A Study of k-Coverage and Measures of Connectivity in 3D Wireless Sensor Networks

Habib M. Ammari, *Member*, *IEEE*, and Sajal K. Das, *Senior Member*, *IEEE*

Abstract—In a wireless sensor network (WSN), *connectivity* enables the sensors to communicate with each other, while sensing *coverage* reflects the quality of surveillance. Although the majority of studies on coverage and connectivity in WSNs consider 2D space, 3D settings represent more accurately the network design for real-world applications. As an example, underwater sensor networks require design in 3D rather than 2D space. In this paper, we focus on the connectivity and k-coverage issues in 3D WSNs, where each point is covered by at least k sensors (the *maximum value* of k is called the *coverage degree*). Precisely, we propose the *Reuleaux tetrahedron* model to characterize k-coverage of a 3D field and investigate the corresponding minimum sensor spatial density. We prove that a 3D field is guaranteed to be k-covered if any Reuleaux tetrahedron region of the field contains at least k sensors. We also compute the connectivity of *3D k-covered* WSNs. Based on the concepts of *conditional connectivity* and *forbidden faulty sensor set*, which cannot include *all* the neighbors of a sensor, we prove that *3D k-covered* WSNs can sustain a large number of sensor failures. Precisely, we prove that *3D k-covered* WSNs have connectivity higher than their coverage degree k. Then, we relax some widely used assumptions in coverage and connectivity in WSNs, such as sensor homogeneity and unit sensing and communication model, so as to promote the practicality of our results in real-world scenarios. Also, we propose a placement strategy of sensors to achieve full k-coverage of a 3D field. This strategy can be used in the design of energy-efficient scheduling protocols for *3D k-covered* WSNs to extend the network lifetime.

Index Terms—*3D k-covered* wireless sensor networks, Reuleaux tetrahedron, coverage, connectivity.

1 Introduction

A wireless sensor network (WSN) consists of a large number of resource-limited (such as CPU, storage, battery power, and communication bandwidth), tiny devices, which are called *sensors*. These sensor nodes can sense task-specific environmental phenomenon, perform in-network processing on the sensed, and communicate wirelessly to other sensor nodes or to a sink (also, called *data gathering node*), usually via multihop communications. WSNs can be used for a variety of applications dealing with monitoring, control, and surveillance.

In the literature, most of the works on WSNs dealt with 2D settings, where sensors are deployed on a planar field. However, there exist applications that cannot be effectively modeled in the 2D space. For instance, WSNs deployed on the trees of different heights in a forest, or in a building with multiple floors, or underwater applications [2], [3] require the design in the 3D space. Oceanographic data collection, pollution monitoring, offshore exploration, disaster prevention, and assisted navigation are typical applications of underwater sensor networks [2]. In [27], different deployment strategies have been proposed for 2D and 3D communication architectures in underwater acoustic sensor networks, where sensors are anchored at the bottom of the ocean for the 2D design and float at different depths of the ocean to cover the entire 3D region. As shown in [31], both coverage of space and energy-efficient data routing for a telepresence application require 3D design.

In this paper, we study the *coverage* and *connectivity* issues in *3D k-covered* WSNs, where sensors are deployed in a 3D *field*, such that every location is covered by at least k sensors (a property known as *k-coverage*, where the maximum value of k is called the *coverage degree*). The limited battery power of sensors and the difficulty of replacing and/or recharging batteries in hostile environments require that the sensors be deployed with high density (up to 20 sensors per cube meter [30]) in order to extend the network lifetime. Moreover, to cope with the problem of *faulty sensors* due to low battery power and achieve high data accuracy, redundant coverage (in particular, k-coverage) of the same region is necessary. As mentioned, the essential function of a WSN is to monitor a field of interest and report the sensed data to the sink for further processing. However, sensing coverage would be meaningless if the sensed data cannot reach the sink due to the absence of communication paths between it and the *source sensors* (or *data generators*). In other words, network connectivity must be guaranteed. A WSN is said to be *fault tolerant* if it remains *functionally connected* in spite of some sensor failures. That is why each source sensor has to be connected to the sink by multiple communication paths. This is why we focus on connectivity. Hence, a WSN will be able to function properly if both coverage and connectivity are maintained simultaneously. This implies that *all* source sensors and the sink should belong to the same connected component.

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Traditional (or *unconditional*) connectivity assumes that any subset of nodes can potentially fail at the same time, including *all* the neighbors of a given node. While such connectivity can measure the fault tolerance of small-scale networks, it is not effective for large-scale dense networks, such as highly dense, 3D *k-covered* WSNs. In such networks with thousands of sensors, it is *highly unlikely* that all the neighbors of a given sensor node fail simultaneously. This is due to high-density deployment and heterogeneity of the sensors. Assuming a uniform sensor distribution, the ratio of the size of the neighbor set of a given sensor to the total number of sensors in a WSN covering a field of volume V is $4\pi R^3/3V$, where $R \ll V^{1/3}$ is the radius of the communication range of the sensors. The probability of failure of all the neighbors of a given sensor could be identified with this ratio, and hence, is very low. Furthermore, in real-world sensing applications [19], WSNs have heterogeneous sensors with unequal energy levels and different sensing, processing, and communication capabilities, thus increasing the network reliability and lifetime [36]. This also implies that the probability that the entire neighbor set of a sensor fails at the same time is very low.

In this study, we use a more general (and realistic) concept of connectivity, called *conditional connectivity* [15] with respect to some property P, defined as follows: The *conditional connectivity* of a connected graph G is the smallest number of nodes of G whose removal disconnects G into components each of which has property P. Another generalization of connectivity, called *restricted connectivity* [12], is based on the concept of *forbidden faulty set*, where *all* neighbors of a node cannot simultaneously fail.

1.1 Contributions

In this paper, we address the following questions concerning coverage and connectivity in 3D *k-covered* WSNs:

1. Given a 3D field, what is the minimum sensor spatial density to guarantee full k-coverage of the field?
2. What is the connectivity and conditional connectivity of 3D *k-covered* WSNs?

To this end, our main contributions are summarized as follows:

1. We characterize k-coverage of 3D WSNs by using the fundamental result of the intersection of convex sets stated in Helly's Theorem. Based on this characterization and the concept of *Reuleaux tetrahedron*, we compute the minimum sensor spatial density required to guarantee full k-coverage of a 3D field. We find that this density is $k/0.422r_0^3$ for *homogeneous* 3D *k-covered* WSNs, where $r_0 = r/1.066$ and r is the sensing radius of sensors. For *heterogeneous* 3D *k-covered* WSNs, r is replaced with r_{min}, the minimum sensing radius of all sensors. Our simulation results match well with the analytical results.
2. Based on the minimum sensor spatial density, we prove that the connectivity of 3D k-covered WSNs is much higher than the degree k of sensing coverage provided by the network. More precisely, this connectivity is $9.926(R/r_0)^3k$ for homogeneous k-covered WSNs, where $r_0 = r/1.066$ and R is the radius of the communication range of sensors. For heterogeneous 3D k-covered WSNs, r is replaced with r_{min} and R with R_{min}, where R_{min} is the minimum radius of the communication range of all sensors.
3. We show that the traditional connectivity metric used to capture network fault-tolerance underestimates the resilience of 3D k-covered WSNs. In particular, we find that the conditional connectivity of 3D k-covered WSNs is equal to $[((r_0 + 2R)^3 - r_0^3)/r_0^3]k$ for homogeneous k-covered WSNs, where $r_0 = r/1.066$. For heterogeneous 3D k-covered WSNs, r and R are replaced with r_{min} and R_{min}, respectively.

The remainder of this paper is organized as follows: Section 2 reviews related work on coverage and connectivity. Section 3 introduces the basic assumptions and definitions. Section 4 discusses coverage and connectivity of 3D k-covered WSNs, while Section 5 focuses on their conditional connectivity. Section 6 shows how to relax some of the assumptions used in our analysis. Section 7 concludes the paper.

2 Related Work

In this section, we review a sample of approaches for coverage and connectivity in 2D and 3D WSNs. For more comprehensive review on coverage and connectivity in WSNs, the interested reader is referred to [8], [13].

2.1 Coverage and Connectivity of 2D WSNs

In [1], a directional sensors-based approach is proposed for WSN coverage, where the coverage region of a directional sensor depends on the location and the orientation of sensors. A differentiated coverage algorithm is presented in [9] for heterogeneous WSNs, where different network areas do not have the same coverage degree. In [14], the authors proposed centralized and distributed algorithms for connected sensor cover, so the network can self-organize its topology in response to a query and activate the necessary sensors to process the query. The relationship between sensing coverage and communication connectivity of WSNs is studied and distributed protocols to guarantee that both coverage and connectivity are proposed in [18]. Based on the Voronoi diagram and graph search algorithms, an optimal polynomial time (worst and average case) algorithm for coverage calculation is proposed in [26].

