



Introduction

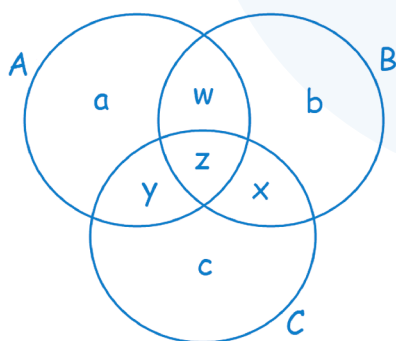
Set theory questions have appeared regularly in the CAT for the past several years. These are a combination of calculation-based questions and maximisation/minimisation-based questions, which actually test a candidate's critical thinking ability.

First, we will go through the concept of set theory, followed by a unique approach to solving maximisation/minimisation questions. Later, we will look at more questions through some solved examples.

In the exercise section, you will come across a good set of questions, which will provide you with sufficient practice on this topic.

Concepts of Set and Venn Diagram

A set is a collection of elements (numbers, letters, objects, people, etc.). A set is commonly represented as a list of all its members enclosed in braces. Venn diagrams are also used to represent a set. Consider the following Venn diagram which represents three sets: A, B, and C.



Important

In the above Venn diagram,

- 'a' represents the number of elements that are part of *only set A*.
- 'b' represents the number of elements that are part of *only set B*.
- 'c' represents the number of elements that are part of *only set C*.

- 'w' represents the number of elements that are part of *only sets A and B but not set C*.
- 'x' represents the number of elements that are part of *only sets B and C but not set A*.
- 'y' represents the number of elements that are part of *only sets A and C but not set B*.
- 'z' represents the number of elements that are part of *all the 3 sets*.

Also,

- The total number of elements in set A is represented by $a + w + y + z$.
- The total number of elements in set B is represented by $b + w + x + z$.
- The total number of elements in set C is represented by $c + x + y + z$.

The set-based questions of DILR rarely require the mathematical formula [i.e., $A \cup B$, $(A \cup B) \cap C$, etc.] to apply. They mostly intend to test your critical thinking ability. For example, over the last few years' CAT exams, most of the questions based on the concepts of sets ask you to find the maximum/minimum value of a certain category, rather than finding a specific value.

So, let's look at a unique approach to tackle the maximum/minimum value questions.

Chocolate Distribution Approach for Maximisation/Minimisation Problems

Consider the following two illustrations to understand this approach.

Example 1:

In a class, there are 50 students. On a special occasion, the math teacher brings 20 eclairs, the English teacher brings 30 mints, and the science teacher brings 40 candies. They wish to distribute the chocolates in



such a way that each student in that class gets at least 1 chocolate and none gets two chocolates of the same variety.

1. What can be the maximum number of students who can get chocolates of three different varieties?

Solution:

Let's assign the roll numbers 1-50 to the students.

First, we can hand over 1 candy to each of the students with roll numbers 1-40. Then, we can hand over 1 mint to each of the students with roll numbers 41-50.

Now, we are left with 20 mints and 20 eclairs. So, at the most, we can assign 2 more varieties of chocolates (i.e., 1 mint and 1 éclair) to 20 students (say roll numbers 1-20).

So, the maximum number of students whom we can give 3 different chocolates can be 20.

Note:

If there is no restriction on distributing at least one chocolate to each student in the class, then we can straightaway say that at most 20 students can get three different varieties of chocolates, as shown below:



2. What can be the minimum number of students who can get chocolates of three different varieties?

Solution:

To minimise the number of students having three different chocolates, we should distribute the chocolates in the following manner:

- 40 candies to Roll no. 1-40 (all candies used)
- 10 mints to Roll no. 41-50
- 20 mints to Roll no. 1-20 (all mints used)
- 20 eclairs to Roll no. 21-40 (all eclairs used)

So, there can be '0' students who got three different chocolates.

3. What can be the minimum number of students who can get chocolates of exactly 1 variety?

Solution:

To minimise the number of students having just 1 chocolate (i.e., exactly one variety), we need to maximise the number of students with more than one chocolate. We have a total of 90 chocolates. So, we will first distribute 1 chocolate to each of the students (so that each gets at least one chocolate), and then remaining 40 chocolates to 40 students. So, a total of 40 students got more than one chocolate. Hence, only 10 students got exactly 1 chocolate.

Hence, the minimum number of students who can get chocolate of just 1 variety is 30.

Now, let's take some more examples to understand other approaches to tackle maximisation/minimisation problems and other set theory-based caselets.

Example 2:

Directions for Questions: Answer the following question on the basis of the information given below.

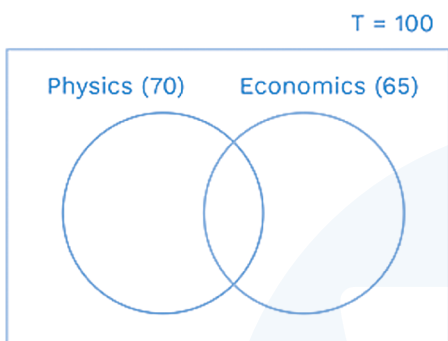


There are 100 students in a class. Of them, 70 students like physics and 65 students like economics.

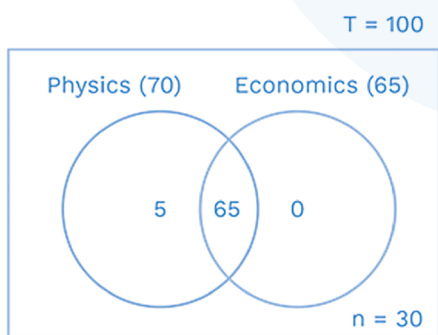
- Find the maximum number of students who like both the subjects.

- (A) 40
(B) 65
(C) 50
(D) 70

Solution: (B)



For the maximum number of students who like both the subjects, we can put students who like only economics as 0. (We cannot put students who like only physics as 0 as then the number of students who like economics will be more than 65.) Hence, our Venn diagram will be as shown below:



Hence, the maximum number of students who like both subjects = 65.

Short trick

Let the number of students who like both the subjects = x . Then,

$(70 - x) \geq 0$ [As the number of persons cannot be negative]

$$\Rightarrow x \leq 70 \text{ (i)}$$

Also, $(65 - x) \geq 0$ [As the number of persons cannot be negative]

$$\Rightarrow x \leq 65 \text{ (ii)}$$

Now, to satisfy both equations (i) and (ii), $x \leq 65$

Hence, the maximum value of x can be 65.



Rack Your Brain

Mr. Advait saw four men while going to a restaurant. Each man saw that two women are going towards the restaurant. Every woman holds one puppy in their hands. What can be the minimum number of living beings going towards the restaurant?



