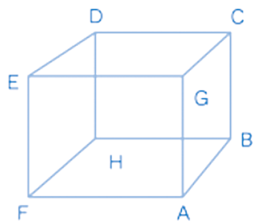
8 Cube Cutting and Dice

Introduction

A cube is a three-dimensional solid with six square faces as shown below:



The CAT and OMETs utilize the cubes to gauge MBA aspirants' reasoning and observation skills by creating questions under various conditions, e.g., colouring a cube with single or multiple colours, cutting a cube by equidistant lines, etc.

A die is a cube with each of its six faces marked with the numbers: 1, 2, 3, ..., 6. In some cases, the numbers might be replaced with dots, as shown in the figure below given. The die is used for playing board games like Ludo. The plural form of the die is dice.

Important Facts about Cubes and Dice

- They have 12 edges.
- They have eight points (vertices).
- They have six square faces or sides.

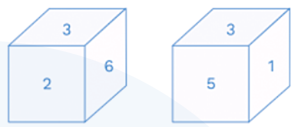
Certain Basic Rules

There are certain rules which govern the thought process of the question creator. Let's analyse them one by one.

Rule 1

Example 1:

Analyse the positions of the following dice.



The faces with numbers 2, 6, 5, and 1 are adjacent to face number 3.

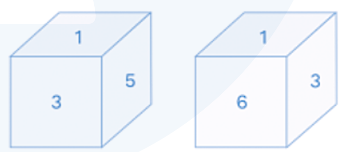
Hence, the numbers 2, 6, 5, and 1 can't be opposite to face number 3.

It can be said that face number 3 is the opposite to face number 4.

Rule 2

Example 2:

Analyse the positions of the following dice.



1 and 3 are the common faces.

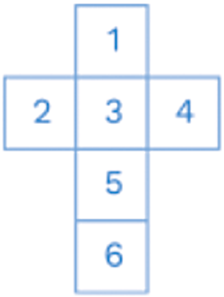
5 and 6 are the remaining two faces.

Construction of Cubes

A cube can be constructed in the following eight ways:

Form I

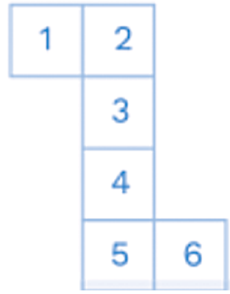
In this case:



1 lies opposite 5;
2 lies opposite 4;
3 lies opposite 6;

Form II

In this case:



1 lies opposite 6;
2 lies opposite 4;
3 lies opposite 5;

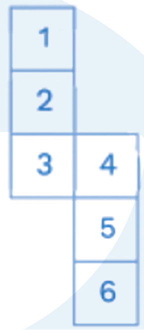
Form III

In this case:

1 lies opposite 4;
2 lies opposite 6;
3 lies opposite 5;

Form IV

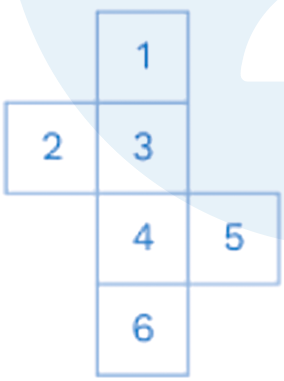
In this case:



1 lies opposite 3;
2 lies opposite 5;
4 lies opposite 6;

Form V

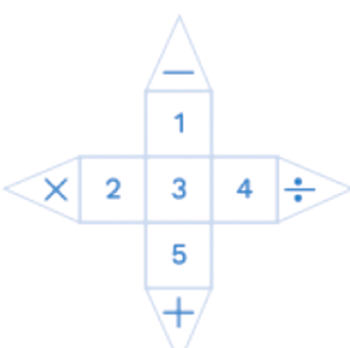
In this case:



1 lies opposite 4;
2 lies opposite 5;
3 lies opposite 6;

Form VI

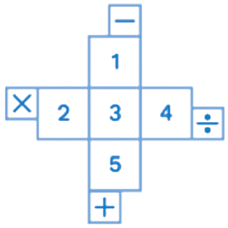
In this case:



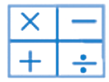
will be the one of the faces of the cube and it lies opposite 3;

2 lies opposite 4;
1 lies opposite 5;

Form VII

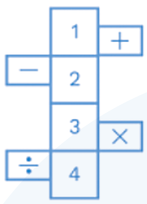


In this case:

 will be the one of the faces of the cube and it lies opposite 3,

2 lies opposite 4;
1 lies opposite 5;

Form VIII



In this case:

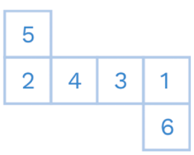
 and  Give two faces of the cube that lie opposite to each other

1 lies opposite 3;
2 lies opposite 4;

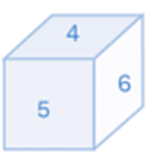
Some examples and their solutions

What kind of box will be formed if the below-given figure is folded?

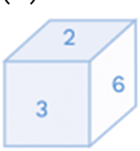
Example 3:



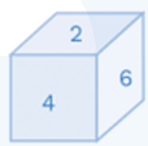
(A)



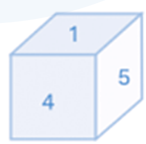
(B)



(C)



(D)



Solution: (C)

2 ⟷ 3

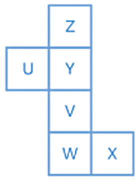
4 ⟷ 1

5 ⟷ 6

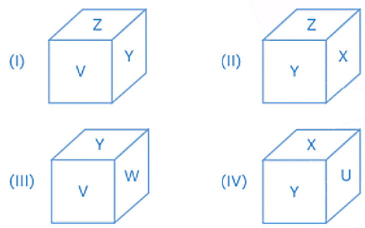
Hence, option C is correct.

Example 4:

What kind of box will be formed if the figure given below is folded?



