5 Coordinate Geometry

Introduction

From the CAT perspective, coordinate geometry itself is not a very conspicuous topic. Usually, we see applied questions from this topic regarding the area of the structure formed or finding missing coordinates.

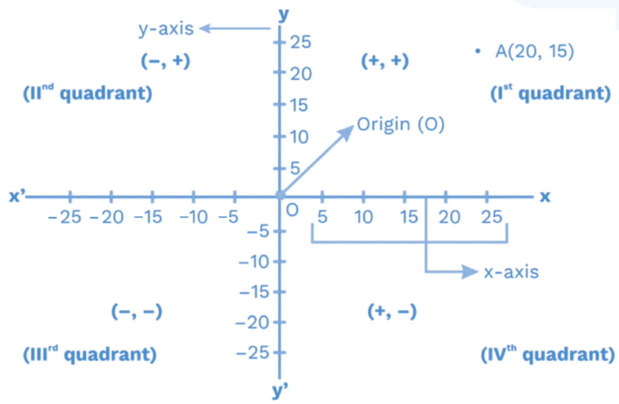
However, going through the basics and practising some questions will prepare you for any surprises from this topic.

Moreover, proper knowledge of this topic will help you understand graphs of functions in algebraic identities.

Coordinate geometry is nothing but a system of denoting points, which make up different geometric shapes and figures and can be represented by equations. These points could also be referred to as Cartesian coordinates.

Coordinates

A coordinate plane is a 2D plane formed by criss-crossing two perpendicular lines known as the x-axis and y-axis. In coordinate geometry, points are pinned on the Cartesian plane, as shown below. The point where the axis intersects is called the origin, where both x and y are zero.



On the x-axis, values to the right of origin are greater than zero, and those to the left are less than zero. Similarly, on the y-axis, values upwards of the origin are greater than zero, and those downwards are less than zero.

Here, the location of point A on the plane is given by the two numbers, the first one tells us where it is on the x-axis, and the second one tells us where it is on the y-axis. In the diagram, point A has values of x and y equal to 20 and 15, respectively, which are the coordinates of the point A.

In the above figure, the regions xoy, $x'oy$, $x'oy'$, and $y'ox$ are the first, second, third, and fourth quadrants, respectively.

The values of x and y in different quadrants are different, i.e.,

Quadrant 1	(+, +)	for (x, y)
Quadrant 2	(–, +)	for (x, y)
Quadrant 3	(–, –)	for (x, y)
Quadrant 4	(+, –)	for, (x, y)

Keynote

The coordinates of a point on the x-axis are of the form $(x, 0)$ and of a point on the y-axis are of the form $(0, y)$.

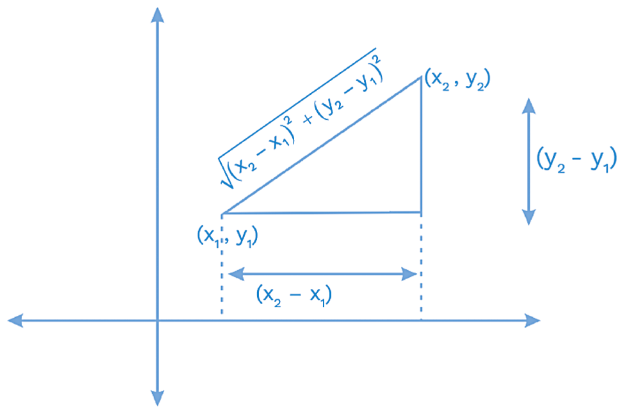
For Example,

- The point (–4, 2) lies in the second quadrant.
- The point (–2, –2) lies in the third quadrant.
- The point (2, –3) lies in the fourth quadrant.

Cartesian Formulae for the Plane

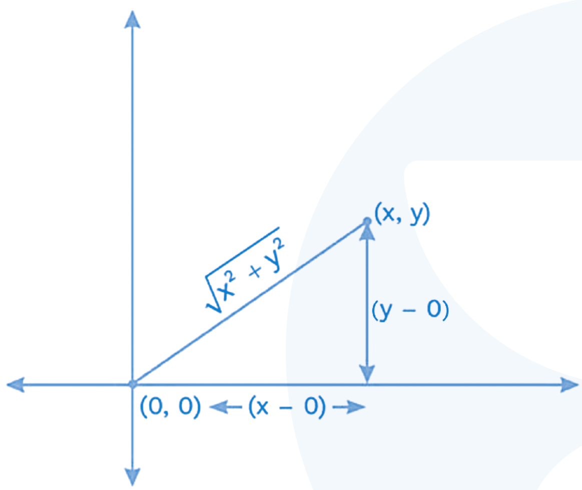
1. **Distance between two points:** If there are two Cartesian coordinates $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the distance between two points P and Q:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



2. Distance from origin:

$$\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2}$$



Keynote

Since distance is always non-negative, we take only the positive square root.

Example 1:

Find the distance between the points (–3, 4) and (2, 1).

(A) $6\sqrt{2}$ (B) $5\sqrt{9}$

(C) $\sqrt{32}$ (D) None of these

Solution: (D)

$(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (2, 1)$

$d = \sqrt{(-3-2)^2+(4-1)^2} = \sqrt{25+9}$

$d = \sqrt{34}$

∴ Option (D) is the correct answer.

Example 2:

If the distance between (4, –2) and (3, a) is $\sqrt{10}$, find the value of a.

Solution:

$x_1 = 4 y_1 = -2$

$x_2 = 3 y_2 = a$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(3-4)^2+(a-(-2))^2}$$

$$\sqrt{10}=\sqrt{1+(a+2)^2}$$

Squaring both sides

$10 = 1 + (a + 2)^2$

$9 = (a + 2)^2$

$a + 2 = \pm 3$

$a = 1, -5.$

Example 3:

Find the coordinates of point P, the ordinate of which is zero and is at a distance of five units from point (5, 3).

Solution:

Let the coordinate of point P be (x, 0).

Now, $(x_1, y_1) = (5, 3)$

$(x_2, y_2) = (x, 0)$

We get, $5=\sqrt{(x-5)^2+3^2}$

$5^2 = (x - 5)^2 + 3^2$

$(x - 5)^2 = 4^2$

$(x - 5) = \pm 4$

$x = 9$ or 1

Therefore, the required points on the x-axis are (9, 0) and (1, 0).

Example 4:

Which point on the y-axis is equidistant from (14, 5) and (–3, 12)?

Solution:

Let the point on the y-axis be (0, y).

Then, $D=\sqrt{(14-0)^2+(5-y)^2}$

Also, $D=\sqrt{(-3-0)^2+(12-y)^2}$

Since the given points are equidistant from (0, y).

$14^2 + (5 - y)^2 = 9 + (12 - y)^2$

$187 = (12 - y)^2 - (5 - y)^2$

$187 = (12 - y - (5 - y)) \times (12 - y + 5 - y)$

$187 = (12 - y - 5 + y) \times (17 - 2y)$

$187 = 7\,(17 - 2y)$

$68 = -14y,$

$y = \frac{34}{7}$

Therefore, coordinates = $\left(0, \frac{34}{7}\right)$.

Example 5:

Find the values of a such that AB = BC where A, B, and C are the points, the coordinates of which are (7, –2), (2, 2), and (a + 1, 7).

Solution:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(7-2)^2 + (-2-2)^2}$$

$$= \sqrt{5^2 + (-4)^2} = \sqrt{41}$$

$$BC = \sqrt{(2-(a+1))^2 + (2-7)^2}$$

$$= \sqrt{(1-a)^2 + (5)^2}$$

$$= \sqrt{(1-a)^2 + 25}$$

Since AB = BC

$$\sqrt{41} = \sqrt{(1-a)^2 + 25}$$

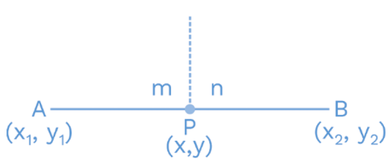
$41 = (1 - a)^2 + 25$

$(1 - a)^2 = 16$

$(1 - a) = \pm 4, \quad a = -3, 5$

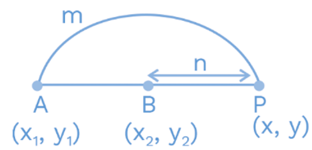
3. Section formula

Internal: If a point P (x, y) divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio m:n, then



$$P(x, y) \equiv \left[x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}\right]$$

External: If P (x, y) divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) externally in the ratio m:n, then

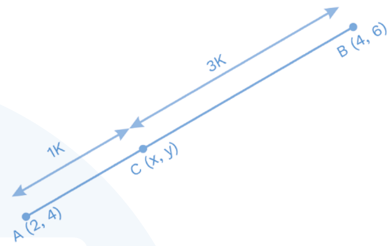


$$P\,(x,y) \equiv \left[x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}\right]$$

Example 6:

Find the coordinates of the point C which divides the line segment joining A (2, 4) and B (4, 6) in the ratio 1:3 internally.

Solution:



Let's apply the section formula for internal division

$(x_1, y_1) = (2, 4), (x_2, y_2) = (4, 6)$

m:n = 1:3

$$P\,(x, y) = \frac{(mx_2 + nx_1)}{m+n}, \frac{(my_2 + ny_1)}{m+n}$$

$$x = \frac{1 \times 4 + 3 \times 2}{1+3} = \frac{10}{4} = \frac{5}{2}$$

$$y = \frac{1 \times 6 + 3 \times 4}{1+3} = \frac{18}{4} = \frac{9}{2}$$

Therefore, the coordinates of the required points are $\left(\frac{5}{2}, \frac{9}{2}\right)$.

Example 7:

In what ratio does the point P (0, y) divide the line segment joining the points Q (7, 3) and R (–2, –5)?

(A) 2:7 (B) 7:2 (C) 5:7 (D) 9:7

Solution: (D)

$$P\,(x, y) = \frac{(mx_2 + nx_1)}{m+n}, \frac{(my_2 + ny_1)}{m+n}$$

x = 0, (given)

$$0 = \frac{-2m + 7n}{m+n}$$

0 = −2m + 7n

$2m = 7n$

$\frac{m}{n} = \frac{7}{2}$

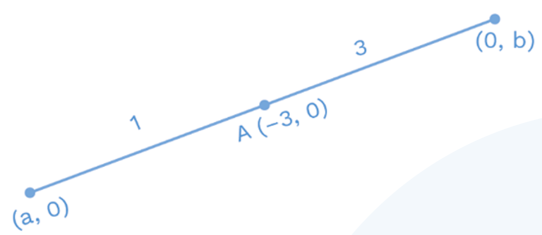
Therefore, option (D) is the correct answer.

Example 8:

The point A (–3, 0) divides the segment joining the points (a, 0) and (0, b) in the ratio 1:3. Find a and b.

Solution:

$$P\,(x, y) = \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}$$



[$x = -3$, $y = 0$], (given)

$$-3 = \frac{1 \times 0 + 3a}{3+1}$$

$3a = -12$

$a = -4$

$$0 = \frac{b \times 1 + 3 \times 0}{3+1}$$

$b = 0$

Values of a and b are –4 and 0, respectively.

Example 9:

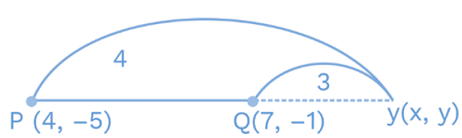
P (4, –5) and Q (7, –1) are two given points, and the point y divides the line segment PQ externally in the ratio of 4:3. Find the coordinates of y.

(A) 16, 11 (B) 11, 15

(C) 15, 18 (D) –11, –13

Solution: (A)

$m = 4$, $n = 3$



$(x_1, y_1) = (4, -5)$ and $(x_2, y_2) = (7, -1)$

$$Y\,(x, y) = \left[x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}\right]$$

$$Y\,(x, y) = \frac{4 \times 7 - 3 \times 4}{4-3}, \frac{4 \times (-1) - 3 \times (-5)}{4-3}$$

$$= \frac{28-12}{1}, \frac{-4+15}{1}$$

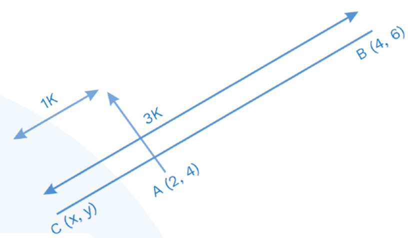
$Y\,(x, y) = 16, 11$

Hence, option (A) is the correct answer.

Example 10:

Find the coordinates of the point which divides the line segment joining A (2, 4) and B (4, 6) in the ratio 1:3 externally.

Solution:



Here, we will apply the section formula for external division.

$(x_1, y_1) = (2, 4)$, $(x_2, y_2) = (4, 6)$, $m{:}n = 1{:}3$,

$$x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$$

$$x = \frac{1 \times 4 - 3 \times 2}{1-3} = \frac{-2}{-2} = 1$$

$$y = \frac{1 \times 6 - 3 \times 4}{1-3} = \frac{-6}{-2} = 3$$

Therefore, the coordinates of the required points are (1, 3).

4. Midpoint formula

The coordinates of the midpoint of the line joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by $x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$.

Example 11:

Find the midpoint of a line segment, the endpoints of which are (4, 5) and (6, 7).

Solution:

Given, $(x_1, y_1) = (4, 5)$ and $(x_2, y_2) = (6, 7)$.

$$\text{Midpoint } (x, y) = \frac{4+6}{2}, \frac{5+7}{2}$$

$(x, y) = 5, 6$

So, the midpoint of the line = (5, 6).

Example 12:

Find the coordinates of the centre of the circle, the endpoints of which of a diameter are (2, 3) and (5, 4).

Solution:

The given points are: $(x_1, y_1) = (2, 3)$, $(x_2, y_2) = (5, 4)$.

The centre of the circle is the midpoint of a diameter.

So, the coordinates of the centre of a circle $= \frac{2+5}{2}, \frac{3+4}{2} = \left(\frac{7}{2}, \frac{7}{2}\right)$.

Therefore, the coordinates of the centre of the circle= (3.5, 3.5).

Example 13:

If the midpoint coordinates of a line segment PQ is $(8K - 3, 7)$. The coordinates of P are $(14K - 6, 10)$ and Q are $(8, K)$. Find the coordinates of the midpoint.

Solution:

$$8K - 3 = \frac{14K - 6 + 8}{2}$$

$$16K - 6 = 14K + 2$$

$$K = 4$$

Midpoint = $(8K - 3, 7) = [(32 - 3), 7] = (29, 7)$.

Hence, the coordinates of the midpoint = (29, 7).

Condition for collinearity of three points

The three given points are collinear, i.e., lie on the same straight line if:

1. The area of a triangle ABC is zero.
2. Slope of AB = slope of BC = slope of AC.
3. Let the distance between A and B is L, distance between B and C is M and distance between C and A is N. So, the sum of any two among L, M and N is equal to the third.
4. Any one point satisfies the line equation formed by using the remaining two points.

