

Set: Sets play a crucial role in almost all branches of mathematics and are being increasingly used in economics, business, and finance. It is sometimes convenient to consider many items together. Such a collective entity is called a set. A set is defined as any well-defined list, collection, or class of objects. The objects in a set can be anything: students, numbers, vehicles, countries, trees, or anything else. Examples of sets include:

The people living in the city of Gopalganj.

The even numbers between 0 and 10.

The odd numbers between 0 and 10.

The numbers 1, 2, 3, 4, and 5.

Definition 1: A set is any collection of well-defined and well-distinguished objects.

Definition 2: A set is any collection of objects such that given an object, it is possible to determine whether that object belongs to the given collection or not.

- ❖ Sets are usually denoted by uppercase letters such as A, B, C, X, Y, Z , etc.
- ❖ The objects in a set are called the elements or members of the set. These objects are usually denoted by lowercase letters such as a, b, c, x, y, z , etc.
- ❖ If x is an object in the set A , then x is called an element of the set and is denoted as $x \in A$, and is read “ x belongs to A ” or “ x is a member of A ”. If x is not an object in A , then we may write it as $x \notin A$, and is read “ x does not belong to A ” or “ x is not a member of A ”.
- ❖ We can represent a set by listing its elements and using $\{ \}$ notation. Assume that the set A consists of numbers 2, 4, 6, 8, and 10. Then we may write the set A as $A = \{2, 4, 6, 8, 10\}$. We call this form of representation of a set the *tabular form*.
- ❖ Sets can also be represented by stating properties that its elements must satisfy. Assume that we want a set B of even numbers. Then we may write it as $B = \{x \mid x \text{ is even}\}$ which we read as “ B is the set of numbers x such that x is even.” This form of representation of a set is called the *set-builder form*.

Subset: Let there be two sets A and B . If every element in A is also an element of B , then A is called a subset of B . In other words, A is a subset of B if $a \in A$ and $a \in B$, and is denoted as $A \subseteq B$. For example, let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$. Since the elements 1, 2, and 3 appear in both sets and since B contains more elements than A does, then $A \subseteq B$.

Proper subset: Let there be two sets A and B . Then A is called a proper subset of B if $A \subseteq B$ and $A \neq B$, and is denoted as $A \subset B$. As an example, if $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$, then $A \subset B$.

Equality of two sets: Two sets A and B are said to be equal if they have the same elements; that is, if every element in A also belongs to B and if every element in B also belongs to A . Let $A = \{9, 8, 7, 6\}$ and $B = \{8, 7, 9, 6\}$. Then $A = B$. Notice that a set does not change if its elements are rearranged. Notice also that the set $\{1, 2, 3, 3, 4\} = \{1, 2, 3, 4\}$.

Null set: Any set which has no element is called null or empty set. It is denoted by the Greek letter ϕ (phi). As an example, let A be a set of people who are neither dead nor alive. We can write this set using the set-builder for as $A=\{x \mid x \text{ is a person who is neither dead nor alive}\}$. We know that this set is a null or empty set. Notice that ϕ is considered to be a subset of all other sets.

Universal set: The universal set consists of all the objects that are being considered in a particular situation. It is generally denoted by U . The set of integers may be considered as a universal set for a set of odd or even integers. i.e,