(X)



(A) Only (II)

(B) Only (I)

(C) (II) and (III) only

(D) (I), (II), (III), and (IV) only

Solution: (A)

Z ⟷ V

Y ⟷ W

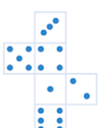
U ⟷ X

Hence, only the cube in Fig. (II) can be formed.

Hence, option (A) is correct.

Example 5:

If the figure below given is folded to form a cube, then how many dots lie opposite the face with two dots?



(A) 2
(B) 4
(C) 5
(D) 6

Solution: (C)

4 dots ⟷ 6 dots

1 dot ⟷ 3 dots

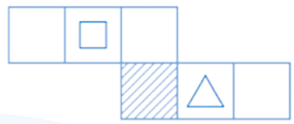
2 dots ⟷ 5 dots

Hence, 5 dots lie opposite the face with 2 dots.

Hence, option (C) is correct.

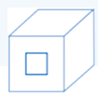
Example 6:

Which of the option figures can be formed from the given sheet of paper (X)?



(X)

(A)



(B)

(C)

(D)

Solution: (A and C)

Fig. (X) is similar to form IV. So, when the sheet in Fig. (X) is folded to form a cube, the face bearing a square lies opposite the face bearing a triangle.

Therefore, the cubes shown in options (B) and (D), which have the faces bearing the square and the triangle adjacent to each other, cannot be formed.

Hence, only the cubes in options (A) and (C) can be formed.

Hence, options (A) and (C) are correct.

Cube Cutting

A cube has 6 faces.

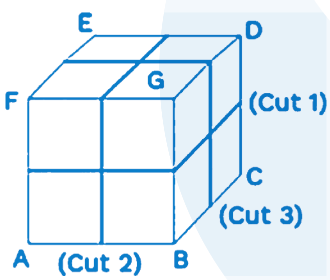
A cube has 12 edges.

A cube has 8 vertices.

Keynote

If and only if $x = y = z$ and all the cuts are equidistant, we get all the pieces as cubes; otherwise, they will be cuboids.

If a cube is cut along any dimension with n straight lines, then there will be $(n + 1)$ pieces of cubes.



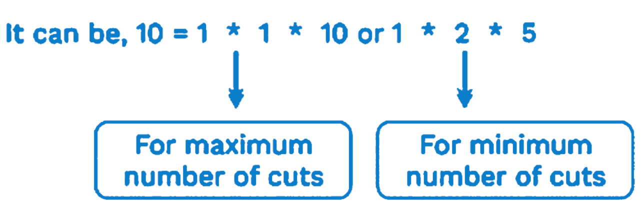
It can be said that X mutually parallel cuts along the x-axis, Y mutually parallel cuts along the y-axis, and Z mutually parallel cuts along the z-axis, would yield $(X + 1) \times (Y + 1) \times (Z + 1)$ pieces of the original cube.

Let's understand this point by an example.

Suppose, a cube is cut into 10 pieces.

Now, $10 = (x + 1) \times (y + 1) \times (z + 1)$

Now, 10 is to be represented as the product of three numbers or factors.



The minimum number of cuts is obtained by factorising the number of pieces into three equal factors. If three equal factors are not possible, then minimise the difference between the numbers. In the case of 10 pieces, $1 \times 2 \times 5$ gives the minimum difference between factors.

To maximise the total number of cuts, all the cuts are made on one axis for instance, for 10 pieces, factors are $10 \times 1 \times 1$, which means the x-axis cuts are 9 and no cut is made in the other two axes.

Example 7:

What is the least number of cuts that should be made to get 24 identical pieces from a single cube?

(A) 6
(B) 5
(C) 7
(D) 8

Solution: (A)

It is given that there are 24 identical pieces. Factorise the number into three such factors so that all the factors are greater than 1.

$\Rightarrow 24 = 2 \times 2 \times 2 \times 3$

$\Rightarrow 24 = 2 \times 3 \times 4$

On comparing the above values with the formula $(x + 1) \times (y + 1) \times (z + 1)$, we get $x = 1$, $y = 2$, and $z = 3$.

So, the least number of cuts = 6.

Hence, option (A) is correct.

Cutting a Big Cube into Smaller Cubes after Colouring the Faces

If a big cube is painted using a single colour on all its faces, and then cut into $(n \times n \times n)$ smaller cubes, then:

The number of cubes with 0 face painted $= (n - 2)^3$.

(i.e., all the unexposed cubes)

The number of cubes with one face painted $= 6\,(n - 2)^2$.

(i.e., all the cubes on the faces, excluding the ones along the edges).

The number of cubes with two faces painted = 12(n - 2) (i.e., all the cubes on the edges excluding the corner ones).

The number of cubes with three faces painted = 8 (i.e., all the corner cubes).

Example 8:

A cube of dimension 8 × 8 × 8 cubic cm is cut into 64 smaller pieces.

1. What is the volume (in cu. cm) of each cube?

Solution: 8

Simply, 512 cu. cm / 64 = 8 cu. cm
(Each smaller cube will be of the dimension 2 × 2 × 2)
Hence, the answer is 8 cu. cm.

2. How many cubes will have exactly two faces painted?

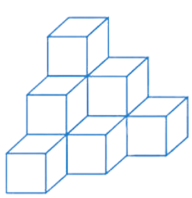
Solution: 24

As there are 64 smaller cubes, $n^3 = 64$, i.e., $n = 4$
Hence, the number of cubes with two faces painted = 12(n - 2) = 12 (4 − 2) = 24
Hence, the answer is 24 cubes.

Counting of Cubes

Example 9:

1. How many cubes are there in the below-given figure?



(A) 6
(B) 8
(C) 10
(D) 12

Solution: (C)

The given figure has one layer containing 6 cubes, the second layer containing 3 cubes and the third layer containing 1 cube. So, the total cubes are 10.
Alternately,
Total cubes in 1 column with 3 cubes = 1 × 3 = 3.
Total cubes in columns with 2 cubes each = 2 × 2 = 4.
Total cubes in 3 columns with 1 cube each = 3 × 1 = 3.
Therefore, total number of cubes = 3 + 4 + 3 = 10.
Hence, option (C) is correct.