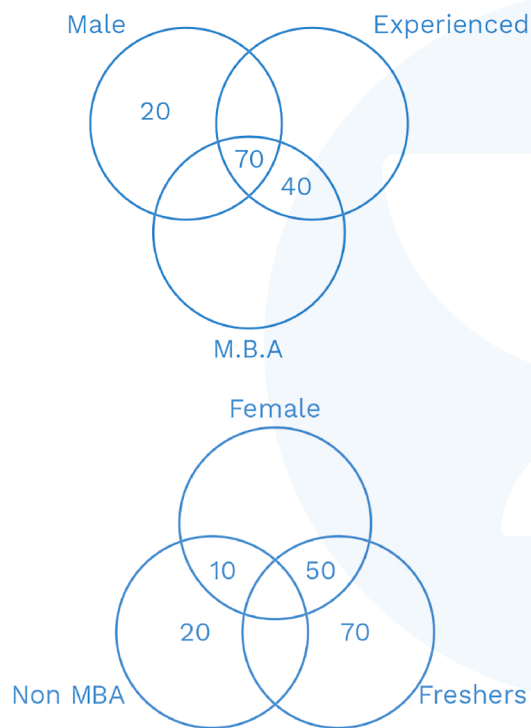
Practice Exercise

Level of Difficulty - 1

Set 1

Directions for Questions 1 to 4: Refer to the data below and answer the questions that follow.

There are 320 employees in a company XYZ at present. Each employee is either a male or a female, a fresher or an experienced, and an MBA or non-MBA. The given two Venn diagrams give the information about the number of employees in the company XYZ.



- What is the total number of freshmen in the company?
(A) 50
(B) 60
(C) 80
(D) 90
- What is the ratio of the number of men who are MBA to the number of men who are non-MBA in the company?

- (A) 5: 2
(B) 7: 2
(C) 4: 3
(D) 5: 3

- What is the difference between the number of men who are MBA and experienced and the number of women who are non-MBA and freshers in the company?
(A) 30
(B) 40
(C) 45
(D) 50
- Which of the following gives the highest value?
(A) Number of non-MBA who are freshers.
(B) Number of women who are MBA.
(C) Number of men who are experienced.
(D) Number of MBA who are experienced.

Set 2

Directions for Questions 5 and 6

A survey was conducted on 70 students about the kids' channels of Pogo TV, Nick, and Hungama, which they watch. All students like at least 1 of the TV channels. It was also known that:

- The number of students who watch Nick and Hungama but not Pogo is 6 times the number of the students who watch only Nick which is not zero.
- The number of students who watch all channels is 20.
- Thirty students watch Pogo only.
- The number of students who watch Pogo and Nick only is twice the number of students who watch only Hungama which is not zero.



5. The number of students who watch Pogo and Hungama but not Nick are least possible, then how many students watch Nick?
6. How many students like at most two channels?

Set 3

Directions for Questions 7 to 11

In an entrance exam, 100 questions were asked. The question paper was divided into four sections: QA, LR, VA, and GA with 25 questions in each of the sections. All sections had objective-type questions with 4 options: A, B, C, and D.

Two marks were awarded for every right answer and 1 mark was deducted for every wrong answer. The following table shows the number of questions attempted by the seven candidates.

Candidate	Sections			
	QA	LR	VA	GA
Abhijeet	14	7	3	19
Daya	21	15	14	09
Praduman	18	5	7	25
Freddy	13	9	18	13
Tarika	11	17	13	25
Salunke	14	11	22	6
Vivek	4	0	17	19

7. If Daya and Tarika gave the same number of correct answers in each section, what could be the maximum number of questions correctly answered by them?
8. If Abhijeet, Daya, Freddy, and Salunke answered all the questions correctly in LR, what could be the minimum number of questions which at least two of them answered correctly?
9. If Freddy, Praduman, and Vivek answered all questions wrongly in QA, what could be the maximum number of questions wrongly answered by at least two in the QA section?
10. Which of the following cannot be the final score of Abhijeet out of the total of 200 marks?
 - (A) 62
 - (B) 47
 - (C) 36
 - (D) -25
11. Since Praduman and Tarika were not prepared for the GA section at all, they decided to mark the answers in a fixed pattern. Praduman answered the questions in pattern A B C D A B C D.....till the last question and Tarika decided to answer the questions in pattern A A B B C C D D A A till the last question. For how many questions do they mark the same option in GA?

Set 4

Directions for Questions 12 to 15

In a survey, it was found that (of the 100 persons surveyed) 20 persons don't play any of the three games: cricket, football, and hockey. Forty people play cricket, 30 people play football and 40 people play hockey.

12. What is the maximum possible number of persons who play exactly one game?
13. What is the maximum possible number of persons who play only cricket?
14. If 15 people play only hockey, what is the maximum possible number of persons playing all three games?
15. How many persons play exactly one game if only five persons play all three games?



Level of Difficulty - 2

Set 1

Directions for Questions 1 to 4: Answer the following questions based on the information given.

The World Health Organisation conducts a survey to finalise the best course of action to eliminate a contagious disease. It is discussing two proposals.

X: A proposal to force complete quarantine on every city in the world.

Y: A proposal to double the number of hospitals in each city.

A city does not necessarily support either of the two proposals. The team of scientists which will eventually oversee the global combat against the disease will be headed by only one of the two countries: the USA and China. Each city prefers one of the two countries. Also, 1,000 cities participate in the survey. Survey highlights are as under:

- i) 620 cities supported proposal X and 450 cities supported proposal Y.
 - ii) Among the 400 cities that preferred China as the lead, 80% supported proposal X.
 - iii) Among those who preferred the USA, 50% supported proposal X.
 - iv) 40% of those who supported proposal Y preferred China as the lead.
 - v) Every city that preferred China and supported proposal Y also supported proposal X.
 - vi) Among those who preferred the USA as the team lead, 40% did not support any of the proposals.
1. How many cities that preferred China as the lead supported both the proposals?
(A) 210
(B) 180

- (C) 150
(D) None of these

2. How many cities supported only proposal X and preferred the USA as the team lead?
3. What percentage of the cities surveyed that did not support proposal X preferred the USA as the team lead? Round off the answer to the nearest integer?.
4. How many of the cities surveyed supported proposal Y, did not support proposal X, and preferred the USA as the team lead?
(A) 0
(B) 40
(C) 60
(D) None of these

Set 2

Directions for Questions 5 to 8: Answer the following questions based on the information given below.

Among the actors and directors who graced the International Film Festival at Cannes in 2021, half of them who invested personally in bitcoins had also invested in bonds. The number of actors and directors who invested only in blue-chip equity was four times those who invested only in bonds. The number of actors and directors who invested in none of the above three options was equal to the number who invested in all the three. The number of actors and directors who invested only in bitcoins and blue-chip equity was equal to the number who invested only in bonds and blue-chip equity, each of which was equal to half the number who invested in bitcoins and bonds. The number of actors and directors who invested only in bitcoins and bonds was the same as the number who invested only in bonds, each of which was twice the number who invested in none of the three.



