Multi Stage Underwater Sensor Localization using Mobile Beacons

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Abstract-Underwater Sensor Networks (USN) are used for harsh oceanographic missions where human operation is dangerous or impossible. Localization is essential for USNs. It is required for data tagging, node tracking and position-based routing algorithms. Localization is challenging because Global Positioning System (GPS) is not available in underwater; at the same time, existing GPS-less schemes based on fixed landmarks have high communication cost. Such cost is critical in Mobile Underwater Sensor Networks (MUSN), since sensor nodes drift with the ocean currents, thus requiring continuous refresh. In this paper, we propose a multi-stage localization scheme using mobile beacons. The beacons periodically ascent and descent in the water column. When they resurface, they receive new GPS coordinates. Then, they dive to the level of the underwater sensors to advertise these coordinates. In turn, localized sensors become proxy beacons and propagate their own coordinates, etc. This iterative, multi-stage localization is the major innovation of this paper. The goal is to localize the nodes with the smallest number of beacons using proxies instead, yet achieving an adequate accuracy. The major benefit is the reduction in operating costs. Mobility is a critical factor in determining performance. In this paper, performance (i.e., the percentage of localized nodes during a cycle, accuracy, delay and communication cost) is tested in a simulation scenario based on a realistic mobility model. The "Meandering Current Mobility with Surface Effect" (MCM-SE) model - a composite model combining surface and subsurface currents.

I. INTRODUCTION

Sensor networks are becoming highly involved in our daily lives as they continuously collect data and monitor the surrounding environment. Raw sensor data are meaningful with the context, i.e., the knowledge of where and when the data is collected. This is known as data tagging. In addition, localization is required for node tracking, target detection and position-based routing algorithms. Sensor networks that operate outdoors are able to benefit from the GPS with some extra cost. Indoor, underground or underwater sensor networks need some specialized solutions for localization.

Underwater Sensor Networks (USNs) can improve ocean exploration, allowing a list of new applications that are presently not possible or very costly to perform, including: oceanographic data collection, ecological applications (e.g. pollution, water quality and biological monitoring), public safety (e.g. disaster prevention, seismic and tsunami monitor-

ing), military underwater surveillance, industrial (offshore exploration), etc. However, before USNs become commercially available or widely used, the networking of sensor nodes in underwater has to be addressed. Medium access and packet forwarding are still active research areas in USNs [1]–[6].

Localization is another challenging task. The use of GPS is restricted to surface nodes because the GPS signal does not propagate through water. Alternative GPS-less positioning schemes have been proposed for terrestrial sensor networks but they have to be revised due to acoustic channel properties. The acoustic channel has low bandwidth, high propagation delay and high bit error rate. Therefore, localization protocols need to work with minimum possible message exchange. This is also dictated by the limited battery power of the sensor nodes and the difficulty of recharging or replacing batteries of the underwater nodes. In Mobile Underwater Sensor Networks (MUSNs), the mobility of free-floating nodes brings up another challenge in localization.

In this paper, we address the localization issue for MUSNs. We propose a multi-stage localization protocol using mobile beacons. Mobile beacons receive absolute time and location information from GPS when they float on the surface. Then, they periodically descent to distribute their coordinates. They are able to dive and rise with volume expansion therefore we name them as Dive and Rise (DNR) beacons. In addition to DNR beacons, an iterative, multi-stage localization is employed. The already localized nodes become active beacons and distribute their coordinates. Our protocol aims to maximize the number of localized nodes while keeping the error, communication overhead and the delay low.

In simulating a mobile network, the ability of the mobility model to capture the real life observations is of major importance. Here, we use the "Meandering Current Mobility with Surface Effect" (MCM-SE) model. The MCM was first suggested by physical oceanographers as a simple model for lagrangian studies of western boundary currents [7] and it is applied to underwater sensor networks in [8]. The MCM describes a sub-surface, jet-like current meandering around recirculating vortices. In this work, we model the surface mobility with a stochastic process superimposed to the MCM. We study the performance of our localization protocol when



the sensor nodes drift according to this composite model.