Some more examples based on the distance and section formulae.

Example 14:

Find the distance between the points (7k + 2, 5 – 3k) and (–3 + 7k, –7 – 3k).

Solution: 13 units

Since we know the distance formula:

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{\left[7k + 2 - (-3 + 7k)\right]^2 + \left[5 - 3k - (-7 - 3k)\right]^2}$$

$$= \sqrt{25 + 144} = 13 \text{ units}$$

Therefore, the distance between the points = 13 units.

Example 15:

If the distance between two points $(K + 1, 9)$ and $(4, -6)$ is 17 units, then find the value of K.

(A) 11, –5 (B) 8, –7

(C) –13, 12 (D) 7, 15

Solution: (A)

Since the distance between two points = 17 units (is given)

Also,

$(x_1, y_1) = (k + 1, 9)$

$(x_2, y_2) = (4, -6)$

$$17 = \sqrt{(k + 1 - 4)^2 + (9 + 6)^2}$$

$$17 = \sqrt{(k - 3)^2 + (15)^2}$$

$(k - 3)^2 = 17^2 - 15^2$

$(k - 3)^2 = 64$

$k - 3 = \pm 8$

$k = -5$ and $k = +11$

Hence, option (A), i.e., 11, –5 is the correct answer.

Example 16:

Point A divides the segment BC in the ratio 5:1 internally. The coordinates of B are (6, –4) and that of C is $\left(\frac{9}{5}, 8\right)$. What are the coordinates of point A?

(A) (2.5, 6) (B) (3, –6)

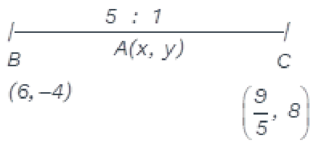
(C) (–3, –6) (D) (3, 6)

Solution: (A)

Since $m = 5$, $n = 1$ is given in the question.

$(x_1, y_1) = (6, -4)$

$(x_2, y_2) = \left(\frac{9}{5}, 8\right)$



$$A\,(x, y) = \frac{\frac{5\times 9}{5}+6\times 1}{5+1}, \frac{5\times 8+1\times(-4)}{5+1}$$

$$= \frac{9+6}{6}, \frac{40-4}{6} \;=\; \frac{5}{2}, 6$$

A (x, y) = 2.5, 6

Therefore, option (A) is the correct answer.

Example 17:

In what ratio the x-axis will divide a line segment which is obtained by the joining the points (9, 14) and (4, –6)?

(A) 9:4 (B) 7:3

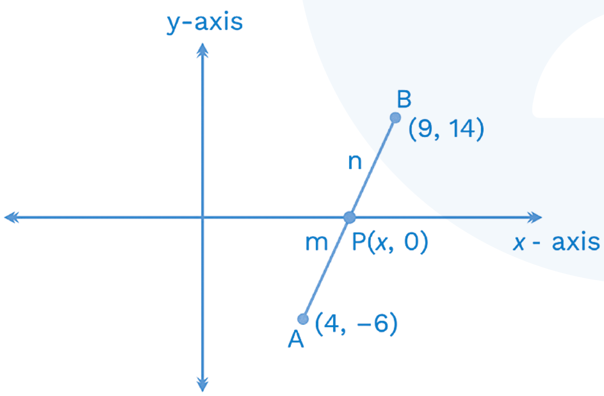
(C) 2:7 (D) 3:2

Solution: (B)

Since the line joining the points (9, 14) and (4, –6) is divided by the x-axis.

Therefore, y = 0 on the x-axis.

Let the line be divided by the x-axis in the ratio of m:n.



Thus,

$$0 = \frac{-6n+14m}{m+n}$$

$6n = 14m$

$3n = 7m$

$n/m = 7/3$ or 7:3

Hence, option (B) is the correct answer.

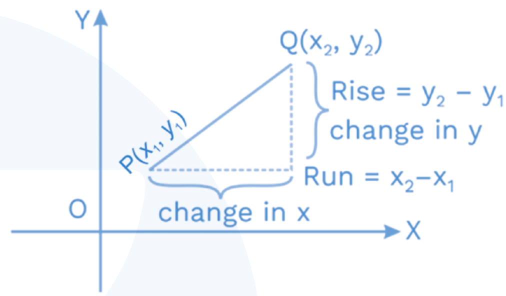
Straight line

A straight line is a set of all points between and stretching beyond two points.

- The general form of the equation of straight line: $ax + by + c = 0$, where a, b, and c are real constants and x and y are two unknowns.
  - The slope of a line: The slope of a line is a number that describes both the direction and the steepness of the line.

To calculate the slope

The change in height divided by the change in horizontal distance.

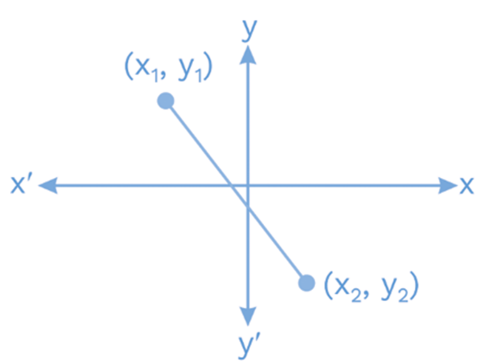


For a non-vertical line, slope = $\dfrac{\text{Change in } y}{\text{Change in } x}$

- Point-slope form: If m is the slope of the line and it passes through the point (x_1, y_1) then the equation of the line is $y - y_1 = m(x - x_1)$.
  - Two-point form: If the lines pass through two points (x_1, y_1) and (x_2, y_2), then the equation is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Using point-slope and two-point forms, we can find $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.



Intercept form

If the line makes an intercept of a unit on the x-axis and b units on the y-axis, then the equation is: $\frac{x}{a}+\frac{y}{b}=1$.

Example 18:

Find the slope of line $3x + 4y = 5$.

Solution: −3/4

Since the given equation of line is: $3x + 4y = 5$.

Therefore, $4y = -3x + 5$

$y = (-3/4)x + 5/4$ ... (i)

If we compare the above equation with the general equation

$y = mx + c$

Then, we will get, $m = -3/4$.

Hence, the slope of the line = −3/4

Example based on the point-slope form

Example 19:

Find the line equation that passes through (2, –3) and has a slope of 3.

Solution:

Since we know that:

$y - y_1 = m(x - x_1)$

$m = 3, (x_1, y_1) = (2, -3)$

$y - (-3) = 3(x - 2)$

$y + 3 = 3x - 6$

$y = 3x - 9$

Example based on the two-point form

Example 20:

Find the equation of the line passing through the points (4, 6) and (5, 7).

Solution:

Since the given points are:

$(x_1, y_1) = (4, 6)$

$(x_2, y_2) = (5, 7)$

Therefore, $y - 6 = \frac{7-6}{5-4}(x-4)$

$y - 6 = \frac{1}{1}(x-4)$

$y - 6 = 1(x - 4)$

$y - 6 = x - 4$

$y = x + 2$

Examples based on intercept form

$$\frac{x}{a}+\frac{y}{b}=1$$

Example 21:

Find x and y intercepts of $11x + 9y - 99 = 0$.

Solution:

Converting the given equation into the intercept form:

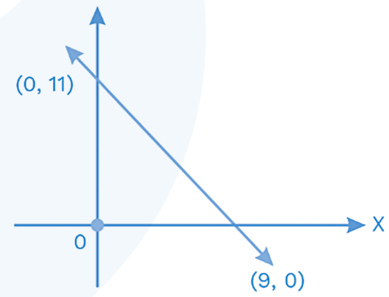
Thus, $11x + 9y = 99$

Divide the equation by 99

$\frac{x}{9}+\frac{y}{11}=1$

$\therefore$ x intercept = 9

y intercept = 11



Example 22:

What is the intercept form of the line, the general form of which is

$7x + 13y - 91 = 0$

Solution:

Putting $x = 0$ in the equation, we get $y = 7$, and also when $y = 0$, then we get $x = 13$.

$\therefore$ The intercept form of the line is $\frac{x}{13}+\frac{y}{7}=1$.

Angle between two lines

If m_1 and m_2 are the slopes of two lines, the acute angle between them is:

$\tan\theta=\left(\frac{m_1-m_2}{1+m_1m_2}\right)$ (where θ is the angle between the lines)

1. Condition for parallel lines:

- Two straight lines can be parallel only if they make an equal inclination with the x-axis.
- The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel:

  if $m_1 = m_2$

$$\Rightarrow -\frac{a_1}{b_1} = -\frac{a_2}{b_2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

Example 23:

Show that the lines $5x + 6y - 12 = 0$ and $20x + 24y - 19 = 0$ are parallel.

Solution:

Since $a_1 = 5$, $a_2 = 20$, $b_1 = 6$, and $b_2 = 24$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \qquad \Rightarrow \frac{5}{20} = \frac{6}{24} \qquad \Rightarrow \frac{1}{4} = \frac{1}{4}$$

$\therefore$ The given lines are parallel to each other.

2. Condition for perpendicular lines:

   If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular, then

   $m_1 \times m_2 = -1$

$$\Rightarrow -\frac{a_1}{b_1} \times \left(-\frac{a_2}{b_2}\right) = -1 \qquad \Rightarrow a_1a_2 + b_1b_2 = 0$$

Example 24:

Show that the lines $5x + 12y - 17 = 0$ and $12x - 5y + 19 = 0$ are perpendicular.

Solution:

$a_1 = 5$, $b_1 = 12$, $a_2 = 12$, $b_2 = -5$

$a_1 \times a_2 + b_1 \times b_2 = 5 \times 12 + 12 \times (-5) = 60 - 60 = 0.$

$\therefore$ The given lines are perpendicular to each other.

Some formulae to remember

1. The general form of the equation of a straight line is $ax + by + c = 0$. Here, the y-intercept is $-\frac{c}{b}$, the x-intercept is $-\frac{c}{a}$, and the Slope is $-\frac{a}{b}$.

2. If $ax + by + c = 0$ is the line equation, the perpendicular distance from a point (x_1, y_1) to this line is given by $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

3. The distance between two parallel straight lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

4. Foot of the perpendicular (h, k) on $ax + by + c = 0$ from (x_1, y_1) is

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}.$$

Conditions for parallelism and perpendicularity of two lines

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are:

1. Coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

2. Parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

3. Intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

4. Perpendicular, if $a_1a_2 + b_1b_2 = 0$

Distance of a point from a given line or length of the perpendicular from the point (x_1, y_1) to the straight line $ax + by + c = 0$.

Let the length of perpendicular be P, then $P = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

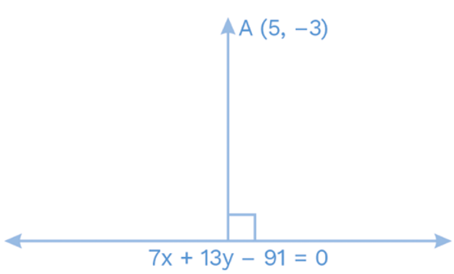
Example 25:

Find the length of the perpendicular from (5, –3) on the line $7x + 13y - 91 = 0$.

Solution:

Let $A\ (x_1, y_1) = (5, -3)$.

Line $7x + 13y - 91 = 0$.



The length of perpendicular from A (x1, y1) on the line $ax + by + c = 0$ is

$$P=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

where $a = 7$ $b = 13$ $c = -91$

$x_1 = 5$ $y_1 = -3$

$$P=\left|\frac{7\times5+13\times(-3)+(-91)}{\sqrt{7^2+13^2}}\right|$$

$$=\left|\frac{35+(-39)+(-91)}{\sqrt{7^2+13^2}}\right|$$

$$=\frac{95}{\sqrt{218}}$$

Point of intersection of two lines: Let the equations of two lines be

$a_1x + b_1y + c_1 = 0$ ...(i)

$a_2x + b_2y + c_2 = 0$ ...(ii)

Let (x_1, y_1) be the coordinates of the point of intersection of the two lines.

Then $a_1x_1 + b_1y_1 + c_1 = 0$ ...(iii)

$a_2x_1 + b_2y_1 + c_2 = 0$ ....(iv)

By the method of cross-multiplication,

$$\frac{x_1}{b_1c_2-b_2c_1}=\frac{y_1}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\therefore\ x_1=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\ y_1=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}.$$

where, $(a_1b_2 - a_2b_1) \neq 0$.

Condition of concurrency of three lines

Three lines are said to be coincident if they pass through a common point, i.e., they meet at a single common point. Thus, three lines are coincident or concurrent when the point of intersection of any two lines lies on the third line.

Let $a_1x + b_1y + c_1 = 0$ ...(i)

$a_2x + b_2y + c_2 = 0$ ...(ii)

$a_3x + b_3y + c_3 = 0$ ...(iii)

If the three lines are concurrent lines then,

$$\Rightarrow\ \begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$$

$a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3) = 0$

Example based on points of intersection of two lines

Example 26:

Find out the points of intersection of two lines $3x + 6y + 3 = 0$ and $6x + 9y + 15 = 0$.

Solution:

Since the given equations are $3x + 6y + 3 = 0$ and $6x + 9y + 15 = 0$.

Here, $a_1 = 3$ $b_1 = 6$ $c_1 = 3$

$a_2 = 6$ $b_2 = 9$ $c_2 = 15$

$$(x,y)=\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}\right)$$

$$(x,y)=\left(\frac{6\times15-9\times3}{3\times9-6\times6},\frac{6\times3-3\times15}{3\times9-6\times6}\right)$$

$$(x,y)=\left(\frac{63}{-9},-\frac{27}{-9}\right)$$

$$\Rightarrow(x,y)=(-7,3).$$

Example based on condition of concurrency of three lines

Example 27:

Show that the lines $4x - 6y + 10 = 0$, $6x + 8y - 14 = 0$, and $18x - 10y + 16 = 0$ are concurrent.

Solution:

We know that if the equations of three straight lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent.

$$\text{Then, }\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$$

Given lines are $4x - 6y + 10 = 0$, $6x + 8y - 14 = 0$, and $18x - 10y + 16 = 0$,

$$\text{we have }\begin{vmatrix}4 & -6 & 10\\ 6 & 8 & -14\\ 18 & -10 & 16\end{vmatrix}$$

$= 4 (128 - 140) - (-6) (96 + 252) + 10 (-60 - 144)$

$= 4 \times (-12) + 6 \times 348 - 10 \times 204$

$= -48 + 2088 - 2040$

$= 2088 - 2088 = 0$

Therefore, the given three lines are concurrent.

Example based on distance between parallel lines

Example 28:

Find the distance between the parallel lines $6x + 8y + 14 = 0$ and $6x + 8y - 10 = 0$.

Solution:

The distance between two parallel lines is given by $d = \dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

Given, $c_1 = 14$, $c_2 = -10$, $a = 6$, $b = 8$

$$d = \left|\frac{-14-(-10)}{\sqrt{6^2+8^2}}\right| = \frac{24}{10} \text{ or } \frac{12}{5}$$

Therefore, the distance between two parallel lines $= \dfrac{12}{5}$ units.