5. If the number of actors and directors who invested in all the options of bit-coins, bonds, and blue-chip equity was 100, how many actors and directors graced the International Film Festival at Cannes in 2021?
- (A) 1650
(B) 1750
(C) 1850
(D) 1950
6. The number who invested only in bit-coins and bonds made up what percent of the number of actors and directors who invested in exactly two options?
- (A) 30%
(B) 35%
(C) 40%
(D) 45%
7. If the number of actors and directors who invested only in bonds is 80, how many actors and directors did not invest in bitcoins?
- (A) 400
(B) 460
(C) 500
(D) 560
8. If the number of actors and directors who invested in both bitcoins and blue-chip equity was 75, how many people invested in bonds?
- (A) 150
(B) 165
(C) 180
(D) 195

Set 3

Directions for Questions 9 and 10

In a theme park, there are 6 types of rides. Last Sunday, a total of 1,000 people visited the park; of them, 5% did not take any of the rides and no person took the same ride twice. It was known that,

- i) 78% of people visited Nitro.
ii) 63% of people visited Twister.

- iii) 57 % of people visited the Alibaba cave.
iv) 79% of people visited Rajasorous.
v) 68% of people visited Mr. India.
vi) 53% of people visited the Titanic.
9. What can be the minimum number of people who took at least five rides?
10. If the maximum possible number of people took all six rides, then what can be the maximum number of people who took exactly four rides?

Set 4

Directions for Questions 11 to 14

In a college, each of the 900 students participated in at least one of the six events of 100 m race, 200 m race, 250 m race, 300 m race, 350 m race, and 400 m race. No student who participated in the 100 m race participated in the 300 m race or the 350 m race. No student who participated in the 400 m race participated in the 200 m or 250 m race. An equal number of students participated in only 100 m race, only 200 m race, only 250 m race, only 300 m race, and only 400 m race events. The same number of students participated in each combination of exactly two events. An equal number of students participated in each combination of exactly three events. Twenty students participated in exactly four events. The number of students who participated in only 200 m race, the number of students who participated in only 200 m and 100 m race and the number of students who participated in only in all three of 200 m, 100 m and 250 m race are in the ratio 1 : 2 : 3

11. How many students participated in the 100 m race?
12. How many students participated in both the 200 m and the 250 m race?
13. How many students did not participate in the 400 m race?
14. How many students participated in at most two events?



Level of Difficulty - 3

Set 1

Directions for Questions 1 to 4: Answer the following questions based on the information given below.

From the city of Kolkata, the three finalists of the top sweets manufacturers: P, Q, and R, manufactured and sent a total of 180, 170, and 150 types of patented specimens of exquisite designer sweets, respectively, to the final round of the reality show '*Kaun Hain Sabse Bada Mithaiwala*'. The basic category of the sweets was either dry or moist in nature.

The following chart shows the number of the two types of sweets that passed the final round on the two parameters, i.e., taste and appearance:

For example, out of 90 types of dry sweets of manufacturer Q, 50 passed the parameter of taste.

The manufacturer whose highest percentage of patented specimens passed either of the parameters in the final round won the crown of '*Sabse Bada Mithaiwala*'.

- What is the total number of patented specimens of manufacturer R which passed either of the parameters in the final round?
(A) 125
(B) 130

- (C) 135
(D) 140

- If p and q are the proportion of dry sweets passing the parameter of taste and appearance, respectively, and if m and n are the proportion of moist sweets passing the parameter of taste and appearance, respectively, then which of the following is true?
(A) $p > m$ and $q > n$
(B) $p < m$ and $q < n$
(C) $p > m$ and $q < n$
(D) $p < m$ and $q > n$
- What is the total number of sweets passing the parameters of both taste and appearance in the final round?
(A) 130
(B) 120
(C) 115
(D) 110
- What is the total number of sweets that failed to pass either of the two parameters of taste and appearance in the final round?
(A) 165
(B) 130
(C) 105
(D) 70

	Taste		Appearance		Taste and Appearance Both	
	Dry sweets	Moist sweets	Dry sweets	Moist sweets	Dry sweets	Moist sweets
P	60 / 100	50 / 80	40 / 100	40 / 80	15 / 100	20 / 80
Q	50 / 90	30 / 80	50 / 90	50 / 80	20 / 90	10 / 80
R	60 / 90	40 / 60	40 / 90	40 / 60	30 / 90	25 / 60



Set 2

Directions for Questions 5 to 9: Answer the questions on the basis of the information given below.

Exactly sixty persons from all over the world participated in a global meet for the betterment of the environment. Each person who participated had at least one pen in black and red and has at least one hobby out of watching sports and watching series.

The following table provides the number of persons, among the 60, for any combination of pens that they used and hobbies that they follow.

	Black Pen	Red Pen	Watching Sports	Watching Series
Black pen	33			
Red pen	10	37		
Watching sports	25	24	41	
Watching series	20	25	19	38

5. How many persons had both black and red pens and watched both—Sports and Series?
6. How many persons used a red pen but not a black pen and watched sports but not watched series?
7. Of the persons who used exactly one pen, how many persons watched only sports?
8. How many persons had both hobbies?
9. How many persons had exactly two pens?

Set 3

Directions for Questions 10 to 15

A reputed MBA College has 300 students who study at least one of the three

specialisations: finance, marketing, and operations. Also, 60% of the total students are males. Of the male students, 120 study operations and 80 study marketing. Of the female students, 45 study operations and 90 study finance.

Further, it is also known that,

- i) The ratio of male and female students studying only finance is 2: 1.
 - ii) The number of female students studying only operations and marketing is half of the number of female students studying only operations and finance.
 - iii) The number of female students studying only marketing is equal to the sum of the number of female students studying only operations and only finance.
 - iv) The number of female students studying only finance and marketing is twice the number of male students studying only those two subjects.
 - v) The number of male students studying all the three specialisations is thrice the number of female students studying the same.
 - vi) The number of male students studying all the three specialisations is twice the number of male students studying only marketing.
 - vii) Of the male students studying operations, one-third do not study any other subjects, and one-third study only finance alongside.
 - viii) Of the male students, 105 study more than one specialisation, and of the female students, 90 study more than one specialisation.
10. How many male students study only finance and marketing?
- (A) 15
 - (B) 25
 - (C) 40
 - (D) None of these



- 11.** How many female students study marketing?
(A) 70
(B) 90
(C) 95
(D) 85
- 12.** How many male students study finance?
(A) 100
(B) 120
(C) 115
(D) 90
- 13.** How many female students study all the three specialisations?
(A) 15
(B) 20
(C) 10
(D) 5
- 14.** How many male students study all the three specialisations?
(A) 15
(B) 20
(C) 25
(D) 30
- 15.** How many female students study only operations?
(A) 10
(B) 15
(C) 5
(D) 20





Solutions

Level of Difficulty - 1

1. (D)

Based on the information given in the Venn diagrams, we can draw the following table:

	Males		Females		Total
	Freshers	Experienced	Freshers	Experienced	
MBA	70	70	50	40	230
Non-MBA	20	20		10	90
Total	90	90		50	320

As the total number of students is 320, the total number of female freshers will be 90.