In Section II, we summarize the related work. We describe our localization protocol and mobility model in Section III and Section IV, respectively. In Section V, we present our simulation results. Section VI concludes the paper.

II. RELATED WORK

Most of the works related to underwater sensor networks have focused on routing protocols [9], [10], energy minimization and MAC [5], [6], [11] issues. Localization is less explored for USNs.

The sensors that are currently used in oceanographic research are localized either with Long or Short Base-Line (LBL/SBL) systems [12]. In both cases, the positions of sensors are determined on the basis of acoustic communications with a set of receivers. In the LBL, acoustic transponders are deployed either on the seafloor or under the surface moorings around the area of operation [13]. In the SBL system, a ship follows the sensors and uses a short-range emitter to enable localization. A commercial SBL localization tool is available for underwater environment [14]. It uses a vessel to localize the underwater equipments. Both techniques are expensive and require long planning for deployment.

Alternatives to these systems have been recently investigated as USNs become more pronounced. These new localization services work as virtual underwater GPS. They aim to address a number of important challenges like: work in a 3-D space, handle mobility and have easy deployment. In [15], the authors consider localization for large-scale USNs. They use surface buoys and two types of underwater nodes: anchor nodes and ordinary sensor nodes. At first, anchor nodes are localized by the help of surface buoys and then the ordinary sensors are localized using these anchor nodes. Anchors are spread among sensors to achieve better localization for large-scale 3D USNs. Though this architecture has benefits, locating anchor nodes is also a challenge by itself. In [16] mobile beacons are used to increase the localization coverage in 3D space. Beacons dive and rise to act as underwater GPS. [16] does not consider multi-stage localization. In this work, we complement the idea of DNR beacons with iterative localization.

In [17], a prediction-based localization scheme is proposed for mobile underwater sensor networks. The same hierarchical USN as in [15] is used. Here, anchor nodes are able to predict their mobility model and they confirm the accuracy of their prediction via measurements with surface buoys. If their model is accurate enough, they do not broadcast updates. This means if the nodes follow a certain mobility pattern they do not receive unnecessary messages, saving from communication cost. In [15] and [17] the authors assume that the anchor nodes are localized by surface buoys but the cost or the contention that will be caused by this operation is not discussed. Their simulation results are not based on an underwater physical layer or a MAC layer.

The mobility models proposed for underwater sensor networks are very few. They are borrowed from oceanography. [8] uses subsurface current model and [17] uses shallow water

model. In [8], the effect of MCM over coverage, connectivity and localization is discussed.

III. MOBILE BEACONS WITH MULTI-HOP LOCALIZATION

We use several mobile beacon nodes and multi-stage process to localize the 3D MUSN. The mobile beacons are called Dive and Rise (DNR) beacons since they are able to move vertically in water [16]. They are equipped with GPS receivers to receive coordinates while floating above the water. We denote the total number of sensor in the network with N and N = B + Swhere B is the number of DNR beacons and S is the number of underwater sensor nodes. We assume the beacons and the sensors do not have surface or ocean bottom wires, i.e. they are free-floating. They may be deployed by dropping from a ship or a plane. We assume that the sensors are built with different densities, such that they distribute uniformly on a number of different depth levels, h. The depth or in other words, the z coordinate depends on the density of each sensor and its value can be learned from an internal pressure sensor. Our localization technique enables sensors to estimate their (x,y) coordinates.

The limited communication range and large deployment volume require mobile beacons. Mobile beacons move up and down to distribute their coordinates. However, diving and rising takes longer time than message propagation. A supplementary "multi-stage phase" can speed up the localization process at the cost of less accuracy and more messaging. In "multi-stage phase", as the nodes become localized, they start to act as beacons. The sensors lying out of the communication range of DNR beacons can be localized by the messages of these new beacons. Similar iterative schemes have been proposed for terrestrial sensor networks [18]–[20]. Here, we make use of these techniques to improve the performance of mobile beacons. Hereafter, we call the sensors that are able to become beacons as "active sensors" and the non-localized sensors as "passive sensors".

