8 Probability

Introduction

Probability means the possibility of happening or non-happening of an event. This is one of the most important topics for most MBA entrance tests. To understand probability properly, one needs to have an in-depth knowledge of permutations and combinations.

In the game of chances, more than one result is possible; therefore, a definite outcome is unpredictable. Probability theory aims to provide a mathematical solution to all such situations arising in the games of chances.

Basic Terminologies

Experiment

An operation that can produce specific results is known as an experiment.

Example: Rolling of fair dice, tossing of a coin, etc.

Random Experiment

Any experiment whose outcome cannot be predicted or determined in advance is known as a random experiment.

Total Number of Outcomes (Sample Space)

The set of all possible outcomes for a particular random experiment is known as sample space. If n denotes the number of outcomes in a single trial and m denotes the number of times the trials or the events are performed, then the number of total possible outcomes = n^m.

For example:

When a die is thrown once, it results in any one of the numbers (1, 2, 3, 4, 5, 6).

Sample space (S) = {1, 2, 3, 4, 5, 6}

When a coin is tossed twice,

Sample space (S) = {HH, TT, HT, TH}

Note: Each element of the sample space (S) is called a sample point.

Example 1:

Two coins are tossed once. Find the sample space.

Solution:

When two coins are tossed, the possible outcomes are:

1. {H, H} = Head on both coins.
2. {HT, TH} = Head on the first coin and tail on the other coin, tail on the first coin and head on the other coin.
3. {TT} = Tail on both coins.

Thus, the sample space is

S = {HH, HT, TH, TT} = four possible outcomes.

Example 2:

From a group of two boys and two girls, we select two children. What should be the sample space for this experiment?

Solution:

Here, total number of children

= 2 + 2 = 4

Now, two children can be selected out of four in $^4C_2$ ways

$$= \frac{4!}{2! \times 2!} = \frac{4 \times 3}{2} = 6 \text{ ways}$$

Let's understand this by the traditional method:

Let the boys be (B_1, B_2) and girls be (G_1, G_2)

$$S = \left\{\begin{matrix} B_1B_2, B_1G_1, B_1G_2 \\ B_2G_1, B_2G_2, \\ G_1G_2 \end{matrix}\right\} = 6 \text{ (ways or outcomes)}$$

Example 3:

A coin is tossed initially; if the tail occurs, a ball is drawn from a box of two red and two white balls, but if the head occurs, we throw a dice. Find the sample space for this experiment.

Solution:

Suppose the two red balls are R_1, R_2 and the two white balls are W_1, W_2.

If it shows tail, the outcomes are

$= TR_1, TR_2, TW_1, TW_2$

If it shows head, the outcomes are

= H1, H2, H3, H4, H5, H6

Total outcomes or (S)

$$= \begin{Bmatrix} TR_1, & TR_2, & TW_1, & TW_2 \\ H_1, H_2, & H_3, H_4, & H_5, & H_6 \end{Bmatrix}$$

Hence, the total number of outcomes is 10.

Events

A subset of a sample space is known as an event.

For example:

Suppose a boy throws a die and reports that the number that appeared on the die is an even number. In this case, he should have got 2, 4, or 6 because these are the only even numbers in the sample space.

Sample space (S) = {1, 2, 3, 4, 5, 6}

Event (E) = {2, 4, 6}

Hence, we can say that event (E) is the subset of sample space (S).

Occurrence of an Event

When an outcome satisfies the condition mentioned in the event completely, then we can say that the event has occurred.

For example:

Suppose we throw a die, and the event (E) = a number less than 3 appears on the die.

If 1 or 2 appear on the die, we will say that event (E) has occurred, and if 3, 4, 5, or 6 appear on the die, we will say that event (E) has not occurred.

Types of Events

Simple Event

If an event has only one sample point of the sample space, it is called a simple (or elementary) event.

For Example:

Suppose a boy throws a coin.

Sample space (S) = {H, T}

There are four subsets of sample space (S) = {H}, {T}, {HT}, {ϕ}

In this case {H} and {T} are simple events.

Compound Events

The events with more than one sample point of the sample space are known as compound events.

In the previous example {HT} is a compound event.

Equally Likely Events

These events have the same chances of occurrence in an experiment.

For example:

When a coin is thrown, the probability of head or tail = $\frac{1}{2}$, then we can say that (H) and (T) are equally likely events.

Similarly, when a dice is rolled, the probability of getting 1, 2, 3, 4, 5, or 6 is the same and equal to $\frac{1}{6}$.

Then {1} {2} {3} {4} {5} {6} are equally likely events.

Sure Event

Let S be a sample space associated with a random experiment of rolling a die.

Here, S = {1, 2, 3, 4, 5, 6}. The set S is also a subset of S ($S \subseteq S$). Since every outcome of the experiment is a member of S, one can say that S is a certain event.

Impossible Event

An event associated with a random experiment is known as an impossible event if it never occurs whenever the experiment is performed.

For example:

Choosing a yellow ball from a bag containing three red balls and five blue balls is an impossible event.

Complement of an Event

The complement of an event E is the set of all elements of sample space (S) that are not in E. The complement of an event is denoted by E′ or $\bar{E}$ or E^c.

Thus,

$$E' = S - E$$

$$E' + E = S \quad \text{(i)}$$

If we divide this throughout by S, we can say

$$\frac{E'}{S} + \frac{E}{S} = \frac{S}{S}$$

Which gives us $P(E') + P(E) = 1$

Now, we can conclude that the sum of the probability of occurrence of an event and the probability of non-occurrence of an event is equal to 1.

For example:

Suppose a person tosses a coin, and E is the event of getting heads $P(E) = \frac{1}{2}$,

Also $P(\bar{E}) = 1 - \frac{1}{2} = \frac{1}{2}$

Now, we can conclude that the complement of getting a head is nothing but getting a tail.

Algebra of Events

Now, we have understood that a sample space is a set of all possible outcomes of an experiment, and the events are subsets of the sample space. In this section, we will learn how new events can be constructed by combining two or more events.

If we assume that sample space is a universal set for these events, then we have the following results.

Let A, B, and C be the events of a sample space S.

- $(A \cup B)$ or (A or B) is an event that means either A or B or both occur.
- $(A \cap B)$ or (A and B) is an event that means both A and B occur.
- A' or A^c or $\bar{A}$ is an event that means A does not occur.
- $(A - B)$ is an event that means A occurs but B does not.

Points to Remember

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$
3. $A \cup (B \cup C) = (A \cup B) \cup C$
4. $A \cap (B \cap C) = (A \cap B) \cap C$
5. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
6. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Mutually Exclusive Events

Two or more than two events that are associated with a random experiment are said to be mutually exclusive when the occurrence of any one of them excludes the occurrence of the other events.

For example:

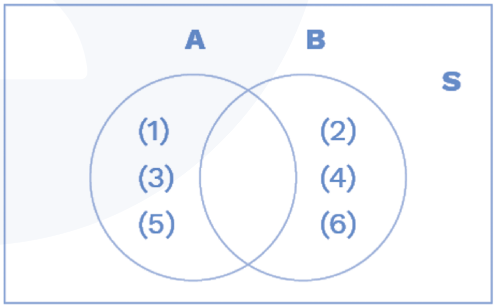
Suppose a boy rolls a die,

Sample space (S) = {1, 2, 3, 4, 5, 6}

Event A = all odd numbers = {1, 3, 5}

Event B = all even numbers = {2, 4, 6}

Pictorially



$A \cap B = \phi$ (null)

Hence, we can say that both events are mutually exclusive.

Let's understand another example.

Event A = all even numbers from 1 to 20 = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}.

Event B = all odd prime numbers from 1 to 20 = {3, 5, 7, 11, 13, 17, 19}.

$A \cap B = \phi$

Mutually Exclusive and Exhaustive Events

Two or more than two events are said to be exhaustive events if their union denotes the entire sample space.

For example:

In a deck of playig cards, if event A is 'card is red', and event B is 'card is black', then both the events are said to be mutually exclusive and exhaustive events.

$\because$ [event A $\cup$ event B = sample space (a deck of 52 cards)]

Further, if event A is 'card of spade' and event B is 'card of diamond', both events are said to be mutually exclusive but not exhaustive.

$\because$ (A $\cup$ B $\neq$ sample space)

Note:

When two or more sets of events are mutually exclusive and collectively exhaustive, then the sum of their probabilities is 1.

Probability

We talk about the chances that a particular event will occur when we perform an experiment. Let S be the sample space, and E be an event, where $E \subseteq S$.

$$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{total outcomes}}$$

or

$$P(\text{E}) = \frac{\text{Number of elements in E}}{\text{Number of elements in S}}$$

Example 4:

If two dices are thrown up simultaneously, what is the probability that one die shows up 3 and the other shows up 4?

Solution: When two dices are thrown.

Total number of outcomes = $6^n \Rightarrow 6^2 \Rightarrow 36$

Favourable outcomes = (3, 4) an (4, 3)

$$P(\text{E}) = \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}} = \frac{2}{36} = \frac{1}{18}$$

Example 5:

If six coins are tossed together, what is the probability of getting exactly two heads?

Solution: When six coins are tossed,

Total number of outcomes = $2^n \Rightarrow 2^6 \Rightarrow 64$

Number of outcomes of an event of getting exactly two heads

= Combination of two heads and four tails.

$$\Rightarrow {}^6C_2 = \frac{6!}{4! \times 2!} \Rightarrow 15 \text{ outcomes}$$

$$P(\text{E}) = \frac{\text{Favourable number of outcomes}}{\text{Total number of outcomes}} \Rightarrow \frac{15}{64}$$

Example 6:

If six coins are tossed together, what is the probability of getting at least two tails?

Solution: When six coins are tossed,

Total number of outcomes = $2^n = 2^6 = 64$

Method 1

Favourable outcomes for an event = getting two tails out of 6 + getting three tails out of 6 + getting four tails out of 6 + getting five tails out of 6 + getting six tails out of 6.

$$P(\text{E}) = \frac{{}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6}{64}$$

$$P(\text{E}) = \frac{15+20+15+6+1}{64} = \frac{57}{64}$$

Method 2

P (at least two tails) = 1 – P (getting no tail or exactly one tail).

P (getting no tail) = P (getting all head) = $\frac{1}{64}$

P (exactly one tail) = $\frac{{}^6C_1}{64} = \frac{6}{64}$

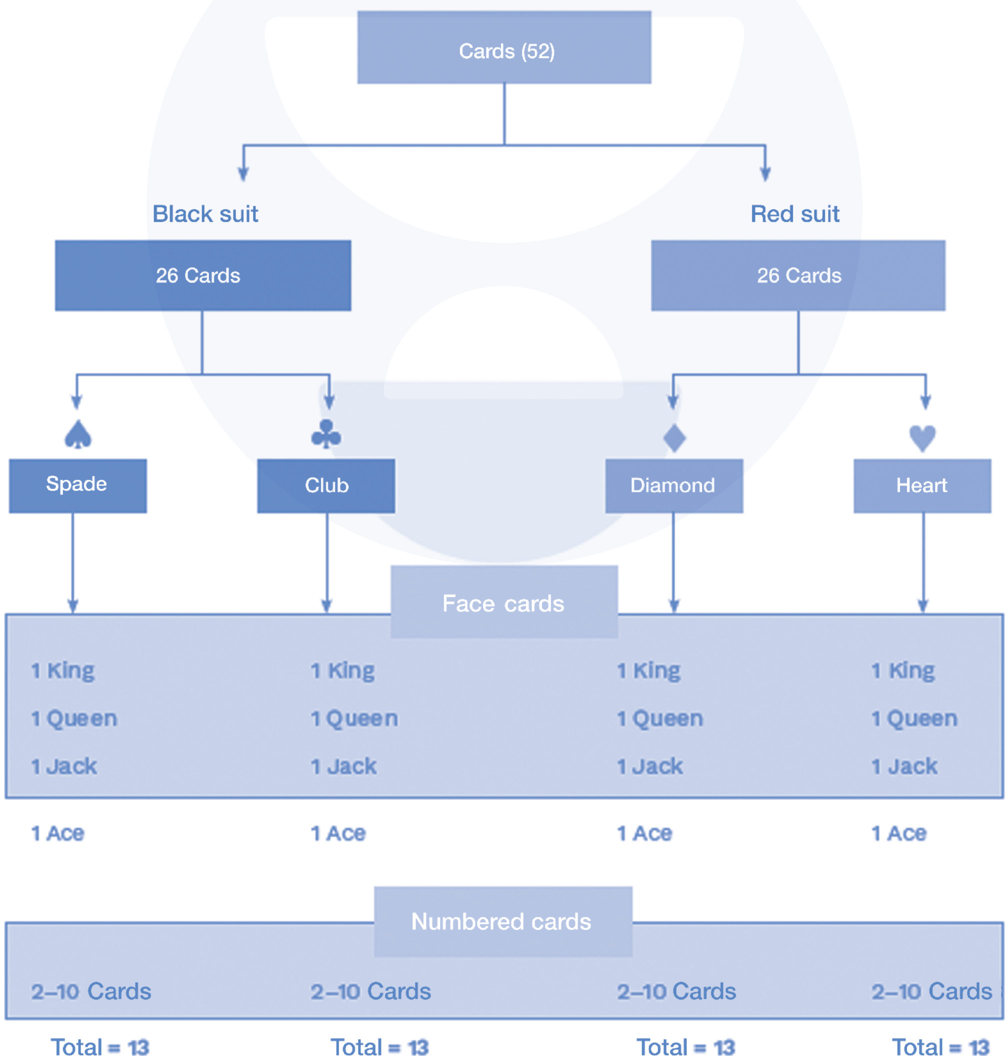
P (getting no tail or exactly one tail)

