Introduction

Average is the mean value which can be calculated simply by dividing the sum of all the values in a set by the total number of values or observations. it is also known as 'mean value' or 'Arithmetic Mean'. Different concepts of averages can be used in different situations and contexts. One or two questions based on the application of averages are asked in CAT and other management entrance exams.

$$\text{Average} = \frac{\text{Sum of quantities}}{\text{Number of quantities}}$$

$$A = \bar{x} = \text{A.M.} = \frac{\sum x}{n}$$

Definitions

Average is the sum of a set of values which are divided by the number of values in the set. Average is also called as arithmetic Mean.

Types of Means

Pythagorean Means

The combination of arithmetic mean (A.M.), geometric mean (G.M.) and harmonic mean (H.M.) are collectively called the classical Pythagorean means.

Arithmetic Mean

The arithmetic mean, also known as the average, or simply the mean, is the sum of a

Key Points

- Average lies between the maximum and minimum value. It will be equal to the maximum or minimum value, if all the values are equal.
- Average is the result of the net surplus + net deficit.
- If the value of each quantity is increased or decreased by the same quantity "q", then the resultant average will also increase or decrease by the quantity "q" respectively.
- If the value of each quantity is multiplied or divided by a quantity "q", then the resultant average also will also be multiplied or divided by "q" respectively.

set of numbers divided by the total count of numbers in the set.

If n numbers are given, each number denoted by a_i (where i = 1, 2, ...n), the arithmetic mean would be

$$\text{A.M.} = \frac{1}{n} \sum_{i=1}^{n} a_i = \frac{a_1 + a_2 + ... + a_n}{n}$$

Example 1:

The average of 1,000, 2,000, 3,000, 4,000, ... 10,000 is 5,500. If each number is divided by 10, then the new average will be equal to:

(A) 450   (B) 1000

(C) 45   (D) None of these

Solution: (D)

Actual average

$$= \frac{(1000 + 2000 + 3000 + \dots 10000)}{10} = 5500$$

The new average = 5500/10 = 550.

Example 2:

The average age of 10 students of a class is 22 years, but if we include the age of a principal and a teacher, then the average age becomes 27 years. It is given that principal's present age is 6 years more than the teacher's present age. Find the present age of teacher (in years).

(A) 49   (B) 55

(C) 59   (D) None of these

Solution: (A)

Total age of 10 Students = 10 × 22 = 220 years

Total age of 12 members (including Principal and Teacher) = 12 × 27 = 324 years

Total age of principal and teacher

= 324 − 220 = 104 years

According to the Question:

Age of Principal = Age of Teacher+ 6

104 − Age of Teacher = Age of Teacher + 6

Age of Teacher = $\dfrac{104 - 6}{2}$

Age of Teacher = $\dfrac{98}{2} = 49$ years

Hence, option (A) is the correct answer.

Example 3:

The average of eight numbers is 20. If the sum of the two numbers is 31, the average of the next three numbers is $21\dfrac{1}{3}$ and the seventh and eighth number exceed the sixth number by 4 and 7 respectively, then find the average of the seventh and eighth numbers.

(A) 22.5   (B) 24.5

(C) 23.5   (D) 24

Solution: (C)

Let the sixth number be 'x'.

So, seventh number = (x + 4)

And eighth number = (x + 7)

$$\text{Average} = \frac{\text{Sum of eight numbers}}{8}$$

$$20 = \frac{\left(31 + \left(21\frac{1}{3} \times 3\right) + x + (x + 4) + (x + 7)\right)}{8}$$

160 = 31 + 64 + 11 + 3x

160 = 106 + 3x

3x = 54

Rack Your Brain

Consider an office of 45 employees, whose average age is 40 years. 'x' new employees join this office, whose average age is 'y' years. If it is known that × + y = 50, then what can be the maximum possible average age of all the employees now?

Rack Your Brain

A set of numbers from 1 to 36 (both inclusive) are split into 6 groups of 6 numbers each. The medians of these 6 groups are P, Q, R, S, T and U. If the average of these medians is x, then what are the smallest and the largest values × can take?

$\therefore \quad x = 18$

So required average

$$= \frac{x + 4 + x + 7}{2} = \frac{(2x + 11)}{2}$$

$$= \frac{2 \times 18 + 11}{2} = \frac{47}{2} = 23.5$$

Geometric Mean

The geometric mean is widely discussed in Sequences and series, we are going to learn the basic application of the geometric mean here in this chapter. Geometric Mean of n positive numbers is obtained by multiplying them all together and then taking its nth root.

In algebraic terms, the geometric mean of $a_1, a_2 \ldots a_n$ is defined as:

$$\text{G.M.} = \sqrt[n]{a_1 \times a_2 \times a_3 \ldots a_n}$$

For example: Geometric mean of 4 and 16 is

$$= \sqrt{4 \times 16} = 8$$

Example 4:

Find the G.M. of the values 15, 20, 5 and 40.

(A) $10(6)^{1/4}$ (B) $4(6)^{1/4}$

(C) 45   (D) None of these

Solution: (A)

Given: 15, 20, 5 and 40

We know that,

$$\text{G.M.} = \sqrt[4]{15 \times 20 \times 5 \times 40}$$

$$= \sqrt[4]{2^4 \times 5^4 \times 6} = \sqrt[4]{10^4 \times 6} = 10\sqrt[4]{6}$$

Therefore, the geometric mean $= 10(6)^{1/4}$

Example 5:

Find the ratio of G.M. and A.M. of the values 8, 20, n and 40, if their G.M. is 20.

(A) $\frac{93}{80}$ (B) $\frac{16}{25}$

(C) $\frac{80}{93}$ (D) None of these

Solution: (C)

Given: 8, 20, n, and 40:

We know that,

$$\text{G.M.} = \sqrt[4]{8 \times 20 \times n \times 40}$$

$$\text{G.M.} = \sqrt[4]{2 \times 2 \times 2 \times 2 \times 2 \times 5 \times n \times 5 \times 2 \times 2 \times 2}$$

$$= \sqrt[4]{2^8 \times 5^2 \times n}$$

$$20 = 4\sqrt[4]{5 \times n \times 5}$$

$$5 = \sqrt[4]{5 \times n \times 5}$$

$$625 = 25 \times n$$

$$\therefore n = 25$$

A.M. will be $= \frac{93}{4}$

$$\frac{\text{G.M.}}{\text{A.M.}} = \frac{20 \times 4}{93} = \frac{80}{93}$$

Harmonic Mean

Harmonic mean for a non-empty collection of numbers $a_1, a_2 \ldots a_n$, all different

from 0, is defined as the reciprocal of the arithmetic mean of the reciprocals of the a_is:

$$\text{H.M.} = \frac{1}{\dfrac{1}{n}\displaystyle\sum_{i=1}^{n}\dfrac{1}{a_1}} = \frac{n}{\dfrac{1}{a_1} + \dfrac{1}{a_2} + \cdots\cdots \dfrac{1}{a_n}}$$

Example 6:

Chetan travels from Mumbai to Bangalore by taxi. He travels one fourth of the distance at a speed of 40 km/hr, in the second quarter of the total, distance his speed is 50 km/hr, in the third quarter his speed is 60 km/hr, and in the fourth, quarter he travels at the speed of 70 km/hr. Find the approximate average speed of Chetan during the whole journey (in km/hr).

Solution: 53

Let's assume the total distance = 4d km

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{4d}{\dfrac{d}{40} + \dfrac{d}{50} + \dfrac{d}{60} + \dfrac{d}{70}} = \frac{4}{\dfrac{1}{40} + \dfrac{1}{50} + \dfrac{1}{60} + \dfrac{1}{70}}$$

X	40	50	60	70	Total
$\dfrac{1}{x}$	0.025	0.020	0.016	0.014	0.075

Average speed

$$\text{H.M.} = \frac{n}{\sum\left(\dfrac{1}{x_i}\right)} = \frac{4}{0.075} = 53 \text{ (Approx)}$$

Average speed of Chetan is 53 km/hr

- A.M. $\geq$ G.M. $\geq$ H.M. is always true. They will be equal, if all elements are equal to each other.

- If we have just two values then:

  G.M.^{2} = A.M. × H.M.

- The sum of deviation (D) of each element with respect to the average is 0.

  $\Rightarrow$ D = $(x_1 - x_{avg}) + (x_2 - x_{avg}) + (x_3 - x_{avg}) + \cdots\cdots(x_N - x_{avg}) = 0$

- $x_{avg} = x_{assumed\ avg} +$

  $\dfrac{\text{The sum of deviations}}{\text{Numbers of observations}}$

- Median is the middle-most value. For a finite list of numbers, we first arrange all the numbers in ascending order. If there are odd number of observations, then median is the middle number in the arrangement. If there are even number of observations, then median is the average of the middle two numbers in that arrangement.

- In a data set, mode is the value that occurs most frequently. A set of data may have no mode, one mode or more than one mode.

One can study in-depth on this in the Sequences and Series chapter.

A CAT aspirant appears for a certain number of tests. His average score increases by 1, if the first 10 tests are not considered and decreases by 1 if the last 10 tests are not considered. If his average scores for the first 10 and the last 10 tests are 20 and 30 respectively, then the total number of tests taken by him is:

Weighted Average

Weighted average is a very useful tool; it is basically the average of different groups or sets. Here different sets have different number of elements and different weightage.