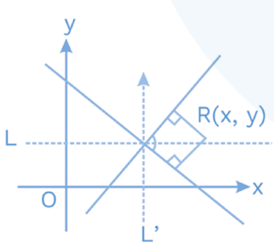
Equation of any line passing through the point of intersection of two given lines

The point where two lines, represented by the equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, respectively, intersect is known as the point of intersection. It can be represented by

$$(a_1x + b_1y + c_1) + \lambda\,(a_2x + b_2y + c_2) = 0$$

Where λ is an arbitrary constant.

Equations of bisectors of the angles between two given lines



Let us have two lines L1: a1x + b1y + c1 = 0 and L2: a2x + b2y + c = 0.

If point $R(x, y)$ lies on the bisector, then the length of the perpendicular from the point R to both the lines should be equal.

$$\text{i.e.} \left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}\right| = \pm\left|\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

Example 29:

Find the equation of the angle bisectors of the straight lines 12x − 9y + 12 = 0 and 3x + 4y − 9 = 0.

Solution:

Let the point on the angle bisector be (x, y), then the distance of this point from both the lines is equal.

$$\left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

$a_1 = 12$ $\quad b_1 = -9$ $\quad c_1 = 12$

$a_2 = 3$ $\quad b_2 = 4$ $\quad c_2 = -9$

$$\Rightarrow \left|\frac{12x + (-9y) + 12}{\sqrt{12^2 + (-9)^2}}\right| = \left|\frac{3x + 4y - 9}{\sqrt{3^2 + 4^2}}\right|$$

$$\Rightarrow \frac{12x - 9y + 12}{15} = \pm\frac{3x + 4y - 9}{5}$$

Taking positive sign we get,

$12x - 9y + 12 = 3(3x + 4y - 9)$

$4x - 3y + 4 = 3x + 4y - 9$

$x = 7y - 13$ or $7y - x - 13 = 0$

Taking negative sign we get,

$12x - 9y + 12 = -3(3x + 4y - 9)$

$4x - 3y + 4 = -3x - 4y + 9$

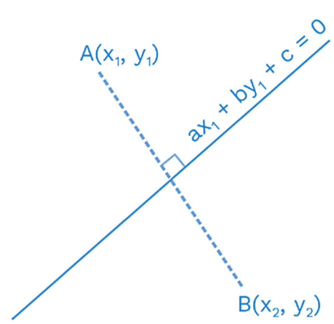
$7x + y - 5 = 0$

Therefore, the equation of the bisectors of the angles between the straight lines $12x - 9y + 12 = 0$ and $3x + 4y - 9 = 0$ are:

$x - 7y + 13 = 0$ and $7x + y - 5 = 0$.

The image of a point along the mirror placed on a straight line

The image of $A(x_1, y_1)$ with respect to the line mirror $ax + by + c = 0$ be $B(x_2, y_2)$ is given by: $\dfrac{x_2 - x_1}{a} = \dfrac{y_2 - y_1}{b} = \dfrac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$



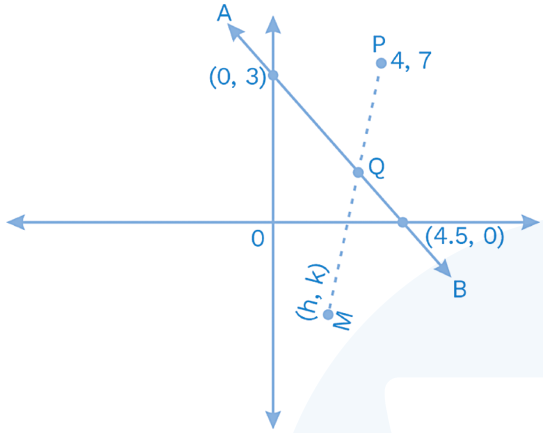
Example 30:

Find the image of the point (4, 7) with respect to the line $2x + 3y = 9$, assuming the line to be a plane mirror.

Solution:

Let M (h, k) be the image of the point (4, 7) with respect to the line $2x + 3y = 9$.

Since line AB is a mirror.



Point P and M are equidistant from line AB, i.e., PQ = QM or Q is the midpoint. Also, the line formed by PM is perpendicular to AB.

Since Q is the midpoint of PM

$$Q = \left(\frac{4+h}{2}, \frac{7+k}{2}\right)$$

Since point Q lies on line AB it must satisfy the equation.

$$\therefore 2\times\left(\frac{4+h}{2}\right) + 3\times\left(\frac{7+k}{2}\right) = 9$$

$2(4 + h) + 3 \times (7 + k) = 18$

$8 + 2h + 21 + 3k = 18$

$2h + 3k = -11$ ...(i)

Also, PM is perpendicular to AB

$\therefore$ Slope of PM × slope of AB = –1

Slope of AB = –2/3

Slope of PM = $\frac{3}{2}$

Now, PM is the line joining the points P (4, 7) and M (h, k)

Slope of PM = $\frac{3}{2} = \frac{k-7}{h-4}$

$3h - 12 = 2k - 14$

$3h - 2k = -2$ ...(ii)

After solving equations (i) and (ii), we will get:

$$h = \frac{28}{13} \text{ and } k = \frac{29}{13}$$

Triangle

Area of a Triangle

The area of a triangle *ABC*, the vertices of which are $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is denoted by Δ.

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Example 31:

Find the area of a triangle ABC, the vertices of which are A (2, 3), B (3, 4), and C (1, 6), respectively.

Solution:

$(x_1, y_1) = (2, 3) = A$

$(x_2, y_2) = (3, 4) = B$ (vertices of a triangle)

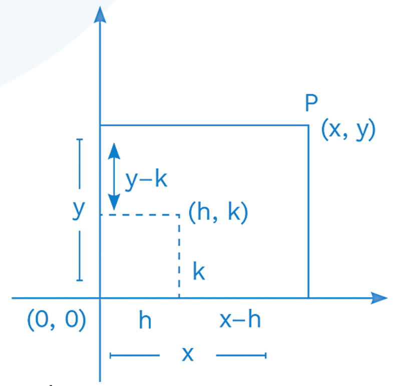
$(x_3, y_3) = (1, 6) = C$

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

So, the area of the triangle = $\frac{1}{2}[2(4-6) + 3(6-3) + 1(3-4)]$

$$= \frac{1}{2}[-4+9-1] = \frac{1}{2}\times(4) = 2 \text{ unit}^2$$

Finding the area of triangle using a shift of origin



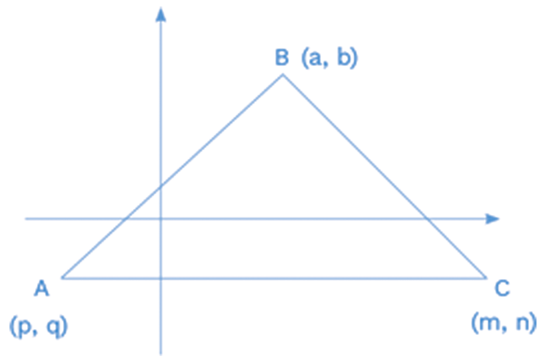
$x_1 = x - h$

$y_1 = y - k$

In the figure, there's point P (x, y), and the origin, the coordinates of which have been shifted to *a* (h, k). Therefore, the distance of P from the new x-axis and y-axis will be x

– h, and y – k, respectively. Therefore, the coordinates of point P will be ($x – h$, $y – k$).

Area of triangle



Here, we shift coordinates of origin to B.

B (a, b) $\rightarrow$ B (0, 0)

So, the coordinate of a and c will be

A $(x_1, y_1) = (p - a, q - b)$

C $(x_2, y_2) = (m - a, n - b)$

Area = $\frac{1}{2}|x_1y_2 - x_2y_1|$

Example 32:

Find the area of the triangle with coordinates A (8, 5), B (2, 2), and C (–3, 8) using the shift of origin method.

Solution:

A (8, 5)	B (2, 2)	C (–3, 8)
–2 ↓ –2	↓ shift this origin	–2 ↓ –2
6, 3	(0, 0)	–5, 6

$$A = \frac{1}{2}|x_1y_2 - x_2y_1|$$

$$= \frac{1}{2}|6 \times 6 - 3 \times (-5)|$$

$$= \frac{51}{2} = 25.5 \text{ unit}^2$$

Example 33:

Find the area of a triangle with coordinates A (10, 7), B (–3, 4), and C (15, 12) using the shift of origin method.

Solution:

Area of D having coordinates. A (10, 7), B (–3, 4), and C (15, 12).

A (10, 7), B (–3, 4), C (15, 12)

(13, 3) (0, 0) (18, 8)

$$A = \frac{1}{2}|x_1y_2 - x_2y_1|$$

$$= \frac{1}{2}|13 \times 8 - 3 \times 18|$$

$$= \frac{1}{2} \times 50 = 25$$

Area of triangle formed by the intersection of three lines

$y = m_1x + c_1$, $y = m_2x + c_2$, $y = m_3x + c_3$

$$\text{Area} = \frac{1}{2}\sum\frac{(c_1 - c_2)^2}{m_1 - m_2}$$

$$\text{Area} = \frac{1}{2}\left[\frac{(c_1 - c_2)^2}{m_1 - m_2} + \frac{(c_2 - c_3)^2}{m_2 - m_3} + \frac{(c_3 - c_1)^2}{m_3 - m_1}\right]$$

Example 34:

Find the area of the triangle formed by the intersection of the following lines.

$y = 2x + 3$, $y = 3x + 1$, $y = x + 2$

Solution:

$$\text{Area} = \frac{1}{2}\left[\frac{(c_1 - c_2)^2}{m_1 - m_2} + \frac{(c_2 - c_3)^2}{m_2 - m_3} + \frac{(c_3 - c_1)^2}{m_3 - m_1}\right]$$

$$\text{Area} = \frac{1}{2}\left[\left[\frac{(3-1)^2}{2-3} + \frac{(1-2)^2}{3-1} + \frac{(2-3)^2}{1-2}\right]\right]$$

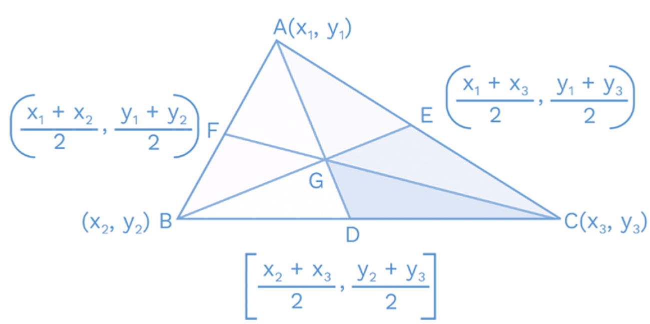
$$\text{Area} = \frac{1}{2}\left|\frac{4}{-1} + \frac{1}{2} + \frac{1}{-1}\right| = \frac{1}{2}\left|-5 + \frac{1}{2}\right| = \frac{9}{4}$$

Important Points in a Triangle

Centroid

Centroid can be defined as the point where all three medians of a triangle meet. It is represented by the letter G.

If (x_1, y_1), (x_2, y_2), and (x_3, y_3) are the vertices of a triangle,



The coordinates of its centroid are $\left(\frac{x_1+x_2+x_3}{3}\right),\left(\frac{y_1+y_2+y_3}{3}\right)$.

Example 35:

If the vertices of a triangle are ABC, (5, 4), (–1, 3), and (2, 2), respectively, then find the centroid of a triangle.

Solution:

Since, (x_1, y_1) = (5, 4)

(x_2, y_2) = (–1, 3)

(x_3, y_3) = (2, 2)

Therefore, the centroid of a triangle = $\frac{5-1+2}{3},\frac{4+3+2}{3}$ = (2, 3).

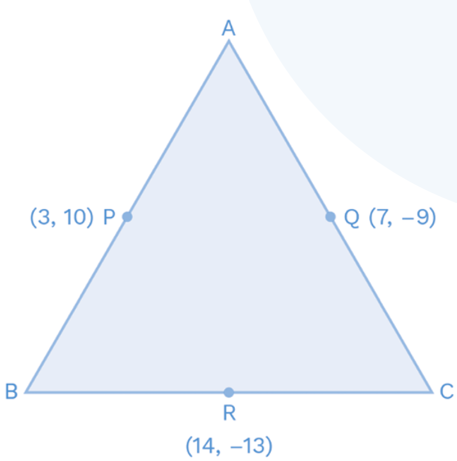
Hence, the coordinates of the centroid of the triangles = (2, 3).

Example 36:

If (3, 10), (7, –9), and (14, –13) are midpoint of the sides of a triangle, then find the centroid of the triangle.

Solution:

Suppose, there is a triangle ABC with P (3, 10), Q (7, –9), and R (14, –13) as midpoint of side AB, AC, and BC, respectively.



Let the coordinates of point A be (x_1, y_1), B be (x_2, y_2), and C be (x_3, y_3).

$$P(3,\ 10)=\left(\frac{x_1+x_2}{2}\right),\ \left(\frac{y_1+y_2}{2}\right)$$

$$Q(7,-9)=\left(\frac{x_1+x_3}{2}\right),\ \left(\frac{y_1+y_3}{2}\right)$$

$$R(14,-13)=\left(\frac{x_2+x_3}{2}\right),\ \left(\frac{y_2+y_3}{2}\right)$$

Comparing all we get,

$$\frac{x_1+x_2}{2}+\frac{x_1+x_3}{2}+\frac{x_2+x_3}{2}=3+7+14$$

$$\frac{2x_1+2x_2+2x_3}{2}=24$$

or $x_1 + x_2 + x_3 = 24$

Similarly, $y_1 + y_2 + y_3 = 10 - 9 - 13 = -12$

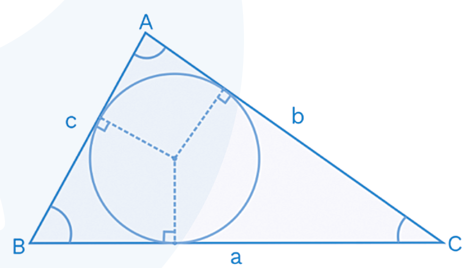
Coordinates for centroid

$$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Centroid = (24/3, –12/3) = (8, –4)

Incentre

It can be defined as the point where all three internal angle bisectors of a triangle meet. If A (x_1, y_1), B (x_2, y_2), and C (x_3, y_3) are the vertices of a triangle ABC such that BC = a, CA = b, and AB = c, then the coordinates of its centre are



$$\left[\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right]$$

Example 37:

Find the coordinates (approximate to one decimal place) of the incentre of the triangle having vertices on the points (1, 3), (2, 7), and (12, –16).