2. How many cubes are there in the below-given figure?

(A) 13
(B) 14
(C) 15
(D) 16

Solution: (C)

The given figure consists of 1 column containing 3 cubes each, 4 columns containing 2, and 4 columns containing 1 cube each.
Number of cubes in column of 3 cubes = 1 × 3 = 3.
Number of cubes in column of 2 cubes = 4 x 2 = 8.
Number of cubes in column of 1 cube = 4 x 1 = 4.
Therefore, total number of cubes = 3 + 8 + 4 = 15.
Hence, option (C) is correct.

Paper Cutting and Folding

Paper folding- and cutting-based questions are slightly less common in exams like SNAP, NMAT, CMAT, and TISSNET. However, they are more frequent in exams like MAH CET and MAT. But you never know when this variety of questions is added in any of the above exams, and hence, it is important to be prepared with the basics of this topic.

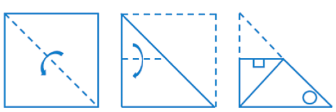
The questions on paper folding and cutting test your spatial ability along with your reasoning ability. Paper folding and cutting involves a *process* in which a sheet of paper is folded and then some cuts and/or folds are made.

In the questions based on paper folding and cutting, a few *figures* are given showing how a piece is to be folded and then cut from a particular section.

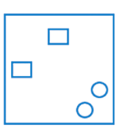
In questions based on paper folding and cutting it can be asked to find either the folded or unfolded pattern of the sheet.

Let us look at some examples to understand the approach to solve such questions.

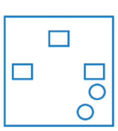
1. A piece of paper is folded and cut marks are made on it. Select the figure which correctly represents the unfolded paper.



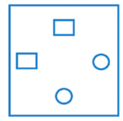
(A)



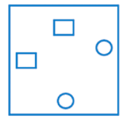
(B)



(C)



(D)



Solution: (A)

When we unfold the paper the figure is shown as:

Hence, option (A) is correct.

2. A piece of paper is folded and punched as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.

(A)

(B)

(C)

(D)

Solution: (A)

Hence, option (A) is correct.

3. A piece of paper is folded and punched as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.

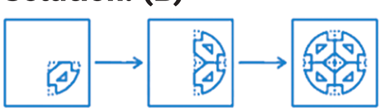
(A)

(B)

(C)

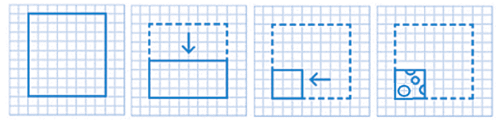
(D)

Solution: (B)

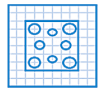


Hence, option (B) is correct.

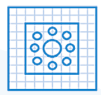
4. A piece of paper is folded and cut as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.



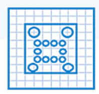
(A)



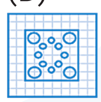
(B)



(C)

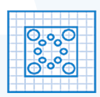


(D)



Solution: (D)

When the paper is unfolded, the figure is shown as:



Hence, option (D) is correct.

5. A piece of paper is folded and punched as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.

(A)

(B)

(C)

(D)

Solution: (A)

When the paper is unfolded, the figure is shown as:

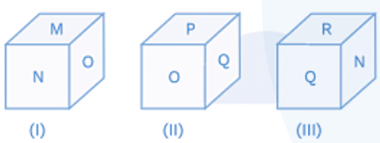
Hence, option (A) is correct.

Level of Difficulty – 1

1. What is the minimum number of pieces that can be obtained with 37 cuts out of a cube without putting the pieces one above another?
(A) 37
(B) 38
(C) 39
(D) 40

2. What is the maximum number of pieces that can be obtained with 26 cuts of a cube without putting the pieces one above another?
(A) 1,000
(B) 729
(C) 920
(D) 900

3. A die is rolled down three times, and the following three positions appear:

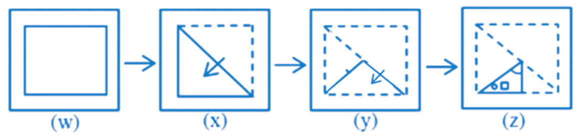


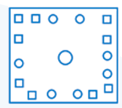
Find the alphabet opposite to M.
(A) O
(B) P
(C) Q
(D) R

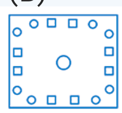
4. Thirty-five cuts are made on a cube. What will be the difference between the maximum and the minimum number of cubes obtained?
(A) 1,992
(B) 512
(C) 2,048
(D) 1,635

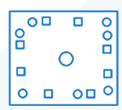
5. Find the least number of cuts required that can cut a cube into 120 identical pieces.
(A) 12
(B) 11
(C) 10
(D) 9

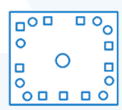
6. A paper is folded and cut as shown in the following problem figure (W, X, Y, and Z) Select the answer figure that depicts how the paper will appear when unfolded.



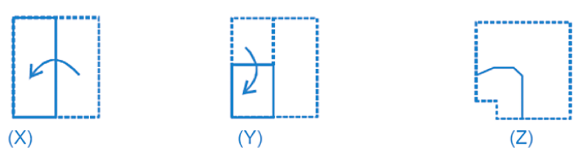
(A)


(B)


(C)


(D)


7. The sequence of folding a piece of square paper and the manner in which the folded paper has been cut is shown in figures X, Y, and Z. How would this paper look when unfolded?



(A)


(B)

(C)

(D)

8. The sequence of folding a piece of paper and the manner in which the folded paper has been cut is shown in the question figures. Select the figure from the answer figures that would most closely resemble the unfolded paper.

(A)

(B)

(C)

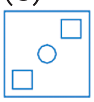
(D)

9. The sequence of folding a piece of paper and the manner in which the folded paper has been cut is shown in the following figures. How would this paper look when unfolded?