All of the sensors are passive at the initial deployment. Beacons send localization messages. Each node stores a limited number of messages in the "localization table". The localization packet format is given in Fig. 1. Coordinates are used in lateration. At least three messages from the beacons at non-collinear locations should be heard for lateration. The distances between the node and the beacons are also required. The distance is measured with the Time of Arrival (ToA) of the messages, assuming the nodes are synchronized. The timestamp field is used in ToA measurement. Maximum dive depth is used in iterative localization. Localized sensors may become active only if they lie below the maximum dive depth of DNR beacons. In this way, we limit the area of multihop phase towards downstream nodes (i.e. the nodes lying in levels deeper than the communication range of the DNR beacons) in order to keep the communication cost low. Hop count field determines the hop distance of the active nodes from the beacons. In iterative localization, error accumulates at the active nodes that are distant from the beacons. Therefore, active nodes with least hop distance to beacons should be

			i i
Coordinates	Timestamp	Max. dive depth	Hop count

Fig. 1. Localization packet format.

preferred in lateration equations. Moreover, an active node may help localizing its neighbour and later, the localized neighbour may send an update to the active node. To prevent the message propagation back and forth and to receive the latest updates, messages with higher timestamp and lower hop count are used in lateration. Lateration estimates the coordinates as follows.

The unknown (x,y) coordinates should satisfy the equations:

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = d_i^2$$
 (1)

where i denotes the beacon id, (x_i, y_i, z_i) are the coordinates and d_i is the measured distance. Note that three equations are sufficient for solving this nonlinear equation system for (x,y). By subtracting the $n+1^{th}$ equation from the first n equations, the system is linearized. We solve $A\varphi=b$ where,

$$A = \begin{bmatrix} 2(x_1 - x_n) & 2(y_1 - y_n) \\ \vdots & \vdots \\ 2(x_{n-1} - x_n) & 2(y_{n-1} - x_n) \end{bmatrix}$$

$$b = \left[\begin{array}{c} x_1^2 - x_n^2 + y_1^2 - y_n^2 + z_1^2 - z_n^2 - 2z(z_1 - z_n) + d_n^2 - d_1^2 \\ \vdots \\ x_{n-1}^2 - x_n^2 + y_{n-1}^2 - y_n^2 + z_{n-1}^2 - z_n^2 - 2z(z_{n-1} - z_n) + d_n^2 - d_{n-1}^2 \end{array}\right]$$

The estimated coordinates $\hat{\varphi} = [\hat{x} \ \hat{y}]^T$ are found by using least-squares approach: $\hat{\varphi} = (A^T A)^{-1} A^T b$.

The lateration error, ϵ is given as:

$$\epsilon = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(x_i - \hat{x})^2 + (y_i - \hat{y})^2 + (z_i - z)^2} - d_i \quad (2)$$

We accept the estimates if ϵ is less than the communication range [21]. Moreover, nodes with high estimation error, even if they are localized, are not allowed to become active to avoid error accumulation.

IV. MEANDERING CURRENT MOBILITY WITH SURFACE EFFECTS

In this paper, we consider a simple mobility model that partitions the ocean into two layers: a surface layer where the motion of the water is directly affected by the local winds and a sub-surface layer where the current is determined by the large-scale, internal dynamics of the ocean. The subsurface model, MCM, is first applied to USNs in [8].

In a typical coastal environment, we may find a dominant current flowing along the coast, which is time-dependent both in intensity and shape, and sheds vortices (see for example [22]). The simplest model for such a current is given by Bower [7]. Here we use the following non-dimensional streamfunction:

$$\psi(x, y, t) = -\tanh \left[\frac{y - B(t)\sin(k(x - ct))}{\sqrt{1 + k^2 B^2(t)\cos^2(k(x - ct))}} \right]$$
(3)

which is a time dependent generalization of Bower's model [23]. From the knowledge of ψ , the two components (u, v) of a divergenceless, horizontal velocity field are recovered as:

$$u = -\frac{\partial \psi}{\partial u}; \quad v = \frac{\partial \psi}{\partial x}.$$
 (4)

A sensor at (x,y,z) is moved by requiring that its horizontal velocity is the same as that of the surrounding flow, that is: $\dot{x}=u,\dot{y}=v$. It is important to note that, vertical movements of the sensors are determined mostly by buoyancy forces, rather than by currents. In our model, sensors are calibrated to follow predefined surfaces of constant density (which can be assumed, in first approximation, as horizontal surfaces). Dive and rise beacons are able to change their volume, thus they can change their vertical position.