Solution:

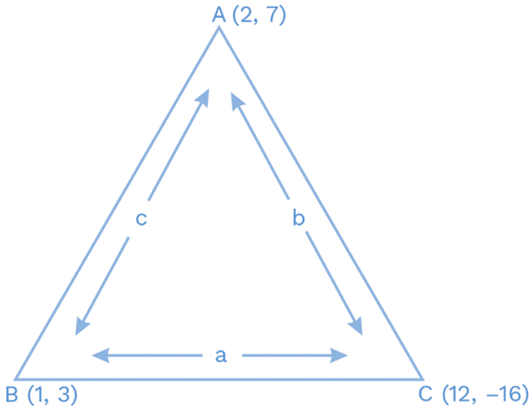
Let the coordinates of the triangle be

A (2, 7), B (1, 3), and C (12, –16)

$$a=BC=\sqrt{(12-1)^2+(-16-3)^2}=\sqrt{11^2+19^2}=22$$

$$b=AC=\sqrt{(12-2)^2+(-16-7)}=\sqrt{100+529}=25$$

$$c=AB=\sqrt{(1-2)^2+(3-7)^2}=\sqrt{1+16}=\sqrt{17}$$



Let (x, y) be the coordinates of incentre of ABC.

Then $x = \dfrac{ax_1 + bx_2 + cx_3}{a+b+c}$

$(x_1, y_1) = (2, 7)$, $(x_2, y_2) = (1, 3)$, $(x_3, y_3) = (12, -16)$

$$\text{So, } x = \frac{22\times 2 + 25\times 1 + 12\times\sqrt{17}}{22+25+\sqrt{17}}$$

$$= \frac{69 + 12\sqrt{17}}{47+\sqrt{17}} = \frac{118.5}{51} = 2.3$$

$$\text{Similarly, } y = \frac{7\times 22 + 25\times 3 + \sqrt{17}\times(-16)}{22+25+\sqrt{17}}$$

$$= \frac{229 - 16\sqrt{17}}{47+\sqrt{17}} = \frac{163}{51} = 3.2$$

So coordinates of incentre = (2.3, 3.2).

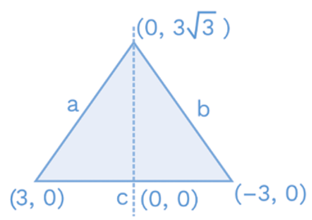
Example 38:

Find the incentre of the equilateral triangle having vertices at $(0, 3\sqrt{3})$, $(3, 0)$, and $(-3, 0)$.

(A) $(3, \sqrt{3})$ (B) (0, 0) (C) $(0, \sqrt{3})$ (D) $(\sqrt{3}, 0)$

Solution: (C)

To find a = b = c, we have to use the distance formula



$$a = b = c = \sqrt{(3-0)^2 + \left(0 - 3\sqrt{3}\right)^2} = \sqrt{36} = 6$$

Coordinates of incentre:

$$x = \frac{(6\times 0) + (6\times(-3)) + (6\times 3)}{6+6+6}$$

$$= \frac{0 - 18 + 18}{18} = 0$$

$$y = \frac{(6\times 3\sqrt{3}) + (6\times 0) + (6\times 0)}{6+6+6} = \frac{18\sqrt{3} + 0 + 0}{18} = \sqrt{3}$$

So, coordinates of incentre of the triangle = $(0, \sqrt{3})$.

Hence, option (C) is the correct answer.

Circumcentre

It is defined as the point where the perpendicular bisectors of the sides of that particular triangle intersect.

If O is the circumcentre of a triangle ABC, then OA = OB = OC, and OA is called the circumradius.

Example 39:

Find the coordinates of the circumcentre of triangle ABC with the vertices A = (0, 2), B = (0, 5), and C = (4, 2).

Solution:

$A = (0, 2) = (x_1, y_1)$

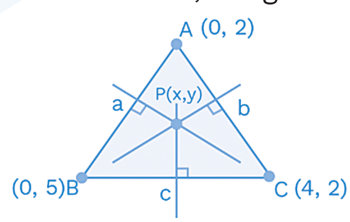
$B = (0, 5) = (x_2, y_2)$

$C = (4, 2) = (x_3, y_3)$

Let (x, y) be the coordinates of the circumcentre.

Also, D_1, D_2, and D_3 be the distance from the circumcentre to vertices A, B, and C, respectively.

Using distance formula, we get:



Since $D_1 = D_2 = D_3$

$D_1 = D_2$ gives,

$(x - 0)^2 + (y - 2)^2 = (x - 0)^2 + (y - 5)^2$

$(y - 2)^2 = (y - 5)^2$

$y - 2 = \pm(y - 5)$

$2y = 7$

$y = 3.5$

$\Rightarrow D_1 = D_3$ gives

$(x - 0)^2 + (y - 2)^2 = (x - 4)^2 + (y - 2)^2$

$x^2 = x^2 + 16 - 8x$

$x = 2$

$\Rightarrow D_2 = D_3$ gives

$(x - 0)^2 + (y - 5)^2 = (x - 4)^2 + (y - 2)^2$

$x^2 + y^2 + 25 - 10y = x^2 + 16 - 8x + y^2 + 4 - 4y$

$5 = 6y - 8x$

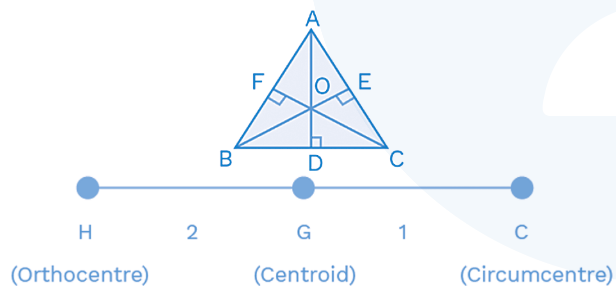
Putting the value of x and y we can see that:

$6 \times 3.5 - 8 \times 2 = 5$ which satisfies this case.

$\therefore (x, y) = (2, 3.5)$.

Orthocentre

- It can be defined as the point where all the perpendicular drawn from the vertices to the opposite side of the triangle intersect with each other.
- The orthocentre of different triangles lies at different positions or places.
- For an acute angle triangle, the orthocentre lies inside the triangle.
- For the obtuse angle triangle, the orthocentre lies outside the triangle.
- For a right triangle, the orthocentre lies on the vertex of the right angle.



The centroid divides the orthocentre and the circumcentre in a 2:1.

Circle

A circle is described by the general and a special equation.

Special equation: $(x - h)^2 + (y - k)^2 = r^2$.

Where r is the radius of the circle and (h, k) is the centre of the circle.

General equation: $x^2 + y^2 + 2gx + 2fy + c = 0$.

Where $h = -g$ and $k = -f$

i.e., centre of the circle = $(-g, -f)$.

Radius of the circle = $\sqrt{g^2 + f^2 - c}$.

Example 40:

Find the radius and centre of the following equation:

$$x^2 + y^2 + 12x + 10y + 12 = 0$$

Solution:

Compare the given equation with the circle's equation.

Here, $2g = 12$

$g = 6$

$2f = 10$

$f = 5$

$\therefore$ Centre of the circle = $(-6, -5)$.

Now, radius = $\sqrt{(6)^2 + (5)^2 - 12}$

$= \sqrt{49} = 7$ units.

Keynote

1. If $g^2 + f^2 - c = 0$, then it is a point circle.
2. If $g^2 + f^2 - c > 0$, then it is a real circle.
3. If $g^2 + f^2 - c < 0$, then it is an imaginary circle.

Example 41:

What does the equation $x^4 - y^4 - 2x^3 + 2y^3 - 2yx^2 + 2xy^2 = 0$ represent?

(A) Two circles

(B) Four straight lines

(C) A pair of straight lines and a circle

(D) Cannot be determined

Solution: (C)

$x^4 - y^4 - 2x^3 + 2y^3 - 2yx^2 + 2xy^2 = 0$ ...(i)

Put $y = x$ in equation (i) we get.

$x^4 - x^4 - 2x^3 + 2x^3 - 2x^3 + 2x^3 = 0$.

Hence, $(x - y)$ is a factor of $x^4 - y^4 - 2x^3 + 2y^3 - 2yx^2 + 2xy^2$.

Similarly, it can be proved that $(x + y)$ is also a factor of

$x^4 - y^4 - 2x^3 + 2y^3 - 2yx^2 + 2xy^2$.

Dividing $x^4 - y^4 - 2x^3 + 2y^3 - 2yx^2 + 2xy^2$ by $x^2 - y^2$,

We get $x^2 + y^2 - 2x - 2y$ as the quotient.

Hence, the given equation can be written as;

$(x - y)(x + y)(x^2 + y^2 - 2x - 2y) = 0$.

$x - y = 0$ and $x + y = 0$ are the equations of straight lines.

$x^2 + y^2 - 2x - 2y = 0$ is the equation of a circle, the centre of which is at (1, 1) and radius is $\sqrt{2}$ units.

Therefore, we get a pair of straight lines and a circle.

Area of a Polygon

We can find the area of the polygon using the Gauss shoelace formula. The area of the polygon, the vertices of which are:

$(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots (x_n, y_n)$ is

$$\Rightarrow \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_n & y_n \\ x_1 & y_1 \end{vmatrix}$$

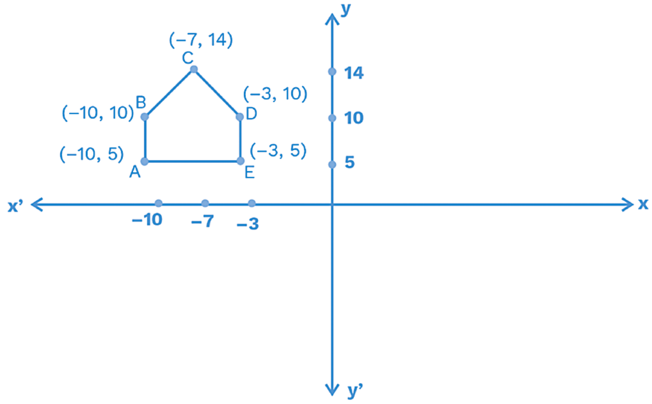
$$\Rightarrow \frac{1}{2}\begin{bmatrix} (x_1y_2 + x_2y_3 + \ldots + x_ny_1) \\ -(y_1x_2 + y_2x_3 + \ldots + y_nx_1) \end{bmatrix}$$

$$\Delta = \frac{1}{2}\begin{bmatrix} (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) \\ + \ldots + (x_ny_1) - (x_1y_n) \end{bmatrix}$$

Example 42:

Find the area of a polygon with vertices on points (–10, 5), (–10, 10), (–7, 14), (–3, 10), and (–3, 5).

Solution:



Let points A $(x_1, y_1) = (-10, 5)$

B $(x_2, y_2) = (-10, 10)$

C $(x_3, y_3) = (-7, 14)$

D $(x_4, y_4) = (-3, 10)$

E $(x_5, y_5) = (-3, 5)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_5 & y_5 \\ x_1 & y_1 \end{vmatrix}$$

$$\text{Area} = \frac{1}{2}\left|[(y_1x_2 + y_2x_3 + y_3x_4 + y_4x_5 + y_5x_1) - (y_2x_1 + y_3x_2 + y_4x_3 + y_5x_4 + y_1x_5)]\right|$$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -10 & 5 \\ -10 & 10 \\ -7 & 14 \\ -3 & 10 \\ -3 & 5 \\ -10 & 5 \end{vmatrix}$$

$$\text{Area} = \frac{1}{2}\left|\begin{bmatrix} (-10\times10) + (-10\times14) + (-7\times10) \\ +(-3\times5) + (-3\times5) \end{bmatrix} - \begin{bmatrix} (-10\times5) + (-10\times7) + (-14\times3) \\ +(-10\times3) + (-5\times10) \end{bmatrix}\right|$$

$$= \frac{1}{2}\begin{vmatrix} (-100 - 140 - 70 - 15 - 15) \\ (-50 - 70 - 42 - 30 - 50) \end{vmatrix}$$

$$= \frac{1}{2}\left|-340 + 242\right| = \frac{1}{2} \times 98 = 49 \text{ sq. units}$$

Practice Exercise – 1

Level of Difficulty – 1

1. If the coordinates of the middle point of a triangle are (6, 4), (5, 5), and (4, 4), then the coordinates of the centroid are:

(A) $5, \frac{13}{3}$
(B) 3, 3
(C) 4, 3
(D) None of these

2. If the ordinate of point $(2P - 3, 5P + 1)$ is twice its abscissa, then find the coordinate of its abscissa.

(A) $\frac{19}{3}$
(B) $-\frac{17}{8}$
(C) –17
(D) –34

3. In a triangle, circumcentre = (7, 10), orthocentre = (5, 7). Find the centroid.

(A) $\left(\frac{17}{2}, 18\right)$
(B) $\left(\frac{11}{3}, 14\right)$
(C) $\left(\frac{10}{3}, 11\right)$
(D) $\left(\frac{19}{3}, 9\right)$

4. If the distance between two points (6, 10) and $(k + 2, -2)$ is 13, then find the value of K.

(A) –1
(B) 8
(C) 9
(D) Both (A) and (C)

5. Find the equation of a straight line that passes through (7, 3) and has intercepts on the axes such that their sum is 20.

(A) $3x + 7y = 42$
(B) $7x + 3y = 58$
(C) $x + y = 10$
(D) Both (A) and (C)

Level of Difficulty – 2

6. What is the area (in sq. units) of the region enclosed between the graphs of the equation $x + y = 3$, $2x + 3y = 12$, and x-axis?

(A) 4.5
(B) 6.5
(C) 7.5
(D) 3.5

7. Let the opposite angular points of a square be (4, 5) and (2, 0). Find the coordinates of the remaining angular points.

(A) $\left(\frac{11}{2}, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, \frac{7}{2}\right)$
(B) $\left(\frac{4}{2}, \frac{5}{2}\right)$ and (1, 0)
(C) $\left(\frac{3}{2}, \frac{1}{2}\right)$ and $\left(\frac{11}{2}, \frac{7}{2}\right)$
(D) $\left(\frac{7}{2}, \frac{3}{2}\right)$ and $\left(\frac{11}{2}, \frac{1}{2}\right)$

8. A line passing through A (4, 3) is tangent to the circle $x^2 + y^2 + 2x + 4y - 2 = 0$ at B. Find AB.

(A) $\sqrt{43}$
(B) $\sqrt{35}$
(C) $\sqrt{51}$
(D) $\sqrt{41}$

9. The line joining two points A (3, 0) and B (4, 1) is rotated about point A in the anticlockwise direction through an angle of 15%. Find the equation of the new line.

(A) $y = \sqrt{3}(x - 3)$
(B) $\sqrt{3}y = (x - 3)$
(C) $3y = \sqrt{3}x - 1$
(D) $y = \sqrt{3}(x - 1)$

10. A circle touches another circle $x^2 + y^2 = 9$ and line $5x + 4y + 20 = 0$, Find area of smallest such circle. Given $\sqrt{41} = 6.4$.

