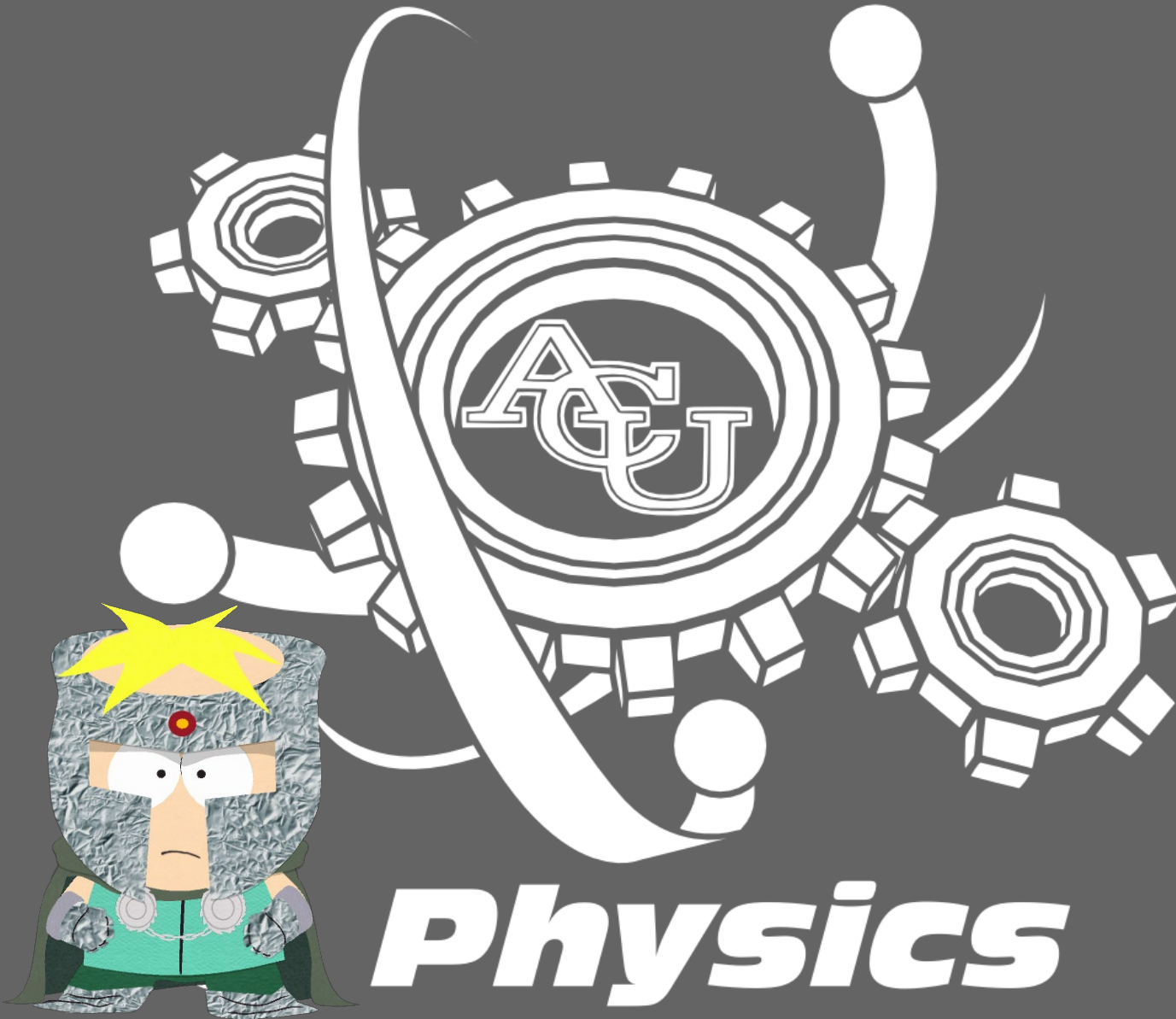


***Engineering***



***Physics***

# PHYS 451

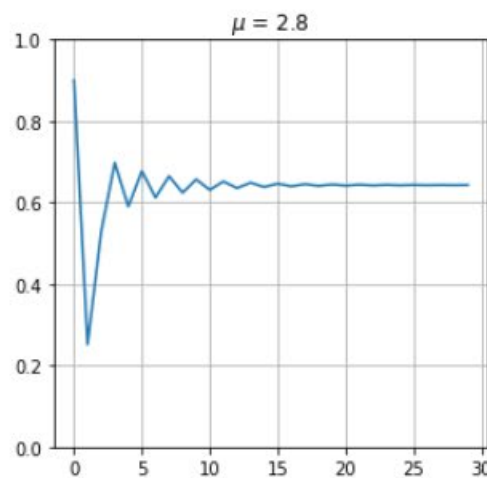
## Special Topic

### CHAOS

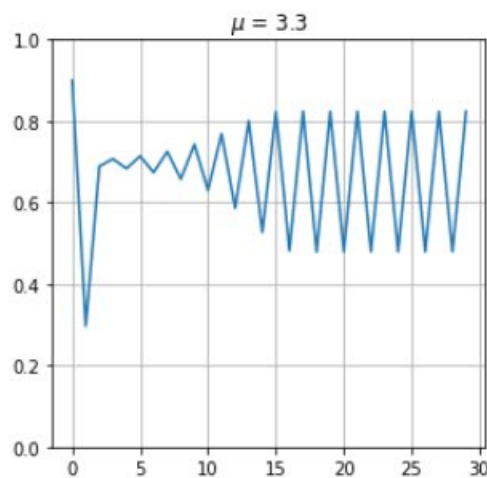
Dr. Daugherty  
Abilene Christian University

