Mathematical modelling

The SIR model
for spread of disease

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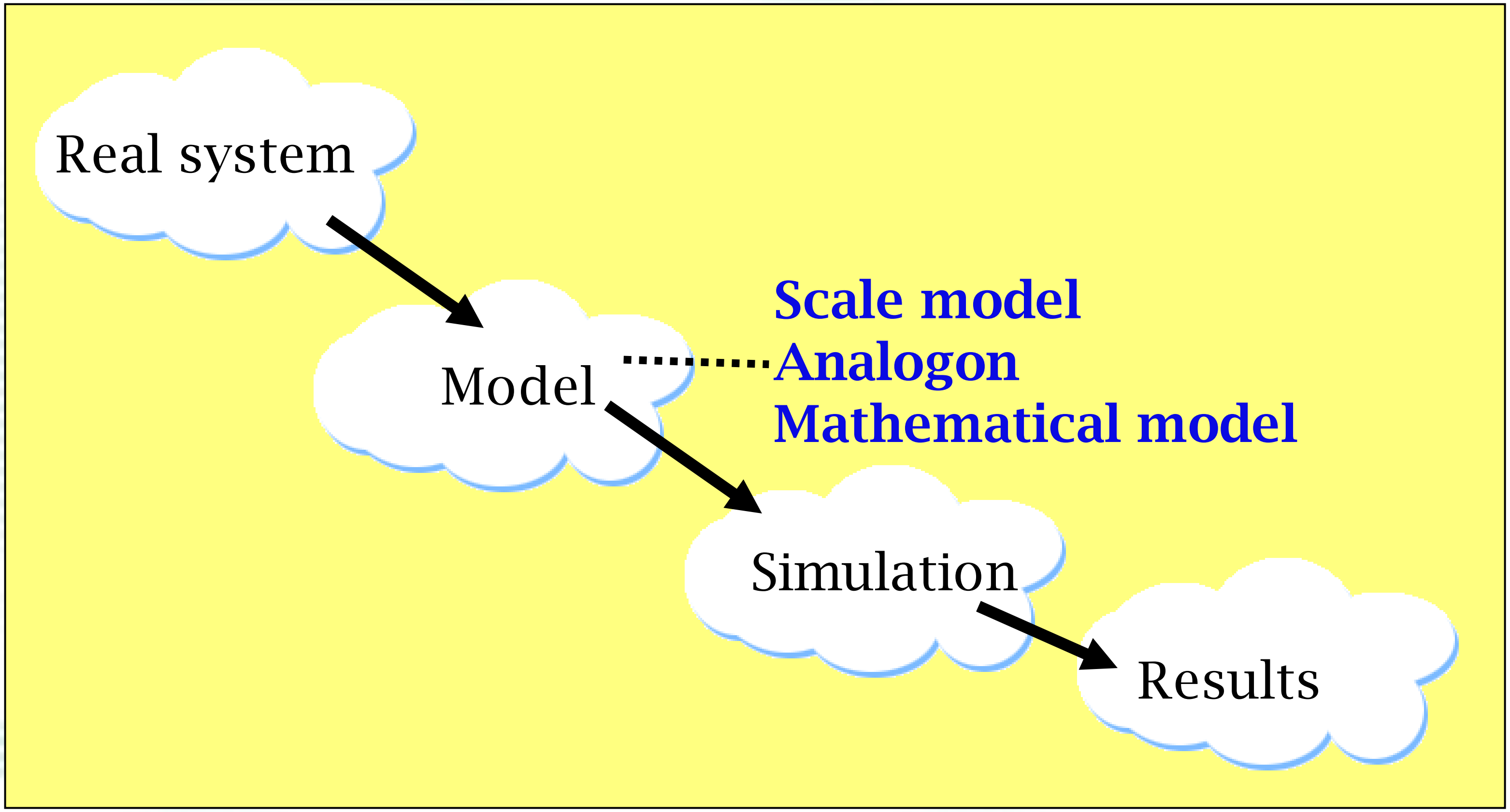
Simulation


Real system
Model
Scale model
Analogon
Mathematical model
Simulation
Results

Models and simulations(I)