(A) $0.042\ \pi$
(B) $0.0039\ \pi$
(C) $0.0049\ \pi$
(D) $0.046\ \pi$

Level of Difficulty – 3

11. If one of the vertices of the equilateral triangle is at (3, 0) and the base is $y=\sqrt{3}x+3\sqrt{3}$, then find the length of each side.
(A) 3
(B) 6
(C) 4
(D) 7

12. Find the number of common tangents that can be drawn to the two circles $(x-\sqrt{3})^2+(y-4\sqrt{5})^2=36$ and $(x-2\sqrt{3})^2+(y-7\sqrt{5})^2=49$.
(A) 2
(B) 4
(C) 6
(D) None of these

13. Find the equation of a line, the intercepts of which are the roots of the quadratic equation $x^2 + (4\sqrt{5}-2\sqrt{3})x-8\sqrt{15}=0$.
(A) $4\sqrt{5}x-2\sqrt{3}y=8\sqrt{15}$
(B) $2\sqrt{3}x-4\sqrt{5}y=8\sqrt{15}$
(C) $4\sqrt{5}x+2\sqrt{3}y=8\sqrt{15}$
(D) CND

14. Find the slope of the chord of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$, if the midpoint of the chord is (–3, 1).
(A) 6
(B) $\frac{1}{6}$
(C) $-\frac{1}{6}$
(D) -6

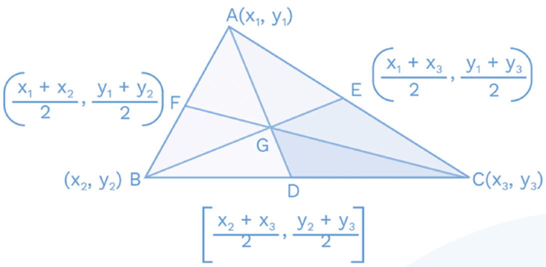
15. What will be the centroid of a triangle with sides?
$y = 2x + 3$
$y = 3x + 1$
$y = x + 2$
(A) (3, 0)
(B) (–3, 0)
(C) (0.5, 3.5)
(D) (–3, 3)

Solutions

1. (A)

We are given the coordinates of D, E, and F. We know that the coordinate of the centroid of a triangle with the vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$



Let the triangle be ABC with points P (6, 4), Q (5, 5), and R (4, 4) as middle points of the sides AB, BC, and CA, respectively.

Let coordinates of A be (x_1, y_1), B be (x_2, y_2), and C be (x_3, y_3)

Then we know that

$\frac{x_1+x_2}{2}=6$ ...(i)

$\frac{x_2+x_3}{2}=5$ ...(ii)

$\frac{x_3+x_1}{2}=4$ ...(iii)

Adding equations (i), (ii), and (iii), we get,

$x_1 + x_2 + x_3 = 15$

Also,

$\frac{y_1+y_2}{2}=4$ ...(iv)

$\frac{y_2+y_3}{2}=5$ ...(v)

$\frac{y_3+y_1}{2}=4$ ...(vi)

Adding equations (iv), (v), and (vi), we get.

$y_1 + y_2 + y_3 = 13$

Also, we know that coordinates of the centroid of a triangle

$$=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{15}{3}, \frac{13}{3}\right)$$

$$=5, \frac{13}{3}$$

2. (C)

$x = 2P - 3$, $y = 5P + 1$

According to the question,

$2(2P - 3) = 5P + 1$

$4P - 6 = 5P + 1$

$P = -7$

By putting the value of P in $(2P - 3)$. So we get, $2 \times (-7) - 3 = -14 - 3 = -17$.

Therefore, option (C) is the correct answer.

3. (D)

As we know, the centroid divides the line by joining circumcentre and orthocentre in 1:2.

$$G = \frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}$$

$$=\left(\frac{1\times5+2\times7}{1+2}\right), \left(\frac{1\times7+2\times10}{1+2}\right)$$

$$=\left(\frac{5+14}{3}\right), \left(\frac{7+20}{3}\right)$$

$$=\left(\frac{19}{3}, 9\right)$$

4. (D)

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$D=\sqrt{(k+2-6)^2+(-2-10)^2}$$

$$13=\sqrt{(k-4)^2+12^2}$$

$13^2 = (k - 4)^2 + 12^2$

$169 - 144 = (k - 4)^2$

$(k - 4)^2 = 25$

$k - 4 = \pm 5$

$k = 9$ or -1

5. (D)

Let the equation of the line in intercept form be $\frac{x}{a}+\frac{y}{b}=1$, where a and b are intercept on the axes.

Here it is given,

$a + b = 20$

$b = 20 - a$ ...(i)

$\therefore \dfrac{7}{a} + \dfrac{3}{20-a} = 1$

$140 - 7a + 3a = a(20 - a)$

$140 - 4a = 20a - a^2$

$a^2 - 24a + 140 = 0$

$a^2 - 14a - 10a + 140 = 0$

$a(a - 14) - 10(a - 14) = 0$

$(a - 14)(a - 10) = 0$

$a = 14$ or 10

Putting the value of a in equation (i)

We will get, $b = 6$ or 10

Required equations (i) $\dfrac{x}{14} + \dfrac{y}{6} = 1$

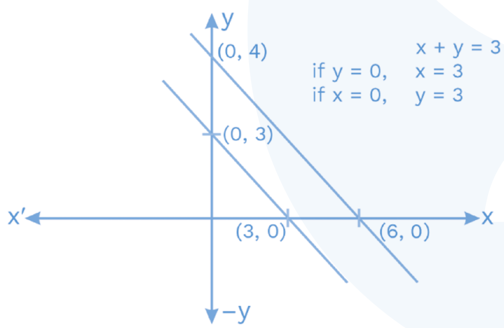
or $6x + 14y = 84$

or $3x + 7y = 42$

(ii) $\dfrac{x}{10} + \dfrac{y}{10} = 1$

$x + y = 10$

6. (C)



Given $= 2x + 3y = 12$.

If $y = 0$, then $2x = 12$, $x = 6$.

If $x = 0$, then $y = 4$.

Therefore, the required area of the region

$= \dfrac{1}{2}a_1b_1 - \dfrac{1}{2}a_2b_2 = \dfrac{1}{2}\times 4\times 6 - \dfrac{1}{2}\times 3\times 3$

$= 12 - \dfrac{9}{2} = 7.5$ sq. unit.

Hence, option (C) is the correct answer.

7. (A)

Let the angular points be (x, y) and (a, b). Since the given figure is a square, then the distance from given points to (x, y) will be the same.

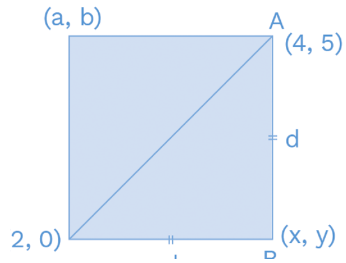
$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\sqrt{(x-4)^2 + (y-5)^2} = \sqrt{(x-2)^2 + (y-0)^2}$

$(x - 4)^2 + (y - 5)^2 = (x - 2)^2 + y^2$ ...(i)

$x^2 + 16 - 8x + y^2 + 25 - 10y = x^2 + 4 - 4x + y^2$

$4x + 10y = 37$



Distance between given angular points

$\Rightarrow \sqrt{(4-2)^2 + (5-0)^2} = \sqrt{4+25} = \sqrt{29}$

In ΔABC, applying 45–45–90 rule

$\because$ AC $= \sqrt{29}$

$\therefore$ AB $=$ BC $= \sqrt{\dfrac{29}{2}}$

$\left(\sqrt{\dfrac{29}{2}}\right)^2 = (x - 2)^2 + (y - 0)^2$

$\dfrac{29}{2} = (x-2)^2 + \left(\dfrac{37-4x}{10}\right)^2$

Solving,

$x = \dfrac{11}{2}, \dfrac{1}{2}$

Putting these values of x in equation ...(i)

$y = \dfrac{3}{2}, \dfrac{7}{2}$

Remaining angular points $= \left(\dfrac{11}{2}, \dfrac{3}{2}\right)$ and $\left(\dfrac{1}{2}, \dfrac{7}{2}\right)$

8. (A)

Equation of circle: $x^2 + y^2 + 2gx + 2fy + c = 0$

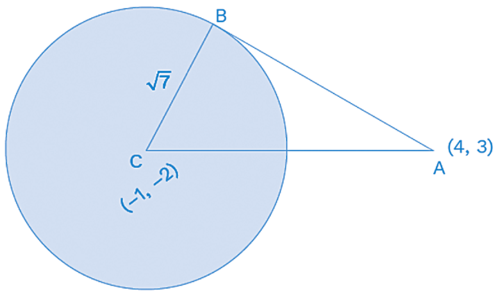
$$x^2 + y^2 + \underbrace{2x}_{2gx} + \underbrace{4y}_{2fy} - 2 = 0$$

Centre $= (-g, -f)$

Radius = $\sqrt{g^2 + f^2 - c}$

Have g = 1 and f = 2.

Centre = (–1, –2)



Radius = $\sqrt{1^2 + 2^2 - (-2)}$ = $\sqrt{7}$

AC = $\sqrt{(4-(-1))^2 + (3-(-2))^2}$

= $\sqrt{25+25}$ = $5\sqrt{2}$

AB = $\sqrt{(5\sqrt{2})^2 - (\sqrt{7})^2}$

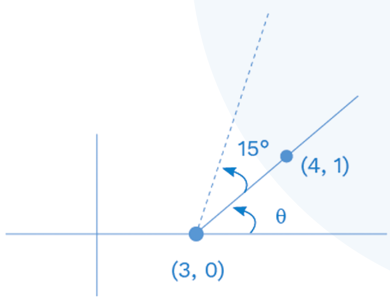
= $\sqrt{50-7}$ = $\sqrt{43}$

9. **(A)**

Slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-0}{4-3} = 1$

tan θ = 1

θ = 45



Now, new angle θ = 45 + 15 = 60.

Equation of line

⇒ (y – 0) = tan 60 (x – 3)

⇒ y = $\sqrt{3}(x-3)$

10. **(B)**

$x^2 + y^2 = 9$

Equation of circle ⇒ $(x - h)^2 + (g - k)^2 = r^2$

Centre = (h, k) radius = r

Here, centre = (0, 0) radius = 3

Also, equation, 5x + 4y + 20 = 0

x = –4 when y = 0

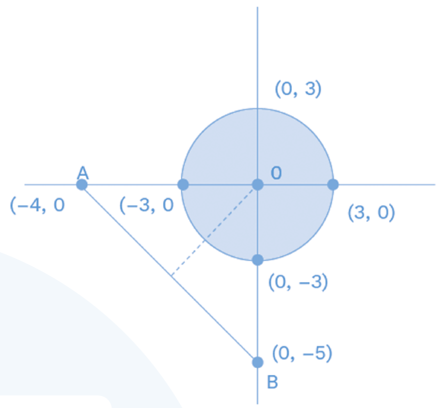
x = 0 when y = –5

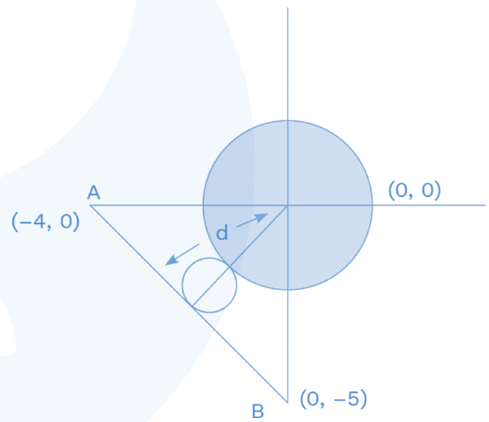
Intercept points are A (–4, 0) and B (0, –5).

Distance between centre and line joining points A and B

$$d = \frac{\frac{1}{2} \times OA \times OB}{\frac{1}{2} AB} = \frac{\frac{1}{2} \times 4 \times 5}{\frac{1}{2} \times \sqrt{4^2 + 5^2}}$$

$$= \frac{20}{\sqrt{41}} = 3.125$$





Diameter of smaller circle = 3.125 – 3 = 0.125.

Radius of smaller circle = 0.0625.

Area = $\pi r^2 = \pi \times (0.0625)^2 = 0.0039\pi$

11. **(B)**

As shown in the figure ABC with A (3, 0) and equation of line BC, as we draw an altitude from A on BC that meets BC at D.

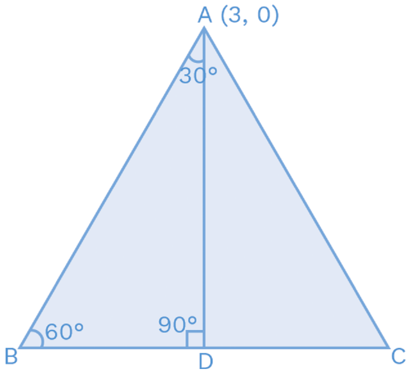
Distance of A from BC,

$$AD = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$AD = \left| \frac{\sqrt{3} \times 3 + 0 + 3\sqrt{3}}{\sqrt{(\sqrt{3})^2 + (-1)^2}} \right|$$

$$AD = \frac{6\sqrt{3}}{2}$$

$$AD = 3\sqrt{3}$$



Now, using the 30 – 60 – 90 rule,

The side opposite 60° = $3\sqrt{3}$ unit.

Side opposite 90° = $3\sqrt{3} \times \frac{2}{\sqrt{3}}$ = 6 unit.

$\therefore$ Side of the equilateral Δ is 6 unit.

12. (A)

We have

$(x-\sqrt{3})^2+(y-4\sqrt{5})^2=36$ ...(i)

and $(x-2\sqrt{3})^2+(y-7\sqrt{5})^2=49$ ...(ii)

$\therefore$ The centre and radius of (i) are $(\sqrt{3}, 4\sqrt{5})$ and 6, respectively; the centre and radius of (ii) are $(2\sqrt{3}, 7\sqrt{5})$ and 7, respectively.

The distance between the two centres is:

$C_1 C_2 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2\sqrt{3}-\sqrt{3})^2+(7\sqrt{5}-4\sqrt{5})^2}$

$= \sqrt{(\sqrt{3})^2+(3\sqrt{5})^2}$

$= \sqrt{3+45}$

$= \sqrt{48}$

$C_1 C_2 = 4\sqrt{3}$

The sum of the radius of both the circles = 6 + 7 = 13.

$C_1C_2 < r_1 + r_2$

Hence, we can draw only two common tangents to the two circles.

