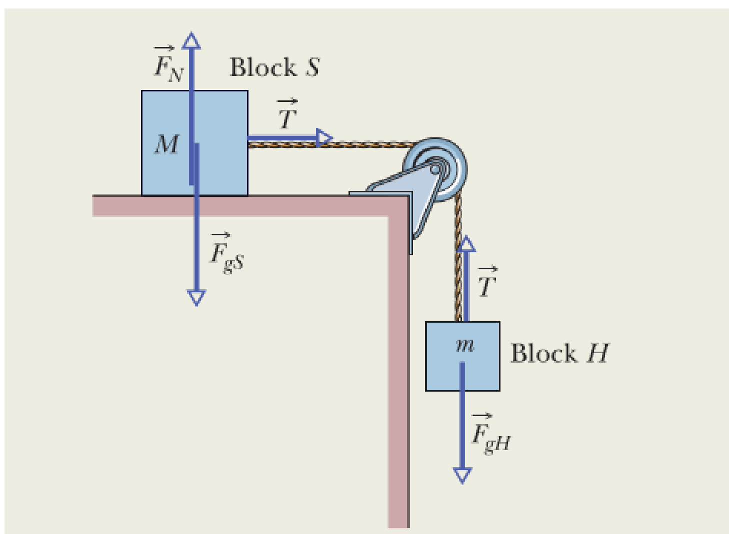
Topic: Problems and Solutions Based on Newton's Laws of Motion

Problem: 1

Figure shows a block S (the sliding block) with mass $M = 3.3$ kg. The block is free to move along a horizontal frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block H (the hanging block), with mass $m = 2.1$ kg. The cord and pulley have negligible masses compared to the blocks (they are "massless"). The hanging block H falls as the sliding block S accelerates to the right.



Find:

(a) The acceleration of block S.

(b) The acceleration of block H.

(c) The tension in the cord.

Solution

Let the acceleration of both blocks be a. Since the blocks are connected by a cord, they have the same magnitude of acceleration.

Equations of Motion

For block S (sliding block): Along x axis,

$$T = Ma \tag{1}$$

Along y axis,

$$F_N - F_g = 0 \tag{2}$$

For block H (hanging block):

$$mg - T = ma \tag{3}$$

Acceleration

Substituting $T = Ma$ into the equation (3):

$$mg - Ma = ma \tag{4}$$

$$mg = Ma + ma \tag{5}$$

$$a = \frac{mg}{M + m} \tag{6}$$

Substituting the given values:

$$a = \frac{(2.1)(9.8)}{3.3 + 2.1} \tag{7}$$

$$a = \frac{20.58}{5.4} = 3.81\,\mathrm{m/s^2} \tag{8}$$

Tension:

Using $T = Ma$:

$$T = (3.3)(3.81) \tag{9}$$

$$T = 12.57\,\mathrm{N} \tag{10}$$

Answers

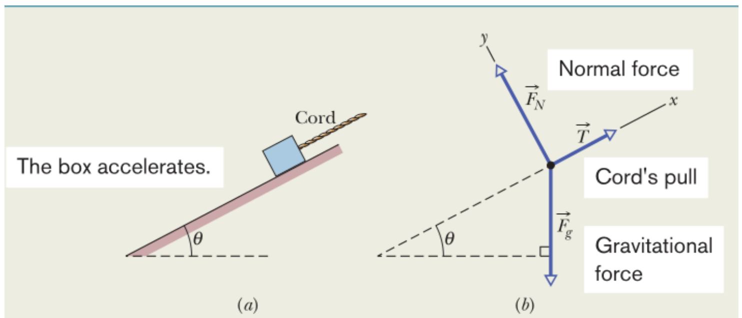
(a) The acceleration of block S is 3.81 $\mathrm{m/s^2}$.

(b) The acceleration of block H is also 3.81 $\mathrm{m/s^2}$.

(c) The tension in the cord is 12.57 N.

Problem 2:

In Figure, a cord pulls a box of biscuits up along a frictionless plane inclined at angle $\theta = 30.03°$. The box has mass $m = 5.00$ kg, and the force from the cord has magnitude $T = 25.0$ N.



(a) What is the box's acceleration a along the inclined plane?

(b) Find normal force F_N.

