1 Linear Equations and Special Equations

Introduction

In this chapter, we will learn about different methods of solving linear equations. Direct questions are asked on this topic in CAT every year. It also helps solve questions from other topics like time, speed and distance problems, time and work problems, pipe and cistern problems, and even ratio proportion related problems. Every year, two or three problems from this chapter are asked in CAT and other examinations.

Linear Equations

Any equation with an exponent or degree of 1 is called a linear equation. It may contain any number of variables, but the degree is always 1.

For example, $3x + 2 = 7 \rightarrow$ Linear equation in one variable

$3x + 7y = 7 \rightarrow$ Linear equation in two variables

Similarly, $3x + 4y + 8z = 10 \rightarrow$ Linear equation in three variables

Solution to Linear Equations

Solution to Linear Equations in One Variable

In any equation with one variable, we may have a constant on one of the sides of the equation, or we may have a linear expression in one variable or both sides of the equation.

Example:

$3x + 7 = 10$

$x = 1$ is a solution.

Example:

$$\frac{2x-3}{2} + 4 = \frac{3x-1}{2}$$

$\Rightarrow \quad 2x - 3 + 8 = 3x - 1$

$\Rightarrow \quad x = 6$

So these types of equations can be easily solved.

Solutions to Linear Equations in Two Variables

To solve a linear equation in two variables, we must have at least two equations. We call them a system of simultaneous linear equations.

Let these equations be:

$$\left.\begin{aligned} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{aligned}\right\} \quad \text{...(i)}$$

We have two major possibilities when we solve linear equations in two variables. The system of equations may possess the solution or may not.

Now, suppose a system of equations is consistent (the solution exists). In that case, it may have a unique solution (one single value for each variable) or infinitely many solutions (infinite values for almost all variables).

Let us consider:

$a_1x + b_1y = c_1$ ...(i)

$a_2x + b_2y = c_2$ ...(ii)

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here lines (i) and (ii) will intersect at one point, which is called the *point of intersection,* and the value of that point will be a *unique solution*.

Let's take an example: $2x + y = 6$ ...(iii)

$x + y = 5$ ...(iv)

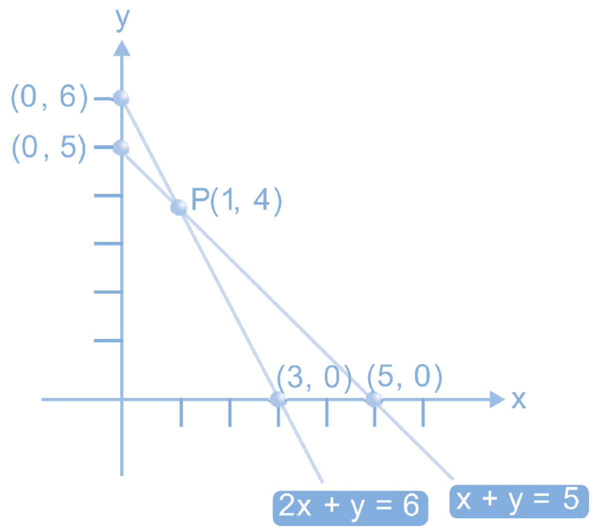
From equation (iii) we will get:

When $y = 0$, $x = 3$ and when $x = 0$, $y = 6$

from equation (iv) we will get-

When $y = 0$, $x = 5$ and when $x = 0$, $y = 5$

After plotting both the lines, we get P as a point of intersection.



- A consistent system with a unique solution is also called an *independent system.*
  Coordinates of P (1, 4)
  $\Rightarrow$ x = 1, y = 4 is the unique solution.

For Infinitely Many Solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Let us take an example:

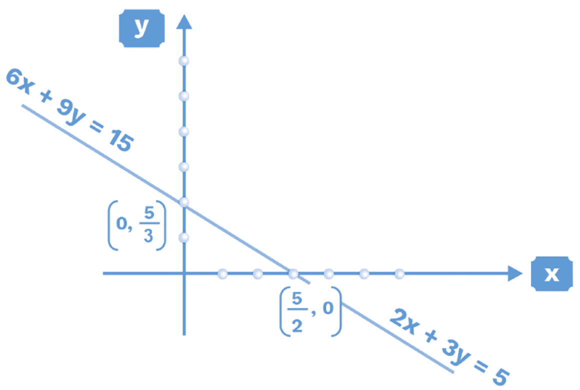
$$\left.\begin{aligned} 2x + 3y &= 5 \qquad &...(v) \\ 6x + 9y &= 15 \qquad &...(vi) \end{aligned}\right\}$$

$$\Rightarrow \frac{2}{6} = \frac{3}{9} = \frac{5}{15} = \left(\frac{1}{3}\right)$$

To draw the line (v): When $x = 0,\ y = \frac{5}{3}$ and when $y = 0,\ x = \frac{5}{2}$

To draw the line (vi): When $x = 0,\ y = \frac{5}{3}$ and when $y = 0,\ x = \frac{5}{2}$

These are coinciding lines.



- A consistent system with infinitely many solutions is also called a *dependent system.*
  $\Rightarrow$ Every solution of equation (v) will be the solution for equation (vi) also.

  Hence, we will have an infinite number of solutions.

For no solution

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the system of equations is inconsistent (No solutions).

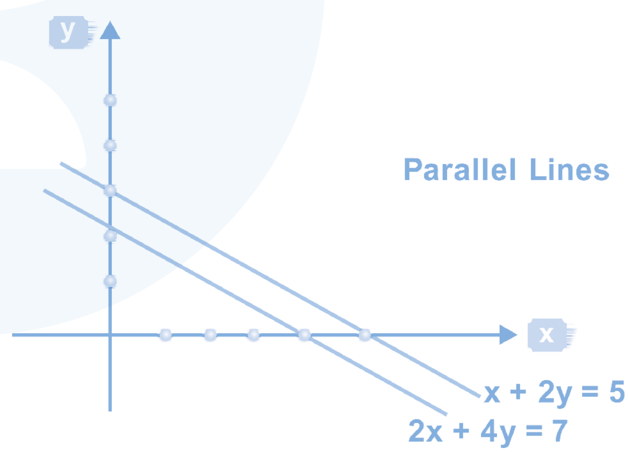
Let us consider one more example:

$$\left.\begin{aligned} x + 2y &= 5 \qquad &...(vii) \\ 2x + 4y &= 7 \qquad &...(viii) \end{aligned}\right\}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{4} \neq \frac{5}{7}$$

To draw line (vii): When $x = 0,\ y = \frac{5}{2}$ and when $y = 0,\ x = 5$

To draw the line (viii): When $x = 0,\ y = \frac{7}{4}$ and when $y = 0,\ x = \frac{7}{2}$



Parallel lines indicate that they will never intersect. That means there is no solution that satisfies both the equations.

Different Methods to Solve a System of Linear Equations

Substitution Method

$$a_1x + b_1y = c_1 \qquad ...(ix)$$

$$a_2x + b_2y = c_2 \qquad ...(x)$$

Firstly, check for consistency. If the given system is consistent, we can use the substitution method.

In the substitution method, we can substitute the value of one variable from one of the equations (in terms of other variables) in another equation.

Therefore, $x = \frac{c_1 - b_1 y}{a_1}$ [from equation (ix)] can be substituted in equation (x).

In this way, we will get the value of y; we can find x.

