Mathematical Modelling – assignments week 2: Complex numbers & Vector and matrix calculations

Assignment 1

Make a row of numbers from 1 to 5 with step size 0.2, and store this in the variable x.
What is the 19th element of this row? How many elements does the row contain? You can include the answers to these two questions as comments in the m-file (a comment line starts with a % sign)

```
x(19) = 4.6000
number_of_elements = 21
```

Assignment 2

Make a row (again use x as the variable) of 100 numbers in ascending order, beginning with 35 and ending with 47, at equal intervals.
What are the 19th up to the 22nd elements of this row? Again, you can include the answers to these questions as comments in the m-file.

```
x(19:22) =   37.1818   37.3030   37.4242   37.5455
```

Assignment 3

Given are the complex numbers $z_1 = 3 + 4j$ and $z_2 = -1 + j$

a. Calculate the modulus and argument of z_1 and z_2 using MATLAB

```
Modulus of z_1  = 5
Argument of z_1 = 0.9273
Modulus of z_2  = 1.4142
Argument of z_2 = 2.3562
```

b. Calculate $z_1 \cdot z_2$ and $\frac{z_1}{z_2}$ using MATLAB

```
product  = -7.0000 - 1.0000i
quotient = 0.5000 - 3.5000i
```

c. $\frac{1}{z_3} = \frac{1}{z_1} + \frac{1}{z_2}$. Calculate z_3 using MATLAB

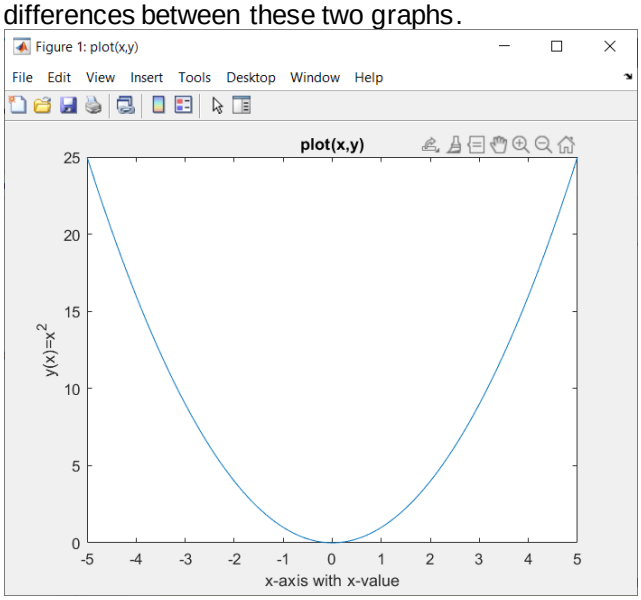
```
z_3 =  -0.6552 + 1.1379i
```

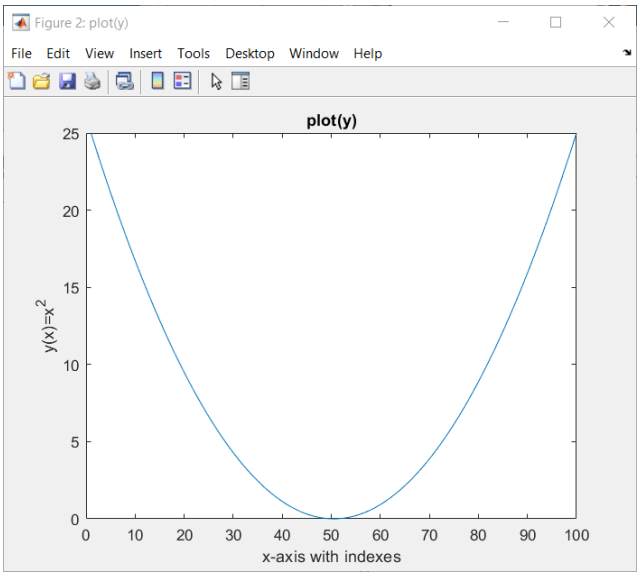
d. Calculate the modulus and argument of z_3 using MATLAB

```
Modulus of z_3  = 1.3131
Argument of z_3 = 2.0932
```

Assignment 4

The variable x contains 100 values between -5 and 5.
$y = x^2$
Make two separate graphs (using `figure`), one with the command `plot(x,y)` and one with the command `plot(y)` instead and compare the two graphs. In your m-file, give an explanation (use a comment) of the differences between these two graphs.





