

# The Node Movement Models Based on Lagrange Motion for 3-D Underwater Acoustic Sensor Network<sup>\*</sup>

Zhaohua Yang<sup>1</sup>, Shaobin Cai<sup>2,3</sup>, Nianmin Yao<sup>2</sup>, Haiwei Pan<sup>2</sup>, and Qilong Han<sup>2</sup>

<sup>1</sup> College of Instrumentation and Opto-electronics Engineering,

Beijing University of Aeronautics & Astronautics, Beijing, China, 100191

<sup>2</sup> College of Computer, Harbin Engineering University, Harbin, China, 150001

<sup>3</sup> College of Acoustic Engineering, Harbin Engineering University, Harbin, China, 150001

Caishaobin@hrbeu.edu.cn

**Abstract.** UWASN (UnderWater Acoustic Sensor Network) is a kind of WSN (Wireless Sensor Network) consisting of underwater acoustic sensor nodes. In its studies, the simulation is a key tool for the research of UWASN. However, the existing node movement models can not reflect the motion characteristics of nodes in a 3-D space, caused by the sea current. So, a new node movement model, based on Lagrange motion, is proposed in this paper to describe the movement of nodes in a 3-D oceanic current. It is proved that the new model can more really describe the 3-D movements of nodes in the current by the performance analysis.

**Keywords:** UnderWater Acoustic Sensor Network, Movement, Model, Lagrange, 3-D.

## 1 Overview

UWASN (UnderWater Acoustic Sensor Network) is a kind of WSN (Wireless Sensor Network) consisting of underwater acoustic sensor nodes, which can be deployed for real-time warship monitoring, oceanographic data collection, environmental monitoring, disaster prevention, etc. There are two ways for the performance analysis of UWASN. One of them is the sea test, the other is the simulation. However, the fee of the sea test is so huge that it can't be used frequently. Therefore, the simulation is a key tool for the research of UWASN. In the performance simulation analysis of underwater acoustic sensor network, a movement model, which can reflect the real movement of the nodes in the ocean, is the base of the performance analysis. Now, simple model and model based on Lagrange motion is the classic movement model of UWASN. However, the existing node movement models can not reflect the motion characteristics of nodes in a 3-D space, caused by the sea current.

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In this paper, a new node movement model based on Lagrange motion is proposed, which can reflect the sea current movement. Compared with the old movement model based on Lagrange motion, the new model has the following characters: (1) the impact of unexpected events on the movement of the nodes is considered; (2) the states of seawater are different under the different pressure.

The rest of the paper is arranged as following: firstly, the simple movement model is introduced in chapter 2; secondly, the model based on Lagrange is introduced in chapter 3; thirdly, a 3-D underwater mobility model based on Lagrange is proposed in chapter 4; fourthly, the performance of the new model is analyzed by mathematics and simulations in chapter 5; finally, a conclusion is drawn in chapter 6.

## 2 Simple Movement Model

The static model is the simplest one [1]. In a static model, the positions of all nodes do not change. The static model can not reflect the node movement characters of UWASN. Hence, a simple movement model was proposed [1]. In this model, the nodes of the same layer do not change, and their movements are caused by the powers, which have the same direction. Now, the model is described as following:

$$\begin{aligned}x(t) &= x(t-1) + v_{cx} \\ y(t) &= y(t-1) + d_t v_{cy}\end{aligned}\tag{1}$$

In the above formula,  $d_t = \begin{cases} -1 & \text{if } d(t-1)=1 \\ 1 & \text{if } d(t-1)=-1 \end{cases}$ ,  $\delta$  is a random number in the interval  $[0, 1]$ ,  $v_{cx}$  and  $v_{cy}$  are two

random numbers in the interval  $[0, v_{\max}]$ ,  $l_{cy}$  is the threshold of nodes oscillatory movement. Therefore, in this model, the nodes move slowly with a state of swing. The model is simple, easy to control, and suitable for the anchored UWASN. This kind of movement model with AUV nodes, whose movement trajectories are sine curve, was used in the performance analysis [1].