Let's understand it through the table given below:

Group No.	No. of members	Avg. age of the group
G1	N_1	A_1
G2	N_2	A_2

In another way, considering that there are only two groups, and both the groups are combined, then the average age of all the members $= \dfrac{(N_1 \times A_1 + N_2 \times A_2)}{N_1 + N_2}$
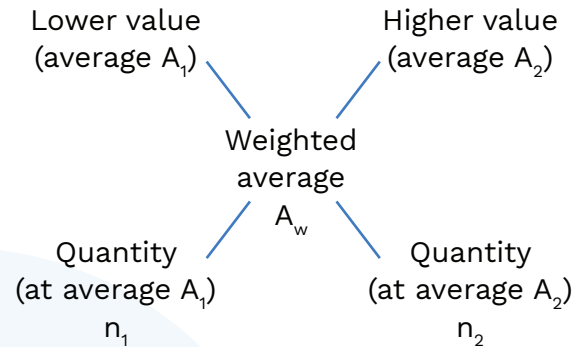
Let's use this concept in an example:

	Class A	Class B
No. of students	10	15
Average age	23 years	18 years

Now, if we combine both these classes, then the average age of all the students

$$= \frac{23 \times 10 + 18 \times 15}{10 + 15} = \frac{500}{25} = 20 \text{ years}$$

Alligation Method

And we write it as: $\dfrac{n_1}{n_2} = \dfrac{A_2 - A_W}{A_W - A_1}$

$$\frac{\text{Quantity (Lower priced)}}{\text{Quantity (Higher priced)}}$$

$$= \frac{\text{Higher price} - \text{Average Price}}{\text{Average Price} - \text{Lower Price}}$$

Example 7:

The average weight of 20 students of section A of class VIIth is 42 kg, whereas the average weight of 25 students of section B of the same class is 60 kg. Find the average weight of all the 45 students in the class.

Solution: 52 kg

$m_1 = 20$, $m_2 = 25$, $x = 42$, $y = 60$

Therefore, average weight of all 45 students

$$= \frac{m_1 x + m_2 y}{m_1 + m_2}$$

$$= \frac{20 \times 42 + 25 \times 60}{20 + 25}$$

$$= \frac{840 + 1500}{45} = 52 \text{ kg}$$

Hence, the average weight of all 45 students
= 52 kg.

Example 8:

There are three categories of jobs P.R.T., T.G.T. and P.G.T. The average salary of the teachers who got the job of P.R.T and T.G.T. categories is 26 lakhs per annum. The average salary of the teachers who got the job of T.G.T. and P.G.T. categories is 44 lakhs per annum and the average salary of those teachers who got the job of P.R.T and P.G.T categories is 34 lakhs per annum. The most appropriate range of average salary (in lakhs per annum) of all the three categories (if it is known that each teacher gets only one category of job i.e. P.R.T. or T.G.T. or P.G.T.):

(A) Lies between 30 and 39

(B) Lies between 28 and 34

(C) Lies between 34 and 43

(D) Lies between 29 and 48

Solution: (A)

Let the number of teachers who got the jobs of P.R.T., T.G.T. and P.G.T categories be a, b, and c respectively,

Then, the average salary of all the three categories of teachers

$$= \frac{(26(a+b)+44(b+c)+34(c+a))}{2(a+b+c)}$$

$$= \frac{(60a+70b+78c)}{2(a+b+c)}$$

$$= \frac{(30a+30b+5b+30c+9c)}{(a+b+c)}$$

$$= \frac{(30(a+b+c)+5b+9c)}{(a+b+c)}$$

= 30 + some positive value

So, the range of the average salary of the three categories lies between 30 and 39.

Also, the average salary will be less than 39, as to make average 39, numerator

must have 9a + 9b + 9c in addition to 30 (a + b + c), which is not there in the numerator. Hence, Option (A) is the correct answer.

Example 9:

Kestone is a digital marketing agency that is assigned with the task of conducting a survey of 4000 households for one of its clients. In the survey, they have to determine the average number of mobile phones with each household. That will help them to determine the digital consumption patterns of people. The data shows that a large number of households have 1 or 2 or 3 mobile phones and small a number of households have 4 or 5 mobile phones. Each household in the sample was found to have at least one mobile phone, and no household has more than 5 phones. What is the average number of phones per household up to two decimal places?

Number of phones per household	Number of households
1	1002
2	1040
3	1028
4	588
5	342

(A) 2.56   (B) 2.65

(C) 2.75   (D) 2.88

Solution: (A)

Average

$$= \frac{1002 \times 1 + 1040 \times 2 + 1028 \times 3 + 588 \times 4 + 342 \times 5}{4000}$$

$$= \frac{1002 + 2080 + 3084 + 2352 + 1710}{4000} = 2.557$$

The average number of phones per household in the sample is 2.56.

Example 10:

The average weight of 20 persons is increased by 2 kg, when one of them whose weight is 50 kg is replaced by a new person. What is the weight of the new person (in kg)?

Solution: 90

Let's assume the average weight of 20 persons = A kg and the weight of new person = y kg

Total new age = 20 × A − 50 + y   ....(1)

Also, total new age = new average × new number of people

$\quad$ = (A + 2) × 20   ....(2)

From (1) and (2)

20 × A − 50 + y = (A + 2) × 20

Solving which, we will get y = 90 kg

Hence, the weight of the new person is 90 kg.

Example 11:

The average age of 20 students and their class teacher is 14 years. If the class teacher's age is excluded, the average age reduces by 2 years. What is the age of the class teacher (in years)?

Solution: 54

Let's assume the age of the teacher = y years.

Total age of students and their teacher

$\quad$ = (21 × 14) years

According to the question

$\quad$ (21 × 14) − y = 20 × 12   ...(1)

(after the teacher's after teacher's age is excluded, 20 students will be left with average age of 20 (22 − 2) years)

Solving (1), we will get y = 54 years

Example 12:

The average score of a batsman in 24 innings, is 40. In the next inning he scored 90 runs. What will be his new average?

(A) 44   (B) 42

(C) 40   (D) 48

Solution: (B)

Step 1: Calculate the difference between the old average and the new score

$\quad$ = 90 − 40 = 50

Step 2: These 50 extra runs spread over 25 innings. So, the innings average will increase by $\frac{50}{25} = 2$

Step 3: Hence, the new average

$\Rightarrow$ 40 + 2 = 42

Alternative Method: $\frac{(24 \times 40 + 90)}{25} = 42$

Example 13:

The average age of 31 students is 22 years. If the age of the teacher is also included, the average age of the class becomes 22.5. Find the age of the teacher.

(A) 28 years   (B) 32 years

(C) 22 years   (D) 38 years

Solution: (D)

Calculate the change in average

$= 22.5 - 22 = 0.5$

This change of 0.5 is reflected over a new number i.e., (31 + 1 teacher = 32)

The total age is increased by

$32 \times 0.5 = 16$ years

22 + 16 = 38 Now is the age of the teacher.

Example 14:

In a school, the number of students in class A is 51, class B is 85 and class C is 102. Average score of the classes A, B and C are 74, 64 and 80 respectively. Find the average score of all the three classes.

(A) 67   (B) 77

(C) 73   (D) 70

Solution: (C)

Assume a value between highest and the lowest average.

Let's take 70 as assumed average.

Class:	A		B		C
Students:	51		85		102
Ratio of students:	3	:	5	:	6
Average:	74	:	64	:	80

Deviation from assumed Average:

Total deviation = (+4)(3) + (-6)(5) + (+10)(6)

Total deviation = 12 − 30 + 60 = +42

Total students will be (3 + 5 + 6) = 14, as we are now using the ratio of students.

Hence, we will get $= \frac{+42}{14} = +3$ this is the extra we have to add to the assumed average to get actual Average.

Hence, the required average = 70 + 3 = 73

Some Special Cases

Average involving Time, Speed and Distance

(a) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

(b) A person travels t_1 hrs at v_1 km/h and t_2 hours at v_2 km/hr

Average speed for the whole journey

$$= \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{t_1 \times v_1 + t_2 \times v_2}{t_1 + t_2} \text{ km/h}$$

(c) A person travels t_1 hrs at v_1 km/h and t_1 hours at v_2 km/hr. If the time taken at two different speeds are the same.

Average speed for the whole journey

$$= \frac{\text{Total distance}}{\text{Total time}} = \frac{t_1 v_1 + t_1 v_2}{t_1 + t_1} \text{ km/h}$$

$$= \frac{v_1 + v_2}{2}$$

Travelling Equal Distance With Different Speeds:

Here you will observe the use of harmonic mean.

Suppose a person is travelling 'd' distance with speeds $v_1, v_2, v_3 \ldots v_n$

Average Speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

Average speed for the whole journey

$$= \frac{d + d + d + d + \ldots + d \text{ (n times)}}{\dfrac{d}{v_1} + \dfrac{d}{v_2} + \dfrac{d}{v_3} + \ldots + \dfrac{d}{v_n}}$$

$$= \frac{n}{\dfrac{1}{v_1} + \dfrac{1}{v_2} + \dfrac{1}{v_3} + \text{.......} + \dfrac{1}{v_n}}$$

Hence we can derive a general formula for the same.