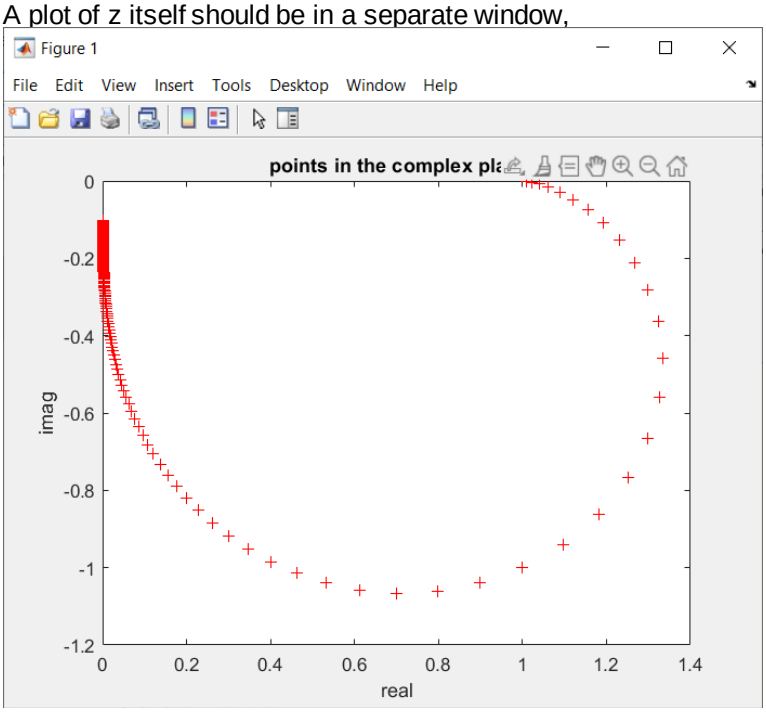

```
% The real x-value is placed at the x-asis when using plot(x,y)
% The index of the array is placed at the x-axis plot(y)
% N.N. the index start at 1 when using Matlab
```

Assignment 5

Given is the complex number $z = \dfrac{1 + j\omega}{1 + j\omega - \omega^2}$.

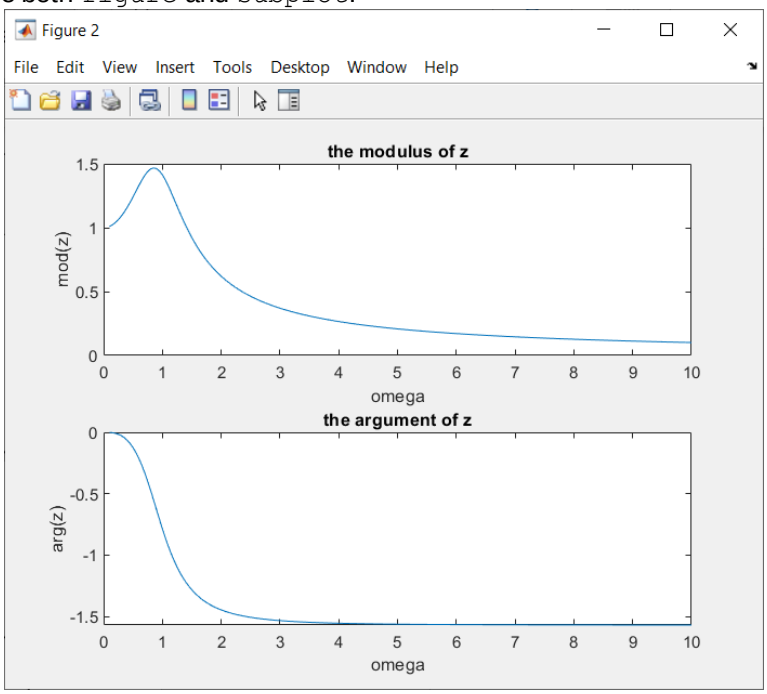
ω consists of values from 0.1 to 10, with a step size of 0.05
Make three different plots in two different windows.

a. A plot of z itself should be in a separate window,



b. A plot of the modulus of z as a function of ω and a plot of the argument of z as a function of ω should be in a separate window, consisting of two sub windows, one on the top and the other on the bottom.

