Probabilistic Connectivity of Underwater Sensor Networks

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Overview

Why UWSNs?

UWSNs fuelled by many important underwater sensing applications and services such as

- **Scientific applications**: e.g., observing geological processes on the ocean floor, determining water characteristics, counting or imaging animal life
- **Industrial applications**: e.g., monitoring and control of commercial activities, determining routes for underwater cables, monitoring underwater equipment and pipelines for oil and mineral extraction, and monitoring commercial fisheries
- **Military and homeland security applications**: e.g., monitoring and securing port facilities
- **Humanitarian applications**: e.g., search and survey missions, disaster prevention tasks, identification of seabed hazards, locating dangerous rocks or shoals, and identifying possible mooring locations

Challenges of the underwater communication channel

Radio Communication

- suffer strong attenuation in salt water
- short distances (6-20 m) and low data rates (1 Kbps)
- require large antennas and high transmission power

Optical Communication

- strongly scattered and absorbed underwater
- limited to short distances (40 m)

Acoustic Communication

- suffers from attenuation, spreading, and noise
- very long delay because of low propagation speed.
- it is most practical method upto now

Node Deployment Strategy

Static Deployment

- nodes attached to underwater ground, anchored buoys, or docks
- are not subject to move

Semi-mobile Deployment

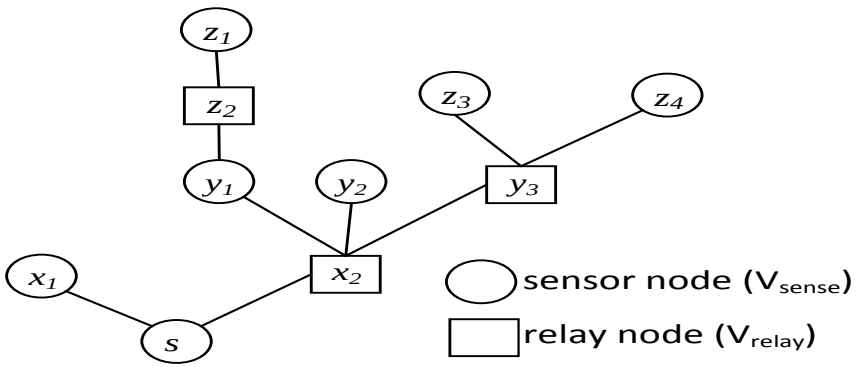
- nodes attached to a free floating buoy
- subject to small scale movement

Mobile Deployment

- composed of drifters with self/noself mobile capability
- are subject to large scale movement
- maintaining connectivity is important to perform localization, routing etc.

Node Locality Sets

- $V = V_{sense} \cup V_{relay}$ the set of nodes in a given UWSN
- the geographic area considered rectangles of a superimposed grid layout.
- at time T, each node x can be in any one of a possible set of grid rectangles denoted $\mathrm{Loc}(x) = \{x[1], x[2], \ldots\}$.
- node x can be grid rectangle $x[i]$ with a certain probability $p_x(i)$.
- truncate some locality sets of low probability for convenience thus, $\sum_{x[i] \in \mathrm{Loc}(x)} p_x(i) \leq 1$, if $\mathrm{Loc}(x)$ is truncated.



x_1	0.2	0.5	0.1	z_1	0.6 0.2
	0.07	0.05	0.08		0.1 0.1
x_2	0.25	0.3		z_2	0.5 0.25 0.2
	0.16	0.29			
y_1	0.2	0.3	0.1	z_3	0.25 0.36
	0.1	0.15	0.15		0.14 0.25
y_2	0.25	0.35		z_4	0.7 0.15 0.15
	0.25	0.15			
y_3	0.16	0.18	0.14	s	0.18 0.1 0.03
	0.3	0.22			0.34 0.1 0.14

Figure 1: Network with Probabilistic locality set

Node Reachability

- node x can reach node y if the acoustic signal strength from x to y (and vice versa) exceeds a certain threshold value.
- if x and y can reach each other then $E_G(x, y) = 1$ otherwise $E_G(x, y) = 0$.
- we compute lower bounds on the likelihood that the network is totally, or partially, connected.
- we set $E_G(x[i], y[j]) = 1$ iff the two nodes x and y can reach each other if they are located anywhere in their respective rectangles $x[i]$ and $y[j]$.
- connectivity between x and y is ignored if they can reach each other at some (but not all) pairs of points in their respective rectangles.
- ignoring connectivity in such cases results in computing lower bounds on the network connectivity, as required.

Kinematic Model

We note that this area is new to networking researchers where the obtained analytical results are rooted in the mathematically deep field of fluid dynamics.

- A particle pathline is a path followed by an individual particle in a flow
- A *stream* function denoted by ψ measures the volume flow rate per unit depth.
- Curves where ψ is constant are called *streamlines*

The stream function can be presented

$$\psi(x, y, t) = -\tanh\left[\frac{y - B(t)\sin(k(x - ct))}{\sqrt{1 + k^2 B^2(t)\cos^2(k(x - ct))}}\right] + cy \tag{1}$$

where $B(t) = A + \epsilon\cos(\omega t)$ and the x and y velocities are given by

$$\dot{x} = -\frac{\partial\psi}{\partial y}; \dot{y} = \frac{\partial\psi}{\partial x} \tag{2}$$

Kinematic Model (cont.)

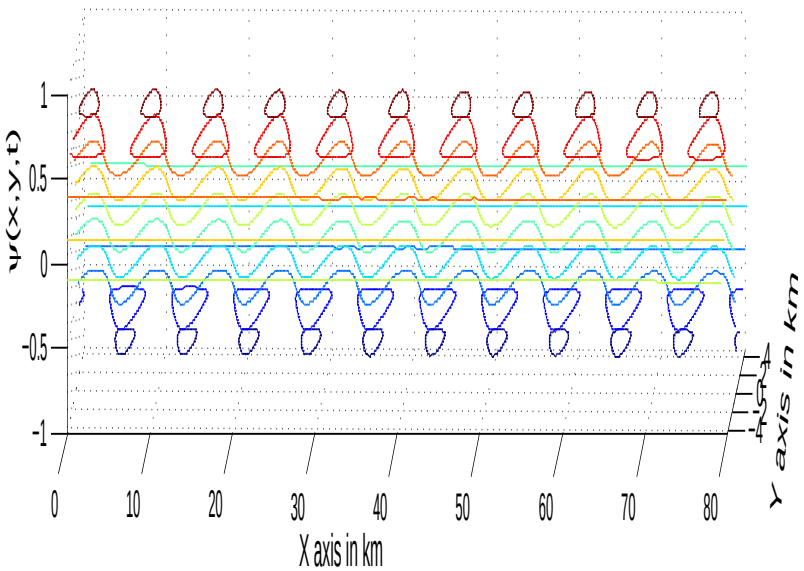


Figure 2: A 3D plot of 1

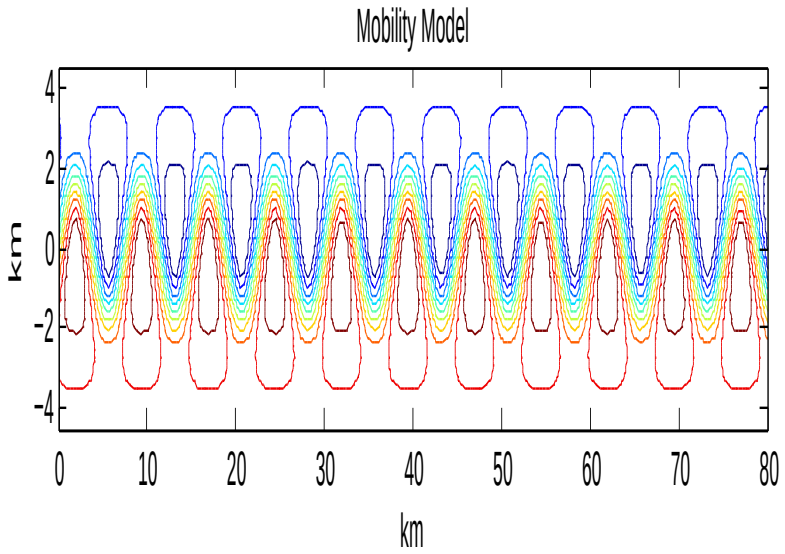


Figure 3: A plot of 1 at $t = 0$.

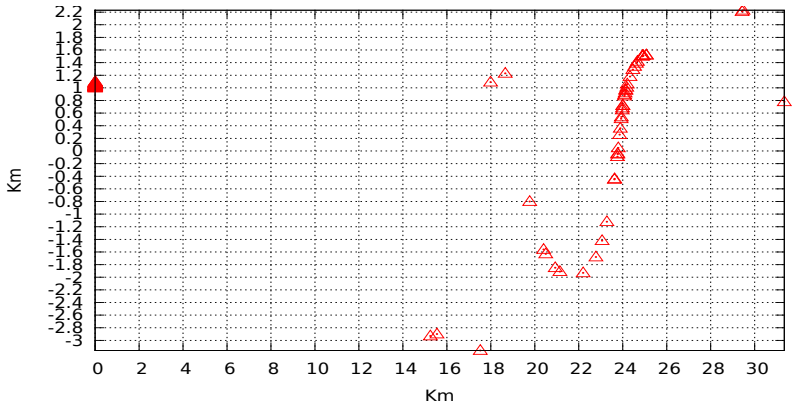


Figure 4: Start and end points of 50 nodes

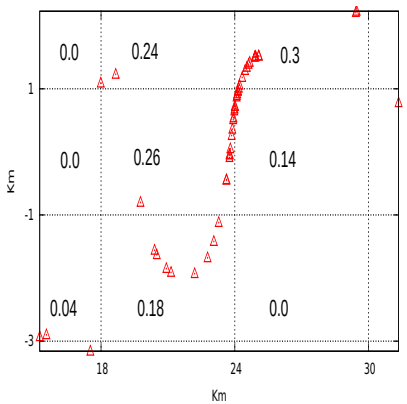


Figure 5: probabilistic distribution

Problem Definition

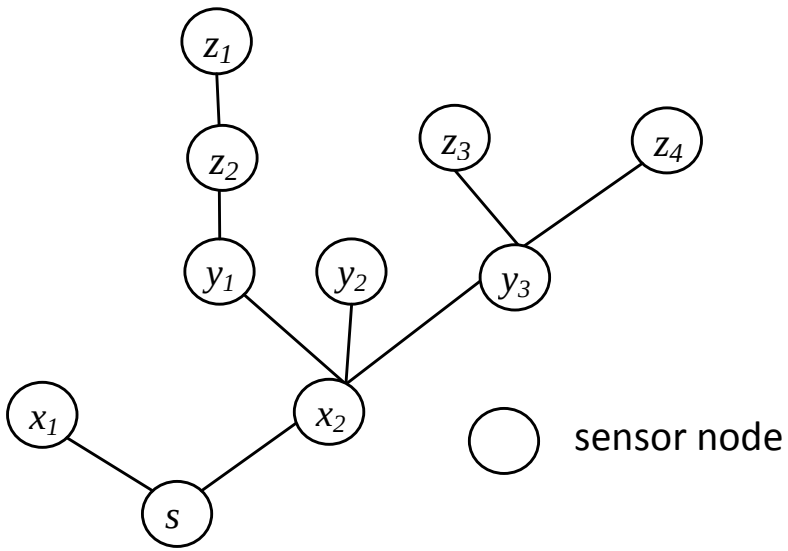
We define four probabilistic connectivity problem. They are