So, the number of Non-MBA fresher females will be 40.

So, the complete table will look as shown below.

	Males		Females		Total
	Freshers	Experienced	Freshers	Experienced	
MBA	70	70	50	40	230
Non-MBA	20	20	40	10	90
Total	90	90	90	50	320

So, the total number of males in the company who are freshers will be 90.

Hence, the answer is (D).

2. (B)

As per the data presented in the table in the previous question, the number of males who are MBA = 140

The number of males who are non-MBA = 40

So, the required ratio = $140:40 = 7:2$

Hence, the answer is (B).

3. (A)

As per the data presented in the table in the previous question, the number of

males who are MBA and experienced = 70.

The number of females who are non-MBA and freshers = 40

So, the required difference = $70 - 40 = 30$

Hence, the option (A).

4. (D)

Let's evaluate each option one by one considering the data presented in the table in the previous question.

Option (A): Number of non-MBA who are freshers = $20 + 40 = 60$

Option (B): Number of females who are MBA = $50 + 40 = 90$

Option (C): Number of males who are experienced = $70 + 20 = 90$

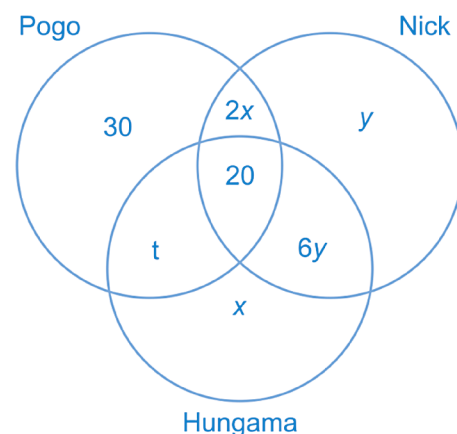
Option (D): Number of MBA who are experienced = $70 + 40 = 110$

Among all the options, option D gives the highest value.

Hence, (D) is the correct answer.

5. 38

Let's put the given information in a Venn diagram, as shown below, where, $x, y \neq 0$.



From the above representation,
 $t + 50 + 3x + 7y = 70$



So, $3x + 7y + t = 20$

Trying values of x , y , and t , we get:

x	Y	T
1	1	10
2	1	7
3	1	4
4	1	1
1	2	3
2	2	0

If the number of students who watch Pogo and Hungama only (i.e. ' t ') is the least possible, then t should be the least, i.e. x , y , and t are 2, 2, and 0, respectively.

So, number of students watching Nick channel = $2x + 20 + 6y + y = 2x + 7y + 20 = 38$

(i.e. sum of all the values inside the 'Nick' set)

Hence, the answer is 38.

6. 50

At most two channels = No channel OR Exactly 1 channel OR Exactly 2 channels
= At least one channel – all three channels (because everyone watches at least 1 channel)

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Person	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	S	S	S	S	S	F	F	F	A	A
Who	S	S	S	S	S	S																			
answered	F	F	F	F	F	F																			
it correctly	A	A	A	A	A																				

As per the above representation, we can say that,

So, as shown in the table, the minimum number of questions which was attempted correctly by at least 2 students can be

$= 70 - 20 = 50$

Hence, the answer is 50.

7. 48

Assuming that Daya attempted 21 questions correctly in the QA section, Tarika attempted only 11 correctly in that section. So, both of them, at the most, could have got 11 correct if it is known that they have got the same number of correct answers in that section.

Similarly, in the other sections, we can get the following figures for the number of questions they answered correctly:

LR - 15

VA - 13

GA - 9

So, at the most, they could have answered $11 + 15 + 13 + 9 = 48$ questions correctly in all the sections put together.

Hence, the answer is 48.

8. 11

Here, we are asked to find the least number of questions which was answered correctly by at least two students.

The best way to get the solution is by using the Table, as shown below.

just 6. This is again an application of the 'Chocolate Distribution Method'.

Hence, the answer is 11.

**Note:**

If the question would have been 'What could be the maximum number of questions answered by at least two of them' then we would have easily answered that by using the chocolate distribution method, as mentioned below.

There are 25 students (No. of questions) in a class and you want to distribute 42 chocolates (number of instances). Also, there is no condition that each should get at least one chocolate. (As it is not compulsory that each question is answered by at least one student.) So, we can directly distribute two chocolates each to 21 students. So, the maximum number of questions that can be answered correctly by at least two students can be 21.

9. 17

Total wrongly answered questions by all of them are $13 + 18 + 4 = 35$

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Person	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P							
Who answered it wrongly	F	F	F	F	F	F	F	F	F	F	F	F	F	V	V	V	V								

as shown in the table above, the maximum number of questions wrongly answered by at least two is: $35/2 = 17$ (a complete quotient)

Hence, the answer is 17.

10. C

Let's say the correctly answered questions for him are x and wrongly answered questions are y .

So, $x + y = 43$ (as he has answered a total of 43 questions for all the subjects put together).

So, his final score will be $2x + y$.

If $y = 0$, then his score would be 86.

If $y = 1$, then his score would be 83.

If $y = 2$, then his score would be 80.

Hence, the pattern of marks he will get is $86 - 3n$, where n is an integer.

From the given options, only 36 is the only value that doesn't follow the above pattern.

Hence, option (C) is correct.

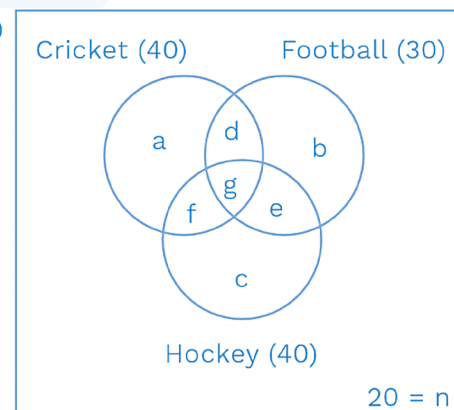
11. 7

As shown in the above table, there will be seven instances where both of the students marked the same option for a particular question.

Hence, the answer is 7.