Model

A description of (some aspects of) a system

Examples

1. a mathematical model of a set of differential equations, describing a dynamical system
2. a electronical network, used as model to describe e.g. a mechanical system
3. Scale models (car, air plane, harbor of Rotterdam)

Models and simulations (II)

Simulation

to mimic a system by means of a model

Simulation tools

E.g. Multisim or PSpice for simulating electrical networks or **Simulink** for simulating e.g. differential equations

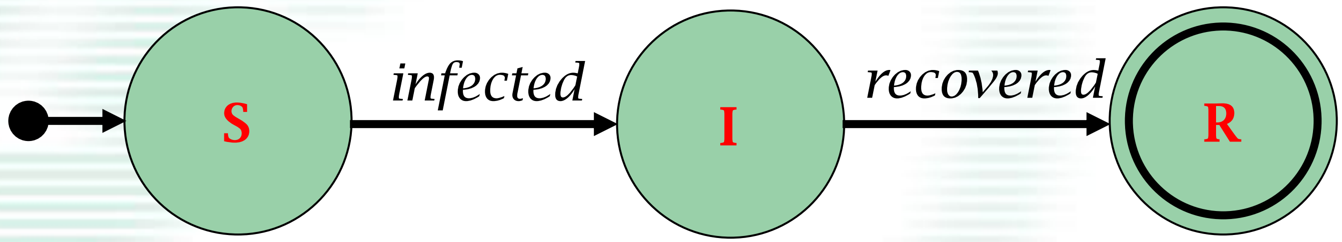
SIR epidemy model (I)

The SIR model has three states:

***S**usceptible ::= # people who still have a chance to become infected*

***I**nfected ::= # people who are infected*

***R**ecovered (also called Removed) ::= # people who are recovered (or deceased); they can't become infected anymore (assuming becoming immune after becoming recovered from an infection)*



Transmission rate ::= multiplication factor related to the chance of becoming infected

Recovery rate ::= multiplication factor related to the chance to become recovered

SIR epidemy model (II)

$$\begin{cases} (1) & \dfrac{dSusceptible}{dt} = -TransmissionRate \cdot Susceptible \cdot Infected \\ (2) & \dfrac{dInfected}{dt} = +TransmissionRate \cdot Susceptible \cdot Infected - RecoveryRate \cdot Infected \\ (3) & \dfrac{dRecovered}{dt} = +RecoveryRate \cdot Infected \end{cases}$$

Ad (1) **Change in # of susceptible people (since, once you are infected you are not susceptible anymore), hence a decrease (– sign)**

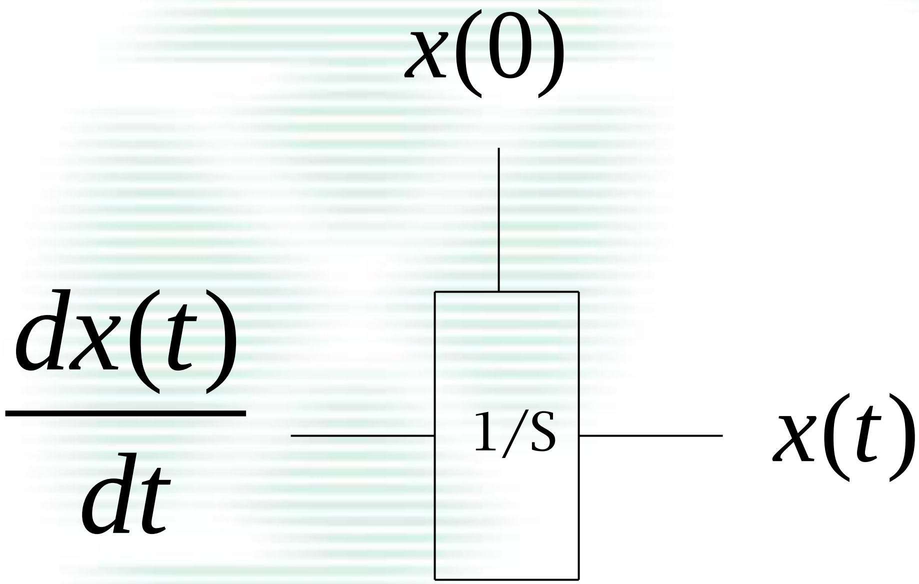
Ad (2) **Change in # of infected people**

