7 Permutations and Combinations

Introduction

Permutations and combinations is one of the most important areas of multiple MBA entrance examinations for two reasons. Firstly, solving questions from this chapter requires examinee's reasoning ability and secondly, solving questions from probability, number system, etc., requires in-depth knowledge of permutations and combinations.

In permutation, one studies the arrangement of a certain number of objects by taking some or all at a time.

In combination, one is not interested in the specific order of objects but interested in selecting a number of objects from the given objects.

As this topic requires an analytical and logical mind, students from non-mathematics backgrounds can efficiently handle this chapter.

Fundamental Principle of Counting

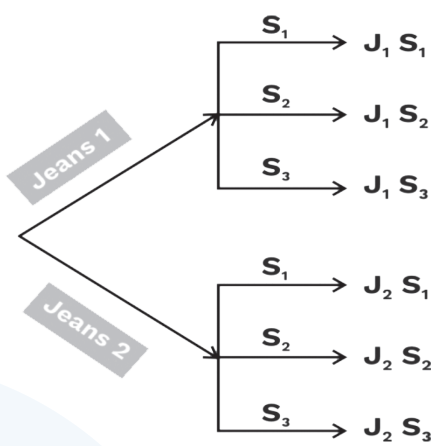
1. **Multiplication Theorem**

   Suppose there are 'm' ways to do a particular task and 'n' ways to do another task, then the total number of ways of doing both the tasks is "m × n".

   Let us understand this with the help of a suitable example.

   Assume that one has 3 shirts and 2 pair of jeans. The total number of ways in which one can decide what to wear is 3 × 2 = 6 ways

Pictorially



Rack Your Brain

There are 5 ways to go from A to B and 4 ways to go from B to C. In order to go from A to C, one has to pass through B. In how many ways one can go

(a) From A to C?

(b) From B to either A or C?

The total number of ways = 6 ways.

CAT Mantra

The formula "m × n" ways is valid if and only if the decisions are independent of each other.

Addition Theorem

Suppose there are 'm' ways to do a particular task and 'n' ways to do another task, then one can perform either of the two tasks (not both) in(m + n) ways.

Let us understand this with the help of a suitable example.

Assume that a person has 3 black shirts and 2 blue shirts. The total number of ways in which one can decide what to wear is 3 + 2 = 5 ways.

In this case, one cannot wear a black shirt and blue shirt at the same time.

Example 1:

You have 35 students in a class and you have to select two out of 35 students for the post of President and Vice-President. Find the total number of ways in which you can do that.

Solution:

The President can be any one of the 35 students, so there are 35 ways to fill the post of President. Vice President can be any of the remaining 34 students, so there are 34 ways to fill the post of Vice-President.

Total number of ways = 35 × 34 = 1,190 ways.

Example 2:

In how many ways can 5 prizes be given away to 6 boys when each boy is eligible for all the prizes?

Solution:

First prize can be given to any of the 6 boys. So, there are 6 ways to distributed first prize.

Again, second prize can also given to any of the 6 boys.

So, there are 6 ways to distribute the second prize.

Similarly, each of the third, fourth and fifth prize can also be distribute in 6 ways.

Total number of ways = 6 × 6 × 6 × 6 × 6 = 6^5 ways.

Example 3:

In a CAT exam, there are 3 sections QA, VARC and DILR containing 34, 34 and 32 questions, respectively. In how many ways can a candidate select one question from each of the three sections?

Solution:

The candidate can select one question from QA in 34 ways, one from VARC in 34 ways and one from DILR in 32 ways.

The total number of ways in which he can select one question from each of the three sections is 34 × 34 × 32 = 36,992 ways.

CAT Mantra

- If it is necessary to do all the tasks, or tasks are mutually inclusive i.e., both the tasks can happen at the same time, then one can use "MULTIPLICATION THEOREM".
- If all the tasks are mutually exclusive i.e., both the tasks cannot happen at the same time then one can use "ADDITION THEOREM".

Permutation

- The arrangement made by taking some or all of a number of items is called permutation. Permutation implies "arrangement where an order of the things is important".
- Permutation means both selection and arrangement, while combination means only selection.
- The permutation of three items P, Q and R, taken two at a time, is "PQ, QP, QR, RQ, PR and RP". Since the order in

which the items are taken is important, PQ and QP are counted as two different arrangements.

Derivation of nPr : The number of permutations of *n* things taking r at a time = ^nP_r

In this statement, one makes two assumptions:

1. All the n things are distinct.

2. Each of them is used at most once.

It is known that the number of Permutations of n different things taken r at a time is the same as the number of ways in which r places in a line can be filled up with n people.

One can fill up the first place in n different ways as any of the n people can be placed there. After filling up the first place in n ways, there are (n–1) different ways of filling up the second place, as any of the (n – 1) people can be placed there. Similarly, one can fill the third place in (n – 2) ways, as any of the remaining (n – 2) people can be placed there. Therefore, one can fill the three places taken together in n (n – 1) (n – 2) ways.

Proceeding this way, one sees that

- Whenever a place is filled up, a new factor is introduced.
- The factor begins with n and goes on diminishing by unity.

  ∴ r^{th} factor = n – (r – 1) = (n – r + 1)

  ∴ Number of ways of filling up r places = n (n – 1) (n – 2) ....(n – r + 1)

  ^nP_r = n(n – 1) (n – 2) ......(n – r + 1)

  Multiply both side by (n – r) ......3 × 2 × 1 one get,

(n – r) .... 3 × 2 × 1 × ^nP_r = n (n – 1) (n – 2) ...... (n – r + 1) × (n – r) ... 3 × 2 × 1

$$^nP_r = \frac{n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)\times(n-r)\ldots\ldots\ldots 3\times 2\times 1}{(n-r)\ldots\ldots\ldots 3\times 2\times 1}$$

$$^nP_r = \frac{n!}{(n-r)!}$$

If one takes n things at a time

Then one get

^nP_n = n(n – 1) (n – 2).......(n – n + 1)

= n (n – 1) (n – 2) 1

= n!

P_n = n!

However, if one substitute r = n in this formula for ^nP_n then one get $^nP_n = \frac{n!}{0!}$;

Since one already found that ^nP_n = n!

One can conclude that 0! = 1

Example 4:

How many 2-letter words can be made using the letters of the word HARMONY?

Solution:

In the word HARMONY

∴ Number of letters = 7

Number of places in which letters are to be placed = 2

∴ Required number of words = $^7P_2$

$$= \frac{7!}{(7-2)!} = \frac{7!}{5!} = 7 \times 6 = 42 \text{ words}$$

Example 5:

Eight chairs are numbered from 1 to 8. 2 men and 3 women wish to occupy one chair each. First, the men choose from chairs 1 to 4, and then the women select from the remaining chairs. Find the total number of possible arrangements.

Solution:

Since 2 men choose the chairs from chairs 1 to 4

∴ Number of arrangements = ${}^4P_2$ = 4 × 3 = 12.

Remaining Chairs = 8 – 2 = 6

∴ 3 women choose the chairs from the remaining 6 chairs.

Number of arrangements = ${}^6P_3$ = 6 × 5 × 4 = 120 ways

∴ Required number of arrangements = 12 × 120 = 1440.

Previous Years' Question

How many four-digit numbers, which are divisible by 6, can be formed using the digits 0, 2, 3, 4 and 6 such that no digit is used more than once and 0 does not occur in the left-most position?

Example 6:

How many words (with or without meaning) can be made from the letters of the word HARMONY, assuming that no letter is repeated, if:

(i) 3 letters are used at a time.

(ii) All letters are used, but the leftmost letter is a vowel.

Solution:

In the word HARMONY.

Number of letters = 7

(i) Number of letters to be taken at a time = 3

∴ required number of words = ${}^7P_3$

= 7 × 6 × 5 = 210

(ii) One can fill the first place with 2 vowels O and A in 2 ways.

Now remaining six places can be filled up with six letters in ${}^6P_6$ ways

Required number of words

= 2 × ${}^6P_6$ = 2 × 720 = 1,440.

Example 7:

In how many ways can 8 girls and 4 boys be seated in a row, such that no two boys are together?

Solution:

Let 8 girls be G_1, G_2, G_3, G_4, G_5, G_6, G_7 and G_8.

Let us arrange the girls first

Note : Always place the majority one first

____ G_1 ____ G_2 ____ G_3 ____ G_4 ____ G_5 ____ G_6 ____ G_7 ____ G_8 ____

∴ No two boys are together

∴ 4 Boys can be arranged in 9 "____" marked places in ${}^9P_4$ ways.

Also, 8 girls can be arranged among themselves in 8! ways.

∴ Required number of ways

= ${}^9P_4$ × 8! = 12, 19, 27, 680 ways.

Rack Your Brain

The English language contains 26 alphabets (21 consonants and 5 vowels). How many distinct words of up to 4 distinct letters can be formed?

Example 8:

How many different signals can be formed with 8 given flags of different colours?

Solution:

Number of flags = 8

A signal may form by hoisting any number of flags at a time.

Number of signals by hoisting one flag at a time = $^8P_1$= 8 signals.

Number of signals by hoisting two flags at a time

= $^8P_2$ = 8 × 7 = 56 signals.

Number of signals by hoisting three flags at a time

= $^8P_3$ = 8 × 7 × 6 = 336 signals.

Number of signals by hoisting four flags at a time

= $^8P_4$ = 8 × 7 × 6 × 5 = 1680 signals.

Number of signals by hoisting five flags at a time

= $^8P_5$ = 8 × 7 × 6 × 5 × 4 = 6720 signals.

Number of signals by hoisting six flags at a time

= $^8P_6$ = 8 × 7 × 6 × 5 × 4 × 3 = 20160 signals.

Number of signals by hoisting seven flags at a time

= $^8P_7$ = 8! = 40,320 signals.

Number of signals by hoisting eight signals at a time

= $^8P_8$ = 8! = 40,320 signals.

The total number of signals formed

= 8 + 56 + 336 + 1680 + 6720 + 20160 + 40320 + 40320 = 1,09,600

Permutation of n things taken r at a time can be divided into two groups

- Those which always include a particular thing.
- Those which do not contain that particular thing.

(a) Number of permutations which includes a particular thing

One is to fill up r places with n person under the condition that one particular person is definitely placed. That particular person can be seated in r different ways as he can be seated in any place. After he has been seated, remaining (r – 1) places can be filled up in $^{(n-1)}P_{(r-1)}$ **ways.**

∴ Number of Permutations containing a particular thing = r × $^{(n-1)}P_{(r-1)}$.

(b) Number of permutations which does not contain that Particular thing

∵ A particular thing is to be excluded.

∴ One has to fill up r places with (n – 1) persons.

∴ Number of permutations which does not contain a particular thing= $^{(n-1)}P_{(r)}$

CAT Mantra

One can conclude that,

$$\underset{\text{(Total Permutations)}}{^nP_r} = \underset{\text{(Include that thing)}}{r \times {}^{(n-1)}P_{(r-1)}} + \underset{\text{(Not include that thing)}}{^{(n-1)}P_r}$$

Permutation of n objects containing some repeated objects :

Let there be some objects (n) of which 'p' number of objects are of one kind, 'q' number of objects are of a second kind, 'r' number of objects are of the third kind and the rest are different, then the number of permutations of the n things taken all together will be equal to

$$\frac{n!}{P! \times q! \times r!}$$

Permutation of 'n' objects when an object can be placed repeatedly :

It is known that the number of permutations of n things taken 'r' at a time is the same as the number of ways in which r places can be filled up with n things.

The first place can be filled up in n ways as any of the n things can be placed there. After filling the first place, the second place can also be filled up in n ways as things can be repeated.

Proceeding in this way, one observes that the power of n is the same as the number of places filled up at this stage.

Therefore, the number of ways of filling up r places = n^r

∴ the required number of permutations = n^r.

Example 9:

How many ways can three prizes be distributed among five boys, when there is no restriction on the number of prizes a boy gets?