Studies on k-coverage to maintain connectivity in 2D WSNs have been the focus of several works. The first to study coverage and connectivity in WSNs in an integrated fashion is due to [34], where the authors proved that if the communication radius of sensors is double their sensing radius, a network is connected given the coverage is provided. They also proposed a coverage configuration protocol based on the degree of coverage of the application. In [17], polynomial-time algorithms are presented for the coverage problem to check whether every point in a field is at least k-covered, depending on whether sensors have the same or different sensing ranges. Efficient distributed algorithms are proposed in [24] to optimally solve the best coverage problem with the least energy consumption. In [37], Zhang and Hou proposed a distributed algorithm in order to keep a small number of active sensors in a WSN regardless of the relationship between sensing and communication ranges. The results in [34] and [37] are improved in [32] by proving that if the original network is connected and the identified active nodes can cover the same region as all the original nodes, then the network formed by the active nodes is connected when the communication range is at least twice the sensing range. An optimal deployment strategy is described in [5] to achieve both full coverage and 2-connectivity regardless of the relationship between communication and sensing radii of sensors. A distributed and localized algorithm using the concept of the kth-order Voronoi diagram is proposed in [40] to provide fault tolerance and extend the network lifetime, while maintaining a required degree of coverage.

Necessary and sufficient conditions for 1-covered, 1-connected wireless sensor grid networks are presented in [29]. A variety of algorithms have been proposed to maintain connectivity and coverage in large WSNs. In [20], Kumar et al. proposed k-barrier coverage for intrusion detection. The authors established an optimal deployment pattern for achieving k-barrier coverage and developed efficient global algorithms for checking such coverage. As shown in [21], the minimum number of sensors needed to achieve k-coverage with high probability is approximately the same regardless of whether sensors are deployed deterministically or randomly, if sensors fail or sleep independently with equal probability. The changes of the probability of k-coverage with the radius of the sensing range of sensors or the number of sensors are studied in [33]. The k-coverage set and the k-connected coverage set problems are formalized in terms of linear programming with two nonglobal solutions in [35]. The coverage problem in heterogeneous planar WSNs as a set intersection problem is formulated in [22] and [23] which also derived analytical expressions to which quantify the stochastic coverage. It is proved in [38] that the required density to k-cover a square region depends on both the side length of the square field and the coverage degree (k).

2.2 Coverage and Connectivity of 3D WSNs

Coverage and connectivity in 3D WSNs have gained relatively less attention in the literature. A placement strategy based on Voronoi tessellation of 3D space is proposed in [4], which creates truncated octahedral cells. Several fundamental characteristics of randomly deployed 3D WSNs for connectivity and coverage are investigated in [28], which compute the required sensing range to guarantee certain degree of coverage of a region, the minimum and maximum network degrees for a given communication range as well as the hop-diameter of the network. Related to our work is the novel result discussed in [43], which proved that the breadth of the Reuleaux tetrahedron is not constant. This shows that the properties of 2D space cannot be directly extended to 3D space. Indeed, the Reuleaux triangle [44] (counterpart of Reuleaux tetrahedron in 2D space) has a constant width. Note that the Reulueax tetrahedron is the symmetric intersection of four congruent spheres such that each sphere passes through the centers of the other three spheres. However, the Reuleaux triangle [44] corresponds to the symmetric intersection of three congruent disks such that each disk passes through the centers of the other two disks. Thus, its constant width is equal to the radius of these disks.

Our work can be viewed as an extension of [4] by considering k-coverage in 3D WSNs. Moreover, existing works on coverage and connectivity in WSNs assumed the notion of traditional connectivity, while our work considers a more realistic measure, namely, *conditional connectivity* [18], which is based on the concept of *forbidden faulty set* [15]. Also, our work exploits the result given in [43] with regard to the breadth of the Reuleaux tetrahedron discussed earlier. This helps us provide correct measures of connectivity and fault tolerance of 3D WSNs based on an accurate characterization of k-coverage of 3D fields.

3 Assumptions and Definitions

In this section, we present the assumptions made in our analysis and define some key concepts. Relaxation of some of the assumptions will be discussed in Section 6 and Appendix B.

We consider static WSNs, which represent the sensing and communication ranges of sensors as *spheres*. From now on, the sensing and communication ranges of a sensor s_i are called *sensing* and *communication spheres* of radii r_i and R_i, respectively, centered at ξ_i which represents the location of s_i. In a homogeneous (*heterogeneous*) WSNs, the sensors have (*do not have*) the same sensing ranges and the same communication ranges. The communication links between sensors are supposed to be perfectly reliable, while sensor nodes can fail or die independently due to low battery power. We consider a cubic field of volume V in 3D euclidean space. Moreover, we assume that the volumes of the sensing and communication spheres of sensors are negligible compared to V such that r_i, $R_i \ll V^{1/3}$.

Let us define the following terms. A location ξ of a field is said to be 1-*covered* (or *sensed*) if it belongs to the sensing sphere of at least one sensor. A 3D convex region C is said to be k-*covered* if each location in C is covered by *at least* k sensors.

The *neighbor set* of a sensor s_i is defined as $N(s_i) = \{s_j : |\xi_i - \xi_j| \leq R_i\}$, where ξ_j is the location of the neighbor s_j of s_i and $|\xi_i - \xi_j|$ is the euclidean distance between ξ_i and ξ_j.

The *breadth* of 3D convex region C is the *maximum distance* between tangential planes on opposing faces or edges of C.

A *communication graph* of a WSN is a graph $G = (S, L)$, where S is a set of sensors and L is a set of communication links between them such that for all s_i, $s_j \in S$, $(s_i, s_j) \in L$ if $|\xi_i - \xi'_j| \leq R_i$.

The *connectivity* of a communication graph G of a WSN is equal to k if G can be disconnected by removing at least k sensors.

4 Unconditional Connectivity of 3D k-Covered WSNs

In this section, we show how to guarantee k-coverage of a 3D field, derive the corresponding minimum sensor spatial density, and compute measures of *unconditional* (or traditional) connectivity for homogeneous 3D k-covered WSNs, where any subset of sensors can fail. The results for heterogeneous WSNs are summarized in Appendix A.

4.1 Minimum Sensor Density for k-Coverage

Lemma 1 characterizes the *breadth* of a *3D k-covered* convex region.

Lemma 1. *A 3D convex region C is guaranteed to be k-covered with exactly k sensors if its breadth is less than or equal to r, the radius of the sensing spheres of the sensors.*

Proof (By contradiction). Assume that the breadth of a 3D convex region C does not exceed r and C is not k-covered when exactly k sensors are deployed in it. Notice that each of these k sensors is located on the boundary or inside of C. Thus, there must be at least one location $\xi \in C$ that is not k-covered. In other words, there is at least one sensor s_i located at ξ' such that $|\xi - \xi'| > r$, which contradicts our hypothesis that the breadth of C does not exceed r. □

It is true that the deployment of k sensors in a 3D convex region, say C, whose breadth is larger than r cannot guarantee its k-coverage, where r is the radius of the sensing range of the sensors. Let p_i and p_j be two points in C such that one sensor s_i is located at p_i and $\delta(p_i, p_j) = b > r$, where b is the breadth of C and $\delta(p_i, p_j)$ is the euclidean distance between p_i and p_j. Given that $b > r$, it is impossible for s_i to sense any event that occurs at p_j. Thus, there is at least one sensor (i.e., s_i) among those k sensors, which cannot cover p_j, and hence, C cannot be k-covered.

Next, we compute the minimum sensor spatial density required for guaranteeing k-coverage of a 3D field. To this end, we compute the maximum volume of a 3D convex region C that is guaranteed to be k-covered when exactly k sensors are deployed in it. First, we state Helly's Theorem [6], a fundamental result that characterizes the intersection of m convex sets in n-dimensional space, where $m \geq n+1$.

Helly's Theorem (Intersection of convex sets) [6]. *Let Ψ be a family of convex sets in IR^n such that for $m \geq n+1$, any m members of Ψ have a nonempty intersection. Then, the intersection of all members of Ψ is nonempty.*

From Helly's Theorem [6], we infer that given $k \geq 4$ sensors, a 3D convex region C is k-covered by those k sensors if and only if C is 4-covered by any four of those k sensors. Given that the breadth of C is r, the network induced by sensors located in C is guaranteed to be connected if $R \geq r$, where r and R are, respectively, the radii of the sensing and communication spheres of sensors. Now, let us address the first question: *What is the minimum sensor spatial density necessary to guarantee full k-coverage of a 3D field?* Theorem 1 computes the minimum sensor density.

Theorem 1. *Let r be the radius of the sensing spheres of sensors and $k \geq 4$. The minimum sensor spatial density required to fully k-cover a 3D field is computed as*

$$\lambda(r,k) = \frac{k}{0.422 r_0^3}, \tag{1}$$

where $r_0 = r/1.066$.