Example 1:

Solve the following system of equations:

$$3x + 5y = 9$$
$$9x + 4y = 5$$

Solution:

$$\boldsymbol{x = -\frac{1}{3},\ y = 2}$$

Firstly, we will use the substitution method $\frac{3}{9} \neq \frac{5}{4}$. Therefore, the system is consistent in solving the given system of equations.

$$3x + 5y = 9 \quad \Rightarrow \quad x = \frac{9 - 5y}{3}$$

Substituting this value in $9x + 4y = 5$,

$$y \Rightarrow \quad 9\left(\frac{9 - 5y}{3}\right) + 4y = 5$$

$$3(9 - 5) + 4y = 5$$

$$27 - 15y + 4y = 5$$

$$-11y = -22$$

$$\Rightarrow \quad y = 2$$

Now, $x = \frac{9 - 5y}{3} = \frac{9 - 5 \times 2}{3} = -\frac{1}{3}$

$$x = -\frac{1}{3},\ y = 2$$

Elimination Method

In the elimination method, we multiply one or both the equations by some non-zero constant to equal the coefficient of any one of the variables. Then add or subtract the equation to eliminate (remove) any one of the variables.

Example 2:

Solve: $3x + 5y = 9$

$9x + 4y = 5$

Solution:

$$\boldsymbol{x = -\frac{1}{3},\ y = 2}$$

The system is consistent. We can see if we multiply the first equation by 3, the coefficient of x becomes equal.

Multiplying the first equation by 3, then subtracting it from the other equation:

$$\begin{array}{r} 9x + 15y = 27 \\ 9x + \ \ 4y = \ \ 5 \\ - \quad - \qquad - \\ \hline 11y = 22 \end{array}$$

$\Rightarrow \quad y = 2$

Putting $y = 2$ in any one of the equation,

$9x + 15(2) = 27$

$\Rightarrow \quad 9x = 27 - 30$

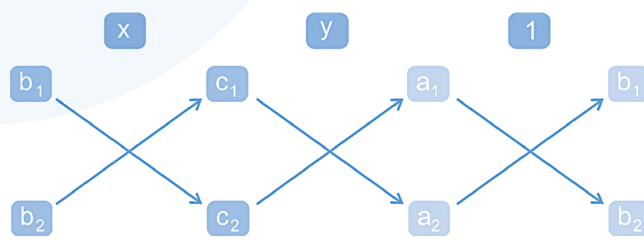
$x = -\frac{1}{3} \quad \Rightarrow \quad x = -\frac{1}{3},\ y = 2$ is the solution.

Cross-Multiplication Method

Let us consider $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

We will make a chart as below:



From the above chart, we can write:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1} \qquad \text{...(i)}$$

Now, we can solve equation (1) for finding x and y provided

$a_1b_2 \neq a_2b_1$ (or $a_1b_2 - a_2b_1 \neq 0$)

Example 3:

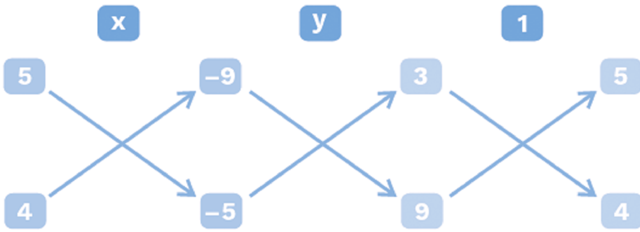
Solve: $3x + 5y = 9$

$9x + 4y = 5$

Solution: $y = 2$

$3x + 5y - 9 = 0$

$9x + 4y - 5 = 0$



$$\Rightarrow \frac{x}{-25+36} = \frac{y}{-81+15} = \frac{1}{12-45}$$

$$\Rightarrow \frac{x}{11} = \frac{y}{-66} = \frac{1}{-33}$$

$$\Rightarrow \frac{x}{11} = \frac{1}{-33}$$

$$\Rightarrow x = -\frac{1}{3}$$

Also $\dfrac{y}{-66} = \dfrac{1}{-33}$

$\Rightarrow \; y = 2$

Keynote

Solving for two variables when two equations are given, or for three variables when three equations are given is not a difficult task. We can use any of the methods to solve equations.

Many times, we deal with problems where we have two variables but only one equation. In such cases, we need to solve by using some other additional information or given condition.

Graphical Method

As we have discussed earlier, we can draw the graph of linear equations, and from the graph, we can find the solution (if it exists).

By looking at the graph, we can identify whether the given system of linear equations is consistent or inconsistent.

First, we will draw the two linear equations, and if

- They are intersecting lines, then the system is consistent with the unique solution.
- If the lines coincide; then, the system is consistent but with infinitely many solutions.
- If the lines are parallel, then the system is inconsistent (no solution).

So, we can use any of the four methods depending upon the form of information given to you, but elimination method is mostly used.

Making Linear Equations from Age-Based Statements

Let's assume the present age of A = a years and the present age of B = b years. Now we will make linear equations from the following age-based statements, which will be very helpful for us to solve these kind of questions.

1. A is twice as old as B, $a = 2b$.
2. 5 years ago, A was thrice as old as B, $(a - 5) = 3\,(b - 5)$.
3. 7 years hence, A will be five times as old as B, $(a + 7) = 5\,(b + 7)$.
4. 3 years ago, A's age was 3 years more than thrice of B's age, $(a - 5) = 3\,(b - 5)$.
5. 5 years hence, A's age will be 6 years more than twice of B's age, $(a + 5) = 2\,(b + 5) + 6$.
6. 7 years ago, A's age was 7 years less than seven times the B's age, $(a - 7) = 7\,(b - 7) - 7$.
7. 4 years hence, A's age will be 5 years less than 5 times the B's age, $(a + 4) = 5\,(b + 4) - 5$.

Practice Exercise – 1

Level of Difficulty – 1

1. Raghav purchased breadsticks, buns, and cheese. In total, he spent ₹33. The cost of breadsticks, bun, and cheese are ₹16, ₹5, and ₹6, respectively. Find all the possible values of breadsticks, buns, and cheese purchases when he has purchased at least one of each item.

2. Find the number of pairs of positive integers x and y that satisfy $4x + 14y = 186$.

3. Find the maximum value of xyz if $x + y + z = 21$, where x, y, and z are positive real numbers.

4. The linear equations $4x + ay = 5$ and $ax + 4y = 6$ (a is a constant) are given. Find the values of 'a' such that the system of equations are inconsistent.

5. Raj is four times his son's age at present. After 5 years, Raj will be thrice the age of his son. What is the present age of Raj?
   (A) 40 years
   (B) 50 years
   (C) 60 years
   (D) 45 years

Level of Difficulty – 2

6. The average age of Ram, Shyam, and Shazia is 47 years. Ram's niece visited their home, and now the average age of all four persons is 37 years. Ram's age is six times the age of his niece, and Shyam's age is two more than the square of the age of Ram's neice. Find the age of Shazia.
   (A) 40 years
   (B) 48 years
   (C) 60 years
   (D) 45 years

7. When the digits of a two-digit number are reversed, the new two-digit number is 63 less than the original number. How many such numbers are possible?
   (A) 4
   (B) 5
   (C) 3
   (D) 2

8. The sum of the original two-digit number and the number formed by reversing the digits is 44. If eight is added to the number, it becomes 13 more than twice the number formed by reversing the digits. Find the number.
   (A) 31
   (B) 63
   (C) 43
   (D) 23

9. Charu's aunt started at 12 km/hr from her home and reached Charu's home at 6 p.m. She stayed there for 30 minutes and then returned to her home at the speed of 15 km/hr. The total time taken by Charu's aunt is 5 hours and 54 minutes. Find the total distance travelled by Charu's aunt.

10. Ashi travelled from Lucknow to Kanpur. She covered $\frac{5}{9}$th of distance via tram, $\frac{1}{20}$th the distance by train, $\frac{1}{4}$th distance by bus, and the remaining 52 km by walking. Find the total distance.

