Lecture 20. Single-phase SPWM inverters

20.1 Sinusoidal Pulse Width Modulation (SPWM)

In this scheme a sinusoidal modulating voltage e_C of the desired output frequency f_o is compared with a higher frequency triangular or saw tooth carrier waveform to generate the switching signals for the inverter. The amplitude of e_c also determines the amplitude (or RMS value) of the fundamental output voltage. The inverter can be half- or full-bridge circuits of figure 19.3 or 19.4. The resulting switching pulses have widths approximately proportional to the sine of the angular position at the center of the pulses as shown in figure 20.5. The widths of these pulses are also proportional to the amplitude of the modulating signal e_c, relative to the amplitude of the carrier. (See figure 20.5 for the half-bridge inverter waveforms).

Half-bridge SPWM inverter

Consider the half-bridge inverter of figure 20.4 in which switches T1 and T2 are turned on and off by the switching pulses of figure 20.5. The switching states are:

when $e_c > v_{tri} \rightarrow$ T1 is on

when $e_c < v_{tri} \rightarrow$ T2 is on

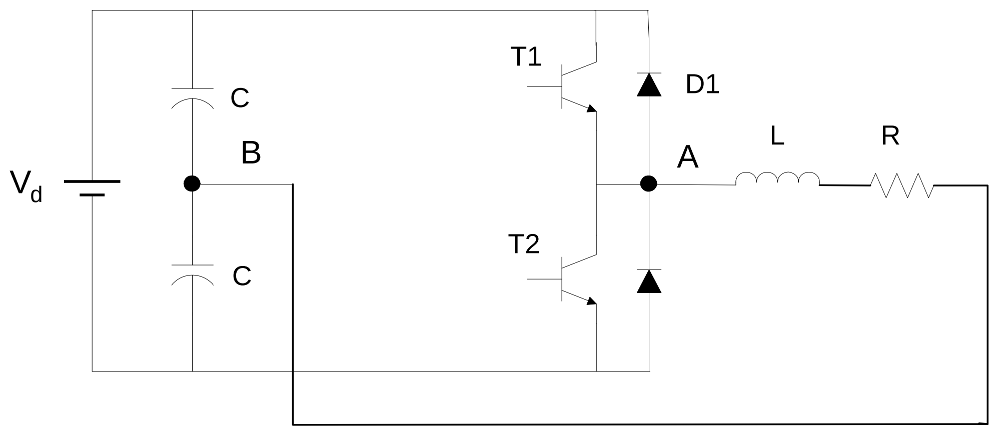

Figure 20.1

Regardless of the direction of current flow through the load, the output voltage waveform of this inverter is determined by the state of the switches, as shown in Figure 20.2.

It will be shown rigorously in a latter section by using the δ, or impulse function, that the amplitude of the fundamental frequency component of the output voltage of a half-bridge SPWM is proportional to the depth of modulation, $m \left(= \dfrac{e_c}{\hat{V}_{tri}} \right)$. This result may be argued also from the fact that the over each switching period T_s ($=1/f_s$), the average output voltage for the inverter is $mV_d/2$. This is based on the assumption that the control voltage remains essentially constant during a switching period. [In other words, we assume that $f_s >> f_1$]. This average voltage during each T_s is represented by the dotted trace of figure 20.5. Note that the *B* terminal of the load is connected to the center-tap of the DC supply and that the amplitude of the output voltage of the half-bridge SPWM inverter is $\frac{V_d}{2}$. Thus,

$$V_{o1max} = m \times \frac{V_d}{2} \quad \text{for } m < 1. \qquad (20.1)$$

Also, $v_{o1} = \frac{mV_d}{2} \sin \omega_1 t$ for $m < 1$, where $\omega_1 = 2\pi f_1$.

Let us first define two important terms associated with SPWM.

$$\text{Depth of Modulation, } m = \frac{e_{c\,max}}{\hat{V}_{tri}} \qquad (20.2)$$

$$\text{Frequency Modulation Ratio, } m_f = \frac{f_s}{f_1} \qquad (20.3)$$

where f_1 is the frequency of the sinusoidal modulating waveform which is also the fundamental output frequency, and f_s is the frequency of the carrier (the inverter switching frequency).

