Course Code: MAT 211

Course Title: Co-ordinate Geometry and Vector Analysis

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Contents:

- Co-ordinates
- Polar Co-ordinates
- The straight line

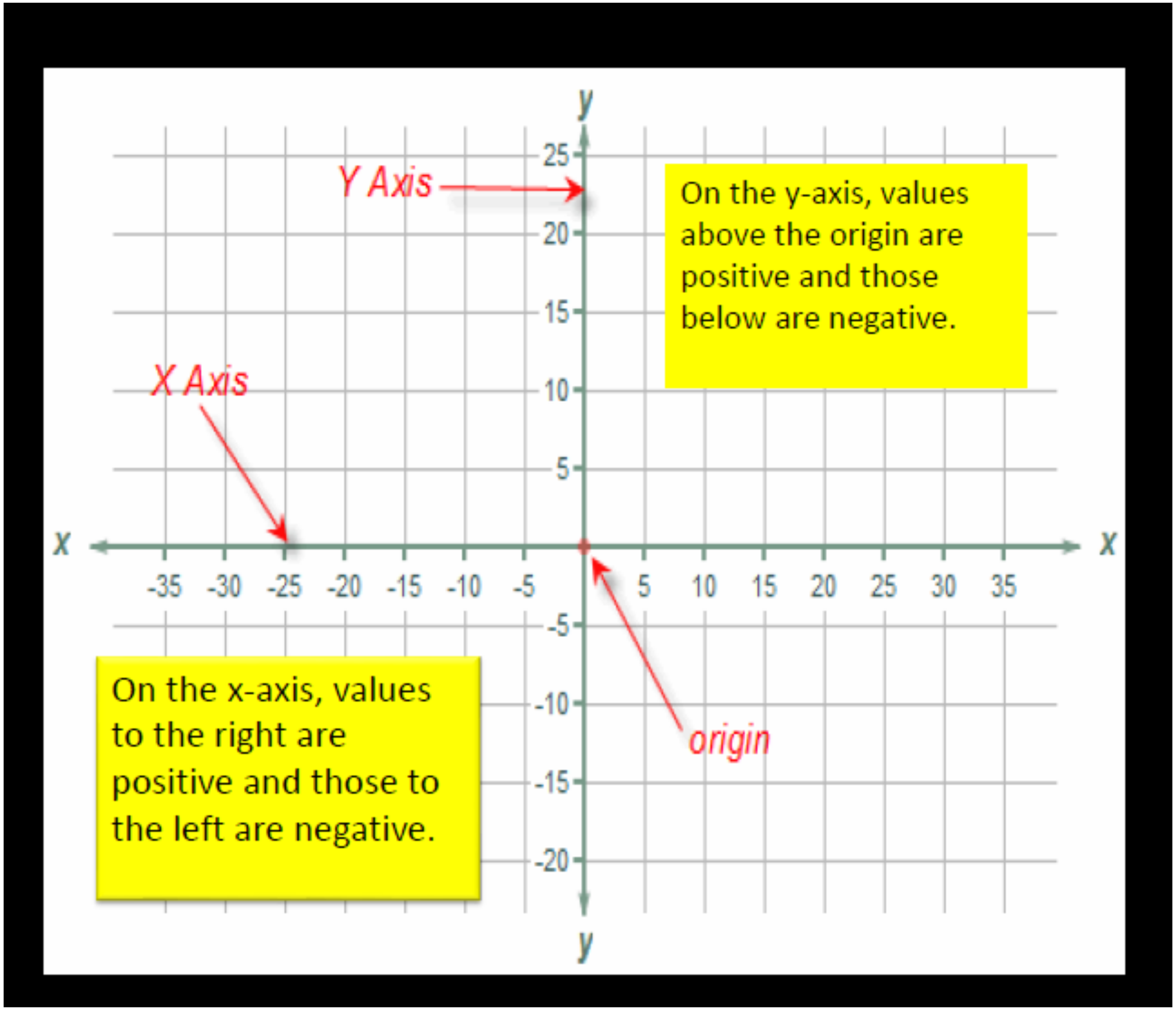
What is Coordinate Geometry?

A system of geometry where the position of points on the plane is described using an ordered pair of numbers.