Level of Difficulty – 3

11. If $2|x| + 7y = 56$, and x, y are integers, how many values (x, y) can take?
   (A) Infinite
   (B) 12
   (C) 8
   (D) 5

12. Check whether the following system of equations will have a unique solution?

$2x + 3y + 4z = 8$

$3x + 5y + 8z = 7$

$6x + 10y + 16z = 14$

(A) No unique solution
(B) Two unique solutions
(C) One unique solution
(D) None of these

13. Akshay and Bunty can do a piece of work in 15 days, Bunty and Chintu can do the same work in 10 days, whereas Chintu and Akshay can do that in 8 days. If Akshay, Bunty, and Chintu can do the work together in x integral number of days, find x.

(A) 7 days
(B) 10 days
(C) 5 days
(D) 9 days

14. If $3x + 2y = 78$, how many pairs of positive integers (x, y) exist that satisfy this equation?

(A) 12
(B) 13
(C) 14
(D) 15

15. In how many ways can you pay a restaurant bill of ₹127 using 1, 10, and 50 notes, given that the number of notes of 1, 10, and 50 is unlimited?

(A) 24
(B) 35
(C) 30
(D) 43

Solutions

1. $x = 1; y = 1; z = 2$

Let the number of breadsticks, buns, and cheese purchased by Raghav be x, y, and z respectively.

According to the question, $16x + 5y + 6z = 33$.

Now, minimum value of each is 1.

$x \geq 1, y \geq 1, z \geq 1$

But Raghav can't purchase more than one breadstick as it is given that he purchased at least one bun and cheese too. The cost of one breadstick is ₹16, and the total amount spent on all three items is ₹33 only. So $x = 1$.

$\Rightarrow \quad 16 + 5y + 6z = 33$

$\Rightarrow \quad 5y + 6z = 17$

Now let's see what values y and z can take:

Units of Buns	Units of Cheese	Total amount
1	1	11
2	1	16
1	2	17*
2	2	22

From the above table, the only possible value is $y = 1$, $z = 2$.

Therefore, $x = 1$, $y = 1$, $z = 2$.

2. 7 values

Given equation:

$4x + 14y = 186$

or $\quad 2x + 7y = 93$

$\Rightarrow \quad 2x = 93 - 7y \qquad ...(i)$

Now, x and y are positive integers.

From equation (i), for x needs to be a positive integer, $(93 - 7y)$ has to be a multiple of 2.

For $(93 - 7y)$ to be an even positive integer, y has to be an odd positive integer such that $7y$ is odd and less than 93.

Y	1	3	5	7	9	11	13
7y	7	21	35	49	63	77	91

We have been asked how many values are possible. Therefore, in total, seven values satisfy the given equation.

3. 7

We know that: Arithmetic Mean ≥ Geometric Mean

$$\Rightarrow \quad \frac{x + y + z}{3} \geq \sqrt[3]{xyz}$$

$$\Rightarrow \quad \frac{21}{3} \geq \sqrt[3]{xyz} \quad \Rightarrow \quad 7 \geq \sqrt[3]{xyz}$$

$$\Rightarrow \quad xyz \leq (7)^3 = 343$$

343 is the maximum value which is only possible when $x = y = z = 7$.

4. $a = \pm 4$ or $|a| = 4$

For any two linear equations of the form:

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

the condition of inconsistency is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Here equations are: $4x + ay = 5$

$ax + 4y = 6$

$$\Rightarrow \quad \frac{4}{a} = \frac{a}{4} \quad \Rightarrow \quad a^2 = 16$$

$\Rightarrow \quad a = \pm 4$ or $|a| = 4$.

5. (A)

Let Raj's present age be x years, and his son's present age be y years.

According to the question,

$x = 4y$ (at present) $\qquad ...(i)$

After 5 years,

$(x + 5) = 3(y + 5)$

$\Rightarrow \quad x + 5 = 3y + 15$

$\Rightarrow \quad 4y + 5 = 3y + 15 \quad$ [Using equation (i)]

$\Rightarrow \quad y = 10$

Now we have to find the present age of Raj.

Therefore, $x = 4y = 40$ years.

6. **(B)**
Let the age of Ram, Shyam, Shazia, and niece be p years, q years, r years, s years, respectively.

Average age of p, q and $r = \frac{p+q+r}{3} = 47$

$\Rightarrow$ $p + q + r = 141$ ...(i)

Also, $\frac{p+q+r+s}{4} = 37$

$\Rightarrow$ $p + q + r + s = 148$ ...(ii)

Subtracting equation (i) from (ii):

$s = 7$ years

Ram is six times the age of his neice.

$\Rightarrow$ $p = 6s = 42$ ...(iii)

Shyam's age 2 more than age of Ram's neice.

$\Rightarrow$ $q = s^2 + 2 = 49 + 2 = 51$ ...(iv)

Substituting equations (iii) and (iv) in (i);

$p + q + r = 141$

$\Rightarrow$ $42 + 51 + r = 141$ $\Rightarrow$ $r = 48$

Therefore, age of Shazia = 48 years.

7. **(D)**
Let x be the digit at the unit place, and y be the number at ten's place.
Therefore, original number = $10y + x$.
After reversing the digits, number = $10x + y$.
According to question, $10y + x - 63 = 10x + y$

$\Rightarrow$ $9y - 9x = 63$

$\Rightarrow$ $y - x = 7$

We will now discuss the possible values such that $y - x = 7$
y or x can't take zero value
If $y = 9$ then $x = 2$ and number will be 92.
If $y = 8$ then $x = 1$ and number will be 81.
If $y = 7$ then $x = 0$ which is not possible.
So, there are only two possible numbers 92 and 81.

8. **(A)**
Let the digit at unit's place be x and let the digit at ten's place be y.
Therefore, original number = $10y + x$.

$10y + x + 10x + y = 44$

$\Rightarrow$ $11x + 11y = 44$

$\Rightarrow$ $x + y = 4$ ...(i)

Also, $10y + x + 8 = 13 + 2(10x + y)$

$10y + x + 8 = 13 + 20x + 2y$

$\Rightarrow$ $19x - 8y = -5$ ...(ii)

Multiplying equation (i) by 8 and then adding to equation (ii) (Elimination method):

$$\begin{array}{r} 19x - 8y = -5 \\ 8x + 8y = 32 \\ \hline 27x = 27 \end{array}$$

$\Rightarrow$ $x = 1$

$x + y = 4$

$\Rightarrow$ $y = 3$

Therefore, original number = $10y + x = 31$.

9. **72 km**
Let the distance between Charu's and her aunt's home be 'd' km.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

According to the question:
Time taken while going + time spent at Charu's home + time taken while returning back = $5\frac{54}{60}$

$$\Rightarrow \frac{d}{12} + \frac{30}{60} + \frac{d}{15} = \frac{59}{10}$$

$$\Rightarrow \frac{d}{12} + \frac{d}{15} = \frac{59}{10} - \frac{1}{2}$$

$$\Rightarrow \frac{5d + 4d}{60} = \frac{54}{10}$$

$$\Rightarrow \frac{9d}{60} = \frac{54}{10}$$

$$\Rightarrow d = \frac{54}{10} \times \frac{60}{9} = 36\,\text{km}$$

Therefore, total distance travelled = $d + d = 2d = 72$ km.

10. **360 km**
Let the total distance be x.

$$\Rightarrow \frac{5}{9} \times x + \frac{1}{20} \times x + \frac{1}{4} \times x + 52 = x$$

$$\Rightarrow x - \frac{5}{9} - \frac{1}{20}x - \frac{1}{4}x = 52$$

$$\Rightarrow \left(\frac{180 - 100 - 9 - 45}{180}\right) x = 52$$

$$\Rightarrow \frac{26}{180}x = 52$$

$$\Rightarrow x = 52 \times \frac{180}{26}$$

$\Rightarrow$ x = 360 km.

