7 Mensuration

Introduction

The chapter on mensuration has its share of importance in the CAT examination. It is also one of the easiest chapters, contributing to almost 6% to 8% of problems in the quantitative aptitude section of CAT. In the CAT examination and several other aptitude exams, many questions are from this topic only.

Therefore, the students who are not so good in other sections like 'algebra' or other logical topics must focus on this chapter. The question asked in this chapter are not much complex.

Definition

Mensuration is the branch of geometry and science of measuring the length, area, or volume in 2D and 3D shapes.

Planes

Planes are two-dimensional that are, namely, length and breadth. Plane mensuration also deals with sides, perimeters, and areas of the plane figure of different shapes and figures.

Solid Mensuration

It deals with the areas and volume of solid objects.

Basic Conversions

1. 1 m = 100 cm = 1,000 mm

   1 km = 1,000 m = $\frac{5}{8}$ miles

   1 inch = 2.54 cm
2. 1 m = 39.37 inches

   1 mile = 1,760 yd = 5,280 ft

   1 nautical mile (knot) = 6,080 ft
3. 100 kg = 1 quintal

   10 quintal = 1 tonne = 1,000 kg

   1 kg = 2.2 pounds (approx.)
4. 1 litre = 1,000 cm^3

   1 acre = 100 m^2

   1 hectare = 10,000 m^2

   1 hectometre^2 = 100 decametre^2
5. $\sqrt{2} = 1.4144$

   $\sqrt{3} = 1.732$

   $\sqrt{5} = 2.236$

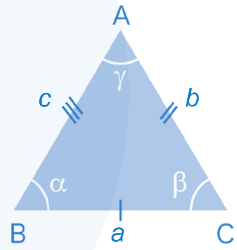
   $\sqrt{6} = 2.45$
6. Weight = volume × density

2D Figures

Scalene Triangle

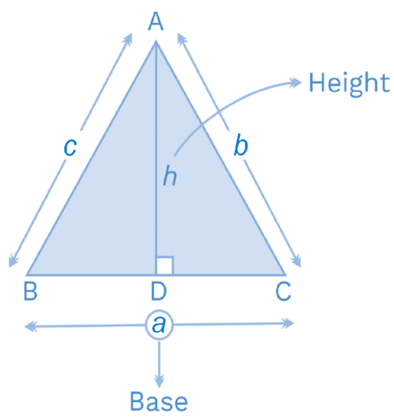
A scalene triangle is a triangle in which all the three sides are of different lengths, and all the three angles are of various measures.

Properties of Scalene Triangle



- It has no equal sides.
- It has no equal angles.
- It has no line of symmetry.
- It has no point of symmetry.
- The angles inside this triangle can be acute, obtuse, or right angles.

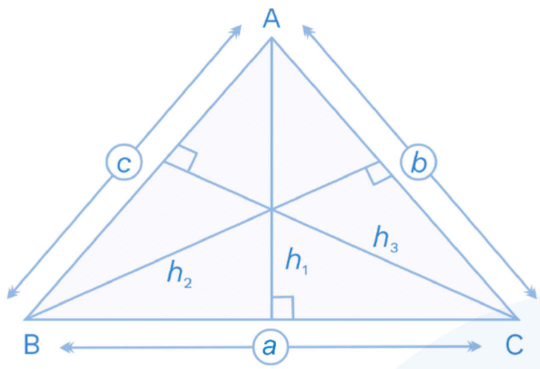
Area and perimeter of a scalene triangle



If the height is given:

Then, the area of the triangle $= \frac{1}{2} \times$ base $\times$ height.

- If all the three sides of a scalene triangle are given. Then, find the relation between heights and sides of the triangle.



Since we know that area of

$$\Delta = \frac{1}{2} \times a \times h_1 = \frac{1}{2} \times b \times h_2 = \frac{1}{2} \times c \times h_3$$

$$\frac{1}{2} \times a \times h_1 = \frac{1}{2} \times b \times h_2 = \frac{1}{2} \times c \times h_3$$

$$a \times h_1 = b \times h_2 = c \times h_3 \Rightarrow \text{constant}$$

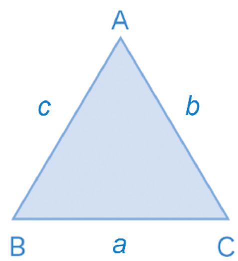
$$h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$$

$$a : b : c = \frac{1}{h_1} : \frac{1}{h_2} : \frac{1}{h_3}$$

If the height is not given in the scalene triangle, we will find the area of the triangle by using Heron's formula.

Heron's formula

If three sides of a triangle are given but height is not given.



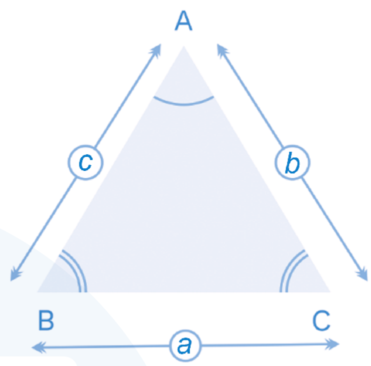
The perimeter (P) of the above given triangle $=$ $a + b + c$ units.

Semi-perimeter (S) $= \frac{P}{2} = \frac{a+b+c}{2}$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

where a, b, and c are the three sides of the triangle and S is the semi-perimeter of the triangle.

- If the angle between two sides of a triangle is given, then:



Area of triangle (A) $= \frac{1}{2} \times c \times b \times \sin A$

$$= \frac{1}{2} \times c \times a \times \sin B$$

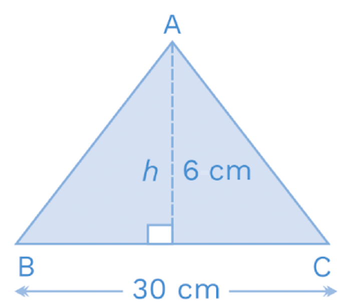
$$= \frac{1}{2} \times a \times b \times \sin C$$

Example 1:

Find the area of the triangle with a base of 30 cm and a height of 6 cm.

Solution: 90 cm²

We know that: the area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$



h = 6 cm

b = 30 cm

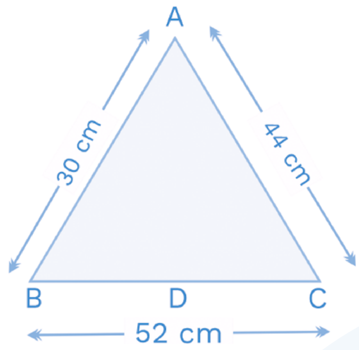
Area of triangle $= \frac{1}{2} \times 30 \times 6 = 90\,\text{cm}^2$

Example 2:

Find the area of the triangle whose sides are 52, 44, and 30 cm, respectively.

Solution: $33\sqrt{399}$ cm^2

Since in this question height is not given.



$P = a + b + c = 52 + 44 + 30 = 126$ cm

$S = \frac{P}{2} = \frac{126}{2} = 63\,\text{cm}$

$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{63(63-52)(63-44)(63-30)}$$

$$= \sqrt{63 \times 11 \times 19 \times 33}$$

$$= \sqrt{3^2 \times 7 \times 11 \times 11 \times 3 \times 19}$$

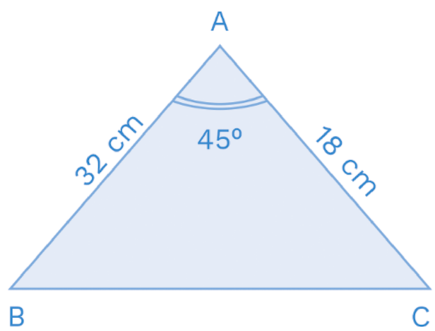
$$= 11 \times 3\sqrt{19 \times 21}$$

$$= 33\sqrt{399}\,\text{cm}^2$$

Example 3:

Find the area of the triangle whose two sides 32 and 18 cm are given and the angle between these two sides is 45°?

Solution: 203.64 cm²



$$\text{Area of the triangle ABC} = \frac{1}{2} \times AB \times AC \times \sin A$$

$$= \frac{1}{2} \times 32 \times 18 \times \sin 45°$$

$$= 288 \times \frac{1}{\sqrt{2}}$$

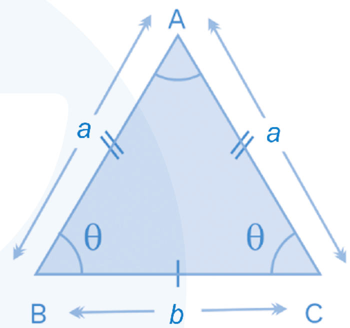
$$= 203.64\ \text{cm}^2$$

Isosceles Triangle

An isosceles triangle has at least two sides of equal length, and their two respective angles are also equal.

AB = AC

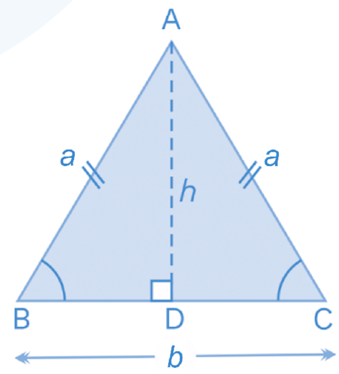
$\angle B = \angle C$



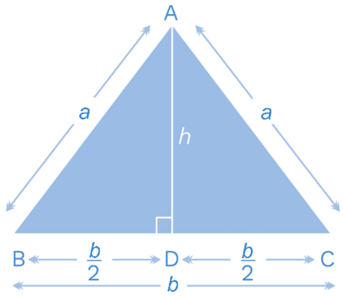
where $a \rightarrow$ equal sides

$b \rightarrow$ base

$h \rightarrow$ height of the altitude

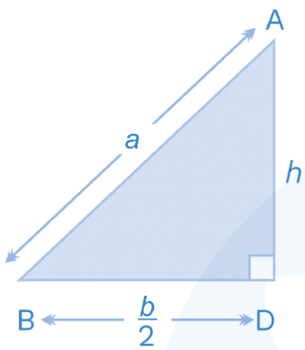


If AD is perpendicular to BC in an isosceles triangle then AD bisects the side BC in two equal parts.



1. Height of an isosceles triangle:

   Since triangle ADB is a right angle triangle.



   By using Pythagoras theorem,

$$a^2 = h^2 + \left(\frac{b}{2}\right)^2$$

$$h^2 = a^2 - \frac{b^2}{4}$$

$$h = \sqrt{\frac{4a^2 - b^2}{4}}$$

$$h = \frac{1}{2}\sqrt{4a^2 - b^2}$$

2. Perimeter (sum of all sides) = $2a + b$
3. Area of an isosceles triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

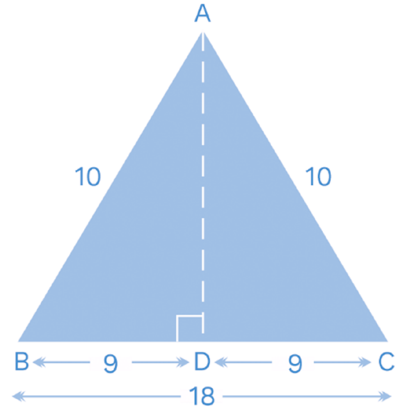
$$= \frac{1}{2} \times b \times \frac{1}{2}\sqrt{4a^2 - b^2}$$

triangle $= \dfrac{b}{4}\sqrt{4a^2 - b^2}$

Example 4:

Find the area, perimeter, and height of an isosceles triangle whose equal sides are 10 cm, and the base is 18 cm.

Solution: 9 × $\sqrt{19}$ cm²



$$\text{Height (AD)} = \frac{1}{2}\sqrt{4a^2 - b^2}$$

$$= \frac{1}{2}\sqrt{4 \times 10^2 - 18^2}$$

$$= \frac{1}{2}\sqrt{400 - 324}$$

$$= \frac{1}{2}\sqrt{76}$$

$$= \frac{1}{2} \times 2\sqrt{19} = \sqrt{19}$$

$$= 4.35 \text{ cm}$$

Perimeter = $2a + b = 2 \times 10 + 18 = 38$ cm

$$\text{Area} = \frac{b}{4}\sqrt{4a^2 - b^2}$$

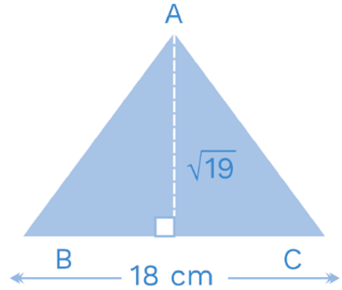
$$= \frac{18}{4}\sqrt{400 - 324}$$

$$= \frac{18}{4}\sqrt{76}$$

$$= \frac{9}{2} \times 2 \times \sqrt{19}$$

$$= 9 \times \sqrt{19} \text{ cm}^2$$

or



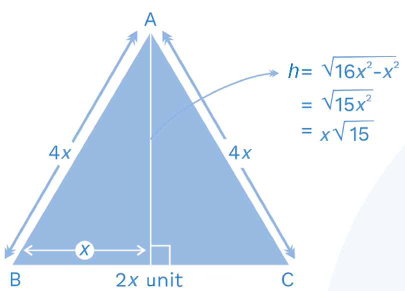
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 18 \times \sqrt{19} = 9\sqrt{19}\ \text{cm}^2$$

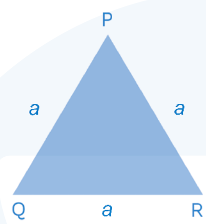
Example 5:

In an isosceles triangle, the length of each equal side is twice the length of the third side. The ratio of areas of the isosceles triangle and an equilateral triangle with the same perimeter is:

Solution: $\dfrac{36\sqrt{5}}{100}$

Let the third side is $2x$ unit then equal sides become $4x$ unit each.





The perimeter of the isosceles triangle = perimeter of equilateral triangle

$(4x \times 4x + 2x) = a + a + a$

$10x = 3a$

$$a = \frac{10x}{3}$$

$$\frac{\text{Area of isosceles triangle}}{\text{Area of equilateral triangle}} = \frac{\frac{1}{2} \times 2x \times x\sqrt{15}}{\frac{\sqrt{3}}{4} \times \left(\frac{10x}{3}\right)^2}$$

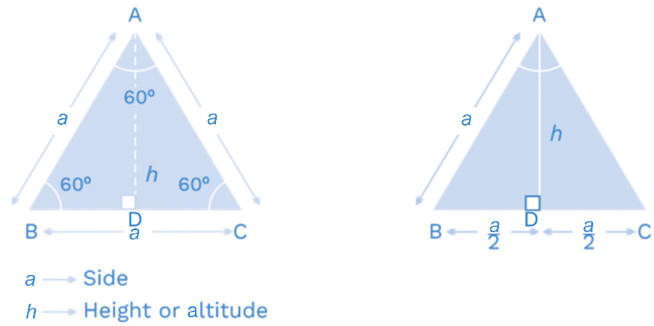
$$= \frac{\frac{1}{2} \times 2x \times x\sqrt{15}}{\frac{\sqrt{3}}{4} \times \frac{100x^2}{9}}$$

$$= \frac{9\sqrt{15}}{25\sqrt{3}} = \frac{9 \times \sqrt{3} \times \sqrt{5}}{25 \times \sqrt{3}}$$

$$= \frac{9\sqrt{5}}{25} = \frac{36\sqrt{5}}{100}$$

Equilateral Triangle

An equilateral triangle is a triangle in which all three sides have the same length and all three internal angles are congruent to each other with an angle of 60° each.



In right-angled triangle ADB.

$$h = \sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{4a^2 - a^2}{4}} = \sqrt{\frac{3a^2}{4}}$$

a) $h = \dfrac{\sqrt{3}a}{2} \rightarrow$ height of an equilateral triangle

b) $\text{Area} = \dfrac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times a \times \frac{\sqrt{3}a}{2} = \frac{\sqrt{3}}{4}a^2$$

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

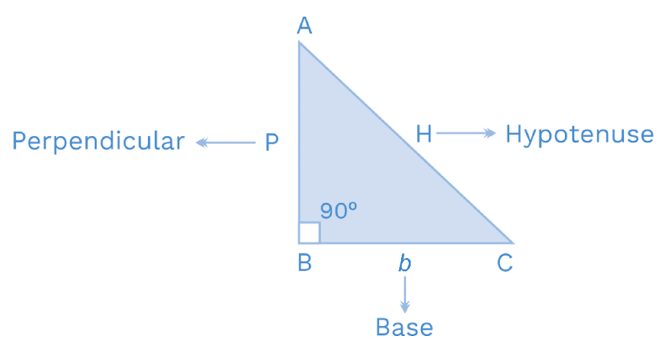
c) Perimeter (P) = sum of all sides

$P = 3a$

Right Angle Triangle

A right triangle is a triangle with one of the angles as 90°.

The 90° angle is called a right angle and hence the triangle with a right angle is called a right triangle. We can easily understand the relationship between the various sides of the right triangle with the help of Pythagoras' rule.



By Pythagoras theorem

1. $H^2 = P^2 + b^2$

$H = \sqrt{P^2 + b^2}$

$\downarrow$

Hypotenuse or diagonal

2. $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

$\text{Area} = \frac{1}{2} \times b \times P$

3. Perimeter = $P + b + H$

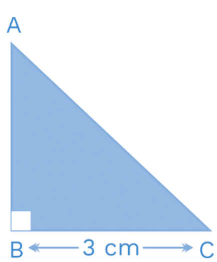
Some important Pythagoras triplet

3, 4, 5 ($2\times$)	5, 12, 13	8, 15, 17
6, 8, 10 ($\times 2$)	10, 24, 26	7, 24, 25
9, 12, 15	15, 36, 39	9, 40, 41
12, 16, 20	⋮	11, 60, 61
15, 20, 25		20, 21, 29
⋮		12, 35, 37
		16, 63, 65
		28, 45, 63

Triplets are nothing but the sides of a right angle Δ and become very helpful when we are solving the problems related to the right angled triangle.

Triplet of the odd number: If one side of a right angle Δ is given. Then we can find the other two sides by using the Pythagorean triplet.

Example: One side of a right angle is given as 3 cm. Find the other two sides of this right-angle triangle.



$3 \rightarrow 3^2 = 9$

$\rightarrow \frac{(9-1)}{2} = 4$

and $\frac{(9+1)}{2} = 5$

Hence, the other two sides are 4 and 5 cm.

Triplet of an even number: 8, 15, and 17 are the sides of a right angle.

$8 \rightarrow 8^2 = 64$

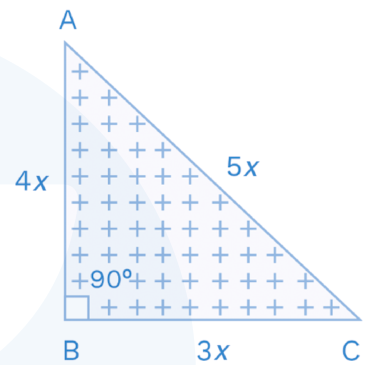
$\rightarrow (64/4) - 1 = 15$

and $(64/4) + 1 = 17$

Example 6:

The perimeter of a right triangular field is 360 m. The three sides of the field are in the ratio of 3:4:5. Find the area of the field.

Solution: 5,400 m²



Perimeter $\Rightarrow 4x + 3x + 5x = 360$ m

$12x = 360$

$x = 30$

Therefore, AB = $4x = 4 \times 30 = 120$ m

BC = $3x = 3 \times 30 = 90$ m

AC = $5x = 5 \times 30 = 150$ m

Hence, area of the triangle ABC $= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 90 \times 120$

$= 45 \times 120$

$= 5{,}400 \text{ m}^2$

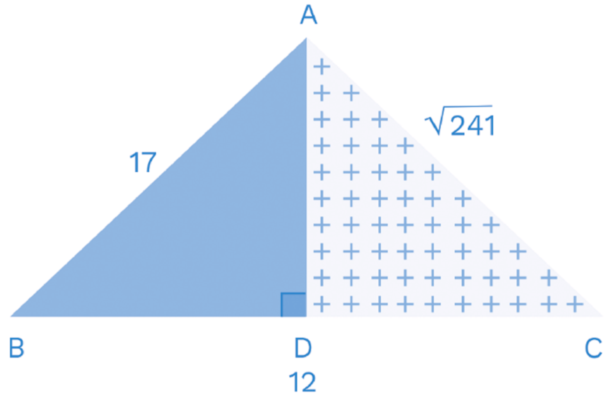
Example 7:

In triangle ABC, AB = 17 cm, BC = 12 cm, AC = $\sqrt{241}$ cm, AD $\perp$ BC, then find the area of the triangle ADC.

(A) 24 cm² (B) 36 cm²

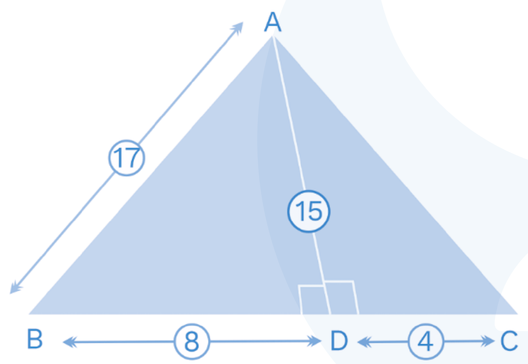
(C) 27 cm² (D) 30 cm²

Solution: (B)



After seeing 17 as the hypotenuse of right angle triangle ABD, the triplet concept must have come once in your mind.

We know that → 8, 15, and 17 are triplets and this triplet also satisfies the condition given in the second triangle.

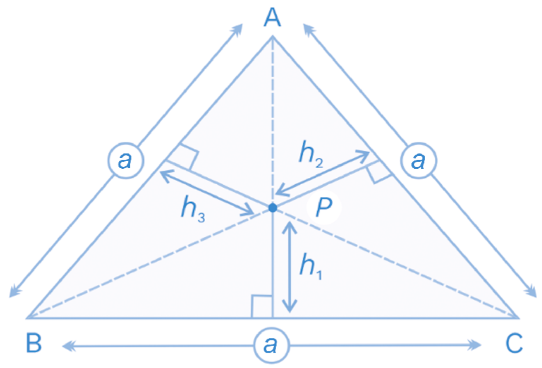


Now, the area of the ΔADC =

$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 15 = 30 \text{ cm}^2$

Hence, option (D) is the correct answer.

Special Case (Equilateral Triangle)



P is any point inside of triangle ABC.

Area of triangle ABC = area of triangle BPC + area of triangle APC + area of triangle APB.

$$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \times a \times h_1 + \frac{1}{2} \times a \times h_2 + \frac{1}{2} \times a \times h_3$$

$$\frac{\sqrt{3}}{2}a = h_1 + h_2 + h_3$$

We know that height of an equilateral Δ,

$$h = \frac{\sqrt{3}a}{2}$$

$\Rightarrow h = h_1 + h_2 + h_3$

where h is the height of the equilateral triangle.

Now, $\frac{\sqrt{3}a}{2} = h_1 + h_2 + h_3$

$$a = \frac{2}{\sqrt{3}}(h_1 + h_2 + h_3)$$

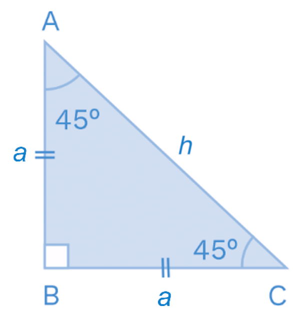
$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times \left[\frac{2}{\sqrt{3}}(h_1 + h_2 + h_3)\right]^2$$

$$\text{Area} = \frac{1}{\sqrt{3}}(h_1 + h_2 + h_3)^2 \text{ or } \frac{1}{\sqrt{3}}h^2$$

where h_1, h_2, and h_3 are the perpendicular distance from the interior point P in an equilateral triangle.

Note: This is applicable the only equilateral triangle.

Isosceles Right Angle



In isosceles right angle triangle, both the sides (base and perpendicular) are equal in magnitude, and their opposite angles are also equal (i.e., 45° each):

Area of an isosceles right angle

$$\text{Triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times a \times a$$

$$\text{Triangle} = \frac{1}{2} \times a^2$$

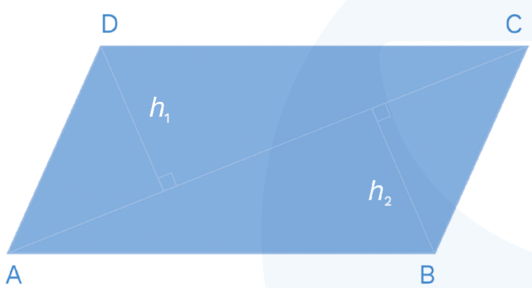
Perimeter = $2a + h$

Hypotenuse (h) or diagonal $= \sqrt{a^2 + a^2}$

$h = \sqrt{2}a$

Quadrilaterals

If no three points out of the four are collinear, joining four points in order is called a quadrilateral. A quadrilateral has four sides, four angles, and four vertices. The sum of the angles of a quadrilateral is 360°.