13. (A)

We have,

$x^2+(4\sqrt{5}-2\sqrt{3})x-8\sqrt{15}=0$

$x^2+4\sqrt{5}x-2\sqrt{3}x-8\sqrt{15}=0$

$x\times(x+4\sqrt{5})-2\sqrt{3}(x+4\sqrt{5})=0$

$(x+4\sqrt{5})(x-2\sqrt{3})=0$

$x=-4\sqrt{5}, 2\sqrt{3}$

The intercept of the lines is $(-4\sqrt{5}, 2\sqrt{3})$

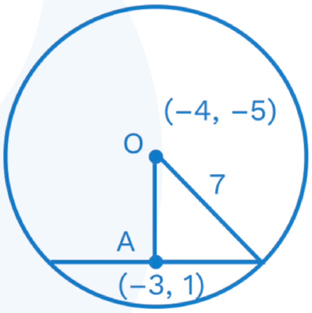
$\therefore$ The line can be $\frac{x}{-4\sqrt{5}}+\frac{y}{2\sqrt{3}}=1$

or $\frac{x}{2\sqrt{3}}+\frac{y}{-4\sqrt{5}}=1$

$\Rightarrow -2\sqrt{3}x+4\sqrt{5}y=8\sqrt{15}$

or $4\sqrt{5}x-2\sqrt{3}y=8\sqrt{15}$

14. (C)



We have, $x^2 + y^2 + 8x + 10y - 8 = 0$

$\Rightarrow x^2 + 8x + y^2 + 10y = 8$

$\Rightarrow x^2 + 2 \times 4 \times x + 16 + y^2 + 2 \times 5 \times y + 25 = 8 + 16 + 25$

$\Rightarrow (x + 4)^2 + (y + 5)^2 = 49$

$\therefore$ The centre is (–4, –5) and radius = 7

(–3, 1) is the midpoint of the chord.

Let O be the centre = (–4, –5) and A be the midpoint of the chord= (–3, 1).

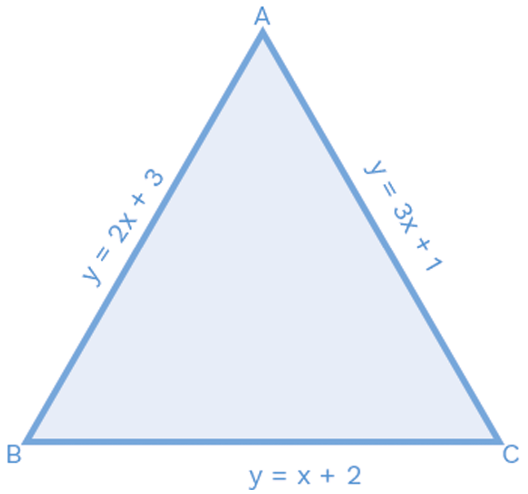
Therefore, the slope of the line OA

$= \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-5-1}{-4-(-3)} = \frac{-6}{-4+3} = \frac{-6}{-1} = 6.$

Slope of the chord = $-\frac{1}{6}$ $[\because M_1 \times M_2 = -1]$.

15. (C)



Given sides

AB $\Rightarrow$ $y = 2x + 3$...(i)

BC $\Rightarrow$ $y = x + 2$ ...(ii)

CA $\Rightarrow$ $y = 3x + 1$...(iii)

Then coordinate of point A is the intersection of line BA and CA.

Solving equations (i) and (iii),

we get $x = 2$ and $y = 7$.

$\therefore$ Coordinates of point A = (2, 7).

Similarly, finding coordinates of point B and point C.

Solving equations (i) and (ii), we get B = (–1, 1).

Similarly, solving equations (ii) and (iii), we get, C = (0.5, 2.5)

A = (2, 7), B = (–1, 1), C = (0.5, 2.5)

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= (0.5, 3.5)$$

Practice Exercise – 2

Level of Difficulty – 1

1. The diagonal of a rhombus is formed by joining (7, 1) and (–1, 3). If $4x - 3y + a = 0$ is the other diagonal, then a is:
(A) –12
(B) –6
(C) 1
(D) 2

2. If two lines $kx + 3y = 20$ and $5x + 2y = 10$ are perpendicular to each other, then find the value of K.
(A) $\frac{-5}{6}$
(B) $\frac{6}{5}$
(C) $\frac{-6}{5}$
(D) $\frac{5}{6}$

3. If the three vertices of a triangle are (3, 5), (–2, –1), and (–1, –4), then find the position of the centroid of this triangle.
(A) Origin
(B) (0, 1)
(C) (1, 0)
(D) (1, 1)

4. P (4, 5) and Q (6, 1) are two given points, and the point y divides the line segment PQ externally in the ratio of 3 : 2. Find the coordinates of y.
(A) (10, –7)
(B) (10, 7)
(C) (–7, 10)
(D) (7, 10)

5. If the distance between two points (K + 2, 14) and (3, –10) is 25 units, then find the value of K.
(A) 8, –6
(B) 8, –7
(C) –13, 12
(D) 7, 15

6. Find the incentre of the equilateral triangle having vertices at $(2, 3\sqrt{3})$, $(5, 0)$, and (–1, 0).
(A) $(3, \sqrt{3})$
(B) (0, 0)
(C) $(2, \sqrt{3})$
(D) $(\sqrt{3}, 0)$

7. Find the coordinates of the point which divides the line segment joining A (2, 4) and B (4, 6) in the ratio 1:2 externally.

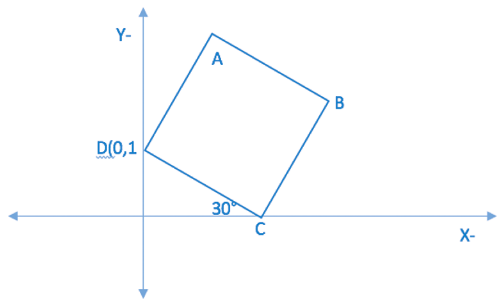
8. What does the equation $2x^3 - 2xy^2 - x^2y + y^3 = 0$ represent?
(A) A circle and a straight lines
(B) A parabola and a straight line
(C) Three straight lines
(D) Cannot be determined

9. Point A divides the segment BC in the ratio 4:1 internally. The coordinates of B are (6, –4) and that of C are $\left(\frac{9}{4}, 8\right)$. What are the coordinates of point A?
(A) (3, 5.6)
(B) (3, –6)
(C) (–3, –6)
(D) 3, 6)

10. Find the area of a triangle with coordinates A (9, 7), B (–3, 6), and C (14, 12) using the shift of origin method.

Level of Difficulty – 2

11. In the given diagram, if ABCD is a square, then what are the coordinates of point B?



(A) $(\sqrt{3}, \sqrt{3})$
(B) $(\sqrt{3}, \sqrt{3} + 1)$
(C) $(\sqrt{3}, 1/\sqrt{3})$
(D) $(\sqrt{3} + 1, \sqrt{3})$

12. In a right-angled triangle QPR with P (0, 6), Q (0, 0), and R (8, 0), what is the distance between the orthocentre and the centroid of this triangle?
(A) 5
(B) 5/3
(C) 10/3
(D) 4

13. The two diagonals of a parallelogram ABCD intersect each other at coordinates (35, 47), and the coordinates of two adjacent points, A and B of the parallelogram, are (11, 15) and (27, 32). Find the measure of the length of diagonals.
(A) 34 and 17
(B) 80 and 34
(C) 19 and 15
(D) 18 and 25

14. Find the distance between the orthocentre and circumcentre of the triangle formed by joining the points (5, 2), (9, 2), and (5, 4).
(A) $2\sqrt{5}$ units
(B) $\sqrt{5}$ units
(C) $2\sqrt{3}$ units
(D) 9 units

15. Find the circumference of the circle circumscribing the triangle formed by the x-axis, y-axis, and the straight line $5x + 12y - 60 = 0$.
(A) 169 π units
(B) 6.5 π units
(C) 13 π units
(D) 26 π units

16. Two diagonals of a parallelogram intersect each other at coordinates (15, 21). The two adjacent points of the parallelogram are (3, 5) and (12, 13.5). Find the length of the diagonals.
(A) 20 and $\sqrt{261}$
(B) 30 and $\sqrt{361}$
(C) 40 and $\sqrt{361}$
(D) 40 and $\sqrt{261}$

17. Find the area of the quadrilateral (in square units) formed by the lines $y = 4$ and $2x + 3y = 20$ with the positive coordinate axes.
(A) 36
(B) 28
(C) 24
(D) 20

18. If (10, 2), (x, 14), and (6, –2) are three consecutive vertices of a rhombus, then find the value of x.
(A) –3
(B) 4
(C) –6
(D) –5

19. The acute angle between the lines $x - y + 5 = 0$ and $3x - 3y + 9 = 0$ is:
(A) 45°
(B) 30°
(C) 15°
(D) 60°

20. A square is inscribed in a circle represented by the equation $x^2 + y^2 + 6x - 10y = 15$. Find the area of the square (in sq.units).

Level of Difficulty – 3

21. Find the area of the quadrilateral (in cm^2) formed by the lines $y = 4$ and $x + 5y = 25$ with the positive x-axis and positive y-axis.
(A) 20 cm^2
(B) 60 cm^2
(C) 72 cm^2
(D) 80 cm^2
(E) 96 cm^2

22. Find the area of the quadrilateral enclosed by the lines $y = 7$, $y = x$, $x + y = 5$, and $x = 0$.

(A) 18.5 sq. units
(B) 9.5 sq. units
(C) 9.25 sq. units
(D) 18.25 sq. units
(E) 18 sq. units

23. Triangle ABC has vertices A (0, 0), B (0, 14), and C (72, 0). Points P and Q lie on AB such that AP:PQ:QB = 1:2:4. Similarly, points R and S lie on side AC such that AR:RS:SC = 5:1:6. If the line segment QR and PS intersect at Z, then find the slope of AZ.

(A) $\frac{1}{25}$

(B) $\frac{2}{31}$

(C) $\frac{1}{15}$

(D) $\frac{1}{10}$

24. Find the distance between the ortho-centre and circumcentre of the triangle formed by joining the points (6, 2), (9, 2), and (6, 4).

(A) $\frac{\sqrt{13}}{2}$ units

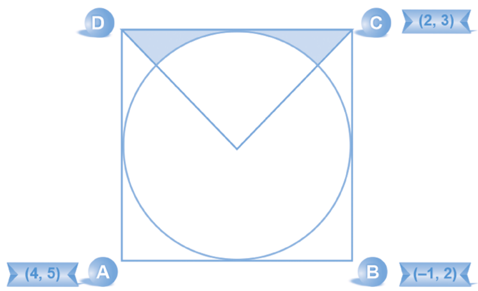
(B) $\frac{\sqrt{17}}{2}$ units

(C) $\sqrt{13}$ units

(D) $\sqrt{31}$ units

25. Find the radius of the circle, the equation of which is $x^2 + y^2 - 14x + 6y - 23 = 0$.

26. A circle is enclosed inside a rhombus ABCD touching its opposite sides. The coordinates of A, B, and C are (4, 5), (–1, 2), and (2, 3), respectively. Calculate the shaded region area.



(A) $\left(\sqrt{10} - \frac{5\pi}{17}\right)$sq. units

(B) $\left(\sqrt{15} + \frac{7\pi}{17}\right)$sq. units

(C) $\left(\sqrt{10} + \frac{5\pi}{17}\right)$sq. units

(D) $\left(\sqrt{10} - \frac{2\pi}{17}\right)$sq. units

(E) $\left(\sqrt{10} - \frac{5\pi}{17}\right)$sq. units

27. Find the minimum value of $\sqrt{x^2 + y^2}$, if $5x + 12y = 60$.

(A) 17/13
(B) 80/19
(C) 60/13
(D) 69/13

28. What is the length of the tangent drawn from the point (2, –1) to the circle $3x^2 + 4x + 2y + 6 = 0$?

(A) 15
(B) 20
(C) 46
(D) CND

29. For how many integral values of k will the intersection of the lines $2x + y = 6$ and $x - 2y = k$ happen strictly in the first quadrant (strictly means not on the two axes but inside the first quadrant)?

30. Find the mirror images of the point {–8, 12} when it is reflected about the line $4x + 7y + 13 = 0$.

(A) (–16, –2)
(B) (–18, –2)
(C) (–20, –2)
(D) (–15, –2)

Solutions

1. (B)

Diagonals of a rhombus bisect each other. Thus, the line $4x - 3y + a = 0$ is bisected by the points joining (7, 1) and (–1, 3).

$\rightarrow$ Midpoint of (7, 1) and (–1, 3) lies on the line $4x - 3y + a = 0$

Midpoint = $\left(\frac{7-1}{2}, \frac{1+3}{2}\right)$ = (3, 2)

$4(3) - 3(2) + a = 0$

$a = -6$

Hence, option (B) is the correct answer.

2. (C)

Line 1: $kx + 3y = 20$

$$y = \frac{-kx}{3} + \frac{20}{3}$$

Slope $m_1 = \frac{-k}{3}$

Line 2: $5x + 2y = 10$

$$y = \frac{-5}{2}x + 5$$

Slope $m_2 = \frac{-5}{2}$

Since lines are perpendicular, therefore $m_1 \times m_2 = -1$

$$\left(-\frac{k}{3}\right) \times \left(\frac{-5}{2}\right) = -1$$

$$k = \frac{-6}{5}$$

Hence, option (C) is correct.

3. (A)

We know that:

(a_1, b_1), (a_2, b_2), and (a_3, b_3) are the three vertices of a triangle, then coordinate of centroid

$$= \left(\frac{a_1 + a_2 + a_3}{3}, \frac{b_1 + b_2 + b_3}{3}\right)$$

Here centroid

$$= \left[\frac{(3)+(-2)+(-1)}{3}, \frac{(5)+(-1)+(-4)}{3}\right]$$

$$= \left(\frac{0}{3}, \frac{0}{3}\right) = (0,0)$$

Hence, option (A) is correct.

4. (A)

Since $m = 3$, n = 2 is given in the question.

$(x_1, y_1) = (4, 5)$ and $(x_2, y_2) = (6, 1)$

$Y(x, y) = [x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}]$

$Y(x, y) = \frac{3\times6 - 2\times4}{3-2}, \frac{3\times1 - 2\times5}{3-2}$

$= \frac{18-8}{1}, \frac{3-10}{1}$

$Y(x, y) = (10, -7)$

Hence, option (A) is the correct answer.

5. (A)

Since the distance between two points = 25 units (given).

Also,

$(x_1, y_1) = (k + 2, 14)$

$(x_2, y_2) = (3, -10)$

$25 = \sqrt{(k+2-3)^2 + (14+10)^2}$

$25 = \sqrt{(k-1)^2 + (24)^2}$

$(k - 1)^2 = 25^2 - 24^2$

$(k - 1)^2 = 49$

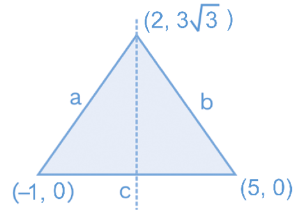
$k - 1 = \pm 7$

$k = -6$ and $k = +8$

Hence, option (A), i.e., 8, –6 is the correct answer.