Therefore, total distance = 360 km.

11. (A)

$2|x| + 7y = 56$

$\Rightarrow$ $7y = 56 - 2|x|$

x, y are integers. It is given in the question.

$|x|$ will always give a positive value, and $2|x|$ is always an even integer, but we need those values of $2|x|$, which are multiple of 7.

56 is divisible by 7, and if $2|x|$ is also a multiple of 7, then $(56 - 2|x|)$ is also divisible by 7. This will give an integer value of y (Positive or negative)

$\Rightarrow$ $7y = 56 - 2|x|$

will give integer value of y when x = 7, –7, 14, –14, 21, –21, ...

So there are infinite integer values possible for (x, y).

12. (A)

Condition for a unique solution is that there should be no linear dependence between any two or more equations.

Here, $\frac{2}{3} \neq \frac{3}{5}$ (so the first two equations are linearly independent).

But if we consider the second and third equations.

$$\frac{3}{6} = \frac{5}{10} = \frac{8}{16} = \frac{7}{14}$$

There is a linear dependence between the second and third equations.

Therefore, the system will not have a unique solution.

13. (A)

7 days

Let Akshay's one day work when he works alone be $\frac{1}{a}$

Let Bunty's one day work be $\frac{1}{b}$

and let Chintu's one day work be $\frac{1}{c}$

Given that $\frac{1}{a} + \frac{1}{b} = \frac{1}{15}$ ...(i)

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{10} \quad \text{...(ii)}$$

$$\frac{1}{c} + \frac{1}{a} = \frac{1}{8} \quad \text{...(iii)}$$

Also, when they work together for x number of days, they will finish that work in x days.

$$x.\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 1$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{x}$$

Multiplying both the sides by 2:

$$\frac{2}{a} + \frac{2}{b} + \frac{2}{c} = \frac{2}{x}$$

$$\Rightarrow \left(\frac{1}{a} + \frac{1}{b}\right) + \left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{c} + \frac{1}{a}\right) = \frac{2}{x}$$

$$\frac{1}{15} + \frac{1}{10} + \frac{1}{8} = \frac{2}{x} \left[\text{Using equations (i), (ii), (iii)}\right]$$

$$\Rightarrow \frac{8 + 12 + 15}{120} = \frac{2}{x} \qquad \Rightarrow \frac{35}{120} = \frac{2}{x}$$

$$\Rightarrow x = \frac{2 \times 120}{35} \qquad \Rightarrow x \cong 7 \text{ days.}$$

14. (B)

Given that $3x + 2y = 78$

$\Rightarrow$ $3x = 78 - 2y$

To get the integer value of x.

$78 - 2y$ has to be divisible by 3.

Now the number 78 is already divisible by 3.

For $78 - 2y$ to be divisible by 3, $2y$ has to be a multiple of 3. y is a positive integer, and for $2y$ to be a multiple of 3. y can take values 3, 6, 9, 12, 15, 18, ...

Remember that $78 - 2y$ can't be less than zero.

$\Rightarrow$ y can only take value 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39 only.

For each value y, there will be one value of x.

Total number of pairs = 13.

15. (A)

₹ 50	₹ 10	₹ 1	
0	0	127	13 ways
	1	117	
	⋮	⋮	
	10	27	
	11	17	
	12	7	
1	0	77	8 ways
	1	67	
	⋮	⋮	
	7	7	
2	0	27	3 ways
	1	17	
	2	7	

Hence in total 24 ways, we can pay the bill of ₹127.

Practice Exercise – 2

Level of Difficulty – 1

1. Find the number of positive integral solutions of the equation $7x + 11y = 273$.
(A) 3
(B) 2
(C) 1
(D) Infinite

2. Ramesh told his son, "My age x years ago was twice your age. My age $4x$ years ago was six times your age at that time". If the difference between their present ages is 30 years. Find the sum of their present ages (in years).

3. Arjun was carrying ₹x and y paise. He spent ₹9 and 30 paise on some candies and was left with ₹$3y$ and $2x$ paise. Find the value of x.

4. Hira went to the fruit market and bought some mangoes and apples for his family members. In total, he spent less than ₹2,500. Had Hira spent the amount he actually spent on buying mangoes to buy apples and vice versa, he would have ended up buying two more fruits in total. What is the maximum number of apples that Hira could have bought if it is known that a mango and an apple cost ₹50 and ₹40, respectively?
(A) 25
(B) 20
(C) 30
(D) 26

5. An online shopping site, 'zip cart', sells three types of machine learning books by different authors. The first book costs ₹6,000, second book costs ₹1,200, and the third book costs ₹300. Three data science edtech institutes, x, y, and z want to buy several books independently. The respective amount spent by each institute is equal. The 'zip cart' sells 1 book of first type, 15 books of third type, and some of the books of second type to these three data science institutes. Find the minimum number of second types of machine learning books that zip cart sells to these institutions.
(A) 7
(B) 5
(C) 6
(D) 8

6. A was born 2 years ago when B was 2 years old. At present, the average age (in years) of A, B, C, and D is a perfect square. Also, the difference between the ages of any two persons is not more than 5 years. Which of the following could be the ages of C and D (in years), respectively?
(A) 8, 2
(B) 10, 9
(C) 7, 3
(D) 6, 3

7. How many pairs of positive integers x and y exist such that $5x + 7y = 185$?
(A) 4
(B) 5
(C) 6
(D) 18

8. The cost of 5 pens, 7 pencils, and 11 erasers is ₹111. The cost of 3 pens, 4 pencils, and 5 erasers is ₹61. Find the cost (in ₹.) of 5 pens, 6 pencils, and 3 erasers.

9. In how many ways can 48 be divided into three positive parts, such that the first part is divisible by 7, the second is divisible by 8, and the third is divisible by 9?
(A) 4
(B) 3
(C) 2
(D) 1

10. The total fare of a railway ticket from station A to station B consists of ticket charges and booking charges. The ticket charges for children (aged 3 years or below) and adult females (aged 21 years and above) are 50% and 80% of ticket charges for adult males (aged 21 years and above), respectively. Booking charges are the same for each railway ticket from station A to station B.

The total fare for an adult male travelling from station A to station B is ₹720, and the total fare for a family of three (an adult male, an adult female, and a 2-year-old child) is ₹1,712 for the same journey. Find the booking charges (in ₹) per ticket.
(A) 120
(B) 100
(C) 90
(D) None of these

Level of Difficulty – 2

11. Jessica is the mother of Wincet. When Wincet was as old as Jackson's present age, the age of Jessica was four times that of Wincet's age. At present, two times the age of Jessica is five times the age of Wincet. Find the ratio of the present age of Jessica to the present age of Jackson.
(A) 4:3
(B) 1:5
(C) 5:1
(D) 2:5

12. A man says that his wife's age is represented by the digits of his age (two-digit number) reversed. She is older than him, and the sum of their ages is 55 years. What is the unit digit of his age if the difference between their ages is not more than 10 years?

13. One thousand members attended a Science exhibition which was organised in Indira Gandhi Planetarium in Lucknow, and ₹1,000 was collected as charges from them in all. The charges were ₹5 per man, ₹ 4 per woman, and 10 paise per child. How many women members were there?

14. A student purchases pens, pencils, and sharpeners by spending a total of ₹104. Each pen, pencil, and sharpener cost ₹20, ₹8, and ₹5, respectively. In how many ways can he purchase them if he buys at least two items of each type?
(A) 1
(B) 2
(C) 3
(D) 4

15. In a four-digit number with distinct digits, the sum of the first two digits is equal to the sum of the last two digits. The sum of digits at the units place and at the thousands place is thrice the sum of the digits at the tens and the hundreds place. If the sum of all the digits is 20, then how many such four-digit numbers are possible?
(A) 2
(B) 4
(C) 6
(D) 8

16. Raju has 262 coins consisting of ₹1, 50 paisa, and 25 paisa coins. The total value of the coins is ₹130. If the 50 paisa and 25 paisa coins are interchanged, the value comes down by ₹6. Find the number of ₹1 coins he has.
(A) 43
(B) 24
(C) 38
(D) 46

17. In a 50-question multiple-choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and –1 point for an incorrect answer. Ranjan's total score in the contest was 99. What is the maximum

number of questions that Ranjan could have answered correctly?

