Course Code: MAT 211

Course Title: Coordinate Geometry and Vector Analysis

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Contents:

- **Pair of Straight Line**

An equation obtained by multiplying two linear equations is called equation of a pair of straight lines. It is the combined form of the equations of two straight lines. It is also called general equation of second degree.

Pair of Straight Line

Let the general equation of 2nd degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Let (α, β) be the point of intersection $\alpha = \frac{hf-bg}{ab-h^2}, \beta = \frac{gh-af}{ab-h^2}$

Let θ be the angle between them $\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$

Problem.1:

Show that the equation $2y^2-xy-x^2+y+2x-1=0$ will represent a pair of straight line. Also finds out the intersecting point and angle between them.

Solution:

Given that ,$2y^2-xy-x^2+y+2x-1=0$..............(i)

comparing this with $ax^2+2hxy+by^2+2gx+2fy+c=0$ we get

$$a=-1,b=2,h=\frac{-1}{2},g=1,f=\frac{1}{2},c=-1$$

$$\Delta=abc+2fgh-af^2-bg^2-ch^2$$

$$=(-1.2.-1)+2(\frac{1}{2}.1.\frac{-1}{2})-(-1)(\frac{1}{2})^2-2(1)^2-(-1)(\frac{-1}{2})^2$$

$$=2-\frac{1}{2}+\frac{1}{4}-2+\frac{1}{4}$$

$$=0$$

The given equation represent a pair of stright line.

Let the intersecting point be (α,β)

$$\alpha = \frac{hf-bg}{ab-h^2} = \frac{(\frac{-1}{2}.\frac{1}{2})-(2.1)}{-2-\frac{1}{4}} = \frac{-2-\frac{1}{4}}{-2-\frac{1}{4}} = 1$$

$$\beta = \frac{gh-af}{ab-h^2} = 0$$

So the intersecting point is (1, 0)

Now the Angle

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$or, \theta = \tan^{-1}(3)$$

Problem.2: Show that the equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$will represent a pair of straight line. Also finds out the intersecting point and angle between them.

Solution:

Comparing with general equation of 2nd degree

$$a = 6, b = -6, h = -\frac{5}{2}, g = 7, f = \frac{5}{2}, c = 4.$$

$$\Delta = abc + 2fgh - bg^2 - af^2 - ch^2$$

$$= 6.(-6).4 + 2.\frac{5}{2}.7.\left(-\frac{5}{2}\right) + 6.49 - 6\frac{25}{4} + 4.\frac{25}{4}$$

$$= -144 - \frac{175}{2} + 294 - \frac{75}{2} - 25$$

$$= \frac{-576 - 350 + 1176 - 150 - 100}{4}$$

$$= 0.$$

So the given equation represents the pair of straight line.

Let the intersecting point be (α,β)

$$\alpha=\frac{hg\text{-}bg}{ab\text{-}h^2}=\frac{(\frac{-5}{2}.\frac{5}{2})\text{-}(-6.7)}{(-6.6)\text{-}(\frac{5}{2})^2}=\frac{-\frac{25}{4}+42}{-36\text{-}\frac{25}{4}}=\frac{\frac{-25+168}{4}}{\frac{-144-25}{4}}=-\frac{-143}{169}$$

$$\beta=\frac{hg\text{-}af}{ab\text{-}h^2}=\frac{(\frac{-5}{2}.7)\text{-}(6.\frac{5}{2})}{-36\text{-}\frac{25}{4}}=\frac{\frac{-35}{2}-\frac{30}{2}}{-2\text{-}\frac{1}{4}}=\frac{\frac{-65}{2}}{\frac{-144-25}{4}}=\frac{130}{169}$$

So the intersecting point is $(\frac{-143}{169},\frac{130}{169})$

Now the Angle

$$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}=\frac{2\sqrt{\frac{25}{4}+36}}{-6+6}=\frac{2\sqrt{\frac{25}{4}+36}}{0}=\infty$$

$$or,\theta=\tan^{-1}(\infty)$$

$$or,\theta=90^\circ$$

Try yourself

Show that the equation $x^2+6xy+9y^2+14x+12-5=0$ will represent a pair of straight line. Also finds out the intersecting point and angle between them.

Problem 1: Find the value of λ so the equation $\lambda x^2+4xy+y^2-4x-2y-3=0$ may represent a pair of straight lines.

Solution: The given equation is $\lambda x^2+4xy+y^2-4x-2y-3=0$………… (1)

Comparing the equation with the second degree equation, we get

a= λ , h=2,b=1,g=-2,f=-1,c=-3

Now

$$\Delta = abc+2fgh-af^2-bg^2-ch^2$$

$$=\lambda.1.(-3)+2(-1)(-2)2-\lambda(1)^2-1(-2)^2+3(2)^2$$

$$=-3\lambda+8-\lambda-4+12$$

$$=-4\lambda+16$$

If the given equation represent a pair of straight lines $\Delta=0$

$$-4\lambda+16=0$$

$$0r, 4\lambda=16$$

$$\therefore \lambda=4$$

Problem 2: Find the value of k so the equation $kx^2+4xy-y^2-2x-2y-3=0$ may represent a pair of straight lines.