AC is one of the diagonals, and h_1, h_2 are the altitudes on AC from the vertices D and B, respectively.

The perimeter of a quadrilateral = sum of all the sides

$$P = AB + BC + CD + DA$$

Area of the quadrilateral = area of triangle ADC + area of triangle ABC.

$$= \frac{1}{2} \times AC \times h_1 + \frac{1}{2} \times AC \times h_2$$

$$\text{Area of the quadrilateral ABCD} = \frac{1}{2} \times AC \times (h_1 + h_2)$$

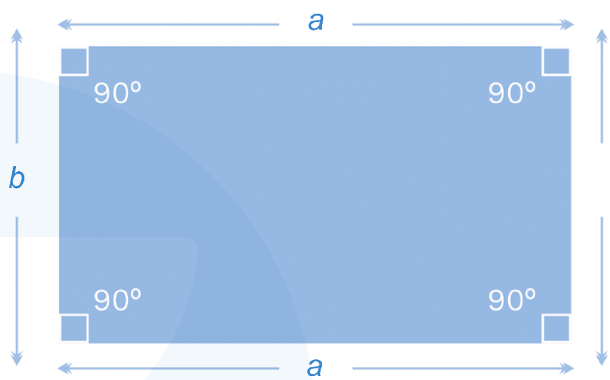
There are five different types of quadrilaterals based on their shape:

1. Rectangle
2. Square
3. Parallelogram
4. Rhombus
5. Trapezium

Rectangle

The word rectangle comes from the Latin word rectangulus, derived from two words, rectus, meaning proper and angulus meaning angle.

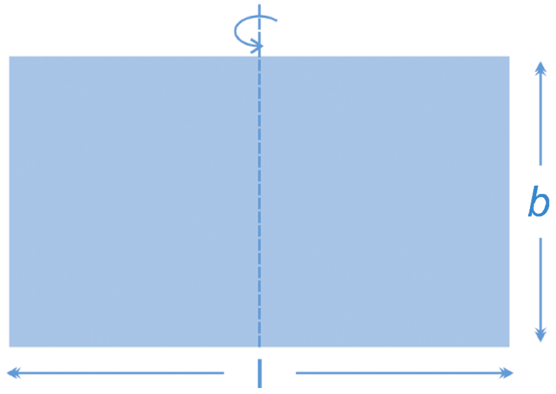
A rectangle is a two-dimensional geometric shape with four sides, four vertices, and four angles. The opposite sides of a rectangle are equal in length and parallel to each other. Moreover, each of the four internal angles of a rectangle measures 90°.



Properties of a rectangle

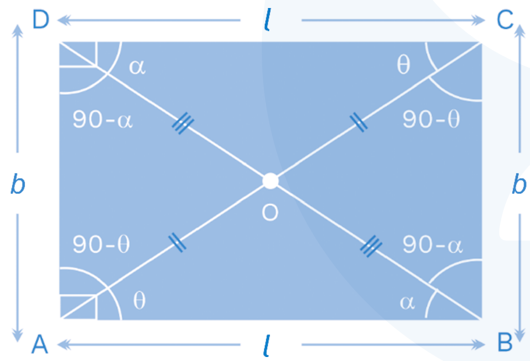
- The opposite sides are parallel and equal to each other.
- Each internal angle of a rectangle measures 90°.
- Since, the opposite angle of a rectangle is equal, then a rectangle also becomes a parallelogram.
- The diagonals of a rectangle bisect each other and are of the same length.
- The two diagonals of a rectangle bisect each other at a different angle – one is an obtuse angle and the other, i.e., an acute angle.
- If the diagonals of a rectangle bisect each other at 90°, then it becomes a square.
- Both the diagonals have the same length.
- The diagonal of a rectangle is the diameter of its circumcircle.
- When a rectangle is rotated along the line joining the midpoint of the longer parallel sides. In this case, the height of the cylinder is equal to the width of the rectangle,

and the diameter of the cylinder is equivalent to the length of the rectangle.



- If the rectangle is rotated along the line joining the midpoint of the shorter parallel sides. In this case, the height of the cylinder is equal to the length of a rectangle and the diameter of the cylinder is equivalent to the width of a rectangle.

Some important formulae of a rectangle



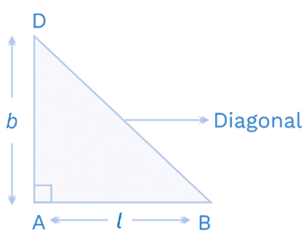
1. Perimeter of a rectangle (sum of all sides)

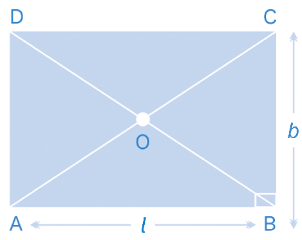
   = 2(length + breadth) = 2(l + b)

2. Area of a rectangle = length × breadth

   $A = l \times b$

3. Diagonal of a rectangle:





- Diagonal $= \sqrt{l^2 + b^2}$
- Diagonals do not bisect vertex angles in a rectangle and also triangle AOB ≅ triangle COD.
- AO = BO = OC = OD $= \frac{AC}{2} = \frac{BD}{2} = \frac{\text{Diagonal}}{2}$

Example 8:

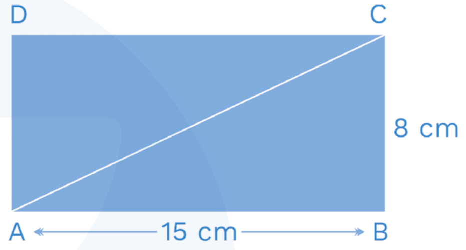
If the length and breadth of a rectangle are 15 and 8 cm, respectively. Then find.

a) The perimeter of the rectangle

b) Area of the rectangle

c) Diagonal of the rectangle

Solution: a) 46 cm; b) 120 cm²; c) 17 cm



a) Perimeter = 2(l + b) = 2(15 + 8) = 46 cm

b) Area of the rectangle $= l \times b = 15 \times 8 = 120\ \text{cm}^2$

c) Diagonal of a rectangle (d), AC

$= \sqrt{15^2 + 8^2} = \sqrt{289} = 17\ \text{cm}$

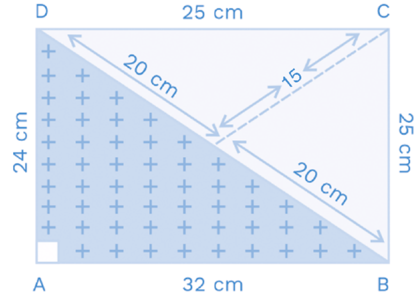
Example 9:

Two sides of a plot measure 32 and 24 cm and the angle between them is a perfect right angle. Then the other two sides measure 25 cm and the other three angles are not right angles. What is the area of the plot?

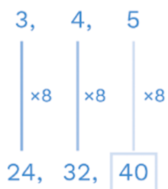
(A) 684 cm² (B) 648 cm²

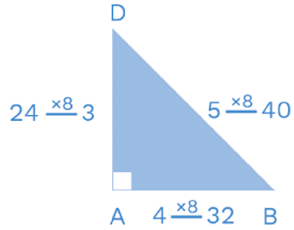
(C) 846 cm² (D) 964 cm²

Solution: (A)



By using triplet





We get the length of BD equivalent to 40 cm.

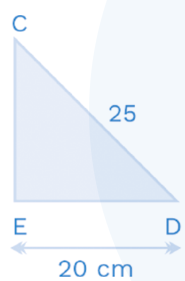
As we can see that Δ BDC is isosceles triangle and if CE is perpendicular to BD then BD is going to be divided into equal parts.

$$BE = DE = \frac{BD}{2} = \frac{40}{2} = 20\,\text{cm}$$

Now, we have to find the length of CE (height).

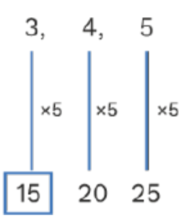
Since triangle CED is a right-angled triangle .

So that by using Pythagoras theorem we can find the length of CE.



$$CE = \sqrt{25^2 - 20^2} = \sqrt{225} = 15 \text{ cm}$$

Or we can use simply Pythagorean triplets



Hence, the area of triangle CDB $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 40 \times 15$$

$$= 20 \times 15 = 300\,\text{cm}^2$$

Therefore, the area of the plot = area of triangle ABD + area of triangle BDC

$$= \frac{1}{2} \times 32 \times 24 + 300$$

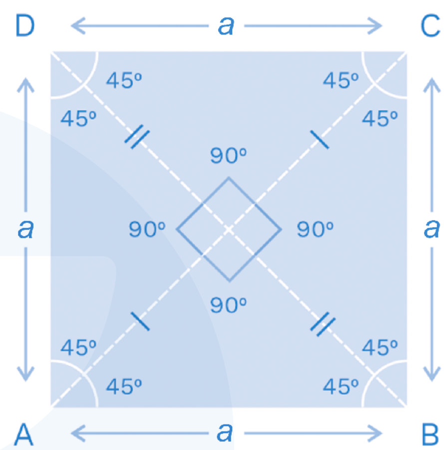
$$= 384 \text{ cm}^2 + 300 \text{ cm}^2$$

$$= 684 \text{ cm}^2$$

Hence, option (A) is the correct answer.

Square

A square is a regular quadrilateral with four equal sides and angles. It has all the angles equal to 90°. We can also see it as a rectangle whose two adjacent sides are equal.



Sides: AB = BC = CD = DA

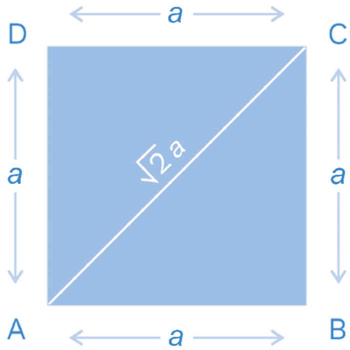
Diagonals: AO = OC and DO = OB

Angles: ∠A = ∠B = ∠C = ∠D

Important properties of a square

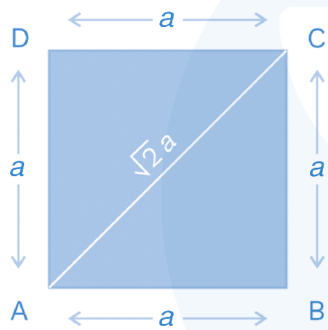
- All the four sides of a square are congruent or equal to each other.
- Each interior angle of a square is equal to 90°.
- The diagonals of a square are equal in length and bisect each other at 90°.
- The diagonal of a square divides the square into two equal, isosceles triangles.
- The diagonals of a square bisect the internal angles.
- The opposite sides of a square are parallel.
- The sum of the two adjacent angles is 180°.
- The square is a highly symmetric figure.
- A square can also be defined as a parallelogram with equal diagonals that bisect the angles.

Important formulae



1. Perimeter of a square (P) = 4 × sides of a square
   (sum of all sides) = $4a$
2. Area of a square (A) = $(\text{side})^2 = a^2$

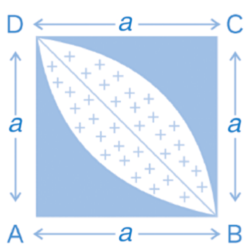
Diagonal of a square



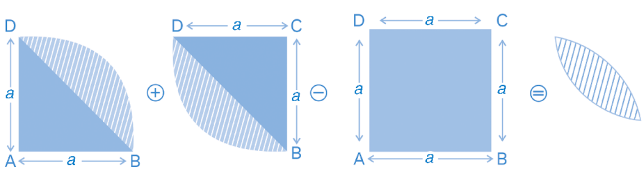
$AC = \sqrt{AB^2 + BC^2} = \sqrt{a^2 + a^2}$

Diagonal $= \sqrt{2}a$

Concept of leaf



Find the area of the leaf



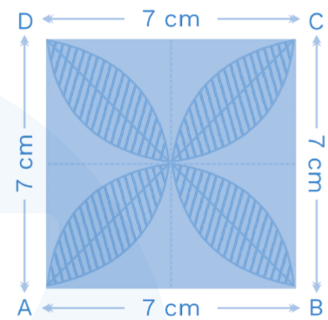
Area of leaf $= \frac{\pi}{4}a^2 + \frac{\pi}{4}a^2 - a^2$

$$= \frac{2\pi}{4}a^2 - a^2 = \frac{\pi}{2}a^2 - a^2$$

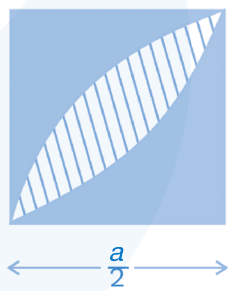
$$= a^2\left[\frac{\pi}{2} - 1\right] = a^2\left[\frac{22}{7} \times \frac{1}{2} - 1\right]$$

$$= a^2\left[\frac{4}{7}\right] = \frac{4}{7}a^2$$

Example: Find the area of the blue-shaded region.



Solution:



We know that area of the one leaf $= \frac{4}{7}a^2$

Therefore, the area of the four-leaf

$$= 4 \times \frac{4}{7} \times \left(\frac{7}{2}\right)^2 = 4 \times \frac{4}{7} \times \frac{49}{4} = 28 \text{ cm}^2$$

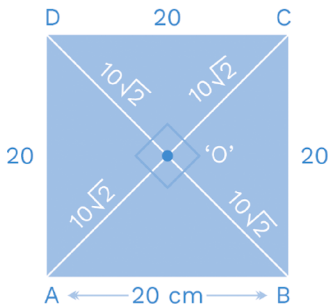
Example 10:

ABCD is a square whose side is 20 cm. By joining opposite vertices of square ABCD, we get four triangles. What is the sum of the perimeter of these four triangles?

(A) $40\sqrt{2}$ (B) $80\sqrt{2} + 80$

(C) $40\sqrt{2} + 40$ (D) $40\sqrt{2} + 80$

Solution: (B)



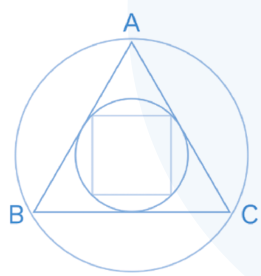
Since the perimeter of one triangle (AOB) $= 20 + 10\sqrt{2} + 10\sqrt{2} = 20 + 20\sqrt{2}$

We know that in a square, all four triangles are congruent to each other.

Therefore, the perimeter of all the triangles $= 4[20 + 20\sqrt{2}] = 80 + 80\sqrt{2}$ cm

Example 11:

As shown, a square is inscribed in the in-circle of an equilateral triangle ABC. What is the ratio of the area of the square and the area of the circumcircle of the triangle ABC?

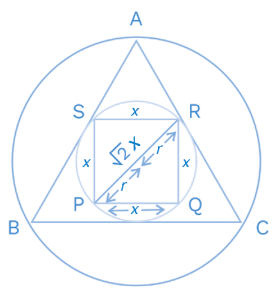


(A) 44:7 (B) 7:44

(C) 46:49 (D) 48:61

Solution: (B)

Let the side of the equilateral triangle ABC be a and the inradius and circumradius of the triangle ABC be r and R, respectively. Moreover, let us suppose the side of the square is x and the diagonal of the square $\sqrt{2}x$.



$\sqrt{2}x = 2r$

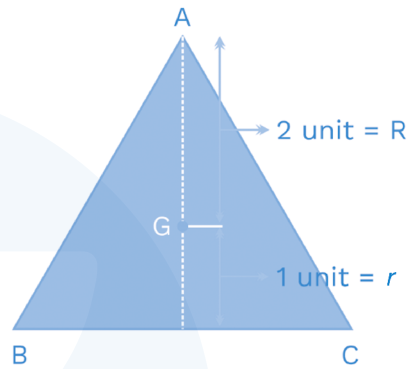
$x = \sqrt{2}r$

Area of the square = (side)$^2 = (\sqrt{2}r)^2 = 2r^2$

The area of the circumcircle of the equilateral triangle is $= \pi R^2$

$$\text{Required ratio} = \frac{\text{Area of the square}}{\text{Area of the circumcircle of } \Delta \text{ABC}} = \frac{2r^2}{\pi R^2}$$

Since we know that in an equilateral triangle $R:r = 2:1$

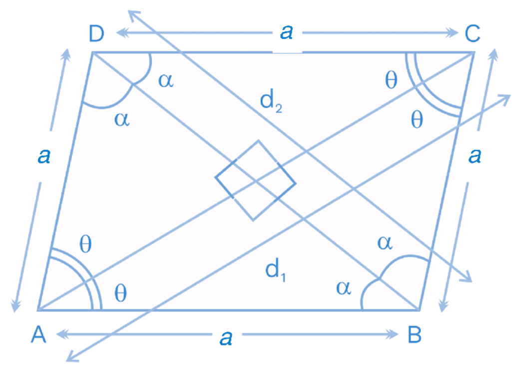


Hence, the required ratio

$$= \frac{2}{\pi}\left(\frac{1}{2}\right)^2 = \frac{2}{\pi} \times \frac{1}{4} = \frac{1}{2\pi} = \frac{1 \times 7}{2 \times 22} = \frac{7}{44}$$

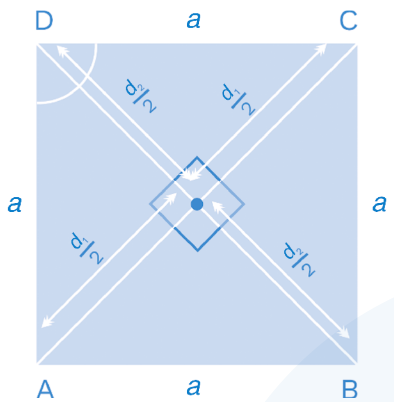
Rhombus

A rhombus is a quadrilateral with four equal-length sides and opposite sides parallel. All rhombuses or rhombi (plural of rhombus) are parallelogram. But not all parallelograms are rhombuses. All squares are rhombuses, but not all rhombuses are squares. The opposite interior angle of rhombuses is congruent. The diagonal of a rhombus always bisect each other at right angles.



Properties of a rhombus

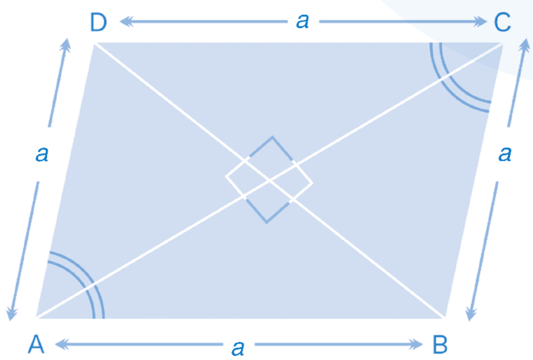
- All sides of a rhombus are equal.
- The opposite sides of a rhombus are parallel.
- Opposite angles are equal.
- In a rhombus, diagonals bisect each other at right angles.



- If you join the midpoints of the sides. Then you will get a rectangle.
- If you join the midpoints of half of the diagonals, you will get another rhombus.
- The sum of the two adjacent angles is 180°.
- Rhombus is a special case of a parallelogram.

Rhombus formulae

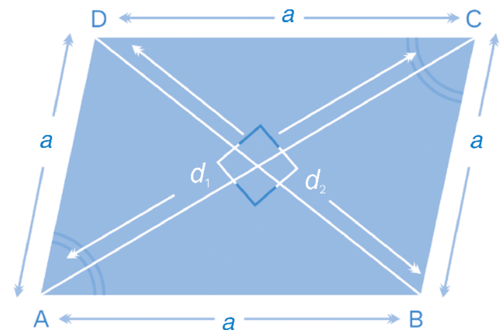
1. **The perimeter of a rhombus:**



Perimeter = sum of all sides = $4a$ units.

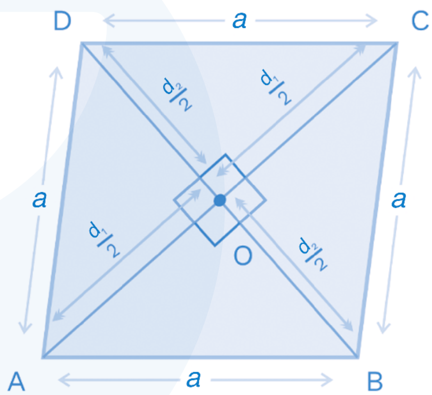
2. **Area of a rhombus:**

The area of a rhombus is the region covered by it in the two-dimensional plane. The formula for the area is equal to the product of diagonals of the rhombus divided by 2.



$$\text{Area} = \frac{1}{2} \times d_1 \times d_2 \text{ units}^2.$$

3. **Relationship between the diagonals and sides of a rhombus:**



Since AOB is a right-angled triangle.

By using Pythagoras theorem, we will get $AB^2 = AO^2 + BO^2$

$$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2; \qquad a^2 = \frac{d_1^2}{4} + \frac{d_2^2}{4}$$

$$a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$$

↓

Side of a rhombus

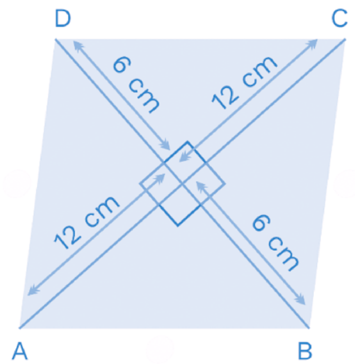
Example 12:

The two diagonals of a rhombus are 12 and 24 cm, respectively. Find:

a) Area of the rhombus.

b) The perimeter of the rhombus.

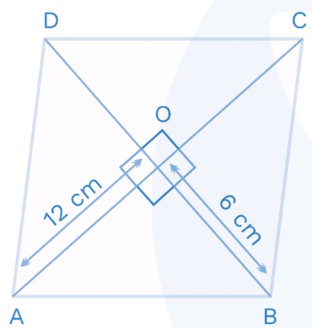
Solution: a) 144 cm²; b) $24\sqrt{5}$ cm



a) Area of rhombus $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 24 \times 12$

$= 144$ cm²

b) Before finding the perimeter, we have to find the side of the rhombus first.



Since we know that AOB is a right-angled triangle.

Hence, $AB^2 = AO^2 + OB^2$

$$AB^2 = 12^2 + 6^2 = 144 + 36$$

$$AB = \sqrt{180} = \sqrt{2 \times 2 \times 3 \times 3 \times 5}$$

$$= 2 \times 3\sqrt{5} = 6\sqrt{5} \text{ cm}$$

Therefore, the perimeter of the rhombus $= 4 \times$ (side)

$= 4 \times 6\sqrt{5} = 24\sqrt{5}$ cm

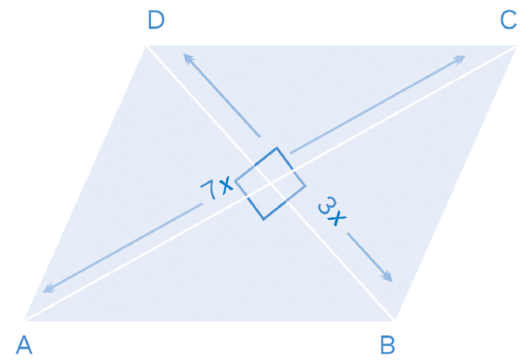
Example 13:

The ratio of the lengths of the diagonal of a rhombus is 3:7. The ratio of the area of the rhombus to the square of the shorter diagonal is:

(A) 7:6 (B) 5:7

(C) 5:3 (D) 3:5

Solution: (A)



Let diagonals of the rhombus are $7x$ unit and $3x$ unit.

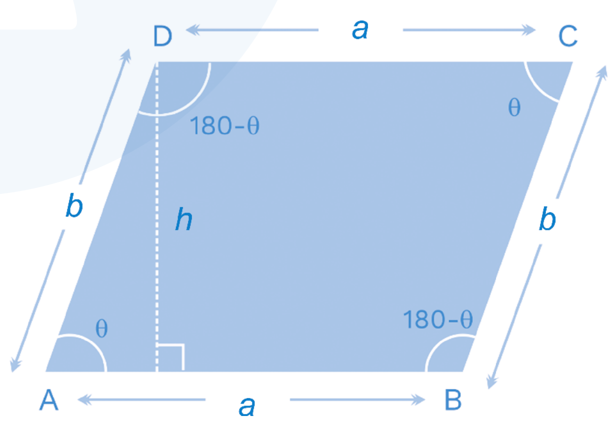
Area of the rhombus $= \frac{1}{2} \times 3x \times 7x = \frac{21x^2}{2}$ units²

Square of the shorter diagonal $= (3x)^2 = 9x^2$

Required ratio $= \dfrac{\frac{21x^2}{2}}{9x^2} = \frac{7}{6}$

Parallelogram

A parallelogram is a simple quadrilateral with two pairs of parallel sides. The opposite or facing sides of a parallelogram are of equal length and the opposite angles of a parallelogram are of equal measure.