(A)

(B)

(C)



(D)

10. A square paper is folded and cut as shown below. How will it appear when unfolded?

(A)

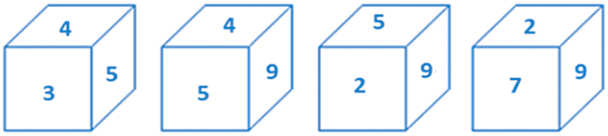
(B)

(C)

(D)

11. What is the least number of cuts required to cut a cube into 28 identical pieces?
(A) 8
(B) 7
(C) 9
(D) 10

12. Which number will be on the face opposite to the face with the number 5?



(A) 1
(B) 7
(C) 4
(D) 10

13. What is the maximum possible number of identical pieces a cube can be cut into by 13 cuts?
(A) 100
(B) 150
(C) 140
(D) 160

Directions for Questions 14 to 17: Read the information carefully and answer the questions that follow.

A bigger cube is painted orange on each of its faces. Now the cube is cut into 216 identical pieces making a minimum number of cuts.

14. How many of the smaller cubes will have exactly three faces painted?
(A) 6
(B) 7
(C) 8
(D) 10

15. How many of the smaller cubes will have exactly two faces painted?
(A) 48
(B) 52
(C) 56
(D) 60

16. How many of the smaller cubes will have exactly one face painted?
(A) 90
(B) 96
(C) 100
(D) 108

17. How many of the smaller cubes will have no face painted?
(A) 50
(B) 56
(C) 64
(D) 72

Level of Difficulty - 2

1. A piece of paper is folded and punched as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.

(A)

(B)

(C)

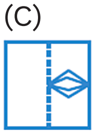
(D)

2. Given here is a square transparent sheet with a pattern on it. How would the pattern appear when the transparent sheet is folded on the dotted line?

(A)

(B)

(C)



(D)

3. Find out which of the figures (A), (B), (C), and (D) can be formed from the pieces given in figure X.

Figure X

(A)

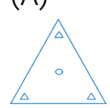
(B)

(C)

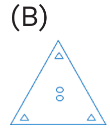
(D)

4. If the paper is folded according to the underlined points, what will it look like when opened?

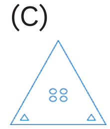
(A)



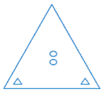
(B)



(C)



(D)



5. A paper is folded and cut as shown in the following problem figures (X, Y, and Z). Select the answer figure that depicts how the paper will appear when unfolded.

X Y Z

(A)

(B)

(C)

(D)

Directions for Questions 6 to 7: Read the information carefully and answer the questions that follow.

Rajeev takes a cube of side 9 cm. He now colours the cube with three different colours, i.e., blue, black and white. After colouring it is seen that three faces of the larger cube are coloured white, two of its faces are coloured black and the remaining one face is coloured blue. Now, he cuts this cube into smaller cubes of side 1 cm.

6. The maximum possible number of small cubes that have only blue and black colours on their faces is:

(A) 10
(B) 15
(C) 20
(D) 25

7. The maximum number of small cubes such that all three colours are on it is:

(A) 2
(B) 4
(C) 6
(D) 8

Directions for Questions 8 to 10

A rectangular block of length 6 cm, breadth 5 cm, and height 3 cm is painted in six different colours. The six faces are coloured with six different colours as follows:

i) One face of dimension 6 cm × 5 cm is coloured green and the other face is coloured blue.
ii) One face of dimension 5 cm × 3 cm is coloured black and the other face is coloured pink.
iii) One face of dimension 6 cm × 3 cm is coloured yellow and the other face is coloured orange.

Now, the rectangular block is cut in such a way that only cubes of side 1 cm are formed.

8. The number of cubes with no colour on them is less than the number of cubes with at least one colour on them by:

(A) 66
(B) 85
(C) 45
(D) None of these

9. The number of cubes with pink colour on it is more or less than the number of cubes with only blue colour on it by:

(A) 1
(B) 2
(C) 3
(D) 4

10. The number of cubes with exactly two colours on it but none of which is pink or black is:

(A) 12
(B) 16
(C) 20
(D) 24

Directions for Questions 11 to 14

Three brothers, Asus, Jasus, and Masus, like playing different games based on cubes. All the three are very smart and every day they try something new so that it keeps them excited as well. Today, they have picked up a cube and want to do some learning on it apart from just playing around with it. They plan to cut the cube into smaller cubes but before that, they also apply colours on each surface of the larger cube. The cube taken by them is of side 12 cm and it is cut by 15 planes such that the number of smaller cubes so formed is maximum in number. The smaller cubes so formed are also all alike in dimensions.

11. If before the larger cube was cut, it was painted with blue colour on all of its faces, then after cutting the number of smaller cubes which do not have blue colour on its faces?

(A) 64
(B) 70
(C) 75
(D) 80

12. If before the larger cube was cut, it was painted such that two of its faces were painted with blue colour and remaining faces was coloured with red colour, then the maximum number of cubes having only blue colour on its faces is:

(A) 30
(B) 36
(C) 42
(D) 46

13. If before the larger cube was cut, it was painted such that three of its faces were painted with blue colour and the other three with green colour, then the minimum number of cubes having only blue colour on their faces is:

(A) 48
(B) 52
(C) 56
(D) 60

14. If before the larger cube was cut, it was painted such that one pair of opposite faces is coloured green, and one pair of adjacent faces is coloured blue. Of the remaining two faces, one is painted black and the other is painted red. Then, the difference between the number of smaller cubes which has only one colour on them and the number of cubes with exactly two colours on them is:

(A) 44
(B) 48
(C) 50
(D) 54

15. A cube is painted with three different colours such that opposite faces of the cube are painted with the same colour. The cube is then cut into 64 identical cubes. What is the ratio of the number of the smaller cubes with all the three colours on three faces to the number of cubes with no colour on any of their faces?