Solution: The given equation is $kx^2+4xy-y^2-2x-2y-3=0$ .... (1)

Comparing the equation with the second degree equation, we get
a=k, h=2,b=-1,g=-1,f=-1,c=-3

$$\Delta = abc+2fgh-af^2-bg^2-ch^2$$
$$=k.-1.(-3)+2(-1)(-1)2-k(-1)^2-(-1)(-1)^2-(-3)(2)^2$$
$$=3k+4-k+1+12$$
$$=2k+17$$

(i) will represent a pair of st line if $\quad 2k+17=0$

$$or, k=-\frac{17}{2}$$

Try yourself : Find the value of λ *or* k so that the following equations may represent pairs of straight lines.

I. $\lambda x^2 + 4xy + y^2 - 4x - 2y - 3 = 0$

II. $2x^2 - y^2 + xy - 2x - 5y + k = 0$

III. $6x^2 + xy + ky^2 - 11x + 43y - 35 = 0$

IV. $x^2 - \lambda xy + 2y^2 + 3x - 5y + 2 = 0$

V. $kxy - 8x + 9y - 12 = 0.$

Problem 1: Find the equations represented by the second degree homogeneous equation $x^2 - 10xy + 9y^2 = 0$.

Solution: the given equation is

$$x^2 - 10xy + 9y^2 = 0$$

$$x^2 - 10y.x + 9y^2 = 0$$

$$x = \frac{-(-10\,\mathrm{y}) \pm \sqrt{(-10\,\mathrm{y})^2 - 4.9.1y^2}}{2.1}$$

$$2x = 10y \pm 8y$$

$$\therefore x - 9y = 0$$

$$x - y = 0$$

Therefore the given equation represents two lines and this are

$$x - 9y = 0$$

and

$$x - y = 0$$

Problem 2: Find the equations represented by the second degree homogeneous

$$2x^2-7xy-5y^2=0$$

Solution: The given equation

$$2x^2-7xy-5y^2=0$$

$$or, x=\frac{-(-3y)\pm\sqrt{(-3y)^2-4.2(-5y^2)}}{2.2}$$

$$or, x=\frac{3y\pm\sqrt{9y^2+40y^2)}}{4}$$

$$or, x=\frac{3y\pm7y}{4}$$

$$or, 4x=3y\pm7y$$

Taking (+)sign we get $4x=3y-7y$

$$or, 4x+4y=0$$

$$or, x+y=0$$

Taking (+)sign we get $4x=3y+7y$

$$or, 4x-10y=0$$

$$or, 4x-10y=0$$

Bisectors:

The equation of bisectors of the angle between the two lines represented by the equation ax2+2hxy+by2=0 with (0,0) as origin is $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$

Problem 1: Find the equation of bisector and angle between the two lines represented by the second degree homogeneous equation $x^2+8xy+7y^2=0$

Solutions:

Comparing the equation $x^2+8xy+7y^2=0$ with the second degree equation

$ax^2+2hxy+by^2=0$ we get

a=1,b=7,h=4

Then the equation of bisector is

$$\frac{x^2-y^2}{1-7}=\frac{xy}{4}$$

$$\text{or, } \frac{x^2-y^2}{-6}=\frac{xy}{4}$$

$$\text{or, } \frac{x^2-y^2}{-3}=\frac{xy}{2}$$

$$\backslash 2x^2+3xy-2y^2=0 \text{(Ans)}$$

Let θ be the angle between the lines represented by equation (1).the we get

$$\therefore \theta = \tan^{-1}\{\frac{2\sqrt{(h^2-ab)}}{a+b}\}$$

$$= \tan^{-1}\{\frac{2\sqrt{16-7)}}{1+7}\}$$

$$= \tan^{-1}(\frac{3}{4})$$

Problem 2: Find the equation of the bisectors and angle between the two lines represented by the second degree homogeneous equation $x^2 - 10xy + 9y^2 = 0$.