1. Area of a parallelogram = base × height $= a \times h$.
2. Perimeter of a parallelogram $= 2(a + b)$ = 2 (sum of adjacent sides).
3. Area of the parallelogram = (product of any two adjacent sides × sine of the included angle).

Properties of parallelogram

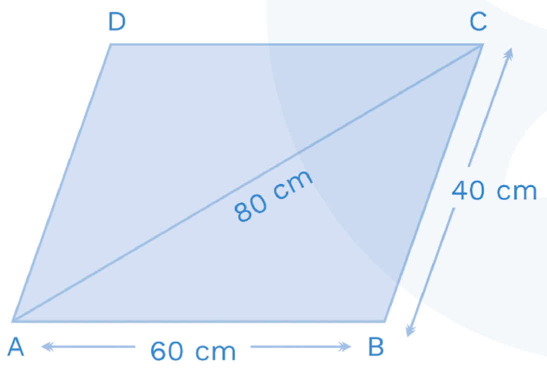
- The opposite sides are parallel.
- The opposite angles are equal.
- The consecutive angles are supplementary.
- If anyone of the angles is a right angle, then all other angles will be right angles.
- Two diagonals bisect each other.
- Each diagonal bisects the parallelogram into two congruent triangles.
- The sum of squares of the sides of a parallelogram is equal to the sum of squares of its diagonals. It is also called parallelogram law.
- Rhombus is a special case of a parallelogram.

Example 14:

A parallelogram has sides of 60 and 40 cm and one of its diagonal is 80-cm long. Then its area is:

Solution: $600\sqrt{15}$ cm^2

To find the area of the parallelogram, first, find the area of ABC by using Heron's formula then double it.



$$s = \frac{a+b+c}{2} = \frac{60+40+80}{2} = 90 \text{ cm}$$

Area of triangle ABC $= \sqrt{S(s-a)(s-b)(s-c)}$

$$= \sqrt{90(90-60)(90-40)(90-80)}$$

$$= \sqrt{90 \times 30 \times 50 \times 10}$$

$$= 100 \times 3\sqrt{3 \times 5}$$

Therefore, the area of parallelogram ABCD

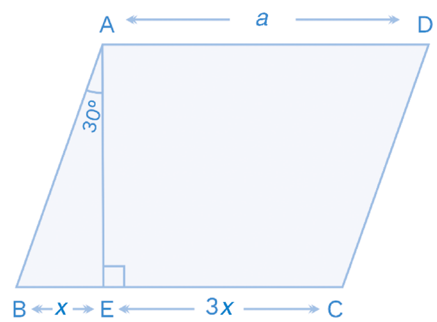
$= 2 \times$ area of triangle ABC

$= 2 \times$ area of triangle ADC

$= 2 \times 300\sqrt{15} = 600\sqrt{15}$ cm^2

Example 15:

In the figure given below, the ratio of the areas of the parallelogram ABCD and that of the triangle ABE is:

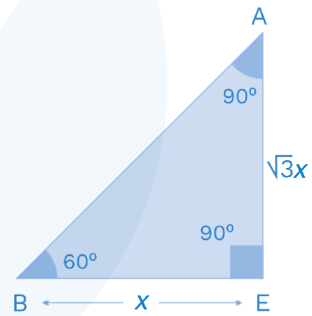


(A) 1:8 (B) 8:1

(C) 1:7 (D) 6:5

Solution: (A)

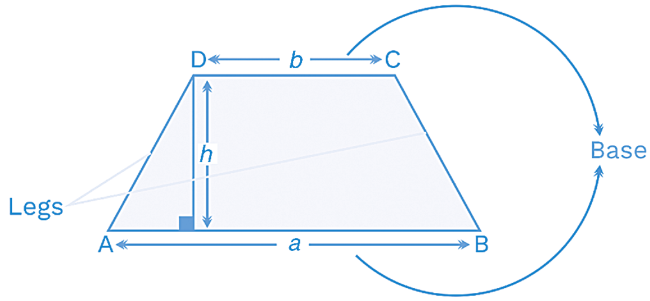
$$\frac{\text{Area of parallelogram ABCD}}{\text{Area of triangle ABE}} = \frac{BC \times AE}{\frac{1}{2} \times BE \times AE}$$



$$= \frac{BC \times AE}{\frac{1}{2} \times BE \times AE} = \frac{4x}{\frac{1}{2} \times x} = \frac{8x}{x} = \frac{8}{1}$$

Trapezium

The trapezium is a quadrilateral with two parallel sides. The parallel sides of a trapezium are called bases and the non-parallel sides of a trapezium are called legs. It is also called a trapezoid.



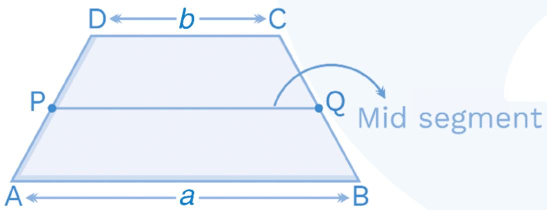
From the above figure, we can see that the sides AB and CD are parallel, whereas AD and BC are non-parallel sides. Moreover, h is the height of the trapezium, or we can say it is the shortest distance between two parallel sides of a trapezium.

Types of a trapezium

1. **Right trapezium:** A right trapezium has at least two right angles.
2. **Scalene trapezium:** A trapezium with all sides and angles are of different measures.
3. **Isosceles trapezium:** A trapezium whose legs or the non-parallel sides are of equal length.

Properties of a trapezium

- In trapezium one pair of opposite sides are parallel.
- The diagonals intersect each other.
- The non-parallel sides in the trapezium are of different measure except in the isosceles trapezium.
- The line that joins the mid-points of the non-parallel sides is always parallel to the bases or parallel sides and is equal to half of the sum of parallel sides.

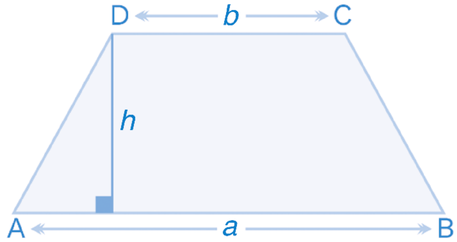


Length of mid-segment (PQ) $= \frac{1}{2}(a+b)$

Formulae

1. Area of the trapezium (A) $= \frac{1}{2}$ (sum of parallel sides)

   $\times$ Perpendicular distance between two parallel sides $= \frac{1}{2}(a+b) \times h$



2. Perimeter = sum of all sides = AB + BC + CD + DA

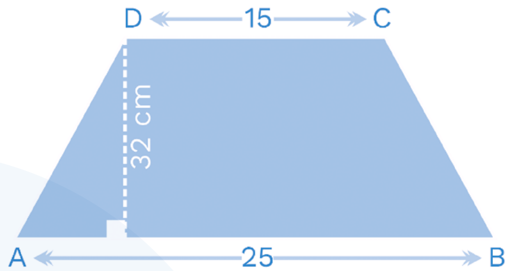
Example 16:

The length of the two parallel sides of a trapezium are 15 and 25 cm and its height is 32 cm. Find its area.

(A) 320 cm^2 (B) 280 cm^2

(C) 540 cm^2 (D) 640 cm^2

Solution: (D)



Area of the trapezium $= \frac{1}{2}(15+25) \times 32 = 20 \times 32 = 640$ cm^2.

Example 17:

The area of the trapezium is 256 cm^2 and the ratio of parallel sides is 9:7. Moreover, the perpendicular distance between them is 16 cm. The longer of parallel sides is:

Solution: 18 cm

Area of trapezium = 256 cm^2 (given)

$$\Rightarrow \frac{1}{2}(9x+7x) \times 16 = 256$$

$$\frac{1}{2} \times 16x \times 16 = 256$$

$$x = 2$$

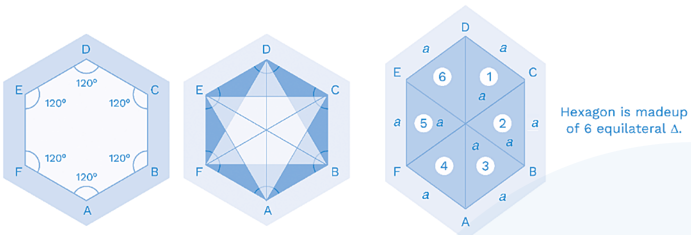
$\therefore$ Longer side $= 9x = 9 \times 2 = 18$ cm

Regular Hexagon

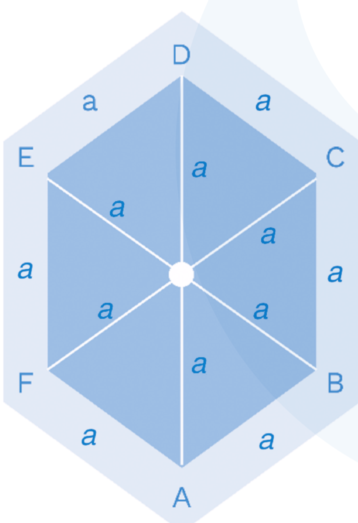
A regular hexagon is a closed shape polygon with six equal sides and six equal angles. In the case of any regular polygon, all its sides and angles are equal. For example, a regular octagon has eight equal sides, and a regular heptagon has seven equal sides. When such conditions are not met, polygons may resemble various irregular shapes. When we arrange six equilateral triangles side by side, a regular hexagon is composed.

Properties of a hexagon

- It has six equal sides and six equal angles.
- It has six vertices.
- Each interior angle is 120°, and each exterior angle is 60°.
- It is made up of six equilateral triangles.
- The sum of all interior angles of a regular hexagon equals 720°.
- One can draw nine diagonals inside a regular hexagon.



Formulae



1. Perimeter of a hexagon = $6a$
2. Area of a hexagon = 6 × area of one equilateral triangle $= 6 \times \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2$

Example 18:

Find the area and perimeter of a hexagon if all its sides have a length equal to 12 cm.

(A) $218\sqrt{3}\ \text{cm}^2$, 72 cm (B) 280 cm^2, 72 cm

(C) $216\sqrt{3}\ \text{cm}^2$, 72 cm (D) 220 cm^2, 72 cm

Solution: (C)

Perimeter = $6a = 6 \times 12 = 72$ cm

Area $= 6 \times \frac{\sqrt{3}}{4}a^2 = 6 \times \frac{\sqrt{3}}{4} \times 12^2 = 6 \times \frac{\sqrt{3}}{4} \times 144 = 216\sqrt{3}\ \text{cm}^2$

Example 19:

Find the area of a hexagon whose one side is 8 cm.

Solution: $96 \times \sqrt{3}\ \text{cm}^2$

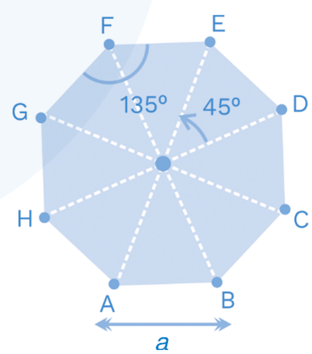
Area of hexagon $= 6 \times \frac{\sqrt{3}}{4}a^2 = 6 \times \frac{\sqrt{3}}{4} \times 8^2$

$= 6 \times \frac{\sqrt{3}}{4} \times 64 = 96 \times \sqrt{3}\ \text{cm}^2$

Regular octagon

A regular octagon is a closed figure with sides of the same length and internal angles of the same size. The sum of the internal angles of any octagon is 1,080°. As we know, in all polygons, the sum of external angles totals 360°. The internal angle at each vertex of a regular octagon is 135°, and the central angle is 45°.

The shape of an octagon



In the above figure, there are eight sides of the polygon and eight vertices as well. This is a regular octagon because all the angles and sides here are equal.

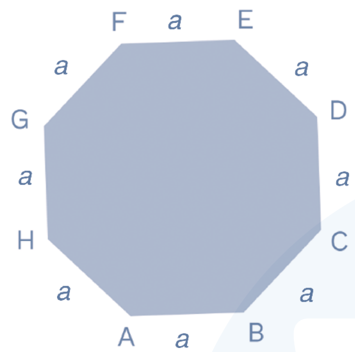
Properties of Octagon

- These have eight sides and eight angles.
- All the sides and all the angles are equal, respectively.
- There is a total of 20 diagonals in a regular octagon.

- The total sum of the interior angle is 1,080°, where each angle is equal to 135° (135 × 8 = 1,080°).
- The sum of all the exterior angles of the octagon is 360°, and each angle is 45° (45 × 8 = 360°).

Formulae

1. Perimeter of an octagon $= 8 \times$ (each side) $=$ $8a$ = (sum of all sides)



2. Area of an octagon $= 2a^2(1+\sqrt{2})$

Length of the longest diagonal of an octagon:

If we join opposite vertices of a regular octagon, then the diagonals formed have a length $(L) = a\sqrt{4+2\sqrt{2}}$.

Example 20:

If the length of the sides of a regular octagon is 10 cm. Find the perimeter and area.

(A) 482 cm^2, 80 cm (B) 475 cm^2, 80 cm

(C) 385 cm^2, 80 cm (D) 280 cm^2, 80 cm

Solution: (A)

Given that a = 10 cm

Therefore, perimeter = 8 × a = 8 × 10 = 80 cm

and, area $= 2a^2(1+\sqrt{2})$

$= 2 \times 10^2(1+\sqrt{2})$

$= 200(1+\sqrt{2})$

= 200(1 + 1.41) = 200(2.41)

≈ 482 cm^2 (approx.)

Example 21:

Find the length of the longest diagonal of a regular octagon whose side length is equal to 20 cm.

Solution: 52.260 cm

We know the formula for the longest diagonals is

$L = a\sqrt{4+2\sqrt{2}}$

$= 20\sqrt{(4+2\times 1.414)}$

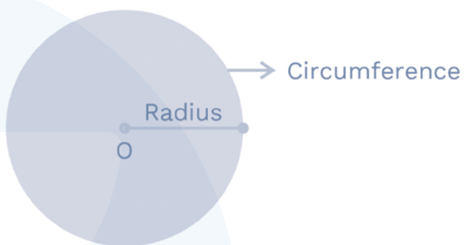
$= 20\sqrt{6.828}$

= 20 × 2.613

L = 52.260 cm

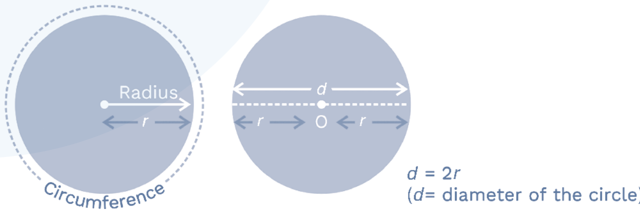
Circle

A circle is a closed two-dimensional figure in which the set of all the points in the plane is equidistant from a given point called the 'centre'.



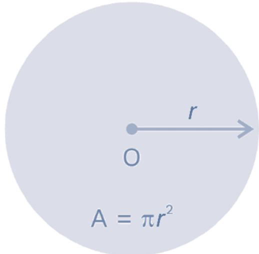
Important formulae

1. **Circumference of a circle (C):** The circumference of a circle is defined as the distance around the circle. It is also called the Perimeter of the circle.



Circumference (C) $= \pi d = 2\pi r$ (where $\pi = \frac{22}{7}$ or $\pi = 3.1415$)

2. **Area of a circle** $= \pi r^2$

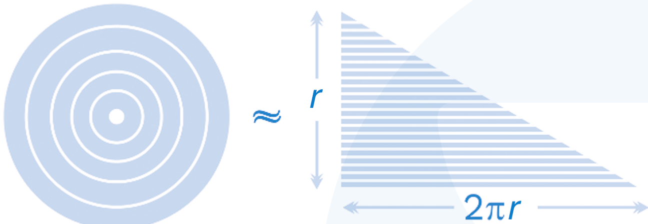


Properties of a circle

- The diameter of the circle divides the circle into two equal parts.
- The outer line of a circle is equidistant from the centre.
- Circles which have equal radii are congruent to each other.
- Circles that are different in size or have different radii are similar.
- The diameter of the circle is the largest chord and is double the radius.

Proof of area of a circle

Consider a concentric circle having an external radius to be r.



Open all the concentric circles to form a right-angle triangle. The outer circle would form a line having a length of $2\pi r$ forming the base. The height would be r.

Therefore, the area of the right-angled triangle would be equal to the area of the circle.

Area of circle = area of a triangle =

$$\left(\frac{1}{2}\right) \times b \times h = \frac{1}{2} \times 2\pi r \times r = \pi r^2$$

Example 22:

A circle of the maximum possible size is cut from a square sheet. Subsequently, a square of the maximum possible size is cut from the resultant circle. What will be the area of the final square?

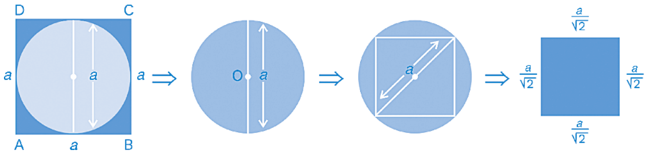
(A) 75% of the size of the original square.

(B) 50% of the size of the original square.

(C) 75% of the size of the circle.

(D) 25% of the size of the original square.

Solution: (B)



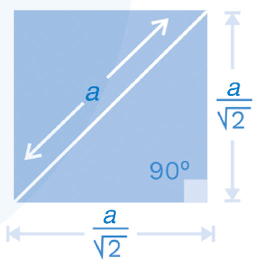
Let the side of the original square be a units.

Therefore, the area of the original square $= a^2$ units.

Concept 1: The diameter of the circle of the maximum possible dimension that is cut from the square will be the side of the square. So, its diameter will be a units (see above figure).

Concept 2: The diagonal of the square of the maximum possible dimension that can be cut from the circle will be the diameter of the circle. So, the diagonal of the final square will be a units (see above figure).

If the diagonal of the final square is a units then its side $= \frac{a}{\sqrt{2}}$ units.



If the side of the final square is $\frac{a}{\sqrt{2}}$ units, then its area $= \frac{a^2}{2}$ units^2.

Original square area	:	Final square area
a^2	:	$\frac{a^2}{2}$
2	:	1

-1

% reduction in the area $= \frac{1}{2} \times 100 = 50\%$

Therefore, the area of the new square will be 50% of the area of the original square.

Example 23:

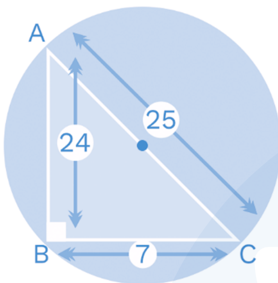
What is the circumradius of a triangle whose sides are 7, 24, and 25 cm, respectively?

(A) 18 (B) 12.5

(C) 12 (D) 14

Solution: (B)

[7, 24, 25] is a Pythagorean triplet, therefore the given triangle is a right angle.

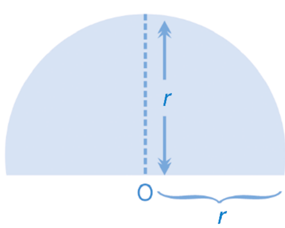


In a right angle triangle, circumradius measures half of the hypotenuse.

$R_C = \frac{\text{Hypotenuse}}{2}$ $\quad R_C = \frac{25}{2} = 12.5$ cm

Semicircle

A semicircle is a one-dimensional locus of points that forms half of a circle. The full arc of a semicircle always measures 180° (equivalently, π radians or a half turn).



Formulae

1. The perimeter of a semicircle $= \pi r + 2r = \frac{22}{7} \times r + 2r$

   $P = \frac{36}{7} r$

2. Area of the semicircle $= \frac{1}{2}\pi r^2$

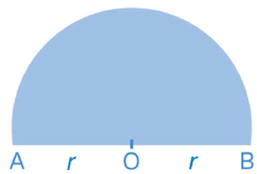
Example 24:

What is the perimeter of a semicircle whose radius is 3.5 cm?

(A) 18 cm (B) 9 cm

(C) 10 cm (D) 12 cm

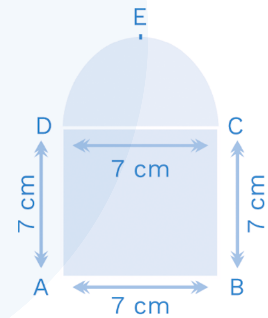
Solution: (A)



The perimeter of a semicircle $= \pi r + 2r = \frac{36}{7} \times 3.5 = 18$ cm

Example 25:

Find the area and perimeter of the figure in which ABCD is a square of side 7 cm, and DEC is a semicircle $\left(\text{Use } \pi = \frac{22}{7}\right)$.



Solution: 32 cm, 68.25 cm²

The required area = area of square ABCD + area of the semicircle DEC

$$= (\text{side})^2 + \frac{1}{2}\pi r^2$$

$$= 7^2 + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 49 + \frac{1}{2} \times \frac{22}{7} \times \frac{49}{4}$$

$$= 49 + \frac{11 \times 7}{4}$$

$$= 49 + \frac{77}{4}$$

$= 49 + 19.25 = 68.25\ cm^2$

The required perimeter = AB + BC + AD + semicircular arc DEC.

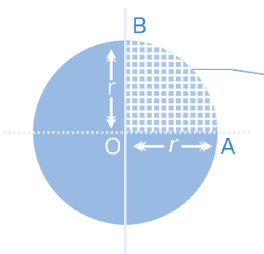
$$= 7 + 7 + 7 + \pi \times \frac{7}{2}$$

$$= 21 + \frac{22}{7} \times \frac{7}{2}$$

$$= 21 + 11 = 32\ cm$$

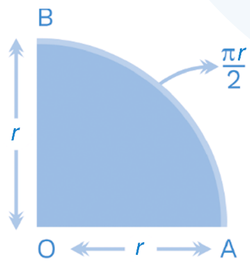
Quadrant

A circle is defined as the locus of all the points that are equidistant from the centre. Now, a quadrant is one-fourth section of a circle obtained when a circle is divided evenly into four sections or rather four quadrants by a set of two lines that are perpendicular to each other.



Formulae

1. The perimeter of a quadrant $= 2r + \frac{\pi r}{2}$



2. Area of a quadrant

   Area $= \frac{1}{4}\pi r^2$

   where r is the radius of the circle.

Example 26:

The radius of a circle is 2.1 cm. Find the area of a quadrant of the circle.

Solution: 3.465 cm²

Area of the quadrant $= \frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (2.1)^2$$

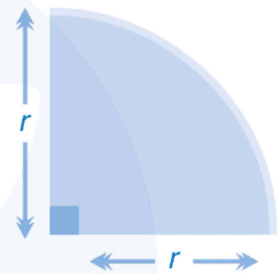
$$= \frac{1}{4} \times \frac{22}{7} \times 2.1 \times 2.1$$

$$= 3.465\ cm^2$$

Example 27:

The perimeter of a sheet of paper in the shape of a quadrant of a circle is 75 cm. Find its area $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Solution: 346.5 cm²



Let the radius be r cm

Then, the perimeter of a quadrant $= 2r + \frac{\pi r}{2}$

$$75 = 2r + \frac{1}{2} \times \frac{22}{7} r$$

$$75 = \frac{25}{7} r$$

$$r = 21\ cm$$

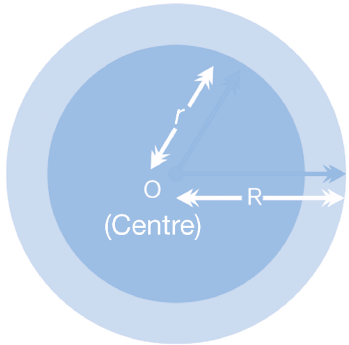
Now, the area of the quadrant $= \frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{11 \times 63}{2} = \frac{693}{2} = 346.5\ cm^2$$

Annulus or Ring

A ring (annulus) shaped object bounded by the circumference of two concentric circles of two different radii. One way to think of it is a circular disk with a circular hole in it. The outer and inner circle circles that define the ring are concentric, i.e., they share a common centre point.



The dimension of a ring is defined by the two radii R, r, which are the radii of the outer ring and the inner whole, respectively. We can find the area of the circular ring by subtracting the area of the small circle from that of the large circle.

Here, are formulas to find the area of the ring or annulus.

1. $A = \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

where, A = area of annulus

R = outer radius

r = inner radius

2. The perimeter of the inner circle = $2\pi r$

The perimeter of the outer circle (ring) = $2\pi R$.

