Topic: Electric potential, Calculation of capacitance, capacitors with dielectric, Energy storage in an electric field

Definition of Electric Potential

The electric potential at a point is defined as the work done by an external force in bringing a unit positive charge from infinity to that point.

$$V = \frac{W}{q} \tag{1}$$

where:

- V: Electric potential (volts, V)
- W: Work done (joules, J)
- q: Charge (coulombs, C)

Electric Potential Difference

The potential difference between two points is the work done in moving a unit charge from one point to another.

$$\Delta V = V_B - V_A = \frac{W_{A \to B}}{q} \tag{2}$$

Electric Potential Due to a Point Charge

For a point charge q, the electric potential at a distance r from the charge is given by:

$$V = \frac{kq}{r} \tag{3}$$

where:

- k: Coulomb's constant ($k = 8.99 \times 10^9 \, N \, m^2/C^2$)
- q: Charge (coulombs, C)
- r: Distance from the charge (meters, m)

Relation to Electric Field

The electric potential and electric field are related by the equation:

$$\mathbf{E} = -\nabla V \tag{4}$$

In one dimension, this simplifies to:

$$E = -\frac{dV}{dx} \tag{5}$$

Units of Electric Potential

The SI unit of electric potential is the volt (V), where:

$$1\,\text{volt} = 1\,\frac{\text{joule}}{\text{coulomb}} \tag{6}$$

Potential Due to a Group of Charged Particles

The electric potential V at a point in space due to a group of charged particles is given by:

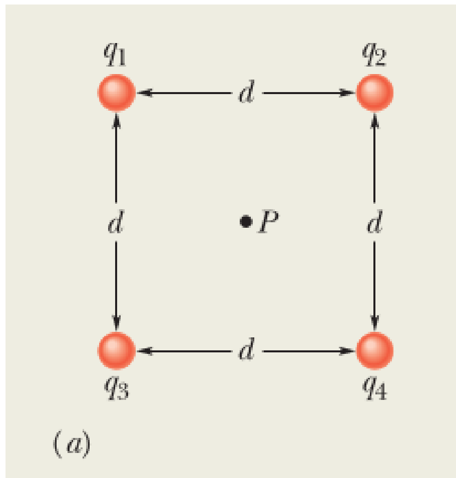
$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}$$

where:

- ϵ_0 is the permittivity of free space,
- q_i is the charge of the i-th particle,
- r_i is the distance from the i-th particle to the point where the potential is being calculated,
- N is the total number of charged particles.

Problem 1: Electric Potential at the Center of a Square of Charges

Calculate the electric potential V_P at point P, located at the center of a square of charged particles.



Given Data

- Distance from each charge to the center: $r = \frac{d}{\sqrt{2}}$, where $d = 1.3m$.
- Charges:

$$q_1 = -12nC,$$
$$q_2 = +24nC,$$
$$q_3 = -31nC,$$
$$q_4 = -17nC.$$

- Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$.

Solution

The electric potential at point P due to a group of charges is given by:

$$V_P = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{4}\frac{q_i}{r}$$

Substituting Known Values

The distance from each charge to the center is:

$$r = \frac{d}{\sqrt{2}} = \frac{1.3m}{\sqrt{2}} = 0.919m.$$

The potential is:

$$V_P = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

Substituting the charges:

$$V_P = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\left(q_1 + q_2 + q_3 + q_4\right).$$

Substitute the numerical values:

$$V_P = \frac{1}{4\pi(8.85\times 10^{-12})}\frac{1}{0.919}\left(-12 + 24 - 31 - 17\right)\times 10^{-9}.$$

V$_P$ =-352.14$V(approximately)$.

Capacitance

Capacitance (C) is a measure of a capacitor's ability to store charge per unit voltage. It is defined as:

$$C = \frac{Q}{V}$$

where:

- C is the capacitance in farads (F),
- Q is the charge stored on the plates in coulombs (C),
- V is the potential difference across the plates in volts (V).

The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\epsilon_0 A}{d}$$

where:

- ϵ_0 is the permittivity of free space (8.85×10^{-12} F/m),
- A is the area of one of the plates in square meters (m^2),
- d is the separation between the plates in meters (m).