$$f(x) = \mu x(1 - x), \quad x_{i+1} = f(x_i)$$

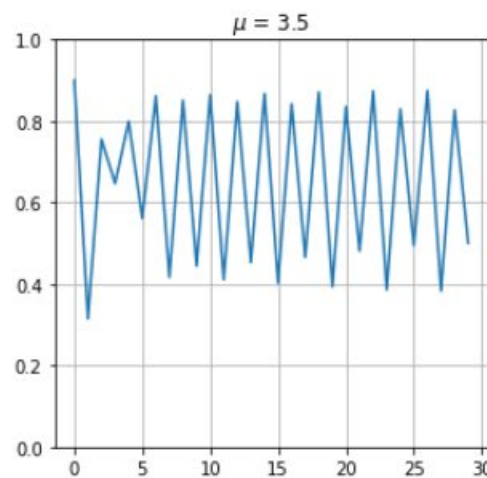
$x_0 = 0.9, N=30$



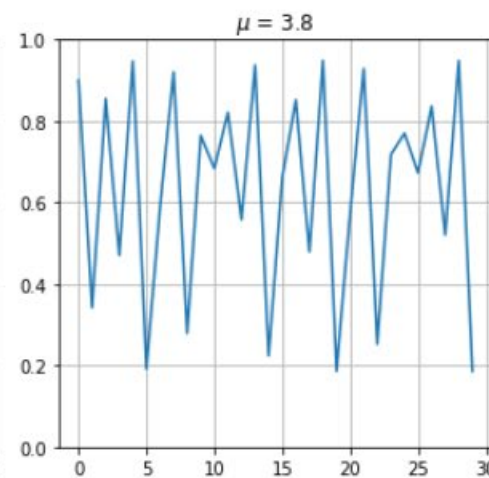
period-one



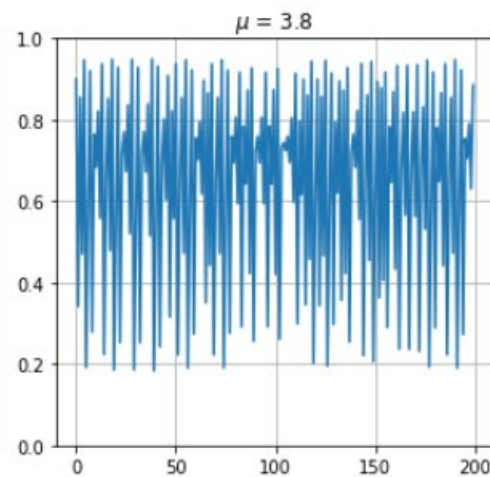
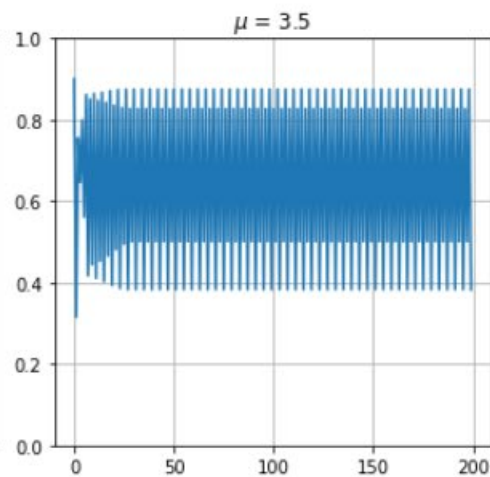
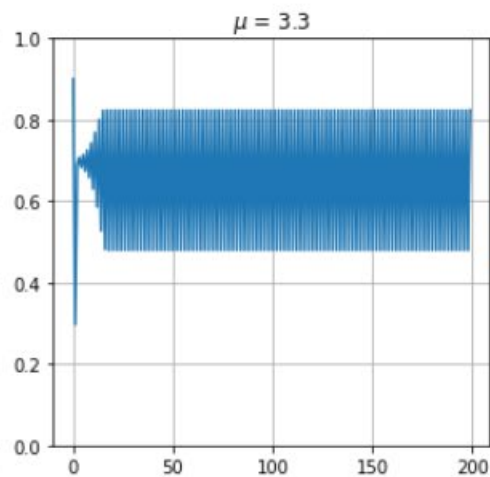
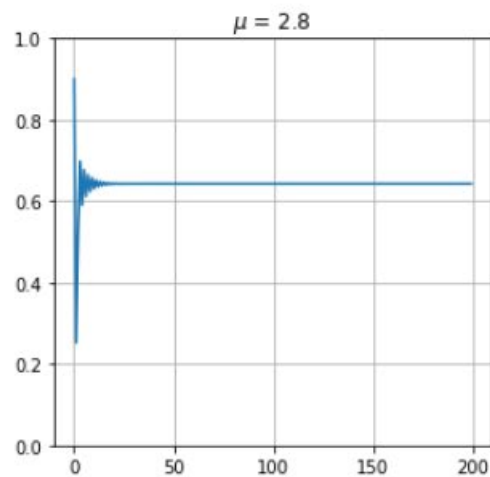
period-two