$$A=\{x \mid x \text{ is an even integer}\}$$

$$B=\{x \mid x \text{ is an odd integer}\}$$

$$U=\{x \mid x \text{ is an integer}\}$$

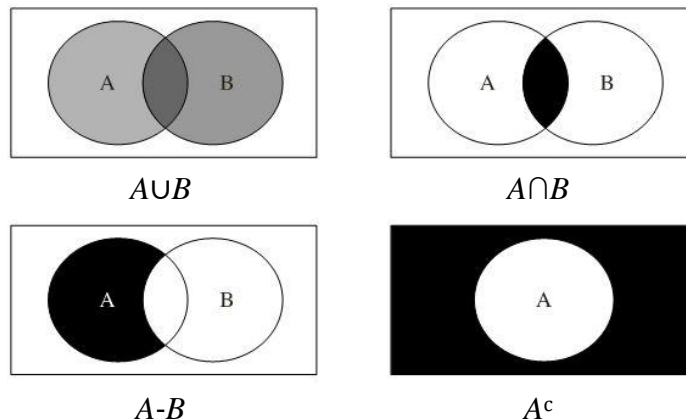
Complement of a set: The complement of a set is the set of all elements that are not the elements of a particular set (say A) but are of U . The complement of a set, say, A is denoted by A' or A^c . For example, if $U=\{1,2,3,4,5\}$ and $A=\{3,4,6\}$, then $A'=\{1,2,5\}$.

Union of sets: The union of two sets A and B is defined as the set of all elements which belong to A , or to B , or to both A and B . We denote the union of sets A and B by $A \cup B$, which is read “ A union B .” Let $A=\{1,2,3,4\}$ and $B=\{4,3,5,6\}$. Then $A \cup B=\{1,2, 3, 4, 5, 6\}$.

Intersection of sets: The intersection of two sets A and B is defined as the set of elements that are common to A and B , and is denoted by $A \cap B$, which is read “ A intersection B .” In our last example, $A \cap B=\{3,4\}$.

Difference between two sets: The difference of two sets A and B is defined as the set of elements which belong to A but not to B and is denoted by $A - B$, which is read “ A difference B ” or “ A minus B .” In our last example, $A - B=\{1,2\}$. Notice that $B - A=\{5,6\}$.

Venn diagram: A useful way of representing sets and their operations is the Venn diagram, named after the English logician and mathematician John Venn. In a Venn diagram, the universal set U is represented by a square or a rectangle within which individual sets are shown as circles. The Venn diagram representations of union, intersection, difference, and complement are illustrated by the shaded areas in the following figures, respectively.



Different laws of sets

1. Associative laws: If A, B, C are any three sets, then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$.

Proof:

<p>(i) Let $x \in (A \cup B) \cup C$ $\Rightarrow x \in A \cup B$ or $x \in C$ $\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$ $\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$ $\Rightarrow x \in A$ or $x \in B \cup C$ $\Rightarrow x \in A \cup (B \cup C)$ $\therefore (A \cup B) \cup C \subseteq A \cup (B \cup C)$</p> <p>Also let $y \in A \cup (B \cup C)$ $\Rightarrow y \in A$ or $(y \in B$ or $y \in C)$ $\Rightarrow (y \in A$ or $y \in B)$ or $y \in C$ $\Rightarrow y \in A \cup B$ or $y \in C$ $\Rightarrow y \in (A \cup B) \cup C$ $\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C$</p> <p>Therefore, $(A \cup B) \cup C = A \cup (B \cup C)$ [Proved]</p>	<p>(ii) Let $x \in (A \cap B) \cap C$ $\Rightarrow (x \in A \cap B)$ and $x \in C$ $\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$ $\Rightarrow x \in A$ and $(x \in B$ and $x \in C)$ $\Rightarrow x \in A$ and $x \in B \cap C$ $\Rightarrow x \in A \cap (B \cap C)$ $\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C)$</p> <p>Also let $y \in A \cap (B \cap C)$ $\Rightarrow y \in A$ and $(y \in B$ and $y \in C)$ $\Rightarrow (y \in A$ and $y \in B)$ and $y \in C$ $\Rightarrow y \in A \cap B$ and $y \in C$ $\Rightarrow y \in (A \cap B) \cap C$ $\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C$</p> <p>Therefore, $(A \cap B) \cap C = A \cap (B \cap C)$ [Proved]</p>
--	--

2. Commutative law: If A and B are any two sets, then (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$.

3. Distributive law: If A, B, C are any three sets, then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. De Morgan's law: If A and B are any two sets, then (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

5. De Morgan's law in case of three sets: If A, B and C are any three sets, then (i) $(A \cup B \cup C)' = A' \cap B' \cap C'$ (ii) $(A \cap B \cap C)' = A' \cup B' \cup C'$.

