Course Code: MAT 211

Course Title: Coordinate Geometry and Vector Analysis

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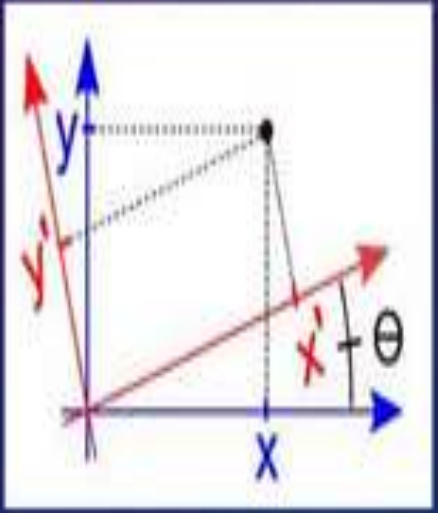
Contents:

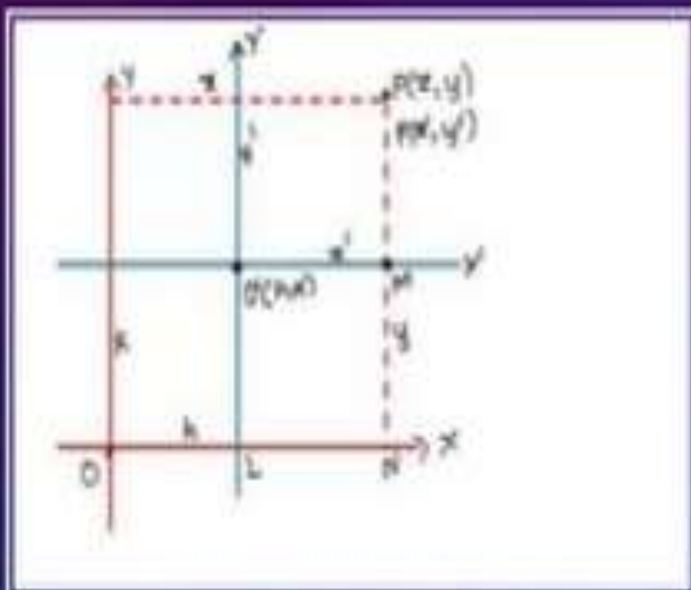
- Change of axes
- Polar Co-ordinates
- The straight line

Defination of Change Of Axes:

- In mathematics, a change of axes in two dimensions is a mapping from an x y - Cartesian co-ordinate system to an x′y′-Cartesian co ordinate system in which the origin is kept fixed and the x′ and y′ axes are obtained by rotating the x and y axes counterclockwise through an angle θ



Translation of axes



- ox &oy is original axes
- ox' &oy' is new axes
- $P \to (x, y)$
- So, $ol \to h$ & $o'l = k$
- $x = on = ol + ln = h + x'$
- $y = pn = pm + mn = y' + k$

Change of origin (Translation of axes)

If the origin o is shifted to o'

Therefore the transformed coordinate are

$$x = \alpha + x$$

$$y = \beta + y$$

***Problems for Change of origin (Translation of axes)

Problem 1: Determine the equation of the curve $2x^2+3y^2-8x+6y-7=0$ when the origin is transferred to the point (2,-1).

Solutions: The given equation is $2x^2+3y^2-8x+6y-7=0$...........(i)

When the origin is transferred to the point (2,-1) then (i)will be

$2(x'+2)^2+3(y'-1)^2-8(x'+2)+6(y'-1)-7=0$

or, $2x'^2+8x'+8+3y'^2-6y'+3-8x'-16+6y'-6-7=0$

or, $2x'^2+3y'^2-18=0$

Now droping the sufiix we get $2x^2+3y^2-18=0$(Ans)

Problem 2: Determine the equation of the curve $x^2+2y^2-6x+7=0$ when the origin is transferred to the point (3,1).