Founder of Coordinate Geometry

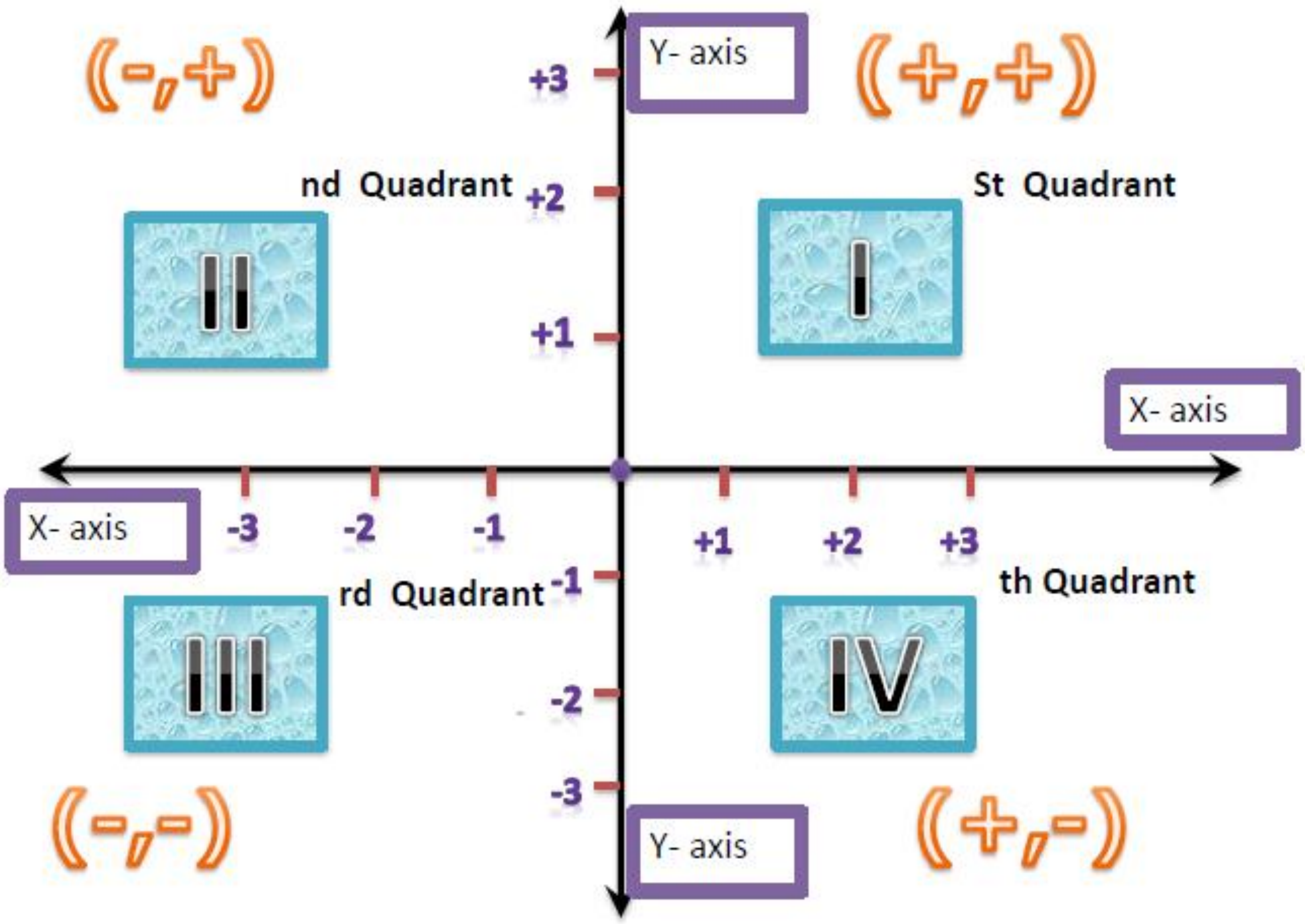
French Mathematician René Descartes
(1596 - 1650).

- He proposed further that curves and lines could be described by equations using this technique, thus being the first to link algebra and geometry.
- In honor of his work, the coordinates of a point are often referred to as its Cartesian coordinates, and the coordinate plane as the Cartesian Coordinate Plane.