Solution

Equations of Motion

Applying Newton's Second Law along the incline:
Along X axis:

$$T - mg\sin\theta = ma \tag{11}$$

Along Y axis:

$$F_N - mg\cos\theta = 0 \tag{12}$$

Solving for acceleration, a:

$$a = \frac{T - mg\sin\theta}{m} \tag{13}$$

Substituting the given values:

$$a = \frac{25.0 - (5.00)(9.8)\sin 30.03°}{5.00} \tag{14}$$

Calculating:

$$a = \frac{25.0 - (5.00)(9.8)(0.5003)}{5.00} \tag{15}$$

$$a = \frac{25.0 - 24.51}{5.00} \tag{16}$$

$$a = \frac{0.49}{5.00} = 0.098\,\text{m/s}^2 \tag{17}$$

Solving for normal force, F_N:

$$F_N = mg\cos\theta \tag{18}$$

Substituting the given values:

$$F_N = (5.00)(9.8)\cos 30.03^\circ \tag{19}$$

$$F_N = (5.00)(9.8)(0.8669) \tag{20}$$

$$F_N = 42.47\,\mathrm{N}$$

Answers:

- The box's acceleration along the inclined plane is 0.098 m/s^2.
- The normal force exerted by the plane on the box is 42.47 N.

Problem 3:

A passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

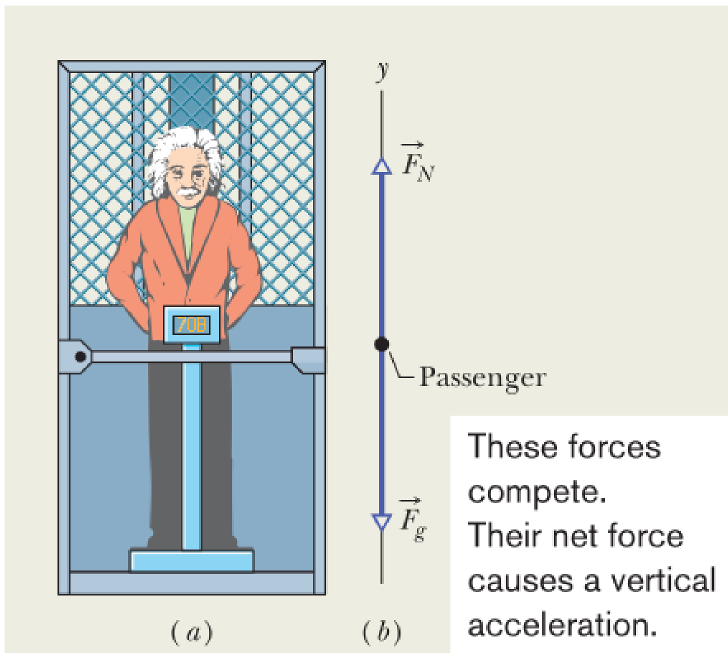


Figure 1: Caption

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

(c) What does the scale read if the cab accelerates upward at 3.20 m/s^2 and downward at 3.20 m/s^2?

Solution

(a) General Solution

The platform scale measures the normal force N exerted on the passenger. Applying Newton's Second Law:

For upward motion:

$$F_N - mg = ma \tag{22}$$

Solving for N:

$$F_N = m(g + a) \tag{23}$$

For downward motion:

$$F_N - mg = -ma \tag{24}$$

Solving for N:

$$F_N = m(g - a) \tag{25}$$

where:

- $g = 9.8$ m/s^2 (acceleration due to gravity),
- a is the acceleration of the elevator (positive for upward motion, negative for downward motion). When the passenger is moving upward he/she will feel heavy and when the passenger is going downward he/she will feel light.

(b) Scale Reading when the Cab is Stationary or Moving at Constant Velocity

If the elevator is stationary or moving at a constant velocity ($a = 0$), then:

$$F_N = mg \tag{26}$$

Substituting the given values:

$$F_N = (72.2)(9.8) \tag{27}$$

$$F_N = 707.6\,\mathrm{N} \tag{28}$$

Thus, the scale reads 707.6 N.

(c) Scale Reading when the Cab Accelerates

Upward Acceleration: If the elevator accelerates upward at $a = 3.20$ m/s^2:

$$F_N = 72.2(9.8 + 3.2) \tag{29}$$

$$F_N = 72.2(13.0) = 938.6\,\mathrm{N} \tag{30}$$

Thus, the scale reads 938.6 N when accelerating upward.

Downward Acceleration: If the elevator accelerates downward at $a = -3.20$ m/s^2:

$$F_N = 72.2(9.8 - 3.2) \tag{31}$$

$$F_N = 72.2(6.6) = 476.5\,\mathrm{N} \tag{32}$$

Thus, the scale reads 476.5 N when accelerating downward.

