Topic: Concept of electric charge, the electric field, dipole in an electric field.

Electric Charge

Definition:

Electric charge is a fundamental property of matter that causes it to experience a force in an electric field. It is a scalar quantity and can be either positive or negative.

Types of Electric Charges

- **Positive Charge:** Associated with protons.
- **Negative Charge:** Associated with electrons.

Key Properties of Charge

1. **Like charges repel, opposite charges attract.**

   Charge Interaction Examples:

   - A positive charge repels another positive charge.
   - A negative charge attracts a positive charge.
   - Two negative charges repel each other.

2. **Quantization of Charge:**

   $$q = n \cdot e, \tag{1}$$

   where $e = 1.6 \times 10^{-19}$ C is the elementary charge, and n is an integer.

3. **Conservation of Charge:** The total charge in a closed system remains constant.

SI Unit of Charge

The SI unit of electric charge is the **Coulomb (C)**. One Coulomb is defined as the charge transported by a current of one Ampere in one second.

Introduction of Electric Force

Two simple demonstrations provide insight into the nature of electric forces and their behavior. These experiments, though appearing magical at first, reveal fundamental properties of electric interactions. Let us explore and make sense of them.

Demonstration 1: Repulsion of Glass Rods

- A glass rod is rubbed with a silk cloth (in low humidity) and suspended by a thread tied around its center.
- Another glass rod, also rubbed with a silk cloth, is brought near the suspended rod.
- Observation: The suspended rod moves away from the nearby rod, indicating a repulsive force.

Explanation: The rubbing process transfers charges to the glass rods, resulting in both rods having the same type of charge. Since like charges repel, the two rods exert a repulsive force on each other. Importantly, this force acts at a distance without any physical contact.

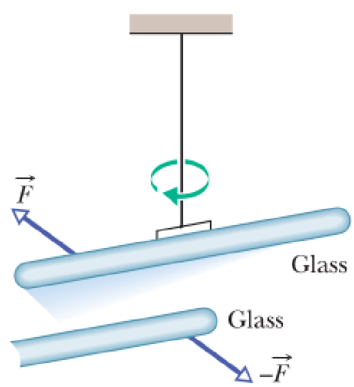


Figure 1: The two glass rods were each rubbed with a silk cloth and one was sus- pended by thread. When they are close to each other, they repel each other

Demonstration 2: Attraction of Glass and Plastic Rods

- The suspended glass rod (rubbed with silk) is left as is.
- A plastic rod, rubbed with fur, is brought near the suspended rod.
- Observation: The suspended rod moves toward the plastic rod, indicating an attractive force.

Explanation: The rubbing process imparts opposite charges to the glass and plastic rods. Since opposite charges attract, the rods experience an attractive force that again acts at a distance without direct contact.

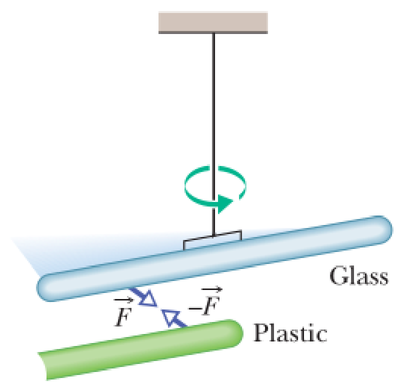


Figure 2: The plastic rod was rubbed with fur. When brought close to the glass rod, the rods attract each other.

Key Notes

- Electric forces can be attractive or repulsive, depending on the types of charges involved.
- These forces act at a distance, requiring no physical contact between the objects.
- The type of charge imparted depends on the material and the method of rubbing (e.g., silk for glass, fur for plastic).

Coulomb's Law

The force between two point charges q_1 and q_2 separated by a distance r is given by:

$$F = k\frac{|q_1 q_2|}{r^2}, \tag{2}$$

where $k = \frac{1}{4\pi\epsilon_0} \approx 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$ is Coulomb's constant, and $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$ is the permittivity of free space.

Problem 1(a)

Two positively charged particles are fixed in place on the x-axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m.

Question: Determine the magnitude and direction of the electrostatic force $\mathbf{F}_{12}$ acting on particle 1 due to particle 2.