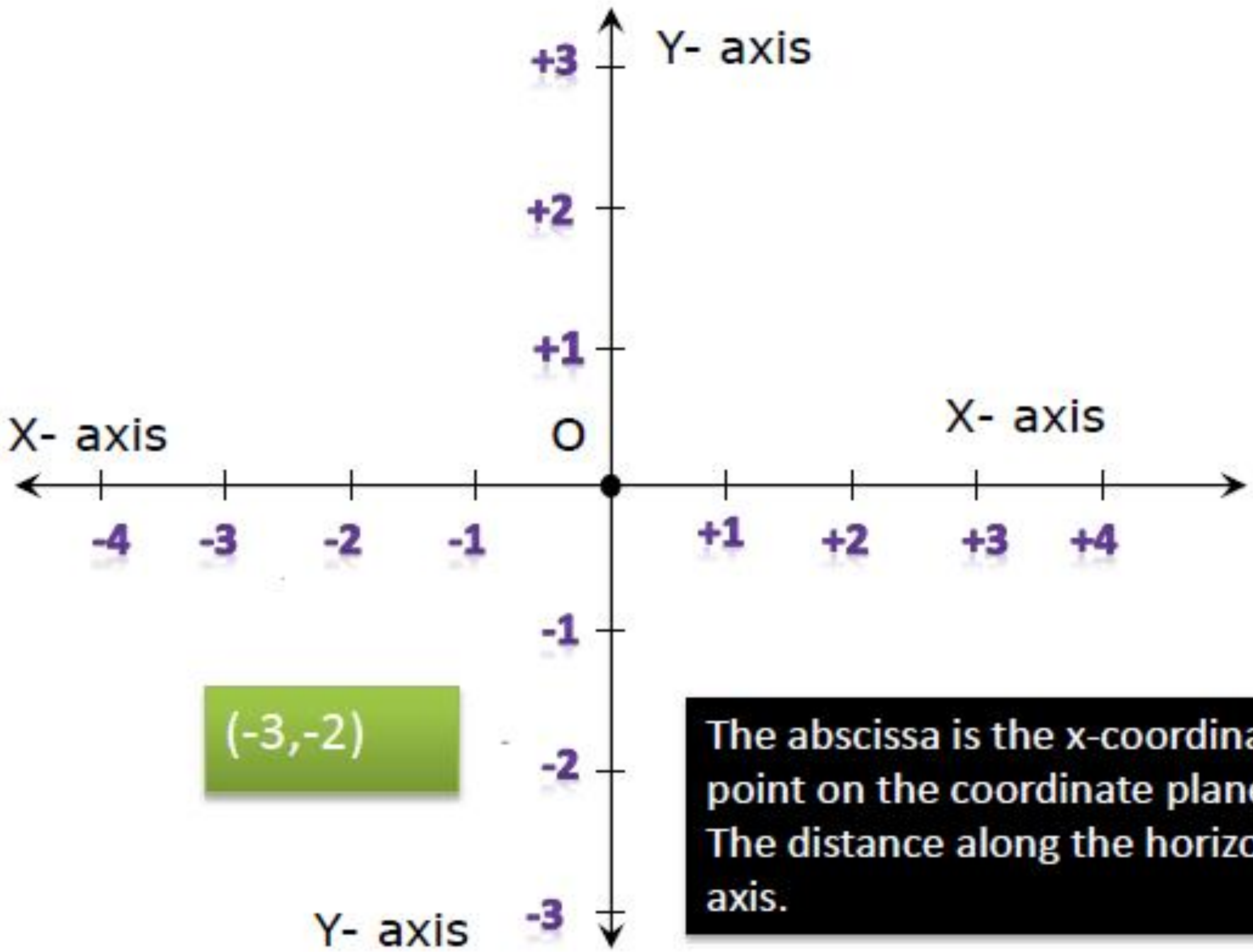
y
Y Axis
On the y-axis, values above the origin are positive and those below are negative.
X Axis
x
x
-35 -30 -25 -20 -15 -10 -5 5 10 15 20 25 30 35
25 20 15 10 5 -5 -10 -15 -20
On the x-axis, values to the right are positive and those to the left are negative.
origin
y

Quadrant



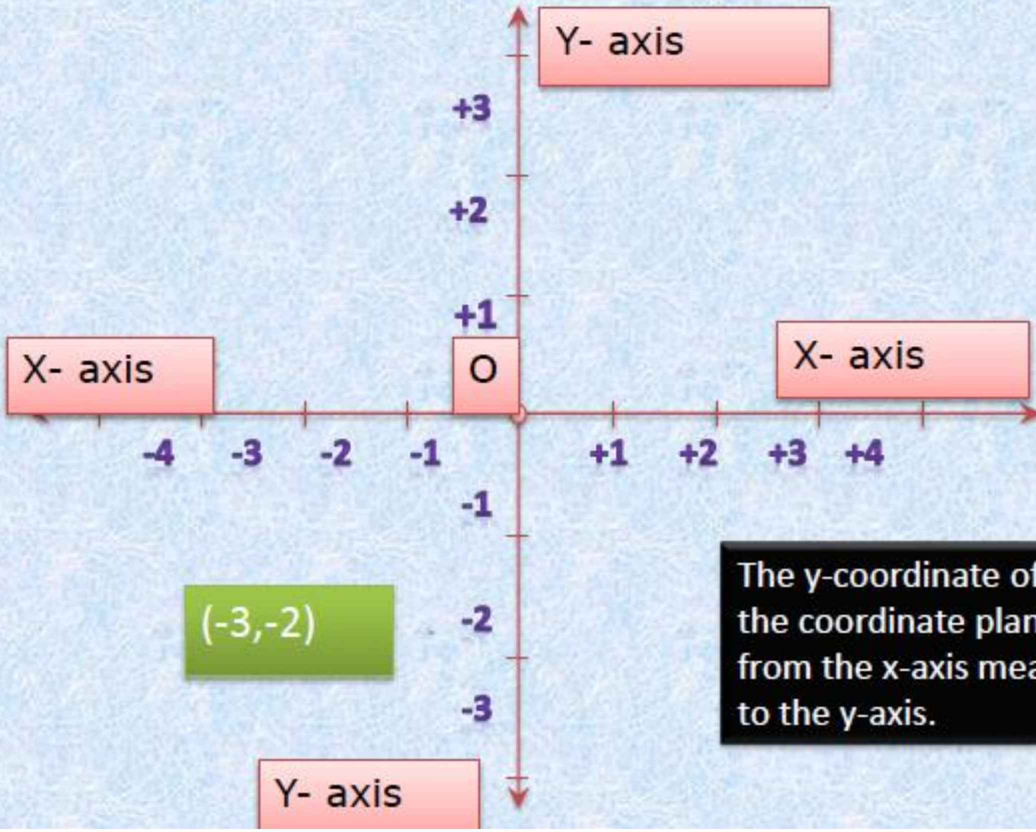
Y- axis
(-,+)
(+,+)
+3
nd Quadrant
+2
St Quadrant
II
I
+1
X- axis
X- axis
-3
-2
-1
+1
+2
+3
rd Quadrant
-1
th Quadrant
III
IV
-2
-3
(-,-)
(+,-)
Y- axis