Answers:

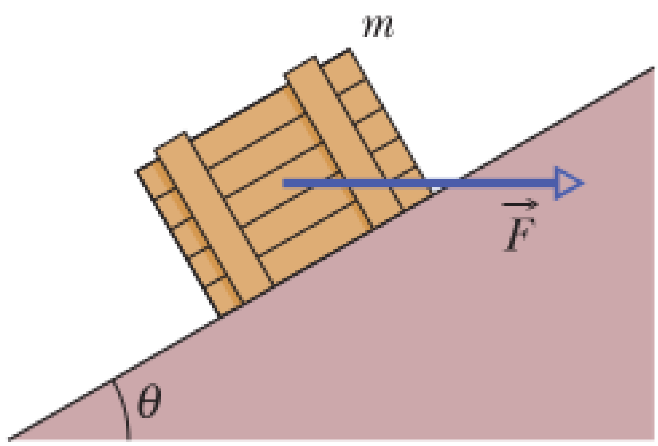
(a) General solution: $F_N = m(g + a)$.

(b) Scale reading when stationary or moving at constant speed: 707.6 N.

(c) Scale reading when accelerating upward: 938.6 N, and when accelerating downward: 476.5 N.

Problem 4:

A crate of mass $m = 100\,\text{kg}$ is pushed at constant speed up a frictionless ramp with an incline angle of $\theta = 30.0°$ by a horizontal force F.



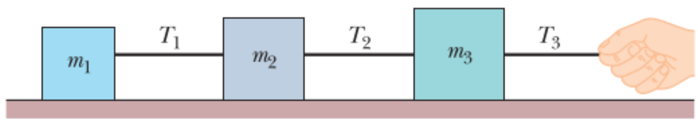
(a) What is the magnitude of F?

(b) What is the magnitude of the force on the crate from the ramp?

Problem 5:

Three connected blocks are pulled to the right on a horizontal frictionless table by a force of magnitude $T_3 = 65.0\,\mathrm{N}$. The masses of the blocks are given as:

- $m_1 = 12.0\,\mathrm{kg}$,
- $m_2 = 24.0\,\mathrm{kg}$,
- $m_3 = 31.0\,\mathrm{kg}$.



Calculate:

(a) The magnitude of the system's acceleration,

(b) The tension T_1 (force exerted by m_2 on m_1),

(c) The tension T_2 (force exerted by m_3 on m_2).

Solution

(a) Calculation of Acceleration

The total mass of the system is the sum of all three masses:

$$m_{\text{total}} = m_1 + m_2 + m_3 = 12.0 + 24.0 + 31.0 = 67.0\,\mathrm{kg}.$$

Since the force T_3 is the only external force acting on the system in the horizontal direction (assuming no friction), Newton's second law gives:

$$F = ma.$$

Substituting the values:

$$65.0 = 67.0a.$$

Solving for acceleration a:

$$a = \frac{65.0}{67.0} \approx 0.970\,\mathrm{m/s^2}.$$

(b) Calculation of Tension T_2

Tension T_2 is the force required to pull the combined mass of m_1 and m_2 with acceleration a. Using Newton's second law:

$$T_2 = (m_1 + m_2)a.$$

Substituting the values:

$$T_2 = (12.0 + 24.0) \times 0.970.$$

$$T_2 = 34.9\,\mathrm{N}.$$

(c) Calculation of Tension T_1

Tension T_1 is the force required to pull mass m_1 with acceleration a. Again, using Newton's second law:

$$T_1 = m_1 a.$$

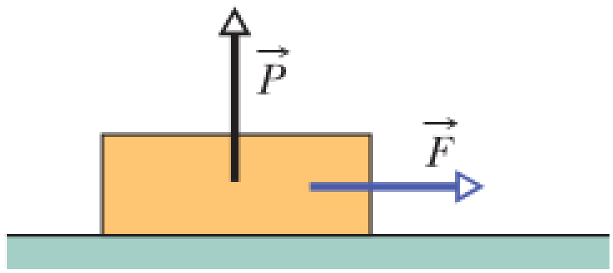
Substituting the values:

$$T_1 = 12.0 \times 0.970.$$

$$T_1 = 11.6\,\mathrm{N}.$$

Answers

- (a) The acceleration of the system is $0.970\,\mathrm{m/s}^2$.
- (b) The tension T_2 is $34.9\,\mathrm{N}$.
- (c) The tension T_1 is $11.6\,\mathrm{N}$.



Problem 6:

[H] A 2.5 kg block is initially at rest on a horizontal surface. A horizontal force F of magnitude 6.0 N and a vertical force P are then applied to the block. The coefficients of friction for the block and surface are:

- Static friction coefficient: $\mu_s = 0.40$,
- Kinetic friction coefficient: $\mu_k = 0.25$.

Determine the magnitude of the frictional force acting on the block if the magnitude of P is:

(a) 8.0 N,

(b) 10.0 N,

(c) 12.0 N.