- A-CONN problem
- AR-CONN problem
- S-CONN problem
- SR-CONN problem

the A-CONN problem

Definition (**the** *A-CONN* **problem**)

Given a probabilistic network G with no relay nodes, compute the probability $Conn(G)$ that the network is in a state where the sink node s can reach all sensor nodes. ■



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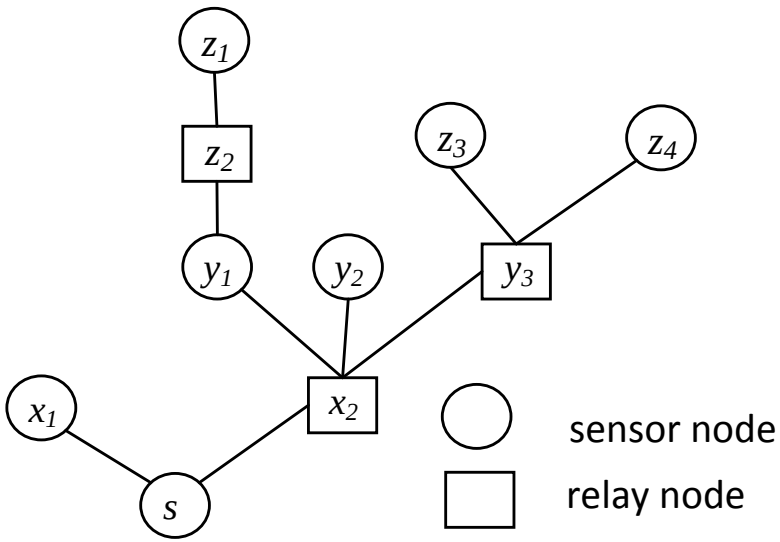
	0.07	0.05	0.08		
x_2	0.25	0.3		z_2	0.5 0.25 0.2
	0.16	0.29			
y_1	0.2	0.3	0.1	z_3	0.25 0.36 / 0.14 0.25
	0.1	0.15	0.15		
y_2	0.25	0.35		z_4	0.7 0.15 0.15
	0.25	0.15			
y_3	0.16	0.18	0.14	**s**	0.18 0.1 0.03 / 0.34 0.1 0.14
	0.3	0.22			

Figure 6: Network with Probabilistic locality set

the AR-CONN problem

Definition (**the** *AR-CONN* **problem**)

Given a probabilistic network G where V_{relay} is possibly non-empty, compute the probability $Conn(G)$ that the network is in a state where the sink node s can reach all sensor nodes. ■



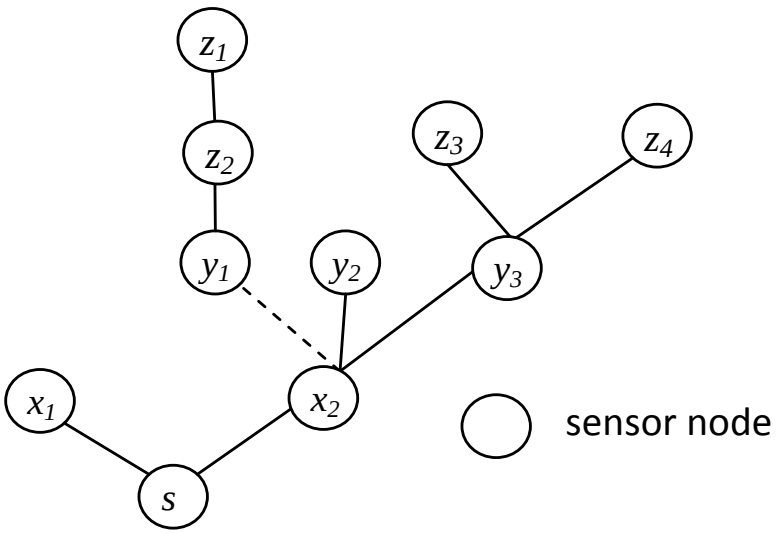
x_1	0.2	0.5	0.1	z_1	0.6 0.2 / 0.1 0.1
	0.07	0.05	0.08		
x_2	0.25	0.3		z_2	0.5 0.25 0.2
	0.16	0.29			
y_1	0.2	0.3	0.1	z_3	0.25 0.36 / 0.14 0.25
	0.1	0.15	0.15		
y_2	0.25	0.35		z_4	0.7 0.15 0.15
	0.25	0.15			
y_3	0.16	0.18	0.14	s	0.18 0.1 0.03 / 0.34 0.1 0.14
	0.3	0.22			

Figure 7: Network with Probabilistic locality set with relay nodes

the S-CONN problem

Definition (**the *S-CONN* problem**)

Given a probabilistic network G with no relay nodes, and a required number of sensor nodes $n_{req} \leq |V_{sense}|$, compute the probability $Conn(G, n_{req})$ that the network is in a state where the sink node s can reach a subset of sensor nodes having at least n_{req} sensor nodes. ■



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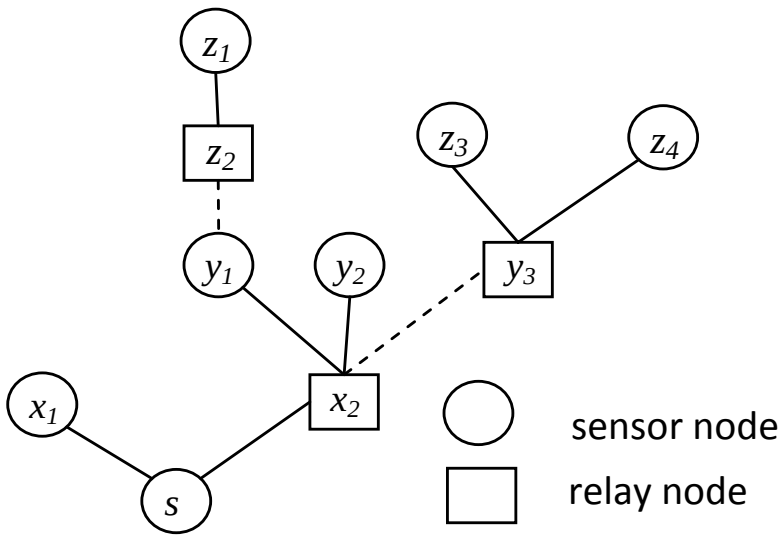
	0.07	0.05	0.08		0.1 0.1
x_2	0.25	0.3		z_2	0.5 0.25 0.2
	0.16	0.29			
y_1	0.2	0.3	0.1	z_3	0.25 0.36
	0.1	0.15	0.15		0.14 0.25
y_2	0.25	0.35		z_4	0.7 0.15 0.15
	0.25	0.15			
y_3	0.16	0.18	0.14	s	0.18 0.1 0.03
	0.3	0.22			0.34 0.1 0.14

Figure 8: Network with Probabilistic locality set and $n_{req} = 7$

the SR-CONN Problem

Definition (**the *SR-CONN* problem**)

Given a probabilistic network G where V_{relay} is possibly non-empty, and a required number of sensor nodes $n_{req} \leq |V_{sense}|$, compute the probability $Conn(G, n_{req})$ that the network is in a state where the sink node s can reach a subset of sensor nodes having at least n_{req} sensor nodes. ■



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	0.07	0.05	0.08		
x_2	0.25	0.3		z_2	0.5 0.25 0.2
	0.16	0.29			
y_1	0.2	0.3	0.1	z_3	0.25 0.36 / 0.14 0.25
	0.1	0.15	0.15		
y_2	0.25	0.35		z_4	0.7 0.15 0.15
	0.25	0.15			
y_3	0.16	0.18	0.14	s	0.18 0.1 0.03 / 0.34 0.1 0.14
	0.3	0.22			

Figure 9: Network with Probabilistic locality set and relay nodes where $n_{req} = 4$

Remarks

We note some of the above problems are special cases of other problems.

- $\text{A-CONN} \leq_p \text{AR-CONN}$ and $\text{S-CONN} \leq_p \text{SR-CONN}$
- $\text{A-CONN} \leq_p \text{S-CONN}$, since $n_{req} = |V_{sense}|$.
- Above problems share some basic aspects with the class of network reliability problems
- A probabilistic graphs arises when the given network is in some particular network states.
- A state S of V can be specified by $\{v_1[i_1], v_2[i_2], ..., v_n[i_n]\}$
- Assuming node locations are independent of each other, we have $Pr(S) = \prod_{v_\alpha \in V} p_{v_\alpha[i_\alpha]}$.
- In the *A-CONN* and *AR-CONN* problem, a state is **operating** if the sink s can reach all sensor nodes in V_{sense} .
- Similarly, in the *S-CONN* and *SR-CONN* problem, a state S is **operating** if the sink node s can reach a subset having at least n_{req} sensor nodes.

Example Probabilistic Network

Example

Figure 10 illustrates a probabilistic graph on 4 nodes where $V = \{s, a, b, c\}$. For the *A-CONN* problem, state $S_1 = \{s[2], a[2], b[2], c[2]\}$ is operating, and state $S_2 = \{s[1], a[1], b[1], c[2]\}$ is failed. ■

Given $G = (V, E_G, Loc, p)$ the *underlying graph* of G is a graph $\tilde{G}$ where

1. $V(\tilde{G}) = V$.
2. $E(\tilde{G})$ has an edge $e = (x, y)$ if for some positions $x[i]$ and $y[j]$ of nodes x and y, respectively, we have $E_G(x[i], y[j]) = 1$.