18. Dinesh, Eswar, Ganesh, and Harish had a total of 360 marbles. The number of marbles with Dinesh was 1/3rd of the total number of marbles with the others. The number of marbles with Eswar was 3/5th of the total number of marbles with the others. The number of marbles with Ganesh is 2/7th of the total number of marbles with the others. Find the number of marbles with Harish.

(A) 45
(B) 55
(C) 60
(D) None of the above

19. A person went to a bank to encash his cheque of A rupees and B paisa. The banker by mistake, gave him the cash of B rupees and A paisa, such that even after spending ₹2 and 40 paisa out of the banker's given cash, the person is left with twice the original amount (amount mentioned on the cheque). Find the value of A + B.

20. Jet airways have a certain free luggage allowance per passenger and charges for excess luggage at a fixed rate per kg. Two colleagues, Sunil and Raghav, have a total of 40 kg of luggage with them and are charged ₹1,200 and ₹2,640, respectively. Had the entire luggage belonged to one of them, the excess luggage charge would have been ₹7,920. What is the difference between Raghav's luggage and the free luggage allowance limit (in kg)?

(A) 6 kg
(B) 5.8 kg
(C) 8.8 kg
(D) 9 kg

Level of Difficulty – 3

21. In a one-day examination, the test paper has 80 questions. Four marks are awarded for every correct answer and –1 for every incorrect answer. A student has scored 132 marks on the test, and the number of questions that are not attempted by him is the lowest possible. If there are no marks awarded for the questions which are not attempted, then find how many questions he has answered correctly?

(A) 41
(B) 32
(C) 42
(D) 40

22. From a four-digit number with distinct digits, Ramesh subtracted the sum of its digits and got the new number 7,119. Find the maximum possible value of the sum of the digits of the original number.

23. There is a fraction P/Q, where P and Q are natural numbers. If the numerator is increased by 2 and the denominator is increased by 3, then the fraction becomes 3/4. Instead, if only the numerator decreased by 1, keeping the denominator the same, the fraction becomes 2/3. Find the value of $(P + Q)$.

24. One month, I kept some money in a magical purse. On every alternate day, starting from the 14th, the money decreased by Rs 10 compared to the closing amount on the evening of the previous day. But on every alternate day, starting from the 15th, the money becomes double than the closing amount on the previous evening. I had ₹500 on the 19th, and the magical purse kept decreasing and multiplying the money for me until the 25th of that month. Had I started with ₹100, how much more money (in ₹) would I have than what I had on the 25th of the month?

25. There were 100 questions in an aptitude exam JMET, where three marks are awarded for every correct answer, and one mark is deducted for every wrong answer. A certain number of students whose total number of attempts were all different got the same total marks of 100 each. Find the maximum number of such students possible.

26. In a four-digit number N with all the digits distinct, the sum of the thousands digit and the hundreds digit is equal to the sum of the tens digit and the units digit. The sum of the thousands digit and the tens digit is three times the sum of the other two digits. The sum of all the digits in the number is denoted by S. If S lies between 11 and 21, then how many different values can N take?

27. In order to impress her wife, Ramu decided to make P sandwiches. For each sandwich, he uses N packs of peanut butter costing ₹4 per pack and K packs of jam costing ₹5 per pack. The total cost of peanut butter and jam to make all P sandwiches is ₹253. Assume that P, N, and K are positive integers with $P > 1$. What is the total expenditure occurred by Ramu on jam to make P sandwiches?

28. Mrs Alpa buys Rs. 249.00 worth of candies for the children of a school. For each girl she gets a strawberry flavoured candy priced at Rs. 3.30 per candy; each boy receives a chocolate flavoured candy priced at Rs. 2.90 per candy.

How many candies of each type did she buy?

(A) 21, 57

(B) 57, 21

(C) 37, 51

(D) 27, 51

29. The equation $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$ has

(A) No solution
(B) More than two solutions
(C) Three solutions
(D) More than three solutions

30. The roses in the garden of Manas started blooming in the month of January. In every alternate month starting from January, 60 more flowers appeared than there were at the end of the previous month. In every alternate month starting from February, the number of flowers became half of the number at the end of the previous month. 120 roses bloomed in the month of June. How many roses bloomed in January?

Solutions

1. (A)

We can write

$$x = \frac{273 - 11y}{7} = 39 - \frac{11}{7}y$$

For x to be an integer, y should be multiple of 7.

Y	7	14	21	28
X	28	17	6	-5

Only three such values are possible as x and y are positive integers.

Hence, option (A) is the correct answer.

2. 106

Let's assume the present age of Ramesh and his son are y and z, respectively.

	Ramesh	Son
Present	Y	Z
x years ago,	$y - x$	$z - x$
$4x$ years ago	$y - 4x$	$z - 4x$

Given $y - x = 2(z - x) \rightarrow y - 2z = -x$

$y - 4x = 6(z - 4x) \rightarrow y - 6z = -20x$

$$\frac{y - 6z}{y - 2z} = \frac{20}{1}$$

$y - 6z = 20y - 40z$

$19y = 34z$

$$\frac{y}{z} = \frac{34}{19}$$

So, let $y = 34K$ and $z = 19K$

Given their age difference

$y - z = 30$

$34K - 19K = 30$

$15K = 30$

$K = 2$

So, sum of theirs ages $= y + z$

$= 34K + 19K$

$= 53K = 106$

3. 40

Converting all the amounts in paisa, initial amount with Arun = $(100x + y)$ paisa.

The amount he spent = (900 + 30) = 930 paisa.

Amount left = $(300y + 2x)$

$\therefore$ $(100x + y) - 930 = (300y + 2x)$

$\therefore$ $98x = 299y + 930$

$$\therefore x = \frac{294y + 5y + 882 + 48}{98}$$

$$\therefore x = 3y + 9 + \frac{5y + 48}{98}$$

Now for $y = 10$, the expression $(5y + 48)$ is completely divisible by 98.

The next value of y, which divides $(5y + 48)$ completely, is 108, which is not possible since y cannot take a value more than 100.

Thus, x = 40 when y = 10.

4. (A)

Let x and y be the number of mangoes and apples bought by Hira initially

$\therefore$ $50x + 40y < 2{,}500$

$\therefore$ $5x + 4y < 250$ ....(i)

According to question

$$\frac{50x}{40} + \frac{40y}{50} = (x + y) + 2$$

$$\Rightarrow \frac{5x}{4} + \frac{4y}{5} = x + y + 2$$

$\Rightarrow$ $25x + 16y = 20x + 20y + 40$

$\Rightarrow$ $5x - 4y = 40$

At $y = 5$, $x = 12$

At $y = 10$, $x = 16$

At $y = 15$, $x = 20$

At $y = 20$, $x = 24$

At $y = 25$, $x = 28$

At $y = 30$, x can't be 32 because in that case $5x + 4y > 250$.

Hence, we can say that $y_{max} = 25$.

5. (A)

Since the zip cart sells one book of the first type, 15 books of third type, and some books of second type.