12. 65

100



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Praduman	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D	A
Tarika	A	A	B	B	C	C	D	D	A	A	B	B	C	C	D	D	A	A	B	B	C	C	D	D	A



Let,

I = No. of persons who play exactly 1 game

II = No. of persons who play exactly 2 games

III = No. of persons who play exactly 3 games

So, the number of persons who play at least 1 game =

$$I + II + III = 100 - 20 = 80 \quad \dots (i)$$

(As 20 do not play any game)

So, sum of the sets:

$$= C + F + H$$

$$= I + 2 II + 3 III$$

$$= 40 + 30 + 40 = 110 \quad \dots (ii)$$

(Because those who play two games are counted twice and those who play 3 games are counted thrice)

$$\text{So, } (2) - (1) \Rightarrow II + 2 III = 30 \quad \dots (iii)$$

So, we can say that at most 30 persons can play exactly 2 games (when exactly three is zero), and at most 15 people can play exactly 3 games (when exactly two is zero).

Now, from $2 \times (i) - (ii)$, we get

$$I - III = 50$$

So, for 'I' to be the maximum, 'III' also has to be the maximum which can be 15, as mentioned above.

So, the maximum value of exactly 1 game will be 65 persons. (as $50 + 15 = 65$)

Hence, the answer is 65.

13. 40

We know that,

$$n(Cric \cup Foot \cup Hock) = 80 \quad (\text{No. of persons playing at least 1 game})$$

$$\text{So, Only } (Cric) + \{(Foot) \cup (Hock)\} = 80$$

As we have to maximize only Cricket, $\{(Foot) \cup (Hock)\}$ must be as minimum as possible.

$$\text{Also, } n(Foot) = 30, \text{ and } n(Hock) = 40$$

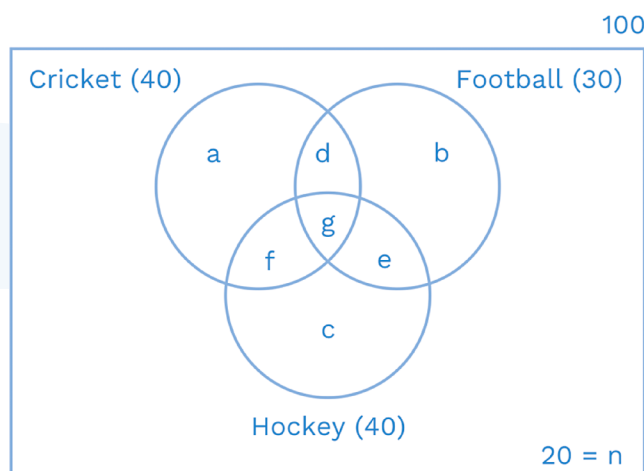
So, $\{(Foot) \cup (Hock)\}$ has to be at least 40. (Assuming all those who play Football, also play Hockey)

$$\text{So, only Cricket is at the most } (80 - 40) = 40$$

Hence, the answer is 40.

14. 5

Let's use a different approach for this.



We know,

$$c = 15 \Rightarrow e + f + g = 25$$

$$II + 2 III = 30 \quad (\text{as explained earlier})$$

$$d + e + f + 2g = 30$$

$$d + g = 5$$

So, the maximum value of 'g' can be 5.

Hence, the answer is 5.

15. 55

As explained in the first question of this set,

$$I - III = 50$$

$$\text{Since } III = 5, I = 55$$

Hence, the answer is 55.



Level of Difficulty - 2

1. (B)

From (i), the number of cities supporting X is 620.

So, the number of cities which support only X + the number of cities which support both X and Y = 620 ... (i) (This is shown in the following table.)

From (ii), among the 400 cities that prefer China, the number of cities which support X = 80% of 400 = 320, which is shown in the table below.

The number of cities which prefer USA = $1,000 - 400 = 600$. So, the number of cities which support X = 50% of 600 = 300.

Using (iv), among the 450 cities which prefer Y , the number of cities which prefer China = 40% of 450 = 180.

Similarly, all the given data can be tabulated as follows:

Using (v): we can deduce that among the cities which prefer China, the number of cities which support only Y = 0. Consequently, the number of cities which support only X = $320 - 180 = 140$.

Among these 400 cities which prefer China, cities which support neither X nor Y = $400 - (140 + 180 + 0) = 80$.

	Only X	Both X and Y	Only Y	Neither X nor Y	Total
USA	300			240	600
China	320				400
		180			
Total	620				1000
		450			

	Only X	Both X and Y	Only Y	Neither X nor Y	Total
USA	300			240	600
China	140	180	0	80	400
Total	620			320	1000
		450			

Consider the 'Total' row: The number of cities that support only Y = $1,000 - (320 + 620) = 60$. Also, the number of cities that support only X = $1,000 - (320 + 450) = 230$.

From equation (i): Number of cities that support both X and Y = $620 - 230 = 390$. The complete table is presented as follows:

	Only X	Both X and Y	Only Y	Neither X nor Y	Total
USA	90	210	60	240	600
China	140	180	0	80	400
Total	230	390	60	320	1000

So, the number of cities that preferred China as the lead and supported both the proposals = 180.

Hence, the answer is (B).

2. 90

Refer to the final table obtained in the solution to the first question.

The number of cities that supported only proposal Y and preferred the USA as the team lead is 90.

Hence, the answer is 90.

3. 79

Refer to the final table obtained in the solution to the first question.

The number of cities that did not support proposal X = $1000 - (230 + 390) = 380$.

Among these 380 cities, the number of cities which preferred the USA is 300.

The required percentage = $(300/380) \times 100 = 78.95 = 79\%$.

Hence, the answer is 79.

4. (C)

Refer to the final table obtained in the solution to the first question.

The number of cities surveyed that supported proposal Y, but did not support proposal X is 60. All of these 60 cities preferred the USA as the team lead.

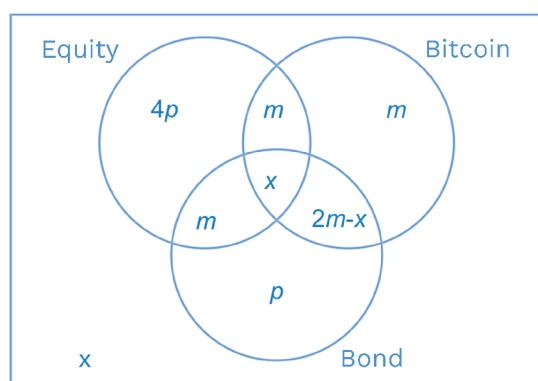
Hence, the answer is (C).