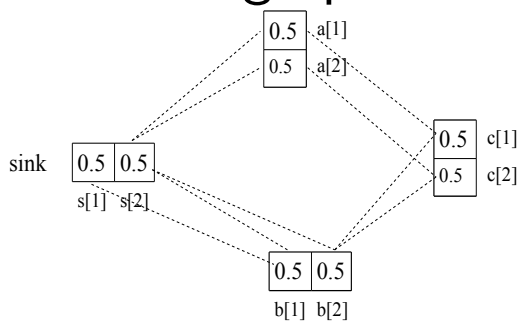


Figure 10: An example network

Example

The underlying graph of the network in figure 10 is the cycle (s, a, b, c). ■

Thesis Contribution

1. At first we review basic definitions, properties, and algorithmic aspects of k-trees and partial k-trees.
2. the first contribution of the thesis: an efficient dynamic programming algorithm to solve the SR-CONN problem on probabilistic networks whose underlying graphs are trees. The algorithm solves the more restricted AR-CONN problem with little overhead compared to a dedicated algorithm to solve the AR-CONN problem.
3. a second contribution of the thesis: a dynamic programming algorithm to solve the A-CONN problem on partial k-trees. The algorithm runs in polynomial time for any fixed k.
4. a third contribution: two dynamic programming algorithms to solve the AR-CONN and SR-CONN problems on partial k-trees. The algorithm runs in polynomial time for any fixed k.

k-Trees and Partial k-Trees

Definition

For a given integer $k \geq 1$, the class of k-trees is defined as follows

1. A k-clique is a k-tree.
2. If G_n is a k-tree on n nodes then the graph G_{n+1} obtained by adding a new node adjacent to every node in k-clique of G_n. ■

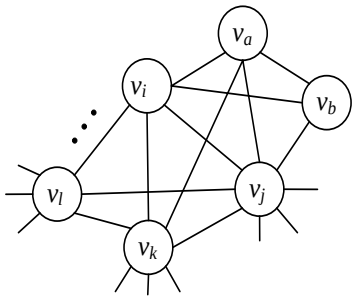


Figure 11: a fragment of a 3-tree

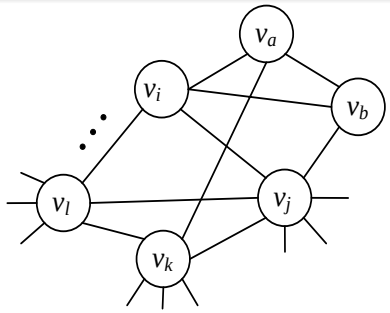


Figure 12: a fragment of partial 3-tree

A partial k-tree is a k-tree possibly missing some edges and a *k-perfect elimination sequence* (k-*PES*) of G is an ordering $(v_1, v_2, \ldots, v_r)$ of $v(G)$

Example

For the graph G in figure 12, $(v_a, v_b, v_i, v_j, v_k, v_l)$ is a 3-*PES*. ■

Graphs with Bounded Tree-width

Definition

A *tree-decomposition* of a graph G is a family $(X_i : i \in I)$ of subsets of $V(G)$, together with a tree T with $V(T) = I$, with the following properties.

1. $\bigcup_{i \in I} X_i = V(G)$.
2. Every edge of G has both its ends in some X_i.
3. For $i, j, k \in I$, if j lies on the path of T from i to k then $X_i \cap X_k \subseteq X_j$. ■

The *width* of a tree-decomposition is $max(|X_i| - 1 : i \in I)$. The *tree-width* of G is the minimum $w \geq 0$ such that G has a tree-decomposition of width $\leq w$. ■

Tree Decomposition example

Example

Figure 13(a) illustrates a graph G having a tree-decomposition shown in figure 13(b). The width of the tree decomposition is 3. One may verify that 3 is actually the tree-width of G. ■

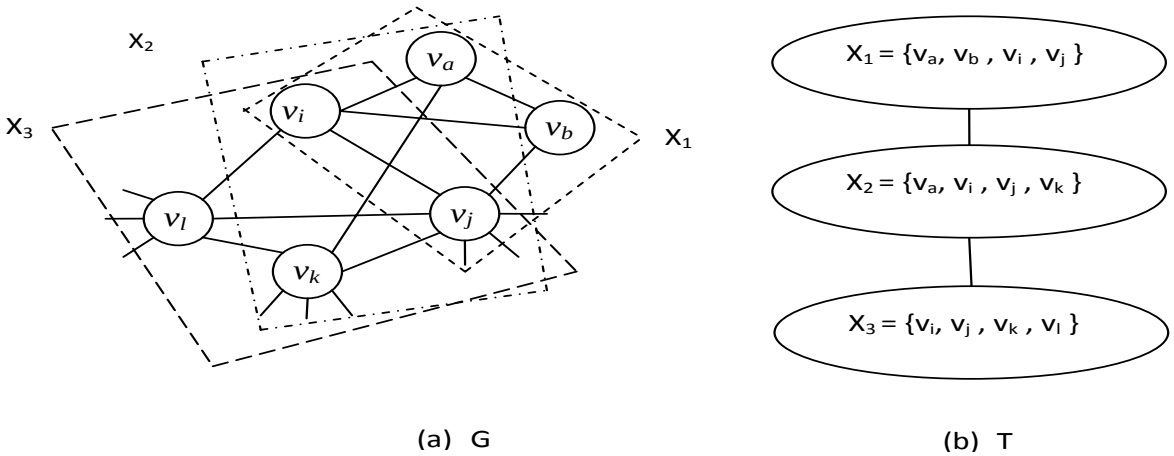


Figure 13: Tree decomposition

Dynamic Programing on Partial k-Trees

- Partial k-trees and graphs with bounded tree-width is rich with dynamic programming algorithms for solving many problems that are NP-complete in general.
- bodlander1988 and arnborg1991 formalizing graph and network properties as logical statements in a particular logic system.

A Steiner tree can be defined as follows.

Definition

Given a graph $G = (V, E)$ with positive integer edge costs, and a set of target nodes $V_{target} \subseteq V$, a Steiner tree, denoted $ST(G, V_{target})$, is a subtree $G' = (V', E')$ satisfying

1. $V_{target} \subseteq V' \subseteq V$,
2. the sum of edge costs in E' is minimum over all subtrees satisfying (1). ■

Solving Steiner tree problem on partial 2-tree

The algorithm devised in wald and colbourn 1983 works as follows. With each edge $\alpha = (x, y)$ of G, the algorithm associates six cost measures, which summarize the cost incurred so far of the subgraph S which has been reduced onto the edge (x, y)

1. $st(\alpha)$ is the minimum cost of a Steiner tree for S, in which x and y appear in the same tree.
2. $dt(\alpha)$ is the minimum cost of two disjoint trees for S including all targets, one tree involving x and the other y.
3. $yn(\alpha)$ is the minimum cost of a Steiner tree for S, which includes x but not y.
4. $ny(\alpha)$ is the minimum cost of a Steiner tree for S, which includes y but not x.
5. $nn(\alpha)$ is the minimum cost of a Steiner tree for S, which includes neither x nor y.
6. $none(\alpha)$ is the cost of omitting all vertices of S from a Steiner tree.

Our Approach

- An alternative approach to present the above algorithm is to store the six measures associated with edge $\alpha = (x, y)$ in a table, denoted $T_{x,y}$.
- The table provides key-value mappings.
- Roughly speaking, the keys replace the use of some of the names st, dt, yn, etc. with set notation.
- Not all names correspond to keys in this formulation.

Examples of names that correspond to keys are:
$st = \{x, y\}_1, dt = \{x\}_1\{y\}_1, yn = \{x\}_1\{y\}_0,$ and $ny = \{x\}_0\{y\}_1$.

Set Partitions

- Given a set X, a partition of X is a set $\{X_1, X_2, \ldots, X_r\}$, $1 \leq r \leq |X|$ such that
  1. $X = \bigcup_{i=1,2,\ldots,r} X_i$.
  2. The X_is are pairwise disjoint.
- In our algorithm, X is a set of nodes in a k-clique, and the partition of X are used as part of states of dynamic programs.
- The number of all possible partition of a set X on n elements are known as Bell numbers, denoted B_n.
- The first few Bell numbers are

$$B_0 = B_1 = 1, B_2 = 2, B_3 = 5, B_4 = 15,$$
$$B_5 = 52, B_6 = 203, B_7 = 877, B_8 = 4140, \ldots$$

- Bell numbers satisfy the following recurrence equation:

$$B_0 = 1, B_1 = 1 \text{ and } B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k.$$

Recognizing and Edge deletion problem on Partial k-tree

1. **Recognizing Partial k-Trees:** Given a graph G, and an integer $k \geq 1$, is G a partial k-tree?
   - The work of arnborg1987 shows $O(n^{k+2})$ algorithm for solving the problem.
   - If k is not fixed, then arnborg1987 shows that the problem is NP-complete.
   - For any fixed k, the work of bodlaender1993 improves on the above result by showing a linear time recognition algorithm.

   The above results produce also a k-*PES* if one exists.

2. **Edge Deletion Problem (EDP) to Obtain a Partial k-Tree:** Given a graph G, and an integer k, find the minimum number of edges whose deletion gives a partial k-tree.
   - For $k = 1$, the problem is simple.
   - For $k \geq 2$, the problem is NP-complete (From the work of elmallah1988 and the references therein)

The Random and Greedy Method

The Random Method.

- **Case** $k = 1$**.** Choose any spanning tree.
- **Case** $k = 2$**.**
  - Choose a spanning tree $G' \subseteq G$.
  - Fix an ordering of the remaining edges not in G'.
  - For each edge e in the fixed order, test whether $G' + e$ is a partial 2-tree.
  - If yes, add e to G', and proceed to the next edge in the fixed order.
- **Case** $k = 3$**.**
  - Build a partial 2-tree $G' \subseteq G$.
  - Fix an ordering of the edges not in G'.
  - For each edge e, test whether $G' + e$ is a partial 3-tree.
  - If yes, add e to G', and proceed to the next edge in the fixed order.

