# Regularized Linear Regression

Andrew Ho

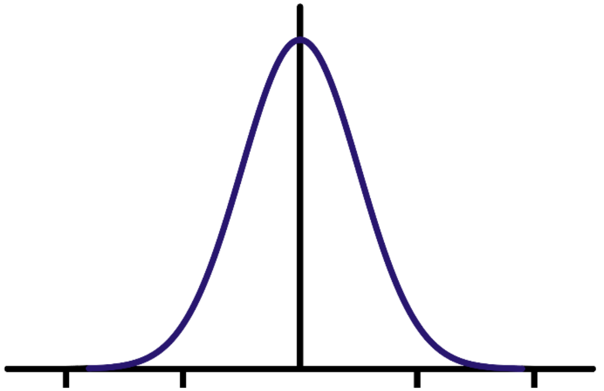
Signal Data Science

# Motivation for regularization

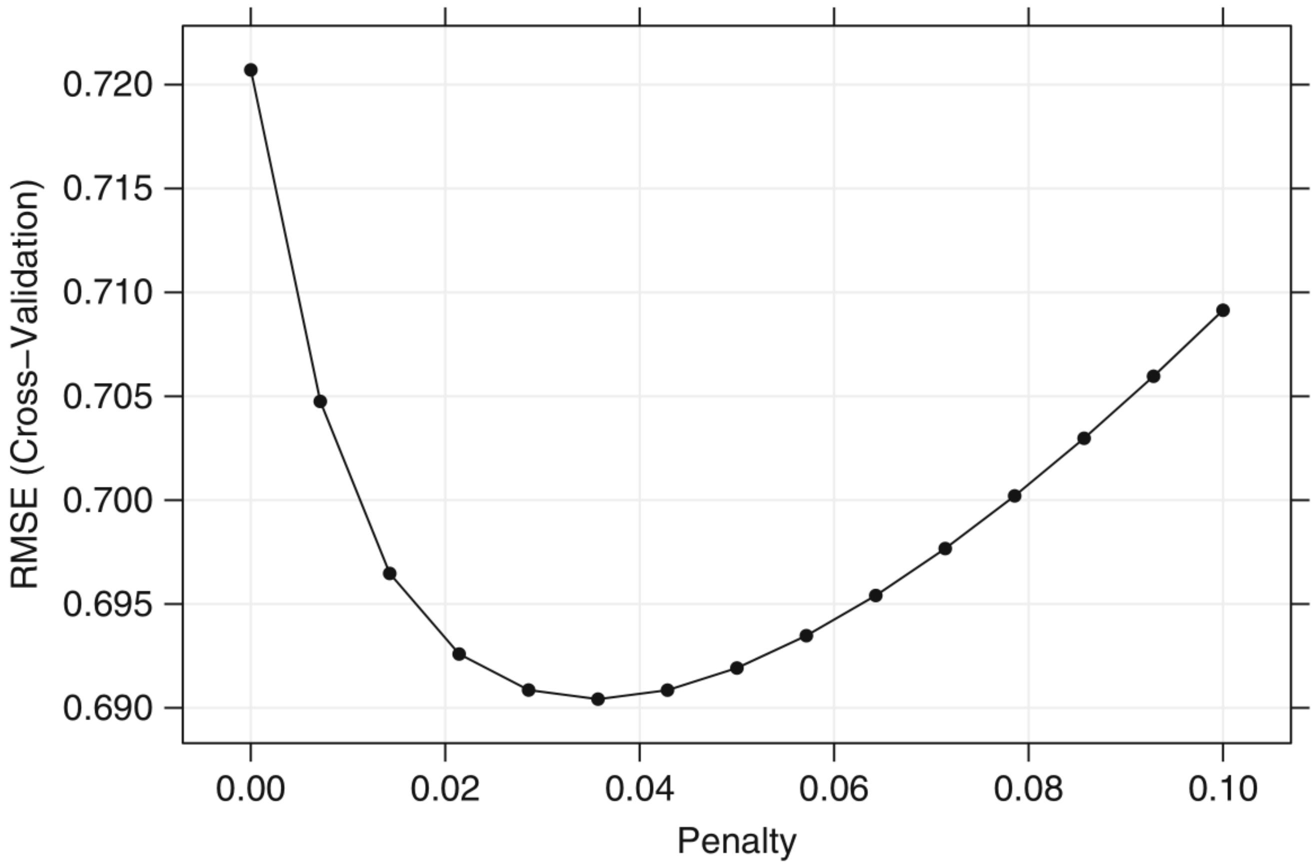
- MSE = (intrinsic error)$^2$ + (bias)$^2$ + variance
- Ordinary least squares is unbiased
  - Isn't the model with the lowest MSE in general
- Introducing some bias can reduce variance
- Two major problems result in inflated coefficients
  - Collinearity among predictors
  - Overfitting