Now, in the marine surveying and mapping, Euler and Lagrange methods are mainly used for independent fundamental measurement research [2]. In the Euler method, the collected data don't change with time; In Lagrange method, the data are collected by the underwater equipments, which move with the ocean current. Hence, a unique description of the ocean current can be given by Lagrange method.

In shallow water area, the water depth is normally between 100m~300 m. The speed of current is relatively slow; the movement is not very complicated. Hence, a motion model based on the Euler method is proposed [3]. In this model, all nodes do swing movement in a certain area, or do clockwise or counterclockwise rotation movement in other areas. The movement is described as following:

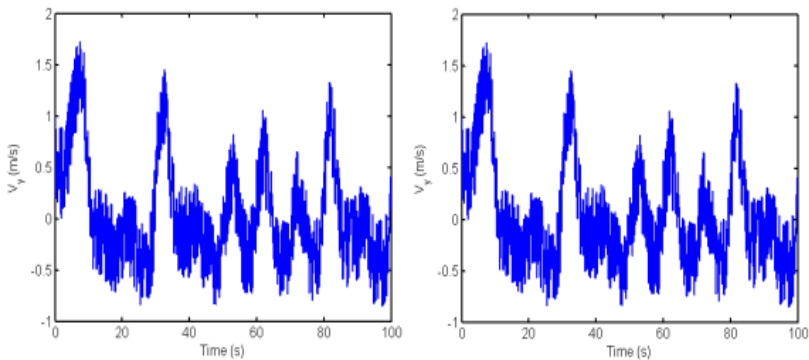
$$\begin{cases} V_x = k_1 \lambda \sin(k_2 x) \cos(k_3 y) + k_1 \lambda \cos(2k_1 t) + k_4 \\ V_y = -\lambda v \cos(k_2 x) \sin(k_3 y) + k_5; \end{cases} \quad (2)$$

In the above formula,  $V_x$  is the speed of x direction,  $V_y$  is the speed of y direction.

$k_1$ 、 $k_2$ 、 $k_3$  and  $\lambda$  are the coefficient closely related to environment, such as, the tide and the depth, which change with the environment;  $k_4$ 、 $k_5$  are random variables related with environment. Therefore, in the model, described in the above formula, the speed of the node is related with time  $t$  and the coordinates (x, y) of node [2] (shown in fig.1).

### 3 Models Based on Lagrange motion

The movements of ocean current particles based on curve model are studied in a miles-long current [4] [5] [6]. The influence of vertexes and flows and of all layers is considered in this model.



**Fig. 1.** The relationship between speed and time

In oceanic surveying, it is usually assumed that movements of particles in the vertical direction normally are floating movement [7]. In other words, the nodes do not initiatively accelerate their movement. In this case, the floating movements of nodes are related to the environmental density. So, their movement is usually like-Damped movement because of the resistance of sea water.

The water of deep or shallow sea area is pushed by strong winds; the up and down stream are formed. So, the internal fluctuations are created in the current. However, according to oceanic models, the phenomenon above can be ignored. In the 70's of last century, a nodes movement model, which can effectually reflected the main characteristics of two-dimensional ocean currents (flow and vortex) is proposed by Dr

Bower[reference]. In this model, if any two-dimensional incompressible currents is represented by  $\varphi$ , then the speed of a node  $U = (u, v)$  is described as following:

$$u = -\frac{\partial \varphi}{\partial y}; \quad v = \frac{\partial \varphi}{\partial x} \tag{3}$$

In the above formula,  $u$  presents x axis direction speed, and  $v$  presents y axis direction speed. Now, the Lagrange movement trajectories of a node can be described as following:

$$\dot{x} = -\partial_y \varphi(x, y, t), \quad \dot{y} = \partial_x \varphi(x, y, t) \tag{4}$$