The Greedy Method.

- **Case** $k = 1$**:** Choose a minimum spanning tree.
- **Case** $k = 2$ **and** 3**:** As in cases $k = 2$ and 3, respectively, of the random method, where we sort the remaining edges in a non-decreasing order of their costs to obtain the fixed order

Tree Network

- $type(x)$: $type(x) = 0$ and 1 if x is a relay node and a sensor node respectively.
- $n_{sense}(X)$: # of sensor nodes in a given subset of nodes $X \subseteq V$.
- $n_{relay}(X)$: # of relay nodes in a given subset of nodes $X \subseteq V$.
- $n(X) = n_{sense} + n_{relay}$.
- $n_{sense,min}(X)$: The minimum number of sensor nodes in a given subset $X \subseteq V$. So, $n_{sense,min}(X) = max(0, n_{req} - n_{sense}(\overline{X}))$

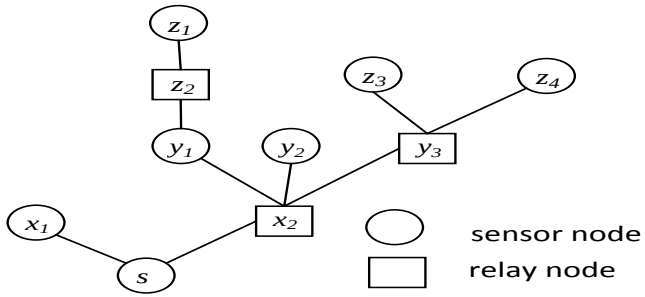


Figure 14: A tree network

Example

In figure 14, consider node x_2. $n(V_{x_2}) = 8$ where $n_{sense}(V_{x_2}) = 5$ and $n_{relay}(V_{x_2}) = 3$. Assuming $n_{req} = 5$ in an instance of the SR-CONN problem then $n_{sense,min}(V_{x_2}) = 3 = n_{req} - n_{sense}(\{s, x_1\}) = 5 - 2$. ■

Pseudo-code for function Conn

Function $\mathrm{Conn}(G, T, n_{req})$
Input: the *SR-CONN* problem where G has a tree topology T with no relay leaves
Output: $Conn(G, n_{req})$

1. **foreach** (node x and a valid location index i)
   set $R_x(i, type(x)) = 1$
2. **while** (T has at least 2 nodes)
   {
3. Let y be a non-sink leaf of T, and $x = parent(y)$
4. **foreach** (key $(i, count) \in R_y$) $R_y(i, count) *= p_y(i)$
5. set $R'_x = \phi$
6. **foreach** (pair of keys $(i_x, count_x) \in R_x$ and $(i_y, count_y) \in R_y$)
   {
7. $count = \min(n_{req}, count_x + count_y)$
8. **if** $(count < n_{sense,min}(\{x\} \cup V_y \cup_{z \in DCH(x)} V_z))$ **continue**
9. $R'_x(i_x, count) += R_y(i_y, count_y) \times R_x(i_x, count_x) \times E_G(x[i_x], y[i_y])$
   }
10. set $R_x = R'_x$; remove y from T
    }
11. return $\sum_{s[i] \in \mathrm{Loc}(s)} R_s(i, n_{req}) * p_s(i)$

Running time

Let n be the number of nodes in G, and ℓ_{max} be the maximum number of locations in the locality set of any node.

Theorem

Function Conn solves the $\mathrm{SR\text{-}CONN}$ problem in $O(n \cdot n_{req}^2 \cdot \ell_{max}^2)$ time

Proof. We note the following.

- Step 1: storing the tree T require $O(n)$ time.
- Step 2: the main loop performs $n-1$ iterations. Each of Steps 3, 5, and 10 can be done in constant time.
- Step 4: this loop requires $O(n_{req} \cdot \ell_{max})$ time.
- Step 6: this loop requires $O(n_{req}^2 \cdot \ell_{max}^2)$ iterations. Steps 7, 8, and 9 can be done in constant time.

Thus, the overall running time is $O(n \cdot n_{req}^2 \cdot \ell_{max}^2)$ time. ■

Theorem

Function Conn solves the $\mathrm{AR\text{-}CONN}$ problem in $O(n \cdot \ell_{max}^2)$ time

Cliques for A-CONN problem

- $K_{i,base}$: the k-clique to which node v_i is attached. For the 3-tree in figure 16, $K_{v_i,base}$ is the triangle (3-clique) on nodes $\{v_j, v_k, v_l\}$.
- $K_{v_i,1}, K_{v_i,2}, \ldots, K_{v_i,k}$: all possible k-cliques involving node v_i when this node becomes a k-leaf. For the 3-tree in the figure 16, we may set $K_{v_i,1}$ = the triangle (v_i, v_j, v_k), $K_{v_i,2}$ = the triangle (v_i, v_j, v_l), $K_{v_i,3}$ = the triangle (v_i, v_k, v_l).

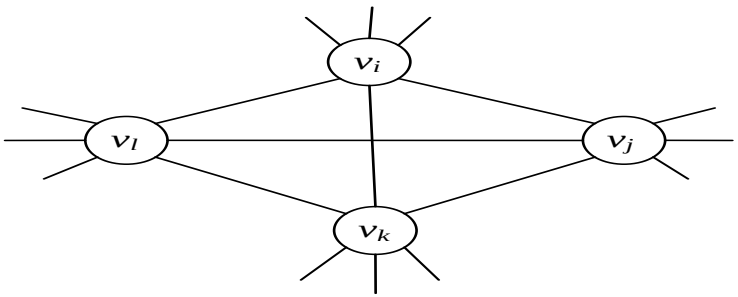


Figure 16: A fragment of a 3-tree

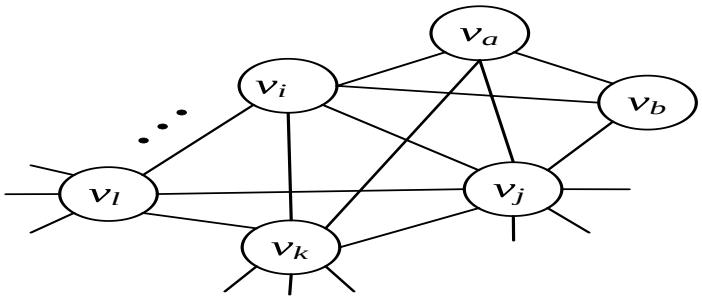


Figure 17: A 3-tree fragment

Example

In figure 17, if v_a and v_b have been deleted to make v_i a simplicial node, the information about the induced subgraph on nodes $(v_a, v_b, v_i, v_j, v_k)$ is summarized in table, say, $T_{v_i,1}$ associated with clique $K_{v_i,1} = (v_i, v_j, v_k)$. ■

State types for A-CONN problem

Definition (**state types of the** *A-CONN*)

Let $G_{v_i,\alpha}$ be a subgraph reduced onto the clique $K_{v_i,\alpha}$. Denote by $V_{v_i,\alpha}$ the set of nodes of the graph $G_{v_i,\alpha}$. Let $S = \{v_a[i_a] : v_a \in V_{v_i,\alpha}, i_a \in Loc(v_a)\}$ be a network state of $G_{v_i,\alpha}$. Then

$$A\text{-}type(S) = (V_1^{\mathrm{Loc}(V_1)}, V_2^{\mathrm{Loc}(V_2)}, \ldots, V_r^{\mathrm{Loc}(V_r)})$$

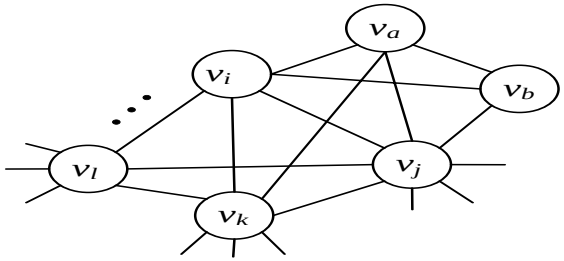


Figure 18: A 3-tree fragment

Example

In figure 17, denote by $G_{v_i,1}$ the graph induced on the set of nodes $\{v_a, v_b, v_i, v_j, v_k\}$. $G_{v_i,1}$ is reduced onto the clique $K_{v_i,1}$ = the triangle (v_i, v_j, v_k). Suppose that $S = \{v_a[1], v_b[2], v_i[1], v_j[2], v_k[3]\}$ is a possible network state of $G_{v_i,1}$. Moreover, suppose that S has 2 connected components : $\{v_a, v_b, v_i\}$ and $\{v_j, v_k\}$. Then state S is good for the A-CONN problem. State S induces the partition $(\{v_i\}, \{v_j, v_k\})$ on the triangle (v_i, v_j, v_k). Thus, $A\text{-}type(S) = (\{v_i\}^{(1)}, \{v_j, v_k\}^{(2,3)})$. ■

A sample Table

A table $T_{v_i,\alpha}$, where $\alpha = base, 1, 2, \ldots, k$ is key-value mapping where

- a key is a network state type of the graph $G_{v_i,\alpha}$
- each value is the probability.