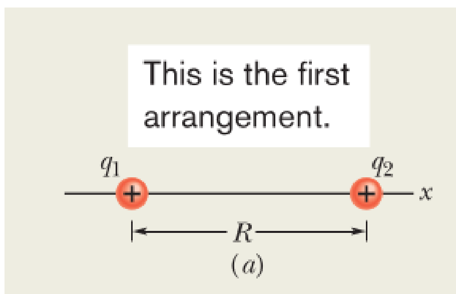


Figure 3: Two charged particles of charges q$_1 andq_2 are fixed in place on an x axis.$

Solution

The magnitude of the electrostatic force between two charges is given by Coulomb's law:

$$F = k\frac{|q_1 q_2|}{R^2},$$

where:

- $k = 8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}$ is the Coulomb constant,
- $q_1 = 1.60 \times 10^{-19}\,\mathrm{C}$,
- $q_2 = 3.20 \times 10^{-19}\,\mathrm{C}$,
- $R = 0.0200\,\mathrm{m}$ is the separation distance between the charges.

Substituting the values:

$$F = (8.99 \times 10^9)\frac{(1.60 \times 10^{-19})(3.20 \times 10^{-19})}{(0.0200)^2}.$$

$$F = (8.99 \times 10^9)\frac{5.12 \times 10^{-38}}{4.00 \times 10^{-4}}.$$

$$F = (8.99 \times 10^9) \times 1.28 \times 10^{-34}.$$

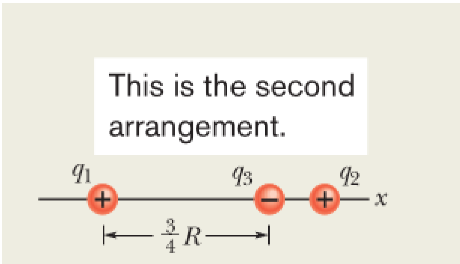
$$F = 1.15 \times 10^{-24}\,\mathrm{N}.$$

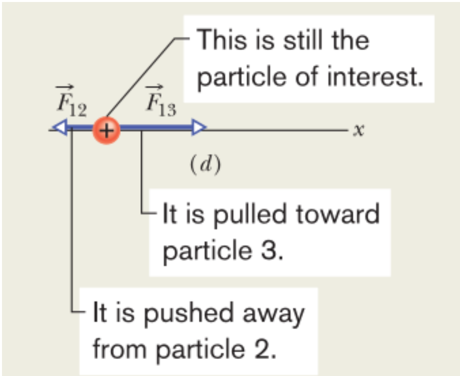
Direction: Since both charges are positive, the force is repulsive. The force $\mathbf{F}_{12}$ on particle 1 due to particle 2 points to the left along the negative x-axis.

Problem 1(b)

Three charged particles lie on the x-axis. Particle 1 has charge $q_1 = 1.60 \times 10^{-19}\,\mathrm{C}$, particle 2 has charge $q_2 = 3.20 \times 10^{-19}\,\mathrm{C}$, and particle 3 has charge $q_3 = -3.20 \times 10^{-19}\,\mathrm{C}$. The separation between particles 1 and 2 is $R = 0.0200\,\mathrm{m}$, and particle 3 lies at a distance $\frac{3}{4}R$ from particle 1.

Question : Determine the net electrostatic force $\mathbf{F}_{1,\mathbf{net}}$ on particle 1 due to particles 2 and 3.





Solution

The net force on particle 1 is the vector sum of the forces exerted by particles 2 and 3:

$$\mathbf{F}_{1,\text{net}} = \mathbf{F}_{12} + \mathbf{F}_{13},$$

where:

- $\mathbf{F}_{12}$ is the force on q_1 due to q_2,
- $\mathbf{F}_{13}$ is the force on q_1 due to q_3.

Step 1: Force $\mathbf{F}_{12}$ From Coulomb's law:

$$F_{12} = k\frac{|q_1 q_2|}{R^2}.$$

Using $k = 8.99 \times 10^9\,\text{N·m}^2/\text{C}^2$, $q_1 = 1.60 \times 10^{-19}\,\text{C}$, $q_2 = 3.20 \times 10^{-19}\,\text{C}$, and $R = 0.0200\,\text{m}$:

$$F_{12} = (8.99 \times 10^9)\frac{(1.60 \times 10^{-19})(3.20 \times 10^{-19})}{(0.0200)^2}.$$

As calculated earlier:

$$F_{12} = 1.15 \times 10^{-24}\,\text{N}.$$

Since both charges are positive, $\mathbf{F}_{12}$ is repulsive and directed to the left along the negative x-axis.