5. (C)

The given points can be summarised (along with the variable used) as mentioned below:

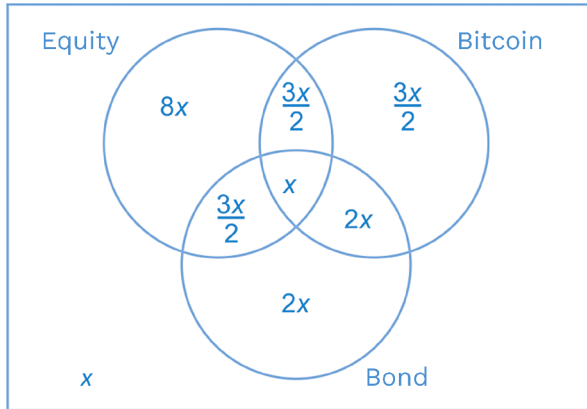
- i) Those who invested only in equity (4p) were four times than those who invested only in bonds (p).
- ii) Those who invested in none of the above three (x) options were equal to those in number who invested in all the three (x).
- iii) Those who invested only in bitcoins and equity (m) were equal to those who invested only in bonds and equity (m), each of which was equal to half the number who invested in bitcoins and bonds (2m).
- iv) The number of those who invested only in bitcoins and bonds (2m-x) was the same as the number of those who invested only in bonds (p), each of which was twice the number who invested in none of the three (x).
- v) Also, as per the very first point of the data set, half of them who invested personally in bitcoins also invested in bonds. So, the number of actors and directors who invested only in bitcoins will be 'm'.

As per the points mentioned above, the following Venn diagram can be created:





Now, as per fourth point, $(2m - x) = p = 2x$
Hence, $p = 2x$, and $m = 3x/2$
So, the final values (in terms of x) will be as shown below:



So, total actors and directors in the festival = $37x/2$

Also, the value of x is given as 100 in the question.

So $37x/2 = 37 \times 50 = 1850$

Hence, (C) is the correct option.

6. (C)

Referring to the second Venn diagram of the previous explanation,

Requisite percentage = $\{2x / (2x + 3x/2 + 3x/2)\} \times 100$

$= (2x/5x) \times 100 = 40\%$

Hence, the correct option is (C).

7. (C)

As shown earlier, the number of actors and directors who invested in only bonds = $2x = 80$ or, $x = 40$.

Now, the number of actors and directors who did not invest in bitcoins

$= (8x + 3x/2 + 2x + x) = 25x/2$

Hence, the number of actors and directors who did not invest in bitcoins

$= 25/2 \times 40 = 500$

Hence, option (C) is correct.

8. (D)

The number of the actors and directors who invested in both bitcoins and blue-chip equity = $3x/2 + x = 5x/2$

Hence, $5x/2 = 75$ or, $x = 30$

Hence, the number of actors and directors who invested in Bonds = $2x + 2x + x + 3x/2 = 13x/2 = 13 \times 15 = 195$

Hence, option (D) is correct.

9. 90

95% of people took at least 1 ride, i.e., 950 people

i) 780 people visited Nitro.

ii) 630 people visited Twister.

iii) 570 people visited Alibaba cave.

iv) 790 people visited Rajasorous.

v) 680 people visited Mr. India.

vii) 530 people visited the Titanic.

That means, people visited the rides a total of 3,980 times. Now, let's consider all 950 people who visited the first 4-4 rides each. So, total rides ($950 \times 4 = 3,800$) are covered.

The remaining 180 rides can be distributed as 2-2 to these people as a maximum of only six rides can be taken. (Here, we are distributing 2-2 rides (instead of 1-1), as we want to minimize the number of people taking five or more rides.)

So, $180/2 = 90$ people took six rides and the remaining people took exactly four rides in this case.

So, the minimum number of people taking at least five rides will at least be 90.

Hence, the answer is 90.

10. 200

As only 530 people took the Titanic (the least number of all), the maximum number of people who took all 6 rides can be 530 only.

So, in total, they took 530×6 rides out of 3,980.



Now, the remaining rides to be distributed are $3,980 - 3,180 = 800$

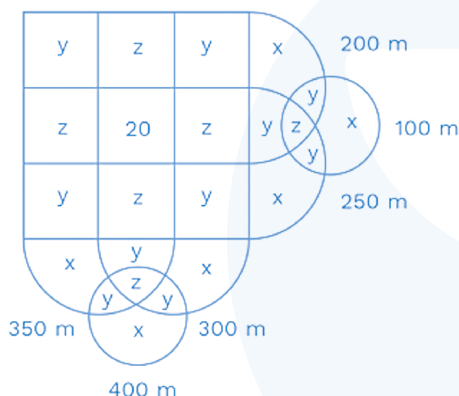
To maximize the number of people who took exactly 4 rides, allot these 800 rides as 4-4 to the remaining people, which means, $800/4 = 200$ people (at most) can take exactly 4 rides.

Hence, the answer is (200).

11. 160

Consider the following diagram.

(It is given that no student who participated in the 100 m race participated in the 300 m race or the 350 m race. Also, no student who participated in the 400 m race participated in the 200 m or 250 m race.)



Where x represents exactly one event, y represents exactly two events and z represents exactly three events.

Also, the number of students participating in exactly four events is 20.

$$\text{So, } 6x + 10y + 6z = 880$$

Given: $x: y: z = 1: 2: 3$ (i.e. $y = 2x$ and $z = 3x$)

$$\therefore 6x + 20x + 18x = 880$$

$$\therefore x = 20$$

The number of students who participated in 100 m race = $x + 2y + z = 8x = 160$.

Hence, the answer is 160.

12. 240

Based on the analysis done in the previous questions, the number of students who participated in both 200 m and 250 m races = $3z + y + 20 = 9x + 2x + 20 = 11x + 20 = 240$

Hence, the answer is 240.

13. 740

Based on the analysis done in the previous questions, the number of students who did not participate in the 400 m race = $900 - (x + 2y + z) = 900 - (20 + 80 + 60) = 900 - 160 = 740$

Hence, the answer is 740.

14. 520

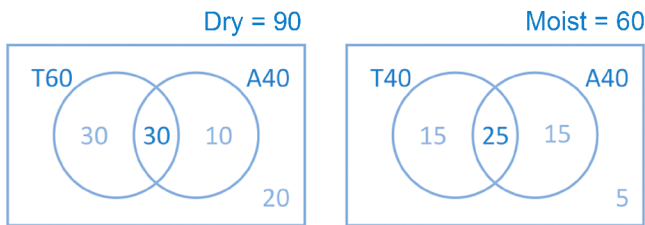
Based on the analysis done in the previous questions, the number of students who participated in at most two events = $6x + 10y = 26x = 520$

Hence, the answer is 520.



Level of Difficulty - 3

1. (A)



The data of the number of dry and moist sweets of manufacturer R passing the parameters of Taste (T) and Appearance (A) is shown in the Venn diagrams given above.

‘Passed either of the parameters’ means at least one of the parameters.

Hence, as shown in the above diagram, the total number of patented specimens of manufacturer R which passed either of the parameters in the final round = $(30+30+10) + (15 + 25 + 15) = 125$

Hence, option (A) is correct.