Abscissa



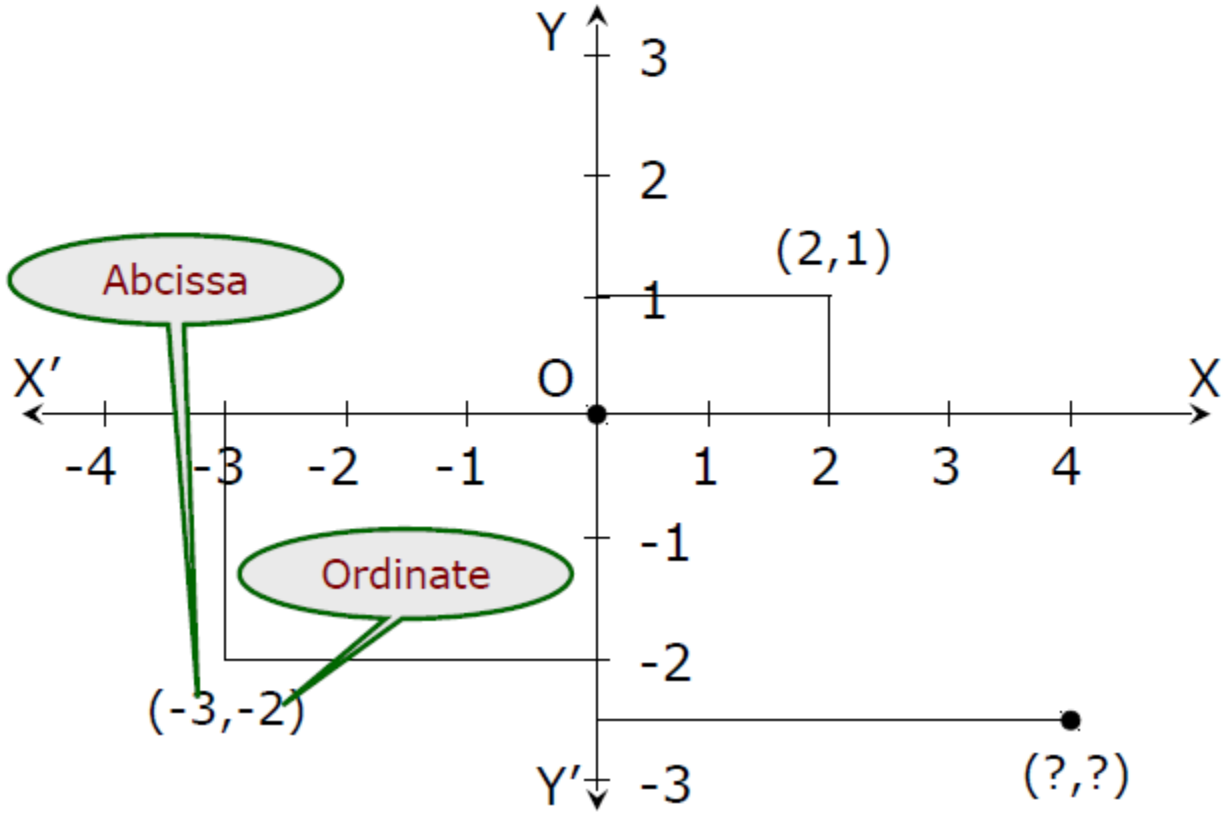
The abscissa is the x-coordinate of a point on the coordinate plane.
The distance along the horizontal axis.

Ordinate



The y-coordinate of a point on the coordinate plane : its distance from the x-axis measured parallel to the y-axis.

Coordinates



2-Dimensinoal Co-ordinate Geometry

Introduction to Coordinate Geometry

A system of geometry where the position of points on the plane is described using an ordered pair of numbers. Recall that a plane is a flat surface that goes on forever in both directions. If we were to place a point on the plane, coordinate geometry gives us a way to describe exactly where it is by using two numbers.

Definition of plane coordinate: Plane coordinate geometry is that branch of geometry in which we use two numbers called coordinates to indicate the position of a point in a plane.

Abscissa: The distance from the y-axis is called the X co-ordinate or abscissa of the point.

Ordinate: The distance from the X-axis is called the Y co-ordinate or ordinate of the point.