Ad (3) **Change in # of recovered people, hence an increase (+ sign)**

Note that in this model # of people in total doesn't change, hence

$$\frac{dSusceptible}{dt} + \frac{dInfected}{dt} + \frac{dRecovered}{dt} = 0$$

The integrator-function



$x(0)$ is the initial condition,

i.e. the value of $x(t)$

at time $t = 0$

$$x(t) = \int_0^t \frac{dx(\tau)}{d\tau} d\tau + x(0)$$

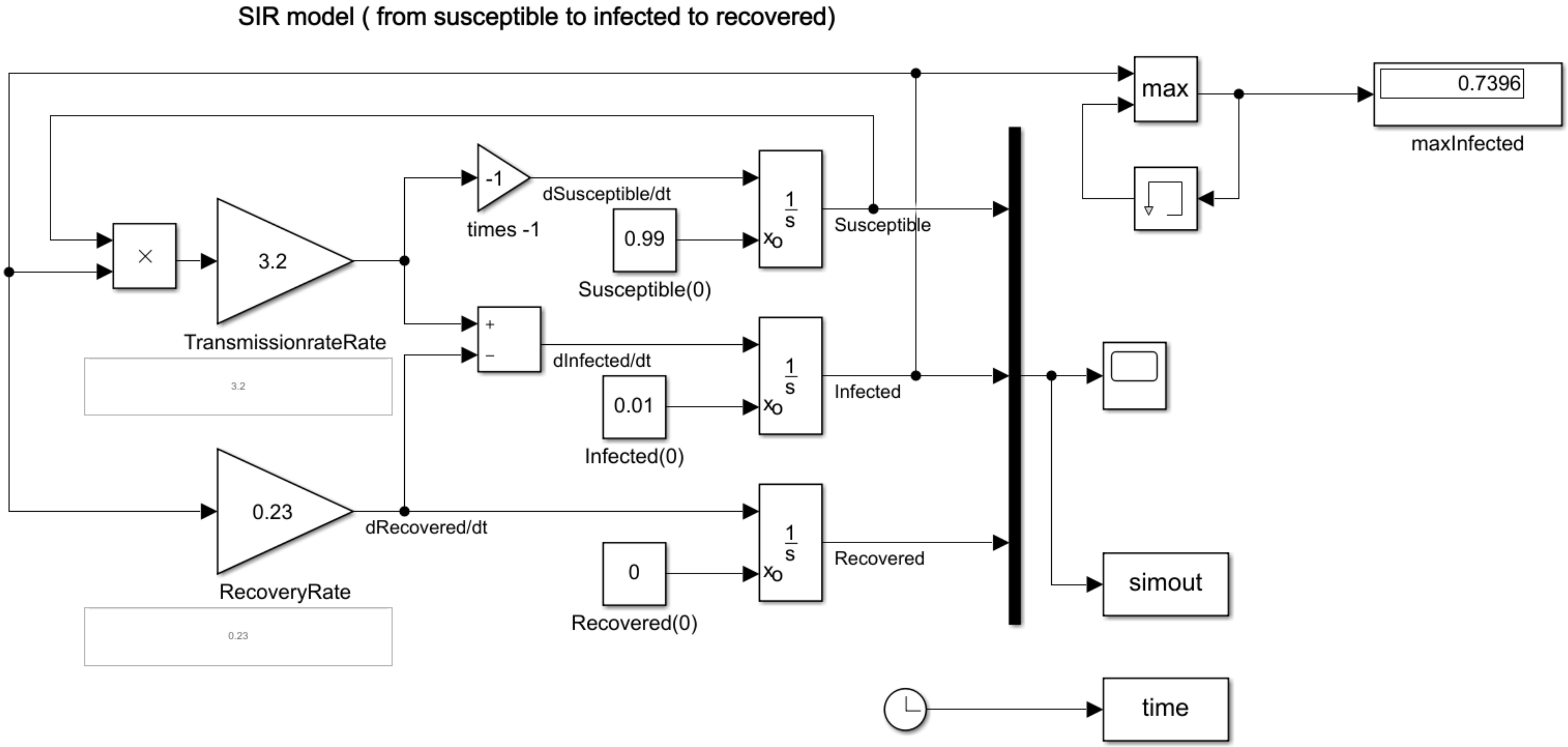
SIR epidemy model (III)