$$= \frac{1}{64} + \frac{6}{64} = \frac{7}{64}$$

P (at least 2 tails) = $1 - \frac{7}{64} = \frac{57}{64}$

Points to Remember

- The probability of an impossible event = 0 i.e., $P(\phi) = 0$.
- The probability of a certain/sure event = 1.
- The probability of an event is greater than equals to zero and less than equal to 1 i.e., $0 \le P(E) \le 1$.
- The probability of occurrence of A but not B is, $P(A - B) = P(A) - P(A \cap B)$.
- Odds in favour of the event = $\dfrac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$
- Odds against the event = $\dfrac{\text{Number of unfavourable outcomes}}{\text{number of favourable outcomes}}$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.
- Last two points are also known as addition theorem of probability.



Let's look at some examples to understand the basics of cards.

Example 7:

From a deck of 52 playing cards, consisting of four suits having 13 cards each.

1. What is the minimum number of cards to be picked to ensure at least two face cards? (King, Queen, Jack)?
2. What is the minimum number of cards to be picked to ensure at least eight cards of the same colour?

Solution 1: To solve such a type of question, always think of the worst-case scenario.

Here, the worst-case scenario is that we pick the first card, and it turns out to be a non-face card; again, we pick a card, and it turns out to be a non-face card.

Now in each trial, we pick all the non-face cards (i.e., 40 cards), which is the maximum without satisfying the condition.

Now, if we pick the 41st card, it turns out to be the first face card, and the 42nd card, then it turns out to be the second face card.

Now the conclusion is to ensure at least 2 face cards. So, we must need to pick at least 42 cards.

Solution 2: Here, the worst-case scenario is that we pick 7 cards of black suit and 7 cards of a red suit, a total of 14 cards without satisfying the condition. Now, the next card we pick it may be of red colour and black colour.

Hence, to ensure at least 8 cards of the same colour, we must pick 15 cards.

Binomial Probability

If n identical and independent trials are conducted, and each trial has a probability of success A and a probability of failure B (where B = 1 – A), then the probability that exactly r out of n trials are successful is given by

$$= {}^{n}C_{r} \times A^{r} \times B^{(n-r)}$$

Properties of Binomial Experiment

- The experiment consists n identical trials.
- The trials must be independent.

Let's understand this concept with the help of examples.

Example 8:

If 100 coins are tossed, what will be the probability of getting exactly 52 heads?

Solution: Here, getting heads can be taken as success, and getting tails can be taken as a failure.

Probability of success (getting head) = A = $\frac{1}{2}$

Probability of failure = (getting tail) = B = $\frac{1}{2}$

Here, n = 100 and r = 52

By using binomial probability, we can directly write this as ${}^{100}C_{52} \times \left[\frac{1}{2}\right]^{52} \times \left[\frac{1}{2}\right]^{48}$

Required probability = $\frac{{}^{100}C_{52}}{(2)^{100}}$

Example 9:

If 10 dice are rolled, what is the probability of getting at least 8 '5's?

Solution: Here, 8 or 9 or 10 '5's can be possible.

By using the binomial probability

Probability of 8 '5's = ${}^{10}C_{8} \times \left(\frac{1}{6}\right)^{8} \times \left(\frac{5}{6}\right)^{2}$

Probability of 9 '5's = ${}^{10}C_{9} \times \left(\frac{1}{6}\right)^{9} \times \left(\frac{5}{6}\right)^{1}$

Probability of 10 '5's = ${}^{10}C_{10} \times \left(\frac{1}{6}\right)^{10} \times \left(\frac{5}{6}\right)^{0}$

Total probability

$$= {}^{10}C_{8} \times \left(\frac{1}{6}\right)^{8} \times \left(\frac{5}{6}\right)^{2} + {}^{10}C_{9} \times \left(\frac{1}{6}\right)^{9} \times \left(\frac{5}{6}\right)^{1} + {}^{10}C_{10} \times \left(\frac{1}{6}\right)^{10} \times \left(\frac{5}{6}\right)^{0}.$$

Conditional Probability

Suppose A and B are two events associated with a random experiment. Then, the probability of an event A, given that an event B has already occurred, where $P(\mathrm{B}) \neq 0$, is known as conditional probability.

It is denoted by $P\left[\frac{A}{B}\right]$ and read as:

1. Probability of A when B has already occurred.
2. Probability of A taking B as a sample space.
3. Probability of A with respect to B.

For example:

Suppose a class containing 25 students, 12 girls (5 brown hair and 7 blue hair) and 13 boys (7 brown hair and 6 blue hair). What is the probability that a student chosen at random will have brown hair, given that it is a boy?

Here, the words given that it is a boy means that our sample space is reduced to 13 boys, and favourable outcomes will be 7 brown hair boys.

Required probability = $\frac{7}{13}$

Now,

$$P\left(\frac{A}{B}\right) = \frac{\text{Number of events favourable to A and B}}{\text{Number of events favourable to B}}$$

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Properties Associated with Conditional Probability

Property 1

When A and B are two events of a sample space (S) and F is an event of S such that $P(F) \neq 0$, then

$$P\left(\frac{A \cup B}{F}\right) = P\left(\frac{A}{F}\right) + \left(\frac{B}{F}\right) - P\left(\frac{A \cap B}{F}\right)$$

If A and B are mutually exclusive events, then P $(A \cap B) = 0$

$$P\left(\frac{A \cup B}{F}\right) = P\left(\frac{A}{F}\right) + P\left(\frac{B}{F}\right)$$

Property 2

If A and B are the events of sample space (S) of a random experiment, then $P\left(\frac{A}{B}\right)' = 1 - P\left(\frac{A}{B}\right)$

Example 10:

15 cards, numbered 1–15, are placed in a box, mixed up thoroughly, and then a card is drawn at random from the box; if it is known that the number on the drawn card is more than 5, then find the probability that it is an even number.

Solution: Let event A = (the number on the card is even).

Let event B = (the number on the card is greater than 5).

Sample space (S) = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

(A) = [2, 4, 6, 8, 10, 12, 14].

(B) = [6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

$(A \cap B)$ = (6, 8, 10, 12, 14).

$P(A) = \frac{7}{15}$, $P(B) = \frac{10}{15}$, and $P(A \cap B) = \frac{5}{15}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{5}{15}}{\frac{10}{15}} = \frac{1}{2}$$

Alternate (Easy) solution

Here, 'given that drawn card is more than 5' means that our sample space is reduced to the number more than 5.

Sample space (S) = (6, 7, 8, 9, 10, 11, 12, 13, 14, 15) = 10 outcomes

Favourable outcomes = (6, 8, 10, 12, 14) = 5 outcomes

$$P(E) = \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{5}{10} = \frac{1}{2}$$

Example 11:

In a class, 55% of the students read English, 40% of students read Hindi, and 20% of the student read both English and Hindi. One student is selected at random. Find the probability that he reads English if it is known that he reads Hindi.

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{4}{5}$ (D) $\frac{1}{4}$

Solution: (B)

Let event A = [reading English]

Event B = [reading Hindi]

$$P(A) = \frac{55}{100} = \frac{11}{20},\ P(B) = \frac{40}{100} = \frac{2}{5}, \text{ and}$$

$$P(A \cap B) = \frac{20}{100} = \frac{1}{5}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \left(\frac{1}{2}\right)$$

Alternate Solution:

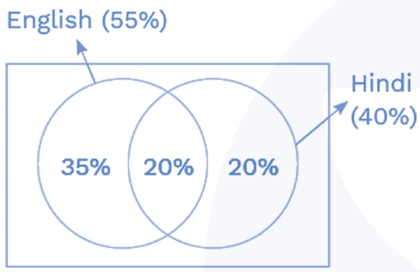
We can solve this question by using the Venn diagram.

Here, it is given that he reads Hindi so our sample space is reduced to Hindi.

Favourable outcomes = 20%

Total outcomes = 40% (reads Hindi)

$$P(E) = \frac{20\%}{40\%} = \frac{1}{2}$$



Multiplication Theorem of Probability

As we know that when A and B are two events associated with the random experiment, then,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \text{ and } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

By cross multiplication

$$P(A \cap B) = P(B) \times P\left(\frac{A}{B}\right) \text{ and}$$

$$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$$

From the above results, we can conclude that

If $E_1, E_2, E_3, E_4, \ldots, E_n$ are the events associated with the same random experiment then the probability of $(E_1 \cap E_2 \cap E_3 \cap E_4 \ldots \cap E_n)$

$$= P(E_1) \times P\left(\frac{E_2}{E_1}\right) \times P\left(\frac{E_3}{E_1 \cap E_2}\right) \ldots$$

$$P\left(\frac{E_n}{E_1 \cap E_2 \cap E_3 \cdots E_{n-1}}\right)$$

Example 12:

A bag contains five black and eight white balls. Two successive drawings of three balls are made without replacement. Find the probability that the first draw results in three black balls and the second draw results in three white balls.

(A) 7/676 (B) 7/429

(C) 5/63 (D) 9/130

Solution: (B)

Let us assume that the event of drawing three black balls is A and the event of drawing three white balls is B.

$$P(A) = P(A) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{10}{286} = \frac{5}{143}$$

$$P\left(\frac{B}{A}\right) = \frac{{}^8C_3}{{}^{10}C_3} = \frac{56}{120} = \frac{7}{15}$$

Total probability = $P(A \cap B)$

$$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right) = \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}$$

Dependent Events

Suppose A and B are two events associated with a random experiment, then the occurrence of one of the events is affected by the occurrence of the other event. Such events are called dependent events.

For example:

Suppose we have 10 black balls and 10 white balls in a bucket. One ball is drawn at random without replacement, which may be black or white. What is the probability that the second ball withdrawn will be white?

Now, if the first ball is white, then the bucket is left with nine white balls out of 19 balls. So, the probability of drawing a white ball on the second draw is $\frac{9}{19}$.

But if the first ball is black, then the bucket is left with 10 white balls out of 19 balls. So, the probability of drawing a white ball on the second draw is $\frac{10}{19}$.

Hence, we can say that a second draw is dependent on the first draw.

Independent Events

Suppose A and B are two events associated with a random experiment, then the occurrence of one of the events is not affected by the occurrence of the other event. Such events are called independent events.

For example:

Consider an experiment of drawing a card from a deck of 52 cards. If event A denotes 'The card drawn is a Heart' and event B denotes 'The card drawn is a Queen', respectively, then

$$P(A) = \frac{13}{52} \Rightarrow \frac{1}{4} \text{ and } P(B) = \frac{4}{52} \Rightarrow \frac{1}{13}$$

$P(A \cap B)$ = [the card drawn is the Queen of Heart].

$$\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{52}}{\frac{1}{13}} = \frac{1}{4}$$

Since $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$

Here, we can say that the probability of event B is not affected by the probability of event A.

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{52}}{\frac{1}{4}} = \frac{1}{13}$$

Since $P(B) = P\left(\frac{B}{A}\right) = \frac{1}{13}$

Here, we can say that the probability of event A is not affected by the probability of event B.

We can clearly say that event A and event B are independent events.