Solutions:

$$\text{The given equation is } x^2+2y^2-6x+7=0 \text{.......(i)}$$

$$\text{when the origin is shifted to the point}(3,1), \text{then (i) become}$$

$$(x'+3)^2+2(y'+1)^2-6(x'+3)+7=0$$

$$or, x'^2+6x'+9+2y'^2+4y'+2-6x'-18+7=0$$

$$or, x'^2+2y'^2+4y'=0$$

$$\text{Now omiting the suffix we get, } x^2+2y^2+4y=0$$

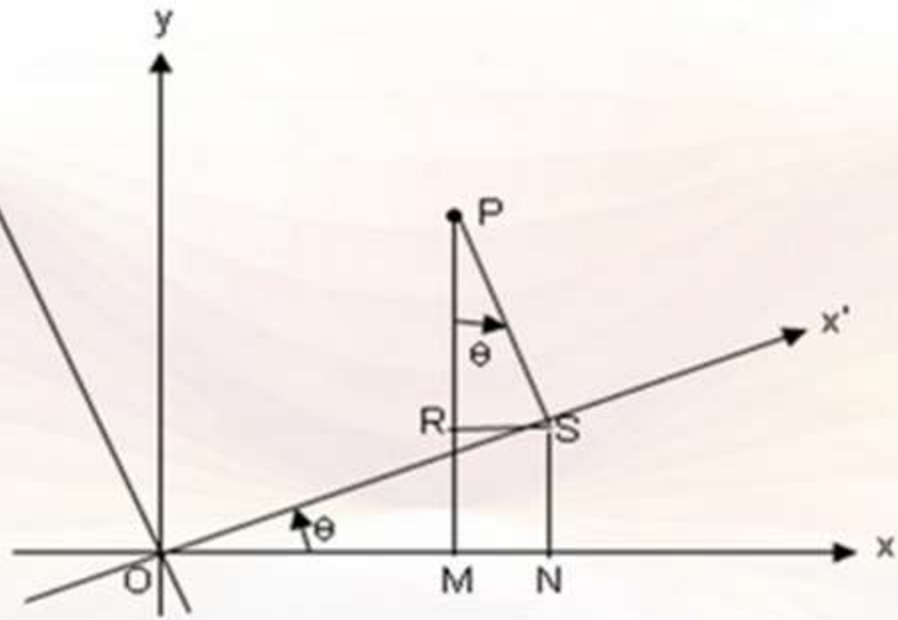
H.W: Determine the equation of the curve $x^2+y^2-x+7=0$ when the origin is transferred to the point (1,1).

Transformation Formulas

For a rotation through an angle θ, the following are the transformation formulas which express the old coordinates (x, y) in terms of θ and the new coordinates (x′, y′).

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$



In the figure, the coordinates P are (x, y) when referring to the old coordinate axes, and are (x′, y′) when referred to the new coordinate axes. Also,

$$x = \overline{OM},\ \ y = \overline{MP},\ \ x' = \overline{OS},\ \ and\ \ y' = \overline{SP}\ .$$

Hence, we have

$$x = \overline{OM} = \overline{ON} - \overline{MN} = \overline{ON} - \overline{RS} = x' \cos\theta - y' \sin\theta$$
$$y = \overline{MP} = \overline{MR} + \overline{RP} = \overline{NS} + \overline{RP} = x' \sin\theta + y' \cos\theta$$

Problems for Rotation of axes(origin fixed):

Problem 1: Transform the equation $x^2-2xy+y^2+2x-4y+3=0$,

when the direction of the axes is turned through an angle 45^o where as the origin of coordinates remains the same.

Solution:

The given equation is $x^2-2xy+y^2+2x-4y+3=0$ ....(i)

Then

$$x = x'\cos 45^o - y'\sin 45^o = \frac{x'-y'}{\sqrt{2}}$$

$$y = x'\sin 45^o + y'\cos 45^o = \frac{x'+y'}{\sqrt{2}}$$

Putting these values in equation (i), we get

$$\left(\frac{x'-y'}{\sqrt{2}}\right)^2-2\left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right)+\left(\frac{x'+y'}{\sqrt{2}}\right)^2+2\left(\frac{x'-y'}{\sqrt{2}}\right)-4\left(\frac{x'+y'}{\sqrt{2}}\right)+3=0$$

$$\text{or, }\frac{x'^2-2x'y'+y'^2}{2}-x'^2+y'^2+\frac{x'^2+2x'y'+y'^2}{2}+\sqrt{2}x'-\sqrt{2}y'-2\sqrt{2}x'-2\sqrt{2}y'+3=0$$

$$\text{or,}2y'^2-\sqrt{2}x'-3\sqrt{2}y'+3=0$$

Now dropping the suffix we get

$$2y^2-\sqrt{2}x-3\sqrt{2}y+3=0$$

Problem 2: Transform the equation $x^2+2\sqrt{3}xy-y^2=2a^2$ after rotating of axes through an angle 30°.