Fourier analysis of the output voltage waveform can be carried out taking a pair of PWM pulse-pairs which repeat in each cycle of the AC output. Note that the output AC cycle consists of many such symmetrical PWM output pulse-pairs for each AC cycle of output voltage. Note also that each pulse pair has a different (sine weighted) width and is shifted from its neighbouring pulse pair by $1/f_s$. Fourier coefficients for each pulse pair can be found by using the familiar formula

$$a_n = \frac{\frac{4V_d}{2}}{n\pi} sin\left(\frac{n\delta}{2}\right) \tag{20.4}$$

where δ is the duration of each pulse. Fourier coefficients of pulse-pairs located at angles α_1, α_2, α_3, and so on from the origin are given by

$$a_{kn} = \frac{4V_d}{n\pi} sin\left(\frac{n\delta}{2}\right)\left(cos\,\alpha_k + j\,sin\,\alpha_k\right) \tag{20.5}$$

where α_k is the angular displacement of the center of the *k*-th pulse from the origin.

Once Fourier coefficients for all pulse-pairs are found, they are added together for all *k*, taking into account their phase displacements α_k from the origin, to obtain the total Fourier coefficients $\sum_{k=1}^{p} a_{kn}$, where *p* is the number of PWM pulses per half cycle of the AC output voltage wavefrom.

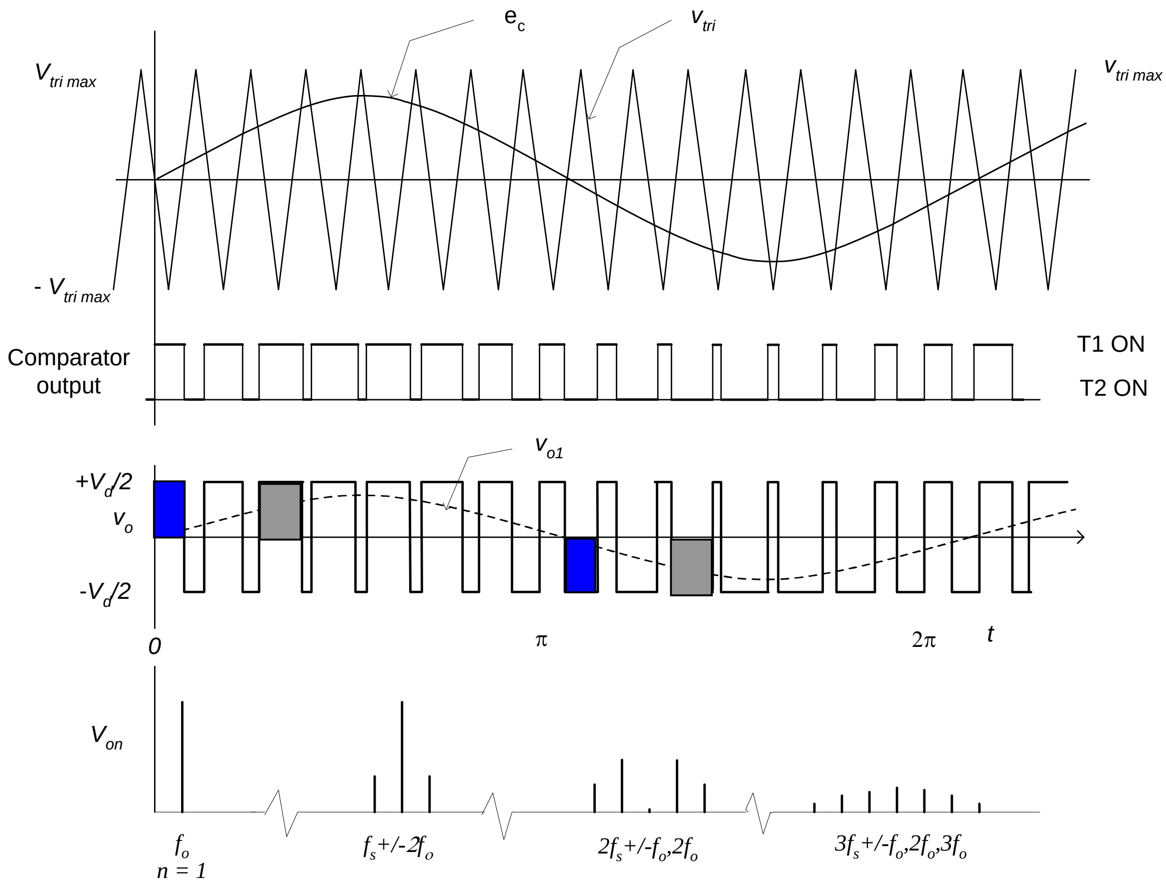

Figure 20.2

It can be shown that the output voltage v_o contains harmonics centered at multiples of the carrier frequency and their side bands given by (20.6)

$$f_n = (j \cdot m_f \pm k) f_1 \tag{20.6}$$

and the harmonic order, $n = j.m_f \pm k$.