Susceptible(0) = 99 %
Infected(0) = 1 %
Recovered(0) = 0 %

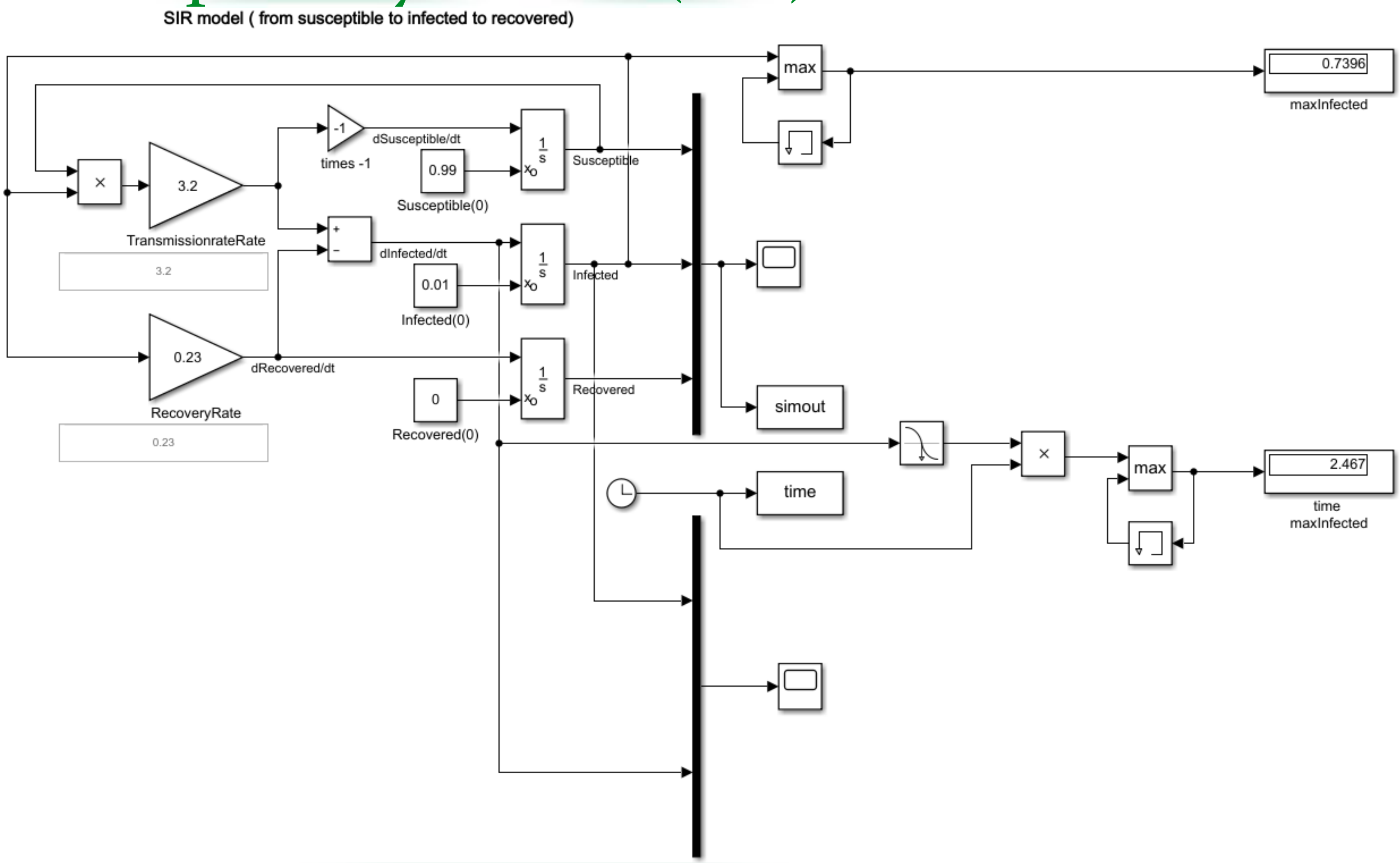
Total population 100%