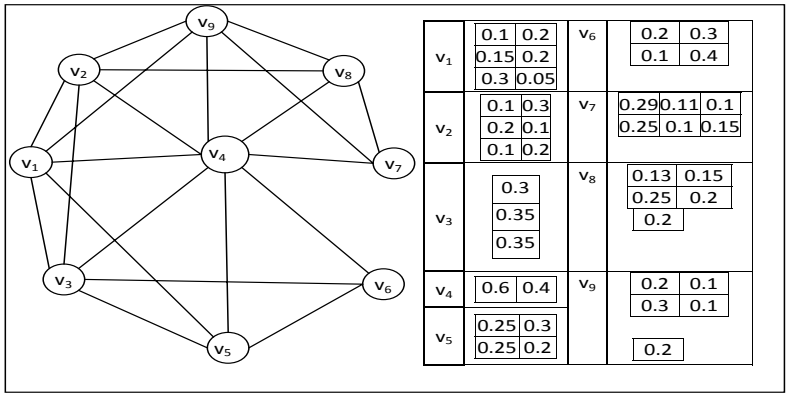


Figure 19: A UWSN modelled by a 3-tree

Example

Figure 19 illustrates a probabilistic network G on 9 nodes. The network has the topology of a 3-tree. The algorithm associates edge with each triangle a table. Each triangle can be partitioned in 5 different ways (since, the Bell number $B_3 = 5$). For example, the 5 partitions of triangle (v_1, v_2, v_3) are $\{v_1, v_2, v_3\}, \{v_1, v_2\}\{v_3\}, \{v_1, v_3\}\{v_2\}, \{v_2, v_3\}\{v_1\}$ and $\{v_1\}\{v_2\}\{v_3\}$. The number of possible keys that appear in the table of triangle (v_1, v_2, v_3) is $5 \times 6 \times 6 \times 3$ since $B_3 = 5, |Loc(v_1)| = 6$, $|Loc(v_2)| = 6$ and $|Loc(v_3)| = 3$. ■

Algorithm Organization

The overall algorithm is organized around 3 functions :

- a top level main function (Algorithm 1),
- a middle level table merge function (Algorithm 2: *t_merge*), and
- a low level partition merge function (Algorithm 3: *p_merge*)

This function reduces the structure of the probabilistic network (and the underlying partial k-tree) G by iteratively processing and then deleting nodes according to the given *PES*, say $(v_1, v_2, \ldots, v_n)$ where $V_n = s$ (the sink node), until the network is reduced to the k-clique $(v_{n-k+1}, \ldots, v_n)$. Processing a node v_i entails merging the tables $\{K_{v_i,\alpha} : \alpha = base, 1, 2, \ldots, k\}$. This merging phase is done by merging a sequence of pairs of tables. The middle level table merge function is used for this purpose. After all iterations are done, the solution is computed from the table associated with the clique on nodes $(v_{n-k+1}, \ldots, v_n)$.

Function Main

Algorithm 1: Function Main(G, PES)

Input: An instance of the $A\text{-}CONN$ problem where G has a partial k-tree topology with a given PES=$(v_1, v_2, \ldots, v_n)$

Output: $Conn(G)$

Notation: $Temp$ is a temporary table.

1 **Initialization:** initialize a table T_H for each k-clique H of G.
T_H contains all possible state types on nodes of H.
2 **for** $(i = 1, 2, \ldots, |V| - k)$ **do**
3 $Temp = T_{v_i,1}$
4 **for** $(j = 2, 3, \ldots, k)$ **do**
5 $Temp = t\_merge(Temp, T_{v_i,j})$
end
6 $T_{v_i,base} = t\_merge(Temp, T_{v_i,base})$
7 **foreach** $(key \in T_{v_i,base})$ **do**
8 **if** (v_i *is a singleton part of* key) **then**
9 delete key from $T_{v_i,base}$
end
else
10 delete v_i and its associated position from key
end
end
end
11 **return** $Conn(G) = \sum$ values in table $T_{v_{n-k},base}$ corresponding to state types that have exactly one connected component

Function Table Merge

Algorithm 2: Function $t\_merge(T_1, T_2)$

Input: Two tables T_1 and T_2 that may share common nodes

Output: A merged table T_{out}

Initialization: Clear table T_{out}
set C = the set of common nodes between T_1 and T_2
foreach (*pair of state types* $key_1 \in T_1$ *and* $key_2 \in T_2$) **do**
    if (*any node in* C *lies in two different positions in* key_1 *and* key_2) **then**
        continue
    end
    set key_{out} = the state type obtained from node positions in key_1 and key_2, and the partition computed by $p\_merge(key_1, key_2)$
    set $p_{out} = T_1(key_1) \times T_2(key_2)$ adjusted to take the effect of common nodes in C into consideration
    if ($key_{out} \in T_{out}$) **then**
        update $T_{out}(k_{out})$ $+\!\!=p_{out}$
    end
    else
        set $T_{out}(key_{out}) = p_{out}$
    end
end
return T_{out}

Function Partition Merge

T_1 on (v_1, v_2, v_3)

.	.
$\{v_1, v_2\}^{(1,1)}\{v_3\}^{(2)}$	0.002
.	.
.	.
.	.

$\times$

T_2 on (v_1, v_3, v_4)

.	.
$\{v_1\}^{(1)}\{v_3, v_4\}^{(2,1)}$	0.008
.	.
.	.
.	.

$\Rightarrow$

$Temp$ on (v_1, v_2, v_3, v_4)

.	.
$\{v_1, v_2\}^{(1,1)}\{v_3, v_4\}^{(2,1)}$	0.0008
.	.
.	.
.	.

$T_{v_1,base}$ on (v_1, v_2, v_3, v_4)

.	.
$\{v_1, v_2\}^{(1,1)}\{v_3, v_4\}^{(2,1)}$	0.0008
.	.
.	.
.	.

$\Rightarrow$

$T_{v_1,base}$ on (v_2, v_3, v_4)

.	.
$\{v_2\}^{(1)}\{v_3, v_4\}^{(2,1)}$	0.0008
.	.
.	.
.	.

Function Partition Merge

$$\boxed{\{v_1, v_2\}\{v_3\}} \times \boxed{\{v_1\}\{v_3, v_4\}} \Rightarrow \boxed{\{v_1, v_2\}\{v_3, v_4\}}$$

Algorithm 3: function $p\_merge(P_1, P_2)$

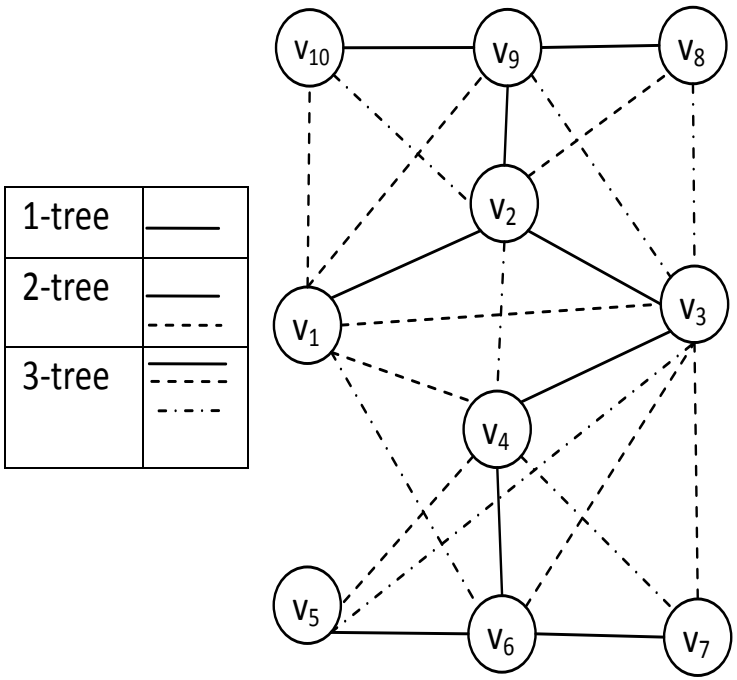
Input: Two partitions P_1 and P_2

Output: A partition P

Notation: s and t are two set iterators and their corresponding set are indicated by s^* and t^*.

```
1  foreach ( set s* in P2) do
2  |  P1.push_back(s*)
   end
3  for (s = P1.begin(); s ≠ P1.end(); ++s) do
4  |  for (t = s.next(); t ≠ P1.end(); ++t) do
5  |  |  if (s* ∩ t* ≠ ∅) then
6  |  |  |  P1.push_front(s* ∪ t*)
7  |  |  |  P1.delete(s*)
8  |  |  |  P1.delete(t*)
9  |  |  |  s = P1.begin()
10 |  |  |  break
   |  |  end
   |  end
   end
11 set P = P1
   return P
```

Test Networks



v_1	0.2	0.5	0.1	v_6	0.6 0.2 / 0.1 0.1
	0.15	0.05			
v_2	0.25	0.3		v_7	0.5 0.25 0.2
	0.16	0.29			
v_3	0.2	0.3	0.1	v_8	0.25 0.36 / 0.14 0.25
	0.1	0.15	0.15		
v_4	0.25	0.35		v_9	0.7 0.15 0.15
	0.25	0.15			
v_5	0.16	0.18	0.14	v_{10}	0.18 0.24 / 0.34 0.1 0.14
	0.3	0.22			

Figure 20: G_{10}

Test Networks(Cont.)