Solution:

First prize can be distributed in 5 ways, as it can be awarded to any boy.

Similarly, each of the remaining prizes can be awarded in 5 different ways.

∴ The required number of ways

$= 5 \times 5 \times 5 = (5)^3 = 125$ ways.

Example 10:

Find the number of permutations of the letters of the word "FATEHABAD".

Solution:

FATEHABAD

Total number of given letters = 9

Number of A's = 3

∴ the required number of permutations,

$$= \frac{9!}{3!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!} = 60{,}480 \text{ ways.}$$

Example 11:

Find the number of arrangements of the letters of the word OMNISCIENT. In how many of these arrangements:

(i) The word starts with M.

(ii) All the vowels always occur together?

Solution:

OMNISCIENT

Number of letters = 10

Number of N = 2

Number of I = 2

∴ the required number of arrangements

$$= \frac{10!}{2! \times 2!} = 9{,}07{,}200$$

(i) Fix M in the beginning

Remaining 9 letters can be arranged in

$$= \frac{9!}{2! \times 2!} = 90{,}720$$

(ii) Consider four vowels as one letter

∴ Letters can be arranged in $= \frac{7!}{2!}$

Also, four vowels (E, I, I, O) can be arranged among themselves in $\frac{4!}{2!}$ ways.

Number of arrangements = $\frac{7!}{2!} \times \frac{4!}{2!}$ = 30,240

Distribution of Objects

Questions asked from this section are basically of four types.

1. **Similar to Similar:** In this, we tend to distribute 'n' identical objects to 'm' identical places.
2. **Similar to Distinct:** In this, we tend to distribute 'n' identical objects to 'm' distinct places.
3. **Distinct to Similar:** In this, we tend to distribute 'n' distinct objects to 'm' identical places.
4. **Distinct to Distinct:** In this, we tend to distribute 'n' distinct objects to 'm' distinct places.

Similar to Similar

In this distribution, we find the number of ways we can write a natural number as a sum of smaller natural numbers.

E.g., P(5) = 5 = 4 + 1 = 3 + 2 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 1 + 1 + 1 = 1 + 1 + 1 + 1 + 1

But this method is inefficient in case of large distribution hence we use the formula stated below.

$$\frac{{}^{n+r-1}C_{r-1} - 3(a, a, b) - (a, a, a)}{3!} + (a, a, b) + (a, a, a)$$

Example 12:

In how many ways can we distribute 4 similar chocolates in 6 similar boxes?

Solution:

B	B	B	B	B	B	
4	0	0	0	0	0	= 1
3	1	0	0	0	0	= 1
2	2	0	0	0	0	= 1
2	1	1	0	0	0	= 1
1	1	1	1	0	0	= 1
					5 ways	

Example 13:

In how many ways 19 identical balls can be distributed in 3 identical boxes?

Solution:

B B B

A + B + C = 19

Step-1 :

Finding the cases of similar to distinct distribution.

${}^{n+r-1}C_{r-1} = {}^{19+3-1}C_{3-1} = {}^{21}C_2$

But here the boxes are similar

Cases possible

$$= \frac{{}^{21}C_2 - 3(a, a, b) - (a, a, a)}{3!} + (a, a, b) + (a, a, a)$$

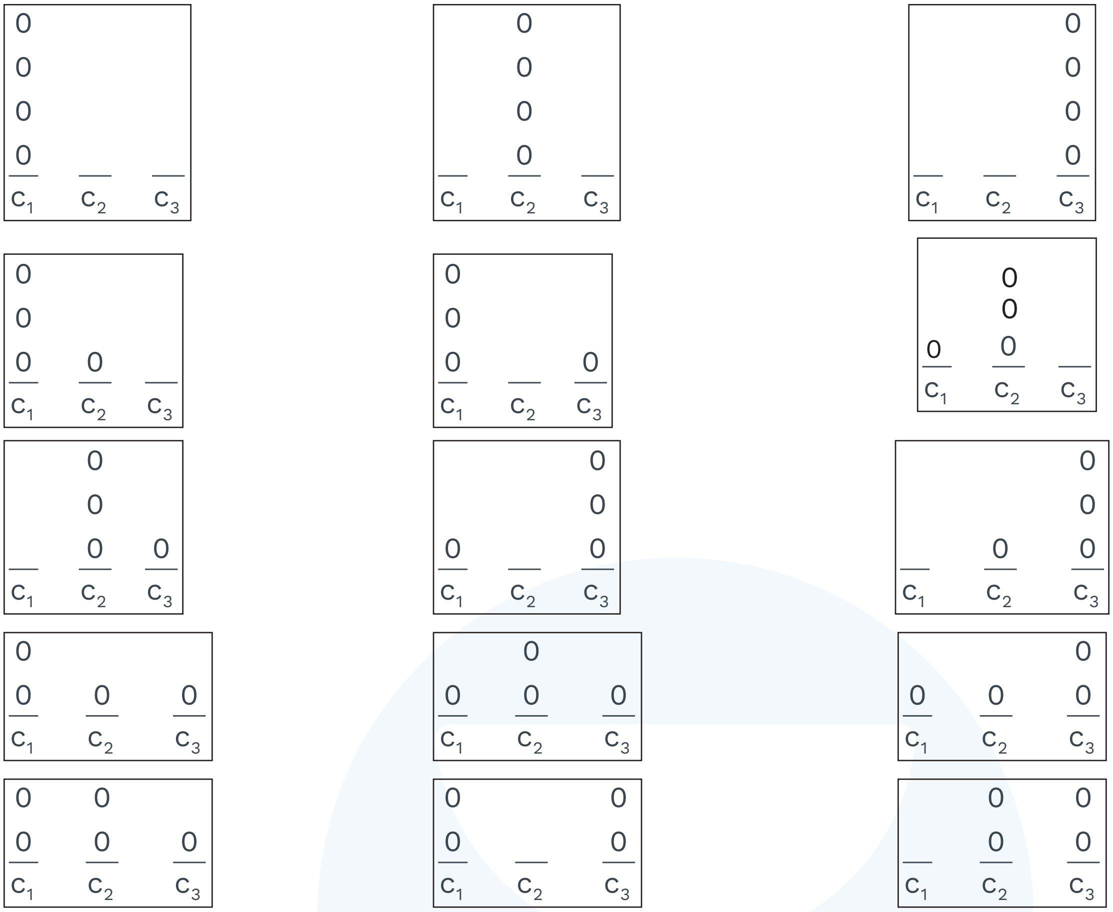
Where a, a, b means distribution of type (0, 0, 19) (1, 1, 17) ....(9, 9, 1) and (a, a, a) means distribution of type when a + a + a = 19 which is not possible in this case.

$$= \frac{210 - 10 \times 3 - 0}{3!} + 10 + 0$$

= 40 ways

Similar to Distinct

In how many ways can we distribute 4 toffees to 3 children?



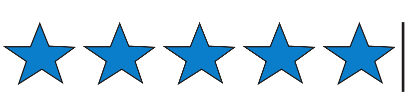
We can see that there are 15 ways by which we can make distributions of the toffees.

In the case of distributing similar objects to distinct places, counting every possibility is not very efficient. Hence, we discover a simple way of counting the number of possibilities.

Partition Method

Let us take five objects in a line, represented as stars here.

We can distribute them to different boxes by placing the bars either between them or at the ends.



Placing a bar creates two different segments. If we place in between it, make two groups, one of the objects on its left and the other on its right. If it's placed at the ends, it makes one single group.

Also, the number of bars decide how many groups will the object be divided into. E.g., placing three bars will create four groups.

By this, we conclude '(r – 1)' bars are required to make 'r' groups.

Finally, the arrangement of all the stars and bars decide the number of possibilities, i.e., for 'n' objects and 'r' groups.

Total possibilities will be ${}^{n+r-1}C_{r-1}$

Example 14:

In how many ways can we distribute 7 identical chocolates among 3 boys?

Solution:

Let the number of Toffees the three boys get be A, B and C.

Now

A + B + C = 7

Applying formula

${}^{n+r-1}C_{r-1} = {}^{7+3-1}C_{3-1} = {}^{9}C_{2} = 36$

Total number of ways in which the given chocolates can be distributed are 36.

Distinct to Distinct

For each object, we have to choose one out of the 'n' given choices. i.e., if we have r objects, then we have n choices for every object, so by multiplication principle, we get n^r possible arrangements.

For example, In how many ways 7 different chocolates can be distributed among 3 boys?

Solution : For each chocolate, we have three options, i.e., we can give it to b_1 or b_2 or b_3.

Total number of cases = 3 × 3 × 3 × 3 × 3 × 3 × 3 = 3^7

Usually, these questions come with certain modifications involving a specified grouping. A person has to give everyone a definite number of objects, like in the example explained below.

Example 15:

In how many ways can seven different chocolates be given to 3 boys such that each gets at least one?

Solution:

B_1	B_2	B_3	Selection				Arrangement		Total
5	1	1	${}^7C_5$	×			3!	=	126
4	2	1	${}^7C_4$	×	${}^3C_2$	×	3!	=	630
3	3	1	${}^7C_3$	×	${}^4C_3$	×	$\frac{3!}{2!}$	=	420
3	2	2	${}^7C_3$	×	${}^4C_2$	×	$\frac{3!}{2!}$	=	630

Total cases = 126 + 630 + 420 + 630 = 1806

Distinct to Similar

In this type of distribution, we have to select objects for distribution and there is no need for arrangement, since the boxes or places to which we distribute objects are similar.

Example 16:

In how many ways can we place seven distinct balls in 3 identical baskets.

Solution:

	B	B	B		
Case-1	7	0	0	${}^7C_7$	= 1
Case-2	6	1	0	${}^7C_6$	= 7
Case-3	5	2	0	${}^7C_5 \times {}^2C_2$	= 21

Case-4	5	1	1	${}^{7}C_{5} = 21$
Case-5	4	3	0	${}^{7}C_{4} \times {}^{3}C_{3} = 35$
Case-6	4	2	1	${}^{7}C_{4} \times {}^{3}C_{2} = 105$
Case-7	3	3	1	$\frac{{}^{7}C_{3} \times {}^{4}C_{3}}{2!} = 70$
Case-8	3	2	2	$\frac{{}^{7}C_{3} \times {}^{4}C_{2}}{2!} = 105$

Total cases = 1 + 7 + 21 + 21 + 35 + 105 + 70 + 105 = 365

Example 17:

In how many ways can we distribute 6 different chocolates to 4 identical boxes?

Solution:

	B	B	B	B	Selection
Case-1	6	0	0	0	${}^{6}C_{6} = 1$
Case-2	5	1	0	0	${}^{6}C_{5} = 6$
Case-3	4	2	0	0	${}^{6}C_{4} \times {}^{2}C_{2} = 15$
Case-4	4	1	1	0	${}^{6}C_{4} \times 1 = 15$
Case-5	3	3	0	0	$\frac{{}^{6}C_{3} \times {}^{3}C_{3}}{2} = 10$
Case-6	3	2	1	0	${}^{6}C_{3} \times {}^{3}C_{2} = 60$
Case-7	3	1	1	1	${}^{6}C_{3} = 20$
Case-8	2	2	2	0	$\frac{{}^{6}C_{2} \times {}^{4}C_{2}}{3!} = 15$
Case-9	2	2	1	1	$\frac{{}^{6}C_{2} \times {}^{4}C_{2}}{2!} = 45$

Total cases = 1 + 6 + 15 + 15 + 10 + 60 + 20 + 15 + 45 = 187

Note : Any number apart from 1 will cause symmetry.

Summary based on the above Explanation

		Objects	
		Distinct	Identical
Boxes	Distinct	r^{n}	${}^{n+r-1}C_{r-1}$
	Identical	Selection (r, n)	Breaking the number as a sum of small numbers

Circular Permutation

- It is known that the arrangement of objects in a line is known as linear permutation, but if one arranges them in the form of a circle i.e., closed-loop, such type of arrangement is known as circular permutation.