Example 28:

The outer diameter and inner diameter of a circular path are 40 and 20 cm, respectively. Find the breadth and the area of the circular path.

(A) 20 cm, 950 cm^2 (B) 10 cm, 942.857 cm^2

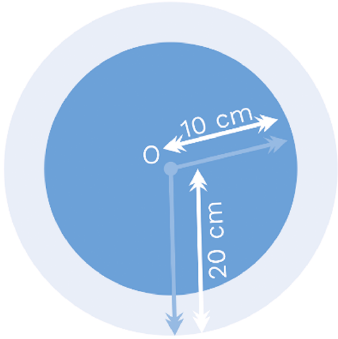
(C) 30 cm, 988 cm^2 (D) 15 cm, 940 cm^2

Solution: (B)

The outer radius of a circular path

$(R) = \frac{40}{2} = 20\,\text{cm}$

The inner radius of a circular path $= \frac{20}{2} = 10$ cm



Therefore, the breadth of the circular path = $R - r = 20 - 10 = 10$ cm.

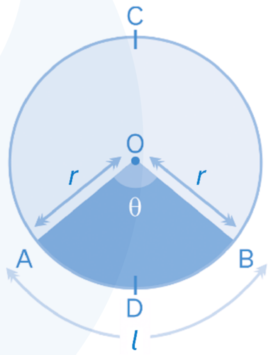
Area of the circular path $= \pi(R^2 - r^2)$

$= \pi(20^2 - 10^2)$

$= \frac{22}{7} \times 30 \times 10$

$= 942.857\ cm^2$

The Sector of a Circle



A sector is said to be a part of a circle made of the arc of the circle along with its two radii. It is a portion of the circle formed by a portion of the circumference (arc) and radii of the circle at both endpoints of the arc.

A circle containing a sector can be further divided into two regions: major and minor sectors. In the above figure, OACB is known as the major sector and OADB is known as the minor sector. As major represents big or large and minor represent small. In a semi-circle, there is no major and minor sector.

Formulae

Length of an arc of a sector $(l) = \frac{\theta}{360} \times 2\pi r$

Area of a sector $= \frac{\theta}{360} \times \pi r^2$

The perimeter of a sector (OADB) = $2r + l$

where l = length of the arc.

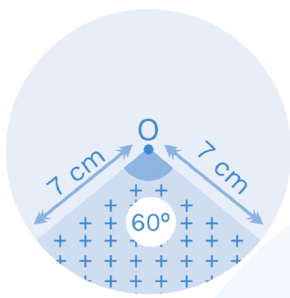
Example 29:

A circle, where the radius is 7 cm and its sector angle is 60°. Find the area of the sector.

Solution: 25.67 cm²

Given, a radius (r) = 7 cm

$\theta = 60°$



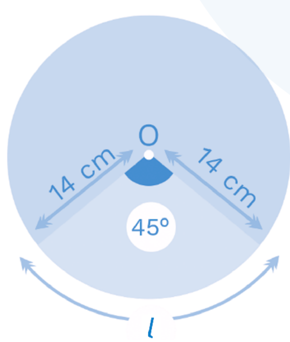
Hence, the area of the sector $= \dfrac{\theta}{360} \times \pi r^2$

$$= \frac{60}{360} \times \frac{22}{7} \times 7^2 = \frac{1}{6} \times \frac{22}{7} \times 49 = 25.67 \text{ cm}^2$$

Example 30:

Find the perimeter of a sector with a central angle of 45° and a radius of 14 cm.

Solution: 39 cm



Given, radius (r) = 14 cm

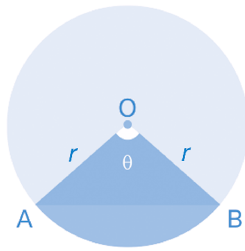
$\theta = 45°$

Length of the arc (l)

$$= \frac{\theta}{360} \times 2\pi r = \frac{45}{360} \times 2 \times \frac{22}{7} \times 14 = 11 \text{ cm}$$

Perimeter of sector = $l + 2r = 11 + 2 \times 14 = 11 + 28 = 39$ cm

A Segment of a Circle



$\theta \Rightarrow$ the angle of the sector

$r \Rightarrow$ radius

AB $\Rightarrow$ chord

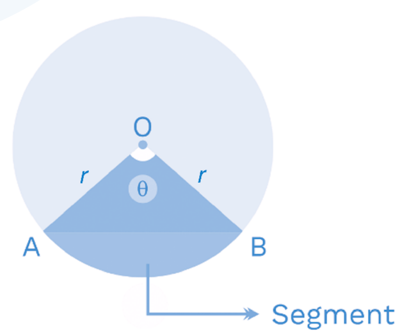
ACB $\Rightarrow$ arc of the circle

A segment of a circle is the region enclosed by a chord and an arc so formed touching the endpoints of the chord.

What is the area of the segment?

The area of a segment is the area enclosed between the chord and the minor or major arc of the circle. We find the area of the segment with the help of a central angle formed by the chord and radius of the circle (denoted as θ, see above figure).

The area of the segment can be found by taking the difference between the area of the sector and the area of the triangle enclosed in it. Or else, it can be found by subtracting the area of the given segment (major or minor) from the area of the circle.



1. Area of the segment = area of sector OAB – area of triangle AOB

$$= \frac{\theta}{360} \times \pi \times r^2 - \frac{1}{2} \times r^2 \sin\theta = r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$$

2. Perimeter of the segment =

$$\left[\frac{\theta}{360} \times 2\pi r + 2r\sin\left(\frac{\theta}{2}\right)\right]$$

Example 31:

Find the area of the minor segment portion if the radius of the wheel is 16 cm and the measurement angle is 30°.

(A) $64\left[\frac{\pi}{3} - 1\right]\text{cm}^2$ (B) $38\left[\frac{\pi}{3} - 1\right]\text{cm}^2$

(C) $520\left[30 - \frac{\pi}{4}\right]\text{cm}^2$ (D) $64\ \text{cm}^2$

Solution: (A)

Area of a segment of the wheel = (A) = $r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]\text{cm}^2$

$$A = 16^2\left[\frac{\pi \times 30^\circ}{360^\circ} - \frac{\sin 30^\circ}{2}\right]$$

$$A = 256\left[\frac{\pi}{12} - \frac{1}{4}\right]$$

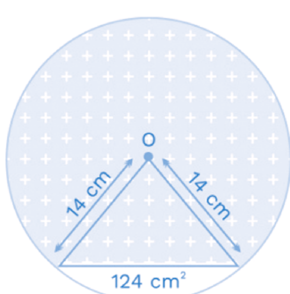
$$A = 64\left[\frac{\pi}{3} - 1\right]\text{cm}^2$$

Hence, option (A) is the correct answer.

Example 32:

Find the area of the major segment of a circle if the area of the corresponding minor segment is 124 cm² and the radius is 14 cm.

Solution: 492 cm²



Area of the major segment = area of the circle – area of the minor segment

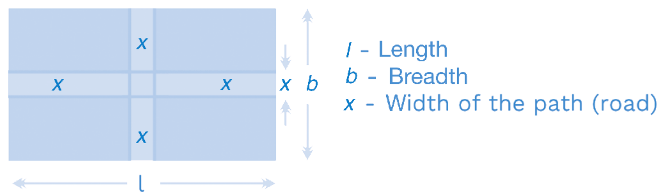
$= \pi r^2 - 124$

$= \frac{22}{7} \times 14 \times 14 - 124\,\text{cm}^2$

$= 22 \times 2 \times 14 - 124$

$= 44 \times 14 - 124 = 492\ \text{cm}^2$

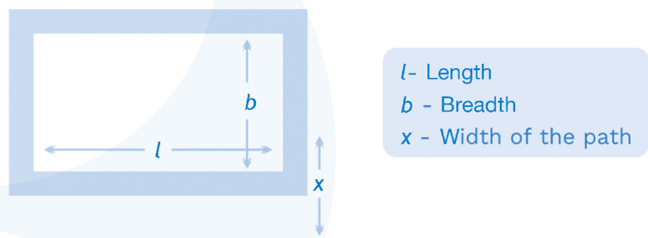
Pathways running across the middle of a rectangle



1. Area path = $l \times x + b \times x - x^2 = (l + b - x) \times x$ unit²

2. Perimeter of the path = $2(l + b) - 4x = 2[l + b - 2x]$

Outer pathways

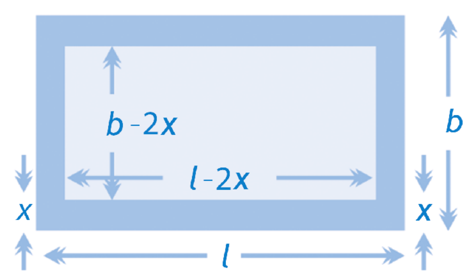


1. Area of the outerpath = $(l + b + 2x) \times 2x$

2. Perimeter → Inner → $2(l+b)$

Perimeter → Outer → $2(l+b+4x)$

Inner pathways



1. Area of the innerpath = $(l + b - 2x) \times 2x$

2. Perimeter → Inner → $2(l+b - 4x)$

Perimeter → Outer → $2(l+b)$

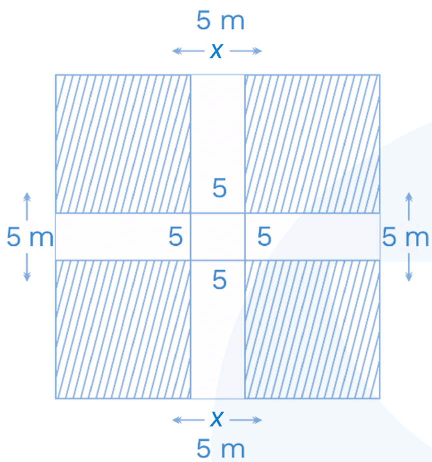
Example 33:

A rectangular lawn 60×40 m^2 has two rods each 5 m wide running between the park. One is parallel to the width and the other is parallel to the length. The cost of gravelling is 60 paise/m^2. Find the total cost of gravelling.

(A) ₹285 (B) ₹300

(C) ₹275 (D) ₹270

Solution: (A)



Area of shaded region $= 60 \times 5 + 40 \times 5 - 5^2 =$ $300 + 200 - 25 = 475$ m^2.

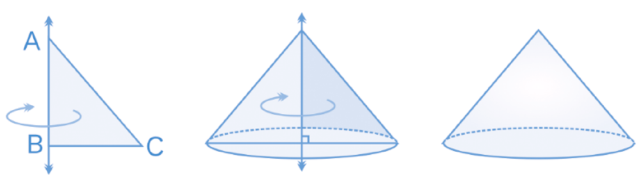
Total cost $= 475 \times 0.6 =$ ₹285

Hence, option (A) is the correct answer.

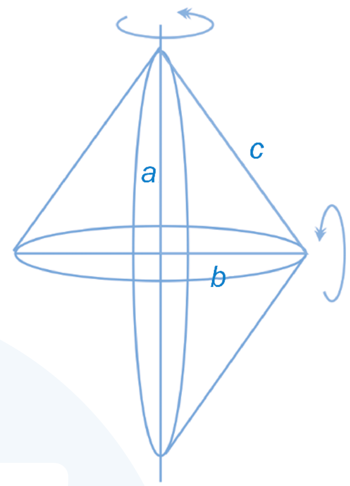
Rotation of 2D figures

When a part of the whole 2D figure is rotated then that portion of the figure will act as a 3D figure when the revolution happens.

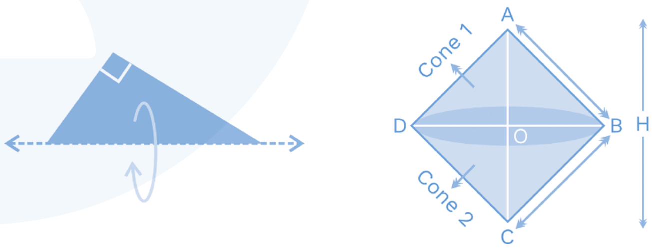
Triangle



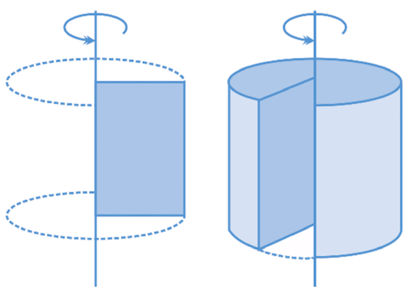
If we rotate a right triangle by its perpendicular we will get a cone with the base of the triangle as the radius of the cone and the height of the triangle as the height of the cone. Moreover, the hypotenuse of the triangle will act as the slant height of the cone. Similarly, if we rotate the triangle by its base, its height will act as the radius and the base will act as the height of the cone. Moreover, the hypotenuse of the triangle will act as the slant height of the cone.



If we revolve a right triangle by its hypotenuse we will get a double-pointed cone with a radius of the cone equal to the height of the triangle and the height of the given figure equal to the length of the hypotenuse of the right triangle.



Rectangle



If we rotate a rectangle by its length, we will get a cylinder with its radius equal to the rectangle's breadth and its length as the height of the cylinder. Similarly, if we rotate a rectangle by its breadth, we will get a cylinder with its radius equal to the rectangle's length and its breadth as the height of the cylinder.

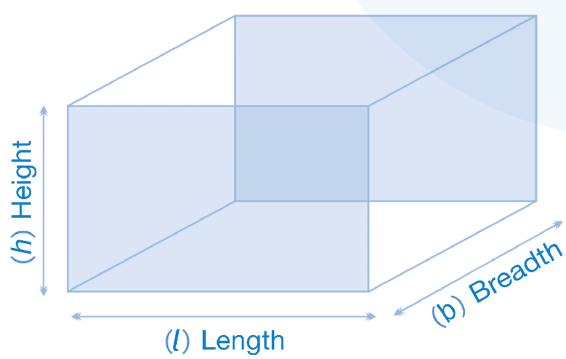
3D Mensuration

If several surfaces or planes surround a shape, then it is a 3D shape. These shapes are called 3-dimensional as they have length, breadth, and depth (height).

3D-mensuration deals with the shape like the cube, cuboid, sphere, hemisphere, cylinder, cone, etc. In this chapter, problems are generally based on curved surface area or lateral surface area, total surface area, and volume.

Cuboid

A cuboid is a 3-dimensional solid shape that has six faces, eight vertices, and 12 edges. This shape is the most commonly seen shape around us which has 3 dimensions, length, width, and height. The base of a cuboid is rectangular and we know about the rectangle that It has four sides and is a two-dimensional shape. Let us imagine a shape that is formed when many congruent rectangles are placed one on top of the other. The shape thus formed is a cuboid.



Properties of a cuboid

- A cuboid has six faces, eight vertices, and 12 edges.
- All the faces of a cuboid are rectangular.
- The opposite edges of a cuboid are parallel to each other.
- The base of a cuboid is rectangular.
- All the angles formed at the vertices of a cuboid are right angles.

Formulae

1. The lateral surface area of a cuboid or area of the four walls = $2(l + b) \times h$.
2. Total surface area of a cuboid = $2(lb + bh + hl)$
3. The volume of the cuboid = length × breadth × height = $l \times b \times h$

   where l = length

   b = breadth

   h = height
4. Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Example 34:

The length, breadth, and height of a box are 4, 3, and 1.6 m, respectively. What would be the cost of canvas to cover it up entirely, if one square meter of canvas costs ₹50?

(A) ₹2,320 (B) ₹2,222

(C) ₹2,830 (D) ₹1,980

Solution: (A)

Since, we know the total surface area of a cuboid or box

$= 2(lb + bh + hl)$

$= 2(4 \times 3 + 3 \times 1.6 + 1.6 \times 4)\ m^2$

$= 2(12 + 4.8 + 6.4)\ m^2$

$= 46.4\ m^2$

Now, it is given in the question that 1 m^2 of canvas costs ₹50.

Therefore, the total expenditure = 46.4×50 = ₹2,320.

Example 35:

The external dimensions of a wooden box closed at both ends are 24, 16, and 10 cm, respectively, and the thickness of the wood is 5 mm. If the empty box weighs 8 kg. Find the weight of 1 cubic cm of wood.

(A) 1.5 g (B) 0.9 g

(C) 10.884 g (D) 1 g

Solution: (C)

The external volume of the box = $24 \times 16 \times 10 = 3,840$ cm^3.

The thickness of wood is given as 5 mm = 0.5 cm.

Therefore, the internal length of the box = $24 - 2 \times 0.5 = 23$ cm.

Internal breadth of the box = $16 - 2 \times 0.5 = 15$ cm, and

The internal height of the box = $10 - 2 \times 0.5 = 9$ cm.

Now, we have to find:

The internal volume of the box = $23 \times 15 \times 9 = 3,105$ cm^3.

$\therefore$ The volume of the wood = $3,840 - 3,105 = 735$ cm^3.

Now, the total weight of the wood = volume × weight of 1 cm^3 of wood

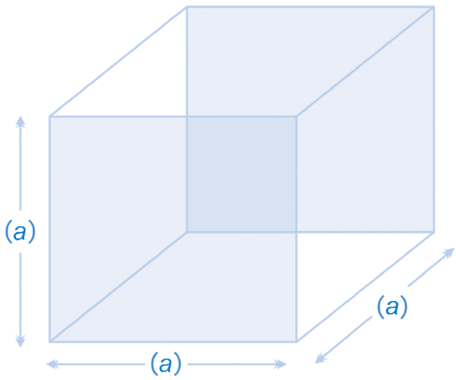
800 g = 735 × weight of 1 cm^3 wood

$\therefore$ Weight of 1 cm^3 wood $= \frac{8,000}{735} = 10.884$ g

Hence, option (C) is the correct answer.

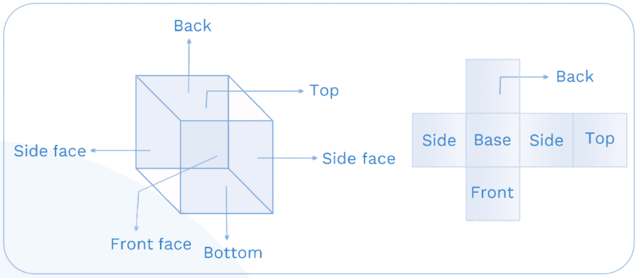
Cube

A cube is a 3-dimensional solid object with six square faces and all the sides of a cube are of the same length. A cube consists of six square faces, eight vertices, and 12 edges. The length, breadth, and height are of the same measurement. In a cube, the faces share a common boundary called the edge which is considered the bounding line of the edge.



Properties of a cube

- A cube has six faces, eight vertices, and 12 edges.
- The angle between two faces or surfaces is 90°.
- The opposite edges of a cube are parallel to each other.
- The base of a cube is a square.
- All the sides of a cube are of equal measurements.



Formulae

1. The lateral surface area of a cube = $4a^2$
2. The total surface area of a cube = $6a^2$
3. The volume of a cube = a^3
4. Diagonal a cube = $\sqrt{3}\,a$

Example 36:

A big cube of side 8 cm is formed by rearranging together 64 small but identical cubes of side 2 cm. Further, if the corner cubes in the topmost layer of the big cube are removed, what is the change in the total surface area of the big cube?

(A) 16 cm^2 (B) 48 cm^2

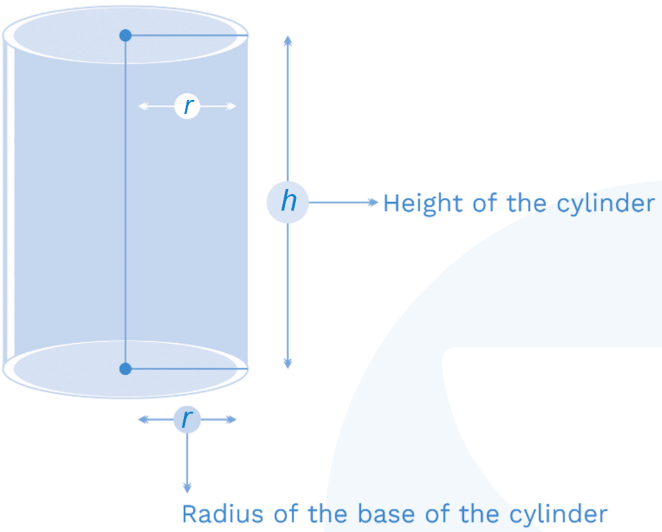
(C) 36 cm^2 (D) Remains the same as previously

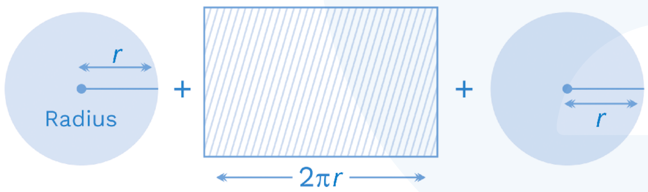
Solution: (D)

Since three faces are visible in a corner cube. When then a cube of a corner is removed then the three faces of other cubes will be visible from the outside. So, there will not be any change in the surface area of this solid cube.

Cylinder

The cylinder is a 3-dimensional solid figure in geometry with two parallel circular bases joined by a curved surface at a particular distance from the centre. A drum, and cold drink cans, are real-life examples of cylinders. The line passing from the centre or joining the centres of two circular bases is called the axis of the cylinder. The distance between the two bases of the cylinder is called perpendicular distance and is represented as height h.



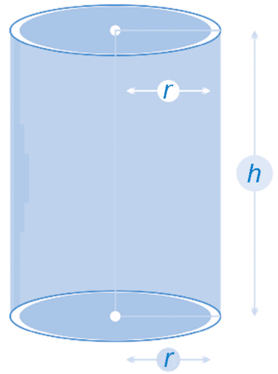


Properties of a cylinder

- The size of the cylinder depends on the radius of the base and the height of the curved sheet.
- A cylinder is called a right circular cylinder, if the axis forms a right angle with the bases, exactly over each other.
- The two bases of a cylinder are congruent to each other.
- The cylinder does not have any vertex like a cube, cone, or cuboid. It means there is no specific corner present in the cylinder.

Formulae

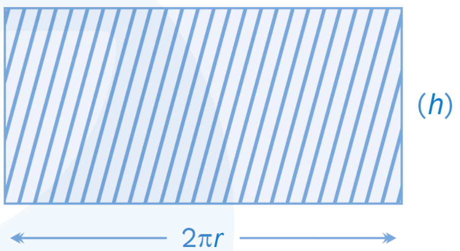
1. Curved surface area or lateral surface area (CSA) = $2\pi rh$



The curved surface area of a cylinder is the space occupied between the two parallel circular bases.

where → r = radius of the base

h = height of the cylinder



Area of a rectangular sheet = $L \times B$ = $2\pi r \times h$

2. Total surface area (TSA) = CSA + $2\pi r^2$ = $2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$.
3. The volume of a cylinder = area of base × height = $\pi r^2 \times h = \pi r^2 h$.

Example 37:

The volume and curved surface area of a right circular cylinder are 4,504.5 and 858 cm^2. Find the height of the cylinder.

(A) 13 cm (B) 17 cm

(C) 19 cm (D) 21 cm

Solution: (A)

Since curved surface area and volume of a cylinder are given.

Therefore, the volume of a right circular cylinder = $\pi r^2 h$ = 4,504.5.

And curved surface area (CSA) = $2\pi rh$ = 858

Divide equation (i) by equation (ii).

$$\therefore \frac{V}{CSA} \Rightarrow \frac{\pi r^2 h}{2\pi rh} = \frac{4,504.5}{858}$$

$$\frac{r}{2} = 5.25$$

$r = 10.5$ cm

Now, we have to find the height of the cylinder.

$\therefore$ CSA = 858

$2\pi rh = 858$

$$2 \times \frac{22}{7} \times \frac{21}{2} \times h = 858$$

$h = 13$ cm.

Hence, option (A) is the correct answer.

Example 38:

The perimeter of a base of a circular cylinder is 35 cm and CSA is 9,600 cm^2. A thread is wound on a cylinder such that it makes exactly 23 turns around the cylinder then, find the length of the string.

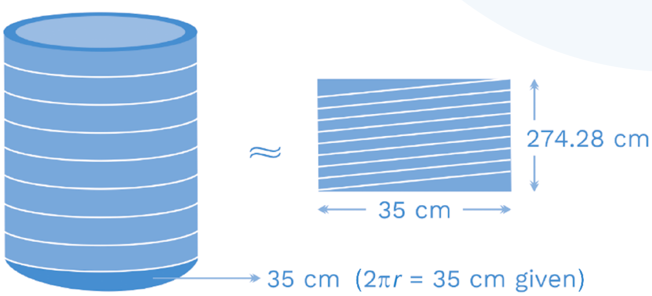
(A) 850.54 cm (B) 892 cm

(C) 752.52 cm (D) 893 cm

Solution: (A)

The perimeter of a base of a circular cylinder = 35 cm.