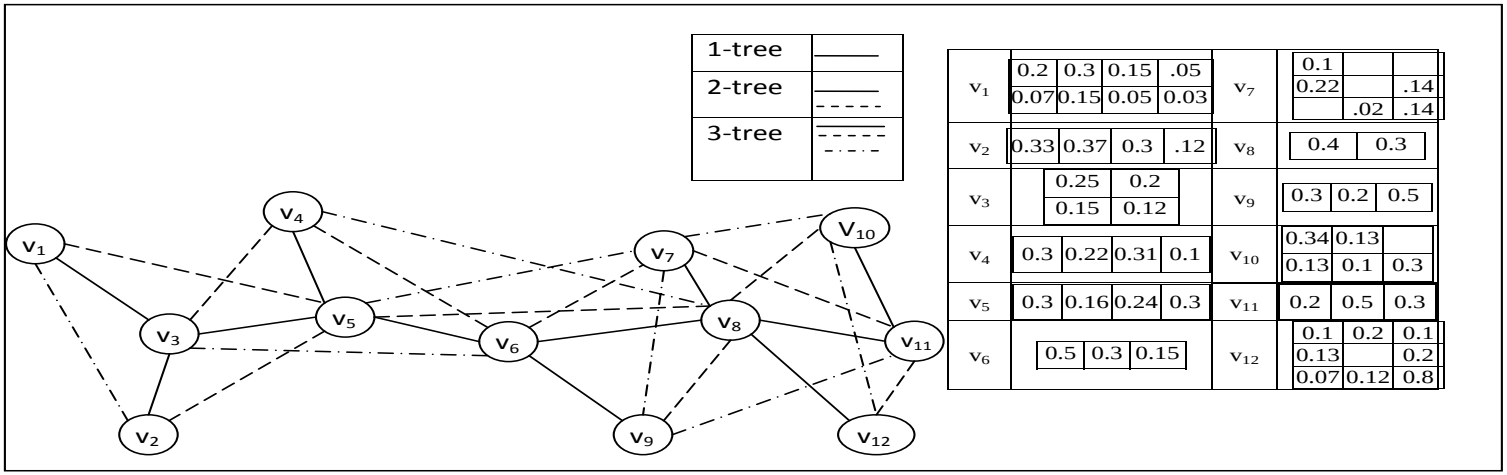


Figure 21: G_{12}

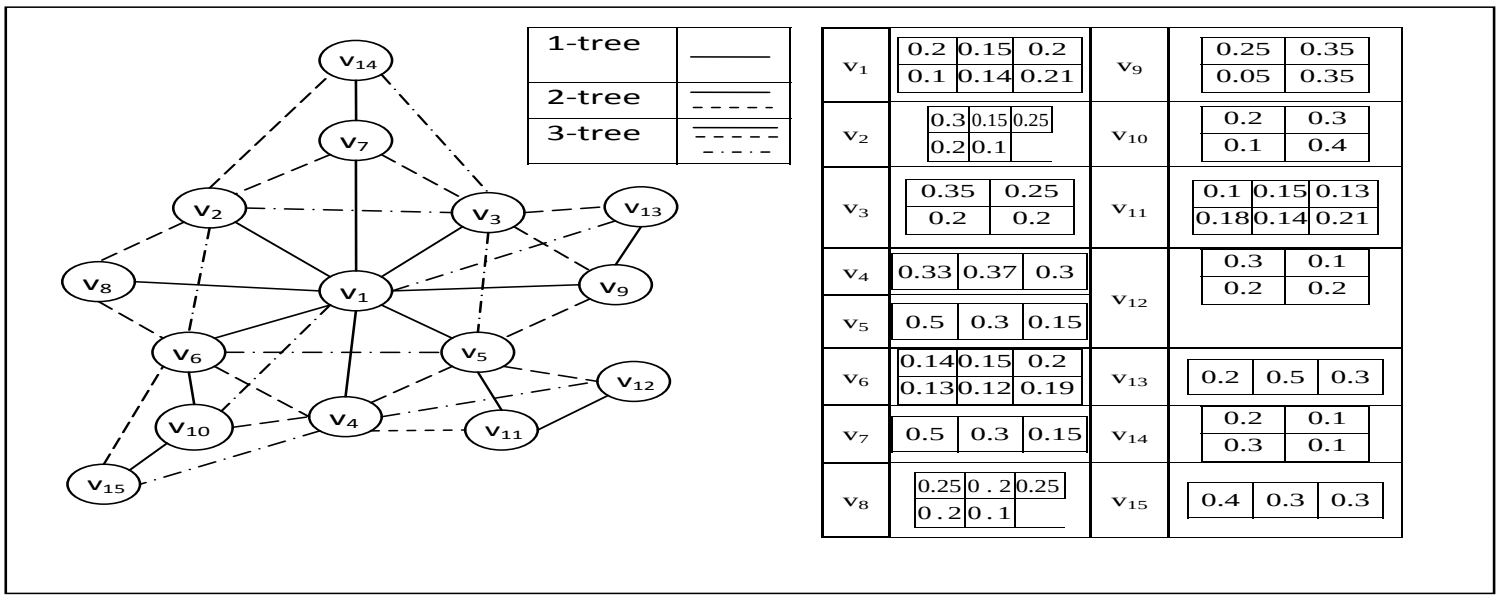


Figure 22: G_{15}

Running Time

k	Network G_{10}	Network G_{12}	Network G_{15}
1	90	130	200
2	1000	1380	1480
3	60000	875000	940000

Table 1: Running time in milliseconds

Connectivity for different partial k-trees

k	Network G_{10}	Network G_{12}	Network G_{15}
1	0.62	0.336	0.82
2	0.71	0.36	0.99
3	0.75	0.37	1

Table 2: Connectivity lower bounds using different partial k-trees

Effect of subgraph selection method

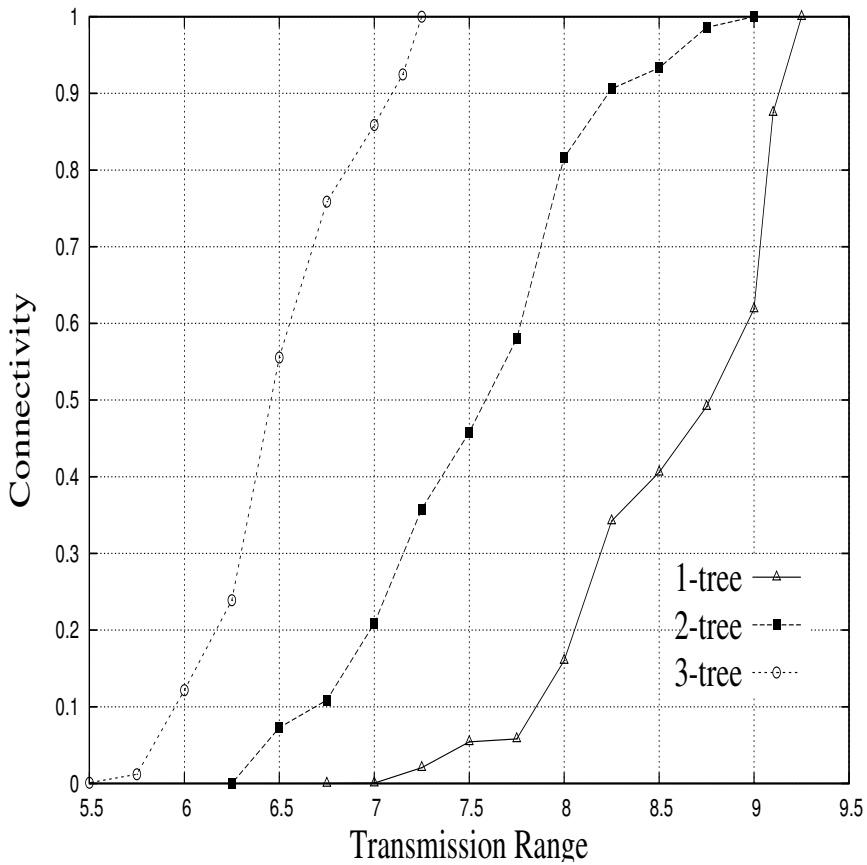


Figure 23: Connectivity versus transmission range with random subgraph selection

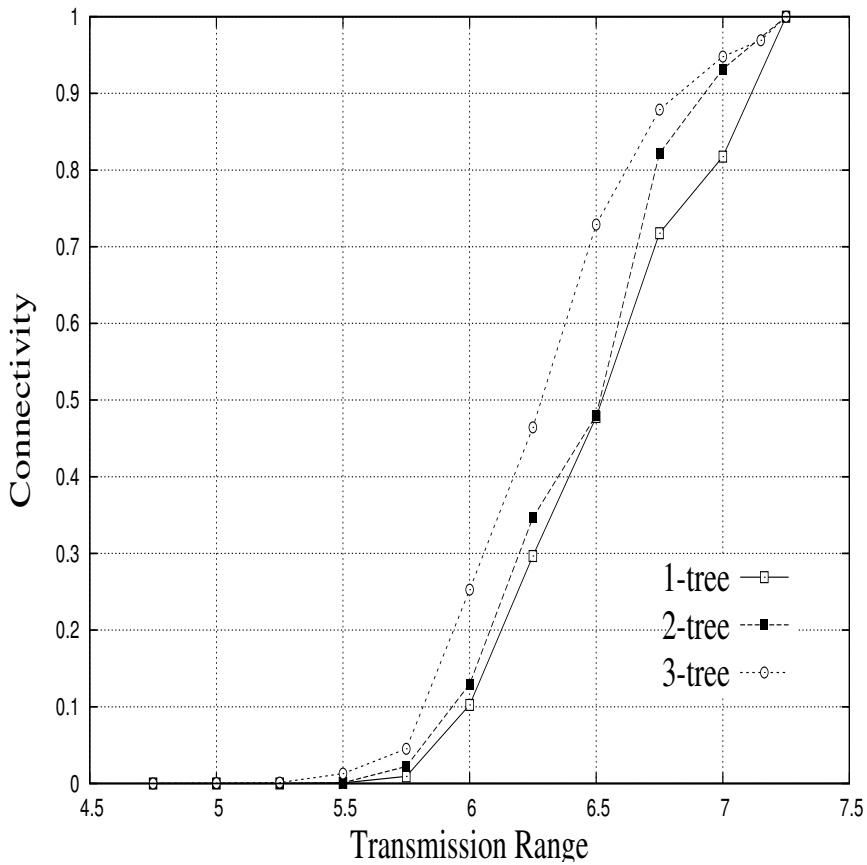


Figure 24: Connectivity versus transmission range with greedy subgraph selection

AR-CONN state types

$$AR\text{-}type(S) = \{V_{1,b(V_1)}^{Loc(V_1)}, V_{2,b(V_2)}^{Loc(V_2)}, \ldots, V_{r,b(V_r)}^{Loc(V_r)}\}$$

Example

In figure 25, denote by $G_{v_i,1}$, the graph induced on nodes $\{v_a, v_b, v_i, v_j, v_k\}$. Assume that $\{v_b\} \in V_{sense}$ and $\{v_a, v_i, v_j, v_k\} \in V_{relay}$. Suppose that state S of figure 26 is a possible network state of $G_{v_i,1}$. Then $AR\text{-}type(S) = \{\{v_i\}_1^{(1)}, \{v_j, v_k\}_0^{(2,3)}\}$. Here, the indicator $b(\{v_i\}) = 1$ since v_b is a sensor node connected to v_i in S. ■