6. De Morgan's law on difference of sets: If A, B, C are any three sets, then

(i) $A - (B \cup C) = (A - B) \cap (A - C)$ (ii) $A - (B \cap C) = (A - B) \cup (A - C)$.

7. If A, B, C are any three finite sets, then (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Cartesian product of two sets: If A and B be any two sets then the set of all ordered pairs whose first members belongs to set A and second members belongs to set B is called the Cartesian product of A and B in that ordered and is denoted by $A \times B$, to be read as "A cross B".

In other words, if A, B are two sets, then the set of all ordered pairs of the form (x, y) , (where $x \in A$ and $y \in B$) is called the Cartesian product of the sets A and B . Mathematically,

$$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$$

Example: If $A = \{a, b, c\}$, $B = \{1, 2\}$ then

$$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$$

$$B \times A = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$$

Note that: $A \times B \neq B \times A$

Solution of some problems

3-38

Chapter-3 : Busi

Example-18 $A = \{a, b\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{3, 5, 7, 9\}$. Find $(A \times B) \cap (A \times C)$

Solution

$$(A \times B) \cap (A \times C) = A \times (B \cap C)$$

$$[\because (A \times B) \cap (A \times C) = A \times (B \cap C)]$$

$$B \cap C = \{1, 2, 3, 4, 5\} \cap \{3, 5, 7, 9\} = \{3, 5\}$$

$$\begin{aligned} \therefore \{A \times B\} \cap \{A \times C\} &= A \times (B \cap C) \\ &= \{a, b\} \times \{3, 5\} \\ &= \{(a, 3), (a, 5), (b, 3), (b, 5)\} \text{ (Ans.)} \end{aligned}$$

Example-19 If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 5\}$, $D = \{7\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, compute $(A' - B') \times (B - C)'$ [NU, BBA (Part-1) 2009]

Solution

Given, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 5\}$, $D = \{7\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

then

$$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 3, 4\} = \{5, 6, 7, 8\}$$

$$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 4, 6\} = \{1, 3, 5, 7, 8\}$$

$$A' - B' = \{5, 6, 7, 8\} - \{1, 3, 5, 7, 8\} = \{6\}$$

$$B - C = \{2, 4, 6\} - \{1, 2, 5\} = \{4, 6\}$$

$$\begin{aligned} \text{so that, } (B - C)' &= U - (B - C) = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 6\} \\ &= \{1, 2, 3, 5, 7, 8\} \end{aligned}$$

$$\text{Now } (A' - B') \times (B - C)' = \{6\} \times \{1, 2, 3, 5, 7, 8\}$$

$$= \{(6, 1), (6, 2), (6, 3), (6, 5), (6, 7), (6, 8)\} \text{ (Ans.)}$$

Example-7 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 3, 4, 5\}$, $B = \{4, 6, 8\}$, $C = \{3, 4, 5, 6, 7\}$. Find $A', B', (A \cap C)', (A \cup B)', (A')', (B - C)'$.

Solution

$$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 4, 5\} \\ = \{1, 6, 7, 8, 9\}$$

$$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{4, 6, 8\} \\ = \{1, 2, 3, 5, 7, 9\}$$

$$A \cap C = \{2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} = \{3, 4, 5\}$$

$$\therefore (A \cap C)' = U - (A \cap C) = \{1, 2, 6, 7, 8, 9\}$$

$$A \cup B = \{2, 3, 4, 5, 6, 8\}$$

$$\therefore (A \cup B)' = U - (A \cup B) = \{1, 7, 9\}$$

$$A' = U - A = \{1, 6, 7, 8, 9\}$$

$$\therefore (A')' = \{2, 3, 4, 5\}$$

It should be noticed that $(A')' = A$

$$B - C = \{x : x \in B, x \notin C\} \\ = \{4, 6, 8\} - \{3, 4, 5, 6, 7\} \\ = \{8\}$$

$$\therefore (B - C)' = U - (B - C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{8\} \\ = \{1, 2, 3, 4, 5, 6, 7, 9\}$$