Capacitance in Series

When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

To derive an expression for the equivalent capacitance C_{eq} , we first use the relationship for capacitance:

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V across the battery is the sum of the potential differences across each capacitor:

$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}.$$

Rearranging, we can write:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Therefore, the equivalent capacitance is:

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}.$$

For n capacitors connected in series, this relationship generalizes to:

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{C_j}.$$

When capacitors are connected in series, the total capacitance (C_{total}) is given by the reciprocal of the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_{\text{total}}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

For two capacitors C_1 and C_2 in series, the total capacitance is:

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

or equivalently,

$$C_{\text{total}} = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors in Parallel

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor.The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

For a parallel combination of capacitors, the total charge is:

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

The equivalent capacitance C_{eq} is defined such that:

$$q = C_{\text{eq}}V$$

Comparing these equations, we find:

$$C_{\text{eq}} = C_1 + C_2 + C_3$$

For n capacitors in parallel, this result generalizes to:

$$C_{\text{eq}} = \sum_{j=1}^{n} C_j$$

When capacitors are connected in parallel, the total capacitance (C_{total}) is the sum of the individual capacitances:

$$C_{\text{total}} = \sum_{i=1}^{n} C_i$$

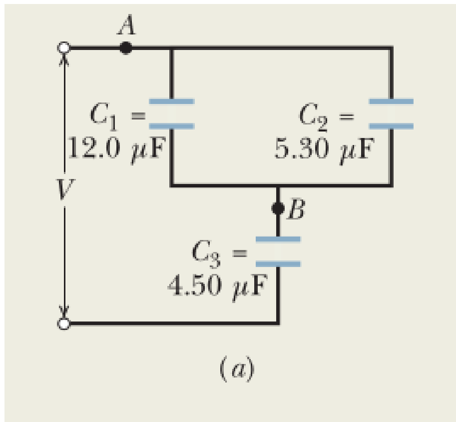
For two capacitors C_1 and C_2 in parallel, the total capacitance is:

$$C_{\text{total}} = C_1 + C_2$$

Problem 2:

Find the equivalent capacitance for the combination of capacitances shown in Fig. 1, across which a potential difference V is applied. Assume the following values for the capacitors:

$$C_1 = 12.0\,\mu\text{F}, \quad C_2 = 5.30\,\mu\text{F}, \quad C_3 = 4.50\,\mu\text{F}.$$



Solution

To calculate the equivalent capacitance C_{123} of the given network, we proceed as follows:

Equivalent Capacitance of Capacitors C_1 and C_2 (Parallel Combination)

Capacitors C_1 and C_2 are in parallel, so their equivalent capacitance is:

$$C_{12} = C_1 + C_2.$$

Substitute the values:

$$C_{12} = 12.0\,\mu\text{F} + 5.30\,\mu\text{F}.$$

$$C_{12} = 17.3\,\mu\text{F}.$$

Equivalent Capacitance of C_{12} and C_3 (Series Combination)

Capacitor C_{12} is in series with C_3. The equivalent capacitance C_{123} is given by:

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}.$$

Substitute the values:

$$\frac{1}{C_{123}} = \frac{1}{17.3\,\mu\text{F}} + \frac{1}{4.50\,\mu\text{F}}.$$

Calculate the reciprocals:

$$\frac{1}{C_{123}} = 0.0578 + 0.2222 = 0.2800\,(\mu\text{F})^{-1}.$$

Take the reciprocal to find C_{123}:

$$C_{123} = \frac{1}{0.2800} \approx 3.57\,\mu\text{F}.$$

Answer

The equivalent capacitance of the network is:

$$\boxed{3.57\,\mu\text{F}}.$$

Question:

(b) The potential difference applied to the input terminals in Fig.is $V = 12.5\,\text{V}$. What is the charge on capacitor C_1?

Solution

To calculate the charge q_1 on capacitor C_1, we proceed as follows:

Equivalent Capacitance for the Entire Combination

The potential difference $V = 12.5\,\text{V}$ is applied across the entire network of capacitors. The equivalent capacitance of the combination is:

$$C_{123} = 3.57\,\mu\text{F}.$$

Using the relationship $q = CV$, the total charge stored in the equivalent capacitance is:

$$q_{123} = C_{123} \cdot V = (3.57\,\mu\text{F}) \cdot (12.5\,\text{V}) = 44.6\,\mu\text{C}.$$

Charge on Series Capacitors

For capacitors in series, the charge is the same across all capacitors. Thus, the charge on C_{12} (the parallel combination of C_1 and C_2) and C_3 is:

$$q_{12} = q_{123} = 44.6\,\mu\text{C}.$$

Potential Difference Across C_{12}

The equivalent capacitance of C_{12} (the parallel combination of C_1 and C_2) is:

$$C_{12} = C_1 + C_2 = 12.0\,\mu\text{F} + 5.30\,\mu\text{F} = 17.3\,\mu\text{F}.$$

The potential difference across C_{12} is:

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\,\mu\text{C}}{17.3\,\mu\text{F}} \approx 2.58\,\text{V}.$$

Potential Difference Across C_1

For capacitors in parallel, the potential difference is the same across each capacitor. Thus, the potential difference across C_1 is:

$$V_1 = V_{12} = 2.58\,\text{V}.$$

Charge on C_1

The charge on C_1 is given by:

$$q_1 = C_1 \cdot V_1 = (12.0\,\mu\text{F}) \cdot (2.58\,\text{V}) = 31.0\,\mu\text{C}.$$

Final Answer

The charge on C_1 is:

$$\boxed{31.0\,\mu\text{C}}.$$

ENERGY STORED IN AN ELECTRIC FIELD

Energy must be supplied by an external agent to charge a capacitor. This work is stored as electric potential energy in the electric field between the plates. We can express the electric potential energy U of a charged capacitor as:

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2,$$

where q is the charge, C is the capacitance, and V is the potential difference across the capacitor.