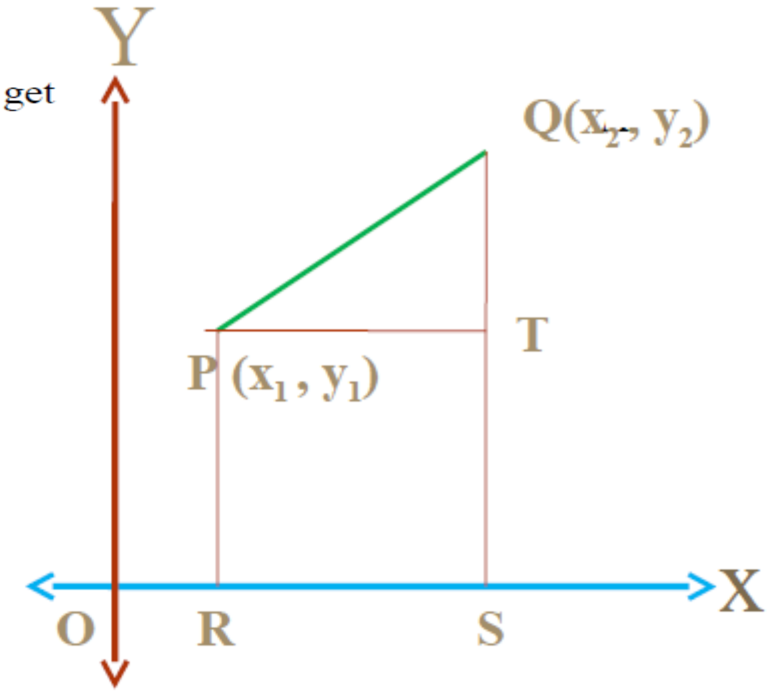
Coordinate: The two distances taken together are called the co-ordinates of the point & are represented by the symbol (x,y)

What is quadratic equation?

Ans: An equation containing a single variable of degree 2, of the form $ax^2 + bx + c = 0$, where x is the variable and a, b, and c are constants $(a \neq 0)$ is called a quadratic equation.

Distance Formula

- Let us now find the distance between any two points **$P(x_1, y_1)$** and **$Q(x_1, y_2)$**



- Now, applying the Pythagoras theorem in **ΔPTQ,** we get

$$PQ^2 = PT^2 + QT^2$$

Therefore

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which is called the **distance formula.**

Section Ratio: The co-ordinate of a point R(x, y) which divides the join of two points P(x_1, y_1) and Q(x_1, y_1) in a given ratio $m_1 : m_2$

Internally are:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_2 + m_1} \qquad y = \frac{m_1 y_2 + m_2 y_1}{m_2 + m_1}$$

Externally are:

$$x = \frac{m_1 x_2 - m_2 x_1}{m_2 - m_1}$$

$$y = \frac{m_1 y_2 - m_2 y_1}{m_2 - m_1}$$

Midpoint

Midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$

$m:n \equiv 1:1$

$$\therefore P \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Find the Mid-Point of P(1,-3) and Q(5,7).

$$Mid-point\ of\ PQ : \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

$$= \left(\frac{5+1}{2}, \frac{7+-3}{2} \right)$$

$$= \left(\frac{6}{2}, \frac{4}{2} \right) = (3, 2)$$

Determine the co-ordinates of the points R(x, y) which divides the join of points P(-2, 1) and Q(3,-4) externally in the ratio 8:3

Solution: R (x, y) must be outside segment PQ

$$X=\frac{8.3-3(-2)}{8-3} \qquad \text{and} \qquad y=\frac{8(-4)-3(1)}{8-3}$$

$=6$ $\qquad\qquad\qquad\qquad$ $= -7$

Hence R(6,-7)

#In what ratio is the straight line joining the points (3, 4) and (8, 1) divided by the x-axis? Find the abscissa of this point on the x-axis.

Solution

The st. line is cut by x-axis at (x_1, o) in the ratio $k : 1$

$$0=\frac{ky_2+y_1}{k+1}=\frac{k1+4}{k+1} \quad \text{or,} \quad k+4=0$$

or, $k=-4$, i e, $-4:1$ is the reqd. ratio.

$$x=\frac{kx_2+x_1}{k+1}=\frac{-4.8+3}{-3}=\frac{29}{3}$$

Ex: The co-ordinates of A, B, C, D are (6,3), (-3,5), (4,-2) and (x,3x) respectively and $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, when Δ is the area of the triangle. Find x.

Solution:

$$\Delta DBC = \frac{1}{2}\begin{bmatrix} x & 3x & 1 \\ -3 & 5 & 1 \\ 4 & -5 & 1 \end{bmatrix} = 14x-7$$

$$\Delta ABC = \frac{1}{2}\begin{bmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{bmatrix} = \frac{49}{2}$$

Given $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$

$$x = \frac{11}{8}$$

. the area of the triangle formed by three given points

Area of triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_2) is :

$$\frac{1}{2}\left\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right\}$$

2 Find the area of the triangle whose vertices are (2, 3), (5, 7), (−3, 4)

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 3 \\ 5 & 7 \\ -3 & 4 \\ 2 & 3 \end{vmatrix} = \frac{1}{2}.\,[2.7 + 5.4 + (-3)\,3 - 2.4 - (-3)\,(7) - 5.3]$$

$= 11.5$ square units.