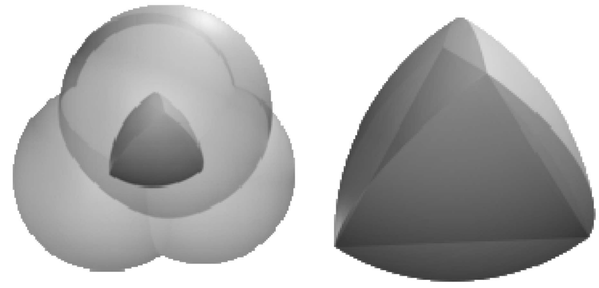
Proof. Let C_k be the intersection of k sensing spheres and assume that their centers *do not* coincide in a 3D field. From Lemma 1, it follows that C_k is guaranteed to be k-covered by exactly k sensors if its breadth does not exceed the radius r of the sensing spheres of sensors. Thus, the maximum volume of C_k is obtained when its breadth is equal to r. From Helly's Theorem [6], it follows that the intersection of k sensing spheres is not empty if the intersection of any four of these k spheres is not empty. On the other hand, the intersection set operator requires that the *maximum* intersection volume of these k sensing spheres be equal to that of four spheres provided that the maximum distance between any pair of these k sensors does not exceed r. Let us focus on the analysis of four sensing spheres. The maximum overlap volume of four sensing spheres such that every point in this overlap volume is 4-covered corresponds to the configuration: the center of each sensing sphere is at distance r from the centers of all other three sensing spheres. Precisely, the sensing sphere of each of the four sensors passes through the centers of the other three sensing spheres, as shown in Fig. 1. The edges between the centers of these four spheres form a *regular tetrahedron* and the shape of the intersection volume of these four spheres is known as the *Reuleaux tetrahedron* [43] and denoted by $RT(r)$. Unfortunately, it was proved that the Reuleaux tetrahedron does not have a constant breadth [43]. Indeed, while the distance between some pairs of points on the boundary of the Reuleaux tetrahedron $RT(r)$ is equal to r, the maximum distance between other pairs of points on the boundary of $RT(r)$ is equal to $1.066r$ [43], i.e., slightly larger than r. This implies that the Reuleaux tetrahedron $RT(r)$ cannot be k-covered with exactly k sensors given that the distance between some pairs of points on the boundary of $RT(r)$ is larger than r, where r is the radius of the sensing spheres of the sensors. Therefore, the Reuleaux tetrahedron that is guaranteed to be k-covered with exactly k sensors should have a side length equal to $r_0 = r/1.066$. The volume of the Reuleaux tetrahedron $RT(r_0)$ is given by [43]

$$vol(r_0) = (3\sqrt{2} - 49\pi + 162\tan^{-1}(\sqrt{2}))r_0^3/12 \approx 0.422 r_0^3.$$

Thus, $RT(r_0)$ is the maximum volume that can be k-covered by exactly k sensors, where $k \geq 4$. We conclude that the *maximum* volume of C_k, denoted by $vol_{max}(C_k)$, is equal to $vol_{max}(C_k) = 0.422 r_0^3$. Given that $vol_{max}(C_k)$ has to contain k sensors to k-cover C_k, we conclude that the minimum sensor spatial density per unit volume required for full k-coverage of a 3D field is computed as

$$\lambda(r,k) = k/vol_{\max}(C_k) = k/0.422 r_0^3.$$

□



(a) (b)

Fig. 1. (a) Intersection of four symmetric spheres and (b) their corresponding Reuleaux tetrahedron.

We should mention that the notion of the *arc length* discussed in [43] corresponds to the (maximum) breadth of the Reuleaux tetrahedron. Indeed, it is possible to find two parallel plans that bound the Reuleaux tetrahedron such that the maximum distance between these two plans is equal to r only.

It is worth emphasizing that the value of $\lambda(r, k)$ is tight in the sense that it is minimum given that k sensors should be located within $vol_{max}(C_k)$, such that C_k is guaranteed to be k-covered with exactly these k sensors. Also, notice that $\lambda(r, k)$ depends only on the coverage degree k dictated by a sensing application and the radius r of the sensing range of sensors. Moreover, $\lambda(r, k)$ increases with k. Indeed, high coverage degree k requires more sensors to be deployed, and hence, a denser WSN is necessary. Also, $\lambda(r, k)$ decreases as r increases. When the sensing range gets larger, a fewer number of sensors is needed to fully k-cover a 3D field. Both of these observations reflect the real behavior of the sensors. Thus, $\lambda(r, k)$ does not depend on the size of the field as opposed to the result of the stochastic approach in [38].

Lemma 2 uses Theorem 1 and states a *sufficient condition* for k-coverage of a 3D field.

Lemma 2. *A 3D field is guaranteed to be k-covered if any Reuleaux tetrahedron region of side r_0 in the field contains at least k sensors, where $r_0 = r/1.066$ and $k \geq 4$.*

It is worth noting that there may be some differences between analytical and simulation/implementation results due to the *boundary effects*. In fact, it is impossible to decompose a 3D field into *complete* Reuleaux tetrahedra such that the Reuleaux tetrahedra close to the border of a 3D field lie entirely inside it. Hence, more than necessary number of sensors is used to k-cover the border of the 3D field.

Next, based on the minimum sensor spatial density (Theorem 1) and another criterion to be specified in the following section, we compute the network connectivity of 3D k-covered WSNs.

4.2 Computing Network Connectivity

Data accuracy depends on the size of the connected component that contains the sink. It reaches the highest value when the sink belongs to the largest connected component of the network. Thus, high-quality coverage requires *all* source sensors be connected to the sink. That is why we focus on the sink to compute the connectivity of 3D k-covered WSNs. In other words, connectivity of WSNs should be so defined as to take into consideration the inherent structure of this type of network. Indeed, sensors have neither the same role nor the same impact on the network performance. Thus, measuring the connectivity of WSNs should account for their specific morphology, where the sink is the most critical node in the network. Hence, we compute the connectivity of 3D k-covered WSNs based on the *size of the connected component* that includes the *sink*. Theorem 2 deals with homogeneous WSNs.

Theorem 2. *Let G be a communication graph of a homogeneous 3D k-covered WSN deployed in a cubic field of volume V. The connectivity of G, which is denoted by $\kappa(G)$, is given by*

$$\kappa_1(G) \leq \kappa(G) \leq \kappa_3(G), \tag{2}$$

where

$$\kappa_1(G) = 12.024\alpha^3 k,$$

$$\kappa_3(G) = \frac{R\mathrm{V}^{2/3}k}{0.422r_0^3},$$

in which $r_0 = r/1.066$, $\alpha = R/r$, and $k \geq 4$.

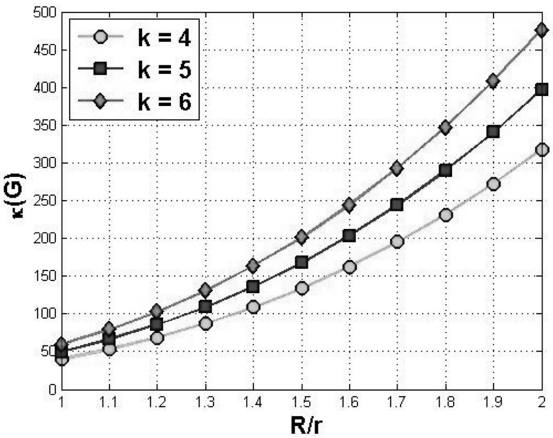


Fig. 2. Plot of the function $\kappa_1(G)$ (fix k and vary α).

Proof. The optimum position of the sink in terms of energy-efficient data gathering is the center of the cubic field [25]. Let ξ_0 be the location of the sink s_0. We consider the following three cases depending on the size of the connected component that includes the sink. Also, given that sensor failure is due to low battery power, we assume that the sink has infinite source of energy, thus excluding the possibility of a faulty sink.

Case 1: Single-node connected component. In this case, there are at least two components, one of them being the single-node component containing the sink. Finding the minimum number of nodes to disconnect the network requires that the disconnected network has only two components. In order to isolate the sink, all its neighbors must fail. Hence, at least the communication sphere of the sink should contain no sensor but the sink.

Let N be a random variable that counts the number of sensor failures to isolate the sink s_0. Given that sensors are randomly and uniformly deployed in a volume V with density $\lambda(r, k)$ per unit volume, where $R \ll V^{1/3}$, the expected number of neighbors of the sink is given by

$$E[\mathrm{N}] = \lambda(r, k)\|B(\xi_0, R)\|, \tag{2a}$$

where $|B(\xi_0, R)| = 4\pi R^3/3$ is the measure of the volume of the communication sphere $B(\xi_0, R)$ of the sink s_0 located at ξ_0. Thus, the expected minimum number of sensor failures to isolate s_0 is equal to $E[\mathrm{N}]$. Substituting (1) in (2a), we find that the network connectivity is given by

$$\kappa_1(G) = E[\mathrm{N}] = 12.024\alpha^3 k, \tag{2b}$$

where $\alpha = R/r$. Figs. 2 and 3 plot the function $\kappa_1(G)$ in (2b). Clearly, $\kappa_1(G)$ increases with the ratio α and the degree of coverage k. More importantly, $\kappa_1(G)$ is much higher than k.