period-four

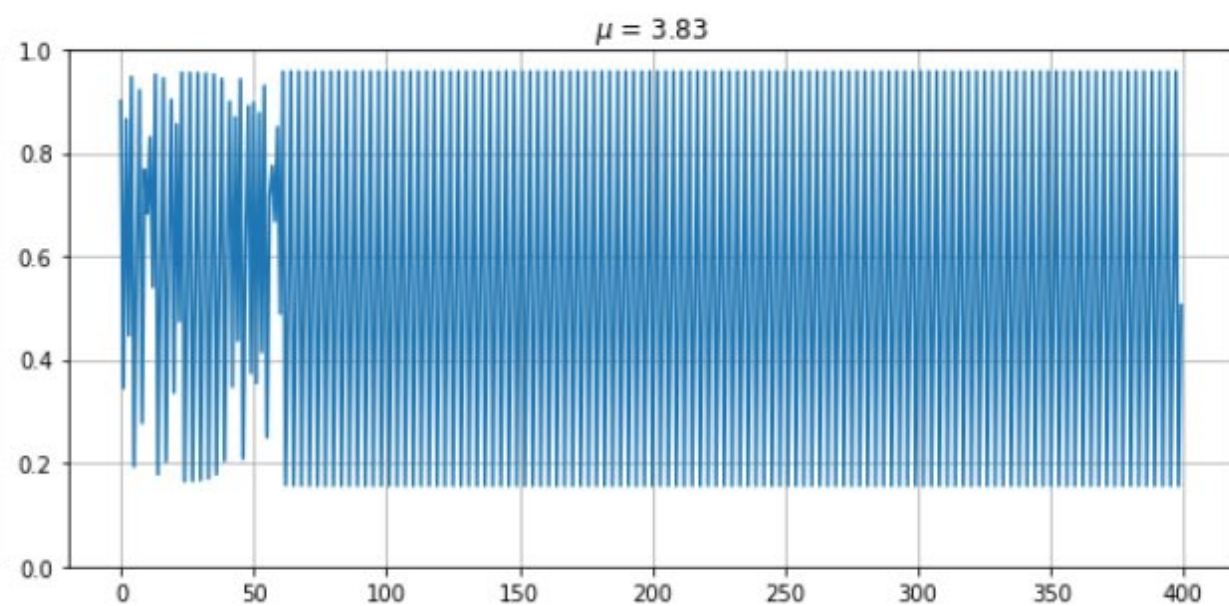
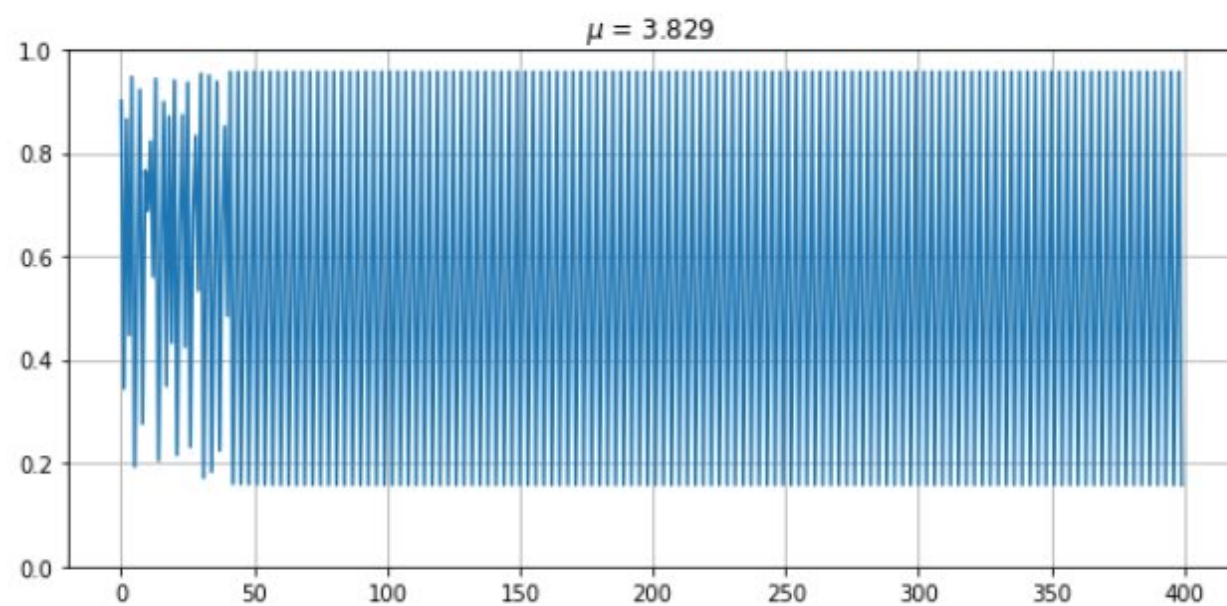
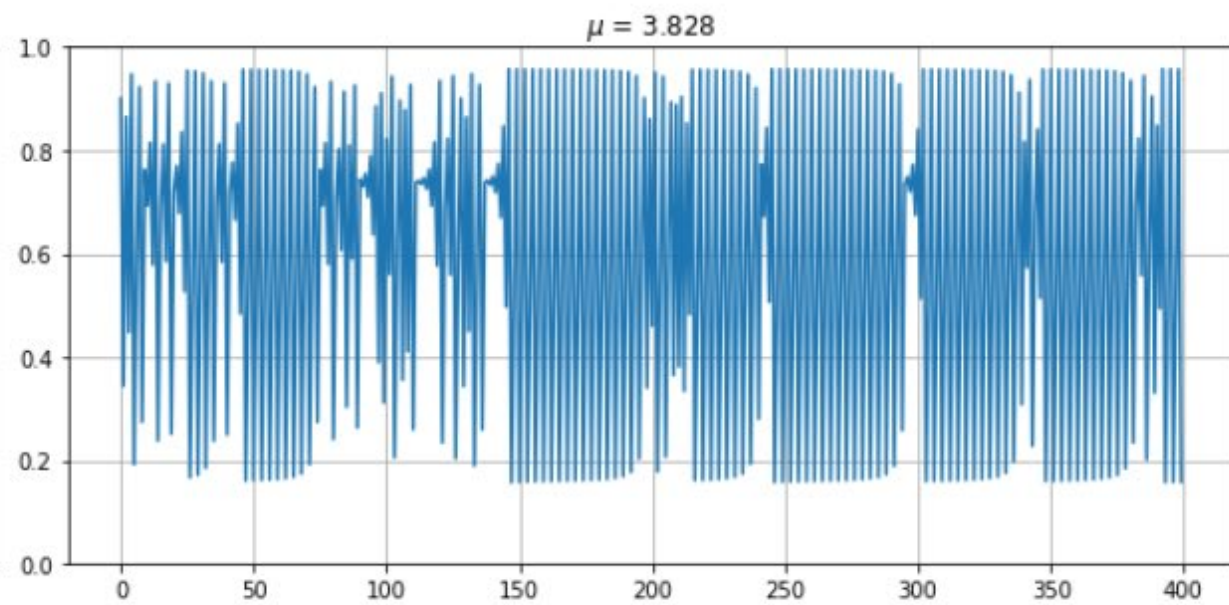
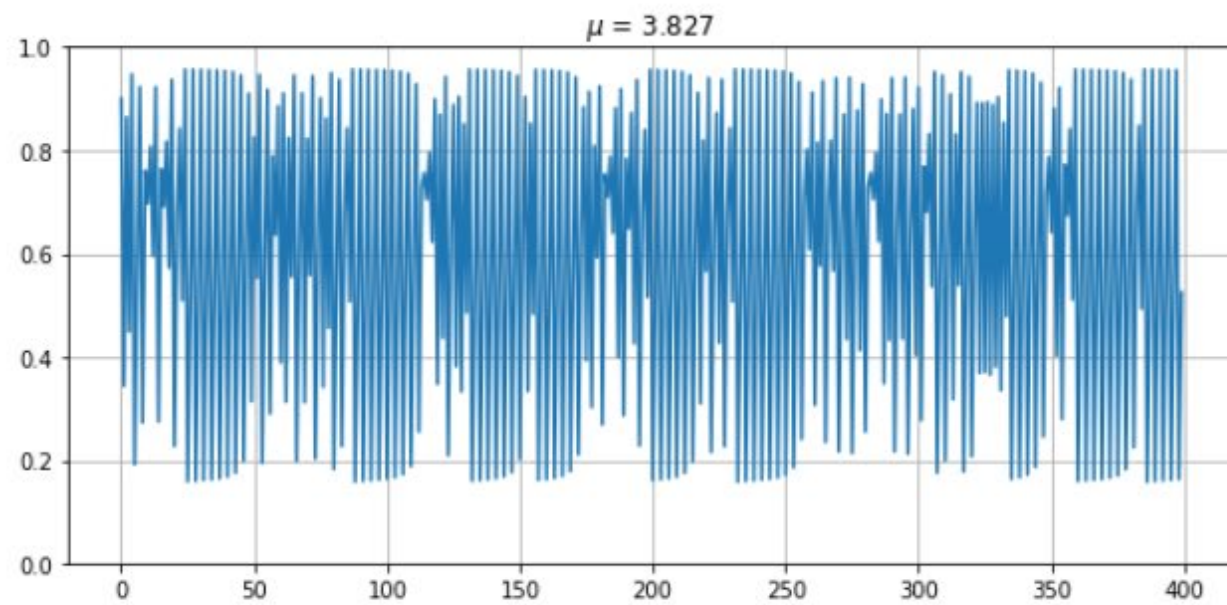