  When n persons are to be arranged in a straight line, one can do this in n! ways. But when n persons are sitting around a circular table, then there is no first person.

  Let us fix the position of one person, the remaining (n–1) person can now be arranged in the remaining (n – 1) places in ${}^{(n-1)}P_{(n-1)}$ i.e., (n–1)! ways.

Circular permutation can be divided into two types:

(i) Clockwise

(ii) Anti-clockwise

- In two such arrangements, each person has the same neighbours though in the reverse order and either of these arrangements can be obtained from the other by just over-turning the circle. If, in this case, there is no difference made between clockwise and anti-clockwise arrangements, then the two such arrangements are considered as only one distinct arrangement.

  Hence, the number of circular permutations in such cases = $\frac{(n-1)!}{2}$

Rack Your Brain

Find the number of ways in which 8 people can sit around an isosceles triangular table with sides having 3, 3 and 2 seats.

CAT Mantra

Questions on necklace with beads of different colours are to be tackled by the above formula. In this case, there is no difference between clockwise and anti-clockwise arrangements.

Example 18:

In how many ways can 7 beads of different colours form a necklace?

Solution:

Number of beads = 7

$\therefore$ required no. of ways = $\frac{(n-1)!}{2}$

$= \frac{(7-1)!}{2} = \frac{6!}{2}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2}$ = 360 ways

Example 19:

Five-persons A, B, C, D and E are to be seated around a circular table. In how many ways they can be seated?

Solution:

Number of person = 5

Number of ways of seating them around a circular table

= (5 – 1)! = 4!

= 4 × 3 × 2 × 1 = 24 ways.

Example 20:

In how many ways can 6 boys and 6 girls be seated around a round table, so that no two girls sit together?

Solution:

Let the boys be seated first, leaving one seat vacant between each of the two boys [∴ no two girls sit together]

This can be done in (6 – 1)! = 5! ways

Now 6 girls can be arranged in 6 vacant seats = ${}^6P_6$ i.e., 6! ways.

Total number of ways = 5! × 6! = 120 × 720

⇒ 86400 ways.

Previous Years' Question

A man, having \$2 in his pocket, goes to play his favourite game at a casino. If he wins, he gets \$2 whereas if he loses, he gets nothing. He plays the game multiple times and pays \$1 for each game as the entry fee. He does not lose more than once and leaves the casino as soon as he has \$4 in his pocket. How many different win-loss sequences are possible for him?

(A) 4 (B) 3

(C) 8 (D) 5

Some Problem based on Number System :

Example 21:

How many three-digit numbers can be formed using digit 1, 2, 3, 4, 5 and 6, if the repetition of digit is allowed?

Solution:

Repetition of digits means that the digit can be used any number of times.

Let the three digit number is abc.

Here, 'a' can take any value from 1, 2, 3, 4, 5 and 6. So, 'a' can be filled in 6 ways.

Since the repetition of digit be allowed, it means 'b' can also take any value from 1, 2, 3, 4, 5 and 6.

So 'b' can be filled in 6 ways.

Similarly, 'c' can take any value from 1, 2, 3, 4, 5 and 6. So 'c' can be filled in 6 ways.

Total number of ways = 6 × 6 × 6 = 216 ways.

Example 22:

How many four digits numbers can be formed using digits 0, 1, 2, 5, 6, 7, 8 if :

(a) Repetition of digits is allowed?

(b) Repetition of digits is not allowed?

Solution:

(a) Repetition of digit is allowed

In a four-digit number, '0' cannot be placed at thousand's place. So, the thousand's place can be filled in 6 ways (1, 2, 5, 6, 7, 8). Since the repetition is allowed and 0 can be filled at hundred's place, so the hundred's place can be filled in 7 ways (0, 1, 2, 5, 6, 7, 8).

Similarly, ten's and one's place each can also be filled in 7 ways

Total number of ways = 6 × 7 × 7 × 7 = 2058

(b) Repetition of digit is not allowed

Repetition is not allowed means a digit is used at most one time.

Here, '0' cannot be placed at thousands place. So, the thousand's place can be filled in 6 ways (1, 2, 5, 6, 7, 8). Since the repetition of digit is not allowed and 0 can be placed at hundred's place, hence the hundreds place can be filled in 6 ways.

Ten's place can be filled by the remaining 5 digits, so ten's place can be filled in 5 ways.

One's place can be filled by the remaining 4 digits, so one's place can be filled in 4 ways.

Total number of ways = 6 × 6 × 5 × 4 = 720 ways.

Example 23:

We have to make a number (multiple of twelve). We can use only 2 digits, which are 7 and 6. Also, the number should be of 9 digits. How many such numbers are possible?

(A) 49 (B) 28 (C) 42 (D) 35

Solution: (C) 42

For any number to be a multiple of 12, it must be divisible by 3 and 4.

So, for divisibility of any number by 4, the last 2 digits should be divisible by 4.

So by using 6 and 7, only possible case is of 76 as its last 2 digit.

Further, the digits 7 and 6 should be used such that the sum of the digits of the number formed has to be multiple of 3.

So possible cases are

CASE-1

When the number has 3 digits as 6 and 6 digits as 7, e.g. 777776676

Total number of ways is = $\frac{7!}{5! \times 2!}$ = 21 ways.

CASE-2

When the number has 3 digits as 7 and 6 digits as 6, e.g. 776666676

Total number of ways = $\frac{7!}{5! \times 2!}$ = 21 ways.

So, 21 cases with each pair means 42 possible number are there.

Example 24:

In how many ways 70 can be written as the sum of three natural numbers?

Solution:

Let the three numbers be x, y and z.

According to the question,

$x + y + z = 70$ where x, y and $z \geq 1$.

It means that x, y and z is at least 1.

Total number of ways = ${}^{n-1}C_{r-1}$

Here, n = 70 and r = 3

Required ways = ${}^{70-1}C_{3-1} = {}^{69}C_2$

$\frac{69 \times 68}{2} = 2346$ ways.

Example 25:

How many numbers from 0 to 999 will give the sum of digits as 17?

Solution:

In the exam, such types of question can be tackled by manually writing all the possible ways. There is no other method to reach to total number of ways.

One digit number doesn't give a sum of 17.

Possible cases for two digit number are

a b = 89 and 98 = 2 ways.

Possible cases for three digit number are

a b c

When c = 0, then a + b = 17

Possible cases are 890 and 980 = 2 ways

When c = 1, then a + b = 16

8 8 1 = $\frac{3!}{2!}$

9 7 1 = 3!

Total number of ways = 3 + 6 = 9 ways.

When c = 2, then a + b = 15

8 7 2 = 3!

9 6 2 = 3!

Total number of ways = 12 ways.

When c = 3, then a + b = 14

7 7 3 = $\frac{4!}{2!}$

8 6 3 = 3!

9 5 3 = 3!

Total number of ways = 15 ways.

When c = 4, then a + b = 13

7 6 4 = 3!

8 5 4 = 3!

9 4 4 = $\frac{4!}{2!}$

Total number of ways = 15 ways.

When c = 5, then a + b = 12

6 6 5 = $\frac{3!}{2!}$

7 5 5 = $\frac{3!}{2!}$

the Total number of ways = 6 ways.

Total number of ways = 2 + 2 + 9 + 12 + 15 + 15 + 6 = 61 ways.

Hence, from 0 to 999, there are 61 numbers whose sum of digits will be 17.

Example 26:

When 4 dice are thrown simultaneously, find total number of cases where the sum of outcomes equal to 19.

Solution:

Required combination for the sum being 19 are = (6, 6,6, 1) (6, 6, 5, 2) (6, 6, 4, 3) (6, 5, 5, 3) (6, 5, 4, 4) (5, 5, 5, 4)

Required number of arrangements for each case =

(6, 6, 6, 1) = $\frac{4!}{2!}$ = 4

(6, 6, 5, 2) = $\frac{4!}{2!}$ = 12

(6, 6, 4, 3) = $\frac{4!}{2!}$ = 12

(6, 5, 5, 3) = $\frac{4!}{2!}$ = 12

(6, 5, 4, 4) = $\frac{4!}{2!}$ = 12

(5, 5, 5, 4) = $\frac{4!}{3!}$ = 4

Total number of ways = 4 + 12 + 12 + 12 + 12 + 4 = 56 ways

Example 27:

In how many ways can four numbers be selected from first 25 consecutive natural numbers so that the selected numbers differs by at least three ?

Solution:

Let the four numbers are 1, 2, 3 and 4.

Gap between them A 1 B 2 C 3 D 4 E

There is a restriction over A and E that it must be ≥ 0.

Differ by at least 3 means there is a gap of 2 between the chosen numbers.

Restriction over B, C and D that it must be ≥ 0

A	+	B	+	C	+	D	+	E = 21
≥		≥		≥		≥		≥
0		2		2		2		0

So, A + B + C + D + E = 21 – 6 = 15

Total number of ways = ${}^{15+5-1}C_{5-1}$ = ${}^{19}C_4$ = 3,876 ways

Example 28:

In how many ways can 144 be written as a product of three natural numbers?

Solution:

144 can be written as a product of three numbers asA × B × C.

144 = A × B × C = $2^4 \times 3^2$

A = $2^a \times 3^x$

B = $2^b \times 3^y$

C = $2^c \times 3^z$

From there one can conclude that,

a + b + c = 4...........(1) (i.e. a, b, c ≥ 0)

x + y + z = 2...........(2) (i.e. x, y, z ≥ 0)

Number of ordered solutions of both the equations is:

$a + b + c = 4 = {}^{4+3-1}C_{3-1} = {}^{6}C_{2} = 15$

$x + y + z = 2 = {}^{2+3-1}C_{3-1} = {}^{4}C_{2} = 6$

Total number of ordered solutions

= 15 × 6 = 90

But here, it is given that “in how many ways” which means unordered solution.

Now, one should find the unordered solution out of 90 ordered solution.

Remove all the cases of (a, a, b) and (a, a, a) type

(a, a, b) = (a^2b) (i.e. write 144 in the product of perfect square and a natural number)

24 = 20, 21, 22, 23, 24 = 3 factors

32 = 30, 31, 32 = 2 factors

Total = 3 × 2 = 6

(a, a, a) = 0 (i.e. 144 is not a perfect cube)

$$\frac{90 - 3(a,a,b) - (a,a,a)}{6} + (a,a,b) + (a,a,a)$$

$$= \frac{90 - 18 - 0}{6} + 6 + 0$$

= 12 + 6 = 18

Hence, 144 can be written as the product of three numbers in 18 ways.

Combination

- The selection which can be made by selecting some or all of a number of items is called a combination. In a combination, one is not interested in arranging but only in selecting r objects from n objects. In fact, one does not want to specify the ordering of these selected objects.

  The combinations of three items P, Q and R, taken two at a time, are PQ, QR and PR. Hence, PQ and QP are not considered separately because the order in which P and Q are taken is not important but it is only required that a combination including P and Q is what is to be counted.

Derivation of ${}^{n}C_{r}$

Let the required number of combinations of n things taken r at a time be x.

∵ combination contains r things, which can be arranged among them in r! ways.

∴ Corresponding 1to each combination, one get r! permutation.

∴ Total number of permutations due to x combinations = $x \times r\,!$

But number of permutation of n things taken ‘r’ at a time = ${}^{n}P_{r}$

∴ It is given that,

$x \times r! = {}^{n}P_{r}$

$$x = \frac{{}^{n}P_{r}}{r!} \Rightarrow {}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} \Rightarrow \frac{n!}{(n-r)! \times r!}$$

Some Important Result

1. ${}^{n}C_{r} = {}^{n}C_{n-r}$

2. ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Division Into Groups

When (p + q) things can be divided into two groups containing p and q things respectively.

When p things are selected out of (p + q) things, then q things are automatically left behind and one gets the required division into two groups.