Example 13:

In a test, two friends, Raghav and Ashwani, appeared. Raghav's chances of clearing the test are $\frac{1}{5}$, whereas Ashwani's chances of clearing the test are $\frac{3}{7}$. What is the probability that both will pass the test if it is given that the two of them do not copy from each other?

CAT Mantra

1. Two events are said to be independent if $P(A \cap B) = P(A) \times P(B)$.
2. Three events are said to be independent if $P(A \cap B \cap C) = P(A) \times P(B) \times P(B)$.
3. For n events.

   $P(E_1 \cap E_2 \cap E_3 \cap E_4 \cap \cdots \cap E_n) = P(E_1) \times P(E_2) \times P(E_3) \times \ldots P(E_n)$
4. If $E_1, E_2, E_3, \ldots, E_n$ are independent events associated with a random experiment, then the probability that at least one happens

   $= 1 - P(\bar{E}_1) \times P(\bar{E}_2) \times P(\bar{E}_3) \times P(\bar{E}_4) \times \cdots P(\bar{E}_u)$

5. If A and B are independent events associated with a random experiment, then

   a) $\bar{A}$ and B are independent events, i.e., $(\bar{A} \cap B) = P(\bar{A}) \times P(B)$

   b) A and $\bar{B}$ are independent events, i.e., $(A \cap \bar{B}) = P(A) \times P(\bar{B})$

   c) $\bar{A}$ and $\bar{B}$ are independent events, i.e., $(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$

Solution:

Here, it is given that the two of them do not copy from each other, which means Raghav's passing or failing will have no bearing on Ashwani's passing or failing.

P (both will pass) = $\frac{1}{5} \times \frac{3}{7} = \frac{3}{35}$

Example 14:

Five friends, Aman, Baman, Chaman, Dheeraj, and Neeraj have their birthdays in the coming week from (Monday to Sunday). What is the probability that at least one of them will have a birthday on the weekend?

Solution:

In this case, the date of birth of one friend will not have any relation with the date of birth of another friend.

There are seven days in a week (five weekdays and two weekends).

The probability that each of them is not on a weekend = $\frac{5}{7}$

$\therefore$ For all the five to be not on the weekend

$$= \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} = \frac{3{,}125}{16{,}807}$$

P (at least once on the weekend) = 1 – none on a weekend

$$= 1 - \frac{3{,}125}{16{,}807} \Rightarrow \frac{13{,}682}{16{,}807}$$

Example 15:

Ankur is a teacher. The probability that he takes an unannounced test during any class meeting is $\frac{1}{7}$. If a student is absent thrice, what is the probability that he will miss at least one test?

(A) $\frac{1}{343}$ (B) $\frac{127}{343}$

(C) $\frac{196}{343}$ (D) $\frac{216}{343}$

Solution: (B)

Suppose E_i be the event that the student misses the ith test (i = 1, 2, 3), then E_1, E_2, E_3 are independent event such that

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{7}$$

$\Rightarrow$ Required probability

$$= 1 - P(\bar{E}_1) \times P(\bar{E}_2) \times P(\bar{E}_3)$$

$$\Rightarrow 1 - \left(1 - \frac{1}{7}\right) \times \left(1 - \frac{1}{7}\right) \times \left(1 - \frac{1}{7}\right)$$

$$\Rightarrow 1 - \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7} = \frac{127}{343}$$

Example 16:

Tarun can hit a target four times in six shots. Abhishek can hit a target five times in six shots, and Ankit can hit a target three times in four shots. Find the probability that any two of Tarun, Abhishek, and Ankit will hit the target.

(A) $\frac{30}{72}$ (B) $\frac{36}{72}$

(C) $\frac{31}{72}$ (D) $\frac{35}{72}$

Solution: (C)

Let's define three events A, B, and C such that

A = Tarun hits the target.

B = Abhishek hits the target.

C = Ankit hits the target.

$P(A) = \frac{4}{6} = \frac{2}{3}$, $P(B) = \frac{5}{6}$, and $P(C) = \frac{3}{4}$

P (any two of A, B, and C will hit the target).

$$= P\left(A \cap B \cap \bar{C}\right) + P\left(A \cap \bar{B} \cap C\right) + P\left(\bar{A} \cap B \cap C\right)$$

$$= P\left[\frac{2}{3} \times \frac{5}{6} \times \left(1 - \frac{3}{4}\right)\right] + P\left[\frac{2}{3} \times \left(1 - \frac{5}{6}\right) \times \frac{3}{4}\right]$$

$$+ P\left[\left(1 - \frac{2}{3}\right) \times \frac{5}{6} \times \frac{3}{4}\right]$$

$$= \frac{2}{3} \times \frac{5}{6} \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{6} \times \frac{3}{4} + \frac{1}{3} \times \frac{5}{6} \times \frac{3}{4}$$

$$= \frac{10}{72} + \frac{6}{72} + \frac{15}{72} = \frac{31}{72}$$

Total Probability

Suppose (A_1, A_2, A_3, ..., A_n) be n mutually exclusive and exhaustive events associated with a random experiment of sample space (S).

If an event 'E' occurs with A_1 or A_2 or A_3, ..., A_n then.

$$P(E) = P(A_1) \times P\left(\frac{E}{A_1}\right) + P(A_2) \times P\left[\frac{E}{A_2}\right] + \cdots + P(A_n) \times P\left(\frac{E}{A_n}\right)$$

$$= \sum_{i=1}^{n} P(A_i) \times P\left(\frac{E}{A_i}\right)$$

Example 17:

A bag contains five gold coins and three silver coins. A second bag contains three gold coins and five silver coins. One bag is selected randomly and from the selected bag one coin is drawn. Find the probability that the coin drawn is of gold.

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Solution: (C)

Let A_1 = selecting bag I

A_2 = selecting bag II

E = drawing a gold coin.

∴ One of the two bags is selected randomly then, $P(A_1) = \frac{1}{2}$

and $P(A_2) = \frac{1}{2}$

Now, $P\left(\frac{E}{A_1}\right)$ = (probability of drawing a gold coin when the first bag has been chosen).

$$P\left[\frac{E}{A_1}\right] = \frac{5}{8}$$

Now, $P\left[\frac{E}{A_2}\right]$ = (probability of drawing a gold coin when the second bag has been chosen).

$$P\left(\frac{E}{A_2}\right) = \frac{3}{8}$$

By using total probability

$$P(E) = P(A_1) \times P\left(\frac{E}{A_1}\right) + P(A_2) \times P\left(\frac{E}{A_2}\right)$$

$$= \frac{1}{2} \times \frac{5}{8} + \frac{1}{2} \times \frac{3}{8}$$

$$= \frac{5}{16} + \frac{3}{16} = \frac{8}{16} = \frac{1}{2}$$

Alternate solution

First, we need to choose a bag, P(choosing a bag) = $\frac{1}{2}$

Now, P (drawing a gold coin from bag 1) = $\frac{5}{8}$

P (drawing a gold coin from bag 2) = $\frac{3}{8}$

$$\text{Required probability} = \frac{1}{2} \times \left[\frac{5}{8} + \frac{3}{8}\right] = \frac{1}{2} \times 1 = \left(\frac{1}{2}\right)$$

Bayes' Theorem

Suppose S be the sample space and A_1, A_2, A_3, ..., A_n be n mutually exclusive and collectively exhaustive events associated with a random experiment. If E is an event that occurs with A_1 or A_2 ... or A_n. Then,

$$P\left[\frac{A_i}{E}\right] = \frac{P(A_i) \times P\left[\frac{E}{A_i}\right]}{\sum_{j=1}^{n} P(A_j) \times P\left[\frac{E}{A_j}\right]}, \ (i = 1, 2, \ldots, n)$$

Proof:

By using the formula of conditional probability, we get

$$P\left[\frac{A_i}{E}\right] = \frac{P(E \cap A_i)}{P(E)}$$

$$\Rightarrow P\left[\frac{A_i}{E}\right] = \frac{P(A_i) \times P\left(\frac{E}{A_i}\right)}{P(E)}$$

(∵ by using the multiplication theorem of probability)

$$P\left[\frac{A_i}{E}\right] = \frac{P(A_i) \times P\left[\frac{E}{A_i}\right]}{\sum_{j=1}^{n} P(A_j) \times P\left[\frac{E}{A_j}\right]}$$

(∵ by using theorem of total probability)

Example 18:

In a toy factory, machines A, B, and C manufacture 30%, 35%, and 35% of the total toys, respectively. Of their output 7%, 5%, and 5% are defective toys, respectively. A toy is drawn at random from the product. If the toy drawn is defective, what is the probability that it is manufactured by machine B?

(A) $\frac{5}{16}$ (B) $\frac{5}{18}$

(C) $\frac{4}{15}$ (D) $\frac{7}{15}$

Solution: (A)

Let A_1 = toy manufactured by machine A

A_2 = toy manufactured by machine B

A_3 = toy manufactured by machine C

E = toy is defective

$$P(A_1)=\frac{30}{100},\ P(A_2)=\frac{35}{100},\text{ and } P(A_3)=\frac{35}{100}$$

$P\left(\frac{E}{A_1}\right)$ = (probability that the defective toy was manufactured by machine A)

$$P\left(\frac{E}{A_1}\right)=\frac{7}{100}$$

Similarly, $P\left(\frac{E}{A_2}\right)=\frac{5}{100}$ and $P\left(\frac{E}{A_3}\right)=\frac{5}{100}$

Required probability = probability that the toy is manufactured by machine B and turns out to be defective = $P\left[\frac{A_2}{E}\right]$

$$P\left[\frac{A_2}{E}\right]=\frac{P(A_2)\times P\left(\frac{E}{A_2}\right)}{P(A_1)\times P\left[\frac{E}{A_1}\right]+P(A_2)\times P\left(\frac{E}{A_2}\right)+[A_3]\times P\left[\frac{E}{A_3}\right]}$$

$$\Rightarrow \frac{\frac{35}{100}\times\frac{5}{100}}{\frac{30}{100}\times\frac{7}{100}+\frac{35}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{5}{100}}=\left(\frac{5}{16}\right)$$

Example 19:

Three boxes contain 6 white, 4 black; 5 white, 6 black, and 5 white, 5 black balls, respectively. Out of these three boxes, one box is selected randomly and a ball is drawn from it. If the ball drawn is white, find the probability that it is drawn from the first box.

(A) $\frac{21}{57}$ (B) $\frac{22}{57}$ (C) $\frac{1}{3}$ (D) $\frac{7}{57}$

Solution: (B)

Let A_1 = first box is chosen.

A_2 = second box is chosen.

A_3 = third box is chosen.

E = ball drawn is white.

$$P(A_1)=P(A_2)=P(A_3)=\frac{1}{3}$$

$$P\left(\frac{E}{A_1}\right)=\frac{6}{10},\ P\left[\frac{E}{A_2}\right]=\frac{5}{11},\text{ and } P\left[\frac{E}{A_3}\right]=\frac{5}{10}$$

Required probability = $P\left[\frac{A_1}{E}\right]$

$$P\left[\frac{A_1}{E}\right]=\frac{P[A_1]\times P\left[\frac{E}{A_1}\right]}{P(A_1)\times P\left(\frac{E}{A_1}\right)+P(A_2)\times P\left[\frac{E}{A_2}\right]+P[A_3]\times P\left[\frac{E}{A_3}\right]}$$

$$P\left[\frac{A_1}{E}\right]=\frac{\frac{1}{3}\times\frac{6}{10}}{\frac{1}{3}\times\frac{6}{10}+\frac{1}{3}\times\frac{5}{11}+\frac{1}{3}\times\frac{5}{10}}$$

$$=\frac{\frac{6}{10}}{\frac{6}{10}+\frac{5}{11}+\frac{5}{10}}=\frac{66}{171}\Rightarrow\frac{22}{57}$$

Alternate solution

Here, it is given that a white ball is drawn from the first box.

P (white ball from first box) = $\frac{6}{10}$

P (white ball from second box) = $\frac{5}{11}$

P (white ball from third box) = $\frac{5}{10}$

P (selection of a box) = $\frac{1}{3}$

Required probability

$$= \frac{\frac{1}{3}\times\frac{6}{10}}{\frac{1}{3}\times\frac{6}{10}+\frac{1}{3}\times\frac{5}{11}+\frac{1}{3}\times\frac{5}{10}}$$

$$= \frac{\frac{6}{10}}{\frac{6}{10}+\frac{5}{11}+\frac{5}{10}} = \frac{22}{57}$$

Expected Value

The concept of expected value is very important in probability. The expected value is usually taken into consideration when an experiment is performed many times. Here, the time period doesn't hold any importance unless or until mentioned.