Moreover, the curved surface area of a cylinder = 9,600.



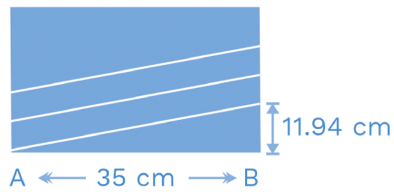
CSA = 9,600 cm^2

$2\pi rh$ = 9,600 cm^2

$35 \times h$ = 9,600 cm^2

$$\therefore h = \frac{9,600}{35} = 274.28 \text{ cm (approx.)}$$

Each wound has been taking place at a height of $= \frac{274.28}{23} = 11.94$ cm.



Now, we have to find the length of one wound of thread.

$$AC = \sqrt{(35)^2 + (11.94)^2}$$

$$= \sqrt{1,225 + 142.56} = \sqrt{1,367.56}$$

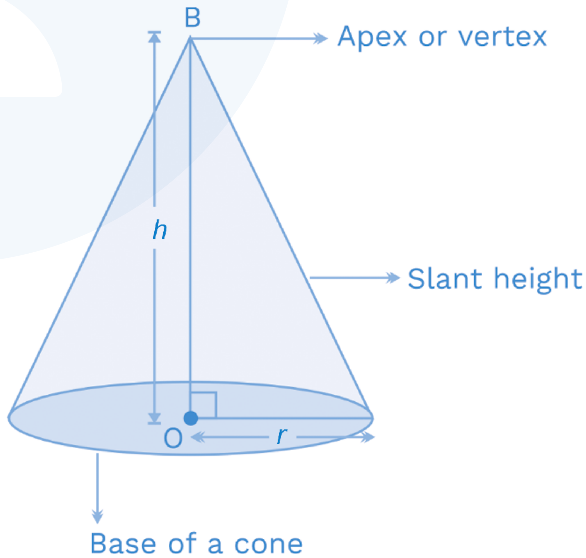
$= 36.98$ cm

Therefore, the total length of the string $= 36.98 \times 23 = 850.54$ cm.

Hence, option (A) is the correct answer.

Cone

A cone is a 3-dimensional shape that has a circular base and it narrows upward to a sharp point called a vertex. There are no edges of a cone.



A cone has three elements

1. Radius
2. Height
3. Slant height

Properties of a cone

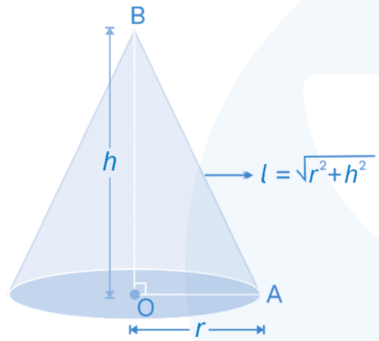
- The base of a cone is circular.
- A cone that has its apex right above the circular base at a perpendicular distance is called a right circular cone.
- There is one face, one vertex, and no edges of a cone.
- The cone is a type of pyramid that's why a cone and a pyramid are just alike.

Formulae

1. The volume of a cone

$$= \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$= \frac{1}{3} \times \pi r^2 \times h$$



2. Curved surface area (CSA) $= \pi rl$

3. Total surface area = area of the base + CSA.

TSA $= \pi r^2 + \pi rl = \pi r\ (r + l)$ unit^2

4. Slant height of a cone (l) $= l = \sqrt{r^2 + h^2}$

Example 39:

A right circular solid cone of radius 3.2 cm and height 7.2 cm is melted by recast into a right circular cylinder of height 9.6 cm. What is the diameter of the base of the cylinder?

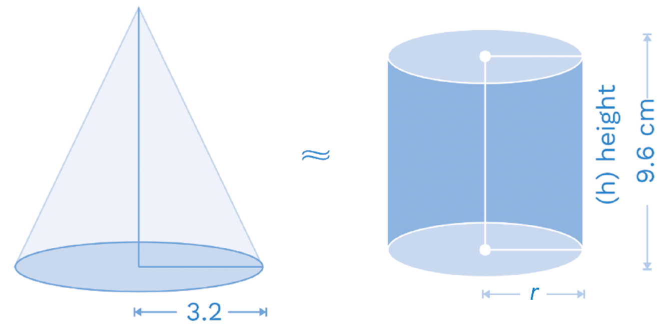
(A) 9.8 cm (B) 32 cm

(C) 40 cm (D) 3.2 cm

Solution: (D)

Since a cone is melted and recast into a cylinder in this case there is no loss of material is given.

Then, the volume of cone = volume of the cylinder



$$\frac{1}{3}\pi R^2 H = \pi \times r^2 \times h$$

$$\frac{1}{3}\pi \times (3.2)^2 \times 7.2 = \pi \times r^2 \times 9.6$$

$$\frac{1}{3} \times 3.2 \times 3.2 \times 7.2 = r^2 \times 9.6$$

$$0.8 \times 3.2 = r^2$$

$$r = \sqrt{0.8 \times 3.2}$$

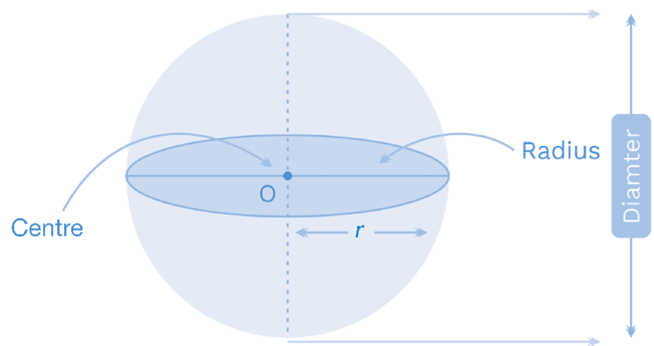
$$= \sqrt{\frac{64 \times 4}{100}}$$

$$r = \frac{16}{10} = 1.6\,\text{cm}$$

$\therefore$ Diameter of the base of the cylinder = $2r$ = 2 × 1.6 = 3.2 cm.

Sphere

A sphere is a 3-dimensional solid figure which has a round shape. If I will say from a mathematical point of view then a sphere is a set of all the points connected with one common point at equal distances. Some examples of a sphere are a soap bubble, a football, etc.

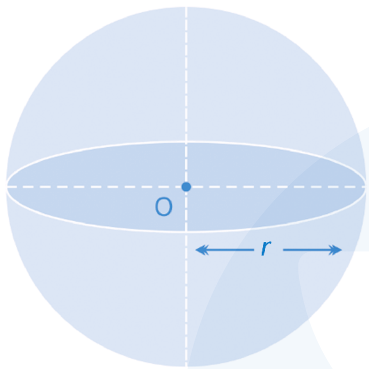


Properties of a sphere

- A sphere is symmetrical in all directions.
- A sphere has only a curved surface area and that is also called the total surface area of the sphere.
- A sphere is not a polyhedron because it does not have vertices, edges, and flat faces.

Note: A polyhedron is an object that should have flat faces.

Formulae



1. The volume of the sphere $= \frac{4}{3}\pi r^3$

2. The curved surface area of a sphere $= 4\pi r^2$

3. The total surface area of a sphere (TSA) $= 4\pi r^2$

Example 40:

A metallic sphere of diameter 40 cm is melted into smaller spheres of radius 0.5 cm. How many such small balls can be made?

(A) 64,000 (B) 32,000

(C) 24,000 (D) 18,000

Solution: (A)

The radius of a bigger sphere $= \frac{40}{2}\text{cm} = 20\,\text{cm}$

Since there is no improper handling of the material takes place. Let the total number of smaller balls be formed by n.

Therefore, the volume of the bigger sphere = volume of all the smaller spheres formed.

$\frac{4}{3}\pi R^3 = n \times \left(\frac{4}{3}\pi r^3\right)$ → The volume of one smaller ball.

$\therefore \frac{4}{3}\pi \times (20)^3 = n \times \frac{4}{3} \times \pi \times (0.5)^3$

$8{,}000 = n \times 0.5 \times 0.5 \times 0.5$

$n = \frac{8 \times 1{,}000 \times 1{,}000}{5 \times 5 \times 5}$

$n = 64{,}000$

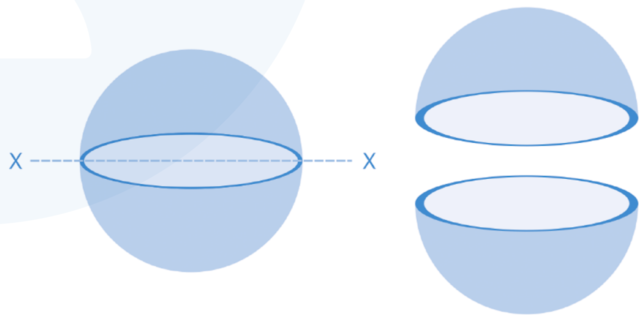
Hence, option (A) is the correct answer.

Alternate solution:

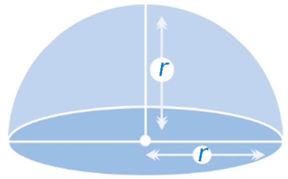
We can directly solve this question by removing some unnecessary steps, so we will get at the end, $\left(\frac{R}{r}\right)^3 = \left(\frac{20}{0.5}\right)^3 = \frac{8{,}000}{5 \times 5 \times 5} \times 1{,}000 = 64{,}000$

Hemisphere

Hemisphere is a 3-dimensional shape, obtained when a sphere is cut along a plane passing through the centre of the sphere, or we can say that the hemisphere is half of a sphere. Hemisphere can be either solid or hollow.



Formulae



1. The curved surface area of a hemisphere = $2\pi r^2$

2. The total surface area of a hemisphere

$= \text{CSA} + \text{area of the base}$

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

3. Volume of the hemisphere $= \dfrac{2}{3}\pi r^3$

Example 41:

If the total surface area of a hemisphere is 924 cm². Find the radius of the hemisphere $\left(\text{Use } \pi = \dfrac{22}{7}\right)$.

(A) $5\sqrt{2}$ cm (B) $8\sqrt{2}$ cm

(C) $9\sqrt{2}$ cm (D) $7\sqrt{2}$ cm

Solution: (D)

Since the total surface area is given in the question. Let r be the radius of the hemisphere.

Therefore, TSA of a hemisphere = 924 cm²

$$3\pi r^2 = 924$$

$$r^2 = \frac{924 \times 7}{3 \times 22}$$

$$r^2 = 98$$

$$r = \sqrt{98} = \sqrt{49 \times 2}$$

$$r = 7\sqrt{2} \text{ cm}$$

Hence, option (D) is the correct answer.

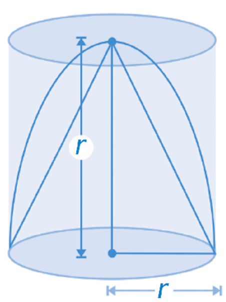
Example 42:

The cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder to have its vertex at the centre of one end and the other end as its base. The volumes of a cylinder, hemisphere, and the cone are, respectively, in the ratio of:

(A) 1:2:3 cm (B) 3:2:1 cm

(C) 9:2:5 cm (D) 1:3:5 cm

Solution: (B)



Let r be the radius of the cylinder and the height of the cylinder also become r unit.

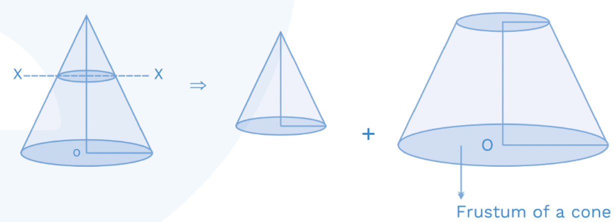
Now, we have to find the ratio of the volume of the cylinder, hemisphere, and cone.

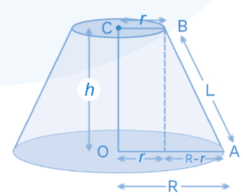
	Cylinder	Hemisphere	Cone
Volume	$\pi r^2 \times r$	$\frac{2}{3}\pi r^3$	$\frac{1}{3}\pi r^2 \times r$
	πr^3	$\frac{2}{3}\pi r^3$	$\frac{1}{3}\pi r^3$
	1	$\frac{2}{3}$	$\frac{1}{3}$
	3	2	1

Hence, option (B) is the correct answer.

Frustum of a Cone

When a cone is cut by a plane into two parts or we can say that when a right circular cone is chopped off. Then we get to the frustum of a right circular cone.





The formula for frustum of a cone:

1. The volume of the frustum

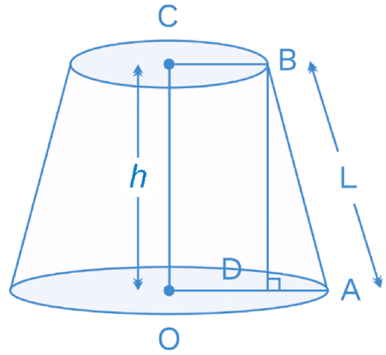
$$= \frac{1}{3}\pi[R^2 + r^2 + rR] \times h$$

when $\rightarrow r, R$ = radius of the bases.

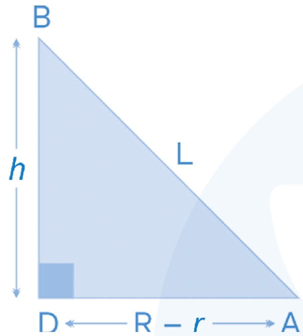
h = height

L = slant height

2. Slant height (L):



$\therefore$ In triangle DAB



By using Pythagoras theorem:

$L^2 = h^2 + (R - r)^2$

$L = \sqrt{(R-r)^2 + h^2}$

where L is a slant height.

3. Lateral surface area of a frustum of cone $= \pi L(R + r)$

(LSA) $= \pi(R + r) \times L$

4. Total surface area of a frustum of a cone $=$ LSA $+ \pi R^2 + \pi r^2$

TSA $= \pi(R + r) \times L + \pi R^2 + \pi r^2$

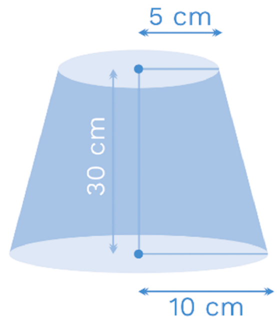
Example 43:

Find the volume of the frustum of a right circular cone whose radius of the bases are 5 and 10 cm, respectively, and height is 30 cm.

(A) $1{,}750\ \pi\,\text{cm}^3$ (B) $812\ \text{cm}^3$

(C) $573\ \pi\,\text{cm}^3$ (D) $910\ \pi\,\text{cm}^3$

Solution: (A)



The volume of the frustum of a cone

$= \frac{1}{3}\pi[R^2 + r^2 + Rr] \times h$

$= \frac{1}{3}\pi[100 + 25 + 10 \times 5] \times 30$

$= \frac{1}{3}\pi[175] \times 30 = 1{,}750\,\pi\,\text{cm}^3$

Example 44:

A bucket in the form of a frustum of a cone height of 60 cm with radii of its lower and upper ends as 10 and 20 cm, respectively. Find the capacity of the bucket. Moreover, find the cost of milk which can fill the bucket at the rate of ₹80 per litre $\left(\text{Use } \pi = \frac{22}{7}\right)$.

(A) ₹3,520, 44 (B) ₹3,820, 47.5

(C) ₹9,280, 116 (D) ₹1,089, 13.6

Solution: (A)

We know the volume of the frustum

$V = \frac{1}{3}\pi h[R^2 + r^2 + rR]$

$= \frac{1}{3}\pi \times 60 \times [100 + 400 + 200]$

$= 20\pi\ [700]$

$= 20 \times \frac{22}{7} \times 700$

$V = 2{,}000 \times 22 = 44{,}000\ \text{cm}^3$

Since 1 litre $= 1{,}000\ \text{cm}^3$

$\therefore 1\ \text{cm}^3 = \frac{1}{1{,}000}$ litre

$\therefore 44{,}000\ \text{cm}^3 = \frac{44{,}000}{1{,}000}$ litre $= 44$ litres

Therefore, the total cost of the milk = price of 1-litre milk × quantity of milk

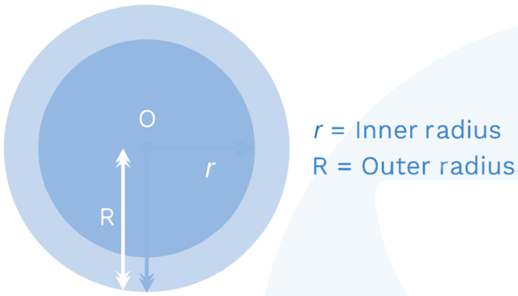
= 80 × 44 = ₹3,520

Hence, option (A) is the correct answer.

Spherical Shells

If one sphere is which radius is r unit is enclosed completely by another sphere of radius R, where $R > r$.

Moreover, the inner sphere is completely hollow and the space between the inner sphere and outer sphere is solid then the solid so formed is a spherical shell.



Formulae

1. The surface area of the shell = $4\pi R^2$

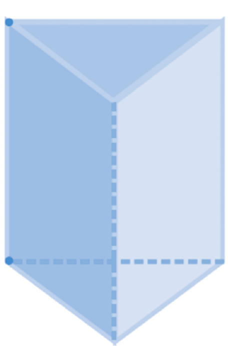
2. The volume of the shell $= \frac{4}{3}\pi[R^3 - r^3]$

Prism

A prism is a solid object bounded by two plane congruent faces in parallel planes and several other plane faces, called side-faces. The two congruent faces in the parallel planes are called the ends.

If the side edges are perpendicular to the ends, the prism is called a right prism.

For example, cylinder



Formulae

1. The volume of a right prism = area of base × height.

2. The curved–surface area of a right prism = perimeter of base × height.

3. The total surface area of a right-prism

= The perimeter of base × height + 2 × area of the base.

Example 45:

A right prism has a triangular base with sides of 13, 20, and 21 cm, if the height of the prism is 9 cm, then find the volume of the prism.

(A) 1,228 cm^3 (B) 1,331 cm^3

(C) 1,234 cm^3 (D) 1,134 cm^3

Solution: (D)

We know that, the volume of the prism = area of base × height.

Since, the base of the prism is triangular with sides of 13, 20, and 21 cm.

By using Heron's formula, we can find the area of the triangle.

$\therefore$ Semi-perimeter (s) $= \frac{\text{Perimeter}}{2} = \frac{54}{2} = 27\,\text{cm}$

Therefore, the area of the

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27 \times (27-13)(27-20)(27-21)}$$

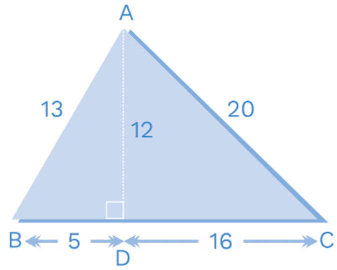
$$= \sqrt{27 \times 14 \times 7 \times 6} = \sqrt{81 \times 14 \times 14}$$

$$\Delta = 9 \times 14 = 126\ cm^2$$

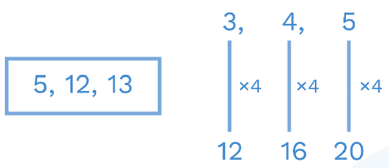
$\therefore$ The volume of the prism $= 126 \times 9 = 1{,}134\ cm^3$

Hence, option (D) is the correct answer.

Alternate solution angle



Since we know about the triplet.



So, by using these triplets we can find the area of triangle very easily.

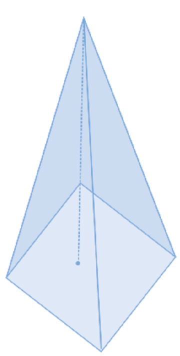
Area of triangle ABC $= \frac{1}{2} \times 21 \times 12 = 126\,\text{cm}^2$

Now, the volume of the prism = 126 × 9 = 1,134 cm^3.

Pyramid

A pyramid is a solid bounded by a polygon at the base and triangles connecting the base to a common vertex that is not in the same plane as the base. The slant height of the pyramid is the height of one of its triangles.

The height of the pyramid is the length of perpendicular from the vertex to the base of the pyramid.



Formulae

1. If the base of the pyramid is a regular polygon, then

   Volume $= \frac{1}{3} \times \text{area of base} \times \text{height}$.

2. The curved surface area of the pyramid

   $= \frac{1}{2} \times \text{perimeter of base} \times \text{slant height}.$

3. The total surface area of the pyramid

   (TSA) $= \frac{1}{2} \times$ the perimeter of the base × slant height + area of the base.

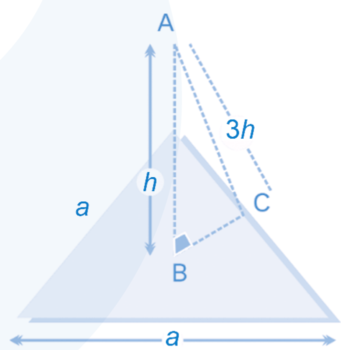
Example 46:

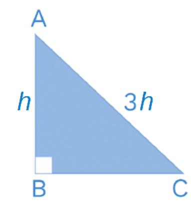
Find the volume of a pyramid whose base is an equilateral triangle of side $16\sqrt{3}$ cm and its slant height is three times its height.

(A) $192\sqrt{3}$ cm^3 (B) $56\sqrt{2}$ cm^3

(C) $128\sqrt{6}$ cm^3 (D) $92\sqrt{2}$ cm^3

Solution: (C)





In triangle ABC

$BC = \sqrt{(3h)^2 - h^2} = \sqrt{8h^2} = 2\sqrt{2}\,h$

Since BC is nothing but it is in the radius of equilateral triangle.

In radius $(r) = \frac{a}{2\sqrt{3}}$

$2\sqrt{2}h = \frac{a}{2\sqrt{3}}$

Moreover, $a = 16\sqrt{3}$ cm is given.

Then, $2\sqrt{2}\,h = \dfrac{16\sqrt{3}}{2\sqrt{3}}$

$$h = 2\sqrt{2}\text{ cm}$$

Therefore, the volume of the pyramid $= \dfrac{1}{3} \times$ area of base $\times$ height

$$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times 16\sqrt{3} \times 16\sqrt{3} \times 2\sqrt{2}$$

Volume $= 128\sqrt{6}$ cm^3

Hence, option (C) is the correct answer.

Example 47:

The base of the pyramid is an equilateral triangle of side $10\sqrt{3}$ cm. If the total surface area of the pyramid is $270\sqrt{3}$ cm^2. Find its height.

(A) 12 cm (B) 13 cm

(C) 18 cm (D) 25 cm

Solution: (A)

The total surface area of a pyramid

(TSA) = LSA + area of the base

$$270\sqrt{3} = \frac{1}{2} \times P_b \times \text{slant height} + \frac{\sqrt{3}}{4} \times a^2$$

$$270\sqrt{3} = \frac{1}{2} \times 3 \times 10\sqrt{3} \times l + \frac{\sqrt{3}}{4} \times (10\sqrt{3})^2$$

$$200\sqrt{3} = 15\sqrt{3}\,l + \frac{\sqrt{3}}{4} \times 100 \times 3$$

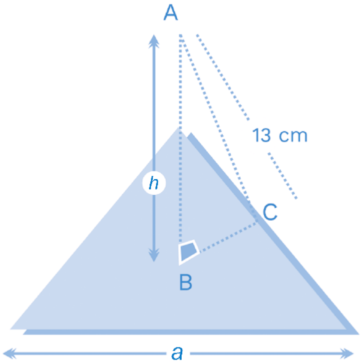
$$270\sqrt{3} = 15\sqrt{3}\,l + 75\sqrt{3}$$

$$\Rightarrow 15\sqrt{3}\,l = 195\sqrt{3}$$

$$l = 13 \text{ cm}$$

Since its base is an equilateral triangle.

Therefore,

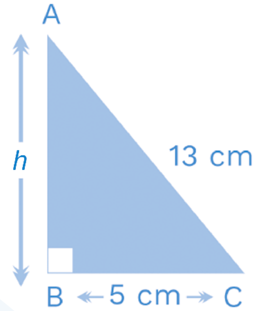


BC is inradius of equilateral triangle.

$$r = \frac{a}{2\sqrt{3}}$$

$$r = \frac{10\sqrt{3}}{2\sqrt{3}} = 5$$

Hence, in triangle ABC

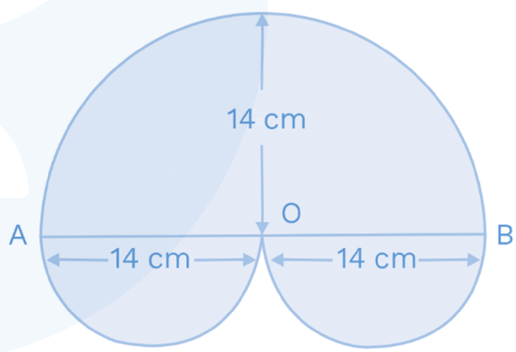


Hence, height (A.B.) $= \sqrt{13^2 - 5^2} = \sqrt{144} = 12$ cm.