Example-8 If $A = \{3, 5, 7, 9\}$, $B = \{5, 6, 7, 8\}$ and $C = \{x \in \mathbb{N}, 2 \leq x \leq 6\}$, find

- (i) $A - B$ (ii) $B - C$ (iii) $A - (B - C)$,
(iv) $(A \cup B) \cap C$, (v) $A - (B \cap C)$.

[NU, BBA (Part-I) 2006, 2008]

Solution Given $A = \{3, 5, 7, 9\}$, $B = \{5, 6, 7, 8\}$ and $C = \{x : x \in \mathbb{N}, 2 \leq x \leq 6\} = \{2, 3, 4, 5, 6\}$

Now,

$$(i) A - B = \{3, 5, 7, 9\} - \{5, 6, 7, 8\} = \{3, 9\} \text{ (Ans.)}$$

$$(ii) B - C = \{5, 6, 7, 8\} - \{2, 3, 4, 5, 6\} = \{7, 8\} \text{ (Ans.)}$$

$$(iii) A - (B - C) = \{3, 5, 7, 9\} - [\{5, 6, 7, 8\} - \{2, 3, 4, 5, 6\}] \\ = \{3, 5, 7, 9\} - \{7, 8\} = \{3, 5, 9\} \text{ (Ans.)}$$

$$(iv) (A \cup B) \cap C = [\{3, 5, 7, 9\} \cup \{5, 6, 7, 8\}] \cap \{2, 3, 4, 5, 6\} \\ = \{3, 5, 6, 7, 8, 9\} \cap \{2, 3, 4, 5, 6\} \\ = \{3, 5, 6\} \text{ (Ans.)}$$

$$(v) A - (B \cap C) = \{3, 5, 7, 9\} - [\{5, 6, 7, 8\} \cap \{2, 3, 4, 5, 6\}] \\ = \{3, 5, 7, 9\} - \{5, 6\} = \{3, 7, 9\} \text{ (Ans.)}$$

Example-9 If $A = \{4, 6, 8, 10\}$, $B = \{x : x \in \mathbb{N}, 6 \leq x \leq 9\}$ and $C = \{x : x \in \mathbb{N}, \frac{5}{2} < x \leq 7\}$, find

- (i) $A - B$; (ii) $A - (B - C)$; (iii) $A \cup (B \cap C)$; (iv) $A - (B \cup C)$

[NU, BBA (Part-I) 2011]

Solution Given $A = \{4, 6, 8, 10\}$;

$$B = \{x : x \in \mathbb{N}, 6 \leq x \leq 9\} = \{6, 7, 8, 9\}$$

$$\text{and } C = \left\{x : x \in \mathbb{N}, \frac{5}{2} < x \leq 7\right\} = \{3, 4, 5, 6, 7\}$$

$$(i) A - B = \{4, 6, 8, 10\} - \{6, 7, 8, 9\} = \{4, 10\} \text{ (Ans.)}$$

$$(ii) A - (B - C) = \{4, 6, 8, 10\} - [\{6, 7, 8, 9\} - \{3, 4, 5, 6, 7\}] \\ = \{4, 6, 8, 10\} - \{8, 9\} = \{4, 6, 10\} \text{ (Ans.)}$$

$$(iii) A \cup (B \cap C) = \{4, 6, 8, 10\} \cup [\{6, 7, 8, 9\} \cap \{3, 4, 5, 6, 7\}] \\ = \{4, 6, 8, 10\} \cup \{6, 7\} \\ = \{4, 6, 7, 8, 10\} \text{ (Ans.)}$$

$$(iv) A - (B \cup C) = \{4, 6, 8, 10\} - [\{6, 7, 8, 9\} \cup \{3, 4, 5, 6, 7\}] \\ = \{4, 6, 8, 10\} - \{3, 4, 5, 6, 7, 8, 9\} \\ = \{10\} \text{ (Ans.)}$$