The concept of expected value is usually used in gambling scenarios where we find gain or loss in the long run.

For example:

Suppose a man is permitted to toss a coin on payment of ₹2 and offered ₹10 as a prize if the throw of the coin results in a head and offered ₹0 if the throw of the coin results in a tail. Now he throws a coin 1,000 times. A head will appear with a probability of $\frac{1}{2}$ i.e., it is expected to appear 500 times.

Similarly, a tail will appear with a probability of $\frac{1}{2}$ i.e., it is expected to appear 500 times.

Expected value in the long run = (500 × 10) – (1,000 × 2) = 3,000.

Gain per throw = ₹ 3

Formula for Expected Value (EV)

$$\text{EV} = \sum_{K}\left[\text{Probability of event } (E_K)\right]\times\left[\text{Monetary value asociated with event } E_K\right]$$

Example 20:

Arav and Reyansh played a game of coin (biased coin) on which the head appears in 72% of the situation. If Arav is paid ₹10 per head by Reyansh and Reyansh is paid ₹15 per tail by Arav, then how much profit per game would Arav earn in the long run?

(A) ₹3 (B) ₹2.251

(C) ₹3.75 (D) ₹3.5

Solution: (A)

Probability of head = $\frac{72}{100} = 0.72$

Probability of tail = 1 – 0.72 = 0.28

Expected profit per game = 0.72 × 10 – 0.28 × 15 = 7.2 – 4.2 = ₹3.

Example 21:

In a Royal casino, there is a game of throwing of a fair die. A casino operator named Radhe promises to pay the customers twice the value of the number showing up on the die if the number appeared is odd and four times the value of the number showing up on the die if the number appeared is even. A customer named Roy enters the casino and is willing to play the game of throwing of a fair die. What is the maximum amount that Roy is willing to pay each time to throw the die if, in the long run, he wants to make an average profit of ₹8.5 per throw of a fair die?

(A) ₹2.5 (B) ₹3.5

(C) ₹3 (D) ₹2.75

Solution: (A)

Here, the probability for each of the turning up is $\frac{1}{6}$ i.e., P (1) = P (2) = P (3) = P (4) = P (5) $= P(6) = \frac{1}{6}$

Amount paid for different numbers are

When 1 occurs = 2 × 1 = ₹2

When 2 occurs = 4 × 2 = ₹8

When 3 occurs = 2 × 3 = ₹6

When 4 occurs = 4 × 4 = ₹16

When 5 occurs = 2 × 5 = ₹10

When 6 occurs = 4 × 6 = ₹24

Expected value = $\frac{1}{6}\left[2+8+6+16+10+24\right]$

$= \frac{1}{6}\times 66$ = ₹11

Maximum amount he is willing to pay

= 11 – 8.5 = ₹2.5.

Practice Exercise – 1

Level of Difficulty – 1

1. Three cards are drawn, one after the other, without replacement from a pack of 52 cards. What is the probability that one is a Jack, one is a King, and one is an Ace?

(A) $\frac{16}{5,525}$ (B) $\frac{24}{5,525}$

(C) $\frac{12}{5,525}$ (D) $\frac{8}{5,525}$

2. The probability that a person will get a road contract is $\frac{6}{11}$ and the probability that he will not get a plumbing contract is $\frac{9}{13}$. If the probability of getting at least one contract is $\frac{4}{5}$, what is the probability that he will get both?

(A) $\frac{38}{715}$ (B) $\frac{39}{715}$

(C) $\frac{41}{715}$ (D) $\frac{37}{715}$

3. Four cards are drawn randomly from a pack of 52 cards. Find the probability that all of them belong to the same suit.

(A) $\frac{11}{5,225}$ (B) $\frac{11}{4,165}$

(C) $\frac{11}{52}$ (D) $\frac{44}{4,165}$

4. The odds against an event are 4:5 and odds in favour of another event are 3:7. Find the probability that at least one of them occurs.

(A) $\frac{11}{45}$ (B) $\frac{31}{45}$

(C) $\frac{28}{45}$ (D) $\frac{15}{31}$

5. There are eight patients, and it is known that exactly three of them are suffering from COVID-19. They are tested one by one in random order, till all the COVID-19 patients are identified. Find the probability that only three tests are required.

(A) $\frac{1}{3}$ (B) $\frac{1}{45}$

(C) $\frac{1}{56}$ (D) $\frac{2}{53}$

Level of Difficulty – 2

6. A person is known to speak the truth 5 out of 9 times. He throws a die and reports that it is a two. Find the probability that it is actually a two.

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{5}{9}$ (D) $\frac{6}{7}$

7. A person throws a die. If the number turned up on the die is even, then he receives four times as many rupees as the number that turns up on the die. If it is odd, then he receives five times as many rupees as the number that turns up on the die. What will be his expected value per throw in the long run?

(A) ₹8.5

(B) ₹12.5

(C) ₹15.5

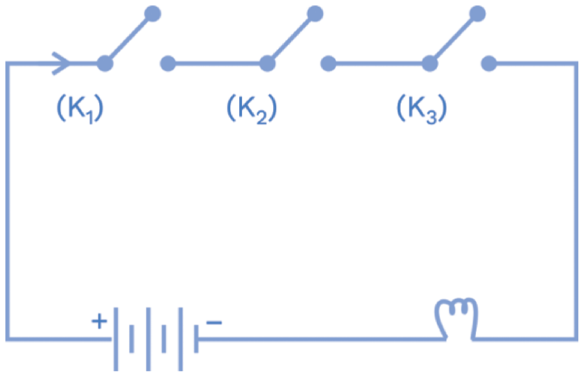
(D) ₹17.5

8. Two friends, Adam and Brat, throw a die alternately till one of them gets a 'one' and wins the game. Find the probability of winning of Adam, if it is given that Adam begins.

(A) $\frac{5}{11}$ (B) $\frac{6}{11}$

(C) $\frac{4}{11}$ (D) $\frac{7}{11}$

9. If three switches (K_1), (K_2), and (K_3) have, respectively, 60%, 45%, and 80% chances of working. Find the probability that the circuit will work.



(A) $\frac{98}{125}$ (B) $\frac{27}{125}$

(C) $\frac{64}{125}$ (D) $\frac{61}{125}$

10. A combination lock on a suitcase has four wheels, each labelled with nine digits from 1 to 9. To open the lock a particular sequence of four digits is required, with no repeats. What is the probability of a person guessing the right combination?

(A) $\frac{24}{3,024}$ (B) $\frac{1}{3,024}$

(C) $\frac{1}{6,561}$ (D) $\frac{24}{6,561}$

Level of Difficulty – 3

11. India's three popular critics review a book. Odds in favour of a positive review of the book are 4:3, 3:2, and 6:5, respectively, for three critics. Find the probability that the majority gives a positive review for the book.

(A) $\frac{223}{385}$ (B) $\frac{162}{385}$

(C) $\frac{235}{385}$ (D) $\frac{234}{385}$

12. If four dice are thrown simultaneously, what is the probability that the sum of the numbers is exactly 19?

(A) $\frac{7}{162}$ (B) $\frac{9}{162}$

(C) $\frac{11}{162}$ (D) $\frac{13}{162}$

13. There are two bags. The first bag contains 6 white and 4 black balls and the second bag contains 2 white and 3 black balls. Two balls are drawn randomly from the first bag and are put into the second bag without noticing their colour, then two balls are drawn from the second bag. Find the probability that the balls are white and black.

(A) $\frac{191}{315}$ (B) $\frac{165}{191}$

(C) $\frac{73}{191}$ (D) $\frac{176}{315}$

14. One day my friend Poplu bought a pack of 52 cards. But unfortunately, a card from a pack of 52 cards is lost from the remaining cards of the pack. Two cards are drawn and are found to be spades. Find the probability of the missing card to be a spade.

(A) $\frac{11}{52}$ (B) $\frac{10}{51}$

(C) $\frac{12}{51}$ (D) $\frac{11}{50}$

15. Of all people having COVID-19, 80% of the test detect the disease but 20% go undetected. Of people free of COVID-19, 99% of the test are judged COVID-ve but 1% are diagnosed as showing COVID+ve. From a large population of which only 0.1% have COVID, one person is selected at random for the COVID-19 test, and the pathologist reports him/her as COVID+ve. What is the probability that the person actually has COVID-19?

(A) $\frac{70}{1,079}$ (B) $\frac{93}{1,079}$

(C) $\frac{80}{1,079}$ (D) $\frac{102}{1,079}$

Solutions

1. (A)

Probability for first Jack = $\frac{4}{52}$

Probability for second King = $\frac{4}{51}$

Probability for third Ace = $\frac{4}{50}$

Since Jack, King, and Ace can be in any order, this gives us 3! cases.

Required probability

$$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} \times 3! = \frac{16}{5,525}$$

2. (A)

Let event A = (person gets a road contract)

Event B = (person gets a plumbing contract)

$P(A) = \frac{6}{11}$, $P(\bar{B}) = \frac{9}{13}$

$P(B) = 1 - \frac{9}{13} = \frac{4}{13}$ and $P(A \cup B) = \frac{4}{5}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = \frac{6}{11} + \frac{4}{13} - \frac{4}{5} = \frac{390 + 220 - 572}{715}$

$P(A \cap B) = \frac{38}{715}$

3. (D)

Four cards can be drawn out of 52 cards in ${}^{52}C_4$ ways.

Total outcomes = ${}^{52}C_4$

Now we need all four cards to be from the same suit. Four cards can be drawn out of 13 cards in ${}^{13}C_4$ ways. But there are four suits for which this can be repeated.

Favourable outcomes = $4 \times {}^{13}C_4$

$$P(E) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{4 \times {}^{13}C_4}{{}^{52}C_4} = \frac{44}{4,165}$$

4. (B)

Let the events be A_1 and A_2

Odds against A_1 are 4 is to 5.

$\therefore P(A_1) = \frac{5}{9}$

Odds in favour of A_2 are 3 is to 7

$P(A_2) = \frac{3}{10}$

P(at least one) = 1 − P(none of them)

P(none of them)

$$= P(\bar{A}_1) \times P(\bar{A}_2) = \frac{4}{9} \times \frac{7}{10} = \frac{28}{90}$$

P(at least one) = $1 - \frac{28}{90} = \frac{62}{90} = \left(\frac{31}{45}\right)$

5. (C)

The total number of ways in which three patients can be chosen out of 8 patients is ${}^8C_3$ = 56 ways.

Only three tests are required to identify COVID-19 patients.

This can be done in only one way, when all the three COVID-19 patients, identified in the first three tests.

Required probability = $\frac{1}{56}$.

6. (A)

Let A_1 = [two occurs] and A_2 = [two does not occur]

E = [person reports that it is a two]

Now, $P(A_1) = \frac{1}{6}$ and $P(A_2) = \frac{5}{6}$

$P\left[\frac{E}{A_1}\right]$ = Probability that the person speaks truth = $\frac{5}{9}$.

$P\left[\frac{E}{A_2}\right]$ = Probability that the person does not speak truth = $\frac{4}{9}$.

Required probability

$$= \frac{P(A_1) \times P\left(\frac{E}{A_1}\right)}{P(A_1) \times P\left(\frac{E}{A_1}\right) + P(A_2) \times P\left[\frac{E}{A_2}\right]}$$

$$= \frac{\frac{1}{6} \times \frac{5}{9}}{\frac{1}{6} \times \frac{5}{9} + \frac{5}{6} \times \frac{4}{9}}$$

$$= \frac{\frac{5}{54}}{\frac{5}{54}+\frac{20}{54}} \Rightarrow \frac{\frac{5}{54}}{\frac{25}{54}}$$

$\therefore$ Required probability $= \frac{1}{5}$

7. **(C)**

Here, the probability for each of the turning up is $\frac{1}{6}$, i.e., $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$.

Amount paid for different numbers are

When 1 occurs = 5 × 1 = ₹5

When 2 occurs = 4 × 2 = ₹8

When 3 occurs = 5 × 3 = ₹15

When 4 occurs = 4 × 4 = ₹16

When 5 occurs = 5 × 5 = ₹25

When 6 occurs = 4 × 6 = ₹24

Expected value per throw in the long run

$$= \frac{1}{6}[5+8+15+16+25+24]$$

$$= \frac{1}{6} \times 93 = ₹15.5.$$

8. **(B)**

Let event [A] = Adam gets a 'one'.