Step 2: Force $\mathbf{F}_{13}$ The distance between particles 1 and 3 is:

$$r_{13} = \frac{3}{4}R = \frac{3}{4} \times 0.0200 = 0.0150\,\text{m}.$$

From Coulomb's law:

$$F_{13} = k\frac{|q_1 q_3|}{r_{13}^2}.$$

Using $q_3 = -3.20 \times 10^{-19}\,\text{C}$:

$$F_{13} = (8.99 \times 10^9)\frac{(1.60 \times 10^{-19})(3.20 \times 10^{-19})}{(0.0150)^2}.$$

Simplify:

$$F_{13} = (8.99 \times 10^9)\frac{5.12 \times 10^{-38}}{2.25 \times 10^{-4}}.$$

$$F_{13} = (8.99 \times 10^9) \times 2.28 \times 10^{-34}.$$

$$F_{13} = 2.05 \times 10^{-24}\,\text{N}.$$

Since q_3 is negative, $\mathbf{F}_{13}$ is attractive and directed to the right along the positive x-axis.

Step 3: Net Force $\mathbf{F}_{1,\text{net}}$ The net force is:

$$F_{1,\text{net}} = F_{13} - F_{12},$$

where F_{13} is positive (right) and F_{12} is negative (left):

$$F_{1,\text{net}} = 2.05 \times 10^{-24} - 1.15 \times 10^{-24}.$$

$$F_{1,\text{net}} = 0.90 \times 10^{-24}\,\text{N}.$$

Direction: The net force is directed to the right along the positive x-axis.

Answer

The net electrostatic force on particle 1 is:

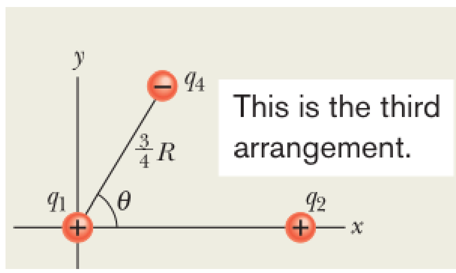
$$F_{1,\text{net}} = 0.90 \times 10^{-24}\,\text{N},$$

directed along the positive x-axis.

Problem 1(c)

Three charged particles lie in a configuration as described:

- **Particle 1 has charge $q_1 = 1.60 \times 10^{-19}$ C and is fixed at the origin.**
- **Particle 2 has charge $q_2 = 3.20 \times 10^{-19}$ C and is fixed on the x-axis at a distance $R = 0.0200$ m from particle 1.**
- **Particle 4 has charge $q_4 = -3.20 \times 10^{-19}$ C and lies at a distance $\frac{3}{4}R = 0.0150$ m from particle 1, making an angle $\theta = 60^\circ$ with the positive x-axis.**



Question: Determine the net electrostatic force $\mathbf{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4.

Solution

The net force on particle 1 is the vector sum of the forces exerted by particles 2 and 4:

$$\mathbf{F}_{1,\text{net}} = \mathbf{F}_{12} + \mathbf{F}_{14}.$$

Force $\mathbf{F}_{12}$ The force $\mathbf{F}_{12}$ due to particle 2 is calculated using Coulomb's law:

$$F_{12} = k\frac{|q_1 q_2|}{R^2}.$$

Substituting:

$$F_{12} = (8.99 \times 10^9)\frac{(1.60 \times 10^{-19})(3.20 \times 10^{-19})}{(0.0200)^2}.$$

As calculated earlier:

$$F_{12} = 1.15 \times 10^{-24}\,\text{N}.$$

Since both charges are positive, $\mathbf{F}_{12}$ is repulsive and directed along the negative x-axis.