The streamfunction (3) represents a jet-like current meandering between recirculating vortices. The amplitude of the meanders is modulated by the time-dependent function $B(t) = A + \epsilon \cos(\omega t)$, and their phase shifts with a speed c. For a wide range of parameters, this flow induces a net mass transport along the current and at the same time, a vigorous chaotic mixing across the current. We use A = 1.2, c = 0.12, $k=2\pi/7.5, \ \omega=0.4, \ \epsilon=0.3$ (For further discussion on these values see [23].) By taking one non-dimensional unit of space to be a kilometer, and one non-dimensional unit of time to be 0.03 days, we have that the size of the meanders is 7.5 km, the typical current speed inside the jet is about 0.3 m/s, and the modulation period is about half a day (a value in agreement with the main tidal period). With these scalings we take the streamfunction (3) as representative of a typical coastal current.

The uppermost layer of the ocean (often called *mixed layer* by the oceanographers) is directly affected by the local winds. The velocity of water parcels in the mixed layer are the result of a delicate and ever-changing balance between the subsurface currents, the wind stress and the Coriolis force. A detailed description of this processes is beyond the scope of this paper (and it is still a subject of active research among physical oceanographers). Here, we will assume that a node floating on the surface has a velocity which is a random perturbation of the subsurface velocity, that is

$$(u, v)_{\text{node}} = (u, v)_{\psi} + (u_s, v_s).$$
 (5)

In order to keep the model simple, we assume that the stochastic component (u_s, v_s) does not have a spatial dependence. This is equivalent to the assumption that the wind blows with the same speed and direction everywhere on the domain. In view of the small domain that we are considering (10x80 km) this is not too far from reality. Furthermore, we observe that the wind does not swing wildly instant after instant, and that its speed has a typical magnitude, which is (on average) rather

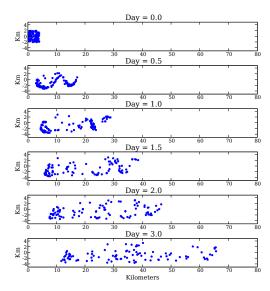


Fig. 2. Time evolution of the position of 100 sensors randomly released in 4km^2 area centered on the axis of the jet-like current.

constant. These observations imply that a stochastic process used to generate (u_s,v_s) needs to have a finite self-correlation in time, and a finite variance. The simplest model that fits both requirements is the Ornstein-Uhlenbeck process described by the Langevin equation

$$du = -\lambda u dt + \sqrt{2\lambda U^2} dw \tag{6}$$

where w(t) is a Wiener process, the positive constants λ and U are the inverse of the decorrelation time and the root-mean-squared speed of the wind [24], [25], respectively. The v component of the velocity is described by the same Langevin equation, with an independent Wiener process.

In practice, the velocities are computed at discrete time intervals, so we use the following discrete expression of (6)

$$\begin{array}{rcl} u_s(t+\Delta t) & = & u_s(t)e^{-\lambda\Delta t} + U\sqrt{1-e^{-2\lambda\Delta t}}\zeta_i \\ v_s(t+\Delta t) & = & v_s(t)e^{-\lambda\Delta t} + U\sqrt{1-e^{-2\lambda\Delta t}}\xi_i \end{array}$$

where ζ_i and ξ_i are independent pseudo-random numbers from a zero-mean, unit-variance gaussian distribution. The parameters are chosen as: $\lambda^{-1}=2{\rm days},\ U=0.5{\rm m/s}.$ The motion of lagrangian devices according to MCM-SE is shown in Fig. 2 for typical mid-latitude conditions.