Event [R] = Brat gets a 'one'.

Now, $P(A) = \frac{1}{6}$, $P(R) = \frac{1}{6}$, $P(\bar{A}) = \frac{5}{6}$, and $P(\bar{R}) = \frac{5}{6}$

Adam wins if he throws a 'one' in first or third or fifth ... throws.

Case 1: Probability of Adam winning in first throw $P(A) = \frac{1}{6}$

Case 2: Probability of Adam winning in the third throw

$= P(\bar{A}) \times P(\bar{R}) \times P(A)$

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{6}\right)^2 \times \frac{1}{6}$$

Case 3: Probability of Adam winning in the fifth throw

$= P(\bar{A}) \times P(\bar{R}) \times P(\bar{A}) \times P(\bar{R}) \times P(A)$

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$= \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$$

Hence, the probability of Adam winning

$$\frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \cdots$$

$$\Rightarrow \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{6}{11} \left[\because \quad a + \frac{a}{r} + \frac{a}{r^2} + \cdots = \frac{a}{1-r}\right]$$

Hence, the probability of Adam winning $= \frac{6}{11}$.

9. **(B)**

Let event A = K_1 works

Event B = K_2 works

Event C = K_3 works

$P(A) = \frac{60}{100}$, $P(B) = \frac{45}{100}$, and $P(C) = \frac{80}{100}$

We can clearly see that the current flows through the circuit if switches K_1, K_2, and K_3 work together.

Required probability = $P(A) \times P(B) \times P(C)$

$$\Rightarrow \frac{60}{100} \times \frac{45}{100} \times \frac{80}{100} \Rightarrow \frac{27}{125}.$$

10. **(B)**

Suppose,

A_1 Event = First wheel occupies the correct position.

A_2 Event = Second wheel occupies the correct position.

A_3 Event = Third wheel occupies the correct position.

A_4 Event = Fourth wheel occupies the correct position.

Required probability = $P(A_1 \cap A_2 \cap A_3 \cap A_4)$

$$= P(A_1) \times P\left(\frac{A_2}{A_1}\right) \times P\left[\frac{A_3}{A_1 \cap A_2}\right] \times P\left[\frac{A_4}{A_1 \cap A_2 \cap A_3}\right]$$

$$= \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7} \times \frac{1}{6} = \left[\frac{1}{3{,}024}\right]$$

11. (D)

Suppose, event A = Positive review of book by the first critic

Event B = Positive review of book by the second critic

Event C = Positive review of book by the third critic

$P(\text{A}) = \frac{4}{7}$, $P(\text{B}) = \frac{3}{5}$, and $P(\text{C}) = \frac{6}{11}$

To find the probability that the majority are in favour of the book. This happens only if at least two review it favourably.

Case 1: First gave a positive review, second gave a positive review, and third gave a negative review.

Required probability = $P(\text{A}) \times P(\text{B}) \times P(\bar{\text{C}})$

$= \frac{4}{7} \times \frac{3}{5} \times \frac{5}{11} = \frac{60}{385}$

Case 2: First gave a positive review, second gave a negative review, and third gave a positive review.

Required probability = $P(\text{A}) \times P(\bar{\text{B}}) \times P(\text{C})$

$= \frac{4}{7} \times \frac{2}{5} \times \frac{6}{11} \Rightarrow \frac{48}{385}$

Case 3: First gave a negative review, second gave a positive review, and third gave a positive review.

Required probability = $P(\bar{\text{A}}) \times P(\text{B}) \times P(\text{C})$

$= \frac{3}{7} \times \frac{3}{5} \times \frac{6}{11} \Rightarrow \frac{54}{385}$

Case 4: All favours

Required probability = $P(\text{A}) \times P(\text{B}) \times P(\text{C})$

$= \frac{4}{7} \times \frac{3}{5} \times \frac{6}{11} = \frac{72}{385}$

Total probability

$= \frac{60}{385} + \frac{48}{385} + \frac{54}{385} + \frac{72}{385} = \frac{234}{385}$

12. (A)

Total outcomes = $6^n = 6^4 = 1{,}296$

Required combination for the sum being 19 are = (6, 6, 6, 1) (6, 6, 5, 2) (6, 6, 4, 3) (6, 5, 5, 3) (6, 5, 4, 4) (5, 5, 5, 4)

Required number of arrangements for each case =

$(6, 6, 6, 1) = \frac{4!}{3!} = 4$

$(6, 6, 5, 2) = \frac{4!}{2!} = 12$

$(6, 6, 4, 3) = \frac{4!}{2!} = 12$

$(6, 5, 5, 3) = \frac{4!}{2!} = 12$

$(6, 5, 4, 4) = \frac{4!}{2!} = 12$

$(5, 5, 5, 4) = \frac{4!}{3!} = 4$

Favourable outcomes = 56

Required probability = $\frac{56}{1{,}296} = \frac{7}{162}$

13. (D)

A white and a black ball can be drawn from the second bag in three ways.

1. When two black balls are drawn from first bag to the second bag and then drawing a white and a black ball from the second bag.
2. When one black and one white ball are drawn from first bag to second bag and then drawing a white and a black ball from the second bag.
3. When two white balls are drawn from first bag to second bag and then drawing a white and a black ball from the second bag.

Suppose, event A = two black balls are drawn from first bag.

Event B = one black and one white ball drawn from first bag

Event C = two white balls are drawn from first bag.

E = a white and a black ball are drawn from second bag.

$$P(\text{A}) = \frac{{}^{4}C_2}{{}^{10}C_2} = \frac{6}{45} = \frac{2}{15}$$

$$P(\text{B}) = \frac{{}^{4}C_1 \times {}^{6}C_1}{{}^{10}C_2} = \frac{4 \times 6}{45} = \frac{8}{15}$$

$$P(\text{C}) = \frac{{}^{6}C_2}{{}^{10}C_2} = \frac{15}{45} = \frac{1}{3}$$

$$P\left[\frac{\text{E}}{\text{A}}\right] = \frac{{}^{5}C_1 \times {}^{2}C_1}{{}^{7}C_2} = \frac{5 \times 2}{21} = \frac{10}{21}$$

$$P\left[\frac{E}{B}\right]=\frac{{}^4C_1\times{}^3C_1}{{}^7C_2}=\frac{4\times3}{21}=\frac{4}{7}$$

$$P\left(\frac{E}{C}\right)=\frac{{}^3C_1\times{}^4C_1}{{}^7C_2}=\frac{3\times4}{21}=\frac{4}{7}$$

Total probability

$$\Rightarrow P(A)\times P\left(\frac{E}{A}\right)+P(B)\times P\left(\frac{E}{B}\right)+P(C)\times P\left(\frac{E}{C}\right)$$

$$=\frac{2}{15}\times\frac{10}{21}+\frac{8}{15}\times\frac{4}{7}+\frac{1}{3}\times\frac{4}{7}=\frac{4}{63}+\frac{32}{105}+\frac{4}{21}$$

$$=\frac{176}{315}$$

14. (D)

Suppose event A = Missing card is a heart card.

Event B = Missing card is a spade card.

Event C = Missing card is a club card.

Event D = Missing card is a diamond card.

and E = Drawing two spade cards from the remaining cards.

$P(A) = P(B) = P(C) = P(D) = \frac{13}{52}=\frac{1}{4}$

$P\left[\frac{E}{A}\right]$ = Probability of drawing two spade cards given that one heart card is missing.

$$=\frac{{}^{13}C_2}{{}^{51}C_2}.$$

$P\left(\frac{E}{B}\right)$ = Probability of drawing two spade cards given that one spade card is missing

$$=\frac{{}^{12}C_2}{{}^{51}C_2}.$$

Similarly, $P\left(\frac{E}{C}\right)=\frac{{}^{13}C_2}{{}^{51}C_2}$ and $P\left(\frac{E}{D}\right)=\frac{{}^{13}C_2}{{}^{51}C_2}$

By Bayes' theorem

Required probability = $P\left[\frac{B}{E}\right]$

$$=\frac{P(B)\times P\left[\frac{E}{B}\right]}{P(A)\times P\left[\frac{E}{A}\right]+P(B)\times P\left[\frac{E}{B}\right]+P(C)\times P\left[\frac{E}{C}\right]+P(D)\times P\left[\frac{E}{D}\right]}$$

$$=\frac{\frac{1}{4}\times\frac{{}^{12}C_2}{{}^{51}C_2}}{\frac{1}{4}\times\frac{{}^{13}C_2}{{}^{51}C_2}+\frac{1}{4}\times\frac{{}^{12}C_2}{{}^{51}C_2}+\frac{1}{4}\times\frac{{}^{13}C_2}{{}^{51}C_2}+\frac{1}{4}\times\frac{{}^{13}C_2}{{}^{51}C_2}}$$

$$=\frac{66}{78+66+78+78}=\frac{66}{300}=\frac{11}{50}$$

Hence, the required probability is $\frac{11}{50}$.

15. (C)

Let Event A = The person selected is actually having COVID.

Event B = The person selected is not having COVID.

Event E = The person's COVID test is diagnosed positive.

Given,

$P(A) = 0.1\% = \frac{0.1}{100}=0.001$

$P(B) = 1 - 0.001 = 0.999$

The probability that the person tested COVID+ve given that he/she is actually having COVID.

$$P\left[\frac{E}{A}\right]=\frac{80}{100}=0.8$$

The probability that the person tested COVID+ive given that he/she is not having COVID. $P\left[\frac{E}{B}\right]=\frac{1}{100}=0.01$

Required probability

$$=P\left(\frac{A}{E}\right)=\frac{P(A)\times P\left(\frac{E}{A}\right)}{P(A)\times P\left(\frac{E}{A}\right)+P(B)\times P\left[\frac{E}{B}\right]}$$

$$\Rightarrow\frac{P(A)\times P\left(\frac{E}{A}\right)}{P(A)\times P\left(\frac{E}{A}\right)+P(B)\times P\left[\frac{E}{B}\right]}=\frac{80}{1{,}079}$$

Practice Exercise – 2

Level of Difficulty – 1

1. A card is drawn from a pack of cards. What is the possibility that it is a black card or Jack?

   (A) $\frac{1}{2}$ (B) $\frac{6}{13}$

   (C) $\frac{7}{13}$ (D) $\frac{15}{26}$

2. Jack tossed a coin five times, one by one. What is the probability that he will get exactly two tails?

   (A) $\frac{3}{16}$ (B) $\frac{5}{16}$

   (C) $\frac{5}{32}$ (D) $\frac{15}{32}$

3. Let A = {1, 2, 3, 4, 5, 6, ..., 98, 99, 100}. Two numbers are selected at random from this set A. What is the possibility that both the numbers are prime numbers and their sum is odd?

   (A) $\frac{25_{C_2}}{100_{C_2}}$ (B) $\frac{24_{C_2}}{100_{C_2}}$

   (C) $\frac{2\times 24_{C_1}}{100_{C_2}}$ (D) None of these

4. Two cards are drawn from the deck of 52 cards one-by-one without replacement. What is the probability that these two cards belong to different suits?

   (A) $\frac{13}{204}$ (B) $\frac{13}{102}$

   (C) $\frac{13}{34}$ (D) $\frac{13}{17}$

5. A coin is tossed 11 times. Find the probability of obtaining at least one tail.

   (A) $1/(2^{11})$

   (B) $2^{10}/2^{11}$

   (C) $(2^{11} - 1)/2^{11}$

   (D) $(2^{10} - 1)/2^{11}$

6. Sonu has two bags. The first bag consists of four white and three black balls. The second bag consists of two white and three black balls. One bag is selected at random and from the selected bag, one ball is chosen at random. Find the probability that the chosen ball is white.

   (A) 13/17

   (B) 4/7

   (C) 5/9

   (D) 17/35

7. There were 8 coins in my pocket, each of a different denomination. If one takes them out one after the other at random, what is the probability that the coins will not be in the ascending order of the denominations?

   (A) $\frac{(8!-1)}{8!}$ (B) $\frac{4\times 7!}{8!}$

   (C) $\frac{7!}{8!}$ (D) $\frac{1}{8!}$

8. A bag contains red, blue, and green colour balls in the ratio 3:6:5, respectively. If two balls are randomly drawn from the bag, the probability that one ball is blue and the other is green is 24/77. Find the total number of balls in the bag:

   (A) 56

   (B) 84

   (C) 98

   (D) 42

9. From a pack of cards, if three cards are drawn in succession without replacement, what is the probability that the first one is a queen, the second is a King, and the third is a number card?