Force $\mathbf{F}_{14}$ The force $\mathbf{F}_{14}$ is calculated similarly:

$$F_{14} = k\frac{|q_1 q_4|}{r_{14}^2},$$

where $r_{14} = \frac{3}{4}R = 0.0150\,\mathrm{m}$. Substituting:

$$F_{14} = (8.99 \times 10^9)\frac{(1.60 \times 10^{-19})(3.20 \times 10^{-19})}{(0.0150)^2}.$$

As calculated earlier:

$$F_{14} = 2.05 \times 10^{-24}\,\mathrm{N}.$$

Since q_4 is negative, $\mathbf{F}_{14}$ is attractive and points toward q_4. This force has components along both the x- and y-axes.

Components of $\mathbf{F}_{14}$ The components of $\mathbf{F}_{14}$ are:

$$F_{14,x} = F_{14}\cos\theta, \quad F_{14,y} = F_{14}\sin\theta.$$

Substituting $\theta = 60°$:

$$F_{14,x} = (2.05 \times 10^{-24})\cos 60° = (2.05 \times 10^{-24})(0.5) = 1.03 \times 10^{-24}\,\mathrm{N},$$

$$F_{14,y} = (2.05 \times 10^{-24})\sin 60° = (2.05 \times 10^{-24})(0.866) = 1.77 \times 10^{-24}\,\mathrm{N}.$$

Net Force on Particle 1 The net force on particle 1 is:

$$\mathbf{F}_{1,\mathrm{net}} = \mathbf{F}_{12} + \mathbf{F}_{14}.$$

x-Component:

$$F_{1,\mathrm{net},x} = F_{12} + F_{14,x} = -1.15 \times 10^{-24} + 1.03 \times 10^{-24} = -0.12 \times 10^{-24}\,\mathrm{N}.$$

y-Component:

$$F_{1,\mathrm{net},y} = F_{14,y} = 1.77 \times 10^{-24}\,\mathrm{N}.$$

Magnitude: The magnitude of the net force is:

$$F_{1,\mathrm{net}} = \sqrt{(F_{1,\mathrm{net},x})^2 + (F_{1,\mathrm{net},y})^2}.$$

$$F_{1,\mathrm{net}} = \sqrt{(-0.12 \times 10^{-24})^2 + (1.77 \times 10^{-24})^2}.$$

$$F_{1,\mathrm{net}} = \sqrt{(0.0144 \times 10^{-48}) + (3.13 \times 10^{-48})}.$$

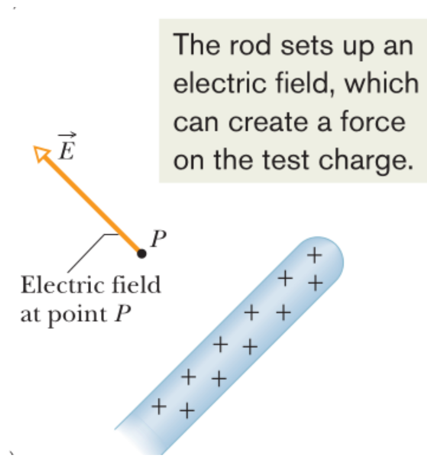
$$F_{1,\mathrm{net}} = \sqrt{3.14 \times 10^{-48}} = 1.77 \times 10^{-24}\,\mathrm{N}.$$

Answer

The net force on particle 1 is:

$$F_{1,\mathrm{net}} = 1.77 \times 10^{-24}\,\mathrm{N},$$

Electric Field:



The electric field is a vector quantity that represents the force per unit charge exerted on a test charge. It is defined as:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where:

- $\mathbf{F}$ is the force experienced by the charge,
- q is the magnitude of the test charge.

Electric Field Due to a Point Charge

For a point charge Q, the electric field at a distance r is given by:

$$\mathbf{E} = k\frac{Q}{r^2}\hat{r},$$

where:

- $k = 8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}$ is Coulomb's constant,
- $\hat{r}$ is the unit vector in the direction of the field.