Note that for odd values of j, k is even and vice versa.

Full-bridge SPWM inverter

The full bridge inverter can be either bipolar or unipolar switched. In the bipolar switching scheme, transistors T1 and T2 are switched ON together, when $e_c > v_{tri}$, as are T3 and T4, when $e_c < v_{tri}$. The control voltage e_c is

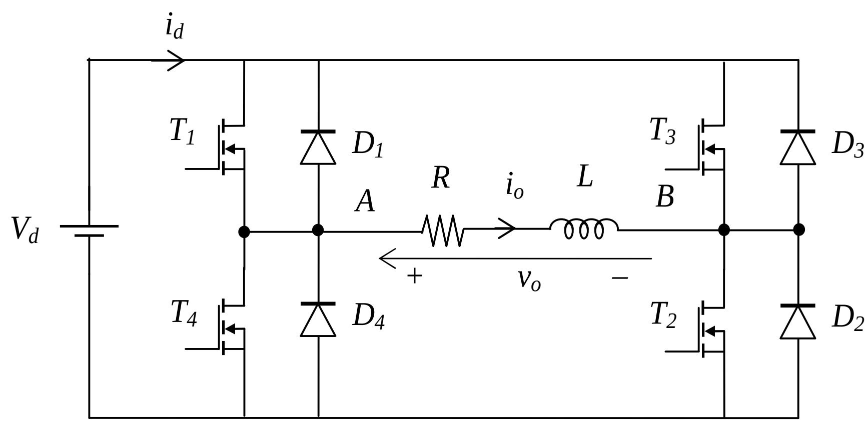

Figure 20.3 Full-bridge inverter

sinusoidal, and of frequency equal to the desired output frequency. Its amplitude is determined by the required RMS output voltage. The carrier frequency is generally much higher than the frequency of the modulating waveform (e_c). Regardless of the direction of current flow in the load, the load voltage waveform is determined by the state of the switches. The amplitude of each SPWM voltage pulse across the load is now $\pm V_d$. Two switches in the same leg of the inverter are never turned on together because that would constitute a short circuit across the DC source.

This switching scheme is called bipolar, as opposed to unipolar in which both switches in a diagonal pair may

NOT be switched on or off simultaneously, as in the bipolar scheme.

In the bipolar scheme, the comparator out which produces the switching signals of the diagonal transistor pairs are based on the following rule :