Average speed

$$= \frac{n(v_1 \times v_2 \times v_3 \times \text{......} v_n)}{\begin{array}{l} v_2 \times v_3 \times \text{...} v_n + v_1 \times v_3 \times \text{...} v_n \\ + \text{...} + v_1 \times v_2 \times v_3 \times \text{...} \times v_{n-1} \end{array}}$$

(d) A person travels 'd' distance at v_1 km/hr and another 'd' distance at v_2 km/h.

In this case, distance travelled at two different speeds is the same.

Average speed for the whole journey

$$= \frac{2 \times v_1 \times v_2}{v_1 + v_2}$$

(e) When a person travels three equal distances 'd' with three different speeds p, q and r.

Average speed for the whole journey

$$= \frac{3 \times p \times q \times r}{pq + qr + pr}$$

> ### Key Points
>
> If the time taken is the same, then the average speed is equal to the Arithmetic mean of the individual

Example 15:

A man travels 5 hours at 28 km/hr and another 5 hours at 32 km/hr, what is the average speed for the whole journey?

Solution: 30 km/hr

Since the time taken is same, average speed = Arithmetic mean of the individual speeds

$$= \frac{28 + 32}{2} = 30 \text{ km/hr}$$

Example 16:

Sonu travels 4 hours at 30 km/hr, another 4 hours at 32 km/hr and finally another 4 hours at 40 km/hr. What is the average speed for the whole journey?

Solution: 34 km/hr

Since the time taken is same, the average speed = Arithmetic mean of the individual speeds $= \dfrac{30 + 32 + 40}{3} = 34$ km/hr

Example 17:

Ambika is travelling from Dehradun to Bangalore. The first 1/3 rd of the journey is covered at a speed of 30 km/hr, the second 1/3rd at 12 km/hr, and the last 1/3rd at 45 km/hr. However, when she returns, from Bangalore to Dehradun, her speed is uniform at 25 km/hr. What is the average speed of Ambika (in km/hr) during the whole journey (approx.)?

(A) 25   (B) 27

(C) 28   (D) 23

Solution: (D)

Average speed when Ambika travels from Dehradun to Bangalore.

Using the formula for equal 3 distances

$$= \frac{3 \times p \times q \times r}{pq + qr + pr}$$

Average speed

$$= \frac{3 \times 30 \times 12 \times 45}{30 \times 12 + 12 \times 45 + 30 \times 45} = \frac{108}{5} = v_1$$

Average speed for the whole journey

$$= \frac{2 \times v_1 \times v_2}{v_1 + v_2}$$

Required Average

$$= \frac{2 \times v_1 \times v_2}{v_1 + v_2} = \frac{2 \times 108 \times 25}{5 \times \left(\dfrac{108}{5} + 25\right)}$$

$$= 23 \text{ km/h approx.}$$

Average Involving Age:

The Average of a group of N persons given at any point in time can be calculated in the following way.

2 years ago   P − 2

5 years ago   P − 5

2 years later P + 2

5 years later P + 5

Where P is the current average age.

Example 18:

The present average age of 6 members of a family is 51 years. A baby was born four years back and 3 old people expired 3, 5, and 7 years back at the ages of 51, 58 and 63 respectively. What was the average ages of the family 10 years back?

(A) 57.125 years   (B) 47.125 years

(C) 51.125 years   (D) 41.125 years

Solution: (C)

Currently, there are 6 people with the average age of 51 years.

Sum of ages is 51 × 6 = 306.

Sum of ages 3 years back

= 306 - 18 + 51 = 339.

Now number of people = 7

Sum of ages 4 years back

= 339 − 7 = 332

Now number of people = 6

Sum of ages 5 years back

= 332 - 6 + 58 = 384

Now number of people = 7

Sum of ages 7 years back

= 384 - 14 + 63 = 433

Now number of people = 8

Sum of ages 10 years back = 433 - 24 = 409 and now there are 8 people.

Average 10 years back = $\dfrac{409}{8} = 51.125$

Average Involving Number System

You will learn all these formulas in-depth in Sequence And Series. We are discussing the basic implementation of these types of formulas here in Average.

1. Average of 1st 'n' consecutive natural numbers = $\dfrac{n+1}{2}$

   The average of 1st five natural numbers
   $$= \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

2. The average of 1st 'n' consecutive even natural numbers = n+1

   Sum of 1st 'n' consecutive even natural numbers = n(n+1)

   The average of 1st five even natural numbers
   $$= \frac{2+4+6+8+10}{5} = 6$$

3. The average of 1st 'n' consecutive odd natural numbers = n

   Sum of 1st 'n' consecutive odd natural numbers = n^2

   The average of 1st five odd natural number = $\dfrac{1+3+5+7+9}{5} = 5$

4. If the series is in Arithmetic progression:

   Average = $\dfrac{\text{First number} + \text{Last number}}{2}$

5. The average of squares of 1st 'n' consecutive natural numbers
   $$= \frac{(n+1)(2n+1)}{6}$$

For example, the average of sum of squares of first 25 natural numbers will be

$$= \frac{26 \times 51}{6} = 221$$

Example 19:

What is the average of the first fifteen multiplies of 3?

Solution: 24

$$\frac{3[1+2+3+4+5+6+7+8+9+10+11+12+13+14+15]}{15} = \frac{3 \times 120}{15} = 24$$

Alternative Method:

Since consecutive multiples will be in Arithmetic progression.

$$\text{Average} = \frac{\text{First number} + \text{Last number}}{2}$$

$\Rightarrow$ 1st number = 1st multiple of 3 = 3

$\Rightarrow$ Last number = 15th multiple of 3 = 45

$\Rightarrow$ $\frac{3+45}{2} = \frac{48}{2} = 24$

Example 20:

The average of 5 observations $x, x+2, x+4, x+6$ and $x + 8$ is 11. What is the average of the last three observations?

Solution: 13

We can see that numbers are in A.P.

$\therefore$ Average = Middle number

$\therefore$ Middle number = Average = x + 4 = 11

    x = 7

    Average of last three observations

    = x + 6 = 7 + 6 = 13

Alternative Method:

$$\frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 11$$

$\therefore$ x = 7

    So, the numbers are = 7, 9, 11, 13, 15

In an apartment complex, the number of people aged 51 years and above is 30, and there are at most 39 people whose ages are below 51 years. The average age of all the people in the apartment complex is 38 years. What is the largest possible average age in years, of the people whose ages are below 51 years?

(A) 25 years   (B) 26 years

(C) 27 years   (D) 28 years

$\therefore$ Required mean $= \frac{11+13+15}{3} = \frac{39}{3} = 13$

Example 21:

In a set of prime and composite numbers, the composite numbers, are thrice the number of prime numbers and the average of all the numbers of the set is 12. If the number of prime numbers and composite numbers are exchanged, then the average of the set of numbers is increased by 4. If ,during the exchange of the numbers, the average of the prime numbers and composite numbers remained constant, then what is the ratio of the average of composite numbers to the average of prime numbers?

(A) 5/9   (B) 7/9

(C) 13/7   (D) None of these

Solution: (A)

Let the average of prime numbers be P and the average of composite numbers be C.

Let the number of prime numbers be x, then the number of composite numbers be 3x

Then, $\left(\dfrac{(Px + 3Cx)}{4x}\right) = 12$

$\left(\dfrac{(P + 3C)}{4}\right) = 12$

P + 3C = 48 ....(1)

According to the question,

$\left(\dfrac{(3Px + Cx)}{4x}\right) = 16$

3P + C = 64 .... (2)

On adding equation (1) and (2) we get,

P + C = 28

And on subtracting equation (1) from equation (2) we get,

P - C = 8

P = 8 + C

Now, putting the value of P in equation (1) we get,

C = 10

Therefore, P = 18

Thus, $\dfrac{C}{P} = \dfrac{10}{18} = \dfrac{5}{9}$

Hence, Option (A) is the correct answer.

Level of Difficulty – 1

1. Vedika goes to Rishikesh from New Delhi at a speed of 80 km/h and returns with a speed of 50 km/h. What is her approximate average speed during the whole journey?
   (A) 62.5 km/h
   (B) 62 km/h
   (C) 61.5 km/h
   (D) 60 km/h

2. The average age of the group of 16 men is 20 years. If 4 more men join the group, then the average age increases by 2 years. What is the average age of the new joiners?
   (A) 29 years
   (B) 30 years
   (C) 44 years
   (D) 35 years

3. The average height of 50 employees in a company is 163 cms. On a particular day, three employees A, B, and C, are absent, and so the average height of the remaining employees becomes 161 cm. If the height of A and B is equal, and the height of C is 5 cm less than A, then find the height of A.
   (A) 195 cm
   (B) 196 cm
   (C) 197 cm
   (D) 198 cm

4. The average age of the group having Amar, Vinod, and Choudhary is 64 years. One more person Dhruv joins the group and now the average becomes 60 years. Now a fifth person Ellen comes, whose age is 3 years more than that of the fourth person, and replaces the first person (Amar). Now, the average age of the group becomes 59. What is the age of the Amar?
   (A) 49 years
   (B) 55 years
   (C) 87 years
   (D) 114 years

5. The average age of Anil and Vimal 15 years ago was 25 years. The average age of Anil, Vimal and Chandan today is 35 years. So what was the age of Chandan 15 years ago?
   (A) 10 years
   (B) 25 years
   (C) 35 years
   (D) 40 years

Level of Difficulty – 2

6. The average weight of 3 friends Parth, Quasim and Raghu is 89 kg. Another friend Shakti joins the group and the average becomes 85 kg. If another person, Tatya whose weight is 4 kg more than that of Shakti, replaces Parth, then the average weight of Quasim, Raghu, Shakti and Tatya become 83 kg. Find the weight of Parth (in kg).