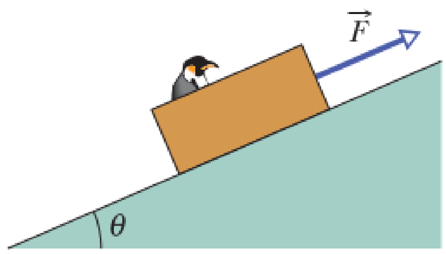
Problem 7:

A loaded penguin sled weighing 80 N rests on a plane inclined at an angle $\theta = 20°$ to the horizontal. Between the sled and the plane, the coefficient of static friction is $\mu_s = 0.25$, and the coefficient of kinetic friction is $\mu_k = 0.15$.

(a) What is the least magnitude of the force F, parallel to the plane, that will prevent the sled from slipping down the plane?

(b) What is the minimum magnitude F that will start the sled moving up the plane?

(c) What value of F is required to move the sled up the plane at constant velocity?



Solution:

Let the weight of the sled be $W = 80$ N, and the angle of inclination be $\theta = 20°$.

The normal force F_N acting on the sled is given by:

$$F_N = W\cos\theta = 80\,\mathrm{N}\cdot\cos(20°) \approx 75.08\,\mathrm{N}$$

Part (a): Preventing the sled from slipping down the plane The force preventing the sled from slipping down the plane is the frictional force, which opposes the motion. The maximum static frictional force is given by:

$$f_s = \mu_s F_N = 0.25\cdot 75.08\,\mathrm{N} \approx 18.77\,\mathrm{N}$$

The component of the weight acting down the plane is:

$$W_{\|} = W\sin\theta = 80\,\mathrm{N}\cdot\sin(20°) \approx 27.47\,\mathrm{N}$$

To prevent the sled from slipping, the applied force F must balance the component of the weight down the plane and the frictional force:

$$F \geq W_{\|} - f_s = 27.47\,\mathrm{N} - 18.77\,\mathrm{N} \approx 8.70\,\mathrm{N}$$

Thus, the least force F that will prevent the sled from slipping down the plane is approximately:

$$F \approx 8.70\,\mathrm{N}$$

Part (b): Minimum force to start the sled moving up the plane To start moving the sled up the plane, the applied force must overcome both the component of the weight down the plane and the maximum static friction:

$$F_{\text{min}} = W_{\parallel} + f_s = 27.47\,\text{N} + 18.77\,\text{N} \approx 46.24\,\text{N}$$

Thus, the minimum force to start the sled moving up the plane is approximately:

$$F_{\text{min}} \approx 46.24\,\text{N}$$

Part (c): Force required to move the sled at constant velocity To move the sled at constant velocity, the applied force must balance the forces acting down the plane, including both the component of the weight and the kinetic frictional force. The kinetic frictional force is given by:

$$f_k = \mu_k N = 0.15 \cdot 75.08\,\text{N} \approx 11.26\,\text{N}$$

The total force required is:

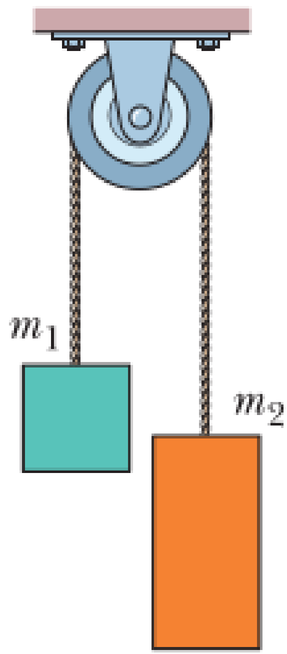
$$F_{\text{constant velocity}} = W_{\parallel} + f_k = 27.47\,\text{N} + 11.26\,\text{N} \approx 38.73\,\text{N}$$

Thus, the force required to move the sled up the plane at constant velocity is approximately:

$$F_{\text{constant velocity}} \approx 38.73\,\text{N}$$

Problem 8:

Two blocks are connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as Atwood's machine. One block has mass $m_1 = 1.30\,\text{kg}$; the other has mass $m_2 = 2.80\,\text{kg}$.



(a) What is the magnitude of the blocks' acceleration?

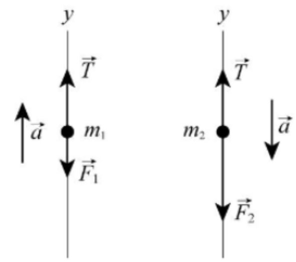
(b) What is the tension in the cord?

Solution:

51. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$



which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

Substituting the result back, we have

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$