Case 2: Nontrivial connected components. Two configurations of the disconnected network are of particular interest where the two connected components of the disconnected network are separated by a vacant region (or gap). Furthermore, any pair of sensors, one from each component, are separated by a distance at least equal to R in order to prohibit any communication between the two components. In the first configuration (Fig. 4a), the component including the sink is reduced to

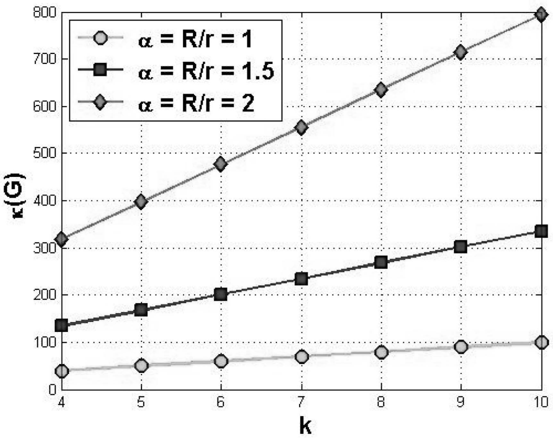


Fig. 3. Plot of the function $\kappa_1(G)$ (fix α and vary k).

its communication sphere. Thus, the volume of the vacant region, denoted by $gap(\xi_0, R)$, and surrounding the component of the sink, is given by

$$\|gap(\xi_0, R)\| = 4\pi(2R)^3/3 - 4\pi R^3/3 = 9.333\pi R^3. \qquad (2c)$$

Thus, the expected minimum number of sensor failures to isolate the component of the sink is given by

$$E[\mathrm{N}] = \lambda(r, k)\|gap(\xi_0, R)\|. \qquad (2d)$$

Substituting (1) in (2d), we find that the network connectivity is equal to

$$\kappa_2(G) = E[\mathrm{N}] = 84.164\alpha^3 k, \qquad (2e)$$

where $\alpha = R/r$.

In the second configuration, the original network is split into two components such that the vacant region forms a *cuboid*, denoted by $cub(R)$, and whose sides are $R, V^{1/3}$ and $V^{1/3}$, as shown in Fig. 4b. Now, this configuration corresponds to the smallest connected component containing the sink if the field has to be divided into two regions such that none of them surrounds the other. Thus, the expected minimum number of sensor failures to isolate the connected component containing the sink is given by

$$E[\mathrm{N}] = \lambda\|cub(R)\|, \qquad (2f)$$

where

$$\|cub(R)\| = RV^{2/3}. \qquad (2g)$$

Substituting (1) and (2g) in (2f), it follows that network connectivity is equal to

$$\kappa_3(G) = E[\mathrm{N}] = \frac{R\mathrm{V}^{2/3}k}{0.422r_0^3}. \qquad (2h)$$

It is easy to check that $\kappa_3(G) > \kappa_2(G)$ since $R \ll V^{1/3}$.

Case 3: Largest connected component. This configuration is totally opposite to the one given in Case 1 and has only one isolated sensor. Since we are interested in k-coverage of the entire field, such a network is considered as disconnected. The network connectivity is the same as in Case 1. Thus,

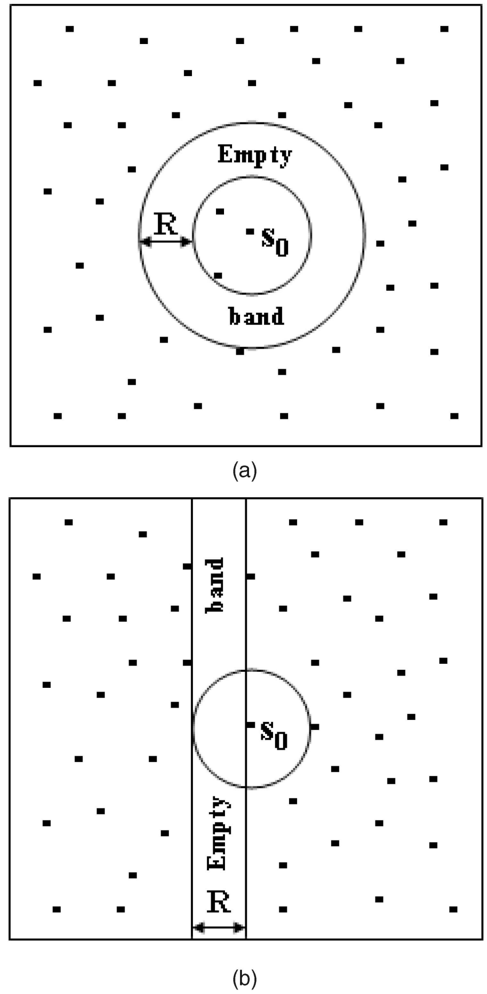


Fig. 4. 2D projection of nontrivial-connected components of the disconnected network.

$$\kappa_1(G) \leq \kappa(G) \leq \kappa_3(G).$$

It is easy to check that $\kappa(G) > k$. □

All these bounds and those to be derived in next sections are based on our fundamental result stated in Theorem 1. Since these bounds are based on the minimum sensor spatial density for full k-coverage of a 3D field, they are lower bounds.

4.3 Boundary Effects

Network connectivity is defined as the *minimum* number of sensors whose failure (or removal) disconnects the network. Given the cubic geometry of a 3D field we considered, sensors located close to the border of the field are affected by the boundary effects. Indeed, the communication ranges of these sensors cover areas outside of the deployment area, and hence, have fewer number of communication neighbors compared to all other sensors (especially the ones whose communication ranges lay entirely inside the field). However, the proposed network connectivity measures consider the sink as the most critical node in the network and whose isolation would definitely kill the entire network. Because the optimum position of the sink in terms of energy-efficient data gathering is the center of the cubic field [25], the boundary effects do not exist at all, and thus, our derived bounds on the connectivity are correct. Even when our approach for computing network connectivity measures considers all sensors as critical and peer-to-peer (see Appendix C for detailed discussion), the boundary effects do not have any impact on the derived bounds on network connectivity. Indeed, we are interested in the *minimum number* of sensor failures to disconnect the network.

As far as sensor deployment to achieve 3D k-coverage is concerned, we should mention that the boundary effects have an impact on the performance of the network. By Lemma 2, a 3D field is guaranteed to be fully k-covered if and only if each Reuleaux tetrahedron region in the field contains at least k sensors. However, it is impossible to randomly decompose a 3D field into an integer number of Reuleaux tetrahedron regions because of the boundary of the field. Indeed, most of the Reuleaux tetrahedron regions close to the border of a 3D field do not entirely lay inside the deployment area. Therefore, more than necessary sensors would be needed to achieve k-coverage of these Reuleaux tetrahedron regions on the border of the field. Simulation results reported in Section 4.1.2 show that the sensor spatial density necessary to fully k-cover a cubic field is a bit higher than the bound given in Theorem 1, mainly due to the boundary effects.

Next, we introduce new measures of connectivity of 3D k-covered WSNs by placing a specific constraint on a subset of sensors that would fail.

5 Conditional Connectivity of 3D k-Covered WSNs

In this section, we use the concepts of *conditional connectivity* [15] and *forbidden faulty set* [12], as a remedy to the above shortcomings of the traditional (unconditional) connectivity metric. Our approach is based on forbidden faulty sensor sets.

Let $G = (S, E)$ be a communication graph representing a 3D k-covered WSN. Define a *forbidden faulty sensor set* of G as a set of faulty sensors that includes the entire neighbor set of a given sensor. Consider the property P: "A faulty sensor set cannot include the entire neighbor set of a given sensor." A faulty sensor set satisfying property P is denoted by $F_{\rm P}$, where $F_{\rm P} \subset S$ and defined by

$$F_{\rm P} = \{U \subset S | \forall_{s_i} \in S : N(s_i) \not\subset U\}.$$

The *conditional connectivity* of G with respect to P, denoted by $\kappa(G : {\rm P})$, is the minimum size of $F_{\rm P}$ such that the graph $G_d = (S - F_{\rm P}, E_{S-F_{\rm P}})$ is disconnected, where $E_{S-F_{\rm P}}$ is a set of remaining communication edges between the nonfaulty sensors.

As we will see, our results prove that 3D k-covered WSNs can sustain a larger number of sensor failures under the restriction imposed on the faulty sensor set. According to Theorem 1, the minimum number of sensors necessary to k-cover a cubic field of volume V is given by $|S_{\min}| = Vk/0.422r^3$, where r is the radius of the sensing spheres of sensors. The probability of the failure of the entire neighbor set of a given sensor can be identified with the ratio of the size of the neighbor set of a given sensor to the total number of sensors $|S_{\min}|$. This ratio is equal to $4\pi R^3/3V$, which is very low given that $R \ll V^{1/3}$. This shows that the traditional connectivity, which does not impose any restriction on the faulty sensor set, is not a useful metric for 3D k-covered WSNs, which are highly dense networks.

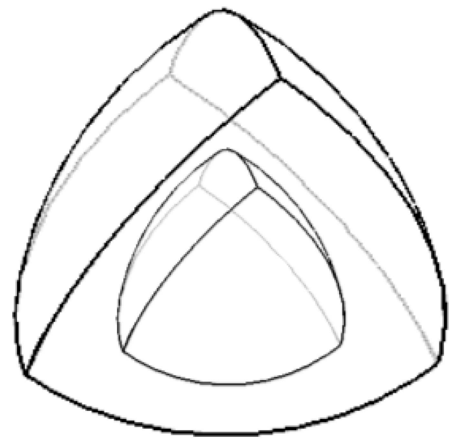


Fig. 5. Two nested concentric Reuleaux tetrahedra.

5.1 Homogeneous 3D k-Covered WSNs

Theorem 3 computes the conditional connectivity of homogeneous 3D k-covered WSNs.