Solutions:

The given equation is $x^2+2\sqrt{3}xy-y^2=2a^2$ ....(i)

when the axes rotated Through an angle 30° *then we get*

$$x=x'\cos 30^\circ-y'\sin 30^\circ=\frac{\sqrt{3}x'}{2}-\frac{y'}{2}=\frac{\sqrt{3}x'-y'}{2}$$

$$y=y'\cos 30^\circ+x'\sin 30^\circ=\frac{x'}{2}+\frac{\sqrt{3}y'}{2}=\frac{\sqrt{3}y'+x'}{2}$$

putting these values in (i) we get

$$\left(\frac{\sqrt{3}x'-y'}{2}\right)^2+2\sqrt{3}\left(\frac{\sqrt{3}x'-y'}{2}\right)\left(\frac{\sqrt{3}y'+x'}{2}\right)-\left(\frac{\sqrt{3}y'+x'}{2}\right)^2=2a^2$$

$$or,\frac{1}{4}(3x'^2-2\sqrt{3}x'y'+y'^2)+\frac{\sqrt{3}}{2}(\sqrt{3}x'^2+2x'y'-\sqrt{3}y'^2)-\frac{1}{4}(x'^2+2\sqrt{3}x'y'+3y'^2)=2\mathrm{a}^2$$

$$or,2x'^2-2y'^2=2\mathrm{a}^2$$

$$or,x'^2-y'^2=\mathrm{a}^2$$

now omiting the suffix we get $x^2-y^2=\mathrm{a}^2$ (Ans)

H.W: Transform the equation $x^2+xy-y^2=0$ after rotating of axes through an angle 45°.

Problems for Removal of the first degree terms:

Problem 1: Remove the first degree terms of the given equation $36x^2+24xy+29y^2-72x+126y+81=0$.

Solution: The given equation is $36x^2+24xy+29y^2-72x+126y+81=0$ ……………(i)

$$f(x,y)=36x^2+24xy+29y^2-27x+126y+81=0$$

$$\therefore \frac{\partial f}{\partial x}=72x+24y-72=0$$

$$\Rightarrow 3x+y-3=0 \text{.........(ii)}$$

Let

$$\text{and, } \frac{\partial f}{\partial y}=24x+58y+126=0$$

$$\Rightarrow 12x+29y+63=0 \text{..........(iii)}$$

$$\left.\begin{aligned}3x+y-3&=0\\12x+29y+63&=0\end{aligned}\right\}$$

$$\frac{x}{63+87}=\frac{y}{-36-189}=\frac{1}{87-12}$$

By cross multiplication, we have

$$\frac{x}{150}=\frac{y}{-225}=\frac{1}{75}$$

$$\therefore x=2$$

$$y=-3$$

To remove the first degree term, shifting the origin to the point (2,-3), equation (i) takes the form

$$36x^2+24xy+29y^2+c_1=0\text{..........(iv)}$$

$$c_1=gx_1+fy_1+c$$

$$=-36.2+63(-3)+81$$

$$=-180$$

The general equation of second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$