7. There are 11 classrooms in a school. An average of 15 students were sitting in each classroom. No classroom has equal number of students, and the number of

students in any classroom does not exceed the number of average students except the 11th classroom. Find the minimum number of students that can be accommodated in the 11th classroom.

(A) 45

(B) 55

(C) 60

(D) 66

8. The average score of an Indian cricketer in a certain number of innings is 48. After that, he played another ten innings in which he scored 95, 10, 28, 60, 12, 0, 45, 72, 69, and 89 runs respectively. Which of the following statements is true regarding his average score after combining all the innings?

(A) New average > old average

(B) New average < old average

(C) New average = old average

(D) Cannot be determined

9. In a hotel, total expenses are partly fixed and partly varying linearly with the number of customers. The average expense per customer is Rs. 1,750, when there are 30 customers and Rs. 1,450 when there are 55 Customers. What is the average expense (in Rs.) per Customer, when there are 100 Customers?

(A) 1,258

(B) 1,288

(C) 1,358

(D) 1,368

10. During a fundraiser event for the elderly people, apart from all the members of the organising committee, a huge number of volunteers also donated some amount for the noble cause. The members of the organising committee made an average contribution of Rs. 1,100 and this sum comprised 88% of the total amount raised through the event. The number of members in the organising committee was 40% more than the number of volunteers. Find the average contribution (in Rs.) per person made by the volunteers.

Level of Difficulty – 3

11. Paramvir is a superintendent jailor in Ramgarh who is assisted by 6 assistant jailors. Each of the seven jailors supervises a certain but distinct number of jail inmates. He has 30 inmates to look after. Sachin Vaje, another jailor and a close aide of Paramvir, supervises 32 inmates, which is also the average number of inmates supervised by each jailor. Each of the seven jailors has to supervise not more than 45 inmates. Once Paramvir heard a ruckus within the jail premises, and he instructed his assistants that half of them go to his left and half of them go to his right and the rest shall follow him. But unfortunately, none was left to follow him. Arnav Goshwami, a jail inmate, was curious to know who went left and who went right. When he asked another inmate Kalia about the same, he got to know that the ones who supervises more inmates than that of Paramvir went to the left and the ones who supervises fewer inmates than that of Paramvir went to the right side. What is the maximum possible difference between the number of inmates supervised by any 2 jailors?

(A) 29

(B) 27

(C) 31

(D) 33

12. Kohli had a cycle shop where along with selling, he would also give his bicycles for rent. Anyone who rented the cycle had to follow the following terms. If a cycle is rented for 3 hours or less, then the charge is Rs. 40 per hour or Rs. 10 per Kilometer, whichever is more. On the other hand, if the cycle is rented for more than 3 hours, then the charge is Rs.30 per hour or Rs. 4.50 per kilometer, whichever is more. Varsha rented a cycle from this shop, rode it for 20 kilometers and ended up paying Rs. 150. For how many hours did she rent the cycle?

(A) 4 hours

(B) 3 hours

(C) 5 hours

(D) 2 hours

13. The average weight of the 6 lightest and the 6 heaviest members of a family of 7 members are 40 kg and 45 kg respectively. If the difference between the overall minimum and maximum average weight is W, then choose the correct option.

(A) 1 < W < 3

(B) 3 < W < 4

(C) 4 < W < 7

(D) None of these

14. A CAT aspirant appears for a certain number of tests. His average score increases by 4 if the first 5 tests are not considered and decreases by 3 if the last 5 tests are not considered. If his average scores for the first 5 and the last 5 tests are 60 and 88 respectively, then the total number of tests taken by him is:

15. In a class of 40 students, the average weight of boys is 10 kg more than the average weight of girls and average weight of total class is 61 kg. Find the maximum possible number of boys in the class.

Solutions

1. (C)

Let's assume the total distance between two cities = 400 km

[By taking the LCM of 80 and 50]

Total time $= \dfrac{400}{80} = 5$ hours

She takes 5 hours to reach Rishikesh.

Now, she comes back with speed of 50 km/hr.

So, time taken by her $= \dfrac{400}{50} = 8$ hours

There, Total time = 8 + 5 = 13 hours

Distance = 400 km

∴ Average speed

$= \dfrac{400 \times 2}{13} = \dfrac{800}{13} \approx 61.5$ km/hr

2. (B)

The total age of 16 men = 16 × 20 = 320

According to the question:

Total age of 20 men (after adding 4 more men) = (16 + 4) (20 + 2)

= 20 × 22 = 440

So the difference between the age

= 440−320 = 120

∴ Average age of new joiners

$= \dfrac{120}{4} = 30$ years

So, Option (B) is the correct answer.

3. (B)

Total sum of height of 50 employees
= 50 × 163 = 8,150 cm
Total sum of the height of 47 employees
= 47 × 161 = 7,567 cm
Sum of the height of A, B and C
= 8150 - 7567 = 583
When the height of A and B is equal, then the equation will be
A + A + (A − 5) = 583
(where the value of C = A-5)
3A = 588
A = 196
∴   The height of A is 196.
Hence, Option (B) is the correct answer.

4. (B)

Total ages of (Amar + Vinod + Choudhary)
= 3 × 64 = 192
Total ages of (Amar + Vinod + Choudhary + Dhruv) = 4 × 60= 240
So, age of Dhruv = 240 − 192 = 48 years
Ellen = Dhruv + 3 = 51 years
According to the question = (Vinod + Choudhary + Dhruv + Ellen)
= 59 × 4 = 236 years
(Amar + Vinod + Choudhary + Dhruv) − (Vinod + Choudhary + Dhruv + Ellen)
= 240 - 236 = 4 Years
Amar − Ellen = 4
Amar = (Ellen + 4)
Amar = 55 years
Hence, Option (B) is the correct answer.

5. (A)

Age of Anil and Vimal (15 years ago)
= 2 x 25 = 50 years
Age of Anil and Vimal (now)
= 50 + 30 = 80 years
Age of Anil, Vimal and Chandan (now)

= 35 × 3 = 105 years
Present age of Chandan
= 105 - 80 = 25 years
Age of Chandan (15 years ago)
= 25 − 15 = 10 years
Hence, 0ption (A) is the correct answer.

6. 85 kg

Let us denote the weights of Parth, Quasim, Raghu, Shakti and Tatya by
P, Q, R, S, and T respectively.
Total weight of 3 friends (P + Q + R)
= 3 × 89 = 267 kg
Total weight of 4 friends P + Q + R + S
= 85 × 4 = 340 kg
Weight of Shakti, S = 340 − 267 = 73 kg
The sum of weight of Q + R + S + T
= 83 × 4 = 332 kg
According to the question-
Weight of Quasim, Raghu, Shakti and Tatya excluding Parth =
Q + R + S + T = 332
Q + R + 2 S + 4 = 332
Q + R + 2 × 73 = 332 − 4
Q + R = 328 − 146
Q + R = 182
So, P + Q + R = 267
P = 267 − 182
P = 85 kg
The weight of Parth is 85 kg.

7. (C)

Number of students in 11 classrooms
= 15 × 11 = 165
According to the question, in each classroom, number of students are unequal so, 15 + 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 = 105
minimum number of students in 11th classroom = 165 − 105 = 60

Hence, a minimum of 60 students can be accomodated in 11th classroom.

Hence, Option (C) is the correct answer.

8. (C)

Since the given average in the starting is 48.

If we take the sum of all deviations, we get,

48 → original average

Ten innings → 95, 10, 28, 60, 12, 0, 45, 72, 69, 89

Deviations from → +47, −38, −20, +12, −36, −48, −3, +24, +21, +41

original average

Sum of deviations → (47 + 12 + 24 + 21 + 41) − (38 + 20 + 36 + 48 + 3)

= (145) − (145) = 0

Thus, the original average will remain the same after 10 more innings also.

Therefore, the new average will be 48 (the same as the old average)

Hence Option (C) is the correct answer.

9. (B)

Let's assume that Fixed expenses

= Rs. x

Variable expenses per person = Rs. y

Then, x + 30 y = 1750 × 30

x + 30 y = 52500 ...(i)

x + 55 y = 1450 × 55

x + 55y = 79750 ...(ii)

Subtracting equation (i) from equation (ii)

25 y = 27250

y = 1090

putting the value of y in equation (i)

30 × 1090 + x = 52500

x = 52500 − 32700 = 19,800

According to the question:

Total expenses of 100 customers

= x + 100 y

Therefore, average expenses per person

$$= \frac{(x + 100\,y)}{100}$$

$$\left(\frac{(x + 100y)}{100} = \frac{19800 + (100 \times 1090)}{100} = \text{Rs. } 1288 \right)$$

Hence, Option (B) is the correct answer.