Hint: use both `figure` and `subplot`.

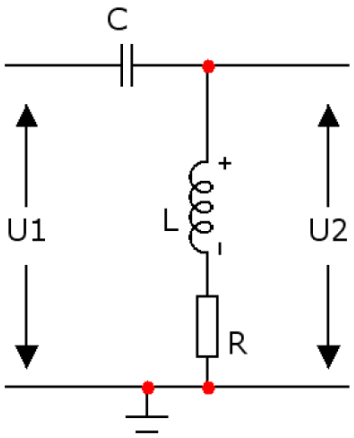


Assignment 6

Search the MATLAB documentation for `logspace` and `semilogx`.

Create a vector f with 1000 values, logarithmic spaced. The smallest value for f must be 10; the biggest vale for f must be 100000.

In the given circuit the following transfer function, $H(f)$, applies:

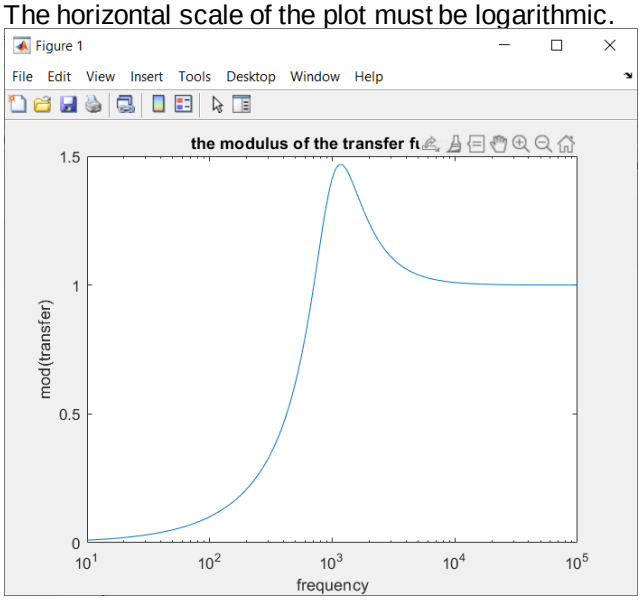


$$H(f) = \frac{z_2(f)}{z_1(f) + z_2(f)} \text{ with } z_1 = \frac{1}{j\omega C},\ z_2 = R + j\omega L,\ \omega = 2\pi f,\ R = 10$$

$$, L = \frac{R}{2000\pi}, \text{ and } C = \frac{1}{2000\pi R}.$$

Make a plot of the **modulus** of the transfer function $H(f)$ as a function of f.

The horizontal scale of the plot must be logarithmic.



Assignment 7

Given are the following alternating signals:

$f(t) = 3\cos\left(5t + \frac{1}{4}\pi\right)$ $g(t) = 2\sin\left(5t - \frac{1}{3}\pi\right)$

$k(t) = f(t) - g(t)$ can be written in the form $r\cos(\omega t + \varphi)$

Calculate the values, in four decimals, of r, ω en φ.