Theorem 3. *The conditional connectivity of a homogeneous 3D k-covered WSN ($k \geq 4$) is given by*

$$\kappa(G : {\rm P}) = \frac{((r_0 + 2R)^3 - r_0^3)k}{r_0^3}, \tag{3}$$

where $r_0 = r/1.066$.

Proof. We consider the following two cases based on the type of connected component that contains the sink.

Case 1: Smallest connected component. According to our conditional connectivity model, no sensor can be isolated, and hence, no trivial component can be part of the disconnected network. Under the assumption of forbidden faulty sensor set, the smallest connected component that is disconnected from the rest of the network and contains the sink can be determined as follows: In order to achieve k-coverage of the cubic field, every location must be k-covered, including the location ξ_0 of the sink s_0. Otherwise, the k-coverage property will not be satisfied. Therefore, the smallest connected component that includes the sink consists of k sensors deployed in the Reuleaux tetrahedron of side r_0 and centered at ξ_0. In order to disconnect the sink under the forbidden faulty sensor set constraint, the Reuleaux tetrahedron should be surrounded by an empty annulus of width equal to R (sensors located in the annulus have failed), as can be seen in Figs. 5 and 6. The Reuleaux tetrahedron together with this annulus forms a larger Reuleaux tetrahedron of side $r_0 + 2R$. The volume of the annulus, denoted by $A(\xi_0, R)$, is equal to

$$\|A(\xi_0, R)\| = 0.422(r_0 + 2R)^3 - 0.422r_0^3.$$

Therefore, the expected conditional minimum number of sensor failures to disconnect the smallest component including the sink can be computed as

$$E[{\rm N} : {\rm P}] = \lambda(r, k)\|A(\xi_0, R)\|. \tag{3a}$$

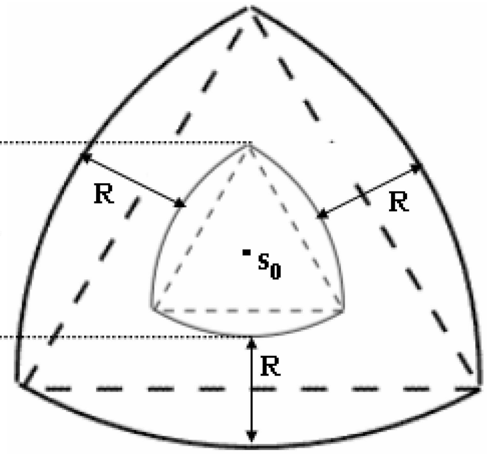


Fig. 6. 2D projection of an annulus of width R surrounding a Reuleaux tetrahedron of side r.

Substituting (1) in (3a), we find that the conditional connectivity is given by

$$\kappa_1(G : \mathrm{P}) = E[\mathrm{N} : \mathrm{P}] = \frac{((r_0 + 2R)^3 - r_0^3)k}{r_0^3}. \quad (3b)$$

It is easy to check that the forbidden faulty set constraint is satisfied for both the faulty sensors (located inside the annulus and which have failed) and nonfaulty sensors (located outside the annulus). Indeed, any sensor in the inner Reuleaux tetrahedron still has nonfaulty neighbors in the inner Reuleaux tetrahedron. Besides, any sensor outside the outer Reuleaux tetrahedron still has nonfaulty neighbors outside the outer Reuleaux tetrahedron. Also, any faulty sensor within the annulus $A(R)$ has nonfaulty neighbors located in the inner Reuleaux tetrahedron and outside the outer Reuleaux tetrahedron.

Case 2: Largest connected component. This case is similar to the previous one. However, the sink belongs to the largest connected component. Hence, the disconnected network consists of two components: the one including the sink and a second component associated with sensors located in a Reuleaux tetrahedron of side r. Using the same analysis as in Case 1, we obtain the same conditional network connectivity:

$$\kappa_2(G : P) = \kappa_1(G : P). \quad (3c)$$

From both cases 1 and 2, the conditional connectivity of homogeneous 3D k-covered WSNs is computed as

$$\kappa(G : P) = \kappa_1(G : P).$$

□

5.2 Heterogeneous 3D k-Covered WSNs

We observe that computing the conditional connectivity of heterogeneous 3D k-covered WSNs is not a straightforward generalization of the approach used previously for homogeneous 3D k-covered WSNs. If we use the same process as previously, we either violate the forbidden faulty sensor set constraint or maintain network connectivity. Precisely, if the width of the annulus containing the faulty sensors is R_{max}, then sensors whose communication radii are less than or equal to $R_{max}/2$ may be located in the annulus.

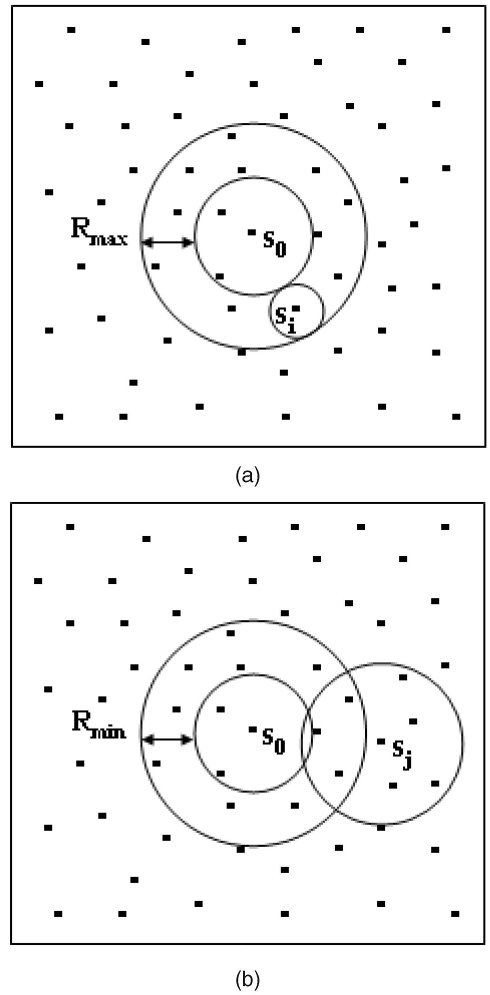


Fig. 7. 2D projection: (a) forbidden fault sensor set constraint violated and (b) connectivity maintained.

Thus, the entire neighbor set of this type of sensors would fail at the same time, and hence, the property P would be violated (Fig. 7a). Now, if the width of the annulus containing the faulty sensors is less than R_{max}, then the nonfaulty sensors of one connected component will be able to communicate with the nonfaulty sensors of the other connected component of the disconnected network. Hence, the network is still connected (Fig. 7b). In this case, it is impossible to find an exact value of conditional connectivity for heterogeneous 3D k-covered WSNs. Next, we compute their lower and upper bounds based on the types of sensors in and around the annulus.

Lemma 3. *The conditional connectivity of the heterogeneous 3D k-covered WSNs is given by*

$$\kappa_1(G : F_P) \leq \kappa(G : \mathrm{P}) \leq \kappa_2(G : F_P), \quad (4)$$

where

$$\kappa_1(G : \mathrm{P}) = \frac{((r_{\min}^0 + 2R_{\min})^3 - r_{\min}^0)^3 k}{{r_{\min}^0}^3},$$

$$\kappa_2(G : \mathrm{P}) = \frac{((r_{\max} + 2R_{\max})^3 - r_{\max}^3)k}{{r_{\min}^0}^3},$$

in which

$$\begin{aligned} k \geq 4, \quad r_{\min}^0 &= \min\{r_j/1.066 : s_j \in S\}, \\ r_{\max} &= \max\{r_j/1.066 : s_j \in S\}, \\ R_{\min} &= \min\{R_j : s_j \in S\}, \text{ and} \\ R_{\max} &= \max\{R_j : s_j \in S\}. \end{aligned}$$

Proof. As above, we consider the following two cases depending on the size of the connected component that includes the sink:

Case 1: Smallest connected component. In order to compute a lower bound on the conditional connectivity of heterogeneous 3D k-covered WSNs, we consider the Reuleaux tetrahedron centered at location ξ_0 of the sink s_0, which will be disconnected from the network. First, we assume that the annulus containing the faulty sensors as well as the volume surrounding it contain only least powerful sensors, and hence, the width of this annulus is equal to $R_{\min}$. Also, to guarantee that the sink will not be isolated, which would violate the forbidden faulty sensor set constraint, the Reuleaux tetrahedron centered at ξ_0 should have a side equal to r_{min}. These two conditions help disconnect the network while satisfying the forbidden faulty sensor set constraint. The volume of the annulus $A(\xi_0, R_{\min})$ is given by

$$\|A(\xi_0, R_{\min})\| = 0.422(r_{\min} + 2R_{\min})^3 - 0.422r_{\min}^3. \tag{4a}$$

Hence, the expected conditional minimum number of sensor failures to disconnect the connected component including the sink (or the inner Reuleaux tetrahedron) from the rest of the network is computed as

$$\mathrm{E[N : P]} = \lambda(r_{\min}, k)\|A(\xi_0, R_{\min})\|, \tag{4b}$$

where

$$\lambda(r_{\min}, k) = \frac{k}{0.422r_{\min}^3}$$

and

$$r_{\min} = \min\{r_j/1.066 : s_j \in S\}.$$

Thus, the conditional network connectivity is given by

$$\kappa_1(G : \mathrm{P}) = \mathrm{E[N : P]} = \frac{((r_{\min} + 2R_{\min})^3 - r_{\min}^3)k}{r_{\min}^3}, \tag{4c}$$

where $k \geq 4$ and $R_{\min} = \min\{R_j : s_j \in S\}$.