when $e_c > v_{tri}$, T1 & T2 are ON and T3 & T4 are OFF

when $e_c < v_{tri}$, T3 & T4 are ON and T1 & T2 are OFF

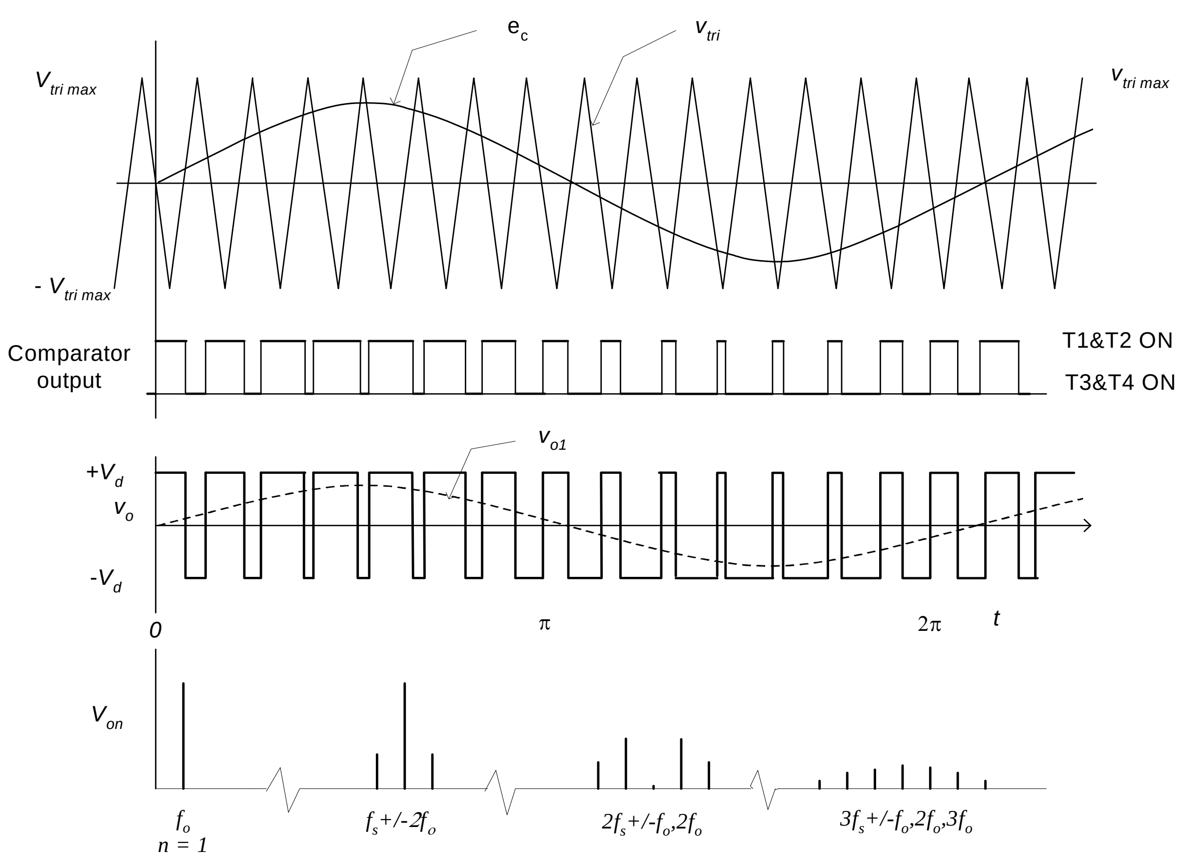

Figure 20.4 Single-phase full bridge inverter output voltage and its spectrum

If the PWM switching or carrier frequency is far higher the frequency of the modulating waveform, it can be assumed that the modulating wave changes little over a switching period. The average output voltage over each switching period is then equal to the depth of modulation (or the effective duty cycle over the switching period) times the supply voltage V_d. It should be expected that the fundamental output voltage waveform should be given by the average voltage voltages during each switching period. This is given the dotted sinusoid of figure above. Thus,

$$V_{o1max} = m \cdot V_d \quad \text{for} \quad 0 \leq m \leq 1. \qquad (20.7)$$

In other words, the amplitude of the fundamental output voltage is proportional to the depth of modulation within the linear range, as shown in figure 20.8. (A rigorous mathematical proof of this given in a latter section with the help of the δ or impulse function).

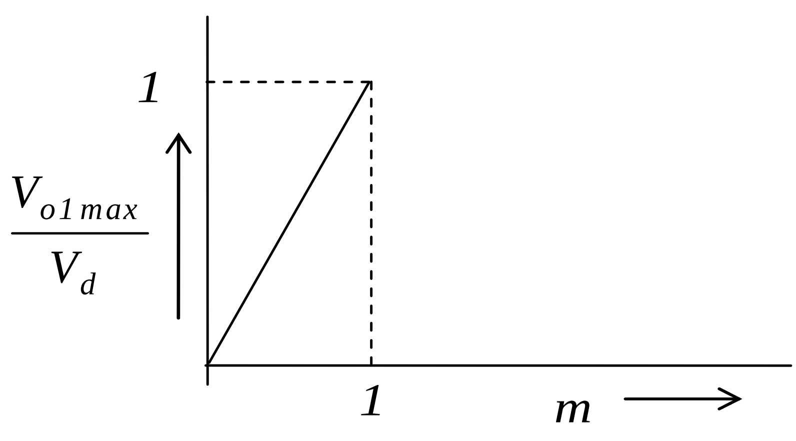

Figure 20.5

The following considerations are normally applied:

- For small m_f (ie., for $m_f < 21$) v_{tri} and e_c should be synchronised and m_f should be an odd integer. Otherwise sub-harmonic voltages would be present in the output.

- The slopes of v_c and v_{tri} should be of opposite polarity at zero crossings of the modulating waveform.