Ex. 3 Find the area of a pentagon whose vertices are (−5, −2), (−2, 5), (2, 7), (5, 1), (2, −4)

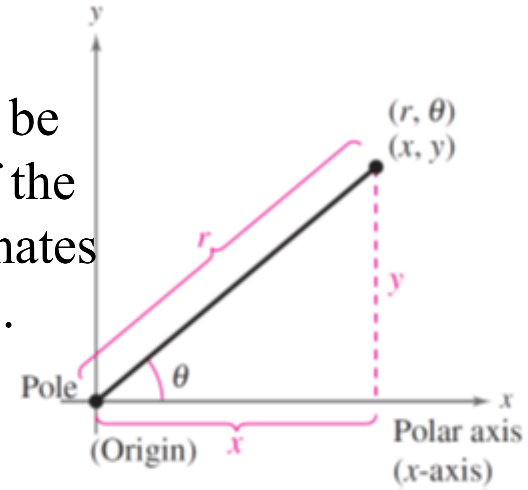
$$S = \frac{1}{2}\begin{vmatrix} -5 & -2 \\ -2 & 5 \\ 2 & 7 \\ 5 & 1 \\ 2 & -4 \\ -5 & -2 \end{vmatrix} = \frac{1}{2}\,[(-5)\,5 + (-2)\,(7) + (2)\,(1) + (5)\,(-4) + (2)\,(-2) - (-5)\,(-4) - (2)\,(1) - (5)\,(7) - (2)\,(5) - (-2)\,(-2)]$$

$$= \frac{1}{2}\,(-132) = -66$$

Ans. 66 square units

Establish the relationship between polar and rectangular coordinates

Let the pole and the initial line of the polar system be the same as the origin and the positive direction of the x-axis of the cartesian system. Suppose the coordinates of a point be (r, θ) and cartesian coordinates (x, y).



Coordinate Conversion

The polar coordinates (r, θ) are related to the rectangular coordinates (x, y) as follows.

Polar-to-Rectangular	*Rectangular-to-Polar*
$x = r\cos\theta$	$\tan\theta = \dfrac{y}{x}$
$y = r\sin\theta$	$r^2 = x^2 + y^2$

Convert the point $(2, \pi/3)$ from polar to Cartesian coordinates.

- Since $r = 2$ and $\theta = \pi/3$, Equations 1 give:

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\cdot\frac{1}{2} = 1$$

$$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2.\frac{\sqrt{3}}{2} = \sqrt{3}$$

- Thus, the point is $(1, \sqrt{3})$ in Cartesian coordinates.

Represent the point with Cartesian coordinates (1, –1) in terms of polar coordinates.

If we choose r to be positive, then Equations 2 give:

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\tan\theta = \frac{y}{x} = -1$$

- As the point (1, –1) lies in the fourth quadrant, we can choose $\theta = -\pi/4$ or $\theta = 7\pi/4$.

Straight line

- A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

Coordinate Geometry in the (x,y) plane

Equation of a straight line

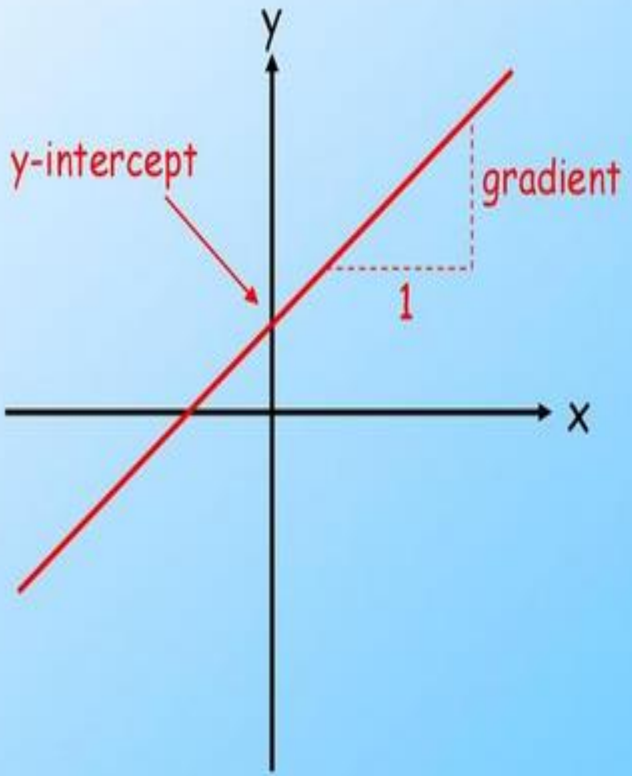
The equation of a straight line is usually written in one of 2 forms. One you will have seen before;

$$y = mx + c$$

Where m is the gradient and c is the y-intercept.