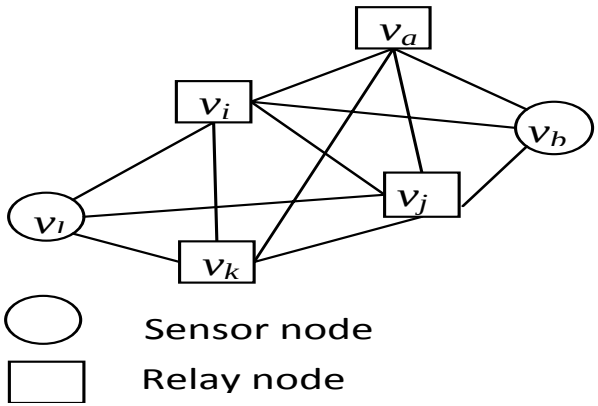


Figure 25: A fragment of a 3-tree

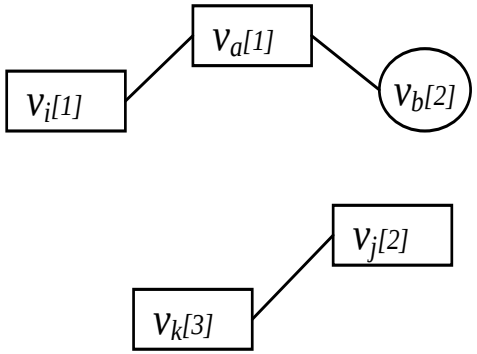


Figure 26: A state S on $\{v_a, v_b, v_i, v_j, v_k\}$

Merging State types

- C is a set of common nodes between two table T_1 and T_2.
- merge two table T_1 and T_2 produces a new table T_{out}.
- each table $T_i, i = 1, 2$, contains summary information about a subgraph G_i that has been reduced onto a clique K_i.
- the merge operation computes a table T_{out} that stores summary information about the graph $G_1 \cup G_2$.
- T_{out} is computed by processing each pair of keys $key_1 \in T_1$ and $key_2 \in T_2$.
- processing key_1 and key_2 results in a new state type, denoted key_{out}, and an associated probability, denoted p_{out}.
- merging S_1 and S_2 generates $key_{out} = AR\text{-}type(S_1 \cup S_2)$.
  To explain the structure of key_{out} (if it exists), let

$$AR\text{-}type(S_1) = \{X_{i,b(X_i)}^{Loc(X_i)} : i = 1, 2, \ldots\}, \quad .$$
$$AR\text{-}type(S_2) = \{Y_{j,b(Y_j)}^{Loc(L_j)} : j = 1, 2, \ldots\}, \textit{and}$$
$$AR\text{-}type(S_1 \cup S_2) = \{Z_{k,b(Z_k)}^{Loc(Z_k)} : k = 1, 2, \ldots\}$$

Removing Bad State Types

- For the AR-CONN problem, a state S of the subgraph $G_{v_i,base}$ reduced onto the k-clique $K_{v_i,base}$.
- S is considered bad if node v_i appears as a singleton part and the indicator $b(v_i) = 1$.
- The algorithm removes all such bad state types from $T_{v_i,base}$.

Simulation Results

Relay nodes are expected to be cheaper than sensor nodes since they do not include sensing devices. In addition, relay nodes are not required to do energy consuming data acquisition tasks as sensor nodes. Hence, their design may enjoy more flexibility than sensor nodes, and their energy supply is expected to last longer.
In this section we utilize the algorithm developed for the AR-CONN problem to investigate the positive effects of deploying relay nodes.

Test Networks

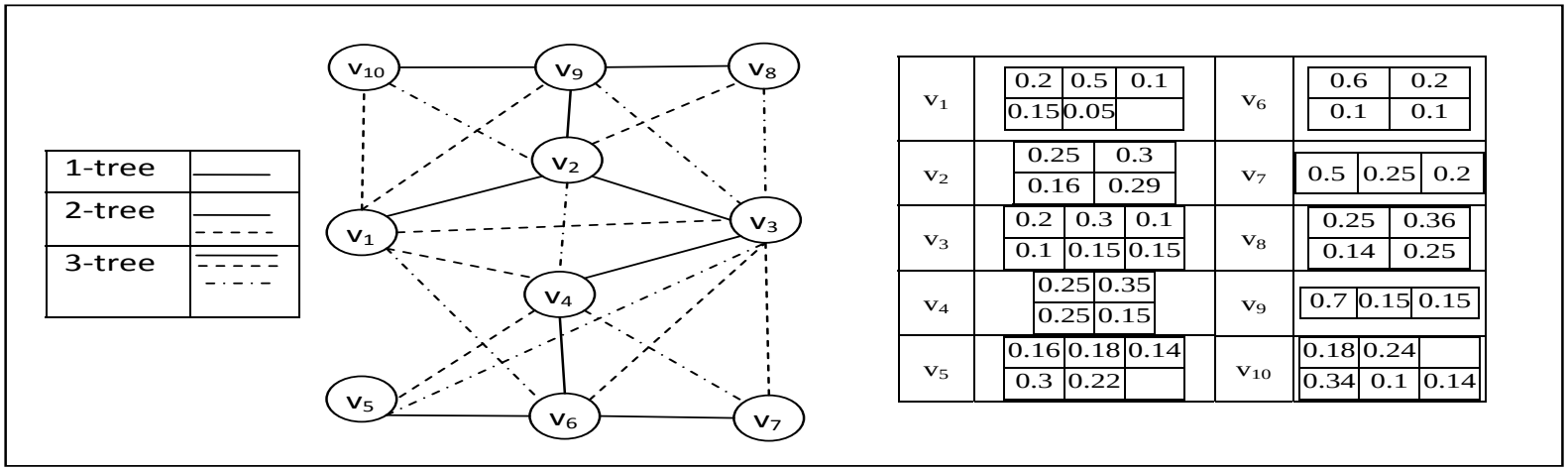


Figure 27: Network G_{10}

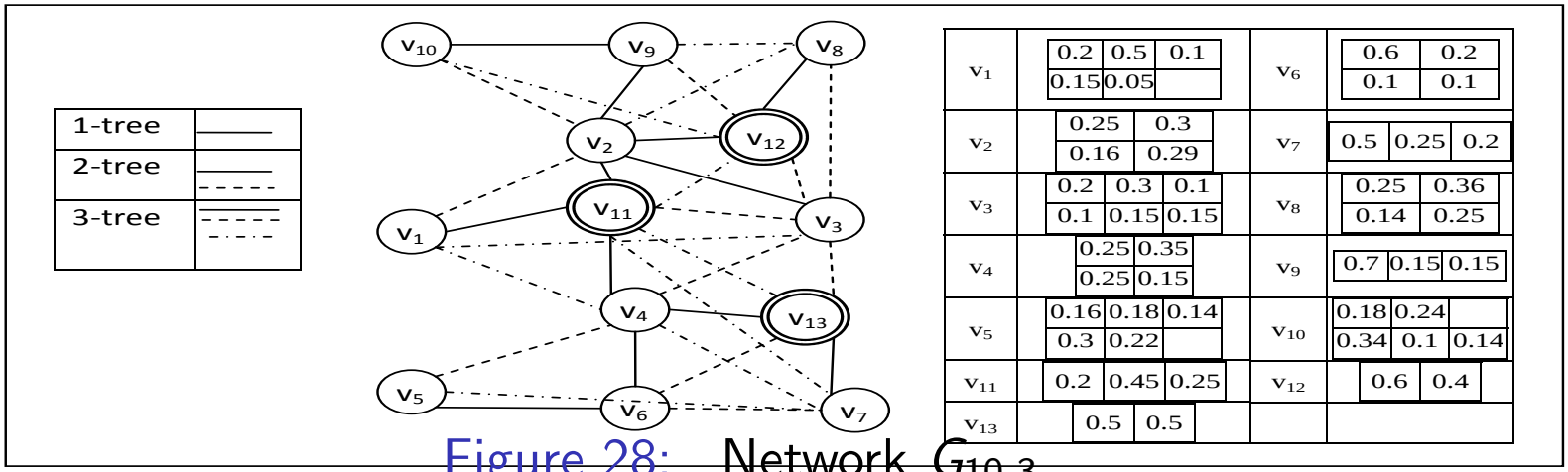


Figure 28: Network $G_{10,3}$

Running Time

k	Network G_{10}	Network $G_{10,3}$
1	90	110
2	1000	6000
3	6000	8000

Table 3: Running time in milliseconds

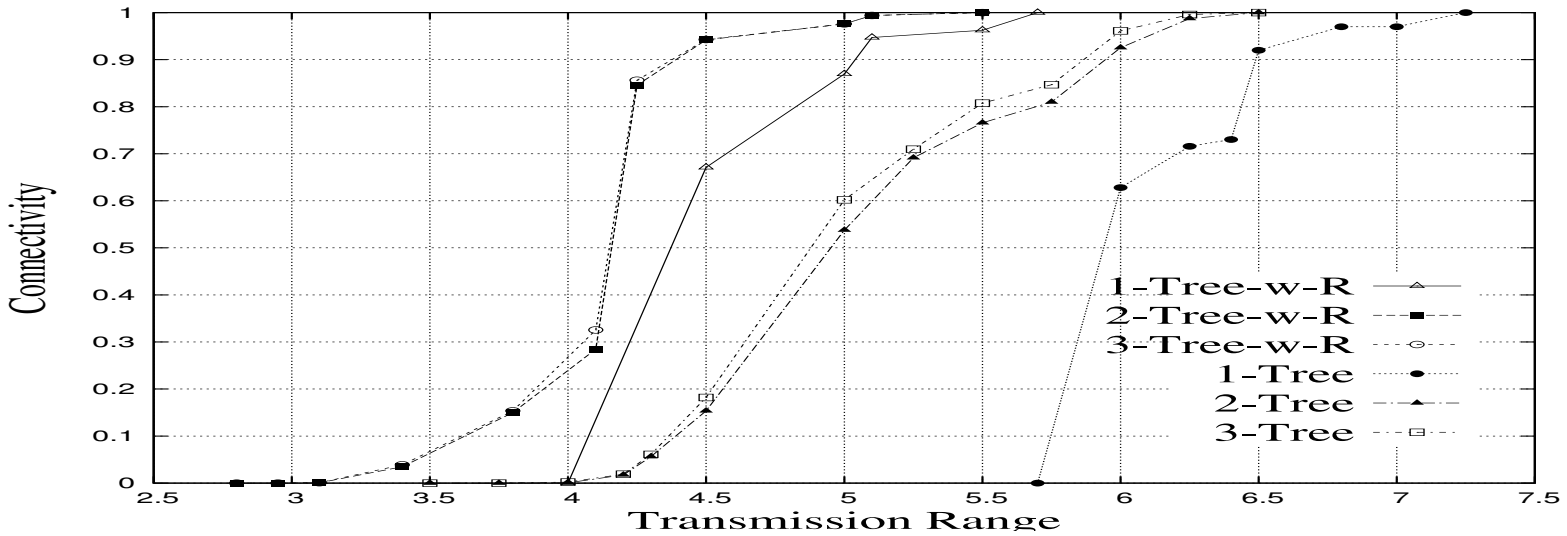
Effect of adding relay nodes

k	Network G_{10}	Network $G_{10,3}$
1	0.30	0.86
2	0.54	0.96
3	0.60	0.98

Table 4: Connectivity with respect to k

Effect of adding relay nodes for various node R_{tr}:

- Each obtained curve exhibits a notable monotonic increasing behaviour as R_{tr} increases.
- This behaviour is due to the appearance of more edges, and the potential increase in the probability of each edge as R_{tr} increases.
- Increasing R_{tr}, however, requires increasing node energy consumption.
- To achieve a desired $Conn(G)$ value, a designer may utilize the obtained curves to assess the merit of increasing R_{tr} versus deploying more relay nodes.