$\therefore$ Option (A) is the correct answer.

Example 48:

In the following figure, the area in (cm^2) is:



(A) 512 cm^2 (B) 462 cm^2

(C) 960 cm^2 (D) 560 cm^2

Solution: (B)

Since the above figure consists of three semicircles with radius 14, 7, and 7 cm, respectively.

$\therefore$ Area of the whole figure

$$= \frac{1}{2}\pi \times (14)^2 + \frac{1}{2} \times \pi \times \left(\frac{14}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{14}{2}\right)^2$$

$$= \frac{1}{2}\pi\left[14^2 + 49 + 49\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times [196 + 98]$$

$$= \frac{1}{2} \times \frac{22}{7} \times 294$$

$$= 11 \times 42 = 462 \text{ cm}^2$$

Hence, option (B) is the correct answer.

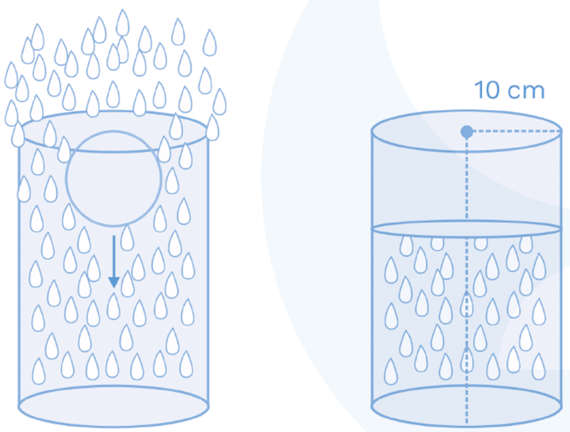
Example 49:

A cylindrical vessel of a radius of 10 cm contains water to the brim. A solid sphere of radius 4 cm is gently dropped into the water until it is completely immersed and removed afterwards. Find the drop in the water level (in cm) in the vessel.

(A) 0.75 cm (B) 0.85 cm

(C) 0.925 cm (D) 0.8533 cm

Solution: (D)



When the sphere is dropped into the cylinder

When the sphere is again taken out

Let the drop in the height of the water when the sphere is removed = h cm.

Let us assume another cylinder of height h cm.

So, the volume of this cylinder with the height (h) = volume of the sphere.

Because the amount of water displaced = the volume of the sphere

Therefore, $\pi \times x^2 \times h = \frac{4}{3} \pi \times (r)^3$

$$\pi \times 10^2 \times h = \frac{4}{3} \pi \times (4)^3$$

$$100 \times h = \frac{4}{3} \times 64$$

$$h = \frac{4 \times 64}{3 \times 100} = \frac{256}{300} \text{ cm.}$$

$$h = 0.8533 \text{ cm}$$

Hence option (D) is the correct answer.

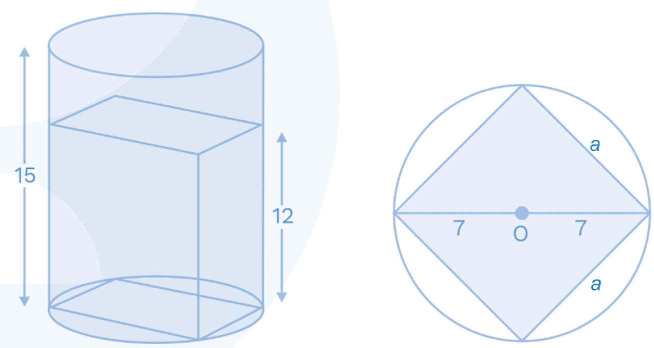
Example 50:

A right circular cylinder has a height of 15 and a radius of 7. A rectangular solid with a height of 12 and a square base, is placed in the cylinder such that each of the corners of the solid is tangent to the cylinder wall. The liquid is then poured into the cylinder such that, it reaches the rim. The volume of the liquid is:

(A) 147 ($5\pi - 8$) (B) 243 ($5\pi - 8$)

(C) 49 ($5\pi - 8$) (D) 49 ($15\pi - 8$)

Solution: (A)



Let the side of the square be a.

The rectangular box has a square base.

Diagonal of a square base = diameter of a circle.

Diagonal of a square = 14

$\sqrt{2}a = 14$

$a = 7\sqrt{2}$

The volume of the rectangular solid = $l \times b \times h$

$= a \times a \times 12 = 7\sqrt{2} \times 7\sqrt{2} \times 12 = 49 \times 24$

Volume of the cylinder = $\pi r^2 h = \pi \times 7^2 \times 15 = 49\pi \times 15$.

Volume of the liquid = $49\pi \times 15 - 49 \times 24 = 49 \times 3 [5\pi - 8] = 147 [5\pi - 8]$.

Example 51:

A cylinder of radius 10.5 cm and height of 22 cm standing on its base is vertically cut into two equal parts. What is the percentage increase in the total surface area of the cylinder?

(A) 42% (B) 43%

(C) 43.08% (D) 41%

Solution: (C)

The total surface area of the cylinder before cutting = $2\pi r\,(h + r)$

$$\Rightarrow 2 \times \frac{22}{7} \times 10.5(10.5 + 22) = 2{,}145\ \text{cm}^2$$

After cutting

T.S.A. of half cylinder part = $\pi r\,(h + r) + 2rh$

$$\Rightarrow \frac{22}{7} \times 10.5(22 + 10.5) + 2 \times 10.5 \times 22 = 1{,}534.5\ \text{cm}^2$$

T.S.A of both cylindrical part $\Rightarrow 2 \times 1{,}534.5 = 3{,}069\ \text{cm}^2$

$$\%\ \text{increased} = \frac{\text{Increased value}}{\text{Initial value}} \times 100$$

$$\Rightarrow \frac{924}{2{,}145} \times 100$$

$$\Rightarrow 43.08$$

Alternate solution:

Increase in areas = $2 \times 2rh$

$\Rightarrow 4rh = 4 \times 10.5 \times 22 = 924\ \text{cm}^2$

Initial total surface area of cylinder = $2\pi r\,(h + r) = 2{,}145\ \text{cm}^2$.

$$\%\ \text{increase} = \frac{924}{2{,}145} \times 100 = 43.08\%$$

Practice Exercise – 1

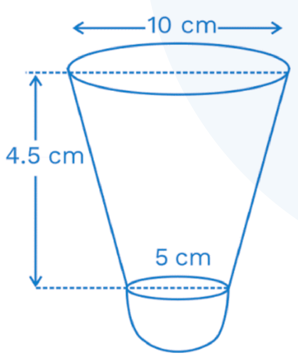
Level of Difficulty – 1

1. Onsquare cardboard of length 10 cm, two identical circles of radius 2 cm are drawn such that the two centres and the two points of intersection of the circle form a square. Find the total area of the card-board covered by the two circles.

(A) $4\pi + 6$
(B) $6\pi - 4$
(C) $4\pi + 4$
(D) $6\pi + 4$

2. A shuttlecock which is used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The diameter of the frustum is 10 and 5 cm and the height of the frustum of the cone is 4.5 cm. Find the external surface area of the shuttlecock.

(A) 160.7 cm^2
(B) 169 cm^2
(C) 165 cm^2
(D) 192.2 cm^2



3. The adjacent side of a parallelogram are 6 and 8 cm and the angle between them is 30°. What is the area of the parallelogram?

(A) 24 cm^2
(B) 12 cm^2
(C) 40 cm^2
(D) $24\sqrt{3}$ cm^2

4. In a quadrilateral ABCD, E is a point on the side AB such that $\angle ADE = \angle DEC = \angle ECB = 45°$. If AD = 16 cm and BC = 24 cm, then find out the area of the triangle ECD.

(A) $100\sqrt{3}$ cm^2
(B) $96\sqrt{2}$ cm^2
(C) $98\sqrt{2}$ cm^2
(D) $95\sqrt{2}$ cm^2

5. At each corner of a triangle field of sides 20, 34, and 42 m, a horse is tied by a rope of 7 m. Find the area of the ungrazed field by the horse.

(A) 259 m^2
(B) 231 m^2
(C) 277 m^2
(D) 247 m^2

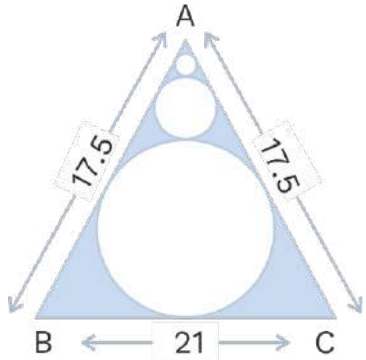
Level of Difficulty – 2

6. Two sheep are tied at the two ends of a diagonal of a square field of length 10 m. The length of both the ropes is equal and they can both just touch each other when stretched to the fullest. What is the maximum area both the sheep can graze in total?

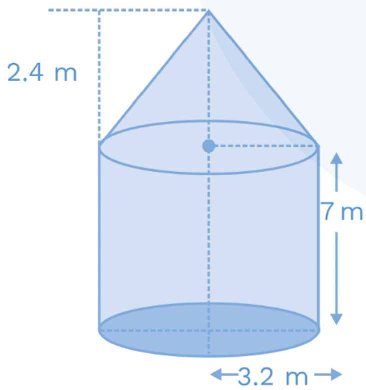
(A) 21π cm^2
(B) 25π cm^2
(C) 24π cm^2
(D) 75π cm^2

7. In an isosceles triangle ABC, AB = AC = 17.5, BC = 21, infinite circles are made inside this triangle, as shown in the figure. Find the sum of the perimeter of all the circles.

(A) 38.5
(B) 44
(C) 15
(D) 42



8. Due to the cyclone Tauktae, being human welfare association jointly requested the government to get 500 tents fixed immediately and offered to contribute 50% of the cost. The lower part of each tent is of the form of a cylinder of radius 3.2 m and a height of 7 m with a conical upper part of the same diameter but a height of 2.4 m. The cost of the canvas used ₹50 per square meter. Find the amount the association will have to pay.
(A) ₹22,62,875
(B) ₹2,62,875
(C) ₹40,000
(D) ₹22,60,800



9. The length of the parallel sides of a trapezium are 51 and 21 cm and that of each of the other two sides is 39 cm. What is the area (in cm^2) of the trapezium?
(A) 1,152
(B) 1,206
(C) 1,296
(D) 1,260

10. If the area of three adjacent faces of a rectangular box that meet in the corner are 32, 24, and 48 cm^2, respectively. Then the volume of the box is:
(A) 192
(B) 216
(C) 144
(D) 256

Level of Difficulty – 3

11. A solid cube of volume 13,824 cm^2 is cut into eight cubes of equal volumes. The ratio of the surface area of the original cube to the sum of the surface areas of three of the smaller cube is:
(A) 1:3
(B) 4:3
(C) 5:2
(D) 6:1

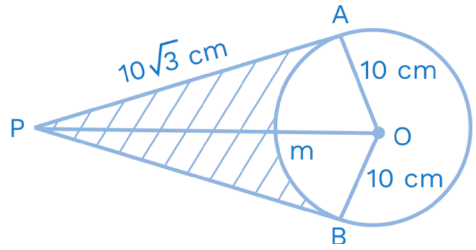
12. A blacksmith has a rectangular sheet of iron. He has to make a cylindrical vessel in which both the circular ends are closed. When he minimises the wastage of the sheet of iron, then what is the ratio of the wastage to the utilised area of a sheet?
(A) $\frac{1}{11}$
(B) $\frac{2}{17}$
(C) $\frac{3}{22}$
(D) None of these

13. A right circular cylinder having a diameter of 21 cm and a height of 38 cm is full of Ice cream. The Ice cream is to be filled in cones of the height of 12 cm and diameter of 7 cm having a hemispherical shape on the top. The number of such cones to be filled with Ice cream is:
(A) 28
(B) 26
(C) 54
(D) 108

14. A ball of diameter 4 cm is kept on top of a hollow cylinder, standing vertically the height of the cylinder is 3 cm, while its volume is $9\pi \text{ cm}^3$. Then the vertical distance, in cm, of the topmost point of the ball from the base of a cylinder is:

(A) 8 cm
(B) 9 cm
(C) 6 cm
(D) 10 cm

15. PA and PB are two tangents of a circle with the centre O. If the radius of the circle is 10 cm and the length of the tangent is $10\sqrt{3}$ cm. What is the area of the shaded region?



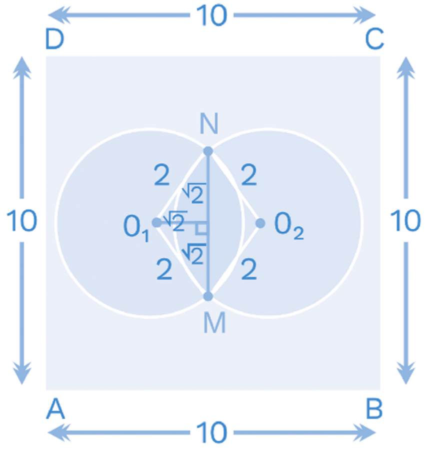
(A) $\dfrac{100(3\sqrt{3}-\pi)}{3}$ cm^2

(B) $\dfrac{100\sqrt{3}}{3}$ cm^2

(C) $\dfrac{100(\sqrt{3}-1)}{3}$ cm^2

(D) $100\sqrt{3}$ cm^2

1. (D)



Area covered by two circles = area of two circles – overlapped area

$= 2(\pi \times (2)^2) - 2$[area of sector O_1 MN

– area of triangle O_1MN]

$= 8\pi - 2\left[\frac{1}{4} \times \pi \times 4 - \frac{1}{2} \times 2\sqrt{2} \times \sqrt{2}\right]$

$= 8\pi - [2\pi - 4]$

$= 6\pi + 4$

2. (A)

Here $R = \frac{10}{2} = 5$ cm and $r = \frac{5}{2}$ cm

$h = 4.5$ cm

Let the slant height of frustum of cone be l cm.

$\therefore\ l^2 = h^2 + (R - r)^2$

$= (4.5)^2 + \left(5 - \frac{5}{2}\right)^2$

$= 20.25 + 6.25$

$l^2 = 26.5$

$\therefore\ l = 5.15$ cm

The external area of the shuttlecock = lateral surface area of frustum + CSA of the hemisphere

$= [\pi l\,(R + r) + 2\pi r^2]$

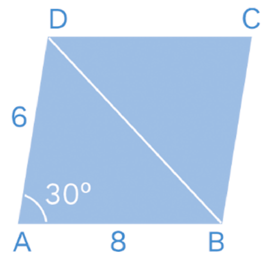
$= \frac{22}{7} \times\ 5.15 \left[5 + \frac{5}{2}\right] + 2 \times \frac{22}{7} \times \left(\frac{5}{2}\right)^2$

$= \frac{22}{7} \times\ 5.15\ \times\ \frac{15}{2} + 2\ \times \frac{22}{7} \times \frac{25}{4}$

$= 121.4 + 39.3$

$= 160.7$ cm^2

3. (A)



Area $= \left(\frac{1}{2} ab \sin\theta\right) \times 2$

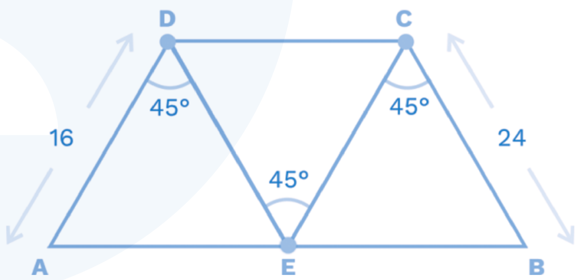
$= \frac{1}{2} \times 6 \times 8 \sin 30° \times 2$

$= 24 \times \frac{1}{2} \times 2 = 24\,\text{cm}^2$

4. (B)

We are given that ∠ADE = ∠DEC therefore, we can say that AD || EC

Similarly, ∠DEC = ∠ECB, DE || BC



In triangle ADE and triangle ECB

DE || BC and AD || EC and ∠ADE = ∠ECB

Hence, we can say that triangle ADE ~ triangle ECB.

Therefore, $\frac{AD}{EC} = \frac{DE}{BC}$

$\Rightarrow$ AD × BC = DE × EC

16 × 24 = DE × EC

$\Rightarrow$ DE × EC = 384 cm^2

We know that area of triangle ECD

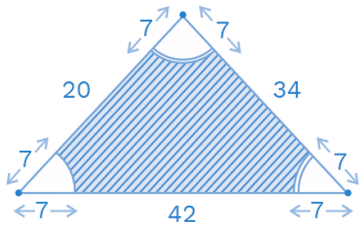
$= \frac{1}{2} \times DE \times EC\ \sin 45°$

$$= \frac{1}{2} \times DE \times EC \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2} \times 384 \times \frac{1}{\sqrt{2}} = 96\sqrt{2}\ \text{cm}^2$$

Hence, option (B) is the correct answer.

5. **(A)**



Semi perimeter of triangle(s)

$$= \frac{20 + 34 + 42}{2} = 48 \text{ m}$$

Area of triangle

$$= \sqrt{48(48-20)\times(48-34)\times(48-42)}$$

$$= \sqrt{16 \times 3 \times 14 \times 14 \times 2 \times 3 \times 2}$$

$$= 4 \times 3 \times 14 \times 2$$

$$= 24 \times 14 = 336\ \text{m}^2$$

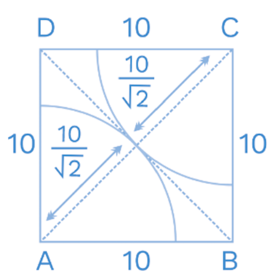
Area grazed by a horse

$$= \frac{180}{360} \times \pi \times 7^2 = \frac{1}{2} \times \frac{22}{7} \times 49 = 77\,\text{m}^2$$

Therefore, the area ungrazed by horses = 336 – 77 = 259 m^2

Hence, option (A) is the correct answer.

6. **(B)**



Diagonal $= 10\sqrt{2}$

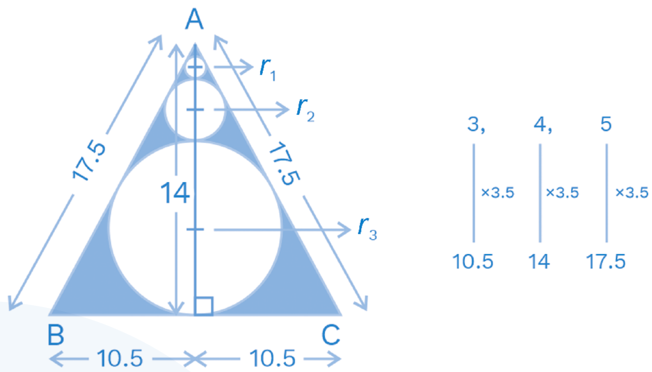
$$\frac{d}{2} = \frac{10\sqrt{2}}{2} = \frac{10}{\sqrt{2}}$$

Area grazed by sheep = area of 2 sector

$$= 2 \times \left[\frac{1}{4} \times \pi \times \left(\frac{10}{\sqrt{2}}\right)^2\right] = 2 \times \left[\frac{1}{4} \times \pi \times \frac{100}{2}\right] = 25\pi\ \text{cm}^2$$

Hence, option (B) is the correct answer.

7. **(B)**



$2r_1 + 2r_2 + 2r_3 \ldots = 14$ cm

$\therefore$ Perimeter is $\pi(2r_1 + 2r_2 + 2r_3 \ldots) = 14 \times \pi$

$$= 14 \times \frac{22}{7} = 44\,\text{cm}$$

Hence, option (B) is the correct answer.

8. **(A)**

The radius of the cylinder = 3.2 m (r).
Height of the cylinder = 7 m (H).
The radius of the cone = radius of cylinder = 3.2 m.
Height of the cone (h) = 2.4 m

The slant height of the cone $l = \sqrt{r^2 + h^2}$

$\Rightarrow \sqrt{(3.2)^2 + (2.4)^2} = 4$ m

Area of the canvas used (1 tent) = CSA of cylinder + CSA of the cone

$\Rightarrow 2\pi rH + \pi rl = 2 \times \frac{22}{7} \times 3.2 \times 7 + \frac{22}{7} \times 3.2 \times 4$

$\Rightarrow 140.8 + 40.23$

Area of the canvas used (1 tent) = 181.03 m^2

The total area of the canvas (500 tents) = 90,515 m^2.

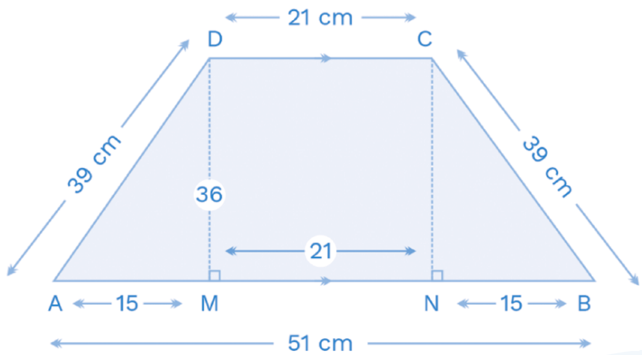
Total cost = area × rate = 90,515 × 50 = ₹45,25,750.

Amount paid by being humanity association = ₹45,25,750 $\times \frac{50}{100}$

$\Rightarrow$ ₹22,62,875

Hence, the association will have to pay ₹22,62,875.
Hence, option (A) is the correct answer.

9. (C)



$DM = \sqrt{39^2 - 15^2} = 36$

Area of the trapezium $= \frac{1}{2}(51 + 21) \times 36$

$= 36 \times 36 = 1{,}296$ cm^2

Hence, option (C) is the correct answer.

10. (A)

$V = \sqrt{x\,y\,z} = \sqrt{32 \times 24 \times 48}$

$= \sqrt{16 \times 2 \times 8 \times 3 \times 16 \times 3}$

$= 16 \times 4 \times 3 = 192$ cm^3

11. (B)

Let the side of the bigger cube is a and the side of the smaller cube be x.

∴ The volume of a bigger cube = volume of eight smaller cubes

$a^3 = 8 \times x^3$

$\frac{a^3}{x^3} = \frac{8}{1}$

$\frac{a}{x} = \frac{2}{1}$

Or $a = 2x$

Now, we have to find the surface area of the bigger cube $= 6a^2$.

Moreover, the surface area of 1 smaller cube $= 6x^2$.

Therefore, the surface area of three smaller cubes $= 3 \times (6x^2)$.

∴	**Bigger cube**	**:**	**Smaller cube**
	Surface $\to 6a^2$	:	$3 \times (6x^2)$

Since, $a = 2x$, put in the above ratios.

$6 \times (2x)^2$	:	$3 \times (6x^2)$
$6 \times 4x^2$	:	$3 \times 6x^2$
4	:	3

Hence, option (B) is the correct answer.

Alternate solution:

Let the volume of a smaller cube is 1 cm^3 and the bigger cube is 8 cm^3.

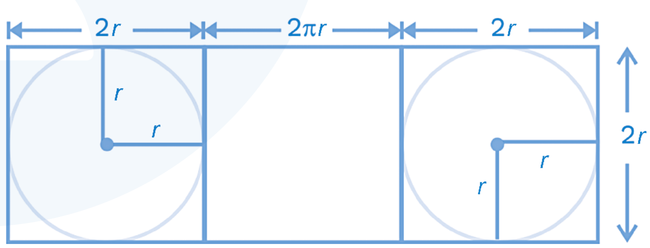
	Bigger cube	**:**	**Smaller cube**
Volume $\to$	8	:	1
Side $\to$	$\sqrt[3]{8}$	:	$\sqrt[3]{1}$
	2	:	1
Surface area $\to$	4	:	1

But we have to find the ratio between the bigger cube's surface area and the sum of the surface areas of three smaller cubes.

∴	4	:	1×3
	4	:	3

12. (A)

For the minimum wastage of the sheet, he has to cut the sheet in the given manner.