Or, the general form:

$$ax + by + c = 0$$



Coordinate Geometry in the (x,y) plane

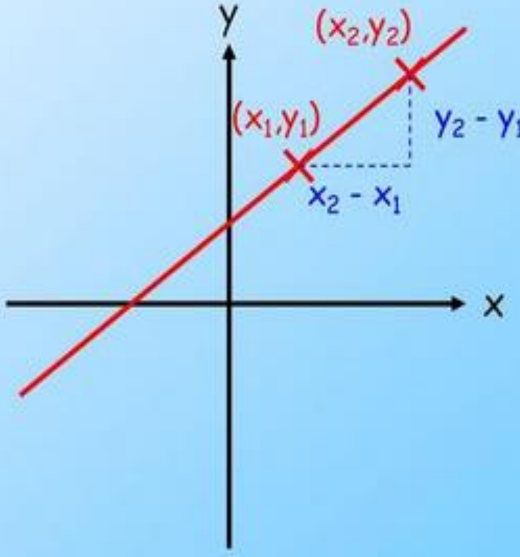
The gradient of a line

You can work out the gradient of a line if you know 2 points on it.

Let the first point be (x_1,y_1) and the second be (x_2,y_2). The following formula gives the gradient:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

'The change in the y values, divided by the change in the x values'



Coordinate Geometry in the (x,y) plane

The gradient of a line

You can work out the gradient of a line if you know 2 points on it.

Let the first point be (x_1,y_1) and the second be (x_2,y_2). The following formula gives the gradient:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

'The change in the y values, divided by the change in the x values'

Example 1

Calculate the gradient of the line which passes through (2,3) and (5,7)

$(x_1, y_1) = (2, 3)$

$(x_2, y_2) = (5, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute numbers in

$$m = \frac{7 - 3}{5 - 2}$$

Work out or leave as a fraction

$$m = \frac{4}{3}$$

Coordinate Geometry in the (x,y) plane

Finding the Equation of a line

You can find the equation of a line from 2 points by using the following formula:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

Example 1

Work out the equation of the line that goes through points (3,-1) and (5, 7). Give your answer in the form y = mx + c.

$(x_1, y_1) = (3, -1)$

$(x_2, y_2) = (5, 7)$

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

Substitute in values

$$\frac{y-(-1)}{7-(-1)}=\frac{x-3}{5-3}$$

Work out any sums

$$\frac{y+1}{8}=\frac{x-3}{2}$$

Multiply the right side by 4 to make fractions the same

$$\frac{y+1}{8}=\frac{4x-12}{8}$$

Multiply by 8

$$y+1=4x-12$$

Subtract 1

$$y=4x-13$$

find the slope of a line which passes through point (3, 2) and (-1, 5).

Sol :- we known that the slope of a line passing through two point (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

here the line is passing through the point (3 ,2) and (-1 , 5). So, Its slope is given by

$$m = \frac{5 - 2}{-1 - 3} = \frac{-3}{4}$$

find the slope of line:

1. passing through (3, -2)and (-1, 4)
2. passing through (3, -2) and (7, -2)

Sol :- 1. slope of line through (3, -2) and (-1, 4)

$$m = \frac{4-(-2)}{-1-3} = \frac{6}{-4} = -\frac{3}{2}$$

2. The slope of line through (3, -2) and (7, -2)

$$m = \frac{-2-(-2)}{7-3} = \frac{0}{4} = 0$$

If A (-2, 1) , B (2, 3) and C (-2, -4) are three points, find the angle between BA and BC

Sol :- let m1 and $m2$ be the slope of BA and BC respectively. then,

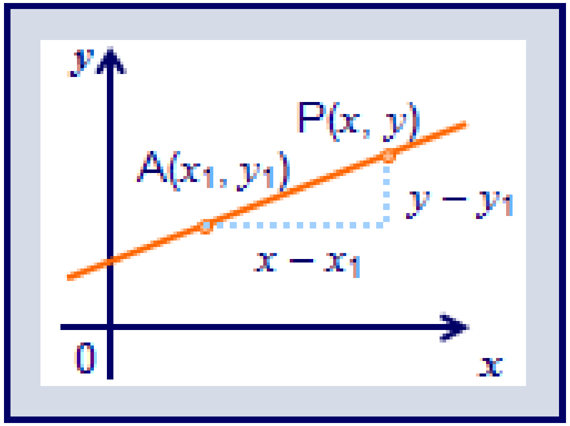
$$m1 = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2} \qquad m2 = \frac{-4-3}{-2-2} = \frac{7}{4}$$

$$\text{Tan}\,\theta = \frac{m2 - m1}{1 + m1\,m2} = \frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{7}{4} \times \frac{1}{2}} = \frac{10/8}{15/8} = \pm\, 2/3$$

$$\Longrightarrow \theta = \tan^{-1}(2/3)$$

Finding the equation of a line

Suppose a line passes through A(x_1, y_1) with gradient m.



Let P(x, y) be any other point on the line.