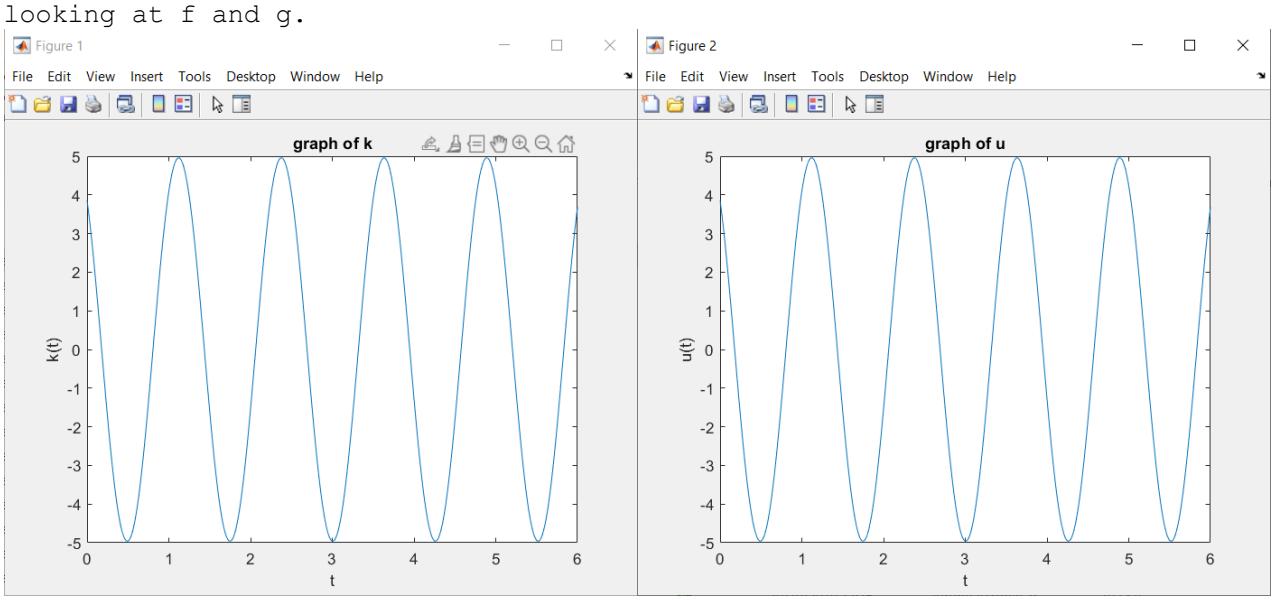
Plot two different graphs in two separate windows:

one graph of $k(t)$ and one graph of the calculated $r\cos(\omega t + \varphi)$

Hint: use $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$ to be able to calculate the subtraction.

r = 4.9589
ω = 5
φ = 0.6808

r and φ need to be calculated by means of Matlab. ω can directly be seen by looking at f and g.



You can choose t freely; a good choice would be a t in which you can see about two or three full periods.

Assignment 8

Given are the points A(2, 1, 4), B(1, -2, 3) and C(4, 5, 2).

Make the vectors $\vec{a} = \overrightarrow{OA}$, $\vec{b} = \overrightarrow{OB}$, and $\vec{c} = \overrightarrow{OC}$.

Calculate the vector $\vec{d} = \vec{a} + \vec{b} - 3\vec{c}$

Calculate the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$

Point M is in the middle of the line AB. Calculate the vector $\overrightarrow{CM}$

Calculate the inner product of the two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (this should give just a number as result!)

```
d =
    -9
   -16
     1

AB =
    -1
    -3
    -1

AC =
     2
     4
    -2

BC =
    -3
    -7
     1

M =
    1.5000
   -0.5000
    3.5000

CM =
   -2.5000
   -5.5000
    1.5000

inproduct = -12
```

Assignment 9

Given are the points A(2, 1, 4), B(1, -2, 3), and C(4, 5, 2).
These three points together form a triangle.
Calculate the area of this triangle.

```
area = 5.4772
```

Assignment 10

Given are the matrices $\mathbf{A} = \begin{pmatrix} 1 & -6 & 5 \\ -2 & 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 4 & 1 \end{pmatrix}$

Calculate (if possible): **AB, BA, $\mathbf{A^TB}$**

If the calculation is not possible, give an explanation in the form of MATLAB comment why the calculation is not possible according to you.

```
%AB = A * B is not possible (dimensions don't fit: 2x3 * 2x2)

BA =
     5   -18    13
     2   -24    22

ATB =
    -5    -3
   -18     6
    23    -3
```

Also give the dimensions of the matrices **A** , **B** , and the dimensions of the results of the previous matrix multiplications (if they were possible).

```
dimensionA =
     2     3

dimensionB =
     2     2

dimensionBA =
     2     3

dimensionATB =
     3     2
```

Assignment 11

Solve the following system of linear equations in MATLAB:

5x + 4y = 4
3x + 6y = 16

```
%solution: x = -2.2222 , y = 3.7778
```

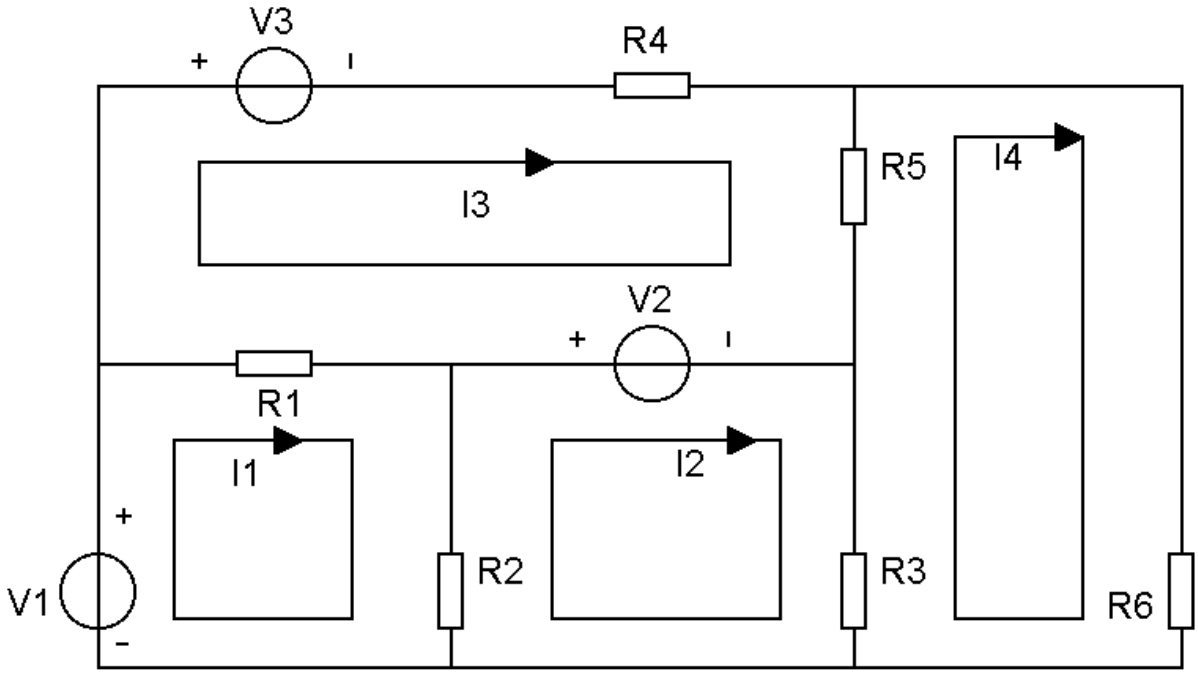
Assignment 12

Solve the following system of linear equations in MATLAB:

5x + 4y - 2z + 6w = 4
3y + 6w – 13 = z + x
6x + 12z – 2y + 16w = 20
42y + 2z – 4w = 6

```
% x = -1.5320, y = 0.3067, w = 1.7771, z = 0.1144
```

Application to electrical engineering



In the electrical system given above, the voltages and resistances are known, but the currents need to be calculated.
This system can be translated into a system of linear equations, which can be solved by MATLAB.
You'll find the corresponding equations by choosing a current direction, see the image.
The voltage will be counted as positive if the current enters the + pole of the voltage supply, the voltage for a resistance will be counted positive in the chosen direction, this is Kirchhoff's Law. Note that resistances and voltage supplies can be used for each current if that current passes through the voltage supply and or resistance.
For example, applying Kirchhoff's voltage law to the top left loop of the system displayed above gives:

$$V3 + R4 \cdot I3 + R5 \cdot I3 - R5 \cdot I4 - V2 + R1 \cdot I3 - R1 \cdot I1 = 0$$

Assignment 13a

Write down the other equations in the form of a MATLAB comment.

You should get a total of four different equations, each corresponding to one of the loops.
Now you'll have to translate these in a way so that you can make a coefficient matrix where the variables are: I1, I2, I3 and I4.
This means you need to combine all the terms with I1 and all terms with I2 etc.
And all constants need to be assigned to the right side of the equation.