chaos



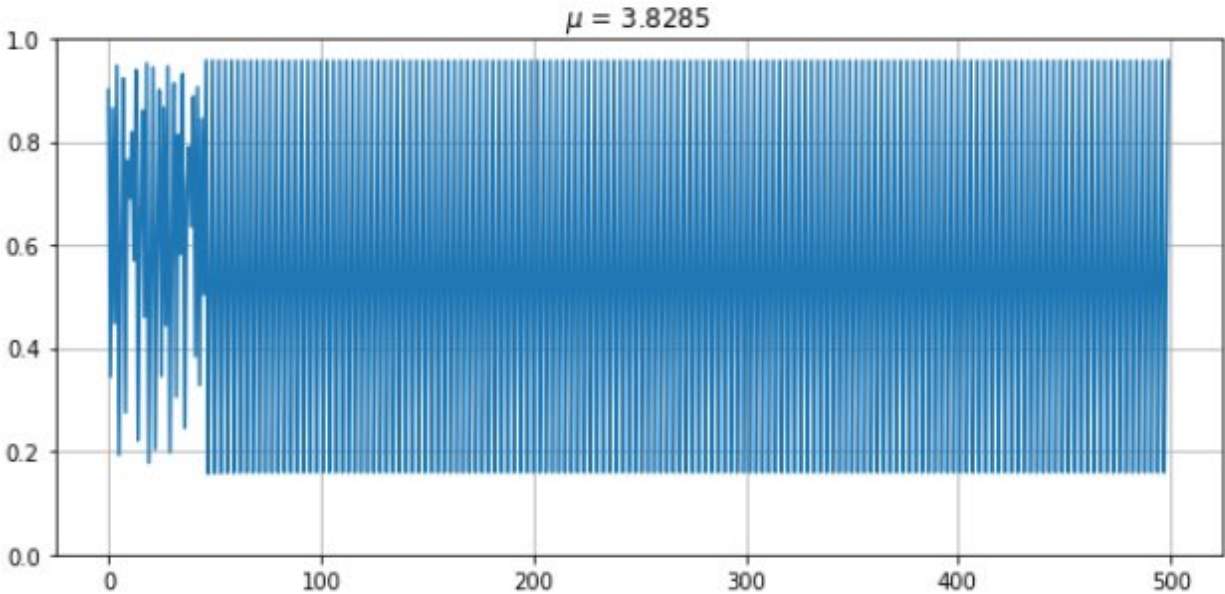
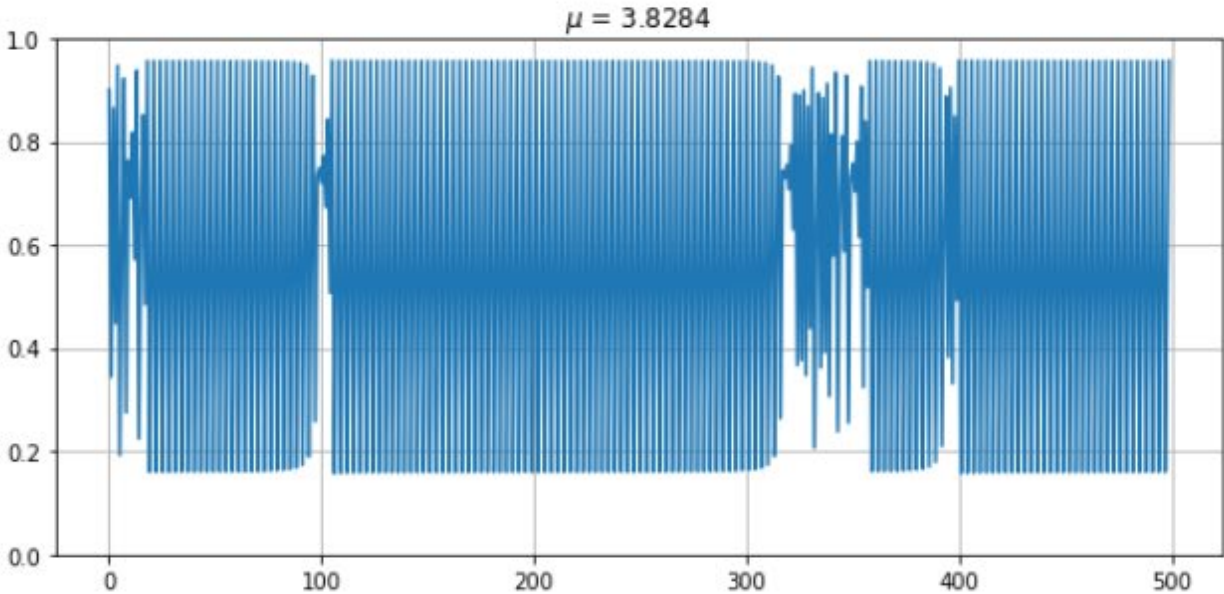
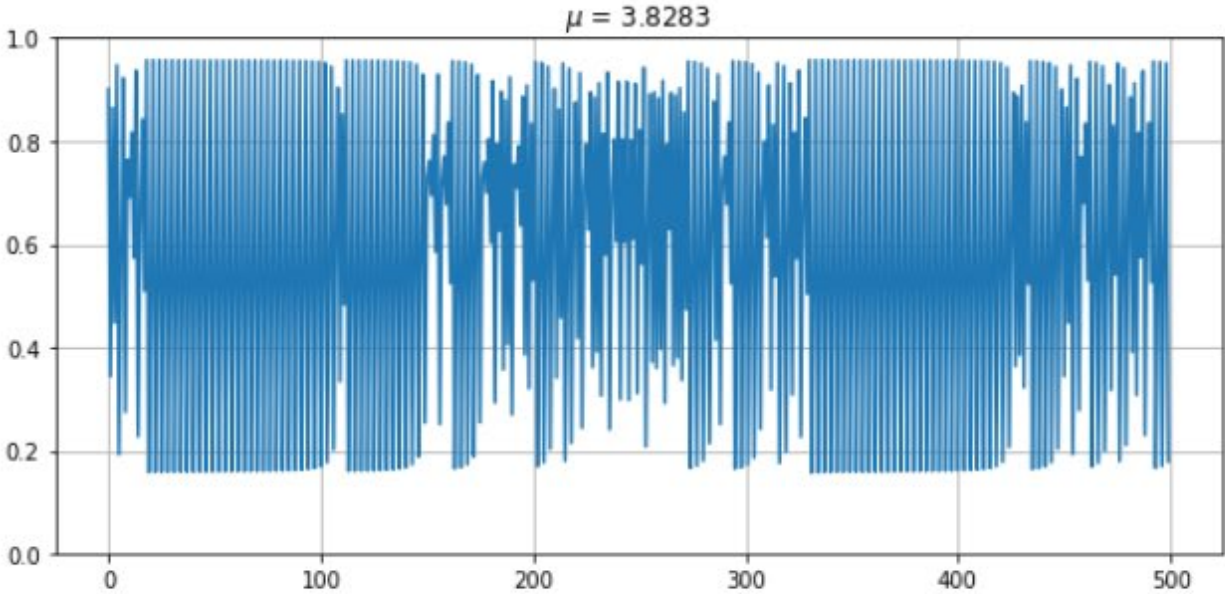
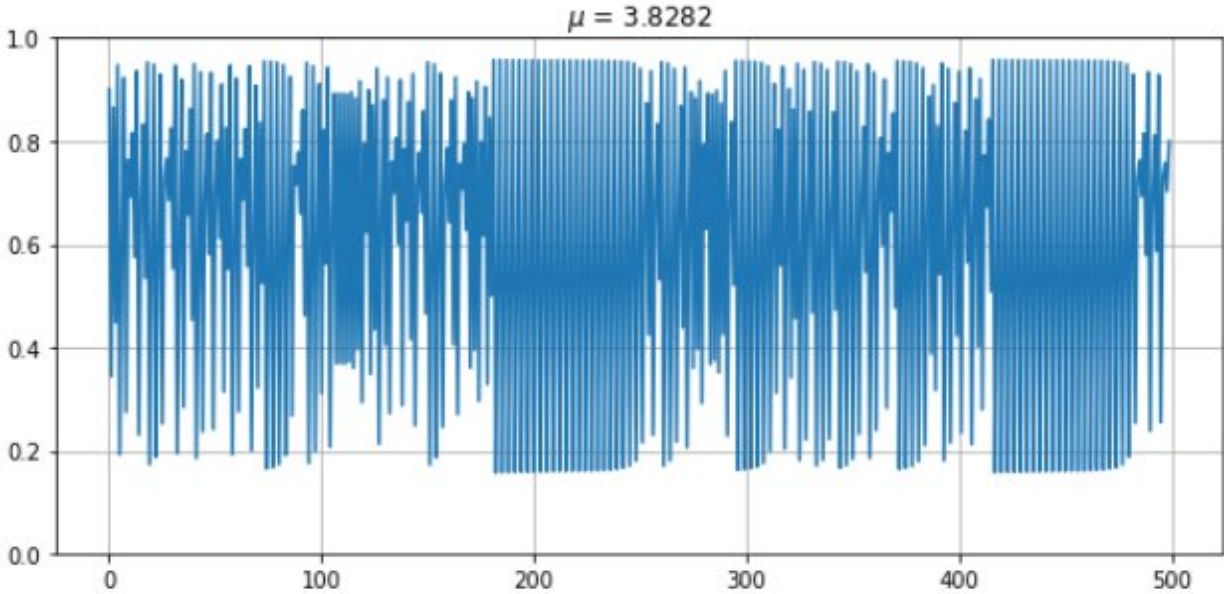
$x_0 = 0.9, N=200$

critical value  $\mu=3.828427$





critical value  $\mu=3.828427$



## ▼ Bifurcation Diagram

Visualize dependence on  $\mu$ . The algorithm is:

For each  $\mu$  value:

- choose  $x_0$
- run  $N$  generations
- plot only the last value  $x_N$
- repeat for many  $x_0$  values

Systemically do this for many  $\mu$ 's and you get a diagram that shows where system is periodic or chaotic.