- For large m_f these restrictions are of lesser importance

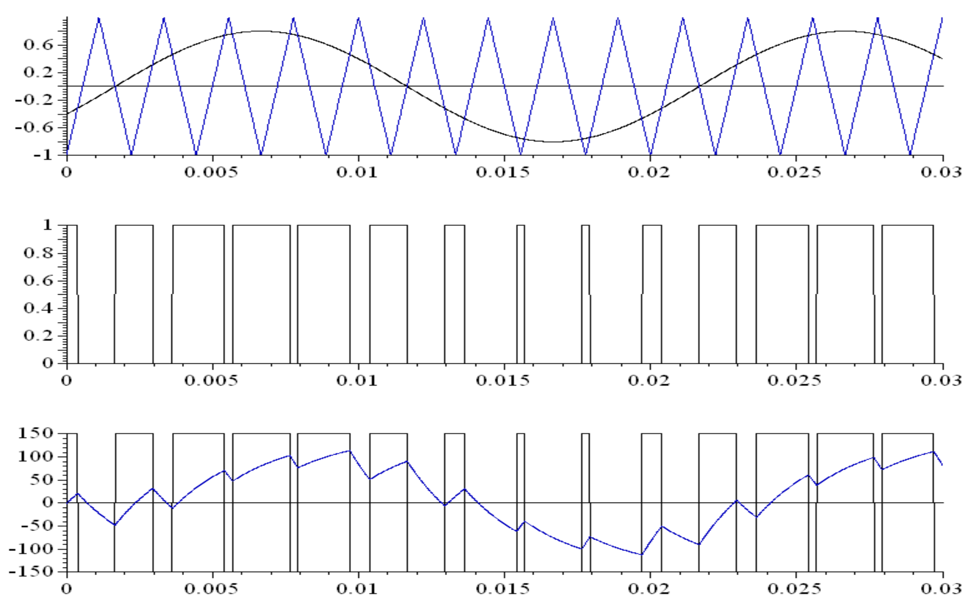

Figure 20.6 $V_d = 150$ VDC, $R = 1\ \Omega$, $L = 2.5$ mH, $f_s = 450$ Hz, $f_o = 50$ Hz, $m = 0.8$, $k = 5$.