Transmission rate = 3.2; 1.82; 0.59
Recovery rate = 0.23

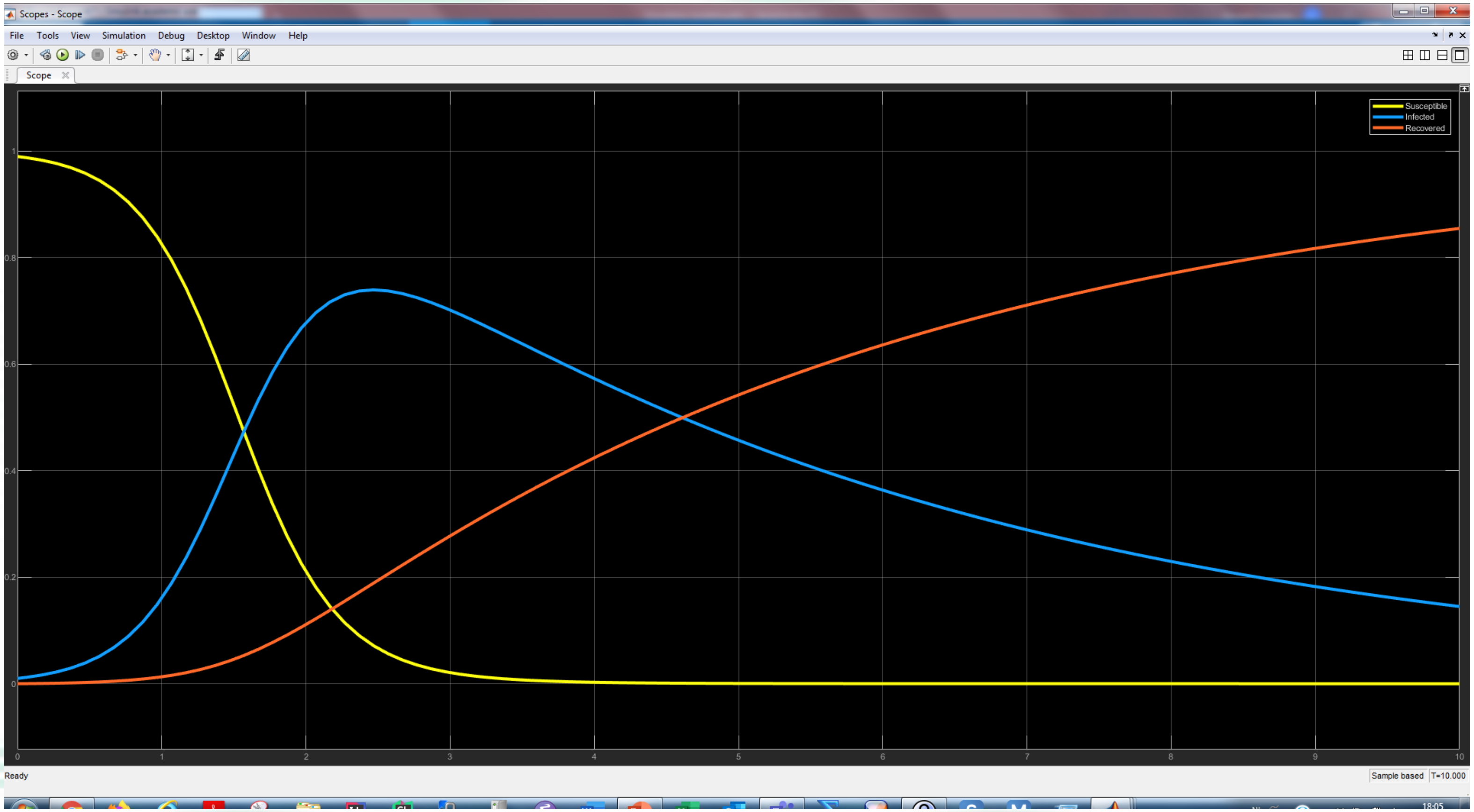
SIR epidemy model (IVa)