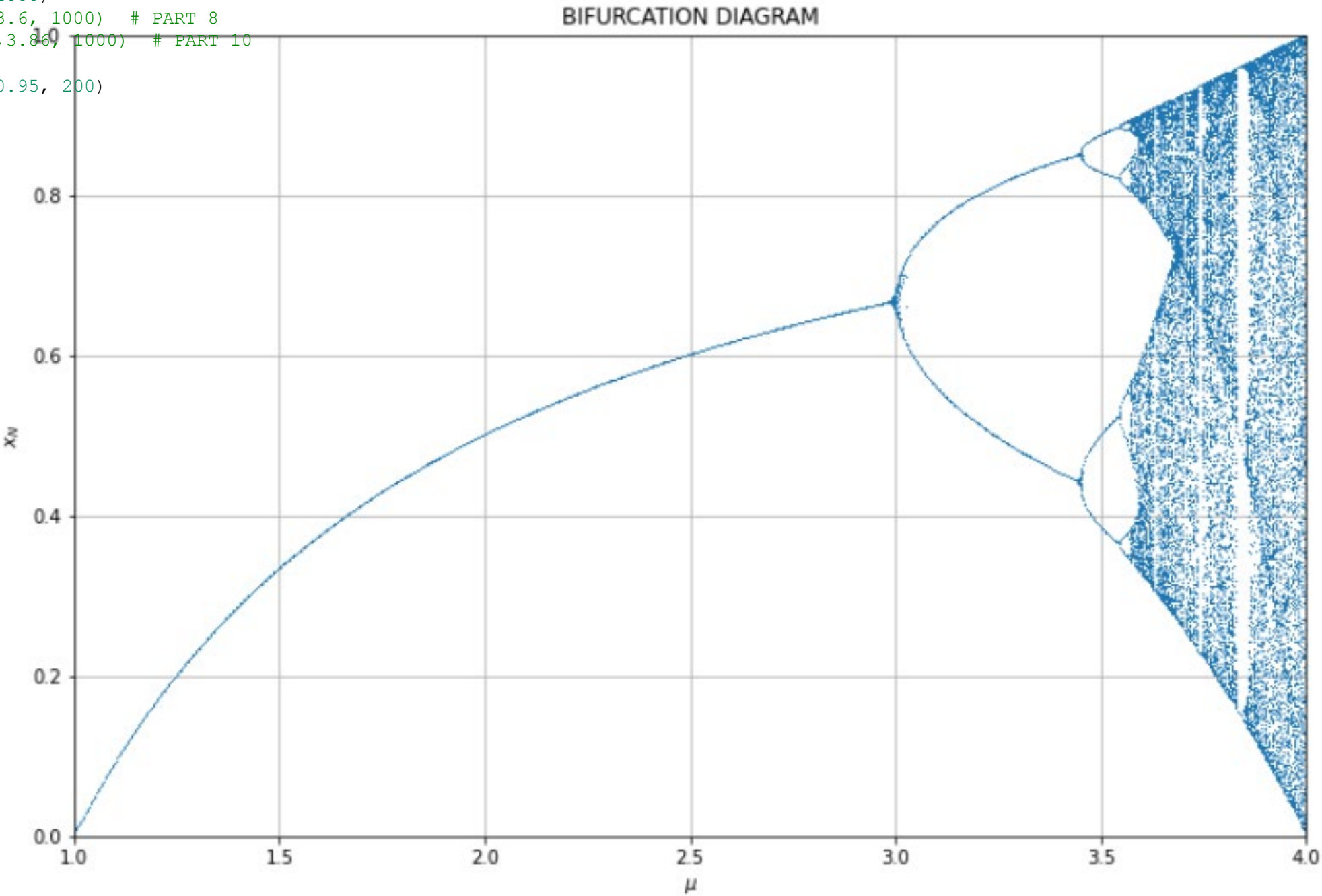
Features to notice:

- period doubling - how a small change in  $\mu$  causes number of outcomes to double
- onset of chaos when period doubles infinitely fast
- fractals from self-similar behavior!

```
MULIST = np.linspace(1,4, 1000)
#MULIST = np.linspace(2.9,3.6, 1000) # PART 8
#MULIST = np.linspace(3.82,3.86, 1000) # PART 10
```

```
X0LIST = np.linspace(0.05,0.95, 200)
N = 200
```

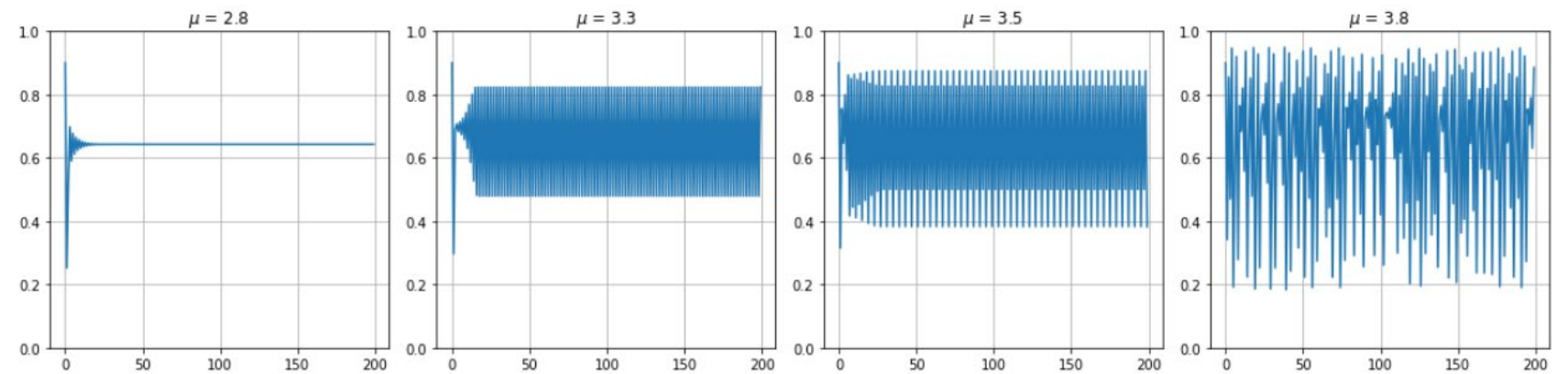
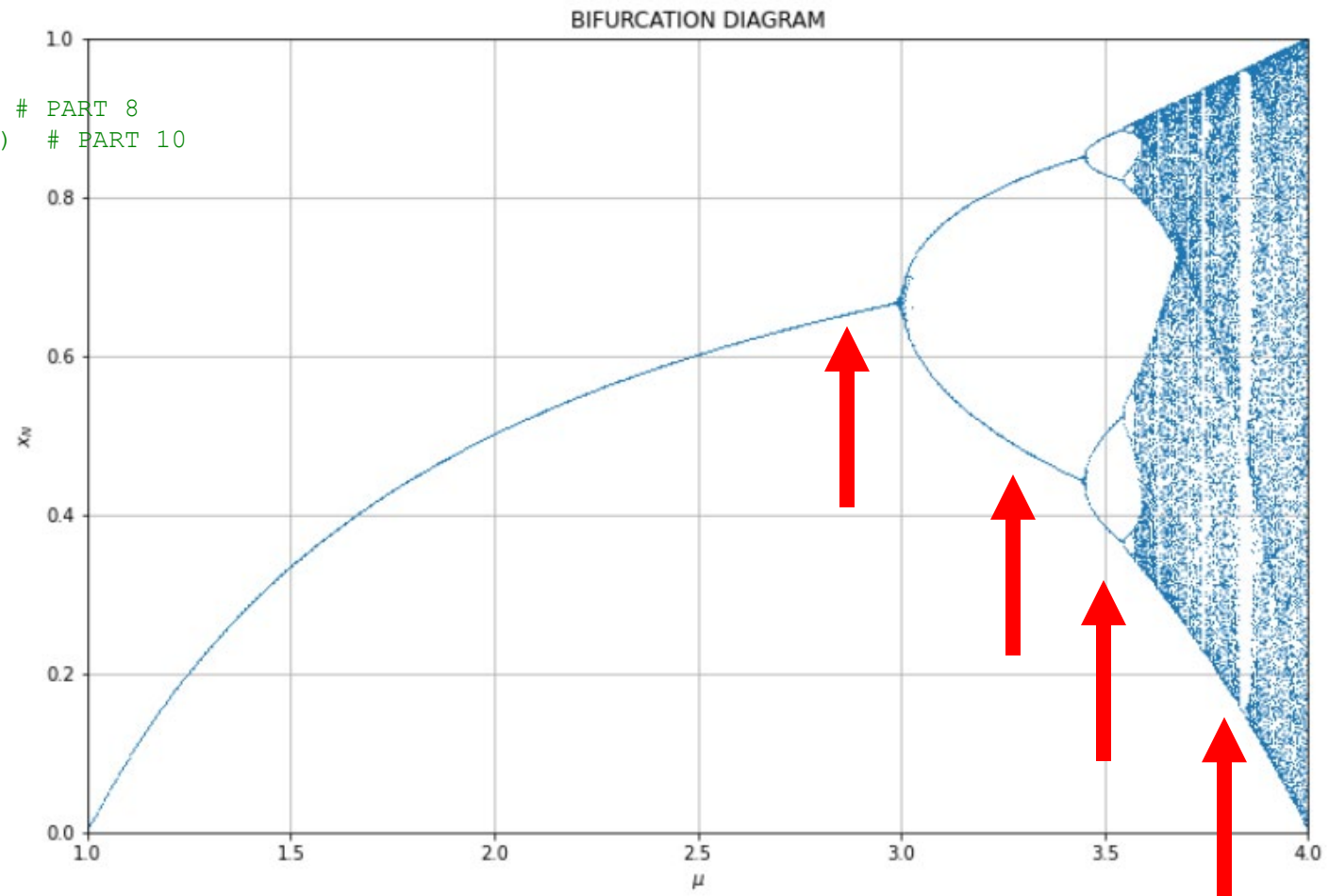
24 seconds