   (A) 1/5,525

   (B) 24/5,525

   (C) 36/5,525

   (D) 9/2,197

10. Odds against for hitting a target by A, B, and C are $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{16}$, respectively.

What is the probability that the target will be hit?

(A) $\frac{3}{8}$ (B) $\frac{7}{8}$

(C) $\frac{13}{28}$ (D) $\frac{27}{28}$

Level of Difficulty – 2

11. In a normal chessboard, each unit square measures 1 cm × 1 cm. If a square is selected at random from this chessboard, what is the probability that it has its area equal to 36 square cm?

(A) 3/68
(B) 9/16
(C) 25/204
(D) 5/18

12. Four dice are thrown simultaneously. The sum of the faces is 19. If the first die has 4 on its face, what is the probability that the second die also has 4?

(A) 1/4
(B) 1/3
(C) 1/5
(D) 2/3

13. Aman is given a set of natural numbers A and B such that set A contains the first four natural numbers and the set B contains the set of the next four natural numbers from set A. A number is chosen at random from set A and another number is chosen from set B. Find the probability that the product of selected numbers is even.

(A) 1/2
(B) 3/4
(C) 1/3
(D) 2/3

14. Three bags A, B, and C contain two types of marbles, black and white, such that bag A contains three white and four black marbles, bag B contains three white and two black marbles and bag C contains four black and four white marbles. One bag is selected at random and then one marble is chosen from it. What is the probability that the marble is neither a black from bag A nor a white from bag C?

(A) 13/14
(B) 11/14
(C) 5/14
(D) None of these

15. There are five artists, eight dancers, some players, and some singers in a locality. If the probability of selecting one player from the locality is 1/5 and the probability of selecting one artist from the locality is 1/4. Find the probability of selecting two artists and two singers from the locality.

(A) 1/323
(B) 11/323
(C) 5/323
(D) 2/323

16. Aisha has two bags A and B with gems. Bag A has three rubies and four emeralds, and bag B has four rubies and five emeralds. She randomly takes out one gem from bag A and places into bag B. She again takes out one gem at random from bag B and places it in bag A. What is the probability that the ratio of rubies to emeralds in both the bags remains unaltered?

(A) $\frac{39}{70}$ (B) $\frac{35}{70}$

(C) $\frac{24}{70}$ (D) $\frac{16}{70}$

17. Two fighter jets A and B are sent to hit a target in succession. The fighter jet B is used to fire the target only if the fighter jet A is not able to hit the target. Jet A is sent out again if Jet B fails to hit the target. If the probability of fighter jet A and fighter jet B hitting the target is 0.6 and 0.7, respectively, then the probability that fighter jet B will hit the target is:

(A) 5/22
(B) 7/22
(C) 9/22
(D) 11/22

18. An unbiased coin is tossed 12 times. What is the probability of getting at most 7 heads?

(A) $\frac{1,707}{2,048}$ (B) $\frac{1}{2}$

(C) $\frac{3,314}{2,048}$ (D) $\frac{1,651}{2,048}$

19. Two dice are tossed once. The probability of getting an even number at the first die or a total of 9 is ____:

(A) 5/12

(B) 8/9

(C) 7/18

(D) 5/9

20. If *P* and *Q* are natural numbers not more than 8, what is the probability that ($P \times Q$) is a perfect square?

(A) 3/32

(B) 5/32

(C) 7/32

(D) 9/32

Level of Difficulty – 3

21. Andrew, a pro gambler, has a biased die in which each even number turns up four times as frequently as any of the odd numbers. If an even number occurs, you are given ₹3, and if an odd number occurs, you are given ₹8. Elon, Andrew's friend, knows that Andrew has a biased die. Elon comes to Andrew's home to play this game. How much would Elon be willing to pay per throw if he does not wish to make any gain or loss?

(A) ₹3

(B) ₹4

(C) ₹4.5

(D) ₹5

22. A biased die has a probability of $\frac{1}{3}$ of showing a 4 and the probability of any of 1, 2, 3, 5, or 6 turning up is the same. If three such dice are rolled, what is the probability of getting a sum of at least 15 without getting a 5 on any dice?

(A) $\frac{8}{1,125}$ (B) $\frac{84}{3,375}$

(C) $\frac{92}{3,375}$ (D) $\frac{28}{1,125}$

23. If one number is selected at random from the list of the first 1,000 natural numbers, what is the probability that it is divisible by 5 or 7 but not 9?

(A) 391/1,000

(B) 157/500

(C) 283/1,000

(D) 7/25

24. If S is defined as a set of the first 30 natural numbers, S = {1, 2, 3, ... 30}. What is the probability of choosing three numbers from the set S such that they are in AP (arithmetic progression)?

(A) $\frac{1}{19}$ (B) $\frac{2}{3}$

(C) $\frac{5}{29}$ (D) $\frac{3}{58}$

25. If four dies are rolled together, find the probability of getting the sum equal to 10.

(A) $\frac{35}{648}$ (B) $\frac{37}{648}$

(C) $\frac{17}{324}$ (D) $\frac{5}{81}$

26. S is a set of the first 20 multiplies of 3. What is the probability of choosing three numbers of set 'S' such that they are in GP (geometric progression)?

(A) $\frac{7}{570}$ (B) $\frac{2}{285}$

(C) $\frac{11}{1,140}$ (D) $\frac{1}{95}$

(E) $\frac{1}{76}$

27. Nine persons are sitting in a room. What is the probability that exactly three of them have birthdays in the same month of the year and no other two persons have birthdays in the same month of the year?

(A) $\frac{16,170}{\left(12^5\right)}$ (B) $\frac{16,170}{\left(12^6\right)}$

(C) $\frac{16,170}{\left(12^8\right)}$ (D) $\frac{16,170}{\left(12^9\right)}$

28. In a test paper, the probability of Rohan scoring a 100/100 is 1/4. In a university examination of 6 test papers, what is the probability of Rohan scoring a 100/100 in at least 2 papers?

(A) $\frac{1,903}{4,096}$ (B) $\frac{1,905}{4,096}$

(C) $\frac{1,907}{4,096}$ (D) $\frac{1,909}{4,096}$

29. Raman uses the following procedure to write down a sequence of numbers. First, he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If the head comes up, he doubles the previous term and subtracts 1. If the tail comes up, he first divides the previous term by 2 and then subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer?

(A) 1/6
(B) 3/4
(C) 1/3
(D) 5/8

30. Four cards are drawn from the deck of playing cards. What is the probability of getting exactly two cards of the same suit?

(A) $\frac{169}{28,025}$ (B) $\frac{4,056}{28,025}$

(C) $\frac{12,168}{20,825}$ (D) $\frac{507}{20,825}$

1. **(C)**

P_R(black or Jack) = P_R(black) + P_R(Jack) – P_R(black and Jack)

P_R(black or Jack)

$$= \frac{^{26}C_1}{^{52}C_1} + \frac{^{4}C_1}{^{52}C_1} - \frac{^{2}C_1}{^{52}C_1}$$

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

2. **(B)**

A coin will either turn up head or tail. Since the coin is tossed five times, therefore:

Total possible outcomes = 2^5 = 32

We need two tails. So, the rest 3 will be heads.

Total favourable outcomes

= distinct arrangement for TTHHH

$$= \frac{5!}{3! \times 2!} = \frac{5 \times 4 \times 3!}{3! \times 2} = 10$$

Therefore, the required probability

$$= \frac{\text{Favourable outcome}}{\text{Total possible outcome}} = \frac{10}{32} = \frac{5}{16}$$

3. **(D)**

$$\text{Possibility} = \frac{\text{Formulae outcomes}}{\text{Total possible outcomes}}$$

Total possible outcomes = $^{100}C_2$

Now, there are 25 prime numbers sum 1–100 and all of these 25 prime numbers are odd except 2.

Thus, the sum of two numbers selected is odd only when and of them is 2 and the other is one among 24 remaining prime numbers.

Favourable outcomes $= 1 \times {}^{24}C_1 = 24$

Required possibility $= \frac{^{24}C_1}{^{100}C_2}$

Hence, option (D) is the correct answer.

4. **(D)**

We know that there are four suits of 13 cards each.

First, we have to select one suit in $^4C_1$ ways

Then we have to select one card from this suit, which can be done in = $^{13}C_1$ ways.

Now, we have to select second suit from the remaining three suits in = $^3C_1$ ways.

So, we can select second card from this suit in = $^{13}C_1$ ways.

Also, the total number of ways of selecting two cards one by one = $^{52}C_1 \times {}^{51}C_1$.

Therefore, required probability

$$= \frac{^4C_1 \times {}^{13}C_1 \times {}^3C_1 \times {}^{13}C_1}{^{52}C_1 \times {}^{51}C_1} = \frac{4 \times 13 \times 3 \times 13}{52 \times 51} = \frac{13}{17}$$

5. **(C)**

Total possible outcomes = 2^{11}

Probability of at least one tail = 1 – [probability of no tail (all heads)]

Probability of all heads = $1/(2^{11})$

Required probability = $1 - \{1/(2^{11})\}$

= $(2^{11} - 1)/2^{11}$

Hence, option (C) is the correct answer.

6. **(D)**

The first bag contains four white and three black balls.

The second bag contains two white and three black balls.

E_1 = First bag is selected

$\Rightarrow P(E_1) = 1/2$

E_2 = second bag is selected

$\Rightarrow P(E_2) = 1/2$

Now,

$P(A/E_1)$ = Probability of picking a white ball when the first bag is selected = 4/7

and

$P(A/E_2)$ = Probability of picking a white ball when the second bag is selected = 2/5.

P(ball picked is white) = $P(E_1)$. $P(A/E_1)$ + $P(E_2)$. $P(A/E_2)$

= $(1/2 \times 4/7) + (1/2 \times 2/5) = 17/35$

Hence, the probability that the ball picked is white is 17/35.

7. **(A)**

Total possible arrangements by which I can take coins from my pockets = 8!

There will be only one case when all the coins will be in ascending order

So, there will be $(8! - 1)$ cases where the coins will not be in ascending order.

Required probability $= \dfrac{(8! - 1)}{8!}$

8. (A)

Let the number of balls of red, blue, and green colour be 3K, 6K, 5K respectively. So that total number of balls in the bag $= 3K + 6K + 5K = 14K$.

Required probability $= (6K \times 5K)/({}^{14K}C_2)$

$= \left(\dfrac{24}{77}\right)$

$\therefore \quad \dfrac{30K^2}{\{7K(14K-1)\}} = \dfrac{24}{77}$

$\Rightarrow \quad 55K = 56K - 4$

$\Rightarrow \quad K = 4$

Total number of balls in the bag $= 14K = 56$

Alternate Solution

The denominator has to be a multiple of 11. So ${}^nC_2$ has to be a multiple of 11, which is only possible when $n = 56$.

9. (B)

P(first Queen and second King and third a number card) =

P(first Queen) × P(King given that the first card was Queen) × P(a number card given that the first two cards were Queen and King, respectively)

$= \left(\dfrac{4}{52}\right) \times \left(\dfrac{4}{51}\right) \times \left(\dfrac{36}{50}\right) = \dfrac{24}{5{,}525}$.

10. (D)

In this case, if at least one among A, B, and C hit the target, then we can say that the target is hit.

Odds against A hitting the target $= \dfrac{2}{3}$

Probability of target not hit by A $= \dfrac{2}{5}$

Odds against B hitting the target $= \dfrac{3}{5}$

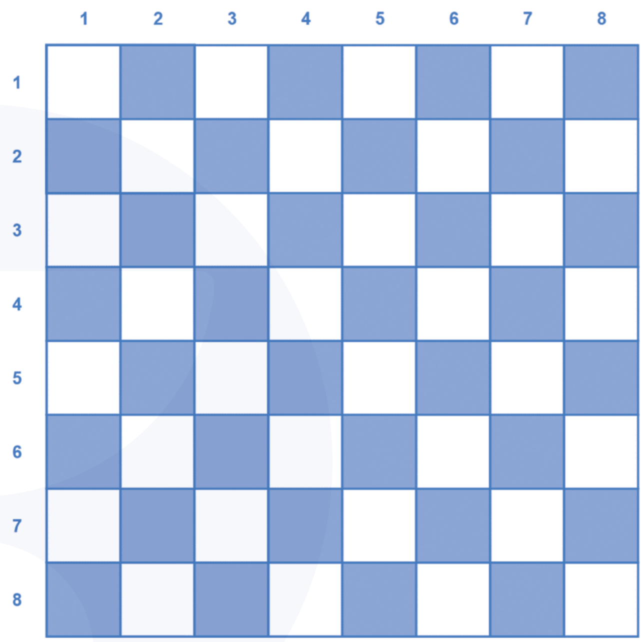
Probability of target not hit by B $= \dfrac{3}{8}$

Similarly, probability of target not hit by C $= \dfrac{5}{21}$.