The total area of the sheet required
= length × breadth
$= (2r + 2\pi r + 2r) \times 2r$
$= (2\pi r + 4r) \times 2r$
$= 4r^2(\pi + 2)$

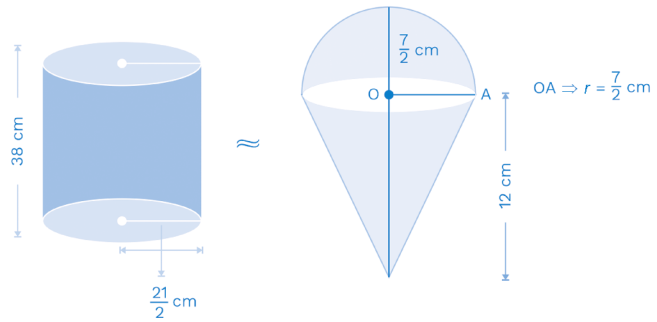
Area of sheet utilised $= (2\pi r \times 2r) + 2\pi r^2$
$= 6\pi r^2$

Therefore, the area of wastage sheet
$= 4r^2(\pi + 2) - 6\pi r^2 = 8r^2 - 2\pi r^2$

∴ Required ratio $= \frac{8r^2 - 2\pi r^2}{6\pi r^2} = \frac{2r^2(4-\pi)}{6r^2\pi} = \frac{1}{11}$

Hence, option (A) is the correct answer.

13. (C)



Let the number of cones with hemisphere to be n.
Therefore, the volume of cylinder = $n \times$ volume of cones which has a hemispherical shape at the top

$$\pi \times R^2 \times H = n \times \left[\frac{1}{3} \times \pi \times r^2 \times h + \frac{2}{3}\pi r^3\right]$$

$$\pi \times \left(\frac{21}{2}\right)^2 \times 38 = n \times \left[\frac{1}{3} \times \pi \times \left(\frac{7}{2}\right)^2 \times 12 + \frac{2}{3}\pi \times \left(\frac{7}{2}\right)^3\right]$$

$$\frac{441 \times 38}{4} = n \times \left[\frac{49}{12} \times 12 + \frac{2}{3} \times \frac{343}{8}\right]$$

$$\frac{441 \times 38}{4} = n \times \left[49 + \frac{343}{12}\right]$$

$$\frac{441 \times 38}{4} = n \times \left[\frac{49 \times 12 + 343}{12}\right]$$

$441 \times 38 \times 3 = n \times 931$

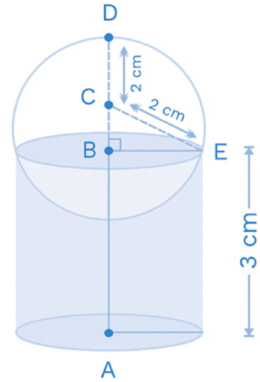
$$\frac{50,274}{931} = n$$

$n = 54$

Hence, option (C) is the correct answer.

14. (C)

Let c be the centre of the sphere.



First of all, we have to find the radius of the cylinder. For that volume of the cylinder and height are given in the question itself.

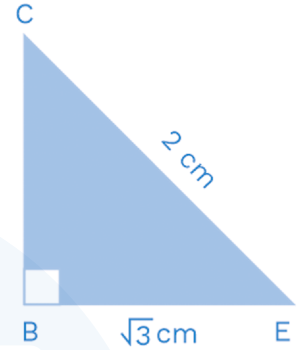
Therefore, $v = \pi r^2 h = 9\pi$

$\Rightarrow \quad r^2 \times 3 = 9$

$r^2 = 3$

$r = \sqrt{3}$ cm

Now, in triangle BCE:



$$\therefore BC = \sqrt{2^2 - \left(\sqrt{3}\right)^2} = 1$$

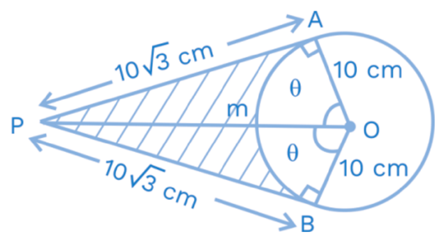
Hence, the vertical distance from the base of the cylinder to the topmost point of the sphere = AB + BC + CD = 3 + 1 + 2 = 6 cm.

Option (C) is the correct answer.

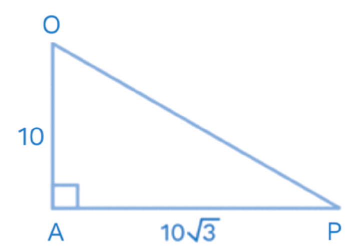
15. (A)

We know that the tangents from the external point on a circle are always equal in length.

$\therefore$ PA = PB = $10\sqrt{3}$ cm.



In triangle APO:



$\therefore \tan\theta = \dfrac{10\sqrt{3}}{10} = \sqrt{3}$

$\theta = 60°$

$\therefore$ Area of the shaded region

= Area of quadrilateral PBOA – 2 × area of sector OAM

= 2 × area of triangle AOP – 2 × area of sector OAM

$$= 2 \times \frac{1}{2} \times 10 \times 10\sqrt{3} - 2 \times \frac{60°}{360°} \times \pi \times (10)^2$$

$$= 100\sqrt{3} - \frac{1}{3}\pi \times 100$$

$$= 100\left(\sqrt{3} - \frac{\pi}{3}\right) \text{ cm}^2$$

$$= 100\frac{(3\sqrt{3} - \pi)}{3} \text{ cm}^2$$

Hence, option (A) is the correct answer.

Practice Exercise – 2

Level of Difficulty – 1

1. A solid circular cylinder of height 104 cm is cut into 15 identical smaller cylinders of the same radius as that of the large original cylinder (without wastage). The total surface area of 15 cylinders is double the surface area of a large cylinder. Find the radius (in cm) of the original cylinder.

2. The ratio of length, breadth, and height of a cuboidal room is 3:2:5, respectively. Find the change in the area of four walls of a cubicle if its length is reduced by two-thirds of its value and height is halved.

(A) Decreases by 20%
(B) Decreases by 50%
(C) Decreases by 60%
(D) Decreases by 70%

3. Given that the six faces of a cuboid have their surface areas as K_1, K_2, K_3, K_4, K_5, and K_6. Find the volume of the cuboid.

(A) $(K_1 \times K_2 \times K_3 \times K_4 \times K_5 \times K_6)^{1/2}$
(B) $(K_1 \times K_2 \times K_3 \times K_4 \times K_5 \times K_6)^{1/3}$
(C) $(K_1 \times K_2 \times K_3 \times K_4 \times K_5 \times K_6)^{1/4}$
(D) None of these

4. Let the dimension of a cuboid a, b, and c. If v is the volume and s is its surface area, then $\frac{1}{v} = \frac{k}{s}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$. The value of k is:

(A) 1
(B) 2
(C) 3
(D) 4

5. If each side of the rhombus is 20 cm, then find the square root of the sum of the square of its diagonals.

(A) $20\sqrt{2}$ cm
(B) 32 cm
(C) 36 cm
(D) 40 cm

6. The curved surface area of a solid cylinder is one-third of its total surface area. Then, find its height if the radius is 7 cm.

(A) 7 cm
(B) 3.5 cm
(C) 1.75 cm
(D) None of these

7. A uniform cylindrical tank is initially filled to 71% of its capacity. The radius of the base of the tank is increased by 7%. By what percent (approx.) of the height of the tank does the level of water will fall?

(A) 8.49%
(B) 8.99%
(C) 9.99%
(D) 7.89%

8. The volume of a right circular cylinder and a sphere are in the ratio 1:4 and the radii of the base of the cylinder and the sphere are in the ratio 1:3. The sum of the height and the radius of the cylinder is 70 cm, and then find the curved surface area of the cylinder is:

(A) 4,158 cm^2
(B) 2,079 cm^2
(C) 1,386 cm^2
(D) 2,772 cm^2

9. If n spherical balls of radius 4 cm are melted together to form a solid disc of radius 8 cm and height 4 cm, then what is the value of n?

(A) 12
(B) 3
(C) 6
(D) Cannot be determined

10. If the two circles touch each other externally such that the sum of the areas of the two circles is 346π cm^2 and the sum of their radii is 26 cm. Find the ratio of the perimeter of the smaller circle to the larger circle.

(A) 7:18
(B) 11:15
(C) 12:14
(D) 9:17

Level of Difficulty – 2

11. If the radius of the cylinder is increased by 9 cm, then the volume of the cylinder increases by a cm^3. Instead, if we increase the height of the cylinder by 9 cm then also the volume of the cylinder increases by a cm^3. If the original height of the cylinder is 3 cm, then find the original radius (in cm) of the cylinder.

12. 100 spheres of radius r are melted to form N cones, each of radius $\frac{r}{100}$ and height which is 66.67% of the radius of the sphere. Find the % change in the curved surface area (approx.).

(A) 8,500%
(B) 8,000%
(C) 9,000%
(D) 9,900%

13. Find the perimeter of a rectangle (in cm), whose length is 25 cm and breadth is b cm. By considering breadth as the diameter, a circle is drawn whose area is equal to the area of the rectangle.

(A) $50\left(1+\frac{4}{\pi}\right)$ cm
(B) $50\left(\frac{4}{\pi}\right)$ cm
(C) $50\left(1+\frac{8}{\pi^2}\right)$ cm
(D) 100 cm

14. The external dimensions of a wooden box closed at both ends are 24, 16, and 10 cm, respectively, and the thickness of the wood is 5 mm. If the empty box weighs 8 kg. Find the approximate weight of 1 cubic cm of wood.

(A) 2.6 g
(B) 9.6 g
(C) 10.9 g
(D) 10.2 g

15. Find the minimum length of the string wound on a cylinder of the height of 48 cm and the base diameter of 56/11 cm. The string makes exactly four complete turns around the cylinder while its two ends touch its top and bottom.

(A) 80
(B) 70
(C) 50
(D) 40

16. If the base of a regular pyramid is a square and each of the other four sides of an equilateral triangle, the length of each side is 15 cm, then the vertical height of the pyramid is:

(A) $15\sqrt{2}$ cm
(B) $12\sqrt{2}$ cm
(C) $\frac{15\sqrt{2}}{2}$ cm
(D) $6\sqrt{2}$ cm

17. A hemisphere and a cylinder have a common base and cylinder, and the cylinder is circumscribing it. If the volume of a hemisphere is 30 cm^2, then the volume of a cylinder is:

(A) 20 cm^2
(B) 30 cm^2
(C) 45 cm^2
(D) 18 cm^2

18. If two fuel cans contain two equal amounts of two different qualities of fuel. The first can has petrol and diesel in a ratio of 4:5, whereas the second can has petrol and diesel in a ratio of 5:4. If these two fuel cans are poured into a larger can. Then, find the ratio of petrol and diesel in a larger can.

(A) 1:1
(B) 4:5
(C) 5:4
(D) Cannot be determined

19. If the perimeter of a square and equilateral triangle is the same. Which of the following can be concluded?

(A) The equilateral triangle and the square have the same area.
(B) The area of the equilateral triangle exceeds that of the square.
(C) The equilateral triangle is less than that of the square.
(D) None of these.

20. The area of a circle inscribed in a regular hexagon is 324π. Then, the area of the hexagon is:

(A) $216\ cm^2$
(B) $648\sqrt{3}\ cm^2$
(C) $1,296\sqrt{3}\ cm^2$
(D) $648\ cm^2$

Level of Difficulty – 3

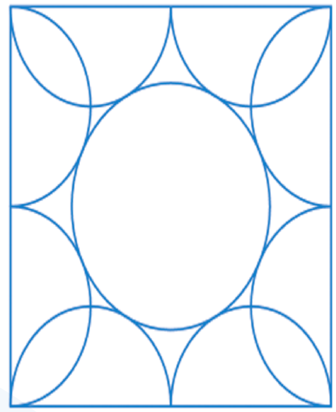
21. A right circular cone is cut (parallel to its base) into seven slices, all of the same height. What is the ratio of the volume of the smallest slice to that of the biggest slice?

(A) 1:7
(B) 1:343
(C) 1:216
(D) 1:127

22. If a sphere of radius r is cut by a plane at a distance of h from its centre, thereby breaking this sphere into two different pieces. The cumulative surface area of these two pieces is 30% more than that of the sphere. Find h.

(A) $0.537r$
(B) $0.7345r$
(C) $0.795r$
(D) $0.6324r$

23. If the eight semicircles lie on the inside of a square with a side length of 8 cm as shown in the below figure. What is the radius of the circle touches to all these semicircles?



(A) $\dfrac{5\left(\sqrt{3}+1\right)}{2}$ cm
(B) $2(\sqrt{5}-1)$ cm
(C) $2(\sqrt{5}+1)$ cm
(D) $\dfrac{3\left(\sqrt{5}-1\right)}{2}$ cm

24. 2 spheres of radii 7 and 3 cm are inscribed in a right circular cone. The bigger sphere touches the smaller sphere and also the base of the cone. What is the height of the cone (in cm)?

(A) 24
(B) 24.5
(C) 25
(D) 25.5

25. What is the ratio of the largest diagonal to the smallest diagonal of a regular hexagon?

(A) 2:1
(B) $2:\sqrt{3}$
(C) $\sqrt{3}:2$
(D) Cannot be determined

26. The volume of a right circular cylinder and a sphere are in the ratio of 1:3, and the radii of the base of the cylinder and the sphere are in the ratio of 1:3. The sum of the height and the radius of the cylinder is 91 cm, and then the height of the cylinder is:

(A) 78 cm
(B) 84 cm
(C) 72 cm
(D) 83 cm

27. Shaun, the sheep, is tied to the corner of a barn, which is a rectangular farm building (with dimensions 40 × 20 feet), with a rope of length of 50 feet. No trees or other obstructions are in the way. What is the approximate maximum area of grass (in feet^2) that Shaun can eat if the barn is surrounded by the grass on all sides? Take the value of π as 3.1428.

(A) 8,000 feet^2
(B) 9,000 feet^2
(C) 6,678 feet^2
(D) 5,200 feet^2

28. The cost of levelling a square field at ₹320 per hectare is ₹13,122. The approximate cost of surrounding it with a metal railing costing ₹1.25 per metre is:

(A) ₹3,100
(B) ₹3,200
(C) ₹3,300
(D) ₹3,400

29. From a circular metal sheet of radius 15 cm, a sector with a central angle of 60° is cut and it is used to form a cone such that the arc subtending the angle 60° at the centre becomes the base of the cone. Find the volume of this cone.

(A) $35\left(\sqrt{35}\right)\pi \text{cm}^3$
(B) $41.67\left(\sqrt{35}\right)\pi \text{cm}^3$
(C) $5.21\sqrt{35}\pi \text{cm}^3$
(D) None of these

30. ABCD is a square. *P* and *Q* are points on AB and BC, respectively. The line through *P* parallel to BC and line through *Q* parallel to AB divide ABCD into two squares and two non-square rectangles. If the sum of the area of two squares is three-fourth of the area of square ABCD, then $\frac{AP}{PB} + \frac{PB}{AP}$ is:

(A) 8
(B) 6
(C) 7
(D) 10

1. 8

h = height of large cylinder = 104 cm.
Let r = radius of larger cylinder.
All 15 cylinders have the same radius r.
Moreover, the sum of heights of all the 15 cylinders will be equal to the height of the large cylinder.
Now surface area of cylinder = surface area of 2 circles + $2\pi rh$.
Let S_1 = total surface area of 15 cylinders
S_2 = total surface area of large cylinders

$\Rightarrow$ $S_1 = 15 \times$ area of two circles + $2\pi r \times$ (sum of height of 15 cylinders)

$= 30\pi r^2 + 2\pi rh$ ....(i)

$S_2 = 2\pi r^2 + 2\pi rh$ ....(ii)

According to question:

$S_1 = 2 \times S_2$

$30\pi r^2 + 2\pi rh = 2\,(2\pi r^2 + 2\pi rh)$

$30\pi r^2 + 2\pi rh = 4\pi r^2 + 4\pi rh$

$26\pi r^2 = 2\pi rh$

$\Rightarrow$ $13\,r = h$

$\Rightarrow$ $h = 13\,r$

$\Rightarrow$ $13r = 104$

$\Rightarrow$ $r = 8$ cm.

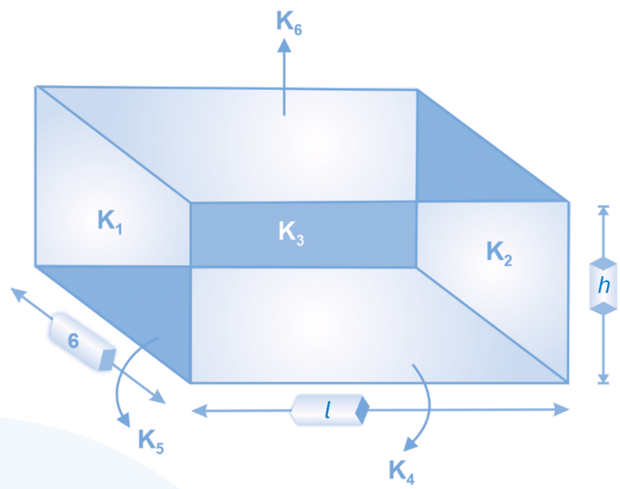
2. (D)

Let length, breadth, and height be $3x$, $2x$, and $5x$, respectively.
Area of four walls of cuboidal room
$= 2h\,(l + b) = 2(5x)(3x + 2x) = 50x^2$.
New length, breadth and height will be x, $2x$ and $\frac{5}{2}x$, respectively.

New area $= 2h\left(l + b\right) = 2\left(\frac{5}{2}x\right)\left(3x\right) = 15x^2$

% decrease in area $= \dfrac{35x^2}{50x^2} \times 100 = 70\%$

3. (C)



Since $K_1 = K_2 = b \times h$

$K_3 = K_4 = l \times h$

$K_5 = K_6 = l \times b$

Therefore, $K_1 \times K_2 \times K_3 \times K_4 \times K_5 \times K_6 = (l \times b \times h)^4$

Since, volume of the cuboid $= l \times b \times h$

Therefore, $(\text{volume})^4 = K_1 \times K_2 \times K_3 \times K_4 \times K_5 \times K_6$

Volume $= (K_1 \times K_2 \times K_3 \times K_4 \times K_5 \times K_6)^{1/4}$

Hence, option (C) is the correct answer.

4. (B)

We have: $\dfrac{1}{v} = \dfrac{k}{s}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$

$\Rightarrow$ $\dfrac{s}{v} = k\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$ ...(i)

We know that:
The volume of cuboid = abc
and the surface area of the cuboid = $2(ab + bc + ca)$

$\therefore$ $\dfrac{s}{v} = \dfrac{2\left(ab + bc + ca\right)}{abc} = 2\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$ ...(ii)

Now, solving equations (i) and (ii), we get:
$K = 2$

5. **(D)**

Given: each side of the rhombus = 20 cm

We know that,

The sum of the square of the diagonal is equal to the sum of the square of its sides.

The sum of the square of diagonal = sum of the square of its sides

$$20^2 + 20^2 + 20^2 + 20^2$$
$$= 400 + 400 + 400 + 400 = 1,600$$

$\therefore$ The required square root

$$= \sqrt{1,600} = \sqrt{40 \times 40} = 40 \text{ cm}$$

Thus, the required option (D) 40 cm is correct.

6. **(B)**

Let r, h be the radius and height of the solid cylinder, respectively.

The curved surface area of a solid cylinder = $2\pi rh$.

The total surface area of a solid cylinder = $2\pi rh + 2\pi r^2$.

According to the question:

$$2\pi rh = \frac{1}{3}\left(2\pi rh + 2\pi r^2\right)$$

$$\Rightarrow \quad 2\pi rh = \frac{1}{3}2\pi r\left(h + r\right)$$

$$\Rightarrow \quad h = \frac{1}{3}\left(h + r\right)$$

$\Rightarrow \quad 3h = h + r$

$\Rightarrow \quad 2h = r$

Given, the radius of a solid cylinder = 7 cm

$$\therefore\ h = \frac{7}{2} = 3.5 \text{ cm}$$

Thus, the required option (B) 3.5 cm is correct.

7. **(B)**

Let r and h be the radius and height of the uniform cylindrical tank, respectively.

The tank is filled to 71% of its capacity.

$\therefore$ The height of the water level = $0.71h$

The volume of water = $\pi r^2\left(0.71h\right)$

The radius is increased by 7%.

$\therefore$ The new radius = $1.07r$

The volume of the water = $\pi(1.07r)^2h'$, where h' is the new water level

$$\pi r^2\left(0.71h\right) = \pi(1.07r)^2h'$$

$\Rightarrow \quad 0.71h = 1.1449h'$

$$\Rightarrow \quad h' = \frac{0.71h}{1.1449}$$

$\therefore$ The percentage of the height of the tank, by which the level of water will fall

$$= \frac{0.71h - \dfrac{0.71h}{1.1449}}{h} \times 100$$

$= (0.71 - 0.62014) \times 100$

$= 0.08986 \times 100$

$= 8.986\% \approx 8.99\%$

Thus, the required option (B) 8.99% is the correct answer.

8. **(D)**

Let r, h be the radius and height of the cylinder.

$\therefore$ The radius of the sphere = $3r$

According to the question:

$$\frac{\pi r^2 h}{\frac{4}{3}\pi\left(3r\right)^3} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{h}{\frac{4}{3}\left(27r\right)} = \frac{1}{4}$$

$\Rightarrow \quad h = 9r$ ...(i)

Also, $r + h = 70$

Using equation (i), we get:

$r + 9r = 70$

$\Rightarrow \quad r = 7$ cm

$\therefore\ h = 70 - 7 = 63$ cm

$\therefore$ The curved surface area of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 63$$

$= 2,772 \text{ cm}^2$

Thus, the required option (D) 2,772 cm^2 is correct.

9. (B)

Given, the radius of the spherical balls = 4 cm

$\therefore$ The volume of the spherical balls

$= \frac{4}{3}\pi \times 4^3$

The volume of n sphere balls $= n\left(\frac{4}{3}\pi \times 4^3\right)$

Moreover, the volume of a solid disc of radius 8 cm and a height of 4 cm $= \pi \times 8^2 \times 4$.

According to the question:

The volume of n spherical balls = the volume of a solid disc of radius 8 cm and height of 4 cm.

$$\therefore n\left(\frac{4}{3}\pi \times 4^3\right) = \pi \times 8^2 \times 4$$

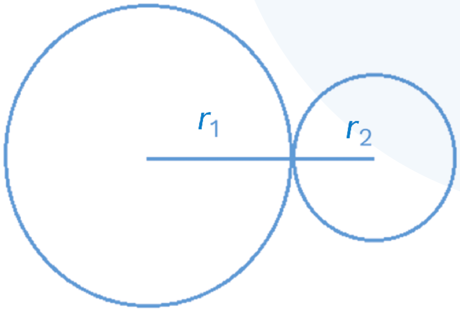
$$\Rightarrow n\left(\frac{4}{3}\pi \times 64\right) = \pi \times 64 \times 4$$

$$\Rightarrow n = 3$$

Thus, option (B) is the correct answer.

10. (B)

Let two circles of radii r_1 and r_2 touch externally such that:



$$r_1 + r_2 = 26 \quad \text{....(i)}$$

and $\pi r_1^2 + \pi r_2^2 = 346\pi$

$$\Rightarrow \pi(r_1^2 + r_2^2) = 346\pi$$

$$\Rightarrow r_1^2 + r_2^2 = 346 \quad \text{....(ii)}$$

$$(r_1 + r_2)^2 - 2r_1r_2 = 346$$

$$\Rightarrow (26)^2 - 2r_1r_2 = 346$$

$$\Rightarrow 676 - 346 = 2r_1r_2$$

$$\Rightarrow 330 = 2r_1r_2 \quad \text{....(iii)}$$

Again, $(r_1 - r_2)^2 = (r_1 + r_2)^2 - 2r_1r_2 - 2r_1r_2$

Using equations (i) and (iii), we get:

$$(r_1 - r_2)^2 = (26)^2 - 2(330)$$

$$\Rightarrow (r_1 - r_2)^2 = 676 - 660 = 16 = 4^2$$

$$\Rightarrow (r_1 - r_2) = 4 \quad \text{...(iv)}$$

Adding equations (i) and (iv), we get

$$r_1 + r_2 + r_2 - r_1 = 26 + 4$$

$$\Rightarrow r_2 = 15 \text{ cm}$$

Put $r_2 = 15$ *cm* in equation (i), we get

$$r_1 = 11 \text{ cm}$$

$$\therefore \frac{\text{Perimeter of smaller circle}}{\text{Perimeter of larger circle}} = \frac{2\pi r_1}{2\pi r_2} = \frac{11}{15}$$

$$= \frac{11}{15} = 11{:}15$$

Thus, option (B) is the correct answer.

11. 9

Let the original radius, height, and volume of the cylinder are r cm, h cm v cm^3, respectively.

