -60^\circ$

CamScanner

Date: 27-08-25

Q1 A load Z draws 12KVA at a power factor of

6 0.856 lagging from a 120 V r.m.s sinusoidal source

(pf) calculate.

a) The average and reactive power delivered to the load.

b) The peak current, and c) the load impedance.

Solve

a) Given, $pf = 0.856$

$S_A = 12KVA$

$V_s = 120$ r.m.s

$pf = \frac{P}{S_A}$

$\Rightarrow P = pf \times S_A$

$= 0.856 \times 12000$

$= 10272W$

$= 10.272 kW$

average power.

* $\theta_v - \theta_i = \theta$

$\therefore pf = \cos\theta = 0.856$

$\therefore \theta = \cos^{-1}(0.856)$

$= 31.12°$

* Reactive power:

$Q = S_A \sin\theta$

$= 12000 \times \sin(31.12°)$

$= 6203.7 VAR$

$= 6.2037 kVAR$

b) ~~peak current~~;

complex power,

$$S = P + jQ$$

$$= 10.27 + j6.20$$

$$S = V_{r.m.s} I_{r.m.s}^{*}$$

$$\Rightarrow 10.27 + j6.20 = 120 \cdot I_{r.m.s}^{*}$$

$$\therefore I^{*}_{r.m.s} = \frac{10.27 + j6.20}{120}$$

$$= 0.09 \angle 31.11 \text{ A}$$

$$\therefore I_{r.m.s} = \cancel{0.09}\ 100 \angle -31.11 \text{ A}$$

c) Impedence,

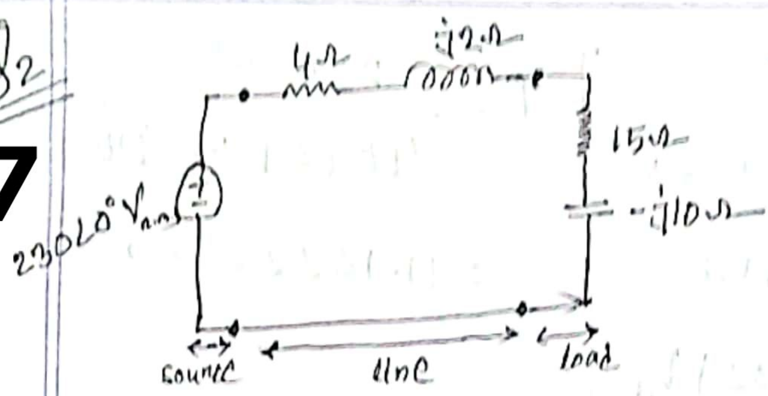
$$Z = \frac{V_{r.m.s}}{I_{r.m.s}}$$

$$= \frac{120}{100 \angle -31.11}$$

$$= 1.2 \angle 31.11^{\circ}\ \Omega\ \text{A}$$

Q2

7



For the following circuit find the real and reactive power absorbed by a) source, b) the line c) the load.

solve

$$I = \frac{230\angle 0^\circ}{4+j2+15-j10} = \cancel{12.04\angle -6.009^\circ \text{ A}}$$
$$= 11.16\angle 22.83^\circ \text{ A}$$

b) $V_{Line} = I\, Z_{Line}$

$= \cancel{12.04\angle -6.009}$ $11.16\angle 22.83^\circ$ $\times (4+j2)$

$\cancel{= 53.84\angle 20.5^\circ\ V_{r.m.s}}$

$= 49.9\angle 49.4^\circ\ V_{r.m.s}$

$s = V_{Line} I^*$

$= 49.9\angle 49.4^\circ \times 11.16\angle -22.83$

$= 555.98\angle 26.56^\circ$

$= 497.29 + j\,248.60$ VA

$P = 497.29,\ Q = 248.60$

c) $V_{Load} = I\, Z_{load}$

$= \cancel{12.04\angle -6.009}$ $11.16\angle 22.83^\circ$ $\times (15 - j10)$

$\cancel{= 217.05\angle -39.69^\circ\ V_{r.m.s}}$

$= 201.189\angle -10.9^\circ$ V

a) $S = V_{r.m.s}\, I_{r.m.s}$

$= 230\angle 0^\circ \times \cancel{12.04\angle 6.009}$ $11.16\angle 22.83^\circ$