Thus equation (iv) becomes

$$36x^2+24xy+29y^2-180=0$$

Problem 2: Remove the first degree terms of the given equation

$$3x^2-4y^2-6x-8y-10=0$$

Solution: The given equation is

$$3x^2-4y^2-6x-8y-10=0$$

$$f(\mathrm{x},\mathrm{y})=3x^2-4y^2-6x-8y-10=0$$

$$\therefore \frac{\partial f}{\partial x}=6x-6=0$$

$$or.x=1$$

$$\text{and,} \frac{\partial f}{\partial y}=-8y-8=0$$

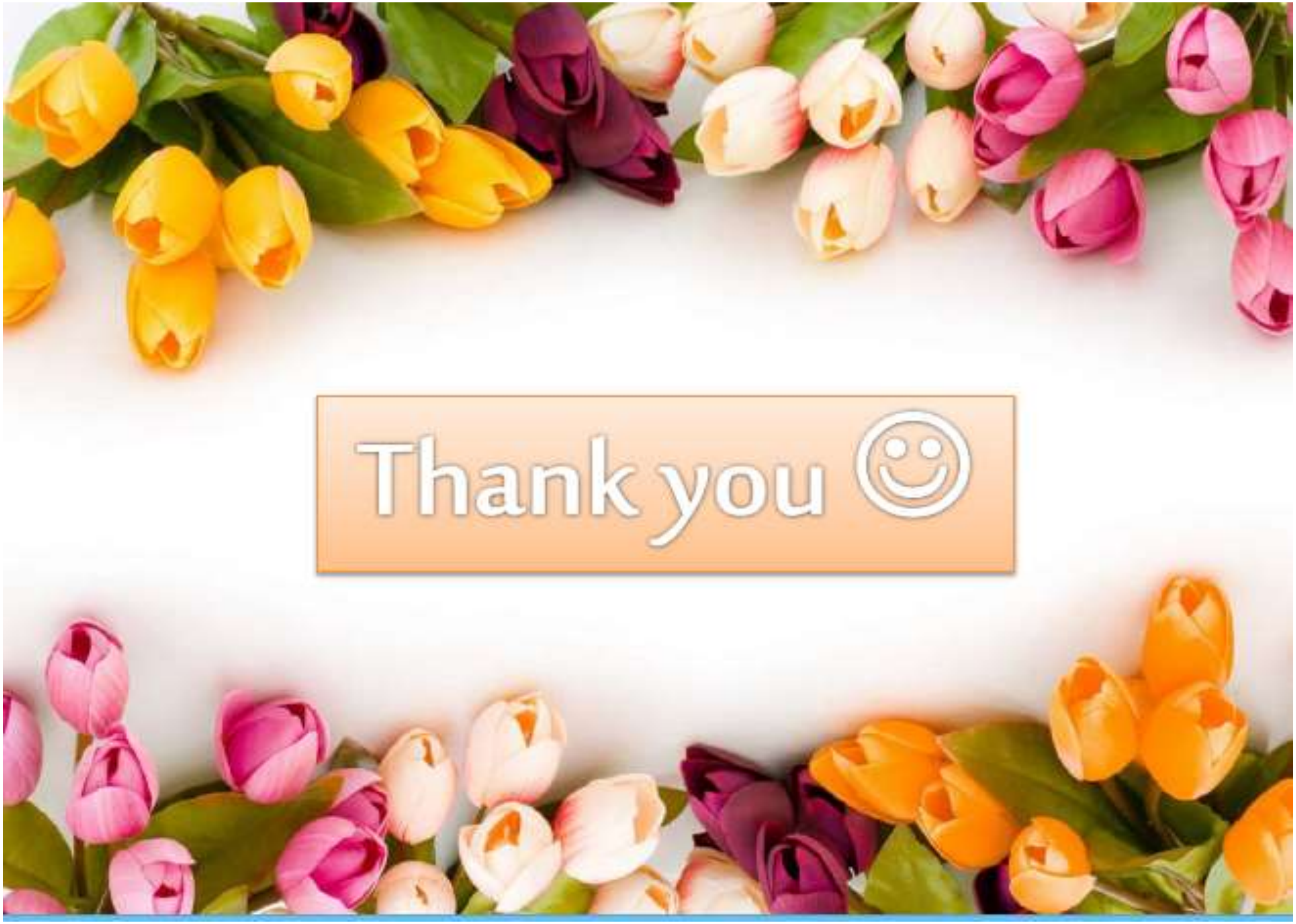
$$or, y=-1$$

To remove the first degree term, shifting the origin to the point (1,-1), equation (i) takes the form

$$3x^2 - 4y^2 + c = 0$$

$$where\, c = -3(1) - 4(-1) - 10 = -9$$

$$Then\, we\, get\; 3x^2 - 4y^2 - 9 = 0 (\text{Ans})$$


Thank you ☺