Problem: 3

An isolated conducting sphere with a radius $R = 6.85\,\mathrm{cm} = 0.0685\,\mathrm{m}$ has a charge $q = 1.25\,\mathrm{nC} = 1.25 \times 10^{-9}\,\mathrm{C}$. How much potential energy is stored in the electric field of this charged conductor?

Solution:

The potential energy U of a charged conducting sphere is given by:

$$U = \frac{q^2}{2C}$$

where C is the capacitance of the sphere, and is given by:

$$C = 4\pi\varepsilon_0 R$$

Substituting the capacitance formula into the potential energy equation:

$$U = \frac{q^2}{2 \times 4\pi\varepsilon_0 R}$$

Substitute the given values of $q = 1.25 \times 10^{-9}\,\mathrm{C}$, $R = 0.0685\,\mathrm{m}$, and $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$:

$$U = \frac{(1.25 \times 10^{-9})^2}{2 \times 4\pi(8.85 \times 10^{-12})(0.0685)}$$

Simplifying the expression:

$$U \approx 1.03 \times 10^{-7}\,\mathrm{J} = 103\,\mathrm{nJ}$$

Thus, the potential energy stored in the electric field of the charged conductor is:

$$U \approx 103\,\mathrm{nJ}$$

CAPACITOR WITH A DIELECTRIC

When the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's dielectric constant k, which is a number greater than 1. The capacitance C' with the dielectric material is given by:

$$C' = kC$$

where: - C' is the capacitance with the dielectric material, - C is the capacitance in vacuum or air, - k is the dielectric constant of the material (with $k > 1$).

Problem 4:

A parallel-plate capacitor with capacitance $C = 13.5\,\text{pF}$ is charged by a battery to a potential difference $V = 12.5\,\text{V}$ between its plates. The charging battery is now disconnected, and a porcelain slab with dielectric constant $k = 6.50$ is inserted between the plates.

(a)What is the potential energy of the capacitor before the slab is inserted?

Solution:

The potential energy U stored in a capacitor is given by:

$$U = \frac{1}{2}CV^2$$

Where: - $C = 13.5\,\text{pF} = 13.5 \times 10^{-12}\,\text{F}$, - $V = 12.5\,\text{V}$.

Substitute the given values into the formula for potential energy:

$$U = \frac{1}{2}(13.5 \times 10^{-12})(12.5)^2$$

Calculating this gives:

$$U \approx 1.05 \times 10^{-9}\,\text{J} = 1.05\,\text{nJ}$$

Thus, the potential energy of the capacitor before the slab is inserted is:

$$U \approx 1.05\,\text{nJ}$$

Problem (b): What is the potential energy of the capacitor–slab device after the slab is inserted?

Solution:

The potential energy of the capacitor with the dielectric inserted is given by:

$$U' = \frac{q^2}{2C'}$$

where: - U' is the potential energy after the dielectric is inserted, - q is the charge on the capacitor (which remains constant), - C' is the capacitance of the capacitor with the dielectric material.

The capacitance with the dielectric is given by:

$$C' = kC$$

where $k = 6.50$ is the dielectric constant of the porcelain slab, and C is the original capacitance.

Since the charge remains constant, the potential energy with the dielectric is:

$$U' = \frac{q^2}{2kC}$$

We can express q in terms of the initial capacitance and potential difference:

$$q = CV$$

Substituting this into the equation for potential energy:

$$U' = \frac{(CV)^2}{2kC}$$

Simplifying:

$$U' = \frac{CV^2}{2k}$$

Substitute the given values of $C = 13.5 \times 10^{-12}\,\text{F}$, $V = 12.5\,\text{V}$, and $k = 6.50$:

$$U' = \frac{(13.5 \times 10^{-12})(12.5)^2}{2 \times 6.50}$$

Calculating this gives:

$$U' \approx 1.62 \times 10^{-9}\,\text{J} = 1.62\,\text{nJ}$$

Thus, the potential energy of the capacitor–slab device after the porcelain slab is inserted is:

$$U' \approx 1.62\,\mathrm{nJ}$$