The required number of ways

$$= {}^{p+q}C_{p} = \frac{(p+q)!}{p! \times q!}$$

CASE-1

If one has to divide 2p things into two equal distinct groups, the required number of ways

$$= {}^{2p}C_p = \frac{2p!}{(p!)^2}$$

CASE 2:

If no distinction is made between the two groups, the groups will be interchanged in 2! ways without giving a new division.

$\therefore$ required number of ways $= \dfrac{2p!}{2! \times (p!)^2}$

- Distribution of distinct items into distinct groups (some groups may be empty)

Distribution of distinct items into distinct groups (some groups may be empty) Is given by $= r^n$

- Distribution of distinct items into distinct groups (arrangement within a group matters)

Distribution of n distinct items into r distinct group (arrangement within a group matter) is given by $= \dfrac{(n+r-1)!}{(r-1)!}$

This concept is also known as the ring and finger concept.

Example 29:

In how many ways you can distribute 6 rings in

(a) 5 boxes (b) 5 fingers

Solution:

There is a difference between distributing in a box and in a finger. In case of fingers, the order in which rings are placed matters. Let the six rings are (R_1, R_2, R_3,R_4,R_5, R_6).

In case of boxes:

R_1 can go in any of the five boxes, so it has five choices. R_2 can also go in any of the five boxes, so it also has five choices. Similarly, for R_3, R_4, R_5 and R_6, there are five choices each. Total number of ways

$$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6 (\ r^n)$$

In case of fingers:

R_1 can go in any of the five fingers, so it has 5 choices. R_2 can go in any of the five fingers but it has six choices (R_2 has two choices to go, above R_1 and below R_1), so the total number of choices for R_2 is 6.

R_3 can go in any of the five fingers, but it now has 7 choices.

R_4 can go in any of the five fingers, but it now has 8 choices.

R_5 can go in any of the five fingers, but it now has 9 choices.

R_6 can go in any of the five fingers, but it now has 10 choices.

So the total number of ways

$$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 1,\ 51{,}200 \text{ ways.}$$

Alternate Solution

One can directly use the formula to find the total number of ways

$$= \frac{(n+r-1)!}{(r-1)!} = \frac{(6+5-1)!}{(5-1)!} = \frac{10!}{4!}$$

$$= 1,51,200 \text{ ways.}$$

Total Number of Combinations

Let us understand this concept with the help of an example.

For example, find the total number of combinations of n different things by taking some or all at a time.

OR

Find the value of ${}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots + {}^nC_n$

- Here, each can be dealt in two ways, either it is included or excluded from a selection.

  $\therefore$ Total number of ways of dealing with n things = 2 × 2 × 2 × ... n times = 2^n

  But this way includes one case when all the things are excluded.

  Rejecting this case, the required number of ways = $2^n - 1$.

Total number of combinations of (a + b + c + ...) things by taking some or all at a time, when 'a' of them are alike and of one kind, 'b' of them are alike and of the second kind and so on.

- 'a' alike things can be dealt in (a + 1) ways.

Similarly, 'b' alike things can be dealt in (b + 1) ways and 'c' alike things can be dealt in (c + 1) way and so on.

Total number of ways of dealing with all the things = (a + 1) (b + 1) (c + 1) ....

But this ways include one case when all the things are excluded. Rejecting this case, the required number of ways = [(a + 1) (b + 1) (c + 1) ....] – 1

Example 30:

${}^{2n}C_3 : {}^nC_3 = 11 : 1$, Determine the value of n.

Solution:

$$\frac{{}^{2n}C_3}{{}^nC_3} = \frac{11}{1}$$

$$\Rightarrow \frac{\frac{(2n)(2n-1)(2n-2)}{3 \times 2 \times 1}}{\frac{n(n-1)(n-2)}{3 \times 2 \times 1}} = \frac{11}{1}$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = \frac{11}{1}$$

$$\therefore \frac{4(2n-1)}{n-2} = \frac{11}{1}$$

$\Rightarrow$ 8n – 4 = 11n – 22

$\Rightarrow$ 3n = 18

$\Rightarrow$ $\boxed{n = 6}$

Example 31:

In how many ways a committee of 2 members can be selected from 15 members?

Solution:

Total number of people = 15

(i) Number of people to be selected = 2

$\therefore$ required number of ways

$$= {}^{15}C_2 = \frac{15 \times 14 \times 13!}{13! \times 2!} = 105 \text{ ways}$$

Example 32:

A group consists of 5 girls and 8 boys. In how many ways can a team of 5 members be selected if the team has?

(i) No boys

(ii) At least 2 boys and 2 girls

Solution:

Number of boys = 8

Number of girls = 5

(i) No boys

We have to select 5 girls out of 5 girls

$= {}^5C_5 = 1$ ways

(ii) At least two boys and two girls

Different possibilities are:

(a) 3 boys 2 girls

(b) 2 boys 3 girls

Required no. of ways

$= {}^8C_3 \times {}^5C_2 + {}^8C_2 \times {}^5C_3$

= 56 × 10 + 28 × 10 = 560 + 280

= 840 ways.

Example 33:

In how many ways can 12 different balls be divided equally, according to the below conditions?

(i) Into three heaps

(ii) among 3 boys

Solution:

(i) Into three heaps

$\because$ no distinction is to be made in the identical groups.

$\therefore$ the required number of ways

$$= \frac{12!}{3! \times (4!)^3}$$

= 5775 ways.

(ii) Among three boys

Here, the groups are distinct

$\therefore$ the required number of ways

$$= \frac{12!}{4! \times 4! \times 4!}$$

= 34,650 ways

Previous Years' Question

In a tournament, there are 43 junior-level and 51 senior-level participants. Each pair of juniors plays one match. Each pair of seniors plays one match. There is no junior versus senior match. The number of girl versus girl matches in junior level is 153, while the number of boy versus boy matches in senior level is 276. The number of matches in which a boy plays against a girl is?

Rack Your Brain

In how many ways can we arrange 6 people P, Q, R, S, T and U in a straight line such that Q always comes in front of T, who comes in front of R?

Algebraic Properties

1. For a natural number n, the given equation is $a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_r = n$

a) If $a_i \geq 0$, the number of integral solutions are ${}^{(n+r-1)}C_{(r-1)}$.

It is also known as the distribution of similar items into distinct groups (some groups may be empty)

b) If $a_i \geq 1$, the number of integral solutions are ${}^{(n-1)}C_{(r-1)}$.

It is also known as the distribution of similar items into distinct groups (when no groups are empty)

2. For a natural number n, the given equation is $|a_1| + |a_2| + |a_3| + \ldots\ldots\ldots\ldots + |a_N| = N$

c) The total number of integral solutions of $|a| + |b| = n$ are $4n$.

d) The total number of integral solutions of $|a| + |b| + |c| = n$ are $4n^2 + 2$

e) The total number of integral solutions of $|a| + |b| + |c| + |d| = n$ are $\left(\frac{8n}{3}\right)(n^2 + 2)$

3. For a natural number n, number of terms in binomial and multinomial expression.

f) Total number of terms in $(a + b)^n = (n + 1)$

g) Total number of terms in $(a_1 + a_2 + \ldots + a_n)^m = {}^{(m+n-1)}C_{(n-1)}$.

Previous Years' Question

In how many ways can 8 identical pens be distributed among Amal, Bimal, and Kamal so that Amal gets at least 1 pen, Bimal gets at least 2 pens, and Kamal gets at least 3 pens?

Example 34:

Find the number of ways in which you can distribute all the 12 identical chocolates among the 4 kids.

Solution:

As one has to distribute the chocolates among 4 kids.

Let the first kid receives 'a' number of chocolates, the second kid receives 'b' number of chocolates, the third kid receives 'c' number of chocolates and the fourth kid receives 'd' number of chocolates.

The algebraic equation would be (a + b + c + d) = 12

n = 12 and r = 4

Required number of solutions = ${}^{(n+r-1)}C_{(r-1)}$

$= {}^{15}C_3$

$= \frac{15!}{12! \times 3!} \Rightarrow \frac{15 \times 14 \times 13}{3 \times 2}$

= 35 × 13 = 455 ways.

Example 35:

Find the number of ways in which you can distribute 12 identical chocolates among 3 boys such that each of them must get at least two chocolates.

Solution:

Here, each child gets at least two chocolates.

Let's distribute 2 chocolates to each child first.

Remaining chocolates

= 12 − 3 × 2 = 6 Chocolates

So, the required algebraic equation would be a + b + c = 6 where (a, b, c ≥ 0).

n = 6 and r = 3

Required number of ways = ${}^{n+r-1}C_{r-1}$

$= {}^{8}C_2 \Rightarrow \frac{8!}{6! \times 2!}$

= 28 ways.

Example 36:

Find the total number of terms in the expansion $(a + b + c)^{12}$.

Solution:

Total number of terms in the expansion

$(a + b + c)^m = {}^{m+n-1}C_{n-1}$

$(a + b + c)^{12}$, where m = 12 and n = 3

Total number of terms = ${}^{(12+3-1)}C_{(3-1)}$

$= {}^{14}C_2 = \frac{14!}{12! \times 2!} = 91$ terms

Geometrical Properties

1. In a plane if there are n points of which no three are collinear, then:
   - **a)** The number of straight lines formed by joining them = ${}^{n}C_2$
   - **b)** The number of triangles formed by joining them = ${}^{n}C_3$
   - **c)** The number of polygons of k sides formed by joining them = ${}^{n}C_k$
2. In a plane if there are n points out of which exactly m points are collinear, then :
   - **a)** The number of straight lines formed by joining them = ${}^{n}C_2 - {}^{m}C_2 + 1$.

b) The number of triangles formed by joining them = ${}^nC_3 - {}^mC_3$.

c) The number of polygons of k sides formed by joining them = ${}^nC_k - {}^mC_k$.

3. If n straight lines are intersecting a circle out of which no two lines are parallel and no three lines are concurrent, then the maximum number of parts/region into which these lines divide the circle is

$$= \frac{n(n+1)}{2} + 1$$

D. Number of rectangle/Square

(1) Total number of squares in a square having n column and n rows.

$$= 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots n^2 = \frac{n(n+1)(2n+1)}{6}$$

(2) Total number of rectangles in a square having n columns and n rows.

$$= 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(3) Total number of squares in a rectangle having 'm' columns and 'n' rows

$= m \times n + (m-1) \times (n-1) + (m-2) \times (n-2) + \ldots\ldots\ldots\ldots + 0$

(4) Total number of rectangles in a rectangle having m columns and n rows.

$= (1 + 2 + 3 + \ldots\ldots\ldots\ldots\ldots + m) \times (1 + 2 + 3 + \ldots\ldots\ldots\ldots n)$

E. Number of Triangles

1. Number of triangles formed by joining the 3 vertices of an n-sided polygon:

$$N = {}^nC_3 = \frac{n(n-1)(n-2)}{6}; \text{ for all } n \geq 3$$

2. Number of triangles that have one side common with that of polygon

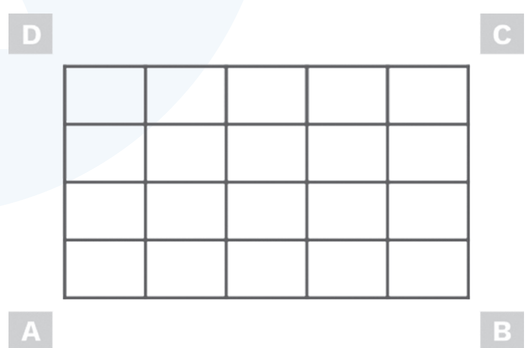
$N_1 = n(n-4)$; for all $n \geq 3$

- Number of triangles that have two sides common with that of the polygon $N_2 = n$; for all $n \geq 3$
- Number of triangles, so that at least one side of the triangle coincides with the side of the polygon = n (n-3)
- Number of triangles having no side common with that of the polygon.

$$= \frac{n(n-4)(n-5)}{6}; \text{ for all } n \geq 6$$

Paths and Grids

- A grid or network is formed when 'm' horizontal lines intersect 'n' vertical lines.
- In the following figure a grid of 4 × 5 dimensions is shown in which there are 5 horizontal lines and 6 vertical lines intersect each other. Using these lines as Road one can go from one corner to another one.