2. (C)

This is an easy question. Based on the values given in the table,

$$\text{Value of } p = (60 + 50 + 60) / (100 + 90 + 90) = 170/280 = 17/28$$

$$\text{Value of } q = (40 + 50 + 40) / (100 + 90 + 90) = 130/280 = 13/28$$

$$\text{Value of } m = (50 + 30 + 40) / (80 + 80 + 60) = 120/220 = 12/22$$

$$\text{Value of } n = (40 + 50 + 40) / (80 + 80 + 60) = 130/220 = 13/22$$

It can clearly be observed that $17/28 > 12/22$,

and $13/28 < 13/22$,

that is $p > m$ and $q < n$

Hence, option (C) is correct.

3. (B)

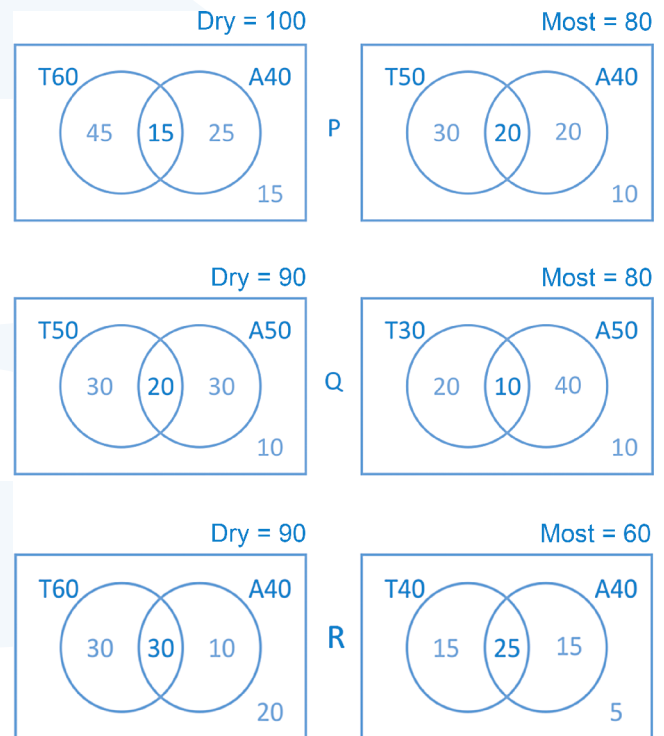
Number of dry sweets of manufacturer P, Q, and R passing both the parameters = $(15+20+30) = 65$

Number of moist sweets of manufacturer P, Q, and R passing both the parameters = $(20 + 10 + 25) = 55$

Hence, the total number of sweets passing both the parameters of taste and appearance in the final round = $65 + 55 = 120$.

Hence, option (B) is correct.

4. (D)



The data of the number of dry and moist sweets of manufacturers P, Q, and R passing the parameters of taste (T) and appearance (A) are as expressed in the above-given Venn Diagrams.

So, the total number of patented specimens of manufacturers P, Q, and R which failed to pass either of the two

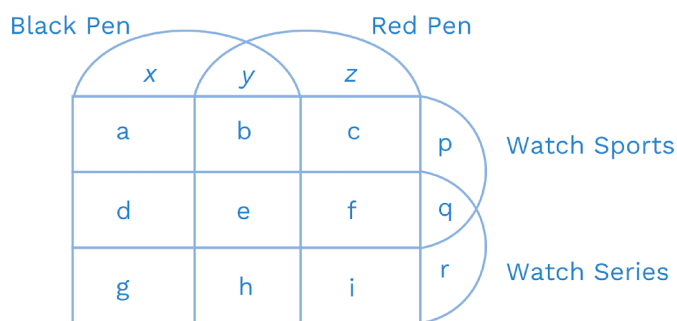


parameters of taste and appearance in the final round = $(15 + 10 + 20) + (10 + 10 + 5) = 70$

Hence, option (D) is correct.

5. 5

The following Venn diagram will help us solve this set easily.



- The union of black pens and red pens must be equal to 60 as everyone is having at least a black pen or a red pen. Similarly, the union of sports and series must be equal to 60 as everyone watches at least one of 2.
- Also, in the Venn diagram, x, y, z, p, q, and r all must be equal to zero. (as each person has at least one pen and also has at least one hobby)
- As per the Venn Diagram above and the values given in the table,

$$\text{Black} \cap \text{Red} = b + e + h = 10 \quad \dots\dots (i)$$

$$\text{Series} \cap \text{Sports} = d + e + f = 19 \quad \dots\dots (ii)$$

$$\text{Black} \cap \text{Sports} = a + b + d + e = 25 \quad \dots\dots (iii)$$

$$\text{Red} \cap \text{Sports} = b + c + e + f = 24 \quad \dots\dots (iv)$$

$$\text{Black} \cap \text{Series} = d + e + g + h = 20 \quad \dots\dots (v)$$

$$\text{Red} \cap \text{Series} = e + f + h + i = 25 \quad \dots\dots (vi)$$
- Also, $\text{Black} \cap \text{Red} \cap \text{Sports} = b + e \quad \dots\dots (vii)$
- Now, using the formula for $A \cup B \cup C$,

$$\text{Black} \cup \text{Red} \cup \text{Sports} = \text{Black} + \text{Red} + \text{Sports} - \text{Black} \cap \text{Red} - \text{Red} \cap \text{Sports} - \text{Black} \cap \text{Sports} + \text{Black} \cap \text{Red} \cap \text{Sports}$$
 Since, $\text{Black} \cup \text{Red} = 60$,

$$\text{Black} \cup \text{Red} \cup \text{Sports} = 60 \quad (\text{as there were a total of 60 participants only})$$

So, $60 = 33 + 37 + 41 - 10 - 25 - 24 + \text{Black} \cap \text{Red} \cap \text{Sports}$

$$\text{Black} \cap \text{Red} \cap \text{Sports} = b + e = 8$$
 So, $h = 2$ [from equation (i)]

- Similarly, solving for black pen, red pen, and series, we get,
- $b = 3$ and $e = 5$
- Similarly, solving for sports, series, and red pen, we get,
- $d = 7$, and $f = 6$.
- Also, from equation (iii), we know $a + b + d + e = 25$, So, $a = 10$.
- Similarly solving for c, g, and i, we get $c = 9$, $g = 6$ and $i = 11$.

So, the final Venn diagram with all the values will be as shown below



Black Pen		Red Pen		
10	3	9	Watch Sports Watch Series	
7	5	7		
6	2	11		

So, the number of persons who used both red pen and black pen and watch sports and series = $e = 5$.

Hence, the answer is 5.

6. 9

As explained earlier, the number of persons who used red pen but not black pen and who watch sports but not watch series is 'c' = 9

Hence, the answer is 9.

7. 19

As explained earlier, the number of persons who used exactly one pen and watch only sports

= $a + c = 10 + 9 = 19$ Hence, the answer is 19.