Example-10 Prove that, [अमान्य करण करें]

$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ when $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 5, 6, 7\}$

Solution $A - B = \{1, 2, 3, 4, 5\} - \{2, 3, 5, 6, 7\} = \{1, 4\}$

$$B - A = \{2, 3, 5, 6, 7\} - \{1, 2, 3, 4, 5\} = \{6, 7\}$$

$$\therefore (A - B) \cup (B - A) = \{1, 4, 6, 7\}$$

Again, $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ and $A \cap B = \{2, 3, 5\}$

$$\therefore (A \cup B) - (A \cap B) = \{1, 2, 3, 4, 5, 6, 7\} - \{2, 3, 5\} \\ = \{1, 4, 6, 7\}$$

Therefore, $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ [Proved]

Example-11 Let the universal set $U = \{a, b, c, d, e, f, g\}$

and $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$, $C = \{b, e, f, g\}$

Find : (i) $B' \cup C$; (ii) $(A - B)'$; (iii) $(A \cap A)'$;

(iv) $(A' - B)$; (v) $(C' \cap A)$ [NU, (Hons) 2000]

Solution Given, $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$,

$$C = \{b, e, f, g\}, U = \{a, b, c, d, e, f, g\}$$

Now, $A' = U - A = \{a, b, c, d, e, f, g\} - \{a, b, c, d, e\} = \{f, g\}$,

$$B' = U - B = \{a, b, c, d, e, f, g\} - \{a, c, e, g\} = \{b, d, f\},$$

$$C' = U - C = \{a, b, c, d, e, f, g\} - \{b, e, f, g\} = \{a, c, d\}$$

$$(i) B' \cup C = \{b, d, f\} \cup \{b, e, f, g\}$$

$$= \{b, d, e, f, g\} \text{ (Ans.)}$$

$$\begin{aligned}\text{Therefore, } n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 400 + 200 - 50 \\ &= 550\end{aligned}$$

This exceeds the total number of car-owners investigated. So, the given data is not correct.

Example-45 In an examination 56% failed in English and 37% failed in Mathematics. If only 17% failed in both the subjects. How many of them passed in the both subject. Determine in percentage. Using Venn diagram.

[কোনো পরীক্ষায় 56% ইংরেজিতে ও 37% গণিতে ফেল করেছে। যদি কেবলমাত্র 17% উভয় বিষয়ে ফেল করে থাকে, তবে শতকরা কতজন উভয় বিষয়ে পাস করেছে তা ভেনচিত্রের সাহায্যে নির্ণয় কর।]

[NU, BBA (Hons) 2003; 2012]

Solution

Let, The set of all examinee = U

The set of students who failed in English = E

The set of students who failed in Mathematics = M

The set of students who failed in both English and Mathematics = $E \cap M$

Given that, $n(U) = 100\%$

$$n(E) = 56\%$$

$$n(M) = 37\%$$

$$n(E \cap M) = 17\%$$

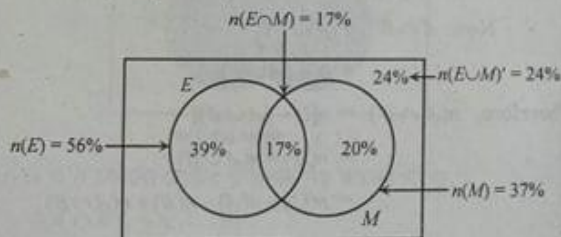


Fig : Venn diagram

We know, $n(E \cup M)' = n(U) - n(E \cup M)$

$$\begin{aligned}&= n(U) - [n(E) + n(M) - n(E \cap M)] \\ &= n(U) - n(E) - n(M) + n(E \cap M) \\ &= 100\% - 56\% - 37\% + 17\% \\ &= 117\% - 93\% = 24\% \text{ (Ans.)}\end{aligned}$$

Example-46 In a class of 25 students, out of them 12 student have taken Economics; 8 students have taken Economics but not Marketing. Find

- The number of students who have taken Economics and marketing?
- Those who have taken marketing but not Economics?
- Those who have taken only one subject?