10. Rs. 210

Let the number of volunteers = x

No. of organising committee members

$$= x + \frac{40x}{100} = x + \frac{2}{5}x = \frac{7}{5}x = 1.4x$$

Total amount contribution by members = 1100 × 1.4x = 1540x = 88% of the contribution

1% of the total contribution $= \dfrac{1540x}{88}$

12% of the total contribution

$$= \frac{1540\,x \times 12}{88} = 210\,x$$

∴ Average contribution per Volunteer

$$= \frac{210x}{x} = \text{Rs. } 210$$

11. (A)

Since the average number of inmates = 32

So, total number of inmates = 32 × 7 = 224

As 3 jailors have more than 30 inmates.

So, 30 is the median number of inmates.

To maximize the difference, we make a table.

Jailor	Jailor	Jailor	Paramvir	Jailor	Jailor	Jailor
X	28	29	30	32	44	45

So, the highest possible total number of inmates supervised by 6 Jailors

= 45 + 44 + 32 + 30 + 29 + 28 = 208

Now, the remaining seventh jailor will be supervising: (224 − 208) = 16 inmates

Thus, the maximum possible difference

= 45 − 16 = 29

∴ Option (A) is the correct answer.

12. (C)

Case I: Let's consider that Varsha rented the cycle for 3 hours or less.

Cost could be either 10 × 20 or 3 × 40 (considering maximum of 3 hours)

Here, 200 > 120,

If Varsha would have taken the cycle for 3 hours or less, she would had paid Rs. 200 for sure.

But she paid only Rs. 150, so, she has not taken the cycle for 3 hours or less.

Case II: Let's consider that Varsha rented the cycle for more than 3 hours.

Then she would have paid

20 × 4.50 = Rs. 90

But she had paid Rs. 150, which means payment was done on per hour bases.

Thus, number of hours for which she rented the cycle would be

= 150 ÷ 30 = 5 hours

Hence, option (C) is the correct answer.

13. (B)

There will be two cases:

Let the members are p, q, r, s, t, u and v and $p \leq q \leq r \leq s \leq t \leq u \leq v$

Case I: For highest average weight:

According to question,

Sum of p, q, r, s, t and u = 6 × 40 = 240

Sum of q, r, s, t, u and v = 6 × 45 = 270

Hence, v − p = 30 or p + 270 = v + 240

So the highest possible value of v would be 30 more than highest value of p.

Value of v will be highest, when all the first 6 weights will be 40, hence v will be 70.

So the highest possible average will be

$$= \frac{240 + 70}{7} = \frac{310}{7} = 44.28$$

Case II: For lowest average weight:

Least possible value of p will give the least average hence v will be least too.

So the least possible value for v will be

$$= \frac{270}{6} = 45$$

Hence p will be:

45 × 5 + p = 240 ⇒ p = 15

Least possible average will be

$$= \frac{15 + 45 \times 6}{7} = \frac{285}{7} = 40.71$$

Hence the required difference will be

= 44.28 − 40.71 = 3.57

14. 25

Let's assume that there are y tests apart from first and last 5 tests and the average of these y tests is z. Also, assume the total average of all y + 10 tests = A

First 5 Tests	y Tests	Last 5 Tests
Avg = 60	Avg = z	Avg = 88

Avg without first 5 Tests

$$= \frac{y \times z + 5 \times 88}{y + 5} = A + 4 \qquad ......(I)$$

Avg without last 5 Tests

$$= \frac{y \times z + 5 \times 60}{y + 5} = A - 3 \qquad ......(II)$$

$$(I - II) \rightarrow \frac{140}{y + 5} = 7 \rightarrow \boxed{y = 15}$$

So, the total number of tests taken by students = y + 10 = 15 + 10 = 25

15. 36

Let's assume the number of boys = B and the number of girls = (40 − B)

Let's assume the average weight of boys
= K

So, the average weight of the girls
= (K − 10)

According to the question

{B × K + (40 − B) × (K − 10)}/40 = 61

On solving the above equation, we will get 4K + B = 284

As 4K and 284 are both multiple of 4, so B must also be a multiple of 4.

Hence, maximum value of B can be 36

Putting B = 36, we will get K = 62 kg.

Accordingly, G = 4 and their average weight will be = (K − 10) = 52 kg.

Hence, the maximum possible number of boys in the class can be 36.

Level of Difficulty – 1

1. The average of n numbers is 40. If each of $(4/5)^{th}$ of the numbers is increased by 5 and each of the remaining $(1/5)^{th}$ of them is decreased by 10, then the new average will be:
   (A) 41
   (B) 41.5
   (C) 42
   (D) 42.5

2. The average weight of 15 students increased by 1.8 kg when one of the students whose weight was 64 kg was replaced by a new man. Find the weight of the new man (in Kg).
   (A) 79
   (B) 84
   (C) 90
   (D) 91

3. If the average of 43 consecutive natural numbers is K, then what is the average of the next 999 consecutive natural numbers?
   (A) K + 498
   (B) K + 520
   (C) K + 521
   (D) K + 518

4. A running track is in the form of a regular pentagon. A runner covers each side of this pentagonal park track with different speeds of 12 km/h, 15 km/h, 18 km/h, 20 km/h and 30 km/h. Find the range of the average speed K of the runner during the entire journey (in km/h)
   (A) 14 < K < 15
   (B) 15 < K < 16
   (C) 16 < K < 17
   (D) 17 < K < 18

5. Three years ago, the average age of a family of 5 members was 17 years. A baby being born; the average of the family is the same today. What is the age of the baby?
   (A) 1 year
   (B) 2 years
   (C) 6 months
   (D) 9 months

6. The average age of a man and his son is 30 years. Four years ago, the ratio of their ages was 10:3. What is the difference (in years) between the present ages of the man and his son?

7. If the average of 9 consecutive even natural numbers, the greatest of which is n, is m. Find the average of 17 consecutive natural numbers, the least of which is m.
   (A) n – 2
   (B) n
   (C) n – 1
   (D) n + 4

8. The average marks of a class of 50 students is 40. Two of them scored 0 and for the rest, the first 32 scored an average of 40, the next fourteen scored an average of 20. If the remaining two scored equal marks, what are their individual marks?
   (A) 220
   (B) 225
   (C) 440
   (D) 222

9. The average weight of a family of 9 people is 4 less than the median weight. If the median weight person is removed, the average weight of the remaining persons will reduce by k. Find the value of k.
   (A) 0.5
   (B) 1
   (C) 3
   (D) 7

10. The average age of A, B and C is 31 years. However, when D joins them, their average age comes down to 28 years. Now a new person E whose age is 51 years more than D, replaces A and the new average of B, C, D and E is 36 years. Find the age of A (in years)?

Level of Difficulty – 2

11. 'A' was born two years ago when B was two years old. At present, the average age (in years) of A, B, C and D is a perfect square. Also, the difference between the ages of any two persons is not more than five years. Which of the following could be the ages of C and D (in years) if all were born in different years?
   (A) 8,2
   (B) 10,9
   (C) 7,3
   (D) 6,3

12. 65 students appeared for an exam. Their average score in that exam is 180 out of 200. If the top three ranker's scores are removed, the average drops by 0.5. The second highest score is less than 190 and all the scores are integers. Find the least possible score that the topper could have obtained?
   (A) 170
   (B) 193
   (C) 185
   (D) 198

13. The sum of five positive numbers is M. The average of the averages of all possible distinct triplets of these 5 numbers is N. Which of the following is true?
   (A) M = 5N
   (B) M = 4N
   (C) M = 3N
   (D) M = 2N

14. Shyam analyzed the monthly salary figures of five managers of his company. All the salary figures are in integer lakhs. The mean and the median salary figures are Rs. 8.6 lakhs and Rs. 9 Lakhs respectively and the only mode is Rs 11 lakhs. Which of the following could be the difference (in lakhs) between the second lowest and the lowest salary?
   (A) 8
   (B) 6
   (C) 4
   (D) 0

15. An NGO has raised 90% of the donation that it needs for school construction by receiving an average donation of Rs. 540 per person from a group of employees of a company. To raise the remaining amount, the NGO now plans to approach another group of employees of another company which has 40% of the number of employees as compared to the first company. Find the average donation per

person (in rupees) from each employee of another company.