But they have to spend a minimum of ₹18,000 altogether because the price of

the first type of book is ₹6,000 and the amount they spend is equal.
Let the total number of books of second type which is sold by zip cart is K.
$\therefore$ Total amount spent (in ₹.) =
$1 \times 6,000 + 15 \times 300 + K \times 1,200 \geq 18,000$
$6,000 + 4,500 + 1,200K \geq 18,000$
$10,500 + 1,200K \geq 18,000$
The minimum value of K for which LHS would be greater than 18,000 would be 7
$10,500 + 1,200 \times 7 = 10,500 + 8,400 =$ ₹18,900

$\therefore$ Each data science institute spent = ₹ $\frac{18,900}{3}$ = ₹6,300.

One first book + one third book = ₹6,300
Five second books + 1 third book = ₹6,300
Two second books + 13 × third books = ₹6,300
These are the possible cases.
Hence, the minimum number of second types of books are sold by zip cart is 7.

6. **(C)**
A's and B's present ages are 2 and 4 years, respectively.
Also $(A + B + C + D)/4$ = a perfect square
1 cannot be considered as the perfect square.
Let it be 4.
$\Rightarrow A + B + C + D = 16$
$\Rightarrow C + D = 10$
Also, the age difference between two persons is not more than 5 years.
Hence the ages of C and D could be 7 and 3 years, satisfying all the conditions.
But when it will be 9 $\Rightarrow (A + B + C + D)/4 = 9$
$A + B + C + D = 36$
$\Rightarrow \quad C + D = 30$
Thus, the age difference between any two persons now becomes more than 5 years. Hence this case is discarded.
Hence, option (C) is the correct answer.

7. **(B)**
It is given that $5x + 7y = 185$
$\Rightarrow 5x = 185 - 7y$
$$\Rightarrow x = \frac{185 - 7y}{5} \quad \text{...(i)}$$
y will take a positive integer value only when $7y < 185$ and $(185 - 7y)$ is divisible by 5.
185 is already divisible by 5. For $(185 - 7y)$ to be divisible by 5. Or we can say that $7y$ has to be a multiple of 5, but at the same time, $7y$ should not exceed 185.
y can take values 5, 10, 15, 20, 25 but not 30 (it will make $7y > 185$).
Corresponding value of x will be 30, 23, 16, 9, 2 [using equation (i)].
Total five number of such pairs are possible.

8. **83**
Let's assume the cost of one pen, one pencil, and one eraser be ₹A, ₹B, and ₹C, respectively.
According to the question
$5A + 7B + 11C = 111$ ...(i)
$3A + 4B + 5C = 61$ ...(ii)
$5A + 6B + 3C$ = K (assumed) ...(iii)
Subtracting equation (i) – (iii), we will get
$B + 8C = 111 - K$ ...(iv)
5 × (ii) – 3 × (iii), we will get $2B + 16C = 305 - 3K$ ...(v)
Equation (iv) can be written as $2B + 16C = 222 - 2K$ ...(vi)
From equation (v) and (vi),
$305 - 3K = 222 - 2K$, solving which we will get $K = 83$.

9. **(C)**
Let's assume the first, second, and third parts be $7A$, $8B$, and $9C$ respectively.
A, B, and C are positive integers.
According to the question
$7A + 8B + 9C = 48$
If $C = 1$, then $7A + 8B = 39$ and one possible solution for the equation would be (A, B) = (1, 4).
If $C = 2$, then $7A + 8B = 30$ and one possible solution for the equation would be (A, B) = (2, 2).
If $C = 3$, then $7A + 8B = 11$ and there won't be any integral solution for the equation.
Also, there won't be any integral solutions for C more than 3.
Thus, a total of two possible solutions for A, B, and C will be (1, 4, 1) and (2, 2, 2).
Hence, option (C) is the correct answer.

10. (D)

Let's assume the ticket charges for an adult male = ₹100K.

So, ticket charges for an adult female and child would be ₹80K and 50K, respectively.

Let's assume the booking charges per ticket = ₹Y.

Now total fare for an adult male travelling between A and B = Y + 100K = 720. ...(i)

Total fare for a family of three, consisting of an adult male, an adult female, and a 2-year old child, would be

= $3Y + 100K + 80K + 50K = 1{,}712$. ...(ii)

On solving equations (i) and (ii), we will get $K = 6.4$.

Putting the value of K in (i), we will get Y = ₹80.

Hence, option (D) would be the correct answer.

11. (C)

Let the present age of Jessica, Wincet and Jackson are x, y, and z years.

Let the difference between the age of Wincet and Jackson be 'a' years.

Therefore, a = $y - z$ ...(i)

Also, it is given in the question that 'a' years ago, the age of Jessica was four times the age of Wincet.

$x - a = 4(y - a)$

$\Rightarrow x - a = 4y - 4a$

$\Rightarrow 3a = 4y - x$

$\Rightarrow a = \dfrac{4y - x}{3}$ ...(ii)

Also, one more condition given in the question is that at present, two times the age of Jessica is five times the age of Wincet.

Thus, $2x = 5y$ ...(iii)

If we equate (i) and (ii).

Then, $y - z = \dfrac{4y - x}{3}$

$\Rightarrow 3y - 3z = 4y - x$

$\Rightarrow y = x - 3z$

If we put the value of y in equation (iii) we will get

$2x = 5 \times (x - 3z)$

$\Rightarrow 2x = 5x - 15z$

$\Rightarrow 3x = 15z$

$\Rightarrow \dfrac{x}{z} = \dfrac{5}{1}$

Hence, option (C) is the correct answer.

12. (3)

Let the age of the husband be $10x + y$.

Then age of wife = $10y + x$.

Since the sum of the ages is given

$10x + y + 10y + x = 55$

$11(x + y) = 55$

$x + y = 5$

Also, it is given that the wife is older than the husband and the difference between their age is not more than 10 years.

$\therefore (10y + x) - (10x + y) \le 10$

$9(y - x) \le 10$

From the above inequality we can conclude that If $y = x$ or $(y - x) = 1$.

Since $y = x$ is not possible because ages are different.

Then only $y - x = 1$ will exist.

Now we can find the value of y and x.

$$\begin{array}{r} y + x = 5 \\ y - x = 1 \\ \hline 2y = 6 \end{array}$$

$$y = \frac{6}{2} = 3$$

$\therefore \quad x + y = 5$

$x = 5 - 3 = 2$

Therefore, the age of the husband = $10x + y = 10 \times 2 + 3 = 23$ years.

Hence, the unit digit of the age of the husband is 3.

13. 80

Let men, women, and children be M, W, and C, respectively.

$5M + 4W + 0.1C = 1{,}000$

Also, $M + W + C = 1{,}000$

Since $5M$ and $4W$ are integers,

therefore, $0.1C$ should be an integer, which is possible only if C is a multiple of 100.

So, C could be any of 100, 200, 300, 400, 500, 600, 700, 800, 900.

Consider, $C = 100 \Rightarrow 5M + 4W = 990$

And $M + W = 900$

This equation gives value M in negative, which is not possible.

Similarly, all the remaining values of C are not possible except $C = 800$.

So, if $C = 800$

$5M + 4W = 920$

$M + W = 200$

Solving above equations

$M = 120$ and $W = 80$

Hence, 80 is the correct answer.

14. (B)

Let's assume the number of pens, pencils, and sharpeners the students bought are $(x + 2)$, $(y + 2)$, and $(z + 2)$, respectively.

$\Rightarrow$ Total cost

$= 20(x + 2) + 8(y + 2) + 5(z + 2) = 104$

$\Rightarrow$ $20x + 8y + 5z = 38$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 1 & 6 \\ 1 & 1 & 2 \end{array}$$

Only two valid solutions are possible.

15. (B)

Let's assume the four-digit number as ABCD, where A, B, C, and D are distinct digits.

