Exercise 1: Paper Submissions *(6 Points)*

Professor Kuhn has to finish writing n different research papers p_i and would like to submit each one of them to a conference. However each paper p_i takes t_i time to be done and submitted by deadline d_i i.e. each deadline is according to the conference that the Professor wants to submit that paper in. Note that when we say by deadline d_i, we mean that at the latest the paper submission should be at time d_i. Moreover, writing any paper is available to be scheduled starting at time s.
Now, the task of writing the full paper p_i and then submitting it needs to be assigned a period from $s_i \geq s$ to $f_i = s_i + t_i$, and doing the same task for different papers should be assigned nonoverlapping intervals. Such an assignment of times will be called a *schedule*.
We consider the case in which writing each paper must either be finished and submitted by its deadline or not at all. We'll say that a subset P of the papers is *schedulable* if there is a schedule where the Professor is able to finish writing each paper in P and submit each of them by its deadline. Your problem is to select a schedulable subset of papers of maximum possible size and give a schedule for this subset that allows each paper to be fully written and submitted by its deadline.

Assume that all deadlines d_i and required times t_i are integers and $d_i \geq t_i$. Give an algorithm to find an optimal solution. Your algorithm should run in time polynomial in the number of papers that needs to be written and submitted n, and the maximum deadline $D = \max_i d_i$. Argue correctness and running time.
Hint: Prove that there is an optimal solution P (i.e., a schedulable set of maximum size) in which the papers in P are scheduled in increasing order of their deadlines.

t d

(3, 8), (4, 10), (1, 3), (2, 2), (8, 9)

3 5 2 1 4

[0, 1, 2, 3, 3, 4]

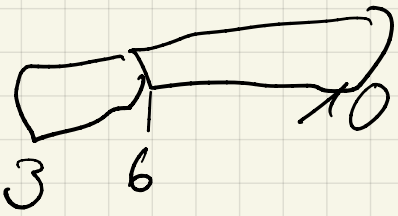
P(4, 0)

P(0, d) = 0

$P(k, d) = \max(P(k-1, d), 1 + P(k', d - t_k))$

max d

$k' \in n : d - t_k - t_{k'} \geq 0$

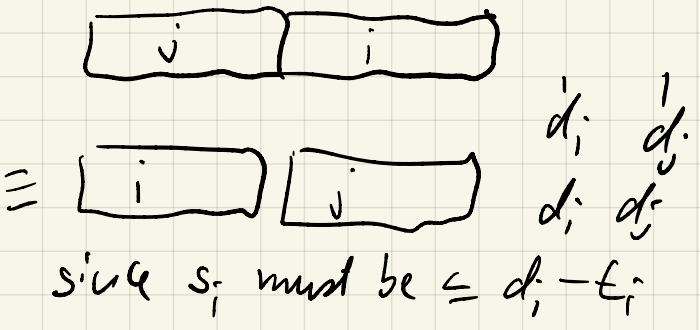


runtime $(d \cdot n + k \log k)$

1 2 .... D

1
2
⋮
k

exchange argument:



since s_i must be $\leq d_i - t_i$