In order to compute an upper bound on the conditional connectivity of heterogeneous 3D k-covered WSNs, we assume that sensors inside the annulus are the most powerful ones. Thus, the side of the inner Reuleaux tetrahedron should be equal to r_{max}, while the width of the annulus surrounding it should be equal to R_{max}. It is easy to check that this setup will disconnect the network, while satisfying the forbidden faulty set constraint. The upper bound on the conditional connectivity is given by

$$\begin{aligned} \kappa_2(G : \mathrm{P}) &= \mathrm{E[N : P]} = \lambda(r_{\min}, k)\|A(\xi_0, R_{\max})\| \\ &= \frac{((r_{\max} + 2R_{\max})^3 - r_{\max}^3)k}{r_{\min}^3}, \end{aligned} \tag{4d}$$

where

$$\begin{aligned} \lambda(r_{\min}, k) &= \frac{k}{0.422r_{\min}^3}, \quad k \geq 4, \\ r_{\min} &= \min\{r_j/1.066 : s_j \in S\}, \\ r_{\max} &= \max\{r_j/1.066 : s_j \in S\}, \quad \text{and} \\ R_{\max} &= \max\{R_j : s_j \in S\}. \end{aligned}$$

Case 2: Largest connected component. In this case, the sink belongs to the largest connected component of the disconnected network. Hence, the previous analysis applies to any sensor in the network instead of the sink. Thus, the conditional connectivity of heterogeneous 3D k-covered WSNs satisfies

$$\kappa_1(G, P) \leq \kappa(G, P) \leq \kappa_2(G, P).$$

□

Next, we relax the assumptions used in our previous analysis to enhance the practicality of our results.

6 Discussions

The analysis of the minimum sensor spatial density necessary for k-coverage of a 3D field and network connectivity of 3D k-covered WSNs is based on the *unit sphere model*, where the sensing and communication ranges of sensors are modeled by spheres. In other words, sensors are supposed to be typically isotropic. Although this assumption is the basis for most of the protocols for coverage and connectivity in WSNs, it may not hold universally, and thus, may not be valid in practice. In Appendix B, we show how to relax this assumption in order to promote the applicability of our results to real-world 3D WSN scenarios, and summarize our results for the convex model, where the sensing and communications ranges of sensors are convex but not necessarily spherical. Moreover, we assumed that our results for network connectivity hold for a degree of sensing coverage k, where $k \geq 4$. In this section, we show how to relax several assumptions to account for more realistic scenarios.

6.1 Relaxing the Assumption of $k \geq 4$

The analysis of k-coverage and connectivity for 3D k-covered WSNs are valid for all $k \geq 4$. Since the breadth of the Reuleaux tetrahedron is equal to r, our results can also be used for $k \leq 3$. That is, a 3D field can be k-covered by deploying k sensors in the Reuleaux tetrahedron, where $k \leq 3$. However, the network would be denser than necessary (especially for $k = 1$) and the coverage degree would be higher than that dictated by the application.

6.2 Sensor Placement Strategy

Notice that under the assumption of spherical model, it is impossible to achieve a degree of coverage exactly equal to k in all the locations of the cube. Thus, a sensor placement strategy to achieve k-coverage should be devised in such a way that every location in the cube is k'-covered, where k' is very close to k. This placement strategy should benefit from the geometry of the Reuleaux tetrahedron. The sensor placement problem can be transformed into a problem of covering a cube with overlapping sets of congruent

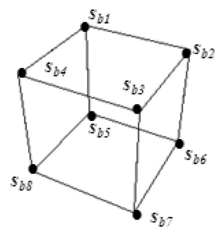


Fig. 8. Eight boundary sensors located on the corners of a cube.

Reuleaux tetrahedra. An optimal covering consists in using a minimum number of Reuleaux tetrahedra by minimizing the overlap volume between them. As described in Section 4.1.2, two adjacent Reuleaux tetrahedra overlap such that the faces of their corresponding regular tetrahedra are entirely coinciding with each other. Thus, the curved edges of the Reuleaux tetrahedra should overlap, so the same subset of sensors deployed on these curved edges could participate in covering the space associated with both Reuleaux tetrahedra, thus minimizing the number of sensors required to k-cover the cube. Notice that sensors located in a 3D lens, which corresponds to the overlap volume of two adjacent Reuleaux tetrahedra of side length r, are at distance r from any point in their volumes, and hence, participate in k-covering both tetrahedra. The design of duty-cycling protocols to k-cover a 3D field with a minimum number of sensors should select sensors based on this observation.

6.3 Sink-Independent Connectivity Measures

Although in centralized algorithms, the concept of sink is well defined, it is likely that distributed algorithms, such as the consensus-based algorithm, will be implemented for WSNs. In this case, the concept of (fixed) sink would lose value. It would be interesting to revise the definition of connectivity to take this concept into account. We suggest that all the nodes be considered as peer-to-peer. Thus, we define connectivity with respect to all sensors in the network. Given the geometry of the deployment field that we consider (cube), the *boundary sensors*, i.e., sensors located at the boundary of the cube, have small neighbor sets. In particular, the sensors located at the eight corners of the cube $s_{b1}, \ldots, s_{b8}$ as shown in Fig. 8 are highly likely to have the smallest neighbor set. Our measures of connectivity will be based on one of these boundary sensors to find the minimum number of sensors of its neighbor set that should fail in order to disconnect the network. It is easy to check that compared to the sink, the size of the neighbor set of a boundary sensor is equal to a *quarter* of that of the neighbor set of the sink. Thus, the previous connectivity measures computed in Sections 4 and 5 with respect to the sink remain the same for a boundary sensor but are weighted by a coefficient equal to $\frac{1}{4}$. For more details, the interested reader is referred to Appendix C.

6.4 Stochastic Sensing and Communication Models

In the *deterministic sensing model*, which has been considered so far, a point ξ in a field is covered by a sensor s_i based on the euclidean distance $\delta(\xi, s_i)$ between ξ and s_i. In this paper, we use "coverage of a point" and "detection of an event" interchangeably. Formally, the coverage $Cov(\xi, s_i)$ of a point ξ by a sensor s_i is equal to 1, if $\delta(\xi, s_i) \leq r$, and 0, else. As can be seen, this sensing model considers the sensing range of a sensor as a *sphere*, and hence, all sensor readings are precise and have no uncertainty. However, given the signal attenuation and the presence of noise associated with sensor readings, it is necessary to consider a more realistic sensing model by defining $Cov(\xi, s_i)$ using some probability function. That is, the sensing capability of a sensor must be modeled as the probability of successful detection of an event, and hence, should depend on the distance between it and the event as well as the type of propagation model being used (free-space versus multipath). Indeed, it has been showed that the probability that an event in a distributed detection application can be detected by an acoustic sensor depends on the distance between the event and the sensor [10]. A realistic sensing model for passive infrared (PIR) sensors that reflect their nonisotropic range was presented in [7]. This sensing irregularity of PIR sensors was verified by simulations [7]. Thus, in our *stochastic sensing model*, the coverage $Cov(\xi, s_i)$ is defined as the *probability of detection* $p(\xi, s_i)$ of an event at point ξ by sensor s_i as follows:

$$p(\xi, s_i) = \begin{cases} e^{-\beta\delta(\xi,s_i)^\alpha}, & \text{if } \delta(\xi, s_i) \leq r, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

where β represents the physical characteristic of the sensors' sensing units and $2 \leq \alpha \leq 4$ is the path-loss exponent. Precisely, we have $\alpha = 2$ for the free-space model and $2 < \alpha \leq 4$ for the multipath model. Our stochastic sensing model is motivated by Elfes' one [11], where the sensing capability of a sonar sensor is modeled by a Gaussian probability density function. A probabilistic sensing model for coverage and target localization in WSNs was proposed in [42]. This sensing model considers $\delta(\xi, s_i) - (r - r_e)$ instead of $\delta(\xi, s_i)$, where r is the detection range of the sensors and $r_e < r$ is a measure of detection uncertainty.

Under the stochastic sensing model, a point ξ in a field is said to be *probabilistically k-covered* if the detection probability of an event occurring at ξ by at least k sensors is at least equal to some *threshold probability* $0 < p_{th} < 1$.

In this section, we exploit the results of Section 4.1 to characterize probabilistic k-coverage in 3D WSNs based on our stochastic sensing model. Theorem 4 computes the *minimum k-coverage probability* $p_{k,min}$ such that every point in a field is probabilistically k-covered.