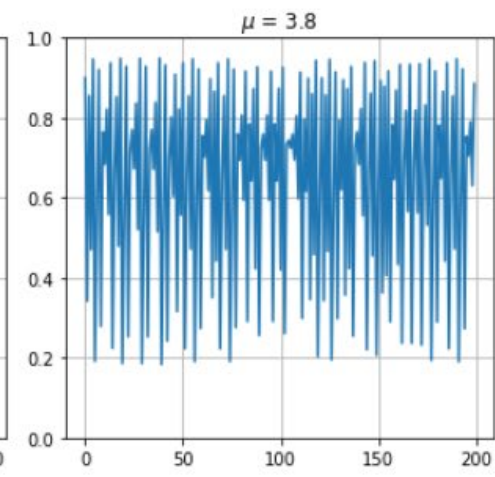
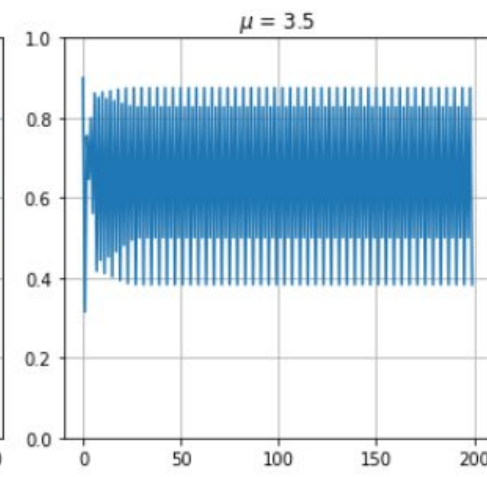
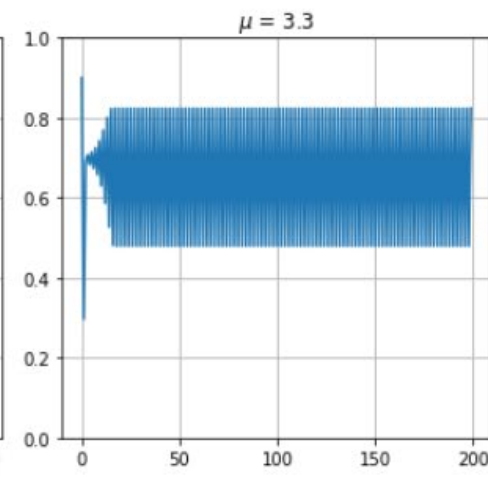
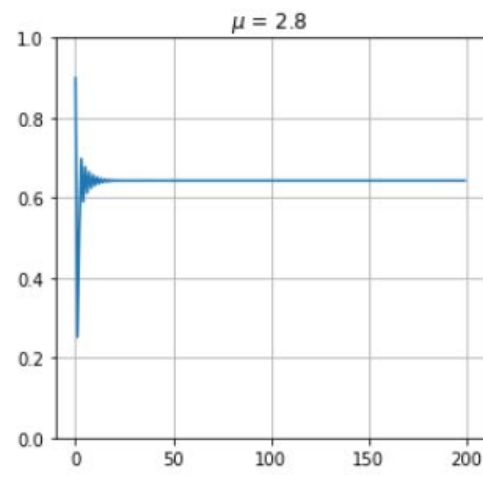