(A) 2:3
(B) 1:1
(C) 8:27
(D) 3:2

16. What is the minimum number of pieces that can be obtained with 37 cuts without putting the pieces one above another?

(A) 37
(B) 38
(C) 39
(D) 40

Level of Difficulty - 3

Set 11

Directions for Questions 1 to 4

A large cube is to be formed from 729 small but identical cubes. The small cubes are numbered 1 to 729 with numbers being etched in a sequence on the surface of the small cube. To form the large cube from the numbered small cubes, a number of vertical layers are constructed using the following instructions:

i) The bottom row of the first vertical layer is constructed, selecting the small cubes in the ascending order of the number etched on the small cubes and placing them from the left-most corner of the cube to the right-most corner.

ii) Once the bottom row is completed, the next row is formed by rearranging small numbered cubes in ascending order immediately above the previously formed row in such a way that the number sequence of smaller cubes continues from left to right. Further, each cube is placed exactly on the top of another identical cube.

iii) This method of arrangement is continued until the vertical layer is formed.

iv) After the formation of the first vertical layer, the second layer is formed which is immediately behind the first layer, and constructed in the same manner as the first layer.

v) The layers are arranged one after the other until all the 729 cubes are used and the larger cube is completely formed.

1. What is the sum of the numbers on the middlemost cubes (i.e., the cubes in the centre-most column) that are vertically aligned and run across different horizontal layers of the larger cube?

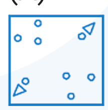
2. What is the maximum possible sum of the numbers forming a column in the third vertical layer?

3. What is the difference in the sum of the numbers on smaller cubes that form the diagonals of the fourth vertical layer?

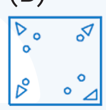
4. What is the maximum sum of the numbers on a single horizontal row of the larger cube?

5. The sequence of folding a piece of square paper (figures X and Y) and the manner in which the folded paper has been cut (figure Z) are shown. How will the paper appear when unfolded?

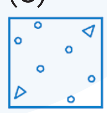
(A)



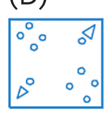
(B)



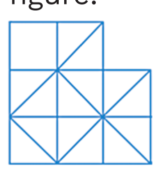
(C)



(D)



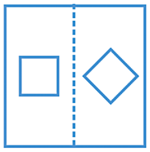
6. Find the number of squares in the given figure.



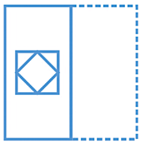
(A) 10
(B) 12
(C) 14
(D) None of these

7. In the following question, a transparent paper of square shape, with some figures is given. When the transparent sheet is folded on the dotted line, what pattern would appear?

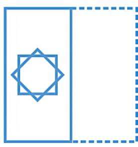
Transparent Sheet



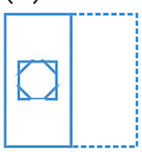
(A)



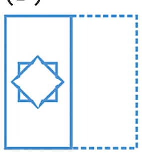
(B)



(C)



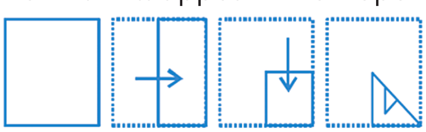
(D)



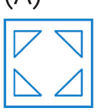
(C)

(D)

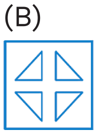
8. A piece of paper is folded and cut as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.



(A)



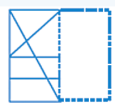
(B)



9. Select the option that depicts how the given transparent sheet of paper would appear if it is folded at the dotted line.

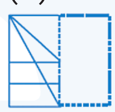
(A)

(B)

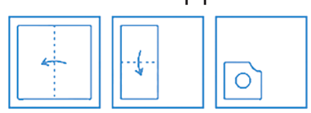


(C)

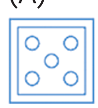
(D)



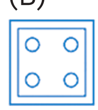
10. A piece of paper is folded and punched as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.



(A)



(B)



(C)

(D)

Directions for Questions 11 to 15

There is a cube having an edge of 8 cm each. It is painted green on two opposite faces, yellow on one other pair of opposite faces, black on one more face, and one face is left unpainted. Then it is cut into smaller cubes of 1 cm each. Answer the following questions:

11. Find the total number of smaller cubes.

12. Find the number of smaller cubes that have three faces painted.

13. Find the number of smaller cubes that have two faces painted.

14. Find the number of smaller cubes that have one face painted.

15. Find the number of smaller cubes that have zero-face painted.

Solutions

Level of Difficulty - 1

1. (B)

There are 37 cuts in total, then all these 37 cuts can be on the x-axis or 18 on the x-axis and 19 on the y-axis, etc. For the given number of cuts, there will be several combinations. To get the minimum number of pieces, all the cuts are made on the same axis.

Now, making all 37 cuts along the x-axis, a total of 38 pieces can be formed.

All the other combinations will result in more than 38 pieces.

Hence, option (B) is correct.

2. (D)

If the total number of cuts is 26, then for the maximum number of pieces, these cuts have to be equally distributed in three planes.

For 26 cuts, 8, 9, and 9 is the distribution of cuts.

Hence, the total number of pieces are (8 + 1) (9 + 1) (9 + 1)

= 9 × 10 × 10 = 900 pieces

Hence, option (D) is correct.

3. (C)

From figures (II) and (III), it is concluded that the alphabets P, O, R, and N appear adjacent to the alphabet Q.

Therefore, the alphabet M appears opposite Q.

Conversely, Q appears opposite M

Hence, option (C) is correct.

4. (A)

Minimum number of pieces with n cuts = $(n + 1)$

= 35 + 1 = 36 pieces

For maximum number of pieces, cuts have to be 11, 12, 12 across X, Y, and Z planes, respectively (as total 35 cuts).

So, the maximum number of pieces = $(x + 1) \times (y + 1) \times (z + 1)$

= (11 + 1) (12 + 1) (12 + 1) = 2,028.

Required difference = 2028 – 36 = 1992.