The gradient of AP = $\dfrac{y - y_1}{x - x_1}$

So $$\frac{y - y_1}{x - x_1} = m$$

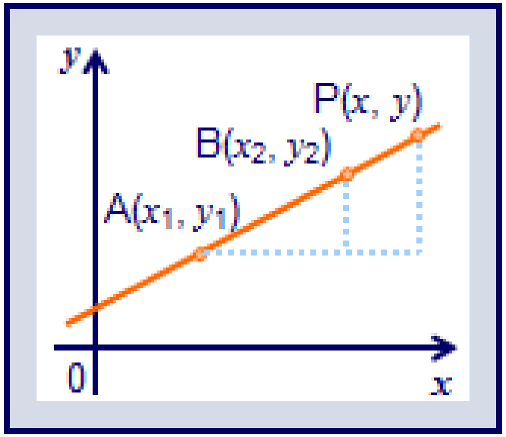
This can be rearranged to give $y - y_1 = m(x - x_1)$.

In general:

> The equation of a line through A(x_1, y_1) with gradient m is
>
> $$y - y_1 = m(x - x_1)$$

Finding the equation of a line

Suppose a straight line passes through the points A(x_1, y_1) and B(x_2, y_2) with another point on the line P(x, y).



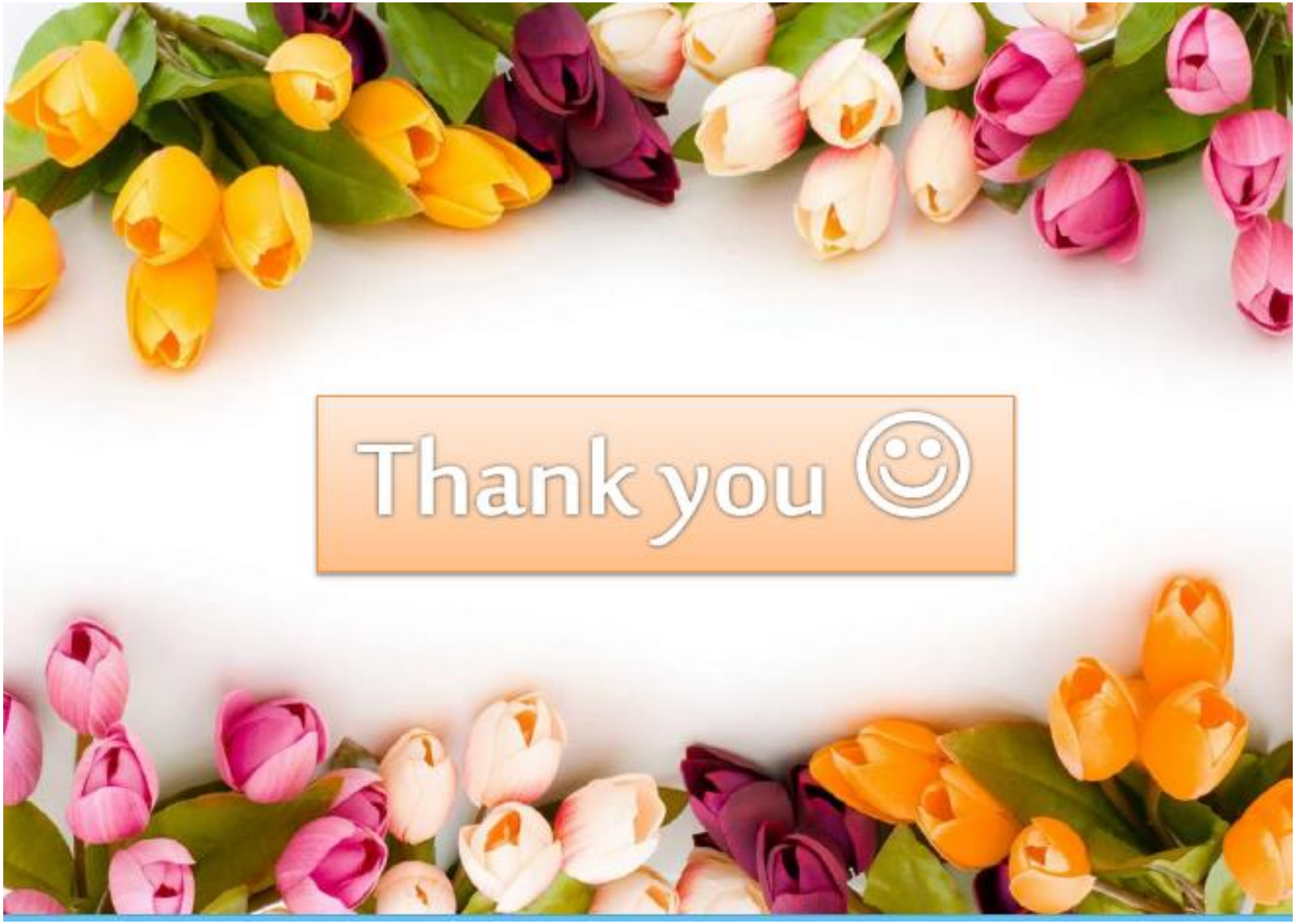
The gradient of AP = the gradient of AB.

So $$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Or $$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

The equation of a line through A(x_1, y_1) and B(x_2, y_2) is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$


Thank you ☺