Ask the teacher to check (especially if you don't have the same result as 13b

Assignment 13b

Work out each of the equations in this way. Then solve the system of equations in MATLAB with the following values:
V1=20, V2=12, V3=40, R1=18, R2=10, R3=16, R4=6, R5=15, R6=8.
What are the four currents you have found?

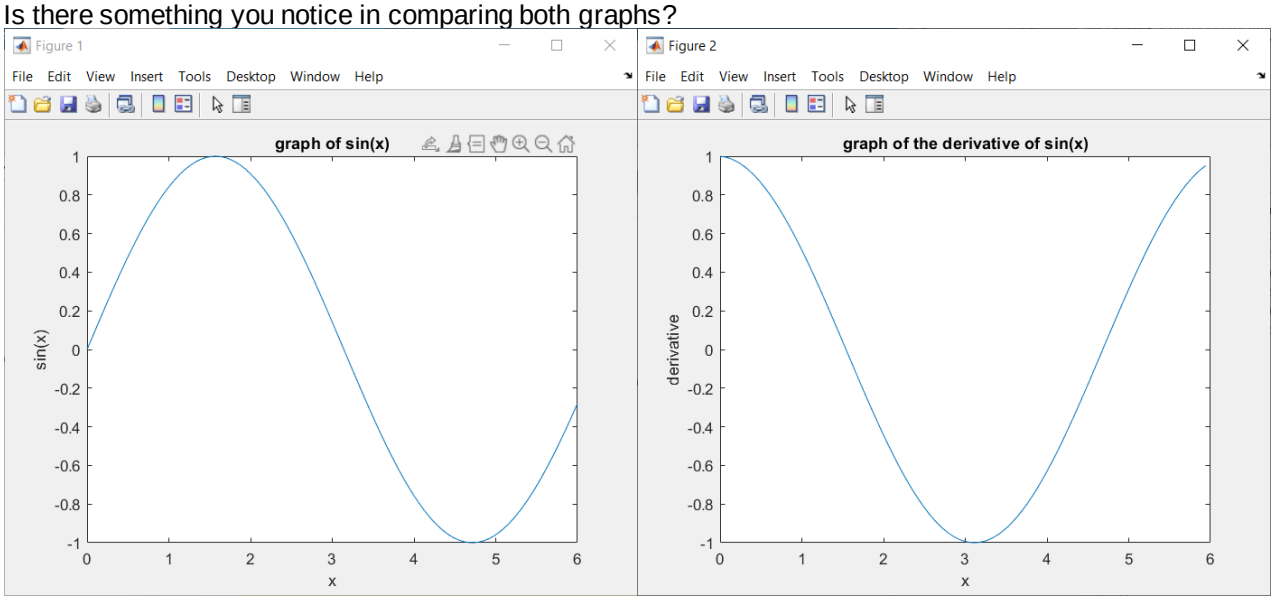
```
% I1 = -0.9012, I2 = -1.5960, I3 = -1.6263, I4 = -1.2803
```

Assignment 14

Make a vector for x of 100 numbers between 0 and 6 using the `linspace` command.
Then calculate the vector y = sin(x)
Calculate the vector dx = diff(x) as well as the vector dy = diff(y).

Calculate the vector y_derivative, being equal to $\frac{dy}{dx}$

This vector y_ derivative is a vector of only 99 numbers.
The graph of this can't just be plotted versus x, because x is a vector of 100 numbers.
That's why you should make a vector x_help that is equal to x, except for the last number.
This can be done for example with the commands: `n = length(x)-1;`
`x_help = x(1:n);`
Make two different plots screens with the graphs of y versus x and of y_ derivative versus x_help.
Make use of the `figure` command and add useful descriptions for the graphs and axes.
Is there something you notice in comparing both graphs?