According to the question

$A + B = C + D$ ...(i)

$A + D = 3\ (B + C)$ ...(ii)

$A + B + C + D = 20$ ...(iii)

From equations (i) and (iii)

$A + B = C + D = 10.$ ...(iv)

From equations (ii) and (iii) only possibility is

$(B + C) = 5$ and $(A + D) = 15$ ...(v)

Now from equations (iv) and (v)

For $(B + C) = 5$, possible cases for the values of B and C are (1, 4), (2, 3), (3, 2), and (4, 1).

Putting the values of B and C in equation (iv) we will get the values of A and D, as (9, 6), (8, 7), (7, 8), and (6, 9), respectively. Hence, four such numbers (9146, 8237, 7328, and 6419) are possible.

16. (D)

Let number of coins of ₹1, 50 paisa, and 25 paisa be x, y, and z.

$x + y + z = 262$ ....(i)

Also, $1 \times x + 0.5 \times y + 0.25 \times z = 130$

$4x + 2y + z = 520$ ....(ii)

One condition is also given if the 50 paisa and 25 paisa coins are interchanged the value comes down by ₹6.

Therefore, $1 \times x + 0.25y + 0.5z = 124$

$4x + y + 2z = 496$ ....(iii)

Since three equations and three variables are present here so; we can solve it by the determinant method.

$x + y + z = 262$ ....(i)

$4x + 2y + z = 520$ ....(ii)

$4x + y + 2z = 496$ ....(iii)

Add equations (ii) and (iii), we will get $8x + 3\ (y + z) = 1{,}016.$ ....(iv)

put the value of $y + z = 262 - x$ from (i) in equation (iv), we will get $x = 46$.

Hence, option (D) is the correct answer.

17. 29

Let a be the number of questions Ranjan answered correctly, b be the number of questions Ranjan left blank, and 'c' be the number of questions Ranjan answered incorrectly.

Since there were 50 questions, $(a + b + c) = 50$.

Since his total score was 99, $4a - c = 99$ or $c = 4a - 99$.

Also, $a + c \le 50$

$\Rightarrow a + (4a - 99) \le 50$

$\Rightarrow 5a \le 149$

$\Rightarrow a \le \frac{149}{5}$

$\Rightarrow a \le 29.8$

Since a is an integer, the maximum value for a is 29.

18. (B)

Given the total number of marbles with Dinesh = 1/3 (total number of marbles with others).

Now, if we assume the total number of marbles with others = $3x$, then the number of marbles with Dinesh would be x, so the total number of marbles = $4x$ = 360, which means x = 90. So, Dinesh has a total of 90 marbles.

Now, the total number of marbles with Eshwar = 3/5 (total number of marbles with others).

Now if we assume the total number of marbles with others = $5y$, then the number of marbles with Eshwar would be $3y$, so the total number of marbles = $8y$ = 360, which means y = 45. So, Eshwar has total of $3y$ marbles = 3 × 45 = 135 marbles.

Similarly, Ganesh will have 80 marbles.

So, number of marbles with Harish

= 360 – 90 – 135 – 80 = 55.

19. 57

Original cheque A rupees and B paisa = (100 A + B) paisa.

Bankers amount to person = B rupees and A paisa = (100 B + A) paisa.

As per the data given in the question

(100 B + A) – 240 = 2(100 A + B)

100 B + A – 240 = 200 A + 2B

98B = 199A + 240

$$B = \frac{199A + 240}{98} = \frac{196A + 3A + 196 + 44}{98}$$

$$B = (2A + 2) + \frac{3A + 44}{98}$$

So 3A + 44 must be divisible by 98.

⇒ 3A + 44 = 98K

3A + 44 = 98 → 3A = 54 → A = 18 and B = 39

3A + 44 = 196 → 3A = 152 → A = 152/3

Now need to check large values of A as B < 100.

(Q paisa cannot be more than 99).

So, only one value of A is possible, which is 18.

Hence, A = 18 and B = 39.

Now A + B = 18 + 39 = 57.

20. (C)

Let Raghav and Sunil carry x and 40 – x kg luggage, respectively, and the free luggage allowance per passenger be m kg, and the excess luggage charge is n ₹/kg.

Excess luggage carried by Sunil = (40 – x – m) kg, and he was charged ₹1,200 and excess luggage carried by Raghav = (x – m) kg, and he was charged ₹2,640.

$$(40 - x - m) \times n = 1,200 \quad \text{...(i)}$$

$$(x - m) \times n = 2640 \quad \text{...(ii)}$$

From equations (i) and (ii), we get,

$$\frac{40 - x - m}{x - m} = \frac{1,200}{2,640} = \frac{5}{11}$$

$$\Rightarrow 440 - 11x - 11m = 5x - 5m$$

$$\Rightarrow 16x + 6m = 440 \quad \text{...(iii)}$$

Had the entire luggage belonged to one of them, then the excess luggage = (40 – m)

$$(40 - m) \times n = 7,920 \quad \text{...(iv)}$$

From equations (ii) and (iv) we get,

$$40 - m = 3(x - m)$$

$$\Rightarrow 40 - m = 3x - 3m$$

$$\Rightarrow 3x - 2m = 40 \quad \text{...(v)}$$

Solving equations (iii) and (v) we get, x = 22.4 kg and m = 13.6 kg.

Thus, x – m = 22.4 – 13.6 = 8.8 kg.

Hence, option (C) is the correct answer.

21. (C)

Let the number of correct, incorrect, and not attempted questions be x, y, and z.

Now according to the question

$x + y + z = 80$ ...(i)

Again $4 \times x + (-1)\, y = 132$

$4x - y = 132$ ...(ii)

If we add equations (i) and (ii) we will get:

$5x + z = 212$

$z = 212 - 5x$

To minimise the value of z, maximise the value of x.

The maximum possible value of x can be 42. Then only we will get the minimum value of z.

Thus, $z = 212 - 5 \times 42$

$z = 212 - 210 = 2$

Hence, the student has answered 42 questions correctly.

Option (C) is the correct answer.

22. 20

Let's assume the original four-digit number = $ABCD = 1{,}000A + 100B + 10C + D$.

According to the question

$(1{,}000A + 100B + 10C + D) - (A + B + C + D) = 999A + 99B + 9C = 7{,}119$

$111A + 11B + C = 791$

Only possible value for A is 7

Putting the value of A in

$111A + 11B + C = 791$, we will get

$11B + C = 14$. For this to be true only possible value of B and C will be 1 and 3, respectively.

So, the original number would be $ABCD$ or $713D$.

Now D is a digit, and its maximum value could be 9.

So, maximum possible original number would be = 7,139.

Hence, maximum possible value of sum of the digits of the original number = 20.

23. 16

According to the first statement

$= \dfrac{P+2}{Q+3} = \dfrac{3}{4}$ ...(i)

According to the second statement

$= \dfrac{P-1}{Q} = \dfrac{2}{3}$ ...(ii)

Solving equations (i) and (ii), we will get $P = 7$ and $Q = 9$.

Hence, $(P + Q) = 16$.

24. 1,280

Case 1: When I had ₹500 on 19th

Dates						19	20	21
Money						500	490	980
Dates	22		23		24		25		
Money	970		1,940		1,930		3,860		

Case 2: When I had ₹100 on 13th

Dates	13	14	15	16	17	18	19	20	21
Money	100	90	180	170	340	330	660	650	1,300
Dates	22		23		24		25		
Money	1,290		2,580		2,570		5,140		

∴ I would have received 5,140 – 3,860, i.e., ₹1,280 more on 25th of the month in the second case as compared to first case.

25. 17

Let's assume the total number of attempts, correct and wrong answers by any student be A, x, and y, respectively.

According to the question

$$3x - y = 100 \Rightarrow x = \frac{100+y}{3}$$

$$\Rightarrow \quad x = 33 + \frac{y+1}{3}.$$

Now number of correct questions will always be an integer, so $(y + 1)$ must be divisible by 3. Minimum value of $y = 2$.