16. Chauhan, Sharma and Kumar scored 45%, 70% and 50% of maximum marks respectively in subject A and 55%, 85% and 70% of maximum marks respectively in subject B respectively. Had Chauhan scored 40 more marks in subject A, his score in subject A would have been equal to his score in subject B. If Kumar scored 60 marks less in subject A as compared to his marks in subject B, then what is the sum of maximum marks of both the subjects?
   (A) 100
   (B) 200
   (C) 300
   (D) 400

17. The management of Oxford Public School proposed to construct an auditorium for which it contributed 15% of the estimated cost. For the remaining amount, the management approached 60% of the parents and all of them contributed Rs. 1200 on an average. The amount thus collected, together with the management's share formed only 75% of the estimated cost. To cover the deficit, they approached the remaining parents. What is the average amount that should be contributed by each of them to meet the estimated cost?
   (A) Rs 600
   (B) Rs 500
   (C) Rs 750
   (D) Rs 400

18. A batsman had an average of 36 runs in the first 8 matches. In the second to the ninth match his average score was 40. If he scored a total of 46 runs in the first and the ninth matches, then find his score in the 9th match.
   (A) 33
   (B) 35
   (C) 37
   (D) 39

19. In a school, the average weight of all the boys is 68 kg and that of all the girls is 52 kg. For the school's annual day, each male teacher is asked to coordinate the activities of exactly 13 boys and each female teacher is asked to coordinate the activities of exactly 12 girls. If the ratio of male to female teachers is 4 : 1, find the average weight of the students of the school (in kg).
   (A) 50
   (B) 55
   (C) 65
   (D) 70

20. Four years ago, the average age of Reena, Sheena and Cheena was 22. The average age of Reena and Cheena today is 25. Also, the present age of Reena is 75% of the present age of Sheena. What is the present age of Cheena?
   (A) 29 years
   (B) 30 years
   (C) 32 years
   (D) 36 years

Level of Difficulty – 3

21. Rohan married 12 years ago at the age of 29 years. His wife was 25 years old then. 8 years after their marriage, the average age of Rohan, his wife and their

son are 26 years. After how many years of Rohan's marriage was his son born?

(A) 2

(B) 3

(C) 4

(D) 5

22. A class consists of 60 boys and 80 girls. In the mid-semester examination, the average score of the girls was 7 higher than that of the boys. In the final exam, however, the average score of the girls dropped by 4 while the average score of the entire class increased by 2. The increase in the average score of the boys in the final exam is?

(A) 12

(B) 10

(C) 8

(D) 6

23. Abhi and Bunny travel from P to Q via R. Abhi travels from P to R in 4 hours and from R to Q in 5 hours. Bunny travels from P to R, then R to P and then P to R in 7 hours and from R to Q in 5 hours. If the average speed of Abhi and Bunny for the whole journey is the same, find the ratio of the distance between P to R to the distance between R to Q.

(A) 1 : 2

(B) 2 : 3

(C) 1 : 3

(D) 1 : 5

24. Gulzar Saab, Mani-Ratnam, Shekhar Kapoor and Vishal Bhardwaj collaborated for a project to write the screenplay of the untitled film by Shubham. All the writers decide that they should share the wages in the proportion of the work they contribute. If Mani Ratnam does 3 units of work per day, then Gulzar Saab does 5 units of work per day. If Shekhar does 1 unit of work per day, then Mani Ratnam does 2 units of work per day and if Vishal does 4 units of work per day, then Shekhar does 3 units of work per day. If they worked together for 10 days and got a total of Rs. 2.07 crores rupees, then find the average daily income (In Rs.) of Gulzar Saab.

25. In the year 2020, Vishal got 20% more incentive in the 3rd month than what he got in the 2nd month. His incentive in the 4th month was $7\frac{61}{77}\%$ more than what he got in the 3rd month. If he got Rs 4500 as an incentive in the first month and the difference in incentives received in the 4th month and 2nd month is Rs 1130, then the average incentive received by Vishal in the first four months is:

(A) Rs 4,472.5

(B) Rs 4,478.5

(C) Rs 4,487.5

(D) Rs 4,497.5

26. Ram went from Delhi to Shimla via Chandigarh by car. The distance from Delhi to Chandigarh is 4/3 times the distance from Chandigarh to Shimla. The average speed from Delhi to Chandigarh was 1.5 times that from Chandigarh to Shimla. If the average speed for the entire journey was 63 km/hr, find the average speed from Chandigarh to Shimla (in km/hr).

(A) 40

(B) 36

(C) 34

(D) 51

27. The average number of runs scored by 6 batsmen including Rohit and Virat in a T-20 cricket match is 30. The average runs by 5 batsmen excluding Rohit is 24 and the average number of runs scored by 5 batsmen excluding Virat is 28. The runs scored by Rohit is what percent higher than the runs scored by Virat?

(A) 20 %

(B) 25 %

(C) 33.33 %

(D) 50%

28. Samuel's present age is 4 times his daughter's present age and 16 times his granddaughter's present age. Samuel's father is 20 years older than him and the average present age of all the members in the family of 4 members is 51.25 years. Find the sum of the ages of all the family members 5 years from now.

(A) 225 years

(B) 200 years

(C) 175 years

(D) 150 years

29. Set S contains 7 integers that have an average value of 65. The mean of all numbers in set S is equal to the median of the numbers. If the smallest number of set S is 20 less than half the largest number, then the minimum value that the smallest number in set S can be:

(A) 8

(B) 12

(C) 20

(D) 15

30. There are N students in the class. Their class teacher gave them a task of finding the average of first N natural numbers and each of them let out a different number and found the average of the remaining numbers. The average of the averages obtained by all the students was 27. Find N.

(A) 25

(B) 53

(C) 60

(D) 78

Solutions

1. (C)

The new average can be found by adding the increased and decreased amounts. Thus, the new sum

= (4n/5) × 45 + (n/5) × 30 = 42n

∴ New average = 42n/n = 42

Hence, option (C) is the correct answer.

2. (D)

Let's assume the weight of the new man = M kg and also the average weight of existing 15 students = A kg

New total weight after 64 kg man is replaced by new man of weight M kg

15 × A − 64 + M   .....(1)

Also new total weight = Number of people × New Average = 15 × (A + 1.8)   .....(2)

From (1) and (2)

15 × A − 64 + M = 15 × (A + 1.8)

Solving which we will get M = 91 kg

Hence, option (D) is the correct answer.

3. (C)

Here, the question is written in a general statement.

For simplicity:

Let 43 consecutive natural numbers are:

1, 2, 3, 4, ..., 43

Average = 22 = K

Now, next 999 terms can be written as:

44, 45, 46, 47, ...1041, 1042, which is an AP with a = 44 and d = 1

Now, their average = 500th term

$\quad$ = 44 + 499 × 1 = 543

This can be written in terms of K as:

$\quad$ K + 521

Hence, option (C) is the correct answer.

4. **(D)**

Let's assume each side of the pentagon = LCM (12, 15, 18, 20 and 30) = 180 km.

Now the time taken by the runners to run 180 km each with the speeds 12 km/h, 15 km/h, 18 km/h, 20 km/h and 30 km/h will be 15 hours, 12 hours, 10 hours, 9 hours and 6 hours respectively.

So, Average Speed

$$= \frac{\text{Total Dis} \tan \text{ce}}{\text{Total time}}$$

$$= \frac{180 \times 5}{(15 + 12 + 10 + 9 + 6)}$$

$$= \frac{900}{52} = 17.30$$

Hence, option (D) is the correct answer.

5. **(B)**

Average Age of The Family

$$= \frac{\text{Sum of The Age of The Family}}{\text{Total Members In The Family}}$$

Sum of the age of the family = Average age of the family × Total members in the family.

Sum of the age of the family

= 17 × 5 = 85 years

Increment in the age of 5 members after 3 years will be = 5 × 3 = 15 years

Now, the sum of age of the family of 5 members after 3 years = 85 + 15 = 100.

New member born in the family

Total members in the family become = 5 members + 1 baby = 6 members

Again, using the same formula,

Average Age of The Family

$$= \frac{\text{Sum of The Age of The Family}}{\text{Total Members In The Family}}$$

$$17 = \frac{\text{Sum of The Age of 5 Members} + \text{Age of the baby}}{6}$$

100 + Age of baby = 102

Age of baby = 102 − 100 = 2 years

Hence, option (B) is the correct answer.

6. **28**

Current average age of the man and son

$\quad$ = 30 years

Average age four years ago = 26 years

Sum of ages four years ago

$\quad$ = 26 × 2 = 52 years

Ratio of ages of man and son = 10: 3

Age of man $= \frac{10}{13} \times 52 = 40$

Similarly,

Age of son $= \frac{3}{13} \times 52 = 12$

Difference in age = 40 - 12 = 28 years

Hence, "28" is the correct answer.

7. **(B)**

Considering a simple case of 9 consecutive even numbers, say from 2 to 18, we get m = average = 10 and n = 18.

Now the average of 17 consecutive natural numbers, starting from m i.e., 10 up to 26, we get average

$$= \frac{(10 + 26)}{2} = 18 \text{ i.e., n itself.}$$

Hence, option (B) is the correct answer.

8. (A)

Sum of marks of 2 students

= 2 × 0 = 0

Students left = 50 − 2 = 48

Sum of marks of 32 students

= 40 × 32 = 1,280 marks

Sum of marks of next 14 students

= 14 × 20 = 280

Let's assume that equal marks of 2 students = 2x

According to the question –

$$\frac{2 \times 0 + 1280 + 280 + 2x}{50} = 40$$

1560 + 2x = 50 × 40

2x = 2,000 - 1,560

2x = 440

x = 220

Hence, option (A) is the correct answer.

9. (A)

Let x' kg be the weight of the median weight person.

∴ The average weight of the 9 people

= (x − 4) kg.

The total weight of 9 people

= 9(x − 4) kg

Now, if median weight person leaves, the total weight will change

= 9(x − 4) − x = 8x − 36

Finally, we have the new average 'k' less than the original average.

∴ $(x-4) - \dfrac{8x-36}{8} = k$

⇒ x − 4 − x + 4.5 = k

⇒ k = 0.5

Hence, option (A) is the correct answer.