SIR epidemy model (IVb)

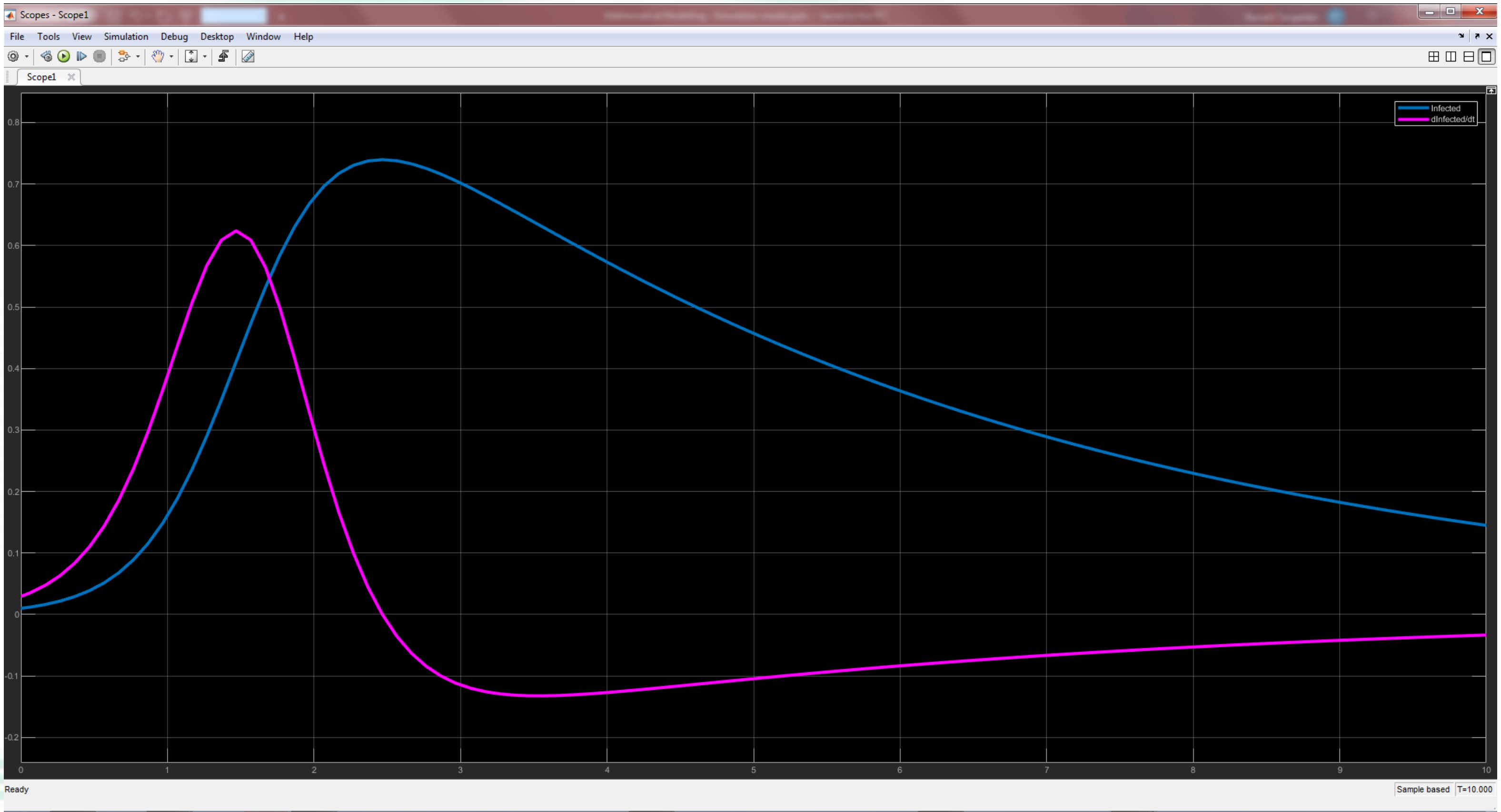


SIR epidemy model (Va1)