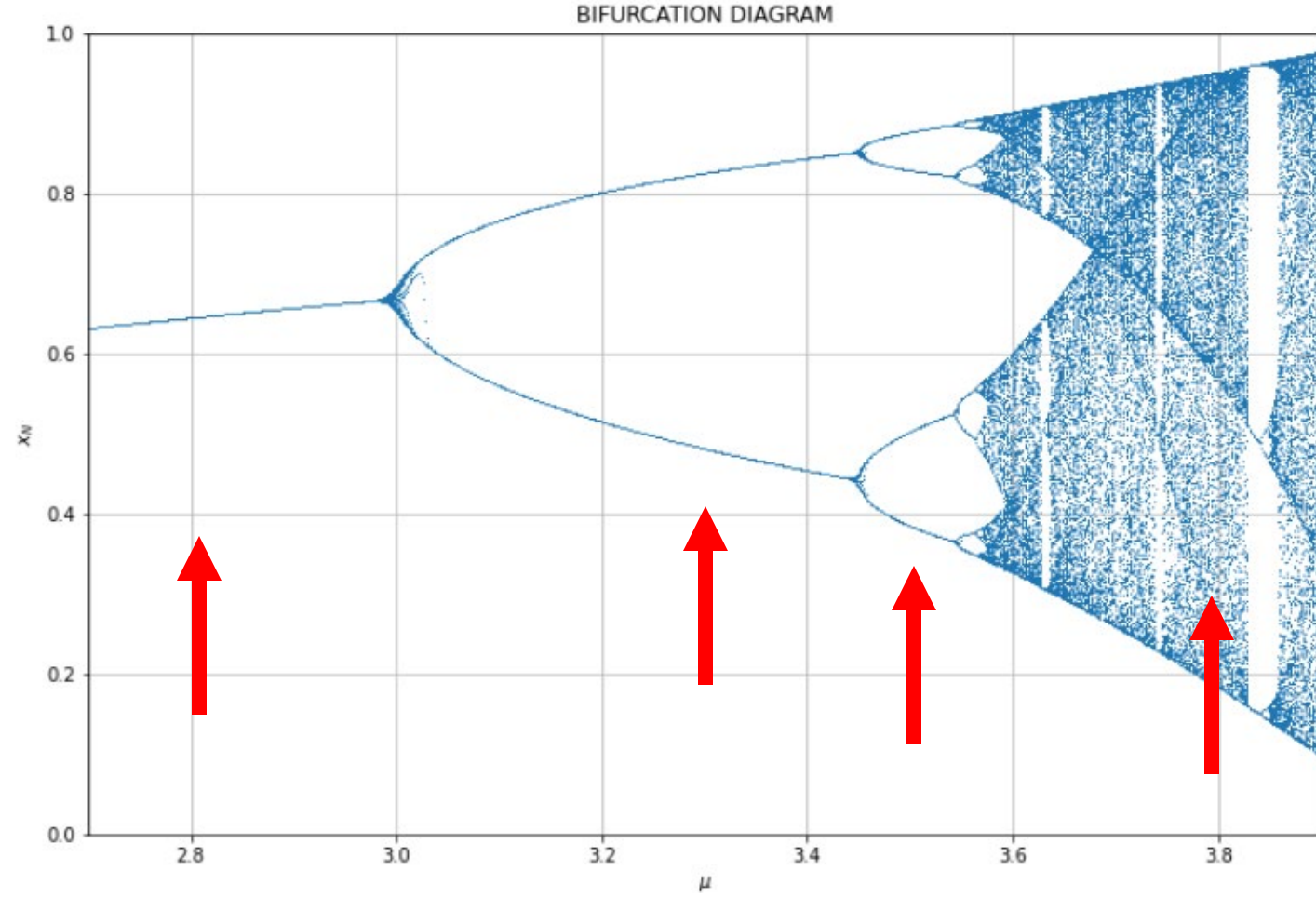
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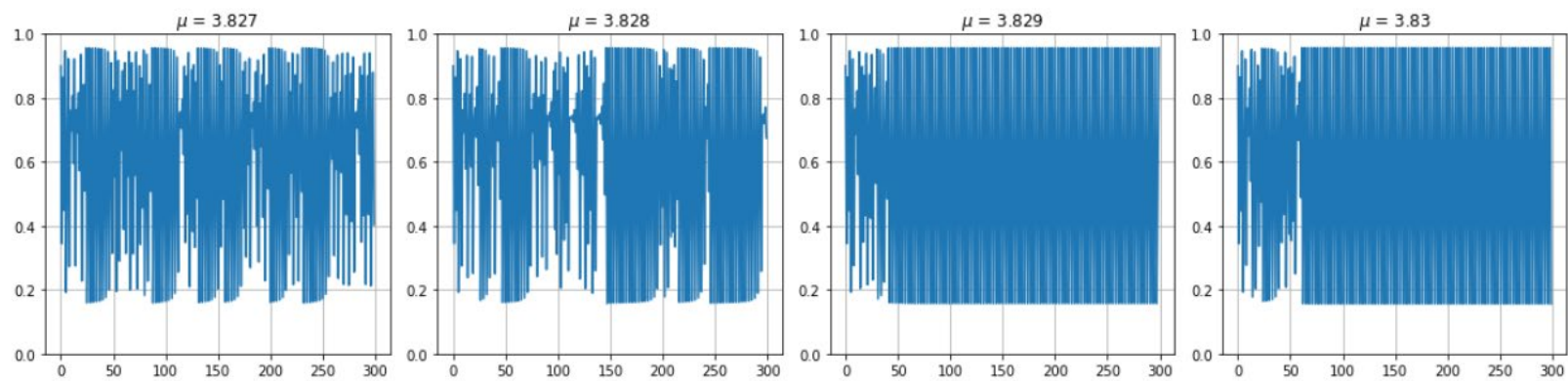
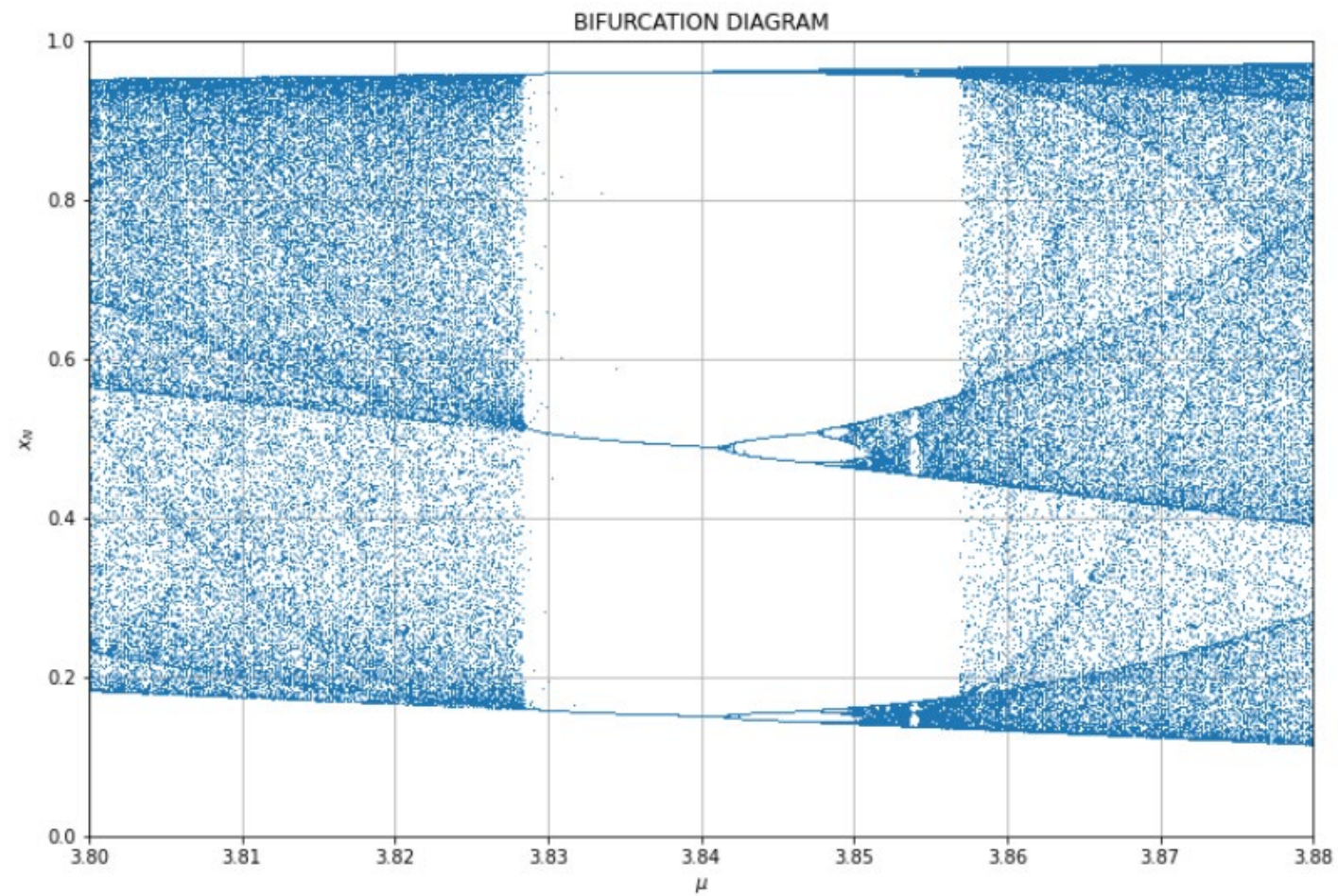