- Assume that one has to go from A to C using the shortest path without backtracking.

Out of every possible way, one must have to cover 5 horizontal steps and 4 vertical steps. It implies that one has to find out that how many ways one can select 5 horizontal steps and 4 vertical steps out of a total of9 steps.

The total number of ways,

$$= \frac{(H+V)!}{H! \times V!} = \frac{9!}{5! \times 4!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 24}$$

The total number of ways = 126

Example 37:

There are 10 points in a plane out of which 3 are collinear find the total number of:

(i) Straight line formed by joining these points.

(ii) Triangle formed by joining these points.

Solution:

(i) Total number of straight lines formed

$= {}^{10}C_2 - {}^{3}C_2 + 1$

$= \frac{10 \times 9}{2} - \frac{3 \times 2}{2} + 1$

45 – 3 + 1 = 43 lines

(ii) Total number of triangles formed

$= {}^{10}C_3 - {}^{3}C_3$

$= \frac{10 \times 9 \times 8}{6} - 1$

= 119 triangles

Example 38:

Find the number of squares on a regular chessboard.

Solution:

A regular chessboard is an

8 × 8 square board.

By using direct formula

$$= \frac{n(n+1)(2n+1)}{6}$$

Where n = 8

$$\frac{8(8+1)(2 \times 8 \times 1)}{6} = \frac{8 \times 9 \times 17}{6}$$

= 204 squares

Example 39:

The number of triangles that can be formed with 10 points as vertices, n of them being collinear, is 110. Find the value of n.

(A) 3 (B) 4 (C) 6 (D) 5

Solution:

According to the question

${}^{10}C_3 - {}^{n}C_3 = 110$

$$\frac{10!}{7! \times 3!} - \frac{n!}{(n-3)! \times 3!} = 110$$

$$\frac{n(n-1)(n-2)}{6} = 10$$

n (n – 1) (n – 2) = 60

n (n – 1) (n – 2) = 5 × 4 × 3

$\boxed{n = 5}$

Option (D) is the correct answer.

DE-Arrangements

De-arrangement of objects means that none of the objects occupies its original place. It implies that if ‘n’ distinct items are arranged in a row, then the number of ways they can be rearranged such that none of them occupies its original position is given by

$$= n!\left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} - \ldots\ldots + \frac{(-1)^n}{n!}\right)$$

CAT Mantra

1. De-arrangement of 1 object is not possible.
2. De-arrangements of 2 objects are = 1
3. De-arrangements of 3 objects are = 2
4. De-arrangements of 4 objects are = 9
5. De-arrangements of 5 objects are = 44
6. De-arrangements of 6 objects are = 265

Example 40:

Find the number of ways in which all the 7 different letters are placed in 7 addressed envelopes, so that all the letters are in the wrong envelopes.

Solution:

By using direct formula

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\dots\dots+(-1)^n\frac{1}{n!}\right]$$

Therefore, the required number of ways

$$=7!\left[1-\frac{1}{1}+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}+\frac{1}{720}-\frac{1}{5040}\right]$$

$$=5040\left[\frac{2520-840+210-42+7-1}{5040}\right]$$

$$=5040\times\frac{1854}{5040}=1854\ \text{ways}.$$

Ranking of a Word

To find the rank of a word out of all possibilities using all the letters given in the word is nothing but the extension of the concept of alphabetically arranging the words in a dictionary.

Let's understand this concept with the help of an example.

Example 41:

All the letters of the word 'QUESTION' are arranged in all possible ways. What will be the rank of the word QUESTION?

Solution:

Alphabetical order of occurrence of letters → E, I, N, O, Q, S, T, U

The number of words starting with E = 7!

The number of words starting with I = 7!

The number of words starting with N = 7!

The number of words starting with O = 7!

All the words starting with QE = 6!

All the words starting with QI = 6!

All the words starting with QN = 6!

All the words starting with QO = 6!

All the words starting with QS = 6!

All the words starting with QT = 6!

All the words starting with QUEI = 4!

All the words starting with QUEN = 4!

All the words starting with QUEO = 4!

All the words starting with QUESI = 3!

All the words starting with QUESN = 3!

All the words starting with QUESO = 3!

Second last word = QUESTINO = 1 word

Last word = QUESTION = 1

Total number of ways = 4 × 7! + 6 × 6! + 3 × 4! + 3 × 3! + 2 = 24,572

Hence, the rank of the word 'QUESTION' is 24,572.

Previous Years' Question

India fielded 'n' (> 3) bowlers in a test match, and they operated in pairs. If a particular bowler did not bowl in pair with at least two other bowlers in the team, then at most how many bowlers could have bowled in pair with every other bowler in the team?

(A) n – 3 (B) n – 1

(C) n – 2 (D) None of these

Practice Exercise – 1

Level of difficulty – 1

1. How many words can be formed using all the letters of the word "PRACTICE"?

2. A five-digit number is formed using 2, 3, 4, 5, 6, 7, 8. How many numbers can be formed which are same when we read from left to right or right to left?

3. How many signals can be made by Hoisting 2 identical Pink, 2 identical Brown and 5 identical Black flags on a pole at the same time?

(A) 343

(B) 756

(C) 1,512

(D) Cannot be determined

4. In how many ways can 6 boys be seated in 7 chairs?

(A) 5,700

(B) 5,720

(C) 5,360

(D) 5,040

5. A girl has to climb 12 steps. She climbs in either a single step or 2 steps simultaneously. In how many ways she can climb the 12 steps?

Level of difficulty – 2

6. There are 6 copies of a book of subject A, 5 copies of a book of subject B and 4 copies of a book of subject C. Find the number of ways in which one or more books can be selected.

7. Find the sum of all numbers that can be formed by taking all the digits at a time from 2, 3, 4, 5, and 6 without repetition.

8. In how many ways can a team of 12 students consisting of 8 boys and 4 girls be selected from 15 boys and 10 girls, if a particular boy A and a particular girl B are never together in the team?

(A) ${}^{14}C_8 \times {}^{9}C_4$

(B) ${}^{15}C_8 \times {}^{10}C_4 - {}^{14}C_8 \times {}^{9}C_4$

(C) ${}^{15}C_8 \times {}^{10}C_4 - {}^{14}C_7 \times {}^{9}C_3$

(D) ${}^{14}C_1 \times {}^{9}C_3$

9. Find the number of triangles whose vertices are at the vertices of an nonagon but none of the sides of such triangle are taken from the sides of the nonagon.

10. Seven people amongst whom Amal, Bimal and Chaman are to speak at a function. The number of ways in which it can be done if Amal wants to speak before Bimal and Bimal wants to speak before Chaman is?

(A) $\frac{7!}{6}$

(B) $\frac{7!}{4}$

(C) $\frac{7!}{3!}$

(D) ${}^{7}C_3$

Level of difficulty – 3

11. Nine states in a country are divided into three-zones. The telephone department of the country intends to connect the states with telephone lines such that every two states in the same zone are connected with four direct lines and every two states in different zones are connected with two direct lines. How many direct lines are required if it is given that there are 3 states per zone?

12. There are three groups of people A, B and C. Group A has 8 people and group B has 6 people. Each people in one group shake hands with each people in the other groups exactly once. No person in any group shook hands with any other person in that group. The total number of handshakes is 216. Find the number of people that group C has:

13. Six friends, Aman, Bimal, Chaman, Dheeraj, Eminem and Faizu came to the marriage of Abdul's sister. He welcomed all his friends and assigned 6 tasks (1 task to each). Abdul assigns six tasks to them in such a way that task 1 cannot be assigned either to Aman or to Bimal; task 2 must be assigned to either Chaman or Dheeraj. In how many ways can the assignment be done?

(A) 276

(B) 140

(C) 144

(D) 196

14. Two numbers are chosen from 1, 3, 5, 7...161 and multiplied together. Find the number of ways that will give us the product as a multiple of 3 if it is given that two numbers can be the same.

15. A person invites his 8 friends to a party and places 4 of them at one round table, and 4 on the other round table. Find the total number of arrangements in which he can arrange the guests.

(A) $\frac{8!}{4!}$

(B) $\frac{8!}{16}$

(C) $\frac{8!}{8}$

(D) $8! \times 4! \times 4!$

Solutions

1. **20,160**

"PRACTICE" has a total of 8 letters with 2C, 1P, 1R, 1A, 1T, 1I, 1E.

The number of words that can be formed

$= \frac{8!}{2!} = 20,160$

2. **343**

A five-digit number that remains the same when read from left to right or right to left will be in the form of A B C B A.

A can take any value out of (2, 3, 4, 5, 6, 7, 8), so the first position can be filled in 7 ways,

B can also take any value out of (2, 3, 4, 5, 6, 7, 8), so the second position can also be filled in 7 ways.

C can also take any value out of (2, 3, 4, 5, 6, 7, 8), so the third position can also be filled in 7 ways.

Each number has the same extremes as well as the same 2^{nd} and 4^{th} digits.

Total such numbers $= 7 \times 7 \times 7 \times 1 \times 1$

= 343 ways.

3. **(B) 756**

Pink flags = 2

Brown flags = 2

Black flags = 5

Total flags = 9 flags

This question says that Permutation of total 9 object, where 2, 2 and 5 are similar.

Total number of signals = $\frac{9!}{2! \times 2! \times 5!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{2 \times 2 \times 5!} = 42 \times 18 = 756$ signals

4. (D) 5,040

6 boys can be seated in 7 chairs in ${}^7P_6$ ways.

${}^7P_6 = \frac{7!}{1!} = 7! = 5040$

5. 233

Single Step	Double step	Number of ways
12	0	$\frac{12!}{12!} = 1$
10	1	$\frac{11!}{10!} = 11$
8	2	$\frac{10!}{8! \times 2!} = 45$
6	3	$\frac{9!}{6! \times 3!} = 84$
4	4	$\frac{8!}{4! \times 4!} = 70$ ways
2	5	$\frac{7!}{2! \times 5!} = 21$ ways
0	6	$\frac{6!}{6!} = 1$ ways

The total number of ways: 1 + 11 + 45 + 84 + 70 + 21 + 1 = 233 ways.

6. 209

The 6 copies of subject A can be dealt in 7 ways i.e., (do not select at all or select 1 or 2 or 3 or 4 or 5 or all 6 copies = 7 ways).

Similarly copies of subject B can be selected in 6 ways and copies of subject C can be selected in 5 ways.

Total ways = 7 × 6 × 5 = 210 ways

But this 210 ways include the one case in which no book is selected.

Hence, the required number of ways = 210 – 1 = 209 ways.

7. 53, 33, 280

It is given that the sum of all numbers that can be formed by using all the digitsof a, b, c, d and e is given by = (n – 1)! × (a + b + c + d + e) × (11111)

$\Rightarrow$ (5 – 1)! × (2 + 3 + 4 + 5 + 6) × 11111

$\Rightarrow$ 4! × 20 × 11111

$\Rightarrow$ 53, 33, 280

8. (C) ${}^{15}C_8 \times {}^{10}C_4 - {}^{14}C_7 \times {}^9C_3$

Total number of ways of selecting 8 boys and 4 girls from 15 boys and10 girls is ${}^{15}C_8 \times {}^{10}C_4$.

Number of ways when A and B are together in the team

$= {}^{14}C_7 \times {}^9C_3$ ways

Number of ways when A and B are not together in the team

$= ({}^{15}C_8 \times {}^{10}C_4 - {}^{14}C_7 \times {}^9C_3)$ ways.

9. 30

Total number of triangles formed by the vertices of a nonagon

$= {}^9C_3$

= 84 triangles

Number of triangles having one side common with the nonagon

= n (n – 4)

= 9 (9 – 4)

= 45 triangles

Number of triangles having two sides common with the nonagon

= n = 9 triangles

Required number of triangles

= 84 – 45 – 9 = 30

10. (A) $\frac{7!}{6}$

Order of speaking is equivalent to the order of placing.