Hence, option (A) is correct.

5. (A)

First of all, if there are 120 identical pieces, factorize the number into three numbers such that all the factors are greater than 1 and close to each other.

120 = 4 × 5 × 6

Number of identical pieces = (X + 1) (Y + 1) (Z + 1).

(X, Y, Z) = number of cuts on each face

So, (X + 1) (Y + 1) (Z + 1) = 120.

(3 + 1) (4 + 1) (5 + 1) = 120

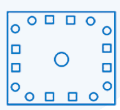
Compare to the above formula, X = 3, Y = 4, Z = 5.

Least number of cuts = 3 + 4 + 5 = 12.

Hence, option (A) is correct.

6. (B)

When unfolding the paper, the figure is shown as:

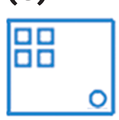


Hence, option (B) is correct.

7. (A)

Hence, option (A) is correct.

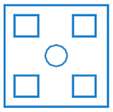
8. (C)



Hence, option (C) is correct.

9. (D)

The figure in option (D) represents how the paper will look when unfolded.



Hence, option (D) is correct.

10. (B)

Hence, option (B) is correct.

11. (A)

When we cut a larger object one time, the number of smaller parts obtained = 2.

Similarly, when we cut a larger object two times, the number of smaller parts obtained = 3.

So, number of parts = number of cuts + 1.

Let the number of cuts made along the x-axis = A.

So, the number of parts obtained = A + 1

Let the number of cuts made along the y-axis = B.

So, the number of parts obtained = (A + 1) × (B + 1).

Let the number of cuts made along the z-axis = C.

So, the number of parts obtained = (A + 1) × (B + 1) × (C + 1).

According to the question,

(A + 1) × (B + 1) × (C + 1) = 28.

We have to find the minimum value of (A + B + C) which satisfies the given equation.

So,

(A + 1) × (B + 1) × (C + 1) = 7 × 4 × 1 or 7 × 2 × 2

A + B + C is minimum when,

A = 6, B = 1, C = 1

Total number of cuts required (minimum) = A + B + C = 8

Hence, option (A) is correct.

12. (B)

According to the diagram, the six faces of the cube are: 2, 3, 4, 5, 7, and 9.

The faces marked as 3, 4, 9, and 2 cannot be opposite to 5 (as they are adjacent to 5). It means 7 must lie opposite to 5.

Hence, option (B) is correct.

13. (B)

The number of pieces obtained would be the maximum possible when the given number of cuts are made as equally as possible in the three directions.

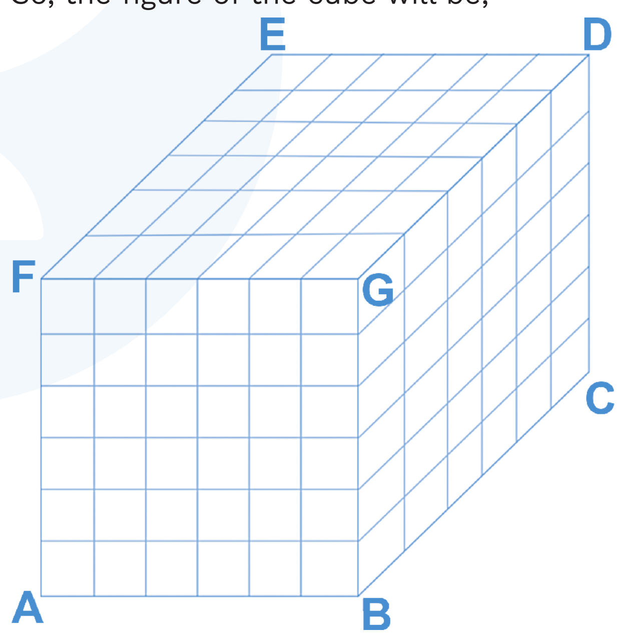
Distribution	Number of pieces
4, 4, 5	5 × 5 × 6 = 150

Hence, option (B) is correct.

14. (C)

First of all, the cube is cut into 216 smaller cubes of equal size.

So, the figure of the cube will be,



Let's start analyzing the figure.

Exactly three faces are painted.

The cubes at the corners will have three faces painted. So, we have 8 of them.

Hence, option (C) is correct.

15. (A)

Exactly two faces painted.

The cubes at the edges (excluding the corners) are the cubes with two faces painted.

For $n \times n \times n$ cube the answer will be 12 $(n - 2)$.

Two painted faces = 12 $(n - 2)$

= 12 (6 − 2)

= 12 × 4 = 48 cubes.

Hence, option (A) is correct.

16. (B)

The cubes at the faces (apart from those on the edges) will have only one face painted.

For an $n \times n \times n$ cubes, the answer will be $6(n - 2)^2$

One painted face = $6(n - 2)^2$

= $6(6 - 2)^2$ = 96 cubes

Hence, option (B) is correct.

17. (C)

All the cubes except the cubes at the faces will have no face painted.

For an $n \times n \times n$ cubes, the answer will be $(n - 2)^3$

No painted face = $(n - 2)^3$

= $(6 - 2)^3$ = 64 cubes.

Hence, option (C) is correct.

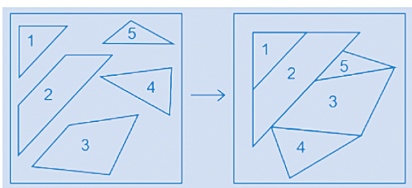
Level of Difficulty - 2

1. **(D)**

Hence, the correct answer is option (D).

2. **(D)**

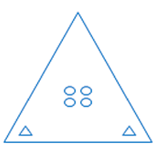
3. **(A)**



Hence, the figure in option (A) is the correct answer.

4. **(C)**

When the paper is unfolded, the figure is shown as:



Hence, option (C) is correct.

5. **(C)**

When unfolding the paper, the figure is shown as:

Hence, option (C) is correct.