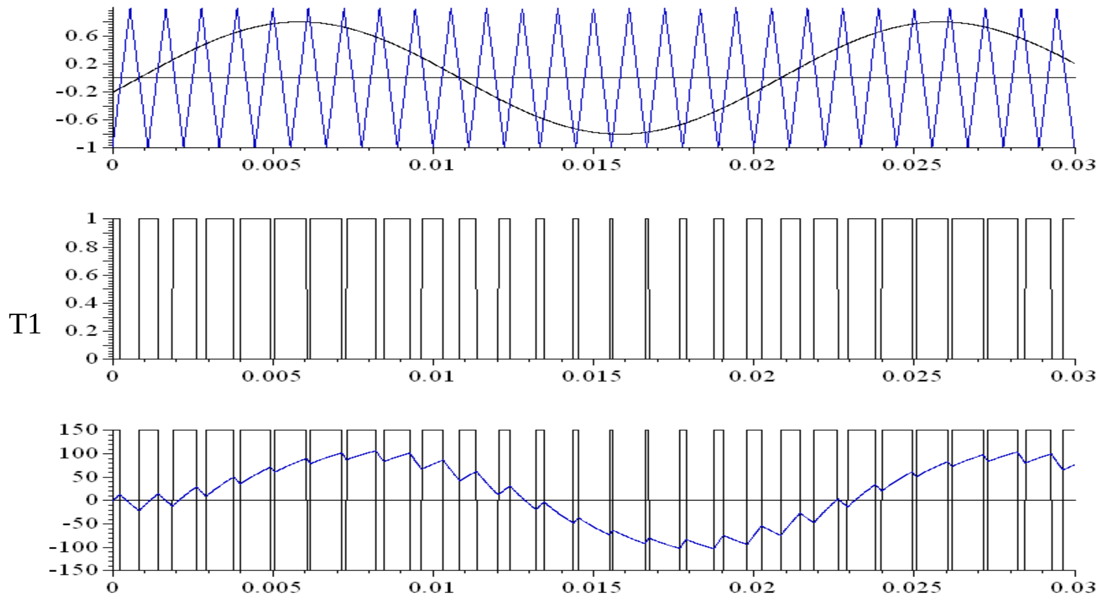

Figure 20.7 f_s = 900 Hz

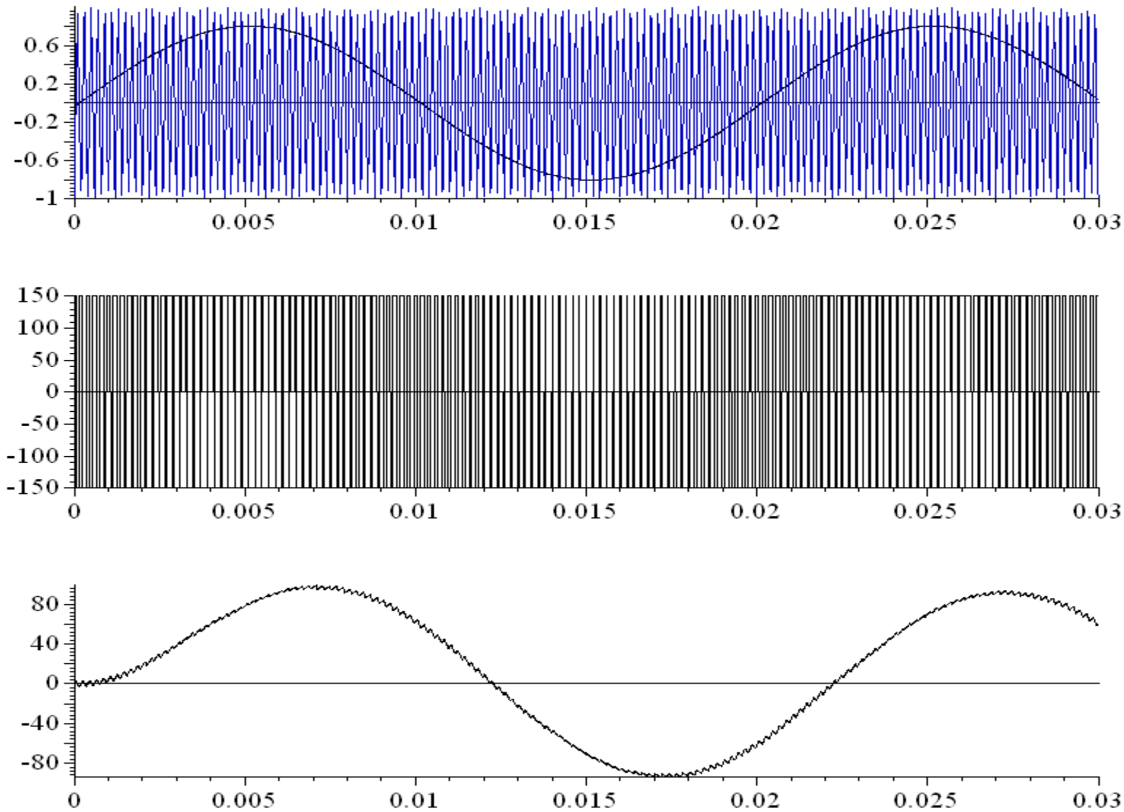

Figure 20.8 f_s = 5 KHz.

Over-modulation

For $m > 1$, over-modulation occurs. This is the so called *modified* SPWM scheme.

- increases the V_{o1} beyond V_d. For $m \to \propto$,
  $$\frac{V_{o1\,max}}{V_d} \to \frac{4}{\pi}.$$
- reduces the switching losses
- reduces the amplitudes of the harmonic voltages. However the number of lower order harmonics in the output voltage is increased.
- Synchronous modulation should be used for the modified SPWM scheme.