Moreover, we know that volume of the cylinder $= \pi r^2 h$

Now, according to the first condition given in the question

$$\pi(r + 9)^2 h = (v + a) \quad \text{....(i)}$$

Again, according to the second condition

$$\pi r^2(h + 9) = (v + a) \quad \text{....(ii)}$$

If we equate equations (i) and (ii).

Then, $\pi(r + 9)^2 \times h = \pi r^2 (h + 9)$

Since $h = 3$ cm (original height is given)

Therefore, $(r + 9)^2 \times 3 = r^2 (3 + 9)$

$(r + 9)^2 \times 3 = r^2 \times 12$

$(r + 9)^2 = 4r^2$

$r + 9 = 2r$

$r = 9$ cm

12. **(D)**

Since there is no wastage of material in melting, therefore the volume of 100 spheres will be equal to the volume of N cones.

Moreover, 66.67% = $\frac{2}{3}$

The volume of 100 spheres = volume N cones

$$100\times\frac{4}{3}\pi r^3 = N\times\frac{1}{3}\pi\times\left(\frac{r}{100}\right)^2\times\frac{2}{3}\times r$$

$$400 = N\times\frac{1}{100}\times\frac{1}{100}\times\frac{2}{3}$$

$$N = \frac{400\times 10{,}000\times 3}{2}$$

$$N = 200 \times 10{,}000 \times 3$$

$$N = 60{,}00{,}000$$

Now, we have found the curved surface area in the starting

⇒ The curved surface area of 100 spheres = 100x 4$x\pi\times r^2$

= $400\pi r^2$

And curved surface area of a cone = πrl

Since l in the slant height.

$$l = \pi\sqrt{r^2 + h^2} = \sqrt{\left(\frac{r}{100}\right)^2+\left(\frac{2r}{3}\right)^2}$$

$$= \sqrt{\left(\frac{r^2}{10{,}000}+\frac{4r^2}{9}\right)} = r\sqrt{\left(\frac{9+40{,}000}{90{,}000}\right)}$$

$$= \frac{r}{3\times 100}\sqrt{4{,}00{,}009} = \frac{r}{300}\times 200.02249$$

$$= 0.6667\ r$$

⇒ Surface area of 60,00,000 smaller cones

$$= 60{,}00{,}000\times\pi\times\frac{r}{100}\times 0.6667\text{r}$$

$$= 40{,}002\pi r^2$$

⇒ Change in curved surface area $r\pi$

$$= 40{,}002\pi r^2 - 400\pi r^2$$

$$= 39{,}602\pi r^2$$

% change in curved surface area

$$= \frac{39{,}602\pi r^2}{400\pi r^2} = 9{,}900.5\%$$

Hence, option (D) is the correct answer.

13. **(A)**

Since the length and breadth of the given rectangle are 25 cm and b cm, respectively.

⇒ Area of rectangle = length × breadth = 25 × b = 25 b

Moreover, the circle is drawn by considering the breadth of the rectangle as the diameter.

⇒ Area of the circle = $\pi\left(\frac{b}{2}\right)^2$

Since the area of the rectangle and the area of the circle are equal.

Then, $25b = \pi\left(\frac{b}{2}\right)^2$

$$25b = \pi\times\frac{b^2}{4}$$

$$b\pi = 100$$

$$b = \frac{100}{\pi}$$

Thus, the perimeter of the rectangle

= 2 (length + breadth)

$$= 2\times\left(25+\frac{100}{\pi}\right)$$

$$= 50\left(1+\frac{4}{\pi}\right)\text{ cm}$$

Hence, option (A) is the correct answer.

14. **(C)**

External volume of the box = 24 × 16 × 10 = 3,840 cm^3.

Thickness of wood is given as 5 mm = 0.5 cm.

Therefore, internal length of the box = 24–2 × 0.5 = 23 cm.

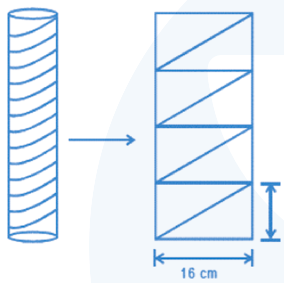
Internal breadth of the box = 16 – 2 × 0.5 = 15 cm.

And, internal height of the box = 10 – 2 × 0.5 = 9 cm
Now, we have to find:
Internal volume of the box = 23 × 15 × 9 = 3,105 cm^3.
∴ Volume of the wood = 3,840–3,105 = 735 cm^3.
Now, the total weight of the wood = volume × weight of 1-cm^3 wood
8 kg = 8,000 g = 735 × weight of 1-cm^3 wood

∴ Weight of 1-cm^3 wood = $\frac{8,000}{735}$ = 10.8843 g

which is approximately equal to 10.9
Hence, option (C) is the correct answer.

15. **(A)**



The base circumference = $\frac{22}{7} \times \frac{56}{11} = 16\,\text{cm}$

The length of one complete tum

$= \sqrt{16^2 + 12^2} = 20\,\text{cm}.$

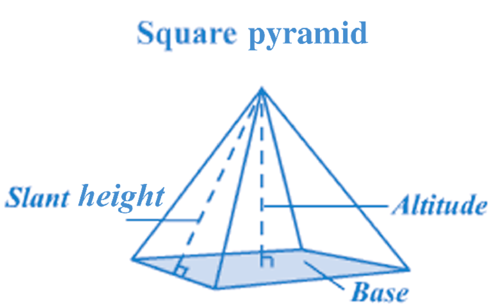
Hence, total length = 80.

16. **(C)**
Given, the length of each side = 15 cm

∴ The diagonal of the square = $15\sqrt{2}$ cm

Let h be the height of the vertical pyramid.

$\therefore\ h^2 + (\text{half of the diagonal})^2 = 15^2$



$$\Rightarrow\ h^2 + \left(\frac{15\sqrt{2}}{2}\right)^2 = 15^2$$

$$\Rightarrow\ h^2 + \frac{450}{4} = 225$$

$$\Rightarrow\ h^2 = 225 - \frac{450}{4} = \frac{900-450}{4}$$

$$\Rightarrow\ h^2 = \frac{450}{4} = \left(\frac{15\sqrt{2}}{2}\right)^2$$

$$\Rightarrow\ h = \frac{15\sqrt{2}}{2}\ \text{cm}$$

Hence, option (C) is the correct answer.

17. **(C)**
Given:
The volume of a hemisphere = 30 cm^2
Let r be the radius of a hemisphere.

∴ The volume of a hemisphere = $\frac{2}{3}\pi r^3$

Let r be the radius of a cylinder.
∴ The height of a cylinder = r
The volume of a cylinder = $\pi r^2 r = \pi r^3$
According to the question:

$$\therefore\ \frac{\text{The volume of a hemisphere}}{\text{The volume of a cylinder}} = \frac{\frac{2}{3}\pi r^3}{\pi r^3}$$

$$= \frac{2}{3} = \frac{30}{45}$$

∴The volume of a cylinder = 45 cm^2
Hence, option (C) is the correct answer.

18. **(A)**
Since both ratios are 4:5 and 5:4.
Assume that both cans contain 9 litres of fuel.
First can:
Petrol = 4 litres and diesel = 5 litres
Second can:
Petrol = 5 litres and diesel = 4 litres
∴ Total petrol = 4 + 5 = 9 litres
And total diesel = 5 + 4 = 9 litres
∴ The ratio of petrol and diesel in larger can = 9:9 = 1:1
Hence, option (A) is the correct answer.

19. (C)

Let a be the side of an equilateral triangle and s be the side of the square.
According to the question:
The perimeter of the square = perimeter of the equilateral triangle

$\therefore 4s = 3a$

$$\Rightarrow \frac{s}{a} = \frac{3}{4}$$

The area of the square $= 3^2 = 9$ sq. units

Moreover, the area of an equilateral triangle

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 16 = 4\sqrt{3} = 6.928 \text{ sq. units}$$

Hence, option (C) is the correct answer.

20. (B)

Given, the area of a circle = 324π

$\therefore \pi r^2 = 324\pi \Rightarrow r = 18$ cm

Now, a regular hexagon is made up of six equilateral triangle's equal areas.

$\therefore$ The height of the equilateral triangle = the radius of the circle.

$\therefore$ The area of one equilateral triangle

$$= \frac{1}{2} \times b \times h$$

$$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \times a \times 18 \Rightarrow \sqrt{3}a = 36$$

$$\Rightarrow \quad a = 12\sqrt{3} \text{ cm}$$

$\therefore$ The area of the regular hexagon

$$= 6 \times \frac{\sqrt{3}}{4}a^2$$

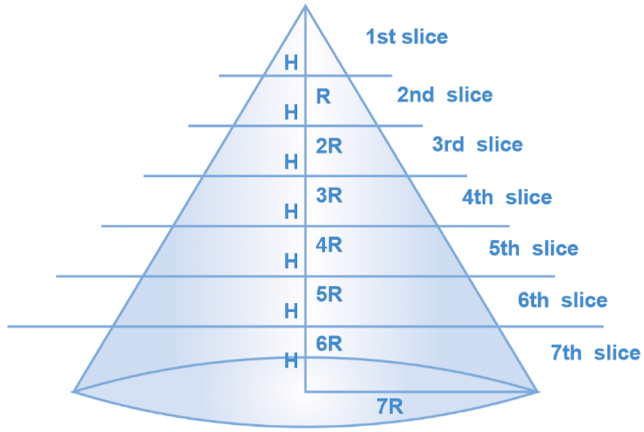
$$= 6 \times \frac{\sqrt{3}}{4}\left(12\sqrt{3}\right)^2 \text{ cm}^2$$

$$= 6 \times \frac{\sqrt{3}}{4} \times 12\sqrt{3} \times 12\sqrt{3} \text{ cm}^2$$

$$= 648\sqrt{3} \text{ cm}^2$$

Hence, option (B) is the correct answer.

21. (D)



By observation and looking at the radius and heights, we can say that the smallest slice in terms of Volume would be the first slice and it will be a cone.

The biggest slice would be the seventh slice and it will be a frustum of the cone.

The volume of the smallest slice $= \frac{1}{3}\pi R^2 H$

The volume of the largest slice = volume of the cone with radius $7R$ and height $7H$ – volume of the cone with radius $6R$ and height $6H$.

The volume of the largest slice.

$$= \frac{1}{3}\pi (7R)^2 \times 7H - \frac{1}{3}\pi (6R)^2\, 6H$$

$$= \frac{1}{3}\pi R^2 H \left[7^3 - 6^3\right]$$

$$= \frac{1}{3}\pi R^2 H [127]$$

Hence, the required ratio = 1:127

22. (D)

Given, the radius of the sphere = r

The area of the sphere = $4\pi r^2$

The cumulative area of the two pieces = 30% more than that of a sphere

The area of two pieces = $1.3 \times 4\pi r^2 = 5.2\pi r^2$

$\therefore$ The extra area of a sphere = $5.2\pi r^2 - 4\pi r^2$

$= 1.2\pi r^2$

Extra area = the area of two new circles that are now created circles.

$\therefore$ The area of each new circle = $\frac{1.2\pi r^2}{2}$

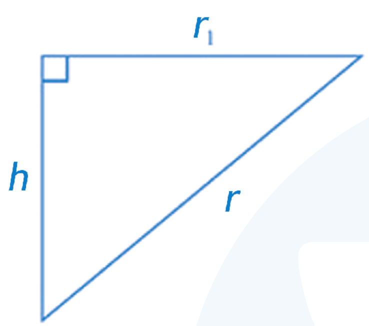
Let r_1 be the radius of the new circle.

$$\pi r_1^2 = \frac{1.2\pi r^2}{2}$$

$$\Rightarrow \quad r_1^2 = 0.6r^2$$

$$\Rightarrow \quad r_1 = 0.7745r$$

Now r_1, h, and r form a right-angled triangle.



$$h = \sqrt{r^2 - r_1^2}$$

$$\Rightarrow \quad h = \sqrt{(r)^2 - (r_1)^2}$$

or, $h = \sqrt{(r)^2 - (0.6r^2)}$

or, $h = \sqrt{0.4r^2}$

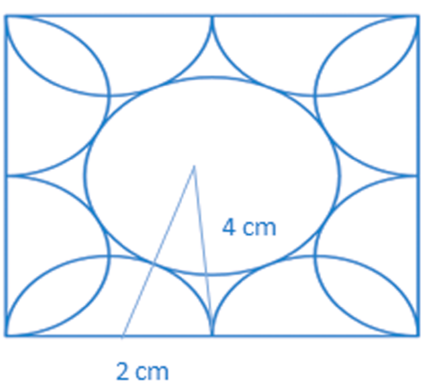
or, $h = 0.6324r$

23. (B)

Given, the side length of square (a) = 8 cm

$\therefore$ The radius of the semicircle (r) = 4 cm

We connect the centres of the circle and one of the semicircles, then draw the perpendicular from the centre of the middle circle to that side, as shown in the given figure:



By using Pythagoras theorem:

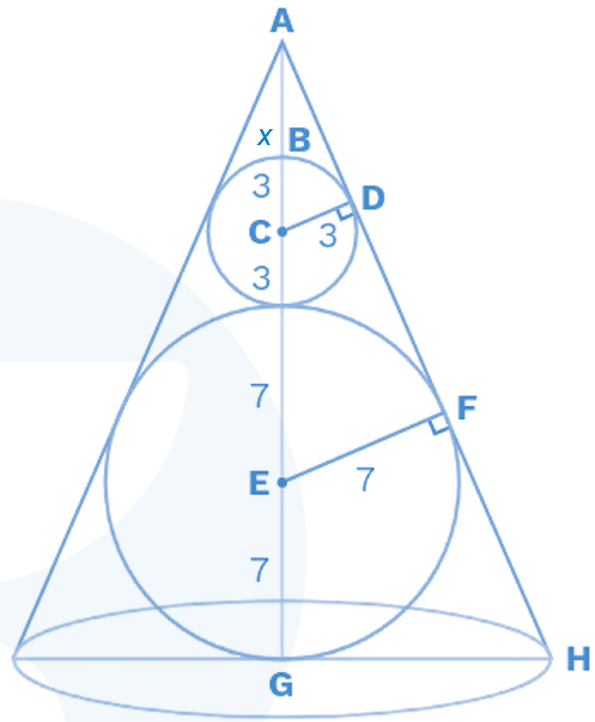
$$\sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \text{ cm}$$

Let r be the radius of the circle.

$\therefore$ $2 + r = 2\sqrt{5}$

$\Rightarrow$ $r = 2\sqrt{5} - 2 = 2(\sqrt{5} - 1)$ cm

24. (B)



The diagram for the question will be as shown above.

Here, AB = x, BC = CD = radius of the smaller sphere = 3 cm, EF = EG = radius of the larger sphere = 7 cm.

Now triangle ACD is similar to triangle AEF (by AAA similarity).

So, AC/AE = CD/EF

$(x + 3)/(x + 3 + 3 + 7) = 3/7$

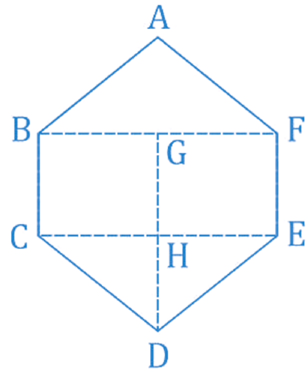
Solving which we will get x = 4.5 cm

Thus, the height of the cone = AE + 7 = (x + 3 + 3 + 7) + 7 = 24.5 cm

Hence, option (B) is the correct answer.

25. (B)

Let ABCDEF be a regular hexagon of side a.



The sum of the total internal angles in hexagon = $(n - 2) \times 180°$

$= (6 - 2) \times 180° = 4 \times 180° = 720°$

$\therefore$ Each internal angle in a hexagon

$= \frac{720°}{6} = 120°$

Let BF and AD be the smallest and largest diagonals of the given hexagon.

Since, BC = FE and BF = CE

Therefore, BCEF is a rectangle.

$\therefore$ CBF = 90°

$\therefore$ ABF = 90° = 120° – 90° = 30°

Now, in triangle ABG :

$$BG = a\cos 30° = \frac{\sqrt{3}a}{2}$$

$$\therefore BF = 2BG = 2 \times \frac{\sqrt{3}a}{2} = \sqrt{3}a \quad ....(i)$$

$$AG = a\sin 30° = \frac{a}{2}$$

$$\therefore AD = AG + GH + HD = \frac{a}{2} + a + \frac{a}{2}$$

$$= 2a \quad ....(ii)$$

Hence, $\frac{BF}{AD} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$ [using (i) and (ii)]

$\therefore$ The ratio of the largest diagonal to the smallest diagonal of a regular hexagon

$= AD:BF = 2:\sqrt{3}$

Hence, option (B) is the correct answer.

26. (B)

Let r, h be the radius and height of the cylinder.

$\therefore$ The radius of the sphere = $3r$

According to the question:

$$\frac{\pi r^2 h}{\frac{4}{3}\pi(3r)^3} = \frac{1}{3}$$

$$\Rightarrow \frac{r^2 h}{\frac{4}{3}(27)r^3} = \frac{1}{3}$$

$\Rightarrow$ $h = 12r$ ...(i)

Also, $r + h = 91$

Using equation (i), we get:

$r + 12r = 91$

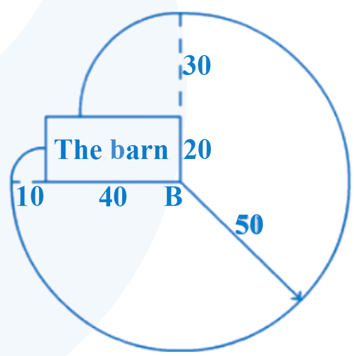
$\Rightarrow$ $r = 7$ cm

$\therefore$ $h = 91 - r = 91 - 7 = 84$ cm

Hence, the height of the cylinder is 84 cm.

Hence, option (B) is the correct answer.

27. (C)



Shaun's rope is anchored at B.

Shaun can range through 3/4th of the circle of radius 50 feet, 1/4th of a circle of radius 30 feet and 1/4th of a circle of radius 10 feet.

Required area = $(3/4) \times \pi(50)^2 + (1/4) \times \pi(30)^2 + (1/4) \times \pi(10)^2$

$= \pi(1{,}875 + 225 + 25)$

$= 2{,}125\pi = 2{,}125 \times 3.1428 = 6{,}678.45$ feet^2

$6{,}678 \approx$ feet^2

28. (B)

Since the cost of levelling the square field per hectare is ₹320 and the total expenditure on levelling is ₹13,122. So from above, we can find the area of the square field.

$\therefore$ Area of the square field = $\frac{13,122}{320}$

= 41.00625 × 10,000 m^2

Note: [1 hectare = 10,000 m^2]

$\therefore$ Area = 41.0062.5 m^2

We know that the area of the square field if side length is $a = a^2$

a^2 = 41.0062.5 m^2

a = 640.3612 m (approx.)

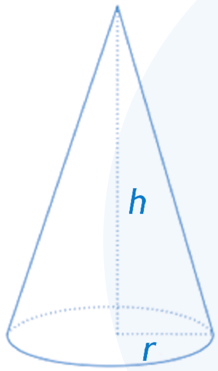
$\therefore$ The perimeter of the square field = $4a$ = 4 × 640.3612 = 2,560 m (approx.)

$\therefore$ The total expenditure in surrounding the square field = 2,560 × 1.25 = ₹3,200

Hence, option (B) is the correct answer.

29. (C)

The length of arc = $\frac{60°}{360°} 2\pi\left(15\right) = 5\pi$



The perimeter of the base of the cone = $2\pi R$

$\therefore\ 2\pi R = 5\pi$

$\Rightarrow\ R = \frac{5}{2}$ cm

$$\therefore h = \sqrt{15^2 - \left(\frac{5}{2}\right)^2} = \sqrt{225 - \frac{25}{4}}$$

$$= \sqrt{\frac{875}{4}} = \frac{5\sqrt{35}}{2} \text{ cm}$$

The volume of the cone = $\frac{1}{3}\pi R^2 h$

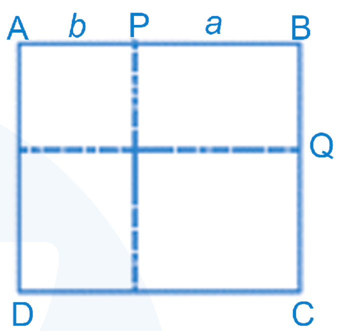
$$= \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 \cdot \frac{5\sqrt{35}}{2}$$

$$= \frac{1}{3}\pi . \frac{25}{4} \cdot \frac{5\sqrt{35}}{2}$$

$$= 5.21\sqrt{35}\pi \text{ cm}^3$$

Hence, option (C) is the correct answer.

30. (B)



Let the side length of square ABCD be p.

Let PB (= a) and AP (= b) be the lengths of sides of larger and smaller inner squares, respectively.

Then, $\frac{AP}{PB} + \frac{PB}{AP} = \frac{b}{a} + \frac{a}{b} = \frac{a^2 + b^2}{ab}$

The sum of the area of the two inner squares = $a^2 + b^2 = 3p^2/4$

The sum of the area of the two inner non-square rectangles = $2ab = p^2/4$

$\Rightarrow ab = p^2/8$

Hence, $\frac{a^2 + b^2}{ab} = \frac{3p^2 / 4}{p^2 / 8} = 6$

Hence, option (B) is the correct answer.

Mind Map

MENSURATION

DEFINITION

Mensuration is the branch of Geometry and Science of measurement of the length, area or volume in 2D and 3D shapes.

2D MENSURATION

TRIANGLES

1. Scalene Δ.
2. Isosceles.
3. Equilateral Δ.
4. Right angle Δ.

SCALENE Δ

Perimeter = a+b+c

If height is given

Area = $\frac{1}{2}$ × base × height

If height is not given

$S = \frac{P}{2} = \frac{a+b+c}{2}$

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

ISOSCELES Δ

P = 2a+b

height = $\frac{1}{2}\sqrt{4a^2-b^2}$

$\Delta = \frac{b}{4}\sqrt{4a^2-b^2}$

EQUILATERAL Δ

Perimeter = 3a

Height $= \frac{\sqrt{3}}{2}a$

Area $= \frac{\sqrt{3}}{4}a^2$

RIGHT ANGLE TRIANGLE Δ

Area = $\frac{1}{2}$ × P × b

Perimeter = P+B+h

QUADRILATERAL

1. Rectangle
2. Square
3. Rhombus
4. Parallelogram
5. Trapizium

HEXAGON

Perimeter = 6a

Area $= 6\times\frac{\sqrt{3}}{4}a$

OCTAGON

P = 8a

Area $= 2a^2(1+\sqrt{2})$

CIRCLE

P $= 2\pi r$

Area $= \pi r^2$

SEMICIRCLE

P $= \pi r+2r$

$= \frac{36}{7}r$

Area $= \frac{1}{2}\pi r^2$

QUADRANT

P $= 2r+\frac{\pi r}{2}$

Area $= \frac{1}{4}\pi r^2$

SECTOR OF A CIRCLE

$l = \frac{\theta}{360^\circ}2\pi r$

Area $= \frac{\theta}{360^\circ}\times\pi r^2$

P = 2r+l

3D MENSURATION

- CUBE
- CUBOID
- CYLINDER
- CONE
- SPHERE
- HEMISPHERE
- FRUSTUM OF CONE
- PRISM
- PYRAMID

2D MENSURATION 3D

CUBE

L.S.A. $= 4a^2$

T.S.A. $= 6a^2$

Vol. $= a^3$

Diagonal $= \sqrt{3}a$

CUBOID

L.S.A. = 2(l+b)×h

T.S.A. = 2(lb+bh+hl)

Vol. = lbh

Diagonal $= \sqrt{l^2+b^2+h^2}$

CYLINDER

C.S.A. $= 2\pi rh$

T.S.A. $= 2\pi r(r+h)$

Vol. $= \pi r^2h$

⇒

CONE

C.S.A. $= \pi rl$

T.S.A. $= \pi r(r+l)$

$l = \sqrt{r^2+h^2}$

Vol. $= \frac{1}{3}\pi r^2h$

SPHERE

Vol. $= \frac{4}{3}\pi r^3$

C.S.A. $= 4\pi r^2$

T.S.A. $= 4\pi r^2$

⇒

HEMISPHERE

Vol. $= \frac{2}{3}\pi r^3$

C.S.A. $= 2\pi r^2$

T.S.A. $= 3\pi r^2$

PRISM

Vol. $= A_b\times ht$

C.S.A. $= P_b\times ht$

T.S.A. = C.S.A.+2 area of base

⇓

PYRAMID

Vol. $= \frac{1}{3}$ × area of base × height

C.S.A. $= \frac{1}{2}\times P_b\times$ slant height

T.S.A. $= \frac{1}{2}\times P_b\times$ slant height + area of base