$\cancel{= 2769.2\angle 6.009^\circ}$

$= 2566.8\angle -22.83$ VA

$= (2363.59 - j995.02)$ VA

Real P:

$P = 2363.59$ W

$Q = -995.02$ VAR

a) $S = V_{r.m.s} \, I_{r.m.s}^*$

$= 230\angle 0^\circ \times 11.16\angle -22.83^\circ$

$= 2566.8\angle -22.83$ VA

$= (2363.59 - j995.02)$ VA

$I = \dfrac{230\angle 0^\circ}{4 + j2 + 15 - j10}$

$= 11.16\angle 22.83$

$P = 2363.59$ W

$Q = -995.02$ VAR

b) $V_{Line} = I \, Z_{Line}$

$= 11.16\angle 22.83 \times (4 + j2)$

$= 49.9\angle 49.4^\circ$ $V_{r.m.s}$

$S = \cancel{49.9}\ V_{Line} \, I^*$

$= 49.9\angle 49.4 \times 11.16\angle -22.83$

$= 555.98\angle 26.56^\circ$

$S = 497.29 + j248.60$ VA

$P = 497.29$ W

$Q = 248.60$ VAR

c) $V_{Load} = 11.16\angle 22.83 \times (15 - j10)$

$= 201.00\angle -10.86$ V

$S = V_{Load} \, I^*$

$= 201.00\angle -10.86 \times 11.16\angle -22.83$

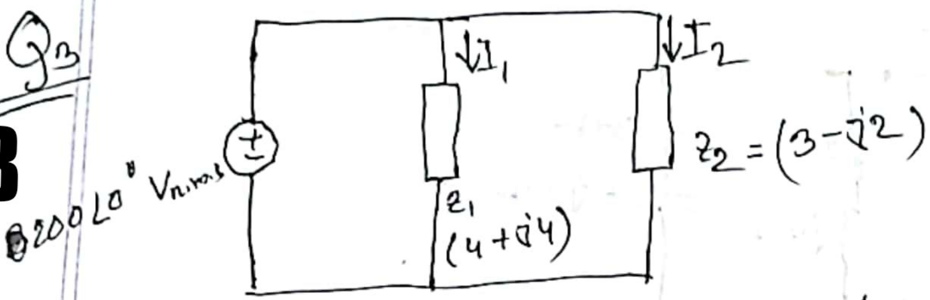
$= 1864.83 - j1243.2$

$P = 1864.83$ W

$Q = -1243.2$ VAR

Q3

8



Find the complex power, real power and the reactive power across all the elements of the following circuit. and find total complex power.

solve

$$I_1 = \frac{200\angle 0^\circ \cancel{\times (3-j2)}}{4+j4 \cancel{+3-j2}} = \cancel{99.05\angle -49.6^\circ}$$
$$= 25 - j25$$

$$I_2 = \frac{200\angle 0^\circ \cancel{\times (4+j4)}}{\cancel{4+j4+} 3-j2} = \cancel{155.4\angle 29.05^\circ}$$
$$= 55.47\angle 33.69^\circ$$

$$S_1 = V_{r.m.s} I_1^* = 200\angle 0^\circ \times (25 - j25) \rightarrow pf_1 = \text{lagging}$$
$$= 4999.24 + j4999.24$$