# Two equivalent formulations

- Penalize large coefficients
  - Instead of minimizing sum of squared errors (SSE), minimize SSE + $\lambda$*sum(|coefficients|)
  - Or minimize SSE + $\lambda$*sum(|coefficients|$^2$)
- Impose Bayesian prior on coefficients
  - Use a Gaussian or Laplacian prior for coefficients being closer to 0



# Penalty leads to lower RMSE

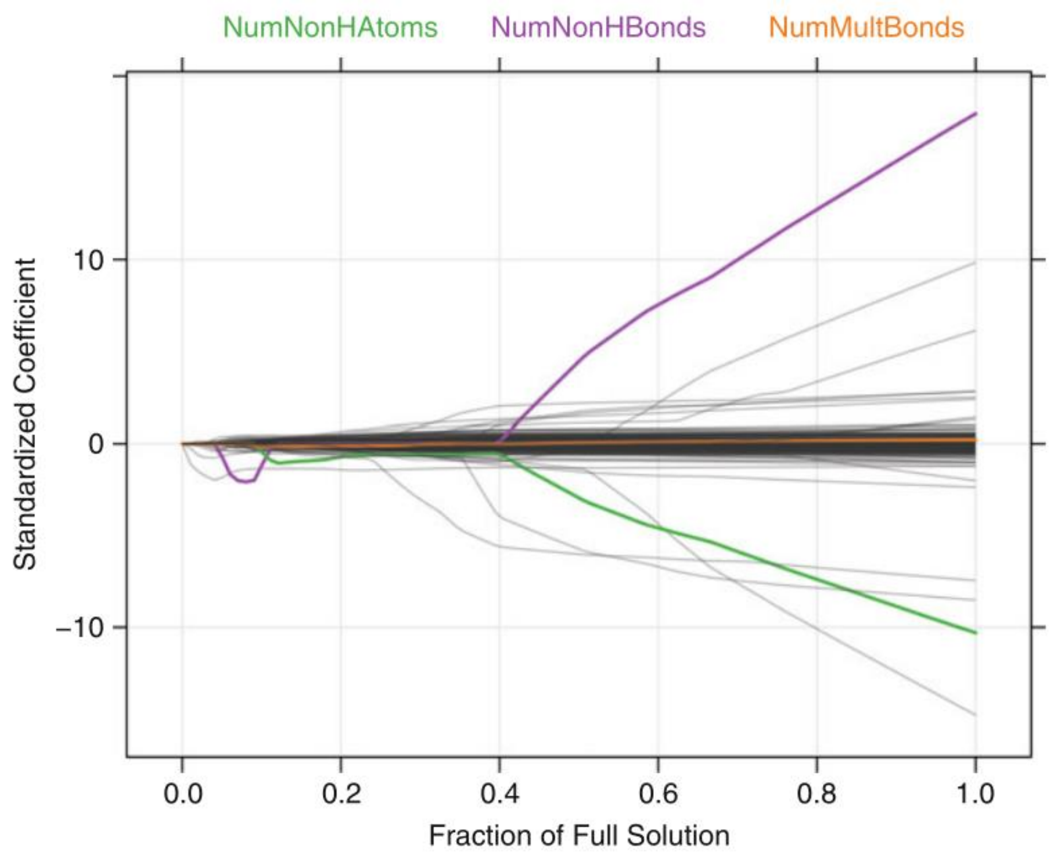



*(Applied Predictive Modeling, p. 125)*