Thus, Amal, Bimal and Chaman can be placed in $^7C_3$ ways and the remaining 4 people can be placed in 4! Ways.

Hence, the number of ways in which they can speak is

$^7C_3 \times 4! = \frac{7!}{4! \times 3!} \times 4! = \frac{7!}{3!} = \frac{7!}{6}$

11. 90

Let the three-zone be Z_1, Z_2 and Z_3.

Each zone contains 3 states.

One can select any two states from the same zone in $^3C_2$ ways and they are connected with 4 direct lines.

∴ The total number of lines that are used in any one zone to connect any two states is $^3C_2 \times 4 = 12$ lines.

∴ the total number of lines that are used in all three zones to connect any two states (internally) = 12 × 3 = 36 lines

Now, two zones can be selected from 3 zones in $^3C_2$ ways and one state from each zone can be selected in $^3C_1 \times {^3C_1}$ ways.

These states are connected by two direct lines.

Total number of lines = $^3C_2 \times {^3C_1} \times {^3C_1} \times 2$

= 54 lines

Total number of lines used = 36 + 54

= 90 lines

12. 12

Let the number of People that group C has be x.

The number of handshakes exchanged between group A to group B,group B to group C, and group A to group C is 216.

$^8C_1 \times {^6C_1} + {^6C_1} \times {^xC_1} + {^xC_1} \times {^8C_1} = 216$

$8 \times 6 + 6 \times x + x \times 8 = 216$

14x = 168

$x = \frac{168}{14} = 12$

Hence, there are 12 people in group C.

13. (C) 144

Suppose there are six tasks T_1, T_2, T_3, T_4, T_5, T_6 and six people Aman (A), Bimal (B) , Chaman (C) , Dheeraj (D), Eminem (E) and Faizu (F).

Case I:

If T_2 is assigned to Chaman (C)

Now, one has 5 tasks and 5 people.

T_1 can't be assigned to Aman on Bimal. So T_1 could be assigned to Dheeraj, Eminem and Faizu (3 ways)

T_3 could be assigned to each of the 4 remaining people.

T_4 could be assigned to each of the 3 remaining people.

T_5 could be assigned to each of the 2 remaining people.

T_6 could be assigned to the only remaining person.

Total number of ways = 3 × 4 × 3 × 2 × 1 = 72 ways.

Case II:

(If T_2 is assigned to Dheeraj).

The result is the same as the first case

Total number of ways = 3 × 4 × 3 × 2 × 1 = 72 ways.

Required number of ways = 2 × 72 = 144 ways.

14. 21,87

In this given number 1, 3, 5, 7 ..... 159, 161

The numbers which are multiples of 3 are 3, 9, 15, 21 159

These numbers are in an arithmetic progression

Where a = 3, d = 9 − 3 = 6 and T_n = 159

$T_n = a + (n - 1)\, d$

$159 = 3 + (n - 1) \times 6$

$n - 1 = 26$

$\boxed{n = 27}$

Total number of terms in the given sequence

1, 3, 5, 7....., 159, 161 is ‘x’, then

$161 = 1 + (x - 1) \times 2$

$x - 1 = 80$

$\boxed{x = 81 \text{ terms}}$

Total number ways to get a multiple of 3

= 27 × 81 ⇒2187 ways.

15. (B) $\frac{8!}{16}$

A person arranges his 8 friends at 2 round tables.

So first, he has to select 4 friends from 8 friends in $^8C_4$ ways and these4 friends have seated at a round table in 3! ways.

The remaining 4 friends have seated at the 2nd round table in 3! ways.

The total number of possible ways

$= {}^8C_4 \times 3! \times 3!$

$= \frac{8!}{4! \times 4!} \times 3! \times 3! = \frac{8!}{16}$

Practice Exercise – 2

Level of Difficulty – 1

1. The value of the expression ${}^{57}C_4$ + is:

(A) ${}^{63}C_3$

(B) ${}^{62}C_4$

(C) ${}^{57}C_5$

(D) ${}^{62}C_5$

2. Find the number of ways in which the alphabets of "SALIENT" can be arranged such that no two vowels are adjacent to each other.

(A) 72

(B) 720

(C) 1,440

(D) 360

3. A reputed paint company plans to award prizes to its top three salespersons, with the highest prize going to the top salesperson, the next highest prize to the next salesperson and a smaller prize to the third-ranking salesperson. If the company has 15 salespersons, how many different arrangements of winners are possible (Assume there are no ties)?

(A) 1,728

(B) 2,730

(C) 3,421

(D) 2,678

4. Eight points lie on the circumference of a circle. The difference between the number of triangles and the number of quadrilaterals that can be formed by connecting these points are?

(A) 7

(B) 14

(C) 32

(D) 84

5. How many positive integers less than 10^5 can be written only with the digits 7 and 9? (Repetition is allowed)

(A) 46

(B) 69

(C) 62

(D) 54

6. A row contains 5 blue balls of different shades and 4 red balls of identical shades. Find the number of ways of arranging them in a row, so that no two red balls are together?

(A) 5!

(B) (5!) x ${}^{6}C_4$

(C) 5! x 4!

(D) None of these

7. How many four-digits numbers are there whose sum of digits is 8?

(A) 37

(B) 120

(C) 165

(D) 200

8. Fifteen girls are to be seated around a circular table for a group discussion. However, it is ensured that Anushka does not sit next to Madhuri and Tabbu must sit next to Aishwarya. In how many ways can they be seated?

(A) 26 × 11!

(B) 22 × 12!

(C) 24 × 12!

(D) 32 × 11!

9. In how many different ways, can the letters of the words "ALIEN" be arranged so that all the vowels are never together?

(A) 14

(B) 75

(C) 84

(D) 94

10. There are 11 apples to be distributed among Aman, Naman and Chaman. If Aman gets at least 3 apples, Naman gets at least two apples and Chaman gets at least one apple, then in how many ways can the apples be distributed?

(A) 24

(B) 23

(C) 21

(D) 20

Level of Difficulty – 2

11. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 28$, then the value of n is:

12. Given that N is an even integer, such that the sum of the digits of N is 2 and $10^7 < N < 10^8$. How many values of N are possible?

(A) 6

(B) 7

(C) 8

(D) 9

13. N = "aabbccddd" is a nine-digit number, where a, b, c & d are distinct digits. Also, a and d are even digits, while b & c are odd digits. If all the digits of N are rearranged to form nine-digit numbers such that the odd digits occupy even positions, then how many such numbers are formed?

(A) 40

(B) 50

(C) 55

(D) 60

14. Twenty points are there on a plane of which 8 points are collinear. How many quadrilaterals can be formed by joining any 4 points?

(A) ${}^{20}C_4$

(B) ${}^{20}C_4 - {}^{8}C_4$

(C) 4,103

(D) 4,106

15. The number of ways in which 37 identical bottles can be distributed among three girls such that each of them receives odd number of bottles is?

(A) 172

(B) 136

(C) 151

(D) 171

16. In how many ways can 14 identical pencils be distributed among 5 kids in such a way that each kid gets at least 2 pencils, but nobody gets more than 5 pencils?

(A) 70

(B) 65

(C) 56

(D) 51

17. A family has a father, a mother, and their six children - A, B, C, D, E, and F. They have to attend a party, for which they decide that only one among the father or the mother will attend, along with any number of children, subject to the following constraints:

(A) 55

(B) 60

(C) 65

(D) 70

18. We have to make a number (multiple of twelve). We can use only 2 digits, which are 5 and 6. Also, the number should be of 10 digits. How many such numbers are possible?

(A) 56

(B) 57

(C) 84

(D) 85

19. A question paper has 5 multiple choice questions and 6 true/false questions. Each multiple-choice question is provided with 3 choices. Find the number of ways in which a student can attempt at least one question of each kind of the given 11 questions:

(A) $3^5 \times 2^6$

(B) $4^5 \times 3^6$

(C) $(4^5 - 1)(3^6 - 1)$

(D) $(4^5 \times 3^6 - 1)$

20. A furniture company assigns a two-digit code with distinct digits to each of its wardrobes using the digits 0, 1, 2, ...9 such that the first digit of the code is not zero. However, the code printed on paper creates confusion when read upside down. The digits 1, 6, 8 and 9 can create confusion, for example, the code 18 may appear as 81. How many codes are there for which no such confusion can arise?

(A) 60

(B) 65

(C) 71

(D) 80

Level of Difficulty – 3

21. How many words can be formed by taking 5 letters at a time out of the letters of the word "PREPLADDER"?

(A) 30,240

(B) 3,840

(C) 3,120

(D) 1,440

22. How many natural numbers (greater than or equal to 10) with distinct digits are there, such that their digits are in descending order from left to right?

23. How many numbers can we form with the help of the digits 3, 4, 5, 6 and 9, where repetition of digits is not allowed and are divisible by 3?

(A) 220

(B) 223

(C) 227

(D) 230

24. In a cricket match, if a batsman can score 0, 2, 4 or 6 runs in a ball, then find the number of different sequences in which he can score exactly 26 runs in an over. (Assume that an over consists of only 6 balls and there were no extras, no run outs and also the batsman has played all the 6 balls)

25. At a farewell party of a certain class, having X students, each student of the class gave a gift to exactly Y other students of the class. The number of students in the class who received at least 1 gift cannot be less than?

(A) X – 1

(B) X – Y

(C) Y + 1

(D) Y – 1

26. Out of 15 objects, 5 are exactly identical to each other and the rest are distinct. The number of ways we can choose 5 objects out of a total 15 objects are?

27. How many 4 letter words can be formed from the letters of the word "MAHARAJA".

28. Find the number of natural number solutions for inequation a + b + c ≤ 10

29. In how many ways can 16 identical and indistinguishable balls be divided into three groups?

30. Rohan wants to open a suitcase, but it has a lock on, and he has forgotten the password for it. As far as he remembers, the password is an odd number between 80 and 420, and he also knows that the digits of the password are from the set {1, 2, 3, 4, 5}. Find the maximum number of tries he has to take to unlock the lock.

(A) 48

(B) 40

(C) 60

(D) 64

Solutions

1. **(B)**

Given $^{57}C_4$ +

$= {}^{57}C_4 + {}^{61}C_3 + {}^{60}C_3 + {}^{59}C_3 + {}^{58}C_3 + {}^{57}C_3$

$= ({}^{57}C_4 + {}^{57}C_3) + {}^{58}C_3 + {}^{59}C_3 + {}^{60}C_3 + {}^{61}C_3$

Using, $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$= ({}^{58}C_4 + {}^{58}C_3) + {}^{59}C_3 + {}^{60}C_3 + {}^{61}C_3$

$= ({}^{59}C_4 + {}^{59}C_3) + {}^{60}C_3 + {}^{61}C_3$

$= ({}^{60}C_4 + {}^{60}C_3) + {}^{61}C_3$

$= ({}^{61}C_4 + {}^{61}C_3)$

$= {}^{62}C_4$

Hence, option (B) is the correct answer.

2. **(C)**

First, we will place 4 consonants

___S___L___N___T______

This can be done in 4! ways = 24 ways

Now remaining 3 vowels can be arranged in 3 places (out of 5 places available) in $^5C_3$ x 3! = 10 x 6 = 60 ways

Total number of ways = 24 × 60 = 1,440

Hence, option (C) is the correct answer.

3. **(B)**

A company has 15 salespersons.

Only three salespersons get the prizes.

So, total number of arrangements of winner = $^{15}C_3$ × 3!

= 15 × 14 × 13 = 2,730

Hence, option (B) is the correct answer.

4. **(B)**

Difference between the number of triangles and the number of quadrilaterals that can be formed by connecting eight points which lie on the circumference of circle = $^8C_4 - {}^8C_3$ = 70 − 56 = 14

Hence, option (B) is the correct answer.