6. (C)

To find $a = b = c$, we have to use the distance formula



$a = b = c = \sqrt{[5-(-1)]^2 + (0-0)^2} = \sqrt{36} = 6$

Coordinates of incentre:

$$x = \frac{6\times2 + 6\times(-1) + 6\times5}{6+6+6}$$

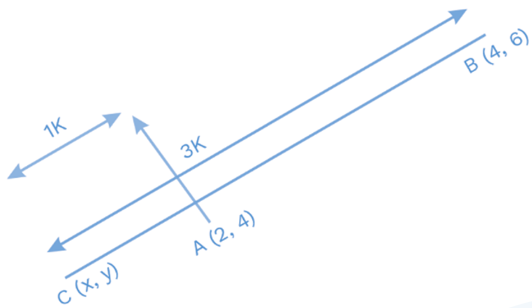
$$= \frac{12 - 6 + 30}{18} = 2$$

$$\frac{(6\times3\sqrt{3})+(6\times0)+(6\times0)}{6+6+6}=\frac{18\sqrt{3}+0+0}{18}=\sqrt{3}$$

So, coordinates of incentre of the triangle = $(2,\sqrt{3})$.

Hence, option (C) is the correct answer.

7. (5, 7)



Here, we will apply the section formula for external division.

$(x_1, y_1) = (2, 4)$, $(x_2, y_2) = (4, 6)$, $m:n = 3:1$,

$$x = \frac{mx_2 - nx_1}{m-n},\ y = \frac{my_2 - ny_1}{m-n}$$

$$x = \frac{|3\times4-1\times2|}{|3-1|}=\frac{10}{2}=5$$

$$y = \frac{|3\times6-1\times4|}{|3-1|}=\frac{14}{2}=7$$

Therefore, the coordinates of the required points are (5, 7).

8. (C)

$2x^3 - 2xy^2 - x^2y + y^3 = 0$...(i)

Put $y = x$ in equation (i) we get:

$2x^3 - 2x^3 - x^3 + x^3 = 0$

Hence, $(x - y)$ is a factor of $2x^3 - 2xy^2 - x^2y + y^3 = 0$.

Similarly, it can be proved that $(x + y)$ is also a factor of

$2x^3 - 2xy^2 - x^2y + y^3 = 0$.

Dividing $2x^3 - 2xy^2 - x^2y + y^3 = 0$ by $x^2 - y^2$,

We get $2x - y$ as the quotient.

Hence, the given equation can be written as

$(x - y)(x + y)(2x - y) = 0$.

All these are equations of straight lines.

Therefore, the equation represents 3 straight lines.

9. (A)

Since $m = 4$, $n = 1$ is given in the question.

$(x_1, y_1) = (6, -4)$

$(x_2, y_2) = \left(\frac{9}{4}, 8\right)$

$$A\ (x, y) = \frac{\frac{4\times9}{4}+6\times1}{4+1}, \frac{4\times8+1\times(-4)}{4+1}$$

$$= \frac{9+6}{5}, \frac{32-4}{5}$$

$$= \frac{15}{5}, \frac{28}{5}$$

$A\ (x, y) = 3, 5.6$

Therefore, option (A) is the correct answer.

10. (27.5 sq. units)

Area of D having coordinates. A (9, 7), B (−3, 6), C (14, 12)

A (9, 7), B (−3, 6), C (14, 12)

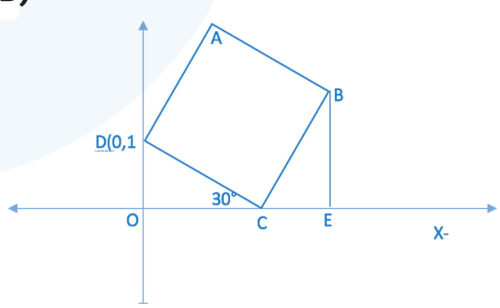
(12, 1) (0, 0) (17, 6)

$$A = \frac{1}{2}|x_1y_2 - x_2y_1|$$

$$= \frac{1}{2}|12\times6-17\times1|$$

$$= \frac{1}{2}\times55=27.5 \text{ sq. units.}$$

11. (D)



Let O be the origin and BE is perpendicular dropped to the x-axis from B.

$$\tan 30 = \frac{OD}{OC} \quad \text{or} \quad OC = \sqrt{3}$$

So, the coordinates of the point C are $(\sqrt{3}, 0)$,

Angle OCE = 180° = angle (OCD + DCB + BCE)

Angle DCB = 90°

Since ABCD is a square.

Therefore, angle BCE

= 180 − 30 − 90 = 60°

Triangle DOC and CEB are congruent (as angles are 30, 60, and 90° in both the triangles and also side DC = BC = side of the square).

Thus, CE = 1 and BE = $\sqrt{3}$

B is (OE, BE) = ($\sqrt{3}$ + 1, $\sqrt{3}$)

12. (C)

In a right-angled triangle, the orthocentre lies at the vertex where the right-angled is formed. So, the coordinates of the orthocentre become (0, 0).

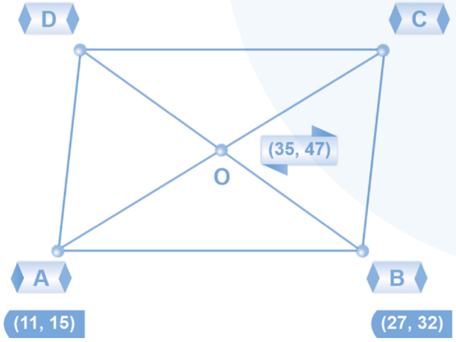
The coordinates of the centroid are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

So, the coordinates of the centroid become (8/3, 2).

Thus, the distance between orthocentre and centroid

$= \sqrt{(\frac{8}{3}-0)^2+(2-0)^2} = 10/3$

13. (B)



Since we know the formula for the distance between the two points.

Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\therefore$ OA = $\sqrt{(35-11)^2+(47-15)^2}$

= $\sqrt{24^2+32^2}$

= $\sqrt{1600}$

OA = 40 cm

AC = 2 × OA = 2 × 40 = 80 cm.

Also, we have to find OB

OB = $\sqrt{(35-27)^2+(4732)^2}$

= $\sqrt{(35-27)^2+(4732)^2}$

= $\sqrt{289}$

= 17 cm.

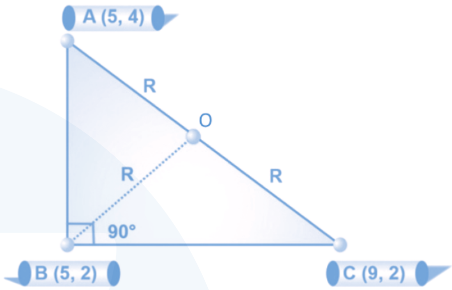
Thus BD = 2 × OB

= 2 × 17 = 34 cm

Hence, option (B) is the correct answer.

14. (B)

Let the points be A (5, 4), B (5, 2), and C (9, 2).



Since point A will be just above point B. Thus, it forms a right-angled triangle.

Also, we know that the orthocentre of a right angle Δ lies at vertex B or right angle and the circumcentre of a right-angled triangle lies at the midpoint of hypotenuse which is at point O.

OA = OB = OC = R = $\frac{AC}{2}$

Now, we have to find the distance between the point A and C.

AC = $\sqrt{(9-5)^2+(2-4)^2}$ = $\sqrt{16+4}$

= $\sqrt{20}$ = $2\sqrt{5}$ units

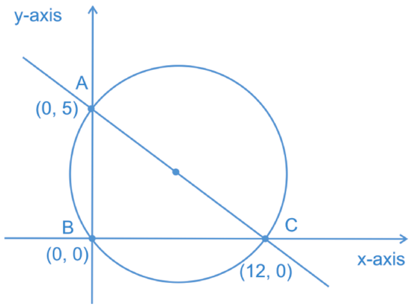
Distance between orthocentre and circumcentre (OB)

= $\frac{AC}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$ units

Hence, option (B) is the correct answer.

15. (C)

The vertices of the triangle will be (0, 0), (12, 0), and (0, 5). It will be a right-angled triangle with side AB = 5 units, BC = 12 units, and AC = 13 units (by Pythagoras theorem).



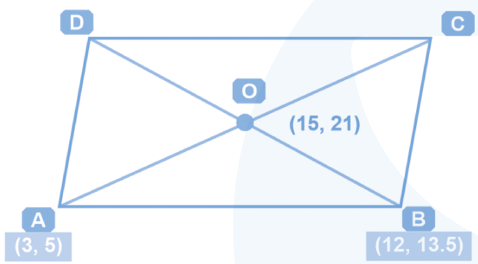
We know that in a right-angled triangle,

Circumradius = $R = \frac{1}{2} \times AC = \frac{1}{2} \times 13 = \frac{13}{2}$

→ Circumference of the circle $= 2\pi R = 2 \times \pi \times \frac{13}{2}$ = 13 π units.

Hence, option (C) is the correct answer.

16. **(D)**



By using the distance formula, we can calculate the length of the diagonals.

$$AO = \sqrt{(15-3)^2 + (21-5)^2}$$

$$= \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256} = 20 \text{ units}$$

The length of AC = 2 × OA = 2 × 20 = 40 units.

Now, we have to find the length of BD

$$OB = \sqrt{(15-12)^2 + (21-13.5)^2}$$

$$= \sqrt{3^2 + (7.5)^2} = \sqrt{65.25}$$

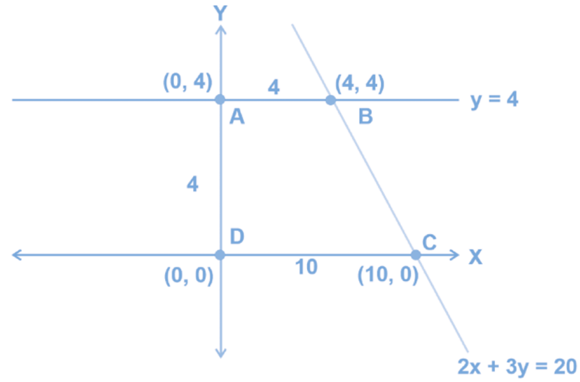
$$= \frac{\sqrt{261}}{2} \text{ units}$$

Length of BD = 2 × OB = $2 \times \frac{\sqrt{261}}{2}$ = $\sqrt{261}$ units.

Hence, option (D) is the correct answer.

17. **(D)**

Line $2x + 3y = 20$ will cut the line $y = 4$ at (4, 4) and x-axis at (10, 0) as shown in the diagram below.



Now, quadrilateral ABCD is a trapezium.

So, area of trapezium ABCD = $\frac{1}{2}(4+10) \times 4$ = 28 sq. units.

18. **(C)**

Since we know that the adjacent sides of a rhombus are equal.

We know the formula for the distance between two points

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(x-10)^2 + (14-2)^2} = \sqrt{(x-6)^2 + (14+2)^2}$$

$$(x - 10)^2 + (12)^2 = (x - 6)^2 + (16)^2$$

$$x^2 + 100 - 20x + 144 = x^2 + 36 - 12x + 256$$

$$8x = 244 - 256 - 36$$

$$8x = -48$$

$$x = -\frac{48}{8} = -6$$

Hence, option (C) is the correct answer.

19. **(C)**

Let the slope of the first line and second line be m_1 and m_2.

$$x - y + 5 = 0$$

$$y = x + 5$$

If we compare above line with

$$y = mx + c$$

Where m = slope of the line $y = mx + c$

$$m_1 = 1$$

Similarly, $\sqrt{3}x - 3y + 9 = 0$

$$3y = \sqrt{3}x + 9$$

$$y = \frac{\sqrt{3}}{3}x + \frac{9}{3}$$

$$y = \frac{1}{\sqrt{3}}x + 3$$

Thus, $m_2 = \frac{1}{\sqrt{3}}$

Let θ be the acute angle between these two lines.

Since

$$\tan\,\theta = \frac{m_1 - m_2}{1 + m_1 \times m_2} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$\tan\,\theta = \tan 15°$, means $\theta = 15°$

Hence option (C) is the correct answer.

20. (98)

We must know that equation of the circle with centre (h, k) and radius r is given by $(x - h)^2 + (y - k)^2 = r^2$.

So, our equation of circle, $x^2 + y^2 + 6x - 10y = 15$ can be rewritten as

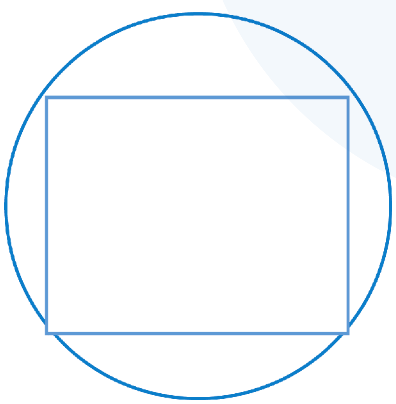
$(x^2 + 2 \times 3x + 3^2) + (y^2 - 2 \times 5y + 5^2) - (3^2 + 5^2) = 15$

$(x^2 + 2 \times 3x + 3^2) + (y^2 - 2 \times 5y + 5^2) = 49$

$(x + 3)^2 + (y - 5)^2 = 7^2$

So, the circle given in the question has a centre (– 3, 5) and a radius of 7 units.

Also, the diameter of the circle (14 units) will become the diagonal of the square as shown in the figure below:

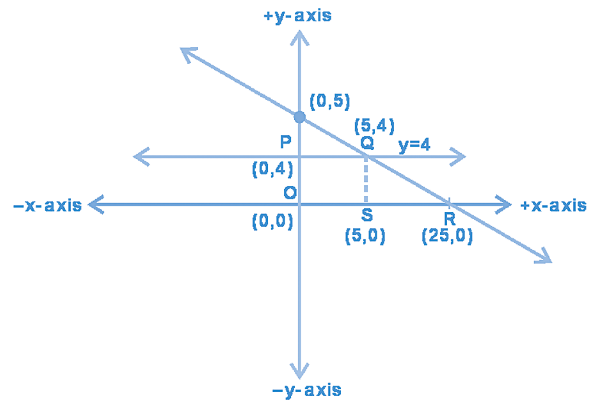


So, the side of the square with diagonal (14 units) $= \frac{14}{\sqrt{2}}$ units.

Hence, the area of the square $= \left(\frac{14}{\sqrt{2}}\right)^2$

= 98 sq. units.

21. (B)



So, when we draw the lines, we have found that a quadrilateral PORQ is formed.

Thus, first of all, we have to find the intersection point at Q.

Therefore, $x + 5y = 25$ (since $y = 4$ is given)

$x + 5 \times 4 = 25$

$x = 25 - 20$

$x = 5$

Therefore, intersection point at Q is (5,4).

If we draw a perpendicular line from point Q to meet the x-axis at S.

Therefore, OS = 5 cm

QS = 4 cm

and SR = 20 cm.