The SR-CONN State Types

$$SR\text{-}type(S) = \{V_{1,c(V_1)}^{Loc(V_1)}, V_{2,c(V_2}^{Loc(V_2)}, \ldots, V_{r,c(V_r}^{Loc(V_r)}\}$$

Example

In figure 30, denote by $G_{v_i,1}$, the graph induced on nodes $\{v_a, v_b, v_i, v_j, v_k\}$. Assume that $\{v_b, v_i\} \in V_{sense}$ and $\{v_a, v_j, v_k\} \in V_{relay}$. Suppose that state S of figure 31 is a possible network state of $G_{v_i,1}$. Then $SR\text{-}type(S) = \{\{v_i\}_2^{(1)}, \{v_j, v_k\}_0^{(2,3)}\}$. Here, the count $c(\{v_i\}) = 2$ since both of v_i and v_b are sensor nodes. ■

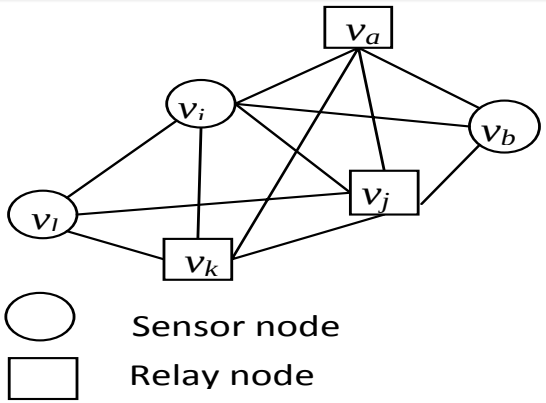


Figure 30: A 3-tree fragment

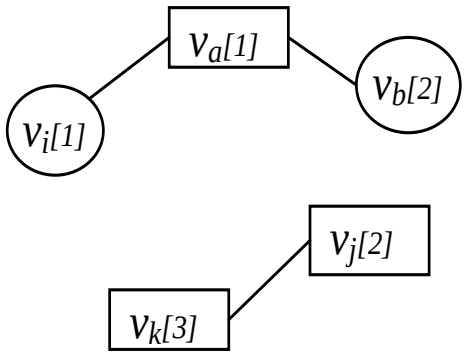


Figure 31: A state S on $\{v_a, v_b, v_i, v_j, v_k\}$

Merging SR-CONN State Types

- we consider two network states: $S_i, i = 1, 2$, where S_i is a network state of the subgraph G_i reduced onto the clique K_i.
- merging S_1 and S_2 generates $key_{out} = SR\text{-}type(S_1 \cup S_2)$.
- the union $S_1 \cup S_2$ is possible only if each common node between S_1 and S_2 takes the same position in both states.

$$SR\text{-}type(S_1) = \{X_{i,c(X_i)}^{Loc(X_i)} : i = 1, 2, \ldots\},$$
$$SR\text{-}type(S_2) = \{Y_{j,c(Y_j)}^{Loc(L_j)} : j = 1, 2, \ldots\}, \text{and}$$
$$SR\text{-}type(S_1 \cup S_2) = \{Z_{k,c(Z_k)}^{Loc(Z_k)} : k = 1, 2, \ldots\}$$

Also

$$c(Z_k) = max(n_{req}, c(X_i) + c(Y_j) - n_{ij}).$$

- the probability p_{out} is the product $T_1(key_1) \times T_2(key_2)$ divided by a correction term $= \prod(p_x(i) : x[i]$ is common between key_1 and $key_2)$.

Bad State Types

For a state S of $G_{v_i,base}$, let $n_{sense}(S)$ be the number of sensor nodes that can reach the rest of the graph by nodes in the separator clique $K_{v_i,base}$. Thus, if $SR\text{-}type(S) = \{X_{i,c(X_i)}^{Loc(X_i)} : i = 1, 2, \ldots, r\}$ then

$$n_{sense}(S) = \begin{cases} \sum_{X_j \neq \{v_i\}} c(X_j) & \text{if node } v_i \text{ appears as a singleton in the} \\ & \text{partition } (X_1, X_2, \ldots, X_r) \\ \sum_{j=1,2,\ldots,r} c(X_j) & \text{otherwise} \end{cases}$$

Now, let us denote by $n_{v_i,base,sense}$ the number of sensor nodes in the subgraph $G_{v_i,base}$ reduced onto the k-clique $K_{v_i,base}$ at the end of the first part of the main loop in function Main. Thus, $|V_{sense}| - n_{v,base,sense}$ is the number of sensor nodes outside the graph $G_{v_i,base}$. By definition of operating states of the SR-CONN problem, $SR\text{-}type(S)$ is bad if

$$n_{sense}(S) + (|V_{sense}| - n_{v_i,base,sense}) \leq n_{req}.$$

We recall that the algorithm removes all such bad state types from $T_{v_i,base}$ in step 9 of function Main.

SR-CONN Simulation Results

A designer has at least 3 options to achieve a minimum required $Conn(G, n_{req})$ value:

- tuning the n_{req} parameter,
- tuning node transmission range R_{tr}, and
- tuning the number of deployed relay nodes.

Effect of varying n_{req}

- Figure 32 illustrates the achieved $Conn(G, n_{req})$ as n_{req} varies in the range $[1, 10]$.
- Figure 33 illustrates the achieved $Conn(G, n_{req})$ as n_{req} varies in the range $[1, 10]$, and R_{tr} varies in the range $[2.5, 5.5]$.

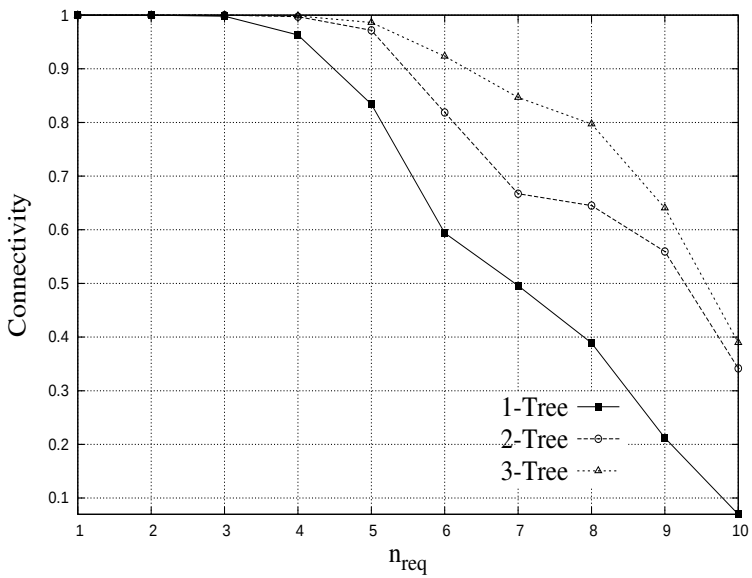


Figure 32: Connectivity versus n_{req}

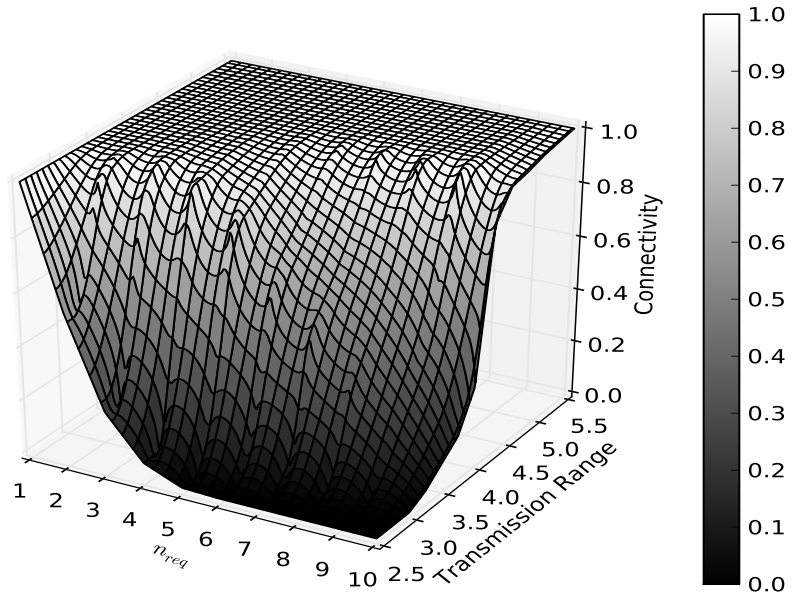


Figure 33: Connectivity versus n_{req} and transmission range

Concluding Remarks

- This thesis is motivated by recent interest in UWSNs as a platform for preforming many useful tasks.
- A challenge arises since sensor nodes incur small scale and large scale movements that can disrupt network connectivity.
- Thus, tools for quantifying the likelihood that a network remains completely or partially connected become of interest.
- To this end, the thesis has formalized 4 probabilistic connectivity problems, denoted $\mathrm{A\text{-}CONN}$, $\mathrm{S\text{-}CONN}$, $\mathrm{AR\text{-}CONN}$, and $\mathrm{SR\text{-}CONN}$.
- The obtained results show that all of the 4 problems admit polynomial time algorithms on k-trees (and their subgraph), for any fixed k.
- The running time of the algorithms, however, increase exponentially as k increases. Thus, more work needs to be done towards obtaining more effective algorithms.

Future Research Directions

- The class of probabilistic connectivity problems discussed in the thesis shares some similarity with the class of network reliability problems.Investigating the applicability of methods discussed in Colbourn 1987 to probabilistic connectivity problems appears to be a worthwhile direction.
- It is interesting to analyze the delays incurred in typical data collection rounds.
- Area coverage analysis is a topic that has received attention in UWSNs. In light of node mobility in UWSNs, it appears worthwhile to investigate area coverage assuming a probabilistic locality model of the nodes.