8. 60

This is a very simple question. Since we know that each of the persons had at least one hobby and at most two hobbies (sports or series or both), the number of persons with at most two hobbies = 60 (i.e. the total number of persons who participated in the global meet)

Hence, the answer is 60.

9. 10

As it can be seen in the final Venn diagram, the number of people having both red and black pens = $3 + 5 + 2 = 10$

Hence, the answer is 10.

Common analysis for Questions 10 to 15

Out of 300 students,

the number of men = 60% of 300 = 180

the number of women = 60% of 300 = 120

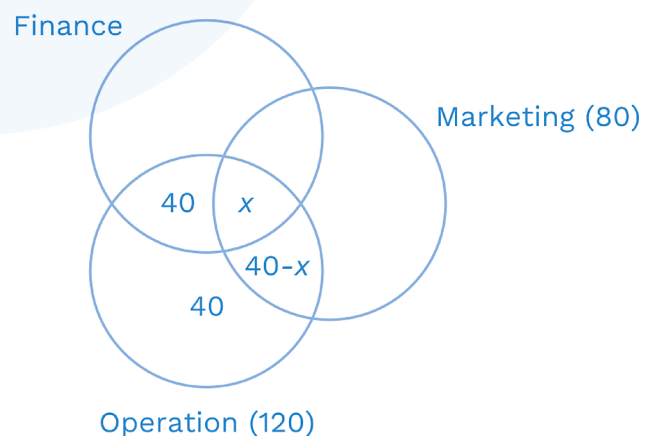
- From (g), the number of male students studying only operations is $\frac{1}{3} \times 120 = 40$, and the number of male students studying only operations and finance = $\frac{1}{3} \times 120 = 40$

- Of the male students, 120 study operations and 80 study marketing.

- Let 'x' be the number of males studying all three subjects (specialisations).

The following Venn diagram can be created for the males.

Total men = 180

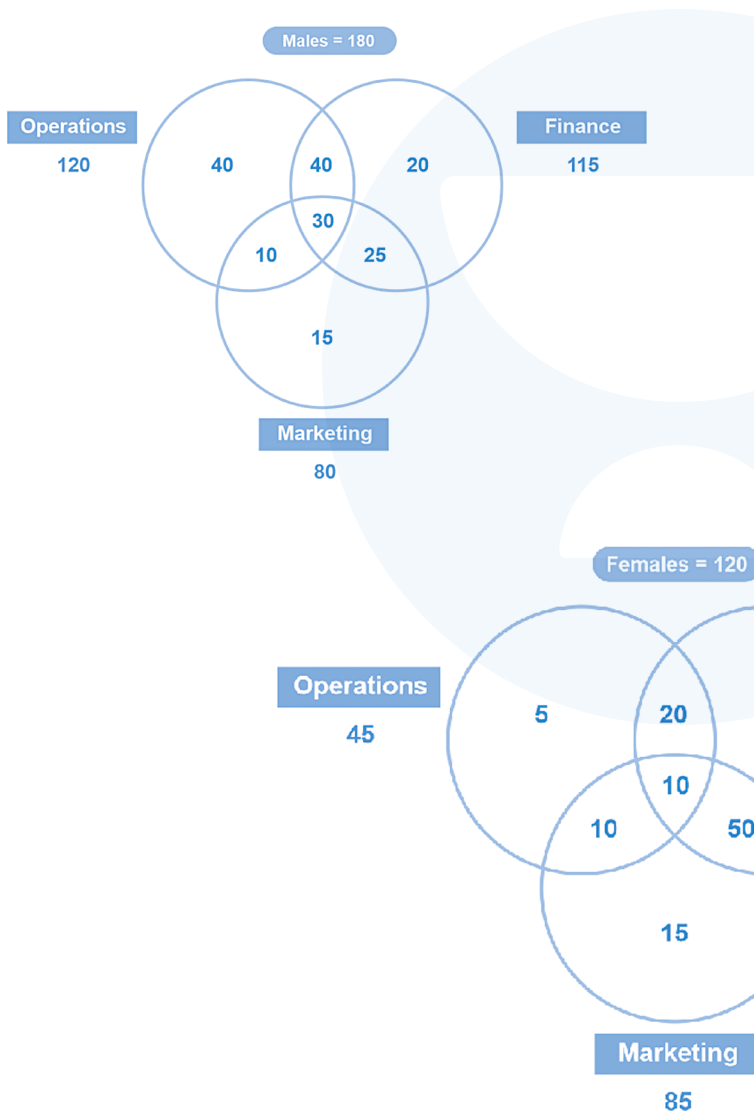
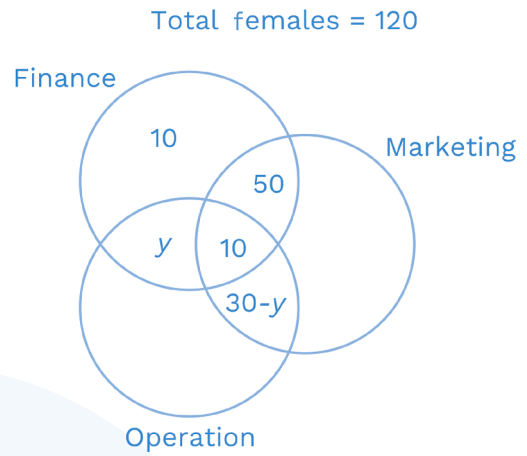


- From (viii), 105 men study more than one subject. So according to the figure, the number of men learning only marketing and finance are $105 - [40 + x + (40 - x)]$ $105 - 80 = 25$



- So, the number of men learning only marketing are $80 - [25 + x + 40 - x] = 15$
- Also, from (vi), we know that the number of male students studying all the three specialisations is twice the number of male students studying only marketing, i.e. $X = 2 \times 15 = 30$.
- Also, the number of males in only finance are $180 - (120 + 25 + 15) = 20$.
- By using the above information, we can fill the following values in the Venn diagram.

For women, using the information given in (i), (iv), (v), and (viii), along with the values obtained for males, we can create the following Venn diagram.





10. (B)

The number of male students studying finance and marketing only is 25.

Hence, option (B) is correct.

11. (D)

As explained earlier, the number of female students who study marketing is 85.

Hence, option (D) is correct.

12. (C)

As explained earlier, the number of male students who study finance is 115.

Hence, the correct option is (C).

13. (C)

As explained earlier, the number of female students who study all three specializations is 10.

Hence, the correct option is (C).

14. (D)

As explained earlier, the number of male students who study all three specializations is 30.

Hence, the correct option is (C).

15. (C)

As explained earlier, the number of female students who study only Operations is 5.

Hence, the correct option is (C).

Solution to Rack Your Brain

Rack Your Brain – 1

The minimum total number of livings can be Advait + 4 men + 2 women + 2 puppies.

(Considering a possibility that each of those four men saw the same two females.)

Hence, the answer is 9.