6. **(B)**

Number of smaller cubes = $9^3/1^3$ = 729

To get the maximum possible number of cubes that have only blue and black colour on them, the larger cube must be coloured such that the two black faces and the blue face must have a common vertex.

So, total small cubes having only blue and black colours on them = 14 (edges) + 1 (corner) = 15 cubes.

Hence, option (B) is correct.

7. **(B)**

Number of smaller cubes = $9^3/1^3$ = 729

This is possible when two white faces are opposite each other and also two black faces are opposite each other. Then one of the other two faces will be white and one would be blue. Then, the maximum number of desired small cubes = 4.

Hence, option (B) is correct.

8. **(A)**

Since the rectangular block of dimensions 6 cm × 5 cm × 3 cm is cut into cubes of dimensions 1 cm × 1 cm × 1 cm. The total number of cubes formed

= 6 × 5 × 3 = 90.

Number of cubes with exactly three faces coloured = 8.

Number of cubes with exactly two faces coloured = 4 × (4 + 3 + 1) = 32.

Number of cubes with exactly one face coloured = 2 × (12 + 4 + 3) = 38.

So, number of cubes with no face coloured = 90 – (8 + 32 + 38) = 12.

Required difference = (8 + 32 + 38) – 12 = 78 – 12 = 66.

Hence, option (A) is correct.

9. **(C)**

As explained earlier, the total number of cubes formed = 90.

Number of cubes with pink colour on it = 5 × 3 = 15.

Number of cubes with only blue colour on it = 4 × 3 = 12.

Required difference = 15 – 12 = 3.

10. (B)

As explained earlier, the total number of cubes formed = 90.

Number of cubes with exactly two colours on it = $4 \times (4 + 3 + 1) = 32$.

However, among these, there will be eight cubes with one colour as pink.

Likewise, there will be eight cubes with one colour as black.

Therefore, required number = $32 - (8 + 8) = 16$.

11. (A)

- To obtain a maximum number of cubes of the same dimensions, an equal number of cuts must be done in each direction (i.e., length, breadth, and height). Thus, the total number of smaller cubes = $6 \times 6 \times 6 = 216$.
- Now, if the larger cube was painted with blue colour on all its faces, then some of these 216 cubes will have blue colour on them.

The cubes with one face painted with blue colour will be = $(4 \times 4) \times 6 = 96$.

The cubes with two faces painted with blue colour will be = $4 \times 12 = 48$.

The cubes with three faces painted with blue colour will be = 8.

Thus, the cubes with no face painted blue = $216 - 96 - 48 - 8 = 64$.
Hence, option (A) is correct.

12. (B)

Now, the larger cube was painted with blue colour on two faces and the remaining faces were coloured red. To maximize the number of cubes with only blue colour on them, the bigger cube should be coloured such that two adjacent faces are coloured blue.
So, the number of cubes with only blue colour (on one face) will be on the face but not around the edges. This would equal $4 \times 4 = 16$ on each face. So, 32 on two faces. Also, the cubes on the edge of the blue-coloured adjacent cubes will only have blue colour on them.

Therefore, total cubes = $32 + 4 = 36$.

Hence, option (B) is correct.

13. (C)

Now, the larger cube was painted with green colour on three faces and the remaining faces were coloured blue. To minimise the number of cubes with only blue colour on it, the bigger cube should be coloured such that two opposite faces are coloured blue and one more face (any face) is coloured blue. This third face would be adjacent to both the previous faces.

So, the number of cubes with only blue colour (on one face) will be on face but not around the edges. This would equal to $4 \times 4 = 16$ on each face. So, 48 on three faces. Also, the four cubes on the edge of the blue-coloured adjacent cubes will only have blue colour on them and there will be two such edges.

Therefore, total cubes = $48 + 8 = 56$.

Hence, option (C) is correct.

14. (D)

In this question, the larger cube was painted with green colour on two faces (opposite each other), with blue on two of the faces (adjacent to each other), and the remaining faces are coloured black and red.

So, the number of cubes with only one colour (on one face) will be on the face but not around the edges. This would equal to $4 \times 4 = 16$ on each face. So, 96 on six faces. Also, the 4 cubes on edge of the blue-coloured adjacent cubes will only have blue colour on them, i.e., total cubes (with only one colour on it) = $96 + 4 = 100$.

The number of cubes with exactly two colours (on two faces) will be along edges and out of 12 edges only cubes on one edge will have the same colour on both its faces. Since there will be four cubes per edge, therefore 4 × 11 = 44 cubes will be formed. Also, there will be two corners with 2 blue faces. Thus, total such cubes = 44 + 2 = 46.

Required difference = 100 − 46 = 54.

Hence, option (D) is correct.

15. (B)

Since the original cube is cut into 64 identical cubes, then $n = 4$.

Number of cubes painted with all the three colours on their faces = 8 (on the eight vertices of the original cube).

Number of cubes painted with no colour on any face = $(n - 2)^3$.

(n = length of original cube)

$(n - 2)^3 = (4 - 2)^3 = 8$

Required ratio = 8:8 = 1:1.

Hence, option (B) is correct.

16. (B)

If one makes a total of 37 cuts, all 37 cuts can be on the x-axis or 18 on the x-axis and 19 on the y-axis, etc. for the given number of cuts, we can have a number of combinations. We get the minimum number of pieces when all the cuts are made on the same axis.

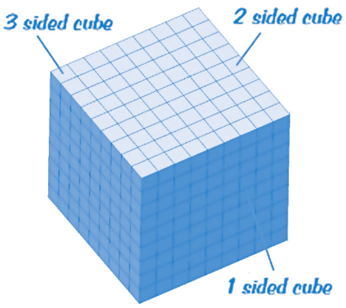
Now, if we make all 37 cuts along the x-axis, a total of 38 pieces are formed. All the other combinations will result in more than 38 pieces.

Hence, option (B) is correct.

Level of Difficulty - 3

1. 3,285

When the smaller cubes are arranged as per the given instructions, 9 vertical and 9 horizontal layers are formed, as shown below.