# Lasso and ridge regression

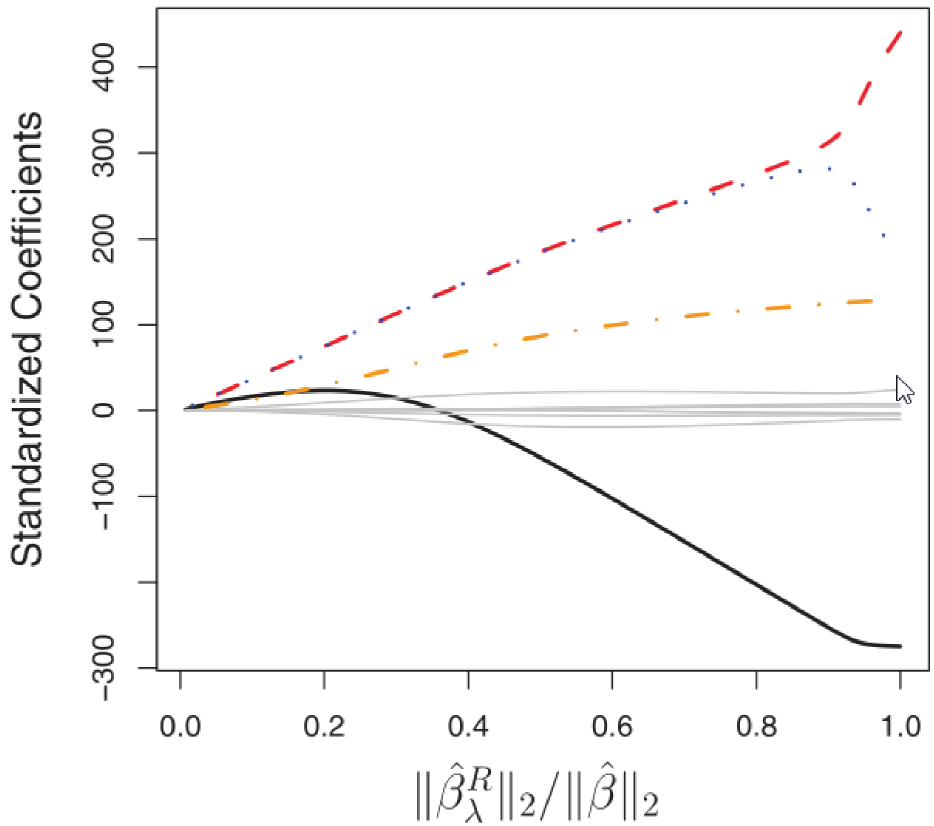
- Minimizing SSE + λ*sum(|coefficients|)
  - Called "lasso" or "$L^1$ penalization"
  - Tends to shrink some coefficients to 0 and leave others
  - Easy to interpret
- Minimizing SSE + λ*sum(|coefficients|$^2$)
  - Called "ridge regression" or "$L^2$ penalization"
  - Tends to shrink coefficients uniformly
- "$L^p$ penalization" comes from notion of $p$-norm
  - $|x|_p = (|x_1|^p + |x_2|^p + \ldots + |x_n|^p)^{1/p}$