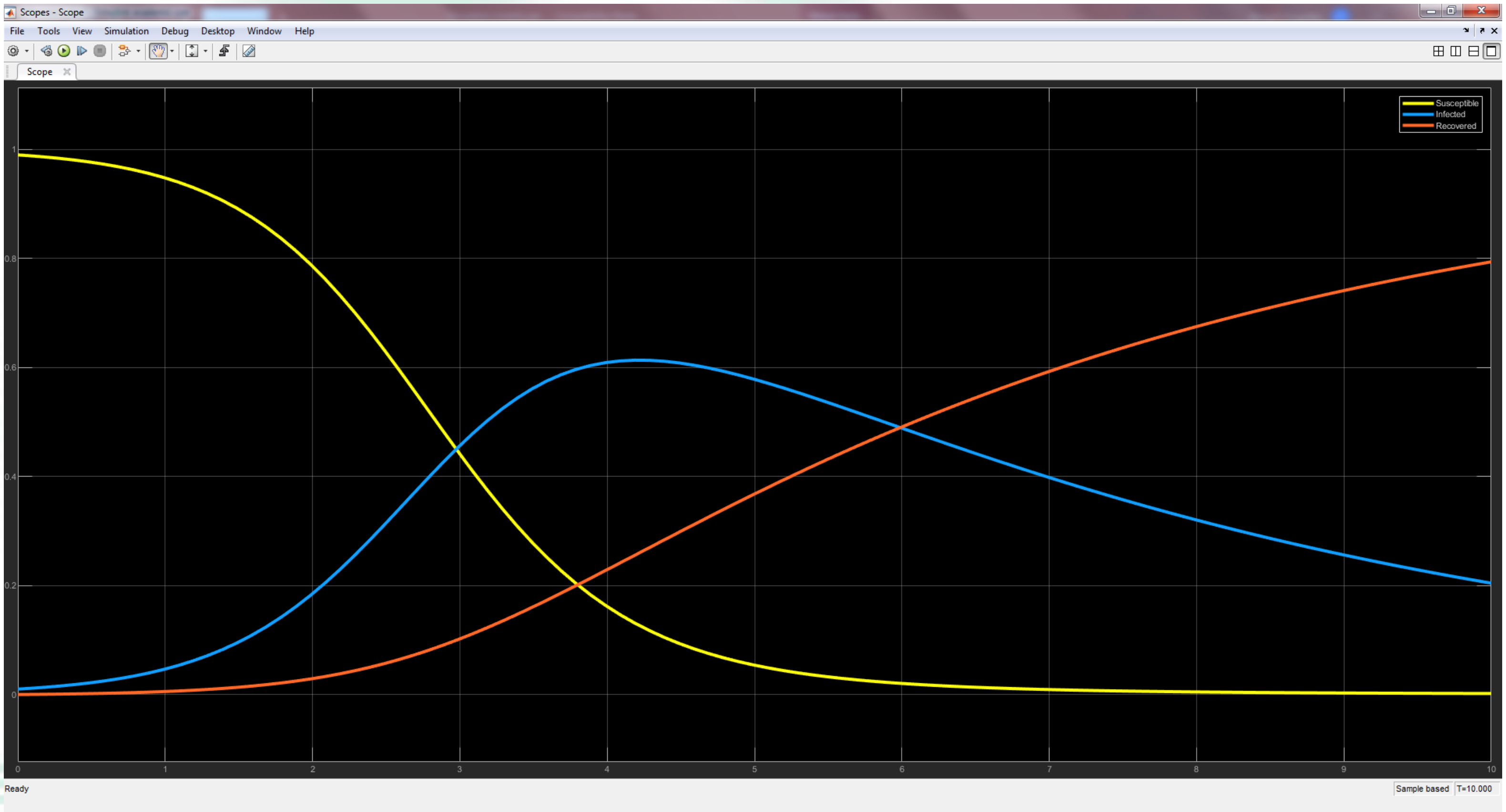
Transmission rate 3.2 gives maximum number 74.0 % after 2.467 days (simulation time 10 days)

SIR epidemy model (Va2)