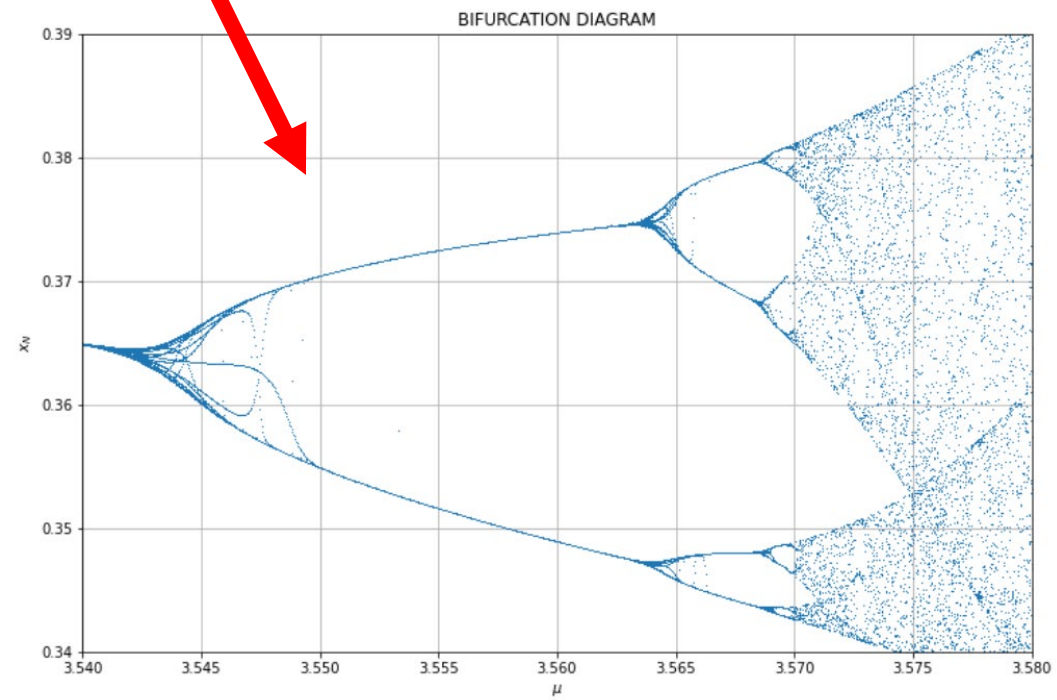
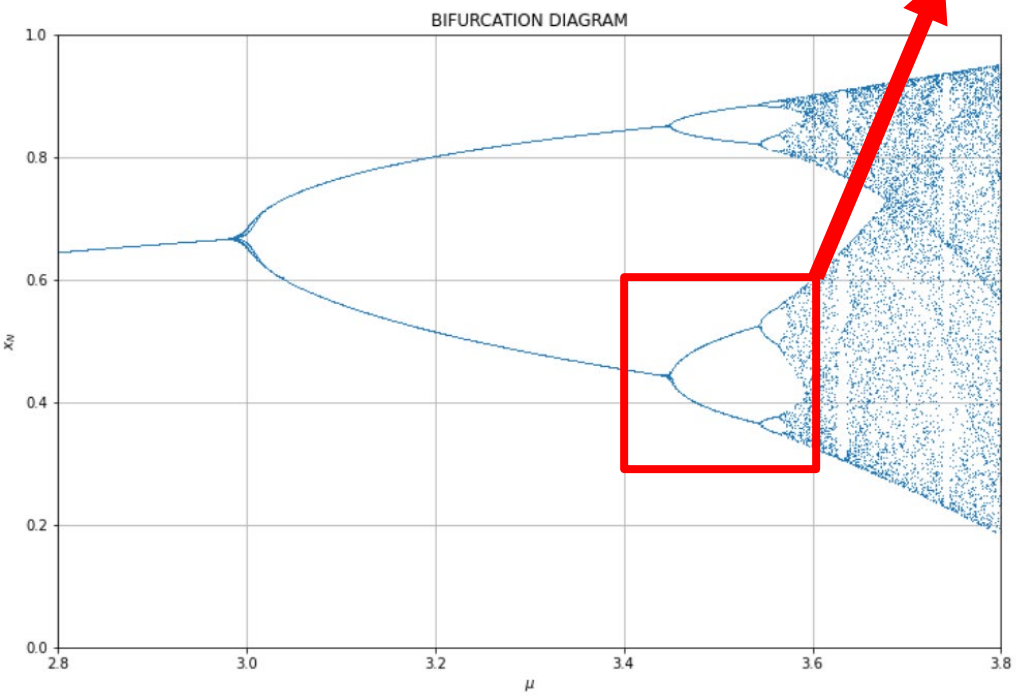
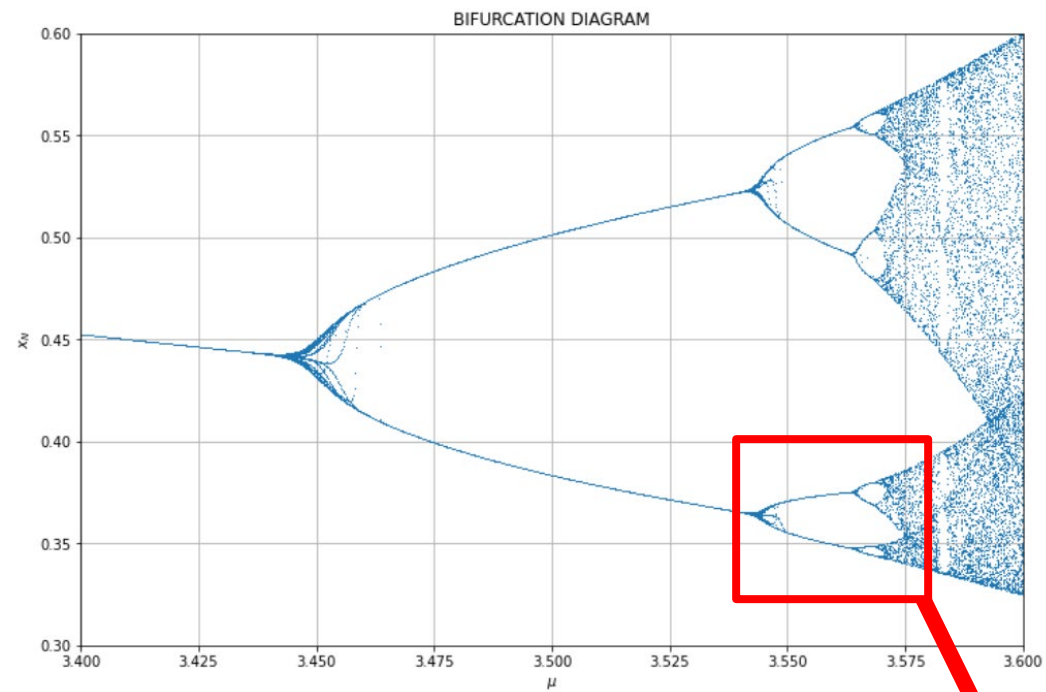






critical value  $\mu=3.828427$





# Things to Know

- ??????