⇒ **A**	**Correct (x)**	**Wrong (y)**
36	34	2
40	35	5
44	36	8
|	|	|
96	49	47
100	50	50

So, a total of 17 (36, 40, 44, ... , 100) different total attempts are possible.

26. 8

Let the four-digit number N be denoted by abcd, where a is the thousands digit, b the hundreds digit, c the tens digit and d the units digit, respectively. It is given that

$a + b = c + d$

$and\ a + c = 3(b + d)$

If $b + d = K$, then $a + c = 3k$

and $a + b + c + d = 4K$ i.e., $S = 4k$

So, 'S' is divisible by 4.

In the given range S can take three values, 12, 16, and 20.

If $S = 12$, then $k = 3$, $b + d = 3$, $a + c = 9$, $a + b = 6$, $c + d = 6$.

N can take the values.

33	60
42	51
51	42
60	33

Since the digits in N are distinct. Only possible values are 4,251 and 5,142.

If $S = 16$, then $b + d = 4$, $a + c = 12$

N can take the values

44	80
53	71
62	62
80	44

With the digits being distinct the only possible values are 5,371 and 7,153.

If $S = 20$ then $b + d = 5$, $a + c = 15$

64	91
73	82
82	73
91	64

N can take four different values if $S = 20$.

Thus, the total number of values that N can take = 2 + 2 + 4 = 8 values.

27. 165

The peanut butter and jam for each sandwich costs = $(4N + 5K)$.

So, the peanut butter and jam for P sandwiches costs = $P(4N + 5K)$.

Now, $P\left(4N + 5K\right) = 253 = 11 \times 23$.

The only possible positive integer pairs $(P,\ 4N + 5K)$ whose product is 253 are (1,253), (11,23), (23, 11), and (253, 1).

The first pair violates $P > 1$, and the third and fourth pairs have no positive integer solutions for N and K.

So, $P = 11$ and $(4N + 5K) = 23$

The only integral solutions for N and K are $N = 2$ and $K = 3$.

Hence, the total cost of jams incurred by Ramu to make P sandwiches

$= P \times \left(5K\right) = 11 \times 5 \times 3 = 165$

28. (B)

Let the number of girls be x and the number of boys be y.

$\therefore\ 3.3x + 2.9y = 249$

Check the options

Option (B) satisfies this equation.

Hence, Mrs. Alpa bought 57 strawberry flavoured and 21 chocolate flavoured candies.

29. (A)

$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

$x + 1 \geq 0, \quad x - 1 \geq 0, \quad 4x - 1 \geq 0$

Squaring both the sides of the equation we get:

$(x + 1) + (x - 1) - 2\sqrt{x^2 - 1} = 4x - 1$

$\Rightarrow 2\sqrt{x^2 - 1} = 1 - 2x$

Squaring again, we get

$4(x^2 - 1) = 1 + 4x^2 - 4x$

Solving which we will get, $x = \dfrac{5}{4}$

For $x = \dfrac{5}{4}$, LHS $= \sqrt{\dfrac{5}{4} + 1} - \sqrt{\dfrac{5}{4} - 1} = 1$

and, RHS $= \sqrt{4\left(\dfrac{5}{4}\right) - 1} = \sqrt{4} = 2$

But LHS ≠ RHS

Thus, $x = \dfrac{5}{4}$ is not the root of the given equation.

Hence, option (A) is the correct answer.

30. 600

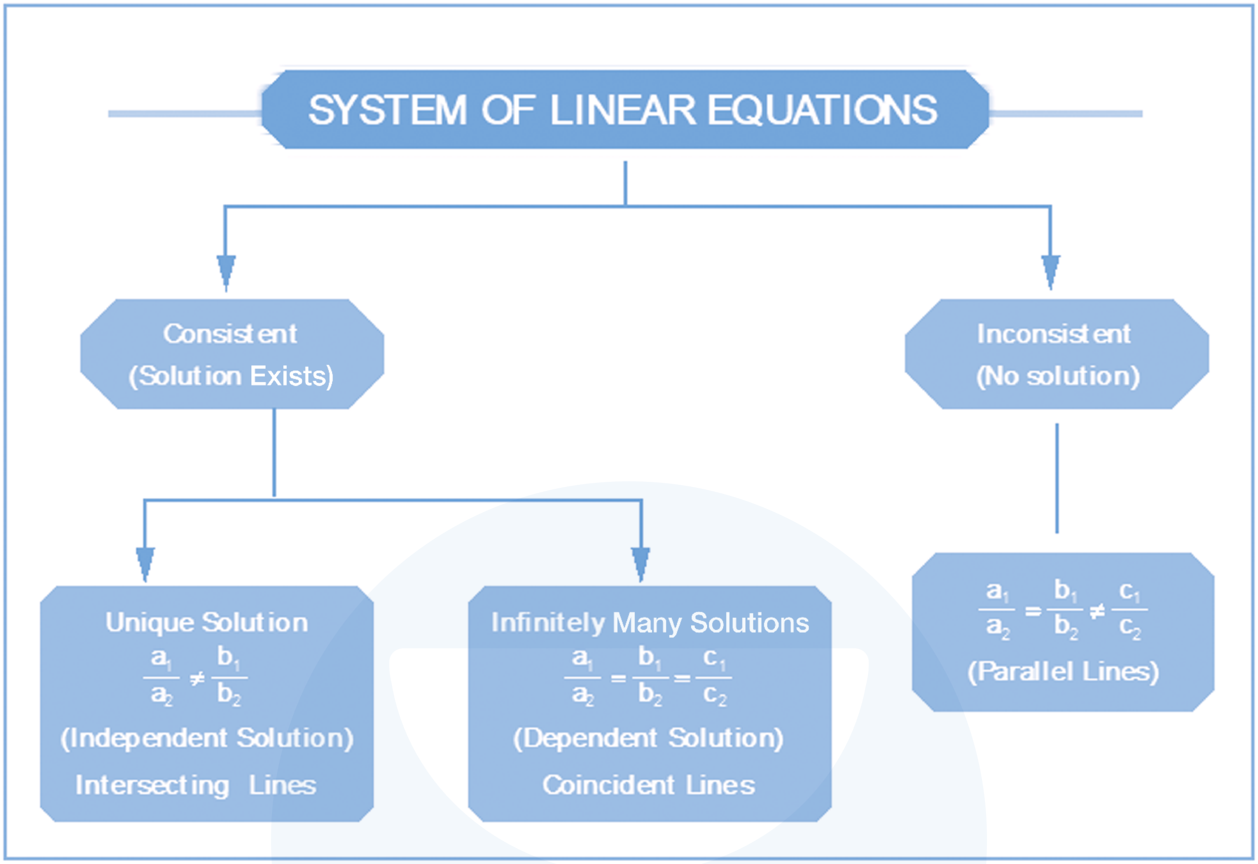
Let the number of roses blooming in the month of January be x. The number of successive months

Month	Number	Month	Number
Jan	x	Apr	$\frac{x}{4} + 30$
Feb	$\frac{x}{2}$	May	$\frac{x}{4} + 90$
Mar	$\frac{x}{2} + 60$	June	$\frac{x}{8} + 45$

$\therefore$ Number of roses blooming in June is

$$\frac{x}{8} + 45 = 120.$$

$$\Rightarrow \quad x = 600$$


SYSTEM OF LINEAR EQUATIONS
Consistent
(Solution Exists)
Inconsistent
(No solution)
Unique Solution
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
(Independent Solution)
Intersecting Lines
Infinitely Many Solutions
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(Dependent Solution)
Coincident Lines
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
(Parallel Lines)