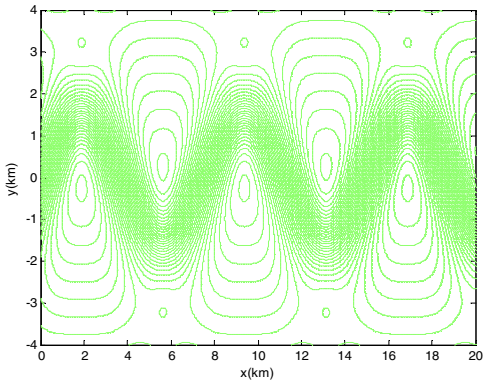
Furthermore, the model also defined the curved injection flow as following:

$$\varphi(x, y, t) = -\tanh\left[\frac{y - B(t) \sin(k(s - ct))}{\sqrt{1 + k^2 B(t)^2 \cos^2(k(x - ct))}}\right] + cy \tag{5}$$

$$B(t) = A + \mathcal{E} \cos(\omega t)$$

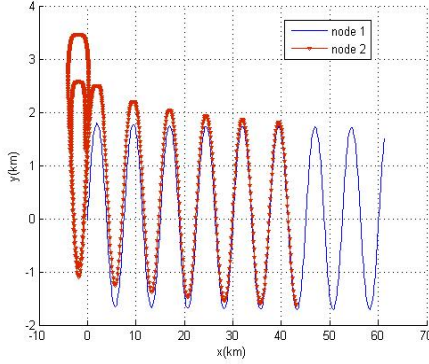
In above model, the curves can shuttle among vortexes. With the parameters change, the particles exchanged between two parts can be seen. Among the parameters,  $k$  represents the number of exchanged flows in the space;  $c$  represents the y axis direction speed;  $B$  represents the width of curve flows;  $\mathcal{E}$  represents the amplitude of the entire flow field; and  $\omega$  represents the movement frequency of the flow field.

When  $A=1.2$ ,  $c = 0.12$ ,  $k = \frac{2\pi}{7.5}$ ,  $\omega=0.4$ ,  $\mathcal{E}=0.3$ , the ocean flow field is shown in Fig. 2.



**Fig. 2.** Environment of flow-field

Fig. 3 depicts the movement of particles (represents nodes) in curved flow model. In Figure 4.3, a cycle is 0.03 days, the size of a curved flow is 7.5 km, the speed of a peak is 0.3m/s, and a simulation cycle is a half day. In Fig. 3, it can be seen that the movement of nodes is a Lagrange movement. In this flow field, particles not only can move from vortex to curve, but also can move from curve to vortex.



**Fig. 3.** The movement of particles in flow-field

Compared with the simple model, this model based on Lagrange motion can reflect the 2-D movement characters of a node in currents can. However, it still can not simulate the movements of nodes in a complicate 3-D space.

#### 4 The 3-D Underwater Mobility Model Based on Lagrange

In order to describe the 3-D movement of the sensor nodes, the formula 3 is improved in this section. Firstly, the impact of unexpected events on the movement of the nodes is considered. So, the speed of a node  $U = (u, v)$  is described as following:

$$\begin{aligned} u &= -\frac{\partial \phi}{\partial y} + \sigma u(t) \\ v &= \frac{\partial \phi}{\partial x} + \sigma v(t) \end{aligned} \quad (6)$$

In the above formula  $\sigma u(t)$  is the variance of the Gaussian noise of X-axis direction speed,  $\sigma v(t)$  is the variance of the Gaussian noise of Y-axis direction speed. Their mean value is 0 and their variance is  $\sigma^2$ .