10. 38

Let's assume the age of A, B, C, D and E be a, b, c, d and e years respectively.

a + b + c = 31 × 3 = 93 .....(1)

a + b + c + d = 28 × 4 = 112 .....(2)

(2) − (1)

d = 19

e = 19 + 51 = 70

b + c + d + e = 36 × 4 = 144

b + c = 144 − 19 − 70 = 55 .....(3)

Now (1) − (3)

a = 38

Hence, "38" is the correct answer.

11. (C)

A's and B's present ages are 2 and 4 years respectively.

Also $\dfrac{(A+B+C+D)}{4}$ = Perfect square

Thus 1 cannot be considered.

Let it be 4.

⇒ A + B + C + D = 16

⇒ C + D = 10

Also given that the age difference of any two persons is not more than five years. Hence the ages of C and D could be 7 and 3 years, satisfying all the conditions. But when it be 9 ⇒ A + B + C + D = 36

⇒ C + D = 30

Thus, the age difference of any two persons now becomes more than five years. Hence this case is discarded.

Hence, option (C) is the correct answer.

12. (B)

Marks scored by top three students

= (62 × 0.5) + (3 × 180) = 571

As it is given that the second highest score is less than 190, at the most, it could be 189. The corresponding least possible score of the topper

= 571 − 189 − 189 = 193.

Hence, option (B) is the correct answer.

13. (A)

Let the five numbers be a, b, c, d and e.

Then, M = (a + b + c + d + e)

Now, there will be $^5C_3 = 10$ triplets i.e. (a, b, c), (a, b, d), (a, b, e),(a, c, d), (a, c, e), (a, d, e) (b, c, d) (b, c, e), (b, d, e), (c, d ,e).

Average of each triplet

$$= \frac{(a+b+c)}{3}, \frac{(a+b+d)}{3}, \frac{(a+b+e)}{3}, \ldots\ldots, \frac{(c+d+e)}{3}$$

Sum averages of all triplets = 6

$$\frac{(a+b+c+d+e)}{3} = 2(a+b+c+d+e)$$

Average of the averages of 10 triplets

$$= N = \frac{2(a+b+c+d+e)}{10}$$

$$= \frac{(a+b+c+d+e)}{5}$$

i.e. M = 5N

Hence, option (A) is the correct answer.

14. (C)

Mean salary = 8.6 Lakhs

Total salary of all 5 Managers

= 8.6 × 5 = 43 Lakhs

$$\underline{} \; \underline{} \; \underset{\text{Median}}{\underline{9}} \; \underset{\text{Mode}}{\underline{11}} \; \underset{\text{Mode}}{\underline{11}}$$

Now, Total salary of remaining 2 Managers

= 43 − (9 + 11 + 11) = 12 Lakhs

Now the possible combinations of lowest 2 salaries having sum 12 would be (4, 8) and (5, 7).

Combinations of (1, 11) are not possible as the median will become 11.

Combinations of (2, 10) are not possible as the median will become 10.

Combinations of (3,9) are not possible as then there will be 2 modes 9 and 11, but in the question, it is given that 11 is the only mode.

Combinations of (6,6) are not possible as then there will be 2 modes 6 and 11, but

in the question, it is given that 11 is the only mode.

Hence, the difference between the lowest and second lowest salaries would be either 4 or 2, but 2 is not mentioned in any of the options,

Hence, option (C) is the correct answer.

15. 150

Let the total donation be Rs. x and the total number of employees from the first company be 'n'.

$\therefore$ 90% × x = 540 × n

$$\frac{x}{n} = \frac{540}{90\%}$$

Also, remaining donation = x × 10%

Let the average donation per person from the 2nd company be Rs y.

$\therefore$ x × 10% = n × 40% × y

$$y = \frac{x}{n} \times \frac{1}{4}$$

$$y = \frac{540}{0.90} \times \frac{1}{4}$$

$$y = 150$$

Hence, "150" is the correct answer.

16. (C)

Let maximum marks in subject A be 100x and in subject B be 100y

From data we get,

	Chauhan	Sharma	Kumar
Subject A	45x	70x	50x
Subject B	55y	85y	70y

Now according to the question,

45x + 40 = 55y

Then, 9x + 8 = 11y   ....(i)

and 70y − 60 = 50x

Then 7y − 6 = 5x   ...(ii)

Solving both equations (i) and (ii),

we get $x = \dfrac{5}{4}$ and $y = \dfrac{7}{4}$

Sum of maximum marks of both the

subjects is $100\,(x + y) = 100\left(\dfrac{5}{4} + \dfrac{7}{4}\right) = 300$

Hence, option (C) is the correct answer.

17. (C)

No. of Donors   Amount Collected

Average Donation (Rs.)

60% of the parents → 60% of total 1,200

Management →15% of total ---

40% of the parents → 100% − (60% + 15%) = 25%

Thus, by the rule of proportions we have:

The average amount to be contributed

$= \dfrac{60\%}{40\%} \times \dfrac{25\%}{60\%} \times 1200 = $ Rs. 750

Alternative

Let the estimated cost be Rs. 100x

Amount contributed by management

= 15% of estimated cost

= 15% of 100x = 15x

Remaining amount left = 100x − 15x = 85x

Average amount contributed by 60% or 3/5 parents of the total parents approached = 1,200

Total amount contributed by these parents = 1200 × 3 = 3600

Now ATQ

15x + 3600 = 75% of 100x

60x = 3,600

X = 60

Deficit amount

= 100x − 75x = 25x = 25 × 60 = 1500

Average amount contributed by

remaining parents $= \dfrac{1500}{2} = 750$

Hence, option (C) is the correct answer.

18. (D)

Let's assume his score in the first match, total score from 2nd to 8th match and score of 9th match be A, B and C respectively.

According to the question

A + B = 36 × 8 = 288   ...(1)

B + C = 40 × 8 = 320   ...(2)

On doing (2) − (1), we will get

C − A = 32   ...(3)

Given in the question that

A + C = 46   ...(4)

On solving (3) and (4), we will get C = 39

Hence, option (D) is the correct answer.

19. (C)

Average weight of the boys = 68 kg and average weight of the girls = 62 kg

To find the average weight of all the students, we need to know the ratio of girls to boys.

Since 13 boys are coordinated by 1 male teacher and 12 girls by one female teacher. Also the ratio of male to female teachers = 4 : 1

Thus the ratio of boys to girls

= (4 × 13) : (1 × 12) = 13 : 3

Hence the average weight of the students $\dfrac{(68 \times 13) + (52 \times 3)}{(13 + 3)}$ = 65 kg, the required answer.

Hence, option (C) is the correct answer.

20. (A)

Let the present age of Reena be × years, the present age of Sheena be y years and the present age of Cheena be z years.

Four years ago:

$\dfrac{x - 4 + y - 4 + z - 4}{3} = 22$

$\Rightarrow$ x + y + z − 12 = 66

$\Rightarrow$ x + y + z = 78   .....(1)

Also, average age of Reena and Cheena now is 25

$\Rightarrow$ $\dfrac{x+z}{2} = 25 \Rightarrow$ x + z = 50   .....(2)

Put x + z = 50 in (1)

$\Rightarrow$ 50 + y = 78 $\Rightarrow$ y = 28   .....(3)

Given that $x = \dfrac{3}{4} \times y$

$\Rightarrow$ $x = \dfrac{3}{4} \times 28 = 21$ .....(4)

Put x = 21 in (2):

$\Rightarrow$ 21 + z = 50

$\Rightarrow$ z = 29

Therefore, the present age of Cheena = 29 years.

Hence, option (A) is the correct answer.

21. (B)

Average age of Rohan and his wife at the time of marriage

$= \dfrac{(29+25)}{2} = 27$ years

Average age of Rohan and his wife 8 years after their marriage

= 27 + 8 = 35 years

Total age of Rohan and his wife 8 years after their marriage = 35 × 2 = 70 years

Average age of Rohan, wife and their son 8 years after their marriage = 25

Total age of Rohan, wife and their son 8 years after their marriage

= 25 × 3 = 75 years

So, Age of their son 8 years after their marriage = 75 − 70 = 5 years

Hence, the son was born 3 years after their marriage.

Hence, option (B) is the correct answer.

22. (B)

	Boys	Girls	Class
Number	60	80	140
Mid semester Average	Y	Y + 7	Y + 4
Final Exams Average	Y + K	Y + 3	Y + 6

Let's assume that in mid semester exams, average marks of boys were Y, so average marks of girls would be (Y + 7)

So, class Average marks in mid semester

$= \dfrac{60 \times Y + 80 \times (Y+7)}{140}$

$= \dfrac{140Y + 560}{140} = Y + 4$

So, according to the question, in the final exams, the class average would be (Y + 6) and girls' average marks would be (Y + 3) respectively.

Now, let's assume that boys average increase by K from mid semester exams to final exams, so boys average in final exams would be (Y + K)

So, $\dfrac{60 \times (Y+K) + 80 \times (Y+3)}{140} = Y + 6$

K = 10

Hence, option (B) is the correct answer.