V. PERFORMANCE EVALUATION

The performance of "multi-stage localization using mobile beacons" is analyzed in terms of localization success, communication cost, mean error and delay. Localization success is the ratio of localized sensor nodes to network size. Communication cost is the number of messages sent by DNR beacons and active sensor nodes in multi-stage phase. Mean error is given in

meters and it is the average difference between the estimated distances and true locations. The ratio of the mean error to terrain size is given. The delay is the average time to get a location estimate for the first time. Location can be updated and refined in time but we consider the delay in estimating the location for the first time.

We use an acoustic physical layer in Qualnet simulator. The communication range is set to 300m. The number of underwater nodes vary from 100 to 250. Nodes are initially deployed over a (1000,1000,600) volume. We assume that sensors are built with different densities so that they lie at several levels, e.g. level=200,400,600 and they are distributed uniformly among the layers. 25 DNR beacons are randomly distributed at level=0. DNR beacons broadcast with period $T_b=100S$. Active ordinary sensors have the same period $T_s=100S$. Each node keeps M=4 entries in the localization table and uses lateration for estimating its location. In MCM-SE model we limit the effect of surface currents to 250m, i.e. below this depth sensors move according to MCM model. The model parameters are given in Section 4. We give the averaged values of 10 simulation runs.

Before discussing simulation results, the role of maximum dive depth, D, and estimated error should be explained. DNR beacons descent until depth D, then they start ascending until they reach the surface. For example, D=0 means DNRs stay on the surface. D, also determines the region of message forwarding for active nodes. Only the nodes at levels deeper than or equal to D are potential active beacons. Again, D=0 means all the underwater nodes may be allowed to become active. However, estimated error also determines if an underwater node can become active. Estimated error is defined as in (2). Nodes are allowed to become active when $\epsilon < 50$ meters. For example, for D=600, DNRs dive until 600m and only the nodes at 600m can become active if their estimated error is less than 50 meters. The nodes above need to wait for DNR beacons.

DNRs are initially deployed on the surface. We test the performance of localization algorithm for the cases when DNRs can dive until depth, D=0,200,400,600 meters. The results are given in Fig. 3. In Fig. 3 (a), for D=0 and D=200, the trend in increase is similar and the localization ratio increases with the number of nodes. In both of the cases, nodes at level=200,400,600, i.e., all the underwater nodes are potential active beacons. For D=400, increasing the number nodes has a slower trend in improving localization ratio.

The communication cost is close for D=0,200,400. The lowest communication cost is at D=600 which is expected because only the nodes at 600m are able to become active and the localization ratio is also lower than it is in the other cases.

The percentage of mean error is the ratio of the mean error to terrain size. For D=600, this is the lowest because the nodes are localized by directly hearing from DNR beacons. In iterative localization it is known that error accumulates, therefore mean error is high for other D values.

For mean delay, at D=0, the nodes do not wait for DNRs

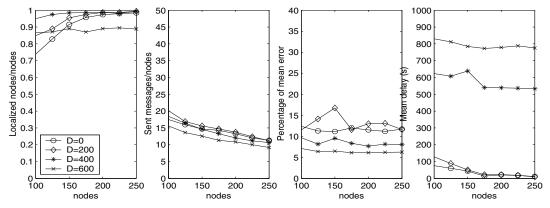


Fig. 3. Performance as a function of max depth and of density. (a) Localization success, (b) communication cost, (c) mean error, (d) delay.

and the delay is due to propagation delay. For D=600, the nodes at level=200,400 actually wait for DNRs to dive. For this reason the dealy is higher for larger D.

VI. CONCLUSION

Multi-stage localization yields more than 85% localization fraction for a 3D mobile underwater sensor network without introducing significant extra communication overhead. Without mobile beacons, localization success and accuracy increase, however the total time to localize the network is longer. When only mobile beacons are used, the total cycle time decreases at the expense of localization success and accuracy.

Communication cost is obviously higher for the mobile beacon case since some of the sensors act as beacons injecting extra messages into the network. By combining the two approaches we can optimize the tradeoff between accuracy, fraction of localized nodes and delay. Using these results, the depth and density of the beacons can be optimized to fit the application requirements.

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