[একটি শ্রেণীতে 25 জন ছাত্রছাত্রী আছে। যাদের মধ্যে 12 জন অর্থনীতি নিয়েছে। 8 জন অর্থনীতি নিয়েছে কিন্তু মার্কেটিং নয়। বের কর—

- অর্থনীতি এবং মার্কেটিং নিয়েছে কতজন ছাত্র ছাত্রী?
- কতজন ছাত্র ছাত্রী মার্কেটিং নিয়েছে কিন্তু অর্থনীতি নয়?
- কতজন ছাত্রছাত্রী শুধু মাত্র একটি বিষয় নিয়েছে?

[NU, BBA (Hons) Mgt. 2015, 2006]

NU, BBA (Part-I) 2012, 2013; JNU, (Mkt.) 2012, 2013]

Solution

Let, The set of all students = U

The set of student who have taken Economics = A

The set of student who have taken Marketing = B

we have to find : (i) $n(A \cap B)$

$$(ii) n(A' \cap B)$$

$$(iii) n(A \cap B') + n(A' \cap B)$$

Given, $n(U) = 25$, $n(A) = 12$, $n(A \cap B') = 8$

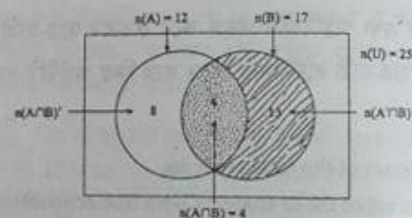


Fig : Venn diagram

(i) We know $n(A \cap B') = n(A) - n(A \cap B)$

$$\Rightarrow 8 = 12 - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 12 - 8$$

$$\therefore n(A \cap B) = 4 \text{ (Ans.)}$$

Again, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\Rightarrow 25 = 12 + n(B) - 4$$

$$\Rightarrow 25 - 12 + 4 = n(B)$$

$$\therefore n(B) = 17$$

(ii) $n(B \cap A') = n(B) - n(A \cap B) = 17 - 4 = 13 \text{ (Ans.)}$

(iii) $n(A \cap B') + n(A' \cap B) = 8 + 13 = 21 \text{ (Ans.)}$

Example-47 On a recent holiday morning 325 persons stopped by a news stand. Of these, 185 bought the Daily Star, 150 bought the Bangladesh Observer and 95 bought both papers. How many persons did not purchase either Newspaper? How many persons purchased the Daily Star but not the Observer? How many persons purchased the Observer but not the Daily Star? How many persons purchased at least one of the Newspaper?

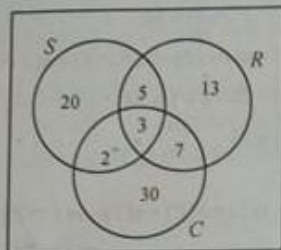


Fig : Venn diagram

(ii) We want to find $n[C - (R \cup S)]$

$$\begin{aligned} \text{Now } n[C - (R \cup S)] &= n(C) - n[C \cap (R \cup S)] \\ &= n(C) - n[(C \cap R) \cup (C \cap S)] \\ &= n(C) - n(C \cap R) - n(C \cap S) \\ &\quad + n(C \cap R \cap S) \\ &= 42 - 10 - 5 + 3 = 30 \text{ (Ans.)} \end{aligned}$$

(iii) We want to find $n[(S \cap C) - R]$

$$\begin{aligned} \text{Now } n[(S \cap C) - R] &= n(S \cap C) - n(S \cap C \cap R) \\ &= 5 - 3 = 2 \text{ (Ans.)} \end{aligned}$$

Example-59 In a survey of 1000 customers, the number of people that buy various grades of coffee seed were found to be as follows :

A grade only	... 180
A grade and C grade	... 80
A grade but not B grade	... 230
Non-grade of the three	... 240
A grade	... 260
C grade	... 480
C grade and B grade	... 80

- (i) How many buy B grade, coffee seeds?
 (ii) How many buy C grade, if and only if they do not buy B grade?
 (iii) How many buy C and B grade but not the A grade?