Electric Field Due to Multiple Charges

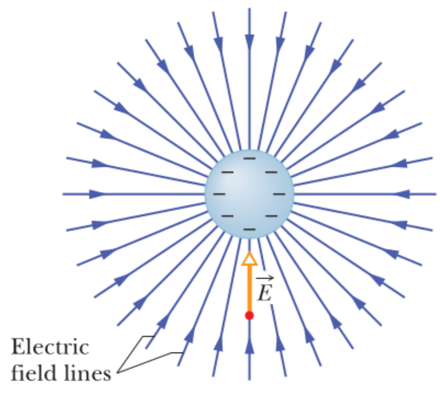
The total electric field due to multiple charges is the vector sum:

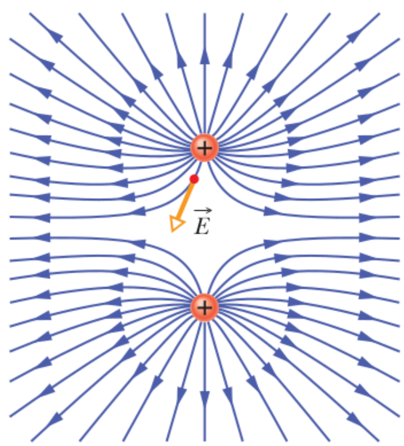
$$\mathbf{E}_{\text{net}} = \sum_i \mathbf{E}_i = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i.$$

Electric Field Lines

Electric field lines represent the direction and strength of the field:

- They start on positive charges and end on negative charges.





- The density of lines indicates the field's strength.

Problem 2:

Two charged particles are attached to an x axis: Particle 1 of charge $+2.00 \times 10^{-7}$ C is at position $x_1 = 6.00$ cm and Particle 2 of charge -2.00×10^{-7} C is at position $x_2 = 21.0$ cm. Midway between the particles, what is their net electric field in unit-vector notation?

Solution

The midpoint is located at:

$$x_{\text{mid}} = \frac{x_1 + x_2}{2} = \frac{6.00 + 21.0}{2} = 13.5 \text{ cm}.$$

The distance from the midpoint to each charge is:

$$r_1 = x_1 - x = 13.5 - 6.00 = 7.5 \text{ cm} = 0.075 \text{ m}.$$

$$r_2 = x - x_2 = 13.5 - 21 = -7.5 \text{ cm} = -0.075 \text{ m}.$$

Calculating the electric field due to each charge:

The electric field magnitude due to a point charge is given by:

$$E = \frac{k|q|}{r^2},$$

where $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ is Coulomb's constant.

Field due to Particle 1:

$$E_1 = \frac{k|q_1|}{r_1^2} = \frac{(8.99 \times 10^9)(2.00 \times 10^{-7})}{(-0.075)^2}.$$

$$E_1 = 3.198 \times 10^4 \text{ N/C}.$$

Field due to Particle 2:

$$E_2 = \frac{k|q_2|}{r_2^2} = \frac{(8.99 \times 10^9)(2.00 \times 10^{-7})}{(0.075)^2}.$$

$$E_2 = 3.198 \times 10^4 \text{ N/C}.$$

Determining the direction of each field:

- Particle 1 has a positive charge, so its field at the midpoint points away from it, i.e., in the $+\hat{\imath}$ direction. - Particle 2 has a negative charge, so its field at the midpoint points toward it, i.e., also in the $+\hat{\imath}$ direction.

Computing the net electric field:

Since both fields point in the same direction, the net electric field is:

$$E_{\text{net}} = E_1 + E_2 = 3.198 \times 10^4 + 3.198 \times 10^4 = 6.396 \times 10^4 \text{ N/C}.$$

In unit-vector notation:

$$\vec{E}_{\text{net}} = (6.396 \times 10^4)\hat{\imath} \text{ N/C}.$$

Answer:

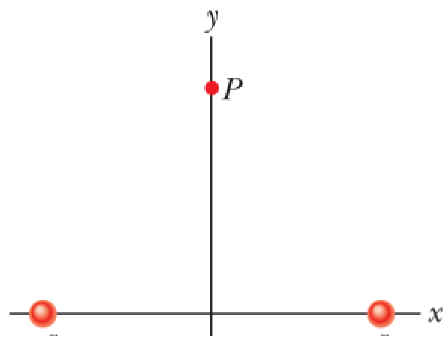
$$\vec{E}_{\text{net}} = 6.40 \times 10^4 \hat{\imath} \text{ N/C}.$$

Problem 3:

Two charges are located on the x axis:

- $q_1 = -3.20 \times 10^{-19}$ **C at** $x = 3.00\,\text{m}$**,**
- $q_2 = +3.20 \times 10^{-19}$ **C at** $x = -3.00\,\text{m}$**.**