Therefore, the area of the quadrilateral PORQ

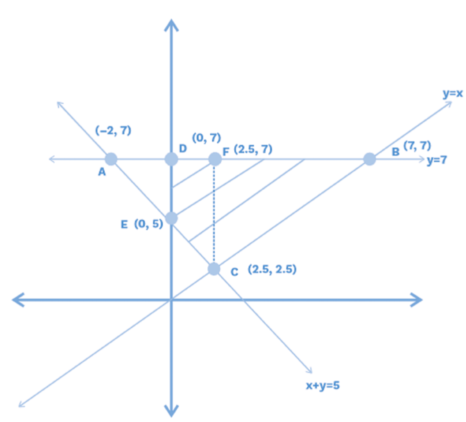
= area of the rectangle PQSO + area of the ΔQSR

$= 5 \times 4 + \frac{1}{2} \times 20 \times 4 = 20 + 40 = 60$ cm^2

Hence, option (B) is the correct answer.

22. (D)

On solving the equations given in the question, we get the following graph:



Here

AB = 7 − (−2) = 9 units

CF = 7 − 2.5 = 4.5 units

AD = 0 − (−2) = 2 units

DE = 7 − 5 = 2 units

To find the area of the quadrilateral BCDE, we can subtract the area of the triangle ABC from the area of the triangle ADE.

Area of triangle ABC = $\frac{1}{2} \times AB \times CF$ = 20.25 sq. units.

Area of triangle ADE = $\frac{1}{2} \times AD \times DE$ = 2 sq. units.

The area of the quadrilateral BCDE = 20.25 − 2 = 18.25 sq. units.

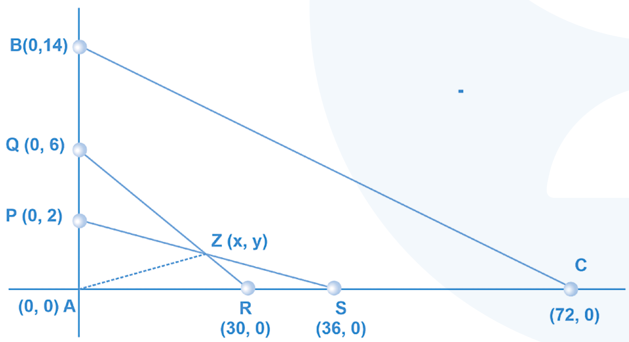
23. (C)

Given AP:PQ:QB = 1:2:4 and AB = 14 units.

This implies, AP = 2 units, PQ = 4 units, and QB = 8 units.

Also AR:RS:SC = 5:1:6 and AC = 72 units.

This implies, AR = 30 units, RS = 6 units, and SC = 36 units.



Now, we know that equation of line is given by $y - y_1 = m(x - x_1)$

where m = slope

Therefore, equation of line RQ

$$y - 0 = \left(\frac{6}{30}\right)(x - 30)$$

$5y = 30 - x$

$x + 5y = 30$ ...(i)

And, equation of line PS:

$$y - 0 = \left(\frac{4}{36}\right)(x - 36)$$

$9y = 36 - x$

$x + 9y = 36$ ...(ii)

Subtracting equation (i) from (ii)

$4y = 6$

$$y = \frac{3}{2}$$

Putting y in equation (i)

$$x = 30 - \frac{15}{2} = \frac{45}{2}$$

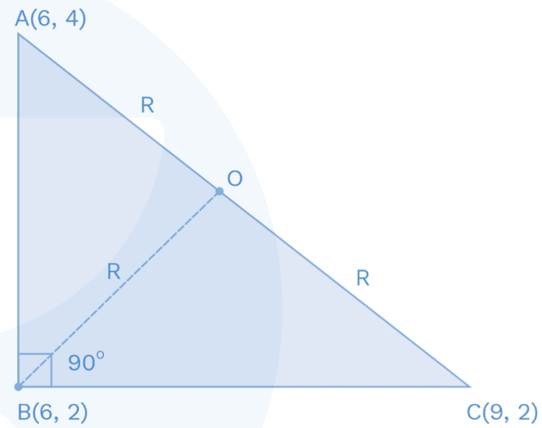
Now, the slope of line AZ = $\dfrac{\left(\frac{3}{2} - 0\right)}{\left(\frac{45}{2} - 0\right)} = \frac{1}{15}$

Hence, option (C) is the correct answer.

24. (A)

Let the points be A (6, 4), B (6, 2), and C (9, 2).

Therefore,



Since point A is just above point B.

Thus, it forms a right angle triangle.

Also, we know that the orthocentre of a right angle triangle lies at vertex B or the right angle.

And circumcentre of a right-angled triangle lies at the midpoint of the hypotenuse, which is at point O.

$\therefore$ Circumradius (R) = $\dfrac{\text{Hypotenuse}}{2} = \dfrac{AC}{2}$

$\therefore$ OA = OB = OC = R $= \dfrac{AC}{2}$

We have to find the distance between the point A and C.

$$\therefore AC = \sqrt{(9-6)^2 + (2-4)^2}$$

$= \sqrt{9+4} = \sqrt{13}$ units

$\therefore$ Distance between orthocenter and circumcenter = (OB) $= \dfrac{AC}{2} = \dfrac{\sqrt{13}}{2}$ units

Hence, option (A) is the correct answer.

25. (9)

The general equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the centre of the circle and r is the radius of the circle.

So, we will convert the given equation into this form and will find the radius.

$x^2 + y^2 - 14x + 6y - 23 = 0$

$x^2 - 14x + y^2 + 6y - 23 = 0$

$x^2 - 2 \times 7x + y^2 + 2 \times 3y - 23 = 0$

$x^2 - 2 \times 7x + 7^2 + y^2 + 2 \times 3y + 3^2 - 23 - 7^2 - 3^2 = 0$

$(x - 7)^2 + (y + 3)^2 = 81$

$(x - 7)^2 + (y + 3)^2 = 9^2$

Thus, the radius of the circle is 9 units.

26. (A)

Given A circle is enclosed inside the rhombus ABCD such that it touches its opposite sides.

O is the centre of the circle, and the rhombus OM is perpendicular from O to CD.

The diagonals of a rhombus bisect each other at 90°.

Using midpoint formula, coordinates of O are {(4 + 2)/2, (5 + 3)/2} = (3, 4).

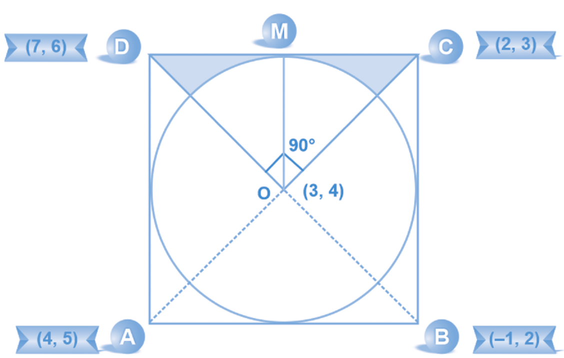
Coordinates of D are (7, 6).

Now,

$$OC = \sqrt{\{(2-3)^2 + (3-4)^2\}} = \sqrt{2}$$

$$OD = \sqrt{\{(7-3)^2 + (6-4)^2\}} = 2\sqrt{5}$$

$$CD = \sqrt{\{(7-2)^2 + (6-3)^2\}} = \sqrt{34}$$



Area of DOCD = 1/2 × OC × OD = 1/2 × $\sqrt{2} \times 2\sqrt{5}$

= $\sqrt{10}$ sq. units ...(i)

Also,

Areas of DOCD = 1/2 × OC × OD = 1/2 × CD × OM

$\Rightarrow \sqrt{2} \times 2\sqrt{5} = \sqrt{34} \times OM$

$\Rightarrow OM = \dfrac{\sqrt{2} \times 2\sqrt{5}}{\sqrt{34}}$

$\Rightarrow OM = 2\sqrt{\dfrac{5}{17}}$

$\Rightarrow$ Radius of circle = OM = $2\sqrt{\dfrac{5}{17}}$ unit

Area of quarter circle = $\dfrac{1}{4} \times \pi \times \left(2 \times \sqrt{\dfrac{5}{17}}\right)^2$

$= \dfrac{5}{17}\pi$ sq. units

Area of shaded region = area of DOCD – area of quarter circle

$= \sqrt{10}$ sq. units $- \dfrac{5\pi}{17}$ sq. units

$= \left(\sqrt{10} - \dfrac{5\pi}{17}\right)$ sq. units

Therefore, the area of the shaded region is $\left(\sqrt{10} - \dfrac{5\pi}{17}\right)$ sq. units.

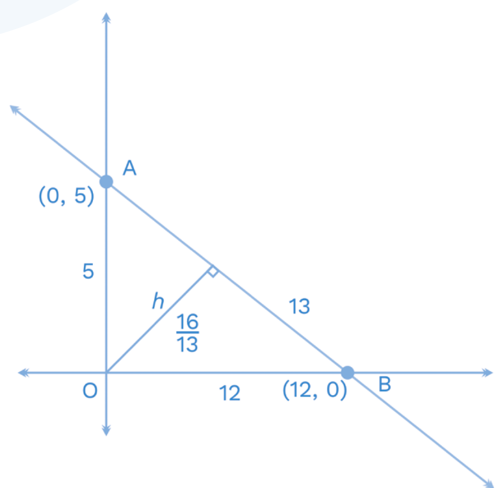
Hence, option (A) is the correct answer.

27. (C)

$\sqrt{x^2 + y^2}$ is the radius of a circle with x and y.

Such that $5x + 12y = 60$

The line $5x = 12y = 60$ is plotted as follows:



All the points on the line AB satisfy $5x + 12y = 60$.

$\sqrt{x^2+y^2}$ will be minimum when the distance between 0 and line AB is minimum.

$$\text{Area of}(\Delta ABC)=\frac{1}{2}\times OB\times AO=\frac{1}{2}\times AB\times h$$

$$\therefore \frac{1}{2}\times 12\times 5=\frac{1}{2}\times 13\times h$$

$$\therefore h=\frac{60}{13}$$

Hence, option (C) is the correct answer.

28. (D)

The standard equation for a circle having a centre (a, b) and radius r is given by the following equation:

$(x-a)^2+(y-b)^2=r^2$

The equation is given in this case $3x^2+3y^2+4x+2y+6=0$.

To put it in the standard form, we divide the entire equation by 3.

Hence, we get,

$$x^2+y^2+\frac{4}{3}x+\frac{2}{3}y+2=0$$

$$\therefore \left\{x^2+2\times\frac{2}{3}x\times 1+\left(\frac{2}{3}\right)^2\right\}$$

$$\left\{y^2+2\times\frac{1}{3}y\times 1+\left(\frac{1}{3}\right)^2\right\}+\frac{13}{9}=0$$

$$\therefore \left(x+\frac{2}{3}\right)^2+\left(y+\frac{1}{3}\right)^2=-\frac{13}{9}$$

$\therefore$ The centre of the circle is $\left(-\frac{2}{3},-\frac{1}{3}\right)$,

and the radius of this circle is $\sqrt{\left(-\frac{13}{9}\right)}$.

Since the radius of the circle is an imaginary number, the circle is not real.

$\because$ The length of the tangent from the point (2, –1) cannot be determined.

29. (14)

Solving $2x+y=6$ and $x-2y=k$ simultaneously, we get

$$x=\frac{12+k}{5} \text{ and } y=\frac{6-2k}{5}$$

For the intersection to be in the first quadrant, both x and y must be positive.

$x>0\Rightarrow(12+k)>0\Rightarrow k>-12$

$y>0\Rightarrow(6-2k)>0\Rightarrow k<3$

For both the conditions to be valid, $-12 < k < 3$.

The integral values in this range are –11 to 2, i.e., 14 in numbers.

30. (A)

If A is (–8, 12) and B is its required reflection, line AB will be perpendicular to $4x + 7y + 13 = 0$.

Since the slop of $4x+7y+13=0$ is $\frac{4}{7}$, the slope of AB will be $\frac{7}{4}$.

Equation of line AB, using the point-slope form, will be

$$y-12=\frac{7}{4}(x-(-8)), \text{ i.e., } 7x-4y+104=0.$$

The point of intersection of $4x + 7y + 13 = 0$ and $7x - 4y + 104 = 0$ can be found by solving the two equations simultaneously (–12, 5).

This point (–12, 5) will be the midpoint of A (–8, 12) and B (p, q),

Say,

Then, $\frac{-8+p}{2}=-12\Rightarrow p=-16$

and $\frac{12+q}{2}=5\Rightarrow q=-2.$

Thus, the required reflection is (–16, –2).

Mind Map

COORDINATE GEOMETRY

Circles

- General equation : $x^2 + y^2 + 2gx + 2fy + c = 0$
- Centre is $(-g, -f)$ and radius $= \sqrt{g^2 + f^2 - c}$
- Centre is (h, k) and radius is $r \Rightarrow \sqrt{(x-h)^2 + (y-k)^2} = r$
- Centre is origin & radius is $r \Rightarrow x^2 + y^2 = r^2$

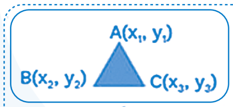
Distance between two points

- $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- If a point divides P and Q internally in the ratio m : n, the coordinates of R(x, y)
  $= \left[\dfrac{mx_2 + nx_1}{m + n}, \dfrac{my_2 + ny_1}{m + n}\right]$
- If a point divides P and Q externally in the ratio m : n, the co-ordinates of R(x, y)
  $= \left[\dfrac{mx_2 - nx_1}{m - n}, \dfrac{my_2 - ny_1}{m - n}\right]$
- Funda-1 : The x-axis divides the line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio of $y_1 : y_2$.
- Funda-2 : The y-axis divides the line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio of $x_1 : x_2$.

Equation of Line

- General form : $ax + by + c = 0$.
- Slope intercept form; $y = mx + c$.
- Point slope form: $(y - y_1) = m(x - x_1)$.
- Point slope form: $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- Intercept form : $\dfrac{x}{a} + \dfrac{y}{b} = 1$
  OR $bx + ay = ab$.
  Some conditions for lines:
  For parallel lines $\theta = 0°$; $m_1 = m_2$
- For perpendicular lines,
  $\theta = 90°$, $m_1 m_2 = -1$
- Acute angle between two lines with slope m_1 and m_2 $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

Triangle



- Incentre $\left\{\dfrac{ax_1 + bx_2 + cx_3}{a + b + c}, \dfrac{ay_1 + by_2 + cy_3}{a + b + c}\right\}$
- Centroid $\left\{\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right\}$
- Area $\dfrac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Distance of a point P from a line ax + by + c = 0

- $d = \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$
- From origin, $d = \left|\dfrac{c}{\sqrt{a^2 + b^2}}\right|$

Distance between Two parallel lines

$ax + by + c_1 = 0$ and $ax + by + c_2 = 0$

$\Rightarrow d = \left|\dfrac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

Funda : If we have three points of parallelogram, the fourth point is $\Rightarrow [(x_1 + x_3 - x_2), (y_1 + y_3 - y_2)]$