23. (D)

Let the distance from P to R be x and from R to Q be y

Total distance traveled by Abhi

= x + y

Total time taken by Abhi

= 4 + 5 = 9hr

Total distance traveled by Bunny

= 3x + y

Total time taken by Bunny

$= 7 + 5 = 12hr$

Average speed of Abhi $= \dfrac{(x+y)}{9}$

Average speed of Bunny $= \dfrac{(3x+y)}{12}$

Average speed of Abhi and Bunny is equal

$\dfrac{(x+y)}{9} = \dfrac{(3x+y)}{12}$

$\Rightarrow \dfrac{(x+y)}{3} = \dfrac{(3x+y)}{4}$

$\Rightarrow 4x + 4y = 9x + 3y$

$\Rightarrow y = 5x$

Gulzar : Mani Ratnam : Shekhar : Vishal

5	3		
	2	1	
		3	4

| 5x2x3 | : | 3x2x3 | : | 3x1x3 | : | 3x1x4 |
| 10 | : | 6 | : | 3 | : | 4 |

Per day share of Gulzar Saab

$= \dfrac{10}{23} \times 20700000 \times \dfrac{1}{10} = \text{Rs. } 900000$

Hence, "900000" is the correct Answer.

25. **(C)**

Let the incentive received by Vishal in the 2nd month $= x$

Then the incentive received by Vishal in the 3rd month $= 1.2x$

Now, $7\dfrac{61}{77}\% = \dfrac{600}{7700} = \dfrac{6}{77}$

Incentive received by Vishal in 4th month

$= 1.2x \times \left(1 + \dfrac{6}{77}\right) = \dfrac{99.6x}{77}$

Ratio of the distance between P to R to the distance between

R to Q $= \dfrac{x}{y} = \dfrac{1}{5}$

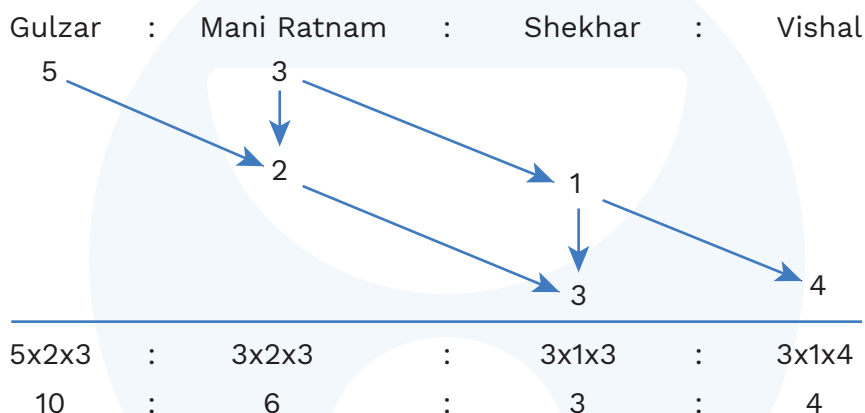Hence, option (D) is the correct answer.

24. **900000**

According to the question, Ratio of work done is given by:

Gulzar : Mani Ratnam = 5 : 3

Shekhar : Mani Ratnam = 1 : 2

Vishal : Shekhar = 4 : 3

Now, the combined ratio can be deduced as:

As per data: $\dfrac{99.6x}{77} - x = 1130$

Solving, x = Rs. 3850

So, Vishal's incentive in 2nd month

$= \text{Rs. } 3850$

Incentive in 3rd month

$= 1.2x = \text{Rs } 4620$

Incentive in 4th month

$= \dfrac{99.6x}{77} = \text{Rs. } 4980$

Therefore, the required sum

$= 4,500 + 3,850 + 4,620 + 4,980$

$= \text{Rs. } 17,950$

Average incentive received by Vishal in first four months

= 17950/4 = Rs. 4487.5

Hence, option (C) is the correct answer.

26. (D)

Let's assume the distance between Delhi to Chandigarh = 4x

Let's assume the distance between Chandigarh to Shimla = 3x

Delhi	Chandigarh	Shimla

$$\begin{array}{|c|c|}\hline 4x & 3x \\ \hline 1.5\,y\ km/hr & y\ km/hr \\ \hline \end{array}$$

Let's assume the speed from Chandigarh to Shimla = y km/hr.

Assume

So, Speed from Delhi to Chandigarh

= 1.5y km/hr.

Average speed for the journey

$$= \frac{\text{Total Distance}}{\text{Total Time}}$$

$$\Rightarrow \frac{7x}{\dfrac{4x}{1.5y}+\dfrac{3x}{y}} = 63$$

$$\Rightarrow \frac{7}{\dfrac{8}{3y}+\dfrac{3}{y}} = \frac{7}{\dfrac{1}{y}\left(\dfrac{8}{3}+3\right)} = \frac{7}{\dfrac{1}{y}\times\dfrac{17}{3}} = 63$$

$$\Rightarrow \frac{7 \times 3y}{17} = 63$$

$$\Rightarrow y = 51$$

Hence, option (D) is the correct answer.

27. (D)

Let's assume the total runs scored by 4 batsmen excluding Virat and Rohit = P

Also, assume that the runs scored by Rohit and Virat are Q and R respectively.

Total runs scored by all 6 batsmen

= P + Q + R = 30 × 6 = 180 ...(1)

Total runs scored by 5 batsmen excluding Rohit

= P + R = 24 × 5 = 120 ...(2)

Total runs scored by 5 batsmen excluding Virat

= P + Q = 28 × 5 = 140 ...(3)

Now (2) + (3) − (1) will give us P = 80

Putting the value of P in (2) and (3) will get R and Q as 40 and 60 respectively.

Now percentage more runs scored by Rohit as compared to Virat

$$\frac{\{(60-40)\}}{40} \times 100 = 50\%$$

Hence, option (D) is the correct answer.

28. (A)

Let the present age of Samuel's grand daughter be × years.

∴ The present age of Samuel

= 16x years

Also, the present age of Samuel's daughter = 4x years.

The age of Samuel's father

= (16x + 20) years

Also, it is given in the question that the average age of all the family members is 51.25 years.

$$\therefore \quad \frac{16x + 4x + x + 16x + 20}{4} = 5,125$$

37x + 20 = 51.25 × 4

37x = 205 − 20

37x = 185

x = 5

Therefore, the present age of Samuel

= 16 × 5 = 80 years

The present age of Samuel's father

= 16x + 20 = 16 × 5 = 20 = 100 years

The present age Samuel's daughter

= 4x = 4 × 5 = 20 years.

Also, the present age of Samuel's grand-daughter = 5 years.

Sum of the ages of all the family members 5 years from now

= (100 + 5) + (80 + 5) + (20 + 5) + (5 + 5)

= 225 years.

Hence, option (A) is the correct answer.

29. (C)

If mean = median, then the number which would be fourth in increasing order = 65

Now, if biggest number of set S = a

Then, smallest number of set

$$S = \frac{a}{2} - 20$$

i.e. second and third smallest number can be 65 each, while the fifth and sixth number can be a.

Then,

$$\frac{\frac{a}{2} - 20 + 65 + 65 + 65 + a + a + a}{7} = 65$$

3.5a = 280

a = 80

Therefore, the minimum value of smallest number in set

$$S = \frac{80}{2} - 20 = 20$$

Hence, option (C) is the correct answer.

30. (B)

We know that

$$1 + 2 + 3 + 4 \ldots\ldots N = \frac{N(N+1)}{2}$$

Now, there are N students and N numbers. Given that each student left out a different number.

Let's assume the first student left out number 1, the second student left out number 2, and so on till N student left out number N.

First student average = $\dfrac{\dfrac{N(N+1)}{2} - 1}{N-1}$

Second student average

$$= \dfrac{\dfrac{N(N+1)}{2} - 2}{N-1}$$

N^{th} student average = $\dfrac{\dfrac{N(N+1)}{2} - N}{N-1}$

Given average of all students' average = 27

$$\dfrac{\left(\dfrac{\dfrac{N(N+1)}{2} - 1}{N-1}\right) + \left(\dfrac{\dfrac{N(N+1)}{2} - 2}{N-1}\right) + \cdots\left(\dfrac{\dfrac{N(N+1)}{2} - N}{N-1}\right)}{N}$$

$$\dfrac{N \times \dfrac{N(N+1)}{2} - (1+2+3+\ldots N)}{N(N-1)}$$

$$\dfrac{\left[N\left(\dfrac{N(N+1)}{2}\right)\right] - \dfrac{N(N+1)}{2}}{N(N-1)} = 27$$

$$\dfrac{\dfrac{N(N+1)}{2} - \dfrac{(N+1)}{2}}{N-1} = 27$$

$$\dfrac{\left(\dfrac{N+1}{2}\right)(N-1)}{(N-1)} = 27$$

$$\dfrac{N+1}{2} = 27 \rightarrow N + 1 = 54$$

N = 53

Hence, option (B) is the correct answer.

$$\frac{\text{Sum of elements}}{\text{Number of elements}}$$

$$\frac{w_1x_1 + w_2x_2 + ...w_nx_n}{w_1 + w_2 + w_3 + ----w_n}$$

Simple Average

Weighted Average

Average

Mean

Arithmetic Mean $\geq$ **Geometric Mean** $\geq$ **Harmonic Mean**

$$\frac{a_1 + a_2 + ----a_n}{n}$$

$$\sqrt[n]{a_1a_2-----a_n}$$

$$\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots \frac{1}{x_n}}$$