Secondly, the model is modified according to that the states of seawater are different under the different pressure. That is, in the normal movement law of ocean current, the speed of the deep flow is lower than speed of the surface flow. So, the new model is defined as following:

$$\phi(x', y', t) = -(0.7 + \frac{1}{|z| + 10}) \tanh \left[ \frac{y' - B(t) \sin(k(s - ct))}{\sqrt{1 + k^2 B(t)^2 \cos^2(k(x' - ct))}} \right] + cy' \quad (7)$$

$$B(t) = A + \varepsilon \cos(\omega t)$$

In the above formula,  $x' = \frac{1}{5} x$ ,  $y' = \frac{1}{4} y$ .

5 Performance Analysis

In the section, MATLAB is used for simulation and analysis. In a 30km×120km×5km 3D underwater space, nodes are randomly placed in the left-middle 2km×4km×5km area. Fig.4 describes the ocean currents state based on formula 7, which has an improved the curve jet flow model. In Fig.4, the upside of vertex is larger than the lower of it, just likes infundibulate. So, the new model can describe the inherence of ocean current better.

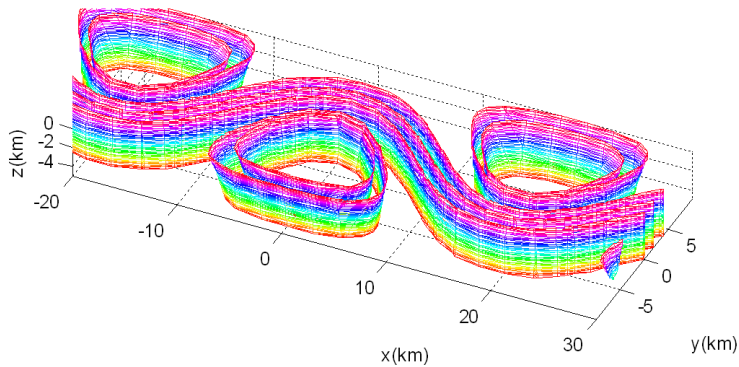
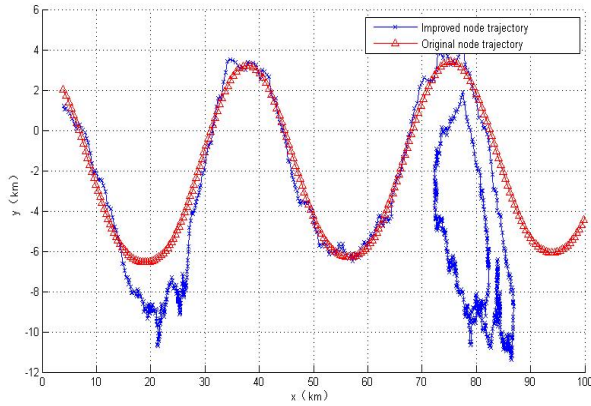


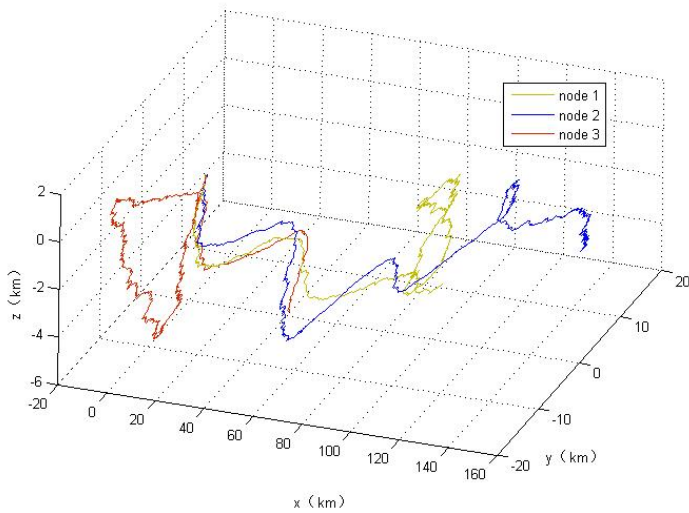
Fig. 4. Sketch of flow field

In Fig.5 the 2-D node movement trajectories of the original model and the improved model are compared when  $6 \sigma u(t) = 0.002$ ,  $\sigma v(t) = 0.008$ . From Fig.5, it can be seen that not only the node shuttle between vertex and curves but also the influence of unexpected events on nodes movement are reflected in the improved model. So, the improved model can describe the ocean current better.