# $L^1$ coefficient shrinkage




*(Applied Predictive Learning, p. 126)*

# $L^2$ coefficient shrinkage

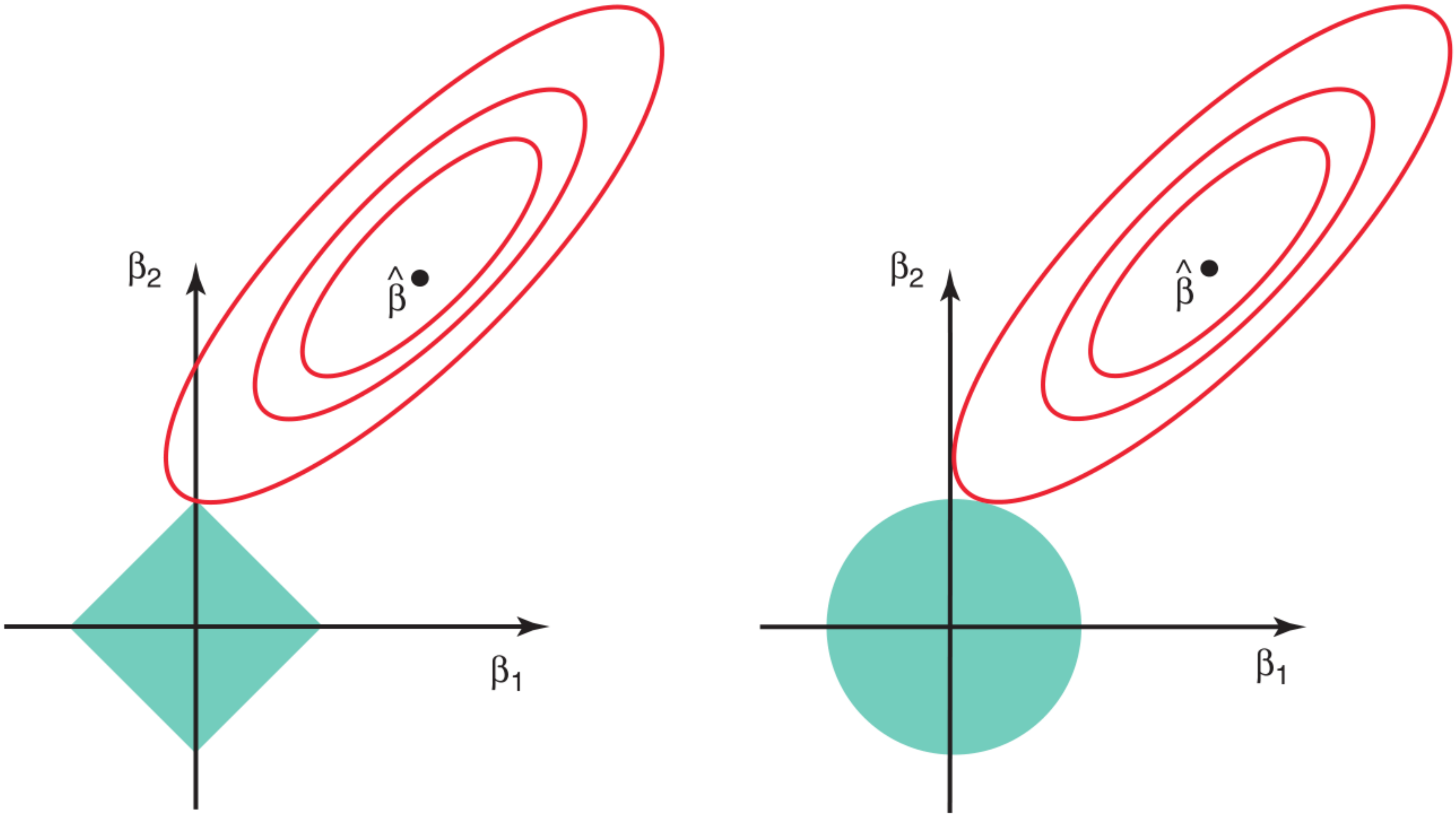



*(Introduction to Statistical Learning, p. 216)*

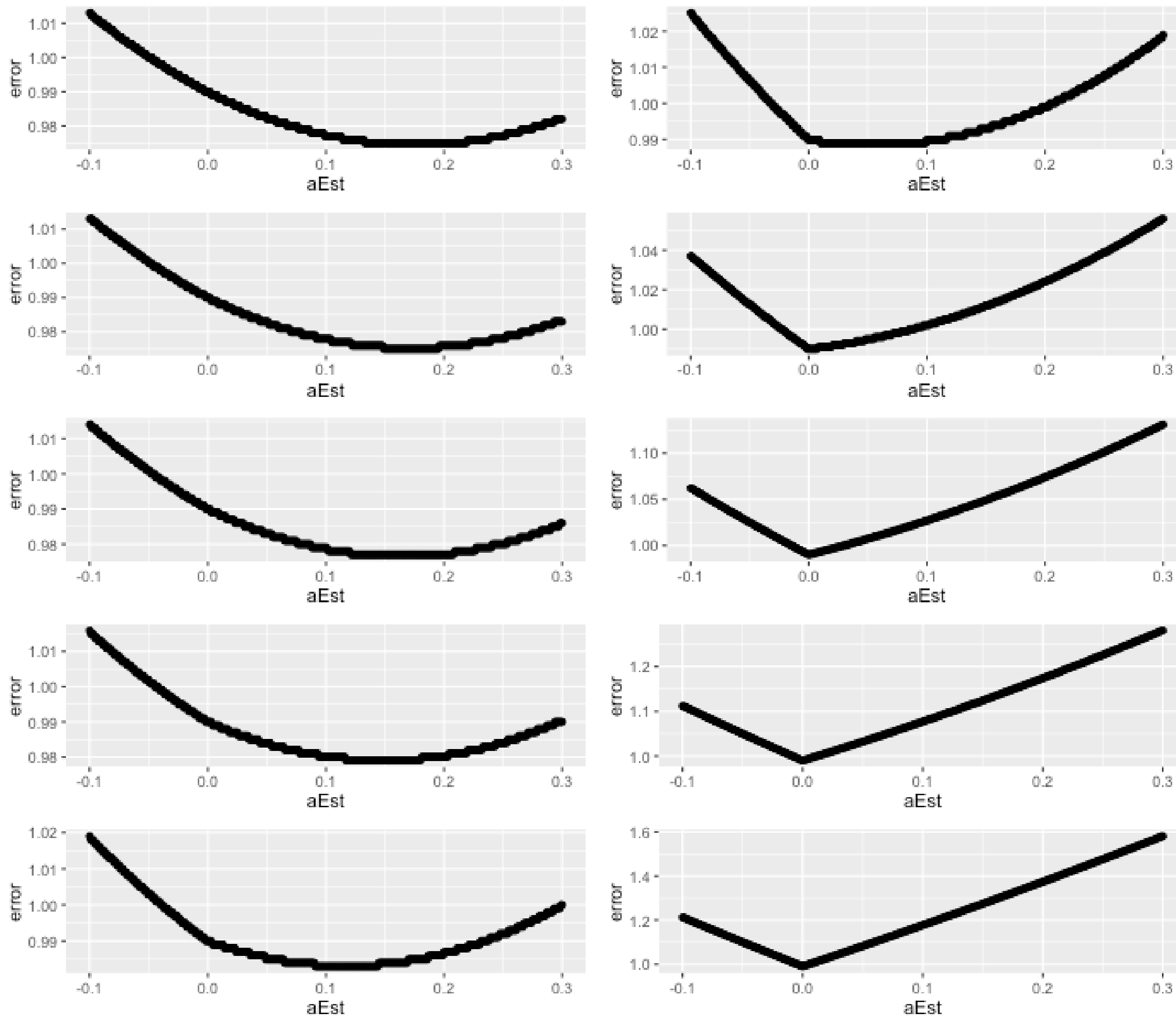
# Duality of optimization

- Two equivalent mathematical formulations
  - Minimizing SSE + λ*sum(|coefficients|)
  - Minimizing SSE *subject to* sum(|coefficients|) ≤ $s_1(\lambda)$
- Same for ridge regression
  - Minimizing SSE + λ*sum(|coefficients|$^2$)
  - Minimizing SSE *subject to* sum(|coefficients|$^2$) ≤ $s_2(\lambda)$

# Visual intuition



# $L^1$ regularization



# $L^1$ regularization