Theorem 4. *Let r be the radius of the nominal sensing range of the sensors, $r_0 = r/1.066$, and $k \geq 4$. The minimum k-coverage probability so that each point in a 3D field is probabilistically k-covered by at least k sensors under our stochastic sensing model defined in (5) is computed as*

$$p_{k,\min} = 1 - \left(1 - e^{-\beta r_0^\alpha}\right)^k. \tag{6}$$

Proof. First, we identify the least k-covered point in a 3D field, so we can compute $p_{k,min}$. By Lemma 2, k sensors should be deployed in a Reuleaux tetrahedron region of side $r_0 = r/1.066$ in the field to achieve k-coverage of a 3D field with a minimum number of sensors. It is easy to check that the least k-covered point ξ_{lc} is the one that corresponds to the configuration where all deployed

k sensors are located at a distance $r_0 = r/1.066$ from ξ_{lc}. In other words, ξ_{lc} is the farthest point from all those k sensors. Hence, the distance between ξ_{lc} and each of these k sensors is equal to $r_0 = r/1.066$. Thus, the *minimum k-coverage probability* for the least k-covered point ξ_{lc} by k sensors under the stochastic sensing model in **(5)** is given by

$$p_{k,\min} = 1 - \prod_{i=1}^{k}(1 - p(\xi, s_i)) = 1 - \left(1 - e^{-\beta r^{\alpha}}\right)^k.$$

□

The stochastic k-coverage problem is to select a minimum subset $S_{min} \subseteq S$ of sensors such that each point in a 3D field is k-covered by at least k sensors and the minimum k-coverage probability of each point is at least equal to some given threshold probability p_{th}, where $0 < p_{th} < 1$. This helps us compute the stochastic sensing range r_s, which provides probabilistic k-coverage of a field with a probability no less than p_{th}. Lemma 4 computes the value of r_s.

Lemma 4. *Let $k \geq 3$ and $2 \leq \alpha \leq 4$. The stochastic sensing range r_s of the sensors that is necessary to probabilistically k-cover a 3D field with a minimum number of sensors and with a probability no lower than $0 < p_{th} < 1$ is given by*

$$r_s = \left(-\frac{1}{\beta}\log(1 - (1 - p_{th})^{1/k})^{1/\alpha}\right), \quad (7)$$

where β represents the physical characteristic of the sensors' sensing units.

Proof. From (6), we deduce that $p_{k,\min} \geq p_{th} \Rightarrow r_s \leq (-\frac{1}{\beta}\log(1 - (1 - p_{th})^{1/k}))^{1/\alpha}$. Since we are interested in the *minimum* number of sensors to probabilistically k-cover a 3D field, we should consider the maximum value of r_s, i.e., the maximum stochastic sensing range of the sensors. This will allow the sensors to probabilistically k-cover as much space of the 3D deployment field as possible. Thus, $r_s = (-\frac{1}{\beta}\log(1 - (1 - p_{th})^{1/k}))^{1/\alpha}$. □

The upper bound on the stochastic sensing range r_s of the sensors computed in (7) will be used to compute our measures of connectivity and fault tolerance of 3D k-covered WSNs under the assumption of more realistic, stochastic sensing, and communication models. Fig. 9 shows r_s for different values of p_{th} and k while considering the free-space model ($\alpha = 2$) (Fig. 9a) and the multipath model ($\alpha = 4$) (Fig. 9b). Note that r_s decreases as a function of p_{th}, k, and α. This is due to the fact that the minimum probability $p_{k,min}$ of k-coverage of the same location by multiple sensors decreases as p_{th}, k, and α increase.

Lemma 5 states a sufficient condition for probabilistic k-coverage of a 3D field based on our stochastic sensing model in (5), the threshold probability p_{th}, and the degree k of coverage.

Lemma 5. *Let $k \geq 4$. A 3D field is probabilistically k-covered with a probability no lower than $0 < p_{th} < 1$ if any Reuleaux tetrahedron of maximum breadth $r_s/1.066$ in the field contains at least k sensors.*

Lemma 6 states a sufficient condition for connectivity between sensors under our stochastic sensing model.

Lemma 6. *Let $k \geq 4$. The sensors that are selected to k-cover a 3D field with a probability no less than $0 \leq p_{th} \leq 1$ under the stochastic sensing model defined in (5) are connected if the*

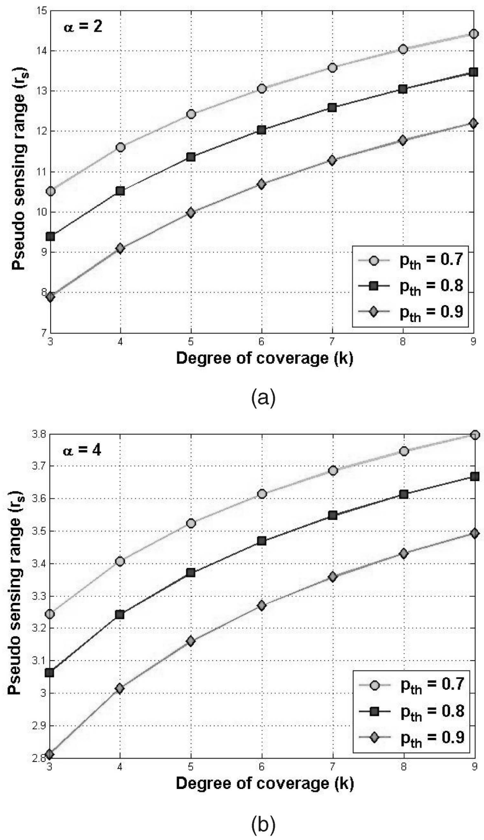


Fig. 9. Upper bound of r_s versus k.

radius of their stochastic communication range R_s is at least equal to their stochastic sensing range r_s., $R_s \geq r_s$.

Given that network connectivity is defined as the *minimum* number of sensors of a neighbor set which need to fail in order to disconnect the network, we should consider the minimum stochastic communication range of the sensors, i.e., $R_s = r_s$, to minimize the size of the neighbor set of each sensor. Therefore, to find our unconditional and conditional measures of network connectivity and fault tolerance for 3D k-covered WSNs using more realistic scenarios, including stochastic models for sensing and communications, we need to replace r by r_s and R by r_s in Sections 4 and 5, and Appendices A, B, and C.

6.4.1 Simulation Results

In this section, we present the simulation results using a high-level simulator written in the C language. We consider a cubic field of side length 1,000 m. All simulations are repeated 200 times and the results are averaged.

Fig. 10 plots the sensor spatial density as a function of the degree of coverage k for different values of the threshold probability p_{th} and for a path-loss exponent $\alpha = 2$. As expected, the density increases with p_{th}. Indeed, as we increase p_{th}, more sensors would be needed to achieve the same degree of coverage k. Recall that the breadth of the Reuleaux tetrahedron that is guaranteed to be covered with exactly k sensors decreases as p_{th} and α increase. Precisely, this breadth is equal to $r_s/1.066$. However, for $p_{th} = 0.8$, the sensor density tends to decrease when k goes from 4 to 5 and increases afterward. This behavior is clearly noticeable for $p_{th} = 0.9$. This is mainly

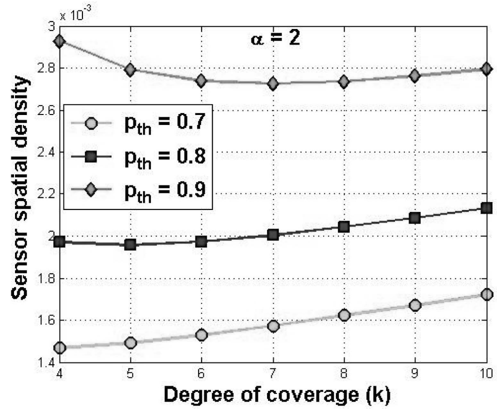


Fig. 10. Sensor spatial density versus k.

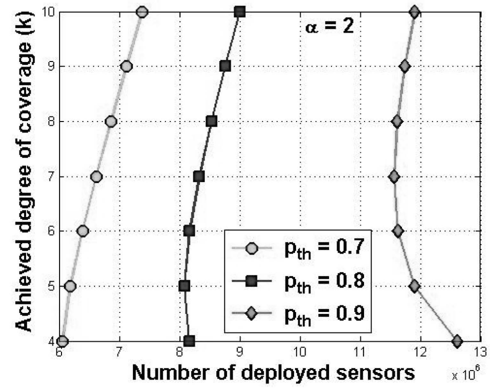


Fig. 11. k versus n_d.

due to the stochastic nature of the sensing range of the sensors, which depends on the logarithm of p_{th}, the threshold probability, k, and α.

Fig. 11 plots the achieved degree of coverage k versus the total number of deployed sensors. Moreover, we vary both p_{th} and fix $\alpha = 2$. Definitely, higher number of deployed sensors would yield higher coverage degree. Here, also, any increase in p_{th} would require a larger number of deployed sensors to provide the same degree of coverage. Notice that the same observation holds for $p_{th} = 0.8$ and $p_{th} = 0.8$ as in the previous experiment.