[NU, BBA (Part-I) 2012]

Solution

Let U = set of all surveyed consumers

A = set of all people who buy A grade coffee

B = set of all people who buy B grade coffee

C = set of all people who buy C grade coffee

Given, $n(A \cap B' \cap C') = 180$, $n(A \cap C) = 80$, $n(A \cap B') = 230$,
 $n(A) = 260$, $n(C) = 480$,

$$n(B \cap C) = 80, n(A' \cap B' \cap C') = n(A \cup B \cup C)' = 240$$

Requirements :

- (i) Number of people who buy B grade coffee seeds, $n(B) = ?$
 (ii) Number of people who buy C grade if and only if they do not buy B grade, $n(C \cap B') = ?$

(iii) Number of people who buy C and B grade but not A grade, $n(B \cap C \cap A') = ?$

$$\begin{aligned} \text{(i) Given } (A \cap B') &= 230 \\ \Rightarrow n(A) - n(A \cap B) &= 230 \\ \Rightarrow n(A \cap B) &= n(A) - 230 \\ &= 260 - 230 \\ \therefore n(A \cap B) &= 30 \end{aligned}$$

$$\begin{aligned} \text{Also given } n(A \cup B \cup C)' &= 240 \\ \Rightarrow n(U) - n(A \cup B \cup C) &= 240 \\ \Rightarrow 1000 - n(A \cup B \cup C) &= 240 \\ \Rightarrow n(A \cup B \cup C) &= 1000 - 240 \\ \therefore n(A \cup B \cup C) &= 760 \end{aligned}$$

$$\text{and } n(A \cup B' \cup C') = 180$$

$$\begin{aligned} \Rightarrow n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) &= 180 \\ \Rightarrow n(A \cap B \cap C) &= 180 - n(A) + n(A \cap B) + n(A \cap C) \\ &= 180 - 260 + 30 + 80 = 30 \\ \therefore n(A \cap B \cap C) &= 30 \end{aligned}$$

$$\begin{aligned} \text{We know, } n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) \\ &\quad - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\ \Rightarrow 760 &= 260 + n(B) + 480 - 30 - 80 - 80 + 30 \\ \Rightarrow 720 &= n(B) + 580 \\ \therefore n(B) &= 140 \text{ (Ans.)} \end{aligned}$$

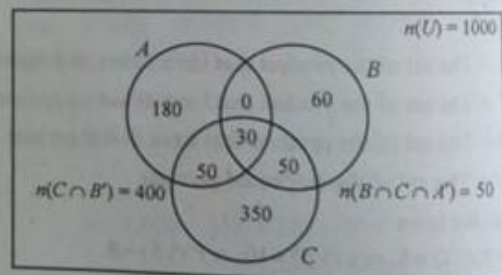


Fig : Venn diagram

$$\text{(ii) } n(C \cap B') = n(C) - n(B \cap C) = 480 - 80 = 400 \text{ (Ans.)}$$

$$\text{(iii) } n(B \cap C \cap A') = n(B \cap C) - n(A \cap B \cap C) = 80 - 30 = 50 \text{ (Ans.)}$$

Example-60 In a survey conducted of women it was found that

- (i) there are more single than married women in South Delhi,
 (ii) there are more married women who own cars than unmarried women without them,
 (iii) there are fewer single women who own cars and homes than married women without cars and without homes.
 Is the number of single women who own cars and do not own homes greater than number of married women who do not own cars but own homes?