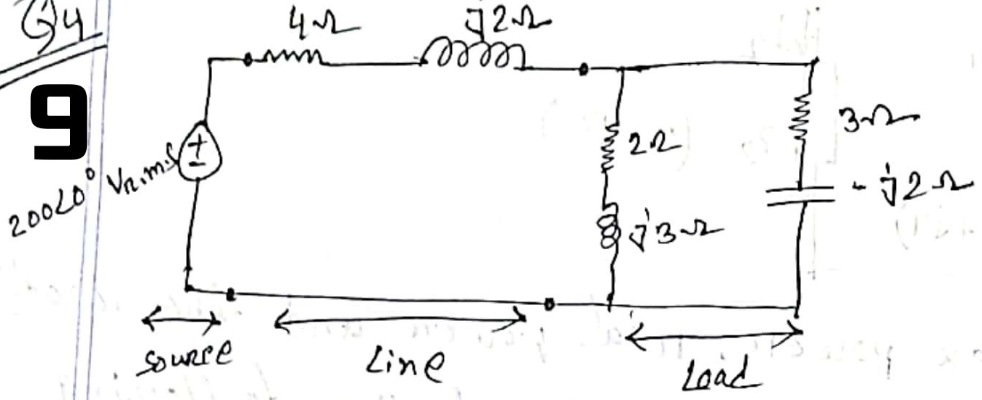
$$S_2 = V_{r.m.s} I_2^* = 200\angle 0^\circ \times (55.47\angle 33.69^\circ)$$
$$= 9229.10 - j6154.72 \rightarrow pf_2 = \text{leading}$$

Total complex power:

$$S = S_1 + S_2 =$$

Q4

9



Find the real and reactive power of the load.

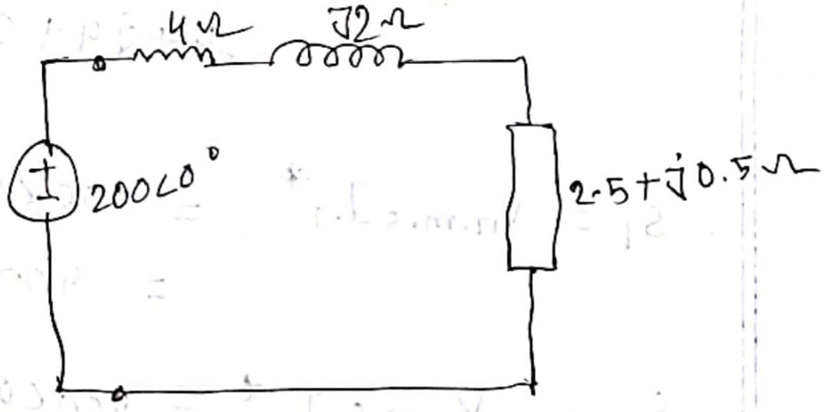
solve

$$Z_{load} = (2+j3) \parallel (3-j2) = \frac{(2+j3)\times(3-j2)}{2+j3+3-j2}$$

$$= 2.5 + j0.5 \ \Omega$$

$$I = \frac{200\angle 0^\circ}{4+j2+(2.5+j0.5)}$$

$$= 29.1\angle -18.7^\circ$$



$$V_{Load} = I \times Z_{load}$$

$$= 29.1\angle -18.7 \times (2.5 - j0.5)$$

$$= 74.19 \angle -30^\circ$$

$S = V_{Load} I^*$

$= 74.19\angle -30^\circ \times 29.1\angle 18.7$

$= 2117.077 - j423.03$

$\therefore P = 2117.077 \, W$

$Q = 423.03 \, VAR$

Practice problem

$\boxed{11.1 \rightarrow 11.14}$

Power

* $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$ [average power]
* $P(t) = V(t) \cdot I(t)$ [Instantaneous Power]
* $S = P + jQ = V_{rms} I^*_{rms}$ [Complex power]
* $S_A = |V_{rms}||I_{rms}| = \sqrt{P^2+Q^2}$ [apparent power]
* [Real power], $P = Re(S) = S_A \cos(\theta_v - \theta_i)$
* [Reactive power], $Q = Im(S) = S_A \sin(\theta_v - \theta_i)$
* [Power factor] $P_f = \frac{P}{S_A} = \cos(\theta_v - \theta_i)$
* [Impedance] $Z = \frac{V}{I}$