Now, P(target will be hit) $= 1 - P$(no one hit the target)

$= 1 - \left(\dfrac{2}{5} \times \dfrac{3}{8} \times \dfrac{5}{21}\right) = 1 - \dfrac{1}{28} = \dfrac{27}{28}$

11. (A)



Total number of squares possible in 8×8 chessboard can be found as below

Number of 1×1 squares $= 8 \times 8 = 64$

Number of 2×2 squares $= 7 \times 7 = 49$

Number of 3×3 squares $= 6 \times 6 = 36$

Number of 4×4 squares $= 5 \times 5 = 25$

Number of 5×5 squares $= 4 \times 4 = 16$

Number of 6×6 squares $= 3 \times 3 = 9$

Number of 7×7 squares $= 2 \times 2 = 4$

Number of 8×8 squares $= 1 \times 1 = 1$

Total number of squares $= 8^2 + 7^2 + 6^2 + \cdots + 2^2 + 1^2 = 204$

Required square should have an area of 36, i.e., its side should be equal to 6 cm.

Number of squares whose side is equal to 6 cm = 9.

Thus, required probability = 9/204 = 3/68

12. (C)

If the first die has 4, then the sum of the remaining dice should be 15.

Case 1: 6, 6, 3

The number of possibilities = 3!/2! = 3

Case 2: 6, 5, 4

The number of possibilities = 3! = 6

Case 3: 5, 5, 5

The number of possibilities = 1

There are a total of 3 + 6 + 1 = 10 possibilities.

There are two possibilities where 4 can occur on the second die, i.e., (4, 5, 6), (4, 6, 5).

Required probability = 2/10 = 1/5

13. (B)

Aman is given sets A and B such that,

A = {1, 2, 3, 4}

B = {5, 6, 7, 8}

Let a and b be the numbers chosen at random from A and B.

$a \times b$ will be even when

(i) both a and b are even

$\Rightarrow$ This can be done in ${}^2C_1 \times {}^2C_1 = 2 \times 2 = 4$

(ii) a is even and b is odd

$\Rightarrow$ This can be done in ${}^2C_1 \times {}^2C_1 = 2 \times 2 = 4$

(iii) a is odd and b is even

$\Rightarrow$ This can be done in ${}^2C_1 \times {}^2C_1 = 2 \times 2 = 4$

Total number of ways in which $a \times b$ is even = 4 + 4 + 4 = 12

Total number of possible ways in which a and b can be selected from A and B,

${}^4C_1 \times {}^4C_1 = 4 \times 4 = 16$

Required probability = 12/16 = 3/4

14. (D)

Total marbles = 3 + 4 + 3 + 2 + 4 + 4 = 20

Probability of selecting a bag = 1/3.

Probability of selecting either a black marble from bag A or a white marble from bag C = (1/3) × [(4/7) + (4/8)] = 5/14

Therefore, the probability of selecting neither a black marble from bag A nor a white marble from bag C = 1 – 5/14 = 9/14

15. (D)

Let's assume the number of players and number of singers be m and n, respectively.

According to the question, probability of selecting one player from the locality is 1/5

$$= \frac{m}{(13+m+n)} = \frac{1}{5}$$

$\Rightarrow 5m = 13 + m + n$

$\Rightarrow 5m - m - n = 13$

$\Rightarrow 4m - n = 13$ (i)

The probability of selecting one artist from the locality is 1/4

$$\frac{5}{(13+m+n)} = \frac{1}{4}$$

$\Rightarrow 20 = 13 + m + n$

$\Rightarrow m + n = 20 - 13$

$\Rightarrow m + n = 7$ (ii)

Adding equations (i) and (ii)

$4m - n + m + n = 13 + 7$

$\Rightarrow 5m = 20$

$\Rightarrow m = 20/5$

$\Rightarrow m = 4$

From equation (ii)

$4 + n = 7$

$\Rightarrow n = 7 - 4 = 3$

Hence, number of players = 4 and number of singers = 3.

Total = 13 + 4 + 3 = 20

Probability of selecting 2 artists and 2 singers

$$= \frac{\left({}^5C_2 \times {}^3C_2\right)}{{}^{20}C_4} = \frac{(10 \times 3)}{4{,}845} = \frac{2}{323}$$

16. (A)

Bag A has three rubies and four emeralds and bag B has four rubies and five emeralds.

The final arrangement should remain the same for the ratio to not change.

Case 1:

1 ruby is transferred from bag A to bag B. 1 ruby is transferred from bag B to bag A.

Probability of picking ruby from bag A = 3/7.

Probability of picking ruby from bag B after one ruby has been added = 5/10.

For case 1, the probability of ratio remaining same = 3/7 × 5/10 = 3/14.

Case 2:

One emerald is transferred from bag A to bag B. One emerald is transferred from bag B to bag A.

Probability of picking emerald from bag A = 4/7.

Probability of picking emerald from bag B after one emerald has been added = 6/10.

For case 2, the probability of ratio remaining same $= \frac{4}{7} \times \frac{6}{10} = \frac{12}{35}$

Total probability = $\frac{3}{14} + \frac{12}{35} = \frac{39}{70}$

17. (B)

Probability that fighter jet A hits the target ⇒ $P(A) = 0.6$.

Probability that fighter jet A doesn't hit the target ⇒ $P(A') = 1 - 0.6 = 0.4$.

Probability that fighter jet B hits the target $P(B) = 0.7$.

Probability that fighter jet B doesn't hit the target $P(B') = 1 - 0.7 = 0.3$

Now, the required probability

$= P(A')\, P(B) + P(A')\, P(B')\, P(A')\, P(B) + P(A')\, P(B')\, P(A')\, P(B')\, P(A')\, P(B) + \cdots$

$= 0.4 \times 0.7 + 0.4 \times 0.3 \times 0.4 \times 0.7 + 0.4 \times 0.3 \times 0.4 \times 0.3 \times 0.4 \times 0.7 + \cdots$

Required probability $= 0.28 \times [1 + 0.12 + (0.12)^2 + \cdots]$

Required probability $= 0.28 \times \frac{1}{1-0.12}$

$= \frac{28}{88} = \frac{7}{22}$

Hence, option (B) is the correct answer.

18. (D)

Favourable outcomes

At most 7 heads means if may have 0, 1, 2, 3, ... 7 heads out of the 12 events.

Cases of at most 7 heads = 0 head + 1 head + 2 head + 3 head + ... + 7 head

$= {}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \cdots + {}^{12}C_7$

$= ({}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \cdots + {}^{12}C_{12}) - ({}^{12}C_8 + {}^{12}C_9 + {}^{12}C_{10} + {}^{12}C_{11} + {}^{12}C_{12})$

$= ({}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \cdots + {}^{12}C_{12}) - ({}^{12}C_4 + {}^{12}C_3 + {}^{12}C_2 + {}^{12}C_1 + {}^{12}C_0)$

$= 2^{12} - \left[\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} + \frac{12 \times 11 \times 10}{3 \times 2 \times 1} + \frac{12 \times 11}{2 \times 1} + 12 + 1\right]$

= 4,096 − [495 + 220 + 66 + 13] = 4,096 − 794 = 3,302

Total possible cases = 2^{12} = 4,096

Therefore, required probability

$= \frac{\text{Number of favourable outcomes}}{\text{Total possible outcomes}}$

$= \frac{3{,}302}{4{,}096} = \frac{1{,}651}{2{,}048}$

19. (D)

Total events, E: {(1, 1), (1, 2), ..., (6, 6)} = 36

Let event A= even number at the first die = {(2, 1), (2, 2), (2, 3), ..., (6, 5), (6, 6)} = 18

⇒ P (A) = 18/36 = 1/2

Let event B = sum is 9 = {(3, 6), (4, 5), (5, 4), (6, 3)} = 4

⇒ P (B) = 4/36 = 1/9

Let event C = getting an even number at the first die and a total of 9 = {(4, 5), (6, 3)}

⇒ P(C) = 2/36 = 1/18

The probability of getting an even number at the first die or a total of 9 = 1/2 + 1/9 − 1/18 = 5/9

Hence, the probability of getting an even number at the first die or a total of 9 is 5/9.

20. (C)

As both P and Q can take any value from 1 to 8, so total number of cases will be 8 × 8 = 64.

Favourable cases will be

$(P \times Q) = 1 = (1 \times 1)$

$(P \times Q) = 4 = (1 \times 4)$, (2×2), and (4×1)

$(P \times Q) = 9 = (1 \times 9)$, (3×3), and (9×1)

$(P \times Q) = 16 = (2 \times 8)$, (4×4), and (8×2)

$(P \times Q) = 25 = (5 \times 5)$

$(P \times Q) = 36 = (6 \times 6)$

$(P \times Q) = 49 = (7 \times 7)$

$(P \times Q) = 64 = (8 \times 8)$

Total favourable cases = 14

Required probability = 14/64 = 7/32

21. (D)

Here, the relationship between the probability of getting an even and an odd number is given.

Let the probability of getting an odd number is x, then the probability of getting an even number is $4x$.
Since all the six events we are considering are mutually exclusive and exhaustive events. So the sum of the probabilities should be equal to 1.

$x + 4x + x + 4x + x + 4x = 1$

$15x = 1$

$x = \frac{1}{15}$

$\therefore$ Probability of getting an odd number is $\left(\frac{1}{15}\right)$ and probability of getting even number is $\left(\frac{4}{15}\right)$.

As we know that to throw the dice without any profit or loss one must be willing to pay an amount equal to the expected value.

Expected value

$= \frac{1}{15}[8+8+8] + \frac{4}{15}[3+3+3]$

$= \frac{24}{15} + \frac{36}{15} = \frac{60}{15} = ₹4$

Alternate solution

Odd probability is 1/5
Even probability is 4/5
Expected outcome is (4/5) × 3 + (1/5) × 8 = ₹4

22. (C)

$P(4) = \frac{1}{3}$

$P(1) = P(2) = P(3) = P(5) = P(6) = x$

Now, $\frac{1}{3} + 5x = 1$ or $5x = 1 - \frac{1}{3} = \frac{2}{3}$ or $x = \frac{2}{15}$

Now, the probability for the sum to be at least 15 means, the sum could be 15 or 16 or 17 or 18. But the sum cannot be 17 as it will involve 5 on one dice which is not possible.