6.5 3D Sensing Applications

In the case of WSNs deployed on the trees of different heights in a forest, the sensors could be seen almost everywhere in the space. Pompili et al. [27] proposed different deployment strategies for 2D and 3D communication architectures for underwater acoustic sensor networks, where the sensors are anchored at the bottom of the ocean for the 2D design and float at different depths of the ocean to cover the entire 3D region. Indeed, oceanographic data collection, pollution monitoring, offshore exploration, disaster prevention, and assisted navigation are all typical applications of underwater sensor networks [2], [3]. For WSNs deployed in buildings with multiple floors, sensors are placed on the ground and/or the wall, but the networks seldom contain sensors floating in the middle of the room. The first examples show that our proposed 3D model is valid and can be applied to choose the sensor density in practical problems. The last example, however, shows the limited validity of our model due to the restriction imposed on the placement of sensors inside buildings or rooms.

7 Conclusion

In this paper, we investigated coverage and connectivity in 3D k-covered WSNs. Indeed, emerging applications, such as underwater acoustic sensor networks, require 3D design. We proposed the Reuleaux tetrahedron model to guarantee k-coverage of a 3D field. Based on the geometric properties of Reuleaux tetrahedron, we derived the minimum sensor spatial density to ensure k-coverage of a 3D space. Also, we computed the connectivity of homogeneous and heterogeneous 3D k-covered WSNs. Our approach takes into account an inherent characteristic of WSNs in that the sink has a critical role in terms of data processing and decision making, compared to the rest of the network. Thus, we computed the connectivity of 3D k-covered WSNs based on the size of the connected component that includes the sink. We conclude that the connectivity of 3D k-covered WSNs is much higher than the degree of sensing coverage k provided by the network. The traditional connectivity metric, however, is defined in an abstract way. More precisely, it does not consider the inherent properties of WSNs because it assumes that any subset of nodes can fail simultaneously. This assumption is not valid for heterogeneous 3D k-covered WSNs. To compensate for these shortcomings, we proposed more realistic measures of connectivity based on the concept of forbidden faulty set. Using this concept, we found that 3D k-covered WSNs can sustain a large number of sensor failures.

We believe that our results have practical significance for sensor network designers to develop 3D applications with prescribed degrees of coverage and connectivity. These connectivity measures can be exploited in the design of fault-tolerant topology control protocols for 3D k-covered WSNs. Our future work will focus on the design of efficient sensor deployment strategies for 3D k-covered WSNs. Also, we are interested in the design of data routing protocols on duty-cycled 3D k-covered WSNs. Indeed, joint k-coverage and routing poses major challenges due to the time-varying connectivity of the network. This is mainly due to the fact that the sensors are turned on or off to save energy and extend the network lifetime.

Appendix A

Heterogeneous 3D k-Covered WSNs

A.1 Minimum Sensor Density for k-Coverage

Achieving k-coverage of a 3D field by heterogeneous sensors would depend on the least powerful ones in terms of their sensing capabilities. Lemmas 7 and 8 correspond to Lemma 1 and Theorem 1, respectively.

Lemma 7. *If the breadth of a 3D convex region C is at most equal to the minimum radius r_{min} of the sensing spheres of sensors, then C is guaranteed to be k-covered if k ($k \geq 4$) sensors are deployed in it, where $r_{min} = min\{r_j/1.066 : s_j \in S\}$.*

From Lemma 7, it follows that connectivity between sensors located in the Reuleaux tetrahedron of side $r_{\min}$ requires that $R_{\min}$ be at least equal to $r_{\min}$, i.e., $R_{\min} \geq r_{\min}$.

Lemma 8. *Let r_{min} be the minimum radius of the sensing spheres of sensors and $k \geq 4$. The minimum sensor spatial density needed for k-coverage of a 3D field by heterogeneous WSNs is given by*

$$\lambda(r_{min}, k) = \frac{k}{0.422 r_{min}^3}, \tag{8}$$

where $r_{min} = min\{r_j/1.066 : s_j \in S\}$.

A.2 Network Connectivity

Lemma 9 computes the connectivity measures for heterogeneous 3D k-covered WSNs.

Lemma 9. *Let G be a communication graph of a heterogeneous 3D k-covered WSN with $R_{min} \geq r_{min}$ and $k \geq 4$. The connectivity of the graph G is given by*

$$\kappa_4(G) \leq \kappa(G) \leq \kappa_3(G), \tag{9}$$

where

$$\kappa_3(G) = \frac{R_{max} \mathrm{V}^{2/3} k}{0.422 r_{min}^3}, \quad \kappa_4(G) = 12.024 \alpha_2^3 k,$$
$$\alpha_2 = R_{min}/r_{min}, \quad k \geq 4, \quad r_{min} = min\{r_j/1.066 : s_j \in S\},$$
$$R_{min} = min\{R_j : s_j \in S\}, \quad \text{and} \quad R_{max} = max\{R_j : s_j \in S\}.$$

Appendix B

Convex Sensing and Communication Model

The assumption of spherical sensing and communication ranges of sensors given in Section 3 may not hold in real-world WSN platforms. It has been observed in [39] that the communication range of MICA motes [16] is asymmetric and depends on the environments. It is also found in [41] that the communication range of radios is highly probabilistic and irregular.

In this Appendix, for problem tractability, we consider a *convex model*, where the sensing and communication ranges of sensors are *convex* but not necessarily spherical.

First, we define the notion of *largest enclosed sphere* of a 3D convex region C as a sphere that lies entirely inside C and whose diameter is equal to the minimum distance between any pair of points on the boundary of the region C.

B.1 Homogeneous 3D k-Covered WSNs

We consider homogeneous sensors that have the same convex sensing ranges and communication ranges. Lemmas 10 and 11 correspond to Lemma 1 and Theorem 1, respectively. Their proof is similar to that in Section 4 by using the largest enclosed sphere instead of the sensing sphere.

Lemma 10. *If $k \geq 4$ homogeneous sensors are deployed in a 3D convex region C, then the region C is k-covered; if its breadth does not exceed r_{led}, the radius of the largest enclosed sphere of the sensing range.*

Lemma 11. *The minimum sensor spatial density required to guarantee k-coverage of a 3D field is given by*

$$\lambda(r_{\text{led}}, k) = \frac{k}{0.422 {r_{led}}^{0^3}}, \tag{10a}$$

where r_{led} stands for the radius of the largest enclosed sphere of the sensing range, $r_{led}^0 = r_{led}/1.066$, and $k \geq 4$.

Now, we discuss how those results can be derived using a convex model, where the sensing and communication ranges of the sensors may not necessarily be spherical.

B.2 Heterogeneous 3D k-Covered WSNs

Lemmas 12 and 13 correspond to Lemma 1 and Theorem 1, respectively. Their proof is also the same as that in Section 4 by using the largest enclosed sphere instead of sensing sphere.

Lemma 12. *A 3D convex region C is guaranteed to be k-covered when exactly k heterogeneous sensors are deployed in it, if the breadth of C does not exceed r_{led}^{min}, where $r_{led}^{min} = min\{r_{led}/1.066 : s_j \in S\}$ and $k \geq 4$.*

Lemma 13. *The minimum sensor spatial density required to k-cover a 3D field is given by*

$$\lambda(r_{led}^{min}, k) = \frac{k}{0.422 (r_{led}^{min})^2}, \tag{10b}$$

where $r_{led}^{min} = \min\{r_{led}/1.066 : s_j \in S\}$ and $k \geq 4$.

The measures of network connectivity can be derived using the same approach as in Section 4.1.2 and Lemma 13. Thus, the assumption of unit sphere model for sensing and communication ranges of sensors can be relaxed with the aid of the largest enclosed sphere of their sensing range.

Appendix C

Case of Underwater WSNs

The results in Sections 4 and 5 and in Appendices A and B are only applicable to the connectivity of sink node. Although the connectivity of sink is critical, in some scenarios, such as underwater WSNs [2], [3], any sensor may be critical due to the high cost, for instance.

In the following, we extend our network connectivity measures for 3D k-covered WSNs to the case where any sensor in the network is critical. Specifically, we consider a *boundary sensor*, i.e., a sensor located at one corner of the cubic field. Such a boundary sensor has the minimum number of communication neighbors given that all sensors are located within the deployment region—the cube, and hence, the actual communication range of a boundary sensor is only a quarter of its communication sphere. In [34], a boundary sensor is considered to compute the connectivity of 2D k-covered WSNs.

Theorem 4 summarizes the connectivity measures with respect to a boundary sensor for homogeneous 3D k-covered WSNs. The case of heterogeneous WSNs and the case of sensors with convex sensing and communication ranges can be treated similar to Sections 4 and 5 as well as Appendices A and B. We omit the proof of Theorem 4 as we use the same analysis in Sections 4 and 5, and Appendices A and B.

Theorem 4. *Let G be a communication graph of a homogeneous 3D k-covered WSN deployed in a cubic field, where the radii of the sensing and communication spheres of sensors are r and R, respectively. The connectivity of G is computed as*

$$\kappa(G) = 3.02 \alpha^3 k, \tag{11}$$

whereas the conditional connectivity of G is given by

$$\kappa(G : \mathrm{P}) = \frac{3.02((r_0 + R)^3 - r_0^3)k}{r_0^3}, \tag{12}$$

where $r_0 = r/1.066$, $\alpha = R/r$, and $k \geq 4$.

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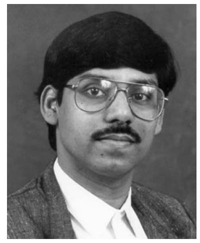

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