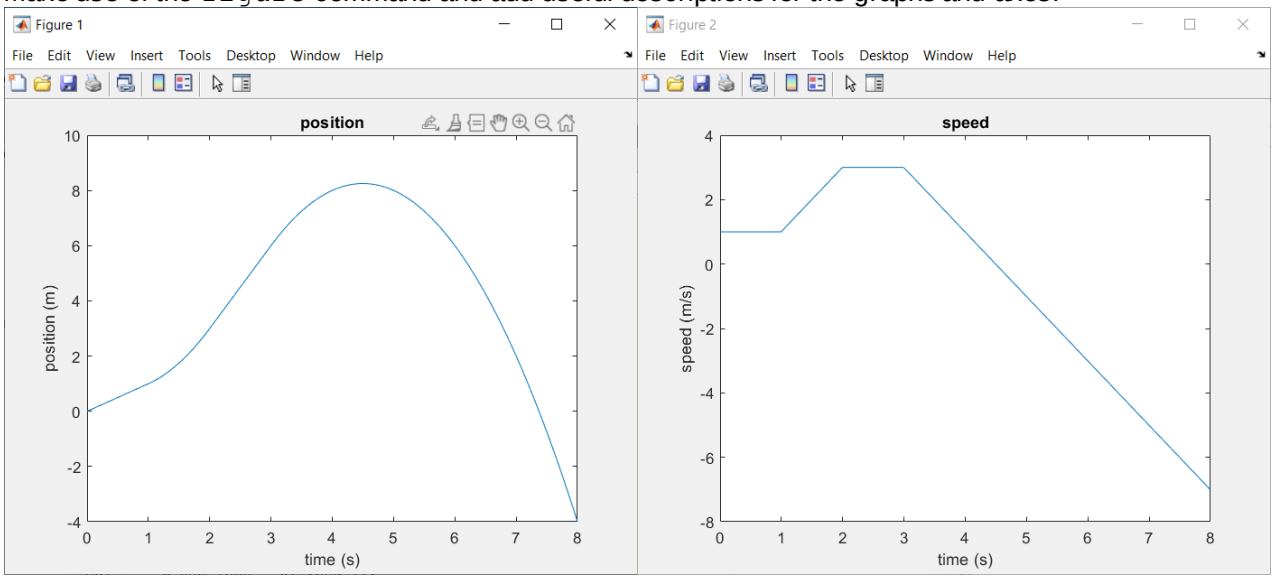
`The second graph has one point less`

Assignment 15

The place-time-function of a given object is given by the following vector:

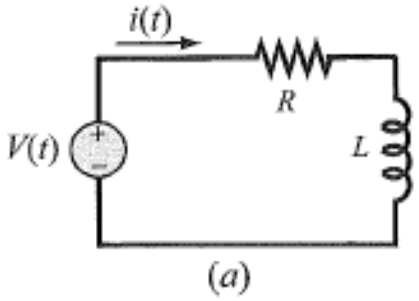
$$s(t) = \begin{cases} t & \text{if} \quad 0 \le t < 1 \\ t^2 - t + 1 & \text{if} \quad 1 \le t < 2 \\ 3t - 3 & \text{if} \quad 2 \le t < 3 \\ -t^2 + 9t - 12 & \text{if} \quad 3 \le t \le 8 \end{cases}$$

Make a vector t of 801 numbers from 0 to 8 using the `linspace` command.
Make a speed-time-function v(t) using differentiation like you also did in the previous assignment.
Make two different plot screens with one graph of s as function of t, and another graph of v as function of t
Make use of the `figure` command and add useful descriptions for the graphs and axes.

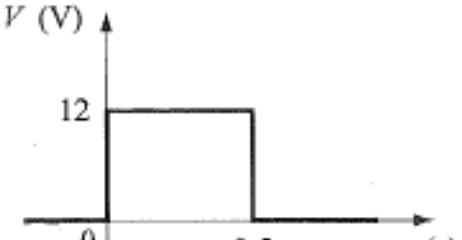


Assignment 16

See figure. R = 4 Ω , L =1.3 H



(a)



(b)

The current i as function of time is given by the following formula:

$$i = \frac{12}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \quad \text{if } 0 \le t \le 0.5$$

$$i = e^{-\frac{Rt}{L}}\frac{12}{R}\left(e^{\frac{R}{2L}} - 1\right) \quad \text{if } 0.5 < t \le 2$$

Make a plot of current *i* as function of time *t* for *t* between 0 and 2 seconds with a step size of 0.01 add a useful description for the graphs and axes.