Solution: the given equation is

$$x^2 - 10xy + 9y^2 = 0 \text{............}(1)$$

Comparing the equation (1) with the second degree equation $ax^2 + 2hxy + by^2 = 0$ we get

a=1, b=9, h=-5

The equation of the bisectors of the angles between the two lines represented by the second degree homogeneous equation $x^2 - 10xy + 9y^2 = 0$ is

$$\frac{x^2-y^2}{1-9}=\frac{xy}{-5}$$

$$or, \frac{x^2-y^2}{-8}=\frac{xy}{-5}$$

$$or, \frac{x^2-y^2}{8}=\frac{xy}{5}$$

$$\therefore 5x^2-8xy-5y^2=0$$

$$\therefore \theta=\tan^{-1}\{\frac{2\sqrt{(h^2-ab)}}{a+b}\}$$

$$=\tan^{-1}\{\frac{2\sqrt{25-9)}}{1+9}\}$$

$$=\tan^{-1}(\frac{4}{5})$$

Problem3: If the pair of straight lines $x^2+2axy-y^2=0$ and $x^2-2bxy-y^2=0$ such that each pair bisects the angle between the other pair, prove that $a+b=0$

Solution:

$$the\ bisector\ of\ 1st\ equation\ is$$

$$\frac{x^2-y^2}{1-(-1)}=\frac{xy}{a}$$

$$or, x^2-y^2=\frac{2xy}{a}............(i)$$

$$the\ bisector\ of\ 2nd\ equation\ is$$

$$\frac{x^2-y^2}{1-(-1)}=\frac{xy}{-b}$$

from (i) and(ii) we get

$$\frac{2xy}{a} = \frac{2xy}{-b}$$

$$or, \frac{1}{a} = \frac{1}{-b}$$

$$or, a = -b$$

$$or, a + b = 0$$

Try yourself

1. Show that $2x^2 - 7xy + 3y^2 + x + 7y - 6$ represent a pair of straight line whose included angle is 45°.

2. Show that the straight lines represented by $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ are perpendicular to each other.

Circle

****The general equation of circle is

$x^2+y^2+2gx+2fy+c=0$,whose center is $(-g,-f)$

and radious is $r=\sqrt{g^2+f^2-c}$

*** The equation of circle whose is center (a,b) and radius is r is $(x-a)^2+(y-b)^2=r^2$

Problem1: Find the equation of the circle whose radius is 10 and the center is (-1,2).

Solutions:

The equation of the circle whose radius is 10 and the center is (-1, 2) is

$(x+1)^2+(y-2)^2=10^2$

$or, x^2+2x+1+y^2-4y+4=100$

$or, x^2+y^2+2x-4y-95=0$(Ans)

Problem2: Find the equation of the circle whose radius is 8 and the centre is (-4, 2).

Solutions:

The equation of the circle whose radius is 8 and the center is (-4, 2) is

$(x+4)^2+(y-2)^2=8^2$

$$x^2+y^2+8x-4y+16+4=64.$$
$$\therefore x^2+y^2+8x-4y-44=0.$$

Problem 4: Find the equation of the circle whose touches both the axes and passes through the point (-2,-1).

Solution:

Let a be the radius of the circle .If it touches both the axes then the co-ordinates of touching point are$(a, 0)$ *and* $(0, a)$ & *centre* (a, a).Since it passes through the point $(-2, -1)$.

$(x-a)^2+(y-a)^2=a^2$
=> $(-2-a)^2+(-1-a)^2=a^2$
=> a^2+6a+5=0
=> $a^2+a+5a+5$=0
=> $(a+1)(a+5)=0$
$a=-1, -5$

Therefore the required equation

$(x+1)^2+(y+1)^2=1.$
$(x+5)^2+(y+5)^2=25$

Problem 5: Show that the locus of the poles of tangents to the circle $x^2+y^2=a^2$ w.r.t the circle $x^2+y^2=2bx$ is the conic. $(a^2-b^2)x+a^2y^2-2a^2bx+a^2b^2=0$

Solution: The polar of any point (x_1, y_1) w.r.to the circle $x^2+y^2-2bx=0$ is $xx_1+yy_1-b(x+x_1)=0$

$=> x(x_1-b)+yy_1-bx_1=0 \ldots\ldots\ldots\ldots\ldots(i)$

(i) is tangent to the circle $x^2+y^2=a^2$

$$\frac{-bx}{\sqrt{\{(x_1-b)^2+{y_1}^2\}}}=\pm a$$

$=> b^2{x^2}_1=a^2(x_1-b)^2+{y^2}_1a^2$

$=> b^2{x^2}_1=a^2{x_1}^2-2a^2{x^2}_1b+a^2b^2+{y_1}^2a^2$

$=> {x_1}^2(a^2-b^2)-2a^2bx_1+a^2b^2+{y^2}_1a^2=0$

Hence the locus of (x_1, y_1) is $(a^2-b^2)x+a^2y^2-2a^2bx+a^2b^2=0$

Problem 6: Show that the equation to the pair of tangent drawn from the origin to the circle $x^2+y^2+2gx+2fy+c=0$ is $(gx+fy)^2+c(x^2+y^2)$.