Fig.6 describes the movement track of a DNR (Dive “N” Rise) node in 3D space. In Fig.6, the bounder is set  $\pm \varphi = \frac{\setminus Z \setminus + 10}{7 \setminus Z \setminus + 80} \times 0.536$  to divide the area. If the



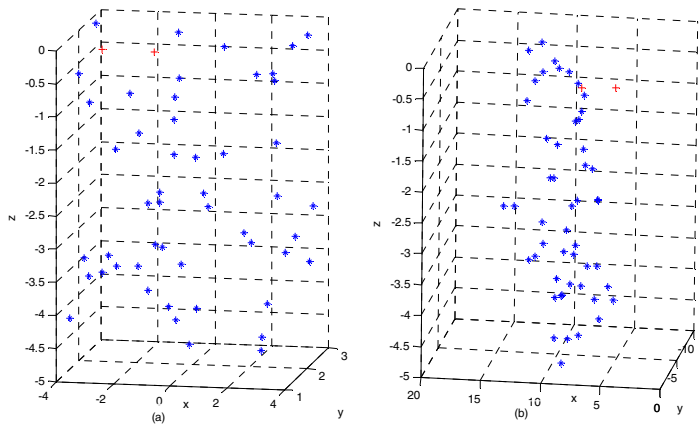
**Fig. 5.** Track of speed model of changed



**Fig. 6.** Track of node with current movement

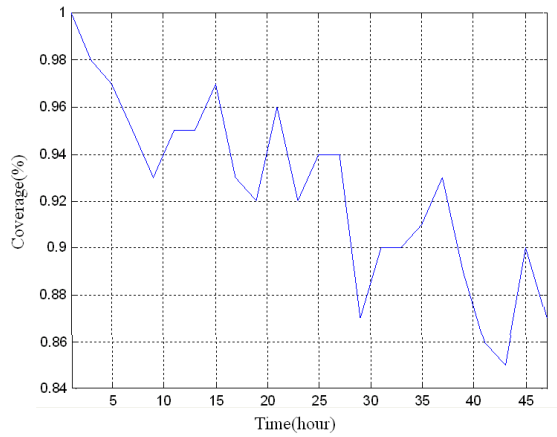
calculation result of formula (7) is in  $[-\varphi, +\varphi]$ , the node is in the vortexes, otherwise, the nodes in the jet stream. So, formula (6) can be used to calculate the real speed of a DNR node in the flow field.

Fig.7 describes the normal stochastic swing in vertical direction of 100 nodes, which are randomly placed in  $8 \times 2 \times 5 \text{ km}$  area (shown in Fig.7 (a)). In Fig.7, red '+' represents AUV node, blue '\*' represents the common node. After 3 hours, nodes move with the current, and their final states are shown as Fig.7 (b). From the figures, it can be seen all normal nodes move as a whole.



**Fig. 7.** Network diagram as the change of time

Fig. 8 describes the coverage of the above network. It can be seen that a node can not move with the others in the curve current when it enters a vortex. So, the coverage of network decreased gradually as a whole. However, the coverage of network regression sometimes because of nodes shuttle among vortexes curve current.



**Fig. 8.** Coverage of node

## 6 Conclusion

As a key tool for the research of UWASN, the simulation plays an important role in the studies of UWSAN. On the basis of analyzing existed models for UWSAN, a modified node movement model based on Lagrange motion for 3-D space is proposed



in this paper. The performance analyses show that the new model is better than the existing ones. It can more closely simulate the movements of nodes in a 3-D oceanic current.

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