5. **(C)**

The number of 1-digit numbers is 2

The number of 2-digit numbers is 2^2

The number of 3-digit numbers is 2^3

The number of 4-digit numbers is 2^4

The number of 5-digit numbers is 2^5

Total = 62

Hence, option (C) is the correct answer.

6. **(B)**

5 blue balls can be arranged in 5! ways.

____ B1___B2___ B3____B4____B5____

Now there are 6 gaps, in these 6 gaps 4 red balls can be arranged in $^6C_4$ ways (there is no arrangement among red balls, as they are all identical)

Required number of ways = 5! X $^6C_4$

Hence, option (B) is the correct answer.

7. **(B)**

Let 'abcd' be a four-digit number.

Now according to the question:

a + b + c + d = 8

But 'a' can't be zero (0).

Therefore, let a = a' + 1

Now, (a' + 1) + (b) + (c) + (d) = 8

a' + b + c + d = 7

Whole numbers solutions

$= {}^{n+r-1}C_{r-1} = {}^{7+4-1}C_{4-1}$

$= {}^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 120$

Hence, option (B) is the correct answer.

8. **(B)**

The number of ways in which Tabbu and Aishwarya are together = 13! × 2!

The number of ways in which Tabbu and Aishwarya are together and Anushka and Madhuri are also together = 12! × 2! × 2!

The number of ways in which Tabbu and Aishwarya are together, but Anushka and Madhuri are not together

= 13! × 2! - 12! × 2! × 2!

= 13 × 12! × 2! – 12! × 2! × 2!

= 12! (26 – 4) = 12! × 22

Hence, option (B) is the correct answer.

9. **(C)**

We will find the number of ways when vowels are always together

Taking the vowels (AIE) as one letter, the given word has the letters LN(AIE) i.e. 3 letters

These letters can be arranged in

3! = 6 ways

The letters AIE can be arranged among themselves in 3! Ways = 6 ways

Thus, the number of arrangements having all the vowels together

= 6 × 6 = 36 ways

The total number of ways in which letters of the word 'ALIEN' can be arranged is

5! = 120 ways

Therefore, the number of arrangements not having vowels together

= (120 – 36) = 84 ways

Hence, option (C) is the correct answer.

10. **(C)**

Let us assume Aman gets 3 + x apples and Naman gets 2 + y apples and Chaman gets 1 + z apples

3 + x + 2 + y + 1 + z = 11

x + y + z = 11 – 6 = 5 (x, y, z being whole numbers)

No. of ways of distribution of n items among r persons = $^{n+r-1}C_{r-1}$

$\therefore$ Required value

$= {}^{5+3-1}C_{3-1} = {}^7C_2 = 21$ ways.

Hence, option (C) is the correct answer.

11. **(8)**

Given: $T_n = {}^nC_3$, $T_{n+1} = {}^{n+1}C_3$

$\therefore \quad T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 28$ [Given]

$\Rightarrow \quad {}^nC_2 + {}^nC_3 - {}^nC_3 = 28$ [Since,

${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$]

$\Rightarrow \quad {}^nC_2 = 28$

$\Rightarrow \quad \frac{n!}{2!\times(n-2)!} = 28$

$\Rightarrow \quad \frac{n(n-1)\,(n-2)!}{2\times(n-2)!} = 28$

$\Rightarrow \quad \frac{n(n-1)}{2} = 28$

$\Rightarrow$ $n^2 - n - 56 = 0$

$\Rightarrow$ $n^2 - 8n + 7n - 56 = 0$

$\Rightarrow$ $(n - 8)(n + 7) = 0$

$\Rightarrow$ $n = 8$ [Because the number of sides of a regular polygon is never negative.]

Hence, "8" is the correct answer.

12. (B)

The digit at the left most place would be either 1 or 2 and also the digit at unit place cannot be 1 as then the number will become an odd number.

Valid possible numbers will be

1. 10,000,010
2. 10,000,100
3. 10,001,000
4. 10,010,000
5. 10,100,000
6. 11,000,000
7. 20,000,000

Hence, option (B) is the correct answer.

13. (D)

The given number has 4 odd digits and 5 even digits.

A 9-digit integer will have 5 odd positions and 4 even positions.

The 4 odd digits can be arranged in the 4 even positions in $\frac{4!}{2! \times 2!} = 6$ ways.

The 5 even digits can be arranged in the 5 odd positions in $\frac{5!}{2! \times 3!} = 10$ ways.

Thus, the total number of numbers formed is $6 \times 10 = 60$.

Hence, option (D) is the correct answer.

14. (C)

This can be done in the following ways:

1. If 4 points of 12 non collinear points are selected, then number of quadrilaterals $= {}^{12}C_4 = 495$
2. If 3 points of 12 non collinear points is selected and 1 point from 8 collinear point is selected, then number of quadrilaterals $= {}^{12}C_3 \times {}^{8}C_1 = 220 \times 8 = 1{,}760$
3. If 2 points of 12 non collinear points is selected and 2 points from 8 collinear point is selected, then number of quadrilaterals $= {}^{12}C_2 \times {}^{8}C_2 = 1{,}848$

Total number of quadrilaterals

$= 495 + 1760 + 1848 = 4{,}103$

Hence, option (C) is the correct answer.

15. (D)

Let $2k + 1$, $2m+1$, $2p+1$ be the numbers of bottles received by each of the three girls.

$2k+ 1 + 2m + 1 + 2p + 1$

$= 37$ $[0 \leq k, m, p \leq 17]$

$2k + 2m + 2p = 34$

$K + m + p = 17$

Required number of ways

$= {}^{17+3-1}C_{3-1} = {}^{19}C_2 = 171.$

Hence, option (D) is the correct answer.

16. (B)

First, give 2 pencils to each of the 5 kids. We can give that in 1 way as all pencils are identical.

Now the number of ways in which the remaining 4 identical pencils can be distributed among 5 kids is the same as the whole number of solutions of

$A + B + C + D + E = 4$

$$\Rightarrow \quad 8_{C_4} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4!} = 70$$

So, there are 70 ways to distribute 14 identical pencils among 5 kids.

But we have to subtract those cases in which any kids get more than 5 pencils.

There will be 5 cases. When any 1 kid gets 6 pencils and the other 4 gets 2 pencils each.

So valid cases = 70 – 5 = 65.

Hence, option (B) is the correct answer.

17. (D)

CASE 1: If the father attends the party.

Each child can either attend the party or not attend the party.

i.e., each child has 2 options.

Therefore, for 6 children the total number of ways we get

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2) = 2^6 = 64$ ways.

CASE 2: If the mother attends the party.

The total numbers of ways = $(2 \times 2 \times 2 \times 2) = 16$ (as B or C won't attend the party with their mother)

Given, F doesn't go when none of A or D attends.

Since F does not go with B, C, E.

8 ways should be removed from 1st case (i.e., FB, FC, FE, FBC, FCE, FBE, FBCE and F)

2 ways should be removed from 2nd case as F will not go with E (i.e., FE, F)

Required number of ways

$= 64 + 16 - 8 - 2 = 70$

Hence, option (D) is the correct answer.

18. (D)

For any number to be a multiple of 12, it has to be divisible by 3 and 4 both.

So, for divisibility of any number by 4, the last 2 digits should be divisible by 4.

So, by using 5 and 6, the only possible case is of 56 as its last 2 digits.

Further the digits 5 and 6 should be used such that the sum of the digits of the number formed has to be multiple of 3.

So possible cases are:

CASE 1:

When the number have 3 digits as 5, and 7 digits as 6, for example, 6665656656

Total numbers of such form

CASE 2:

When the number have 4 digits as 6, and 6 digits as 5, for example, 6555656556

Total numbers of such form

CASE 2:

When the number have 9 digits as 5, and 1 digit as 6, for example, 5555555556

Total numbers of such form

So, a total of 28 + 56 + 1 = 85, possible numbers are there.

Hence, option (D) is the correct answer.

19. (C)

There are 4 ways of dealing with each multiple-choice question, i.e., 3 ways of attempting and 1 way of not attempting.

No. of ways of attempting at least 1 multiple choice question = $(4^5 - 1)$

There are 3 ways of dealing with each true false, i.e., 2 ways of attempting and 1 way of not attempting.

No. of ways of attempting at least 1 true-false = $(3^6 - 1)$

Total number of ways = $(4^5 - 1) \times (3^6 - 1)$

Hence, option (C) is the correct answer.

20. (C)

As the first digit of the code is not '0' then the possible number of codes = $9 \times 9 = 81$ codes.

The digits which create confusion are 1, 6, 8 and 9. The number of codes that can be formed out of these digits

$= 4 \times 3 = 12$ codes.

But out of these 12 codes, 69 and 96 do not cause confusion.

Thus only 10 of the possible 81 codes cause confusion.

The other 71 do not cause any confusion.

Hence, option (C) is the correct answer.

21. (B)

The word contains 10 letters with repetition of letters. We can choose 5 letters out of 10 in the following ways:

(a) All the 5 letters are distinct

(b) 3 distinct and 2 alike

(c) 2 alike of one kind, 2 alike of other kind and 1 distinct

All the 5 letters are distinct:

There are 6 distinct letters P, R, E, D, L and A out of which 5 can be chosen by $^6C_5$ ways.

Number of words formed in this case

$= {}^6C_5 \times 5! = 6 \times 120 = 720$

3 distinct and 2 alike:

There are 4 types of 2 alike i.e., PP, RR, EE and DD out of which one alike can be chosen in $^4C_1$ ways and the remaining 3 distinct can be chosen in $^5C_3$ ways.

So, total number of ways to select 3 distinct and 2 alike letters

$= {}^4C_1 \times {}^5C_3 = 40$

Number of words formed in this case

$= 40 \times (5! / 2!) = 2{,}400$

2 alike of one kind, 2 alike of other kind and 1 distinct:

There are 4 types of 2 alike i.e., PP, RR, EE and DD out of which two alike can be chosen in $^4C_2$ ways and rest 1 distinct can be chosen in $^4C_1$ ways

So, total number of ways to select 2 alike of one kind, 2 alike of other kind and 1 distinct $= {}^4C_2 \times {}^4C_1 = 24$

Number of words formed in this case

$$= 24 \times \left\{\frac{5!}{(2! \times 2!)}\right\} = 720$$

Total words formed = 720 + 2,400 + 720 = 3,840

Hence, option (B) is the correct answer.

22. 1,013

Whenever we select any number of digits out of 10 digits, then there is only one arrangement of digits where digits are in descending order from left to right.

For example, select any 3 digits out of 10 digits, let's say 3, 8 and 5, then 853 is the only number where digits are in descending order

Number of 2-digit numbers = $^{10}C_2$

Number of 3-digit numbers = $^{10}C_3$

Number of 4-digit numbers = $^{10}C_4$

And so, on till

Number of 10-digit numbers = $^{10}C_{10}$

Therefore, total numbers

$= {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + \ldots\ldots {}^{10}C_{10}$

$= ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \ldots {}^{10}C_{10}) - ({}^{10}C_0 + {}^{10}C_1)$

$= 2^{10} - 1 - 10 = 1{,}013$

Hence, "1013" is the correct answer.

23. (C)

We know that, for a number which is divisible by 3, its sum of digits must be divisible by 3.

One-digit Number:

Only 3, 6 and 9 is possible = 3 ways

Two-digit Numbers:

Case 1: When '4' and '5' both are included

$\underline{\boxed{4}}$ $\underline{\boxed{5}}$

Also, the sum of digits will remain the same on different arrangements of digits.

Therefore, total 2 digits number

$= 2! = 2$ ways

Case 2: When '4' and '5', both are excluded, we can choose 2 numbers out of 3 choices and then we can arrange them too.

Therefore, total ways $= {}^3C_2 \times 2! = 6$ ways

Total two-digit numbers $= 6 + 2 = 8$

Three-digit Numbers:

Case 1: When '4' and '5', both are included.