Solution: The equation to the pair tangent is $SS_1 = T^2$

$=>(x^2+y^2+2gx+2fy+c)c=(x.0+y.0+g(x+0)+f(y+0)+c)^2$

$=>x^2+y^2+2gx+2fy+c=(gx+fy+c)^2$

$=>c(x^2+y^2)=(gx+fy)^2$(Showed)

Try yourself

Problem 1: Find the equation of the circle whose radius is 5 and the center is (-4,-3

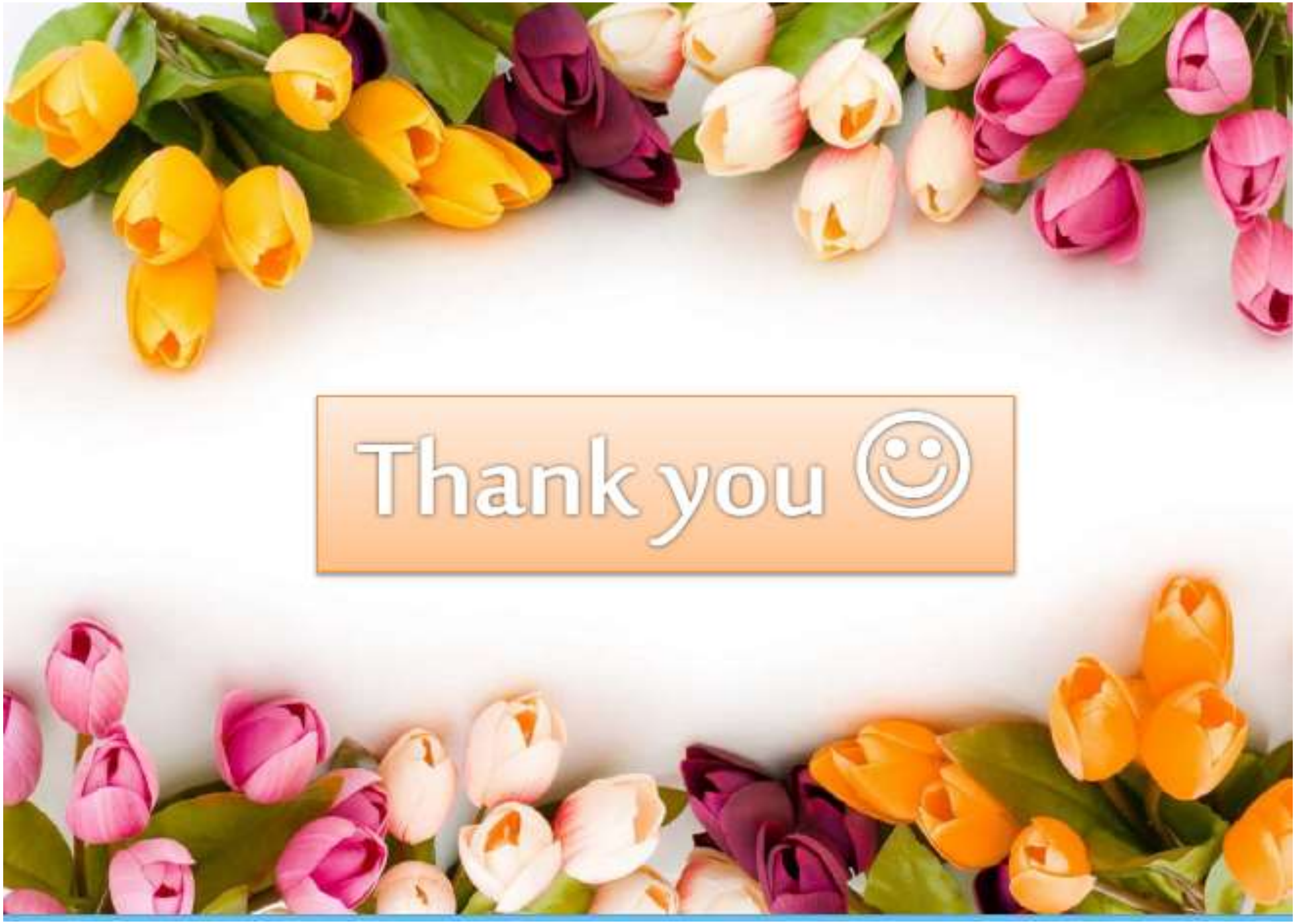
Problem 2: Find the equation of the circle whose radius is 4 and the center is (1,2)

Problem 3: Find the equation of the circle whose radius is 12 and the center is (7,-8 **).**

Problem 4: Find the equation of the circle whose radius is 15 and the center is (1,-5).

Problem 5: Find the equation of the circle whose touches both the axes and passes through the point (2,-1).

Problem 6: Find the equation of the circle whose touches both the axes and passes through the point (3,-5).


Thank you ☺