The 9 vertical layers consist of cubes numbered,

(1 – 81), (82 – 162), (163 – 243), and so on.

Thus, the middlemost vertical layer (i.e., the fifth layer from the front) will consist of cubes numbers (325 – 405). (As, 81 × 5 = 405, so the last cube will have the number 405.)

The idle most column of this layer will consist of cubes numbers (329, 338, 347, ... 401)

So, the sum of the numbers on middle most cubes

= 329 + (329 + 1 × 9) + (329 + 2 × 9) + (329 + 3 × 9) + (329 + 4 × 9) + ... + (329 + 8 × 9)

= 9 × 329 + 9 × (1 + 2 + 3 + ... + 8)

= 2,961 + 324 = 3,285

Hence, the answer is 3,285.

2. 1,863

The third vertical layer will consist of cubes numbered (163 – 243).

So, the column in this layer which will give maximum sum will consist of cubes numbered 171, 180, ... 243.

So, the maximum possible sum of the numbers forming a column in the third vertical layer = 171 + (171 + 1 × 9) + (171 + 2 × 9) + (171 + 3 × 9) + ... + (171 + 8 × 9)

= 9 × 171 + 9 × (1 + 2 + 3 + ... + 8)

= 1,539 + 324 = 1,863

Hence, 1,863 is the answer.

3. 0

The sum of one of the diagonals that starts from the bottom most left cube and extends to the topmost right cube

= 244 + (245 + 1 × 9) + (246 + 2 × 9) + (247 + 3 × 9) + (248 + 4 × 9) + (249 + 5 × 9) + (250 + 6 × 9) + (251 + 7 × 9) + (252 + 8 × 9)

= (244 + 245 + ... + 252) + 9 × (1 + 2 + ... + 8)

$= \frac{9}{2}[244 + 252] + 9 \times (1 + \ldots + 8)$

= 2,232 + 324 = 2,556

The sum of the second diagonal that starts from the bottom most right cube and extends to the topmost left cube

= 252 + (251 + 1 × 9) + (250 + 2 × 9) + (249 + 3 × 9) + (248 + 4 × 9) + (247 + 5 × 9) + (246 + 6 × 9) + (245 + 7 × 9) + (244 + 8 × 9)

$= \frac{9}{2}[244 + 252] + 9 \times (1 + \ldots + 8)$

$= 2{,}232 + 9 \times \frac{8 \times 9}{2}$

= 2,232 + 324 = 2,556

Hence, the required difference

= 2,556 – 2,556 = 0.

Hence, the answer is 0.

4. 32,481

The maximum sum on a single horizontal row will be on the top row.

Here, the difference between numbers on cubes of two successive layers will be 81.

The sum of the numbers of top cubes on the first vertical layer is 693.

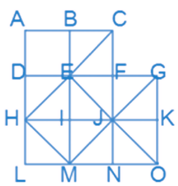
The sum of the numbers of top cubes on the next vertical layer will be 693 + 81 × 9.

Similarly for other layers, the sum of the numbers will be as shown below:
693 + (81 × 9) 2, 693 + (81 × 9) 3, ... , 693 + (81 × 9) 8.
These numbers are in AP.
Hence, the required sum
$= \frac{9}{2}[2 \times 693 + (81 \times 9)8] = 32{,}481$
Hence, the answer is 32,481.

5. (D)
The figure in option (D) represents the paper when unfolded.

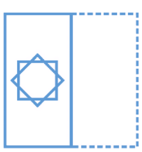
Hence, option (D) is correct.

6. (B)



ABED, BCFE, DEIH, EFJI, FGKJ, HIML, IJNM, JKON, ACJH, DFNL, EGOM, HEJM
Hence, there are 12 squares.
Hence, option (B) is correct.

7. (B)
When the transparent paper is folded on the dotted line, the figure will be as shown below.

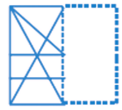


Hence, option (B) is correct.

8. (B)

Hence, option (B) is correct.

9. (A)



Hence, option (A) is correct.

10. (A)

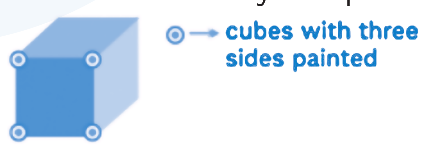
On close observation, we find that option (A) is the correct answer.
Hence, option (A) is correct.

11. 512

Total number of cubes = (8 × 8 × 8)/ (1 × 1 × 1) = 512 cubes.
Hence, the answer is 512 cubes.

12. 4

For a cube with all sides painted, 8 cubes with three faces coloured. But here one side is unpainted.
Hence, you can obtain four cubes which will have three painted faces and the rest four with only two painted faces.



Hence, the answer is 4.

13. 52

Notice the edges for the problems involving two painted faces.

A cube has 12 edges. Of the 8 edges, each edge having 6 cubes will have two faces painted. (Four edges of an unpainted face won't be included).

Those four cubes (which did not count while counting three coloured faces, as they have two faces painted) should also be included.

Cubes on four edges of the unpainted face of the cube will have one face painted.

Therefore, total cubes with two faces painted = 8 × 6 + 4 = 52 cubes.

Hence, the answer is 52 cubes.

14. 204

For one face painted, look for the faces of the cube.

A cube has six faces.

Five faces each having (8 – 2) × (8 – 2) = 36 cubes will have one side painted.

Those cubes on the edges linked with an unpainted face should be included.

Six cubes on each of those edges will have one side painted.

Therefore, total cubes with one face painted = 5 × 36 + 6 × 4 = 204 cubes.

Hence, the answer is 204 cubes.

15. 252

According to the formula, cubes with no face painted = $(8 - 2)^3 = 216$.

But to include the cubes from the unpainted face to them, we need 6 × 6 = 36 cubes.

So, total number of unpainted cubes = 216 + 36 = 252 cubes.

Hence, the answer is 252 cubes.