So, the valid sum would be 15, 16, and 17
The cases for sum 15, 16, and 17 are shown below along with their probabilities:

$P(6,6,3) \rightarrow \frac{2}{15} \times \frac{2}{15} \times \frac{2}{15} \times \frac{3!}{2!}$

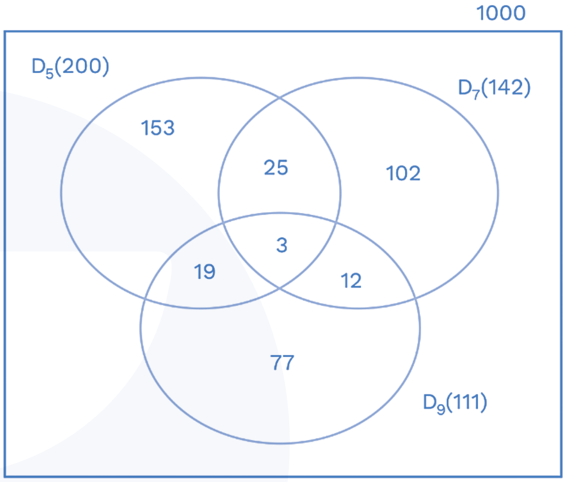
$P(6,6,4) \rightarrow \frac{2}{15} \times \frac{2}{15} \times \frac{1}{3} \times \frac{3!}{2!}$

$P(6,6,6) \rightarrow \frac{2}{15} \times \frac{2}{15} \times \frac{2}{15} \times \frac{3!}{3!}$

Now, required probability

$= \frac{2}{15} \times \frac{2}{15}\left[\frac{6}{15} + \frac{2}{15} + 1\right] = \frac{92}{3{,}375}$

23. (D)



Number divided by 5 = D_5 = 200
Number divided by 7 = D_7 = 142
Number divided by 9 = D_9 = 111
Number divided by all of 5, 7, and 9 = Number divided by LCM (5, 7, 9)
= Number divided by 315 = 3
Number divided by 5 and 7 = Number divided by 315 = 28
Number divided by 5 and 7 but not 9 = 28 − 3 = 25
Similarly, Number divided by 7 and 9 = Number divided by 63 = 15
Number divided by 7 and 9 but not 5 = 15 − 3 = 12
Number divided by 5 and 9 = Number divided by 45 = 22
Number divided By 5 and 9 but not 7 = 22 − 3 = 19
Hence the Number divided by 5 or 7 but not 9 = 153 + 25 + 102 = 280.
Required probability = 280/1,000 = 7/25

24. (D)

We have:

S = {1, 2, 3, 4, 5, ⋯, 28, 29, 30}

Now, number of A.P.'s we can form

If d = 1 ⇒ (1, 2, 3), (2, 3, 4), (3, 4, 5), --- (27, 28, 29) (28, 29, 30) = 28 cases

If d = 2 ⇒ (1, 3, 5) (2, 4, 6), (3, 5, 7), --- (26, 28, 30) = 26 cases

If d = 3 ⇒ (1, 4, 7), (2, 5, 8), (3, 6, 9), --- (24, 27, 30) = 24 cases

If d = 14 ⇒ (1, 15, 29) (2, 16, 30) = 2 cases

Note: d = 15 and onwards it is not possible

Therefore, total AP's = 2 + 4 + 6 + ⋯ 28

$$= 2\ (1 + 2 + \cdots + 14) = 2 \times \frac{14 \times 15}{2} = 14 \times 15$$

Now, the total number of ways in which 3 numbers can be selected

$$= {}^{30}C_3 = \frac{30 \times 29 \times 28}{3 \times 2 \times 1}$$

Therefore, required probability

$$= \frac{(14 \times 15) \times (3 \times 2 \times 1)}{30 \times 29 \times 28} = \frac{3}{58}$$

25. (D)

Here, the total no. of possible outcomes = 6 × 6 × 6 × 6 = 1,296.

Now, sum 10 can be obtain following ways:

$$(1, 1, 2, 6) = \frac{4!}{2!} = 12 \text{ ways}$$

$$(1, 1, 3, 5) = \frac{4!}{2!} = 12 \text{ ways}$$

$$(1, 1, 4, 4) = \frac{4!}{2! \times 2!} = 6 \text{ ways}$$

$$(1, 2, 2, 5) = \frac{4!}{2!} = 12 \text{ ways}$$

$$(1, 2, 3, 4) = 4! = 24 \text{ ways}$$

$$(1, 3, 3, 3) = \frac{4!}{3!} = 4 \text{ ways}$$

$$(2, 2, 2, 4) = \frac{4!}{3!} = 4 \text{ ways}$$

$$(2, 2, 3, 3) = \frac{4!}{2! \times 2!} = 6 \text{ ways}$$

Now, total favourable cases = 80 ways

Therefore, required probability $= \frac{80}{1,296}$ $= \frac{40}{648} = \frac{5}{81}$.

26. (C)

Given, S = {3, 6, 9, 12, 15, ..., 60}

If r = 2, possible GP's = (3, 6, 12), (6, 12, 24), (9, 18, 36) , (12, 24, 48), (15, 30, 60)

If r = 3, possible GP's = (3, 9, 27), (6, 18, 54)

If r = 4, possible GP's = (3, 12, 48)

Note: There is a possibility of GP's in which r (common ratio) is in fraction.

If $r = \frac{3}{2}$, possible GP's = (12, 18, 27), (24, 26, 54)

If $r = \frac{4}{3}$, possible GP's = (27, 36, 48)

Total possible of GP's = 11

Therefore, required probability

$$= \frac{11}{20C_3} = \frac{11 \times 3 \times 2 \times 1}{20 \times 19 \times 18} = \frac{11}{1{,}140}$$

Hence, option (C) is correct.

27. (A)

Probability = Favourable cases/Total possible cases.

Total possible cases = 12^9 (as any person can have a birthday in any of the 12 months of the year).

Favourable cases:

First, we will select three persons whose birthday will be in the same month of the year and this can be done in ${}^9C_3$ ways.

Now as the three persons are selected, then we have to select which month of the year, that they have a birthday and this can be done in ${}^{12}C_1$ way. (For example, let's assume that these three persons have a birthday in the month of January) Also, the birthdays of all others (except the 3) must be on the different months. 4th person can have a birthday on any of the remaining 11 months (except January), the 5th person can have a birthday on any of the remaining 10 months, and so on till the 9th person can have a birthday in any of the remaining 6 months.

Thus, total favourable cases = ${}^9C_3 \times {}^{12}C_1 \times$ (11 × 10 × 9 × 8 × 7 × 6).

Required probability = Favourable cases/ Total possible cases.

Required probability

$$= \frac{\left[{}^9C_3 \times {}^{12}C_1 \times (11 \times 10 \times 9 \times 8 \times 7 \times 6)\right]}{\left[12^9\right]} = \frac{16,170}{\left(12^5\right)}$$

Hence, option (A) is the correct answer.

28. (D)

Probability that Rohan scores $\frac{100}{100}$ in exactly two papers

$$= {}^6C_2 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^4 = \frac{(15 \times 81)}{4,096}$$

Probability that Rohan scores $\frac{100}{100}$ in exactly three papers

$$= {}^6C_3 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^3 = \frac{(20 \times 27)}{4,096}$$

Probability that Rohan scores $\frac{100}{100}$ in exactly four papers

$$= {}^6C_4 \times \left(\frac{1}{4}\right)^4 \times \left(\frac{3}{4}\right)^4 = \frac{(15 \times 9)}{4,096}$$

Probability that Rohan scores $\frac{100}{100}$ in exactly five papers

$$= {}^6C_5 \times \left(\frac{1}{4}\right)^5 \times \left(\frac{3}{4}\right)^1 = \frac{(6 \times 3)}{4,096}$$

Probability that Rohan scores $\frac{100}{100}$ in exactly six papers

$$= {}^6C_6 \times \left(\frac{1}{4}\right)^6 \times \left(\frac{3}{4}\right)^0 = \frac{1}{4,096}$$

Required answer

$$= \frac{(15 \times 81)}{4,096} + \frac{(20 \times 27)}{4,096} + \frac{(15 \times 9)}{4,096} + \frac{(6 \times 3)}{4,096} + \frac{1}{4,096}$$

$$= \frac{1,909}{4,096}$$

29. (D)

We construct a tree showing all possible outcomes that Raman may get after three flips;

We can do this because there are only eight possibilities:

$$6 \begin{cases} H:11 \begin{cases} H:21 \begin{cases} H:\boxed{41} \\ T:9.5 \end{cases} \\ T:4.5 \begin{cases} H:\boxed{8} \\ T:1.25 \end{cases} \end{cases} \\ T:2 \begin{cases} H:3 \begin{cases} H:\boxed{5} \\ T:0.5 \end{cases} \\ T:0 \begin{cases} H:\boxed{-1} \\ T:\boxed{-1} \end{cases} \end{cases} \end{cases}$$

So, you can see that out of total eight cases for the fourth term, five are integers. Hence, there is 5/8 chance that Raman's fourth term is an integer

30. (C)

First, we need to select one suit from four suits can be done in = ${}^4C_1$ ways.

Then we two have two cards from this suit can be selected in = ${}^{13}C_2$ ways.

Now the remaining two cards must be from two different suits, So, we will first select two suits out of three suits, this can be done in = ${}^3C_2$ ways.

Now two two cards (one card from each suit) from these two suits can be selected in

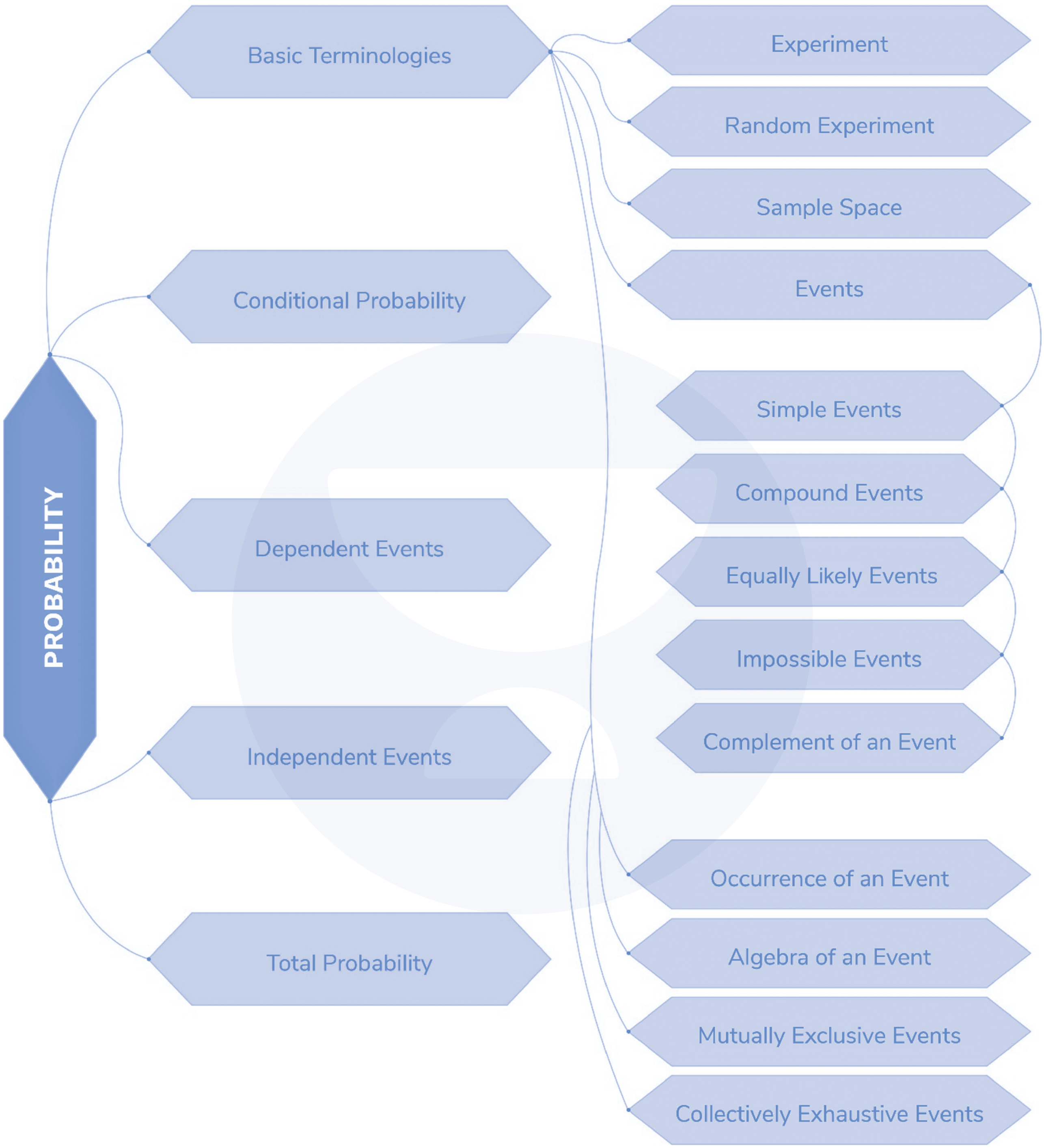
= ${}^{13}C_1 \times {}^{13}C_1$ ways.

Therefore,

Required probability

$$= \frac{{}^4C_1 \times {}^{13}C_2 \times {}^3C_2 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4} = \frac{12,168}{20,825}$$

Mind Map


PROBABILITY
Basic Terminologies
Conditional Probability
Dependent Events
Independent Events
Total Probability
Experiment
Random Experiment
Sample Space
Events
Simple Events
Compound Events
Equally Likely Events
Impossible Events
Complement of an Event
Occurrence of an Event
Algebra of an Event
Mutually Exclusive Events
Collectively Exhaustive Events