$\boxed{4}$ $\boxed{5}$ $\downarrow$

3 possibilities
$(3,6,9) = {}^3C_1$

Also sum will remain constant on different arrangement

Therefore, total 3-digit number

$= {}^3C_1 \times 3! = 3 \times 6 = 18$

Case 2: When '4' and '5', both are excluded

We can choose three numbers out of 3 choices and then arrange them.

Therefore, total ways $= {}^3C_3 \times 3! = 6$

Total 3-digit numbers $= 18 + 6 = 24$

Four-digit Numbers:

The only possible way is two include both '4' and '5' and then choose any two digits from 3, 6 and 9.

Therefore, total ways $= {}^3C_2 \times 4! = 3 \times 24 = 72$

Five- digit Numbers:

All the 5 digits can be arranged at 5 places in 5! Ways

Total 5-digit number $= 5! = 120$

Therefore, total possible number

$= 3 + 8 + 24 + 72 + 120 = 227.$

Hence, option (C) is the correct answer.

24. 216

26 runs in 6 balls can be scored in the following ways

6, 6, 6, 6, 2, 0 ---- total arrangements

$= \dfrac{6!}{4!} = 30$

6, 6, 6, 4, 4, 0 ---- total arrangements

$= \dfrac{6!}{(3! \times 2!)} = 60$

6, 6, 6, 4, 2, 2 ---- total arrangements

$= \dfrac{6!}{(3! \times 2!)} = 60$

6, 6, 4, 4, 4, 2 ---- total arrangements

$= \dfrac{6!}{(3! \times 2!)} = 60$

6, 4, 4, 4, 4, 4 ---- total arrangements

$= \dfrac{6!}{5!} = 6$

Hence, total number of ways to score 26 as per given conditions

$= 30 + 60 + 60 + 60 + 6 = 216$

Hence, "216" is the correct answer.

25. (C)

Rather than variables, it would be better if we solve this question by assumptions

Let's take X = 10 and Y = 1

If each of the 9 students gives their gift to the 10th student and the 10th student gives his gift to the 1st student, there will be 2 students (1^{st} and 10^{th}) who will receive all the gifts and remaining 8 students will have no gift. So, in this case there will be 2 students (Y + 1) who are receiving gifts.

Let's take X = 10 and Y = 2

If each of the 8 students gives their gift to 9^{th} and 10^{th} student, 9^{th} student gives his gift to 10^{th} & 1^{st} student and 10^{th} student gives his gift to 9^{th} & 1^{st} student, then there will be 3 students (1^{st}, 9^{th}, and 10^{th}) who will receive all the gifts and remaining 7 students will have no gift. So, in this case there will be 3 students (Y + 1) who are receiving gifts

Hence, whatever be the value of X and Y, our answer would be (Y + 1).
Hence, option (C) is the correct answer.

26. 638

We can choose all 5 identical objects and none of the distinct objects in $^{10}C_0 = 1$ way
We can choose all 4 identical objects and 1 distinct object in $^{10}C_1 = 10$ ways
We can choose all 3 identical objects and 2 distinct objects in $^{10}C_2 = 45$ ways
We can choose all 2 identical objects and 3 distinct objects in $^{10}C_3 = 120$ ways
We can choose all 1 identical object and 4 distinct objects in $^{10}C_4 = 210$ ways
We can choose all 0 identical objects and 5 distinct objects in $^{10}C_5 = 252$ ways
So, total number of ways of selecting 5 objects out of 15 objects
$= 1 + 10 + 45 + 120 + 210 + 252 = 638$
Hence, "638" is the correct answer.

27. 209

Case 1: all the 4 letters are distinct
There are 5 distinct letters M, H, R, J and A in the word "MAHARAJA".
So, 4 letters out of these 5 can be selected in $^5C_4 = 5$ ways
Words formed from these selected 4 distinct letters = 4!
Total words formed = 5 × 4! = 120
Case 2: 2 letters are same (AA) and 2 letters are distinct
2 same letters would be AA and they can be selected in 1 way
2 distinct out of 4 remaining 4 distinct letters (M, H, R and J) can be selected in $^4C_2 = 6$ ways.
Words formed from these 4 selected letters $= 6 \times \left(\frac{4!}{2!}\right) = 72$
Case 3: 3 letters are same (AAA) and 1 letter is distinct
3 same letters would be A, A and A and they can be selected in 1 way
1 distinct letter out of 4 remaining 4 distinct letters (M, H, R and J) can be selected in $^4C_1 = 4$ ways.
Words formed from these 4 selected letters $= 4 \times \left(\frac{4!}{3!}\right) = 16$
Case 4: all 4 letters are same (AAAA)
4 same letters A, A, A and A can be selected in 1 way
Words formed from these 4 same selected letters = 1
Total words formed = 120 + 72 + 16 + 1 = 209. Hence, "209" is the correct answer.

28. 120

We know that natural number solutions for the expression
$a_1 + a_2 + a_3 + a_4 + \dots\dots\dots + a_r = n$ is given by $^{n-1}C_{r-1}$.
Now, solutions for $a + b + c = 10 \Rightarrow {}^9C_2$
solutions for $a + b + c = 9 \Rightarrow {}^8C_2$
solutions for $a + b + c = 8 \Rightarrow {}^7C_2$
solutions for $a + b + c = 7 \Rightarrow {}^6C_2$
solutions for $a + b + c = 6 \Rightarrow {}^5C_2$
solutions for $a + b + c = 5 \Rightarrow {}^4C_2$
solutions for $a + b + c = 4 \Rightarrow {}^3C_2$
solutions for $a + b + c = 3 \Rightarrow {}^2C_2$
Therefore, total number of solutions
$= {}^9C_2 + {}^8C_2 + {}^7C_2 + {}^6C_2 + {}^5C_2 + {}^4C_2 + {}^3C_2 + {}^2C_2$
$= 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 120$
Hence, "120" is the correct answer.

29. 21

If one group has 1 ball, other 2 groups can have 15 balls in 7 ways as follows:

(1,14), (2,13), (3,12), (4,11), (5,10), (6,9) and (7,8) --- **7 cases**

(Remember that order does not matter while putting balls in groups as (1, 1, 14) is same as (1, 14, 1) and (14, 1, 1) as all there have same meaning that two groups each having 1 ball and 1 group having 14 balls)

If one group has 2 balls, other 2 groups can have 14 balls in 6 ways as follows:

(2,12), (3,11), (4,10), (5,9), (6,8) and (7,7) --- **6 cases**

If one group has 3 balls, other 2 groups can have 13 balls in 4 ways as follows:

(3,10), (4,9), (5,8) and (6,7) --- **4 cases**

If the one group has 4 balls, other 2 groups can have 12 balls in 3 ways as follows:

(4,8) (5,7) and (6, 6) --- **3 cases**

If one group has 5 balls, other 2 groups can have 11 balls in 1 way as follows:

(5,6) --- **1 case**

So, total cases = 7 + 6 + 4 + 3 + 1 = 21

Hence, "21" is the correct answer.

30. (A)

Case 1: digit number: we cannot choose 8 as it is not from the given set (Since, the number is greater than 80). The unit place can be filled in 3 ways. Either 1 or 3 or 5 (as the number is an odd number). Any two-digit code is not possible.

Case 2: digit numbers: the number less than 400 - 0 cannot be in the hundred's place. So, the hundreds' place can be filled in 3 ways (either 1 or 2 or 3). Any digit can occupy the tenth place. Therefore, it can be filled in 5 ways. The unit place can be filled in 3 ways. Either 1 or 3 or 5. (as the number is an odd number.) Therefore, number of ways

= 3 x 5 x 3= 45.

Case 3: A number greater than 400 but less than 420 - only 4 can come in the hundreds' place. At the tenth place, we have only on choice i.e., digit 1

Number should be less than 420. can be filled in only 1 way

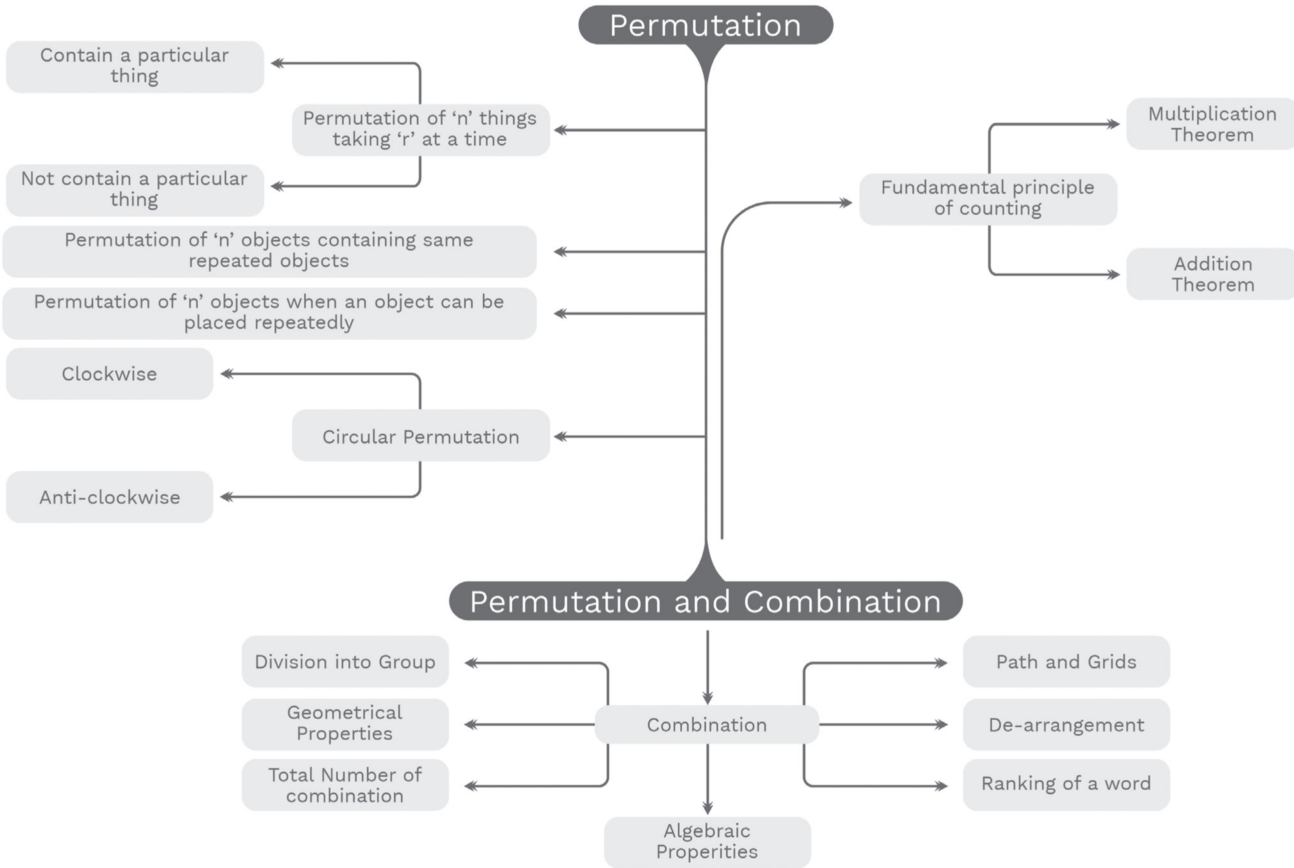
The unit's place can be filled in three ways.

The number of ways = 1 x 1 x 3 = 3

Total number of ways = 0 + 45 + 3 = 48

Hence, option (A) is the correct answer.

MIND MAP



Permutation
Contain a particular thing
Permutation of 'n' things taking 'r' at a time
Not contain a particular thing
Permutation of 'n' objects containing same repeated objects
Permutation of 'n' objects when an object can be placed repeatedly
Clockwise
Circular Permutation
Anti-clockwise
Fundamental principle of counting
Multiplication Theorem
Addition Theorem
Permutation and Combination
Combination
Division into Group
Geometrical Properties
Total Number of combination
Path and Grids
De-arrangement
Ranking of a word
Algebraic Properities