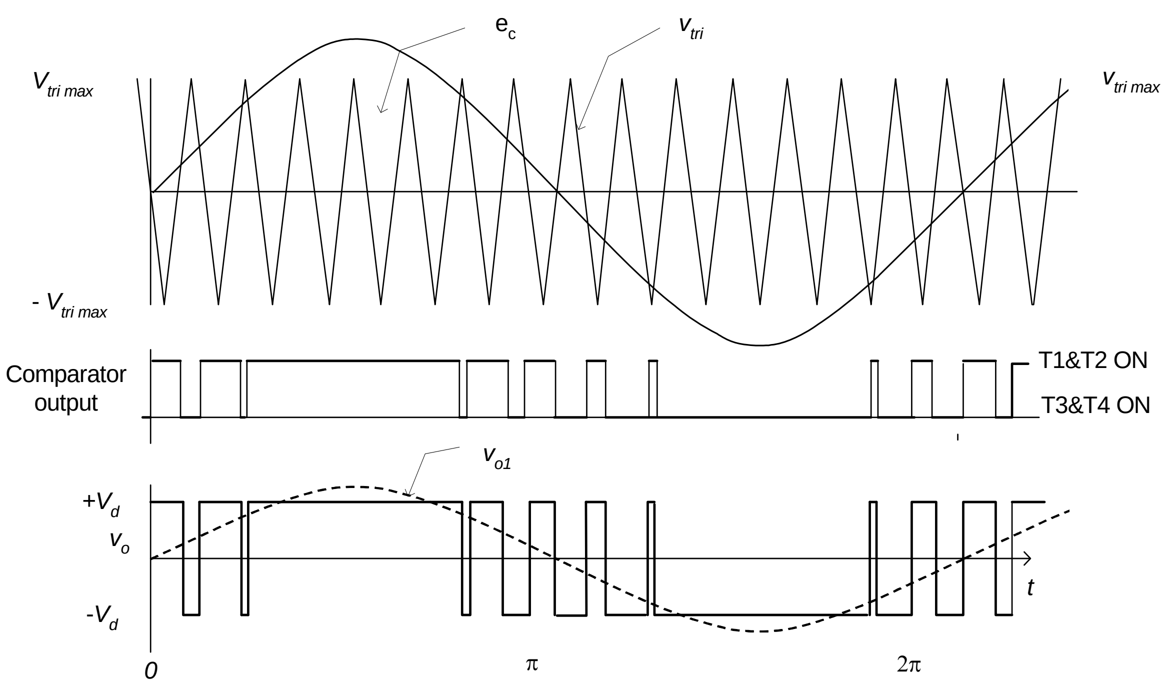

Figure 20.9

For $m > 1$, the fundamental output voltage does not increase proportionately with m, as in the linear range (for $m < 1$). Note that for $m = \infty$, the output voltage waveform becomes a squarewave, with 180°of conduction in each half cycle. The output voltage harmonics are now more spread out.