Point P **is located at** $y = 4.00\,\text{m}$**.**



What is:

(a) the magnitude and direction of the net electric field relative to the positive x**-axis at point P?**

Solution

Distance between each charge and point P **:**
The distance r from each charge to point P is:

$$r = \sqrt{x^2 + y^2} = \sqrt{(3.00)^2 + (4.00)^2} = \sqrt{9 + 16} = 5.00\,\text{m}.$$

Electric field magnitude due to a single charge:
The electric field magnitude at distance r from a point charge q is given by:

$$E = \frac{k|q|}{r^2},$$

where $k = 8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$. Substituting the given values:

$$E = \frac{(8.99 \times 10^9)(3.20 \times 10^{-19})}{(5.00)^2} = \frac{(8.99)(3.20)}{25} \times 10^{-10} = 1.15 \times 10^{-10}\,\text{N/C}.$$

Horizontal component of the electric field:
The horizontal component of the electric field is given by:

$$E_x = E\cos\theta,$$

where $\cos\theta = \frac{x}{r}$. Substituting $x = 3.00\,\text{m}$ and $r = 5.00\,\text{m}$:

$$\cos\theta = \frac{3.00}{5.00} = 0.6.$$

Thus:

$$E_x = (1.15 \times 10^{-10})(0.6) = 6.90 \times 10^{-11}\,\text{N/C}.$$

Net horizontal electric field:

The two charges produce electric fields in opposite horizontal directions, but their magnitudes are equal. Hence, the net horizontal field is:

$$E_{x,\text{net}} = 2 \times E_x = 2 \times (6.90 \times 10^{-11}) = 1.38 \times 10^{-10}\,\text{N/C}.$$

Direction of the net field :

Both horizontal components point toward the positive x-direction, so the net electric field points in the x-direction.

Answer:

(a) Magnitude:

$$E_{x,\text{net}} = 1.38 \times 10^{-10}\,\text{N/C}.$$

(b) Direction:

The net electric field is in the positive x-direction.

Electric Dipole

An **electric dipole** consists of two equal and opposite charges separated by a small distance. The electric dipole moment $\vec{\mathbf{p}}$ is defined as:

$$\vec{\mathbf{p}} = q \cdot \vec{\mathbf{d}}, \tag{3}$$

where:

- q: Magnitude of each charge,
- $\vec{\mathbf{d}}$: Displacement vector from the negative to the positive charge.

The dipole moment is a vector quantity with units of coulomb-meter (C·m).

Dipole in a Uniform Electric Field

When an electric dipole is placed in a uniform electric field $\vec{\mathbf{E}}$, it experiences:

Force on the Charges

Each charge in the dipole experiences a force due to the electric field:

$$\vec{F}_{\pm} = \pm q\vec{\mathbf{E}}, \tag{4}$$

where $\vec{F}_+$ and $\vec{F}_-$ are the forces on the positive and negative charges, respectively. Since the forces are equal in magnitude and opposite in direction, the net force on the dipole is zero in a uniform field.

Torque on the Dipole

Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions and with the same magnitude $F = qE$. Thus, the net force on the dipole from the field is zero, and the center of mass of the dipole does not move. However, the forces produce a net torque $\vec{\boldsymbol{\tau}}$ on the dipole about its center of mass.

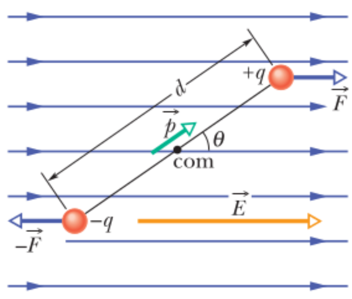


Figure 4: An electric dipole in a uniform external electric field E

From the torque equation $\tau = rF\sin\phi$, the magnitude of the torque on the dipole is:

$$\tau = Fd\sin\theta, \tag{5}$$

where d is the separation between the charges and θ is the angle between $\vec{\mathbf{p}}$ and $\vec{\mathbf{E}}$.

Substituting $F = qE$ and $p = qd$, the torque can also be expressed as:

$$\tau = pE\sin\theta. \tag{6}$$

In vector form, the torque is:

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}. \tag{7}$$

This torque tends to align the dipole moment with the electric field.