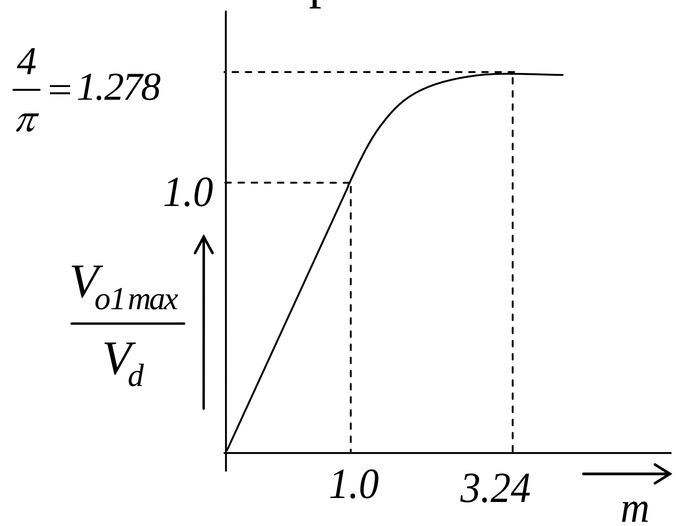

Figure 20.10

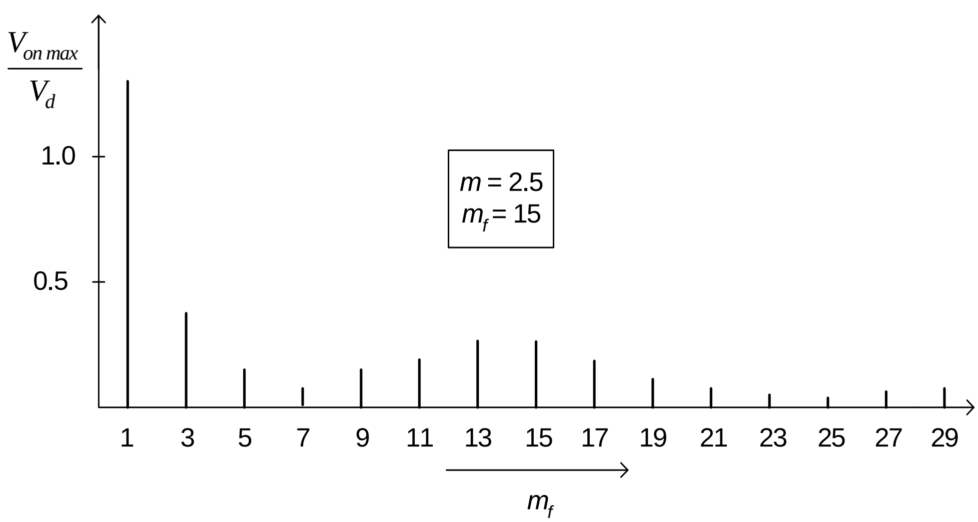

Figure 20.11

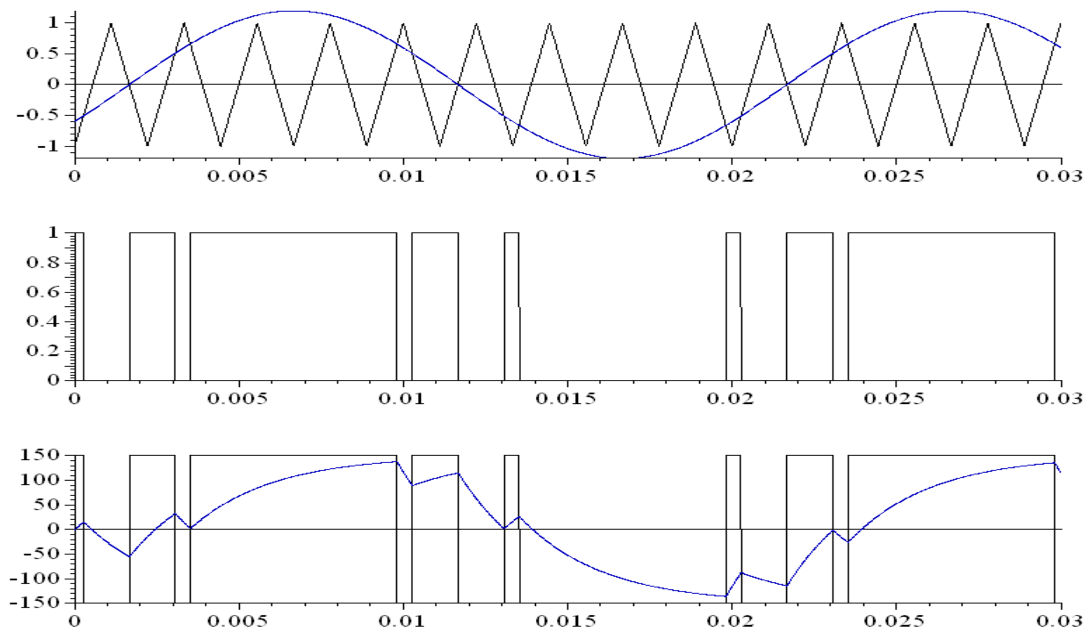

Figure 20.12 $m = 1.2$, $f_s = 450$ Hz, $f_o = 50$ Hz

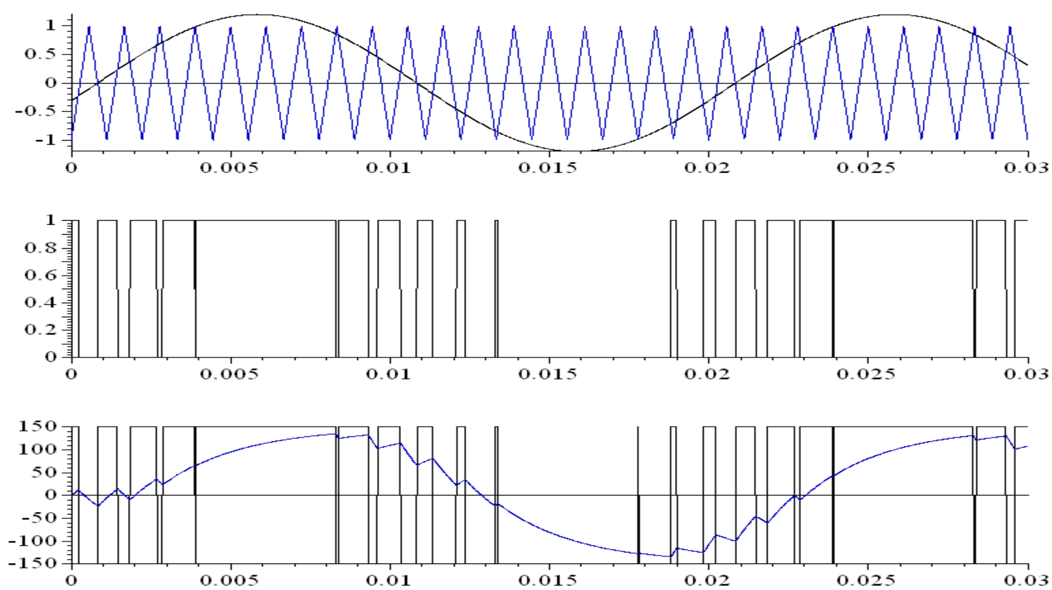

Figure 20.13 $m = 1.2$, $f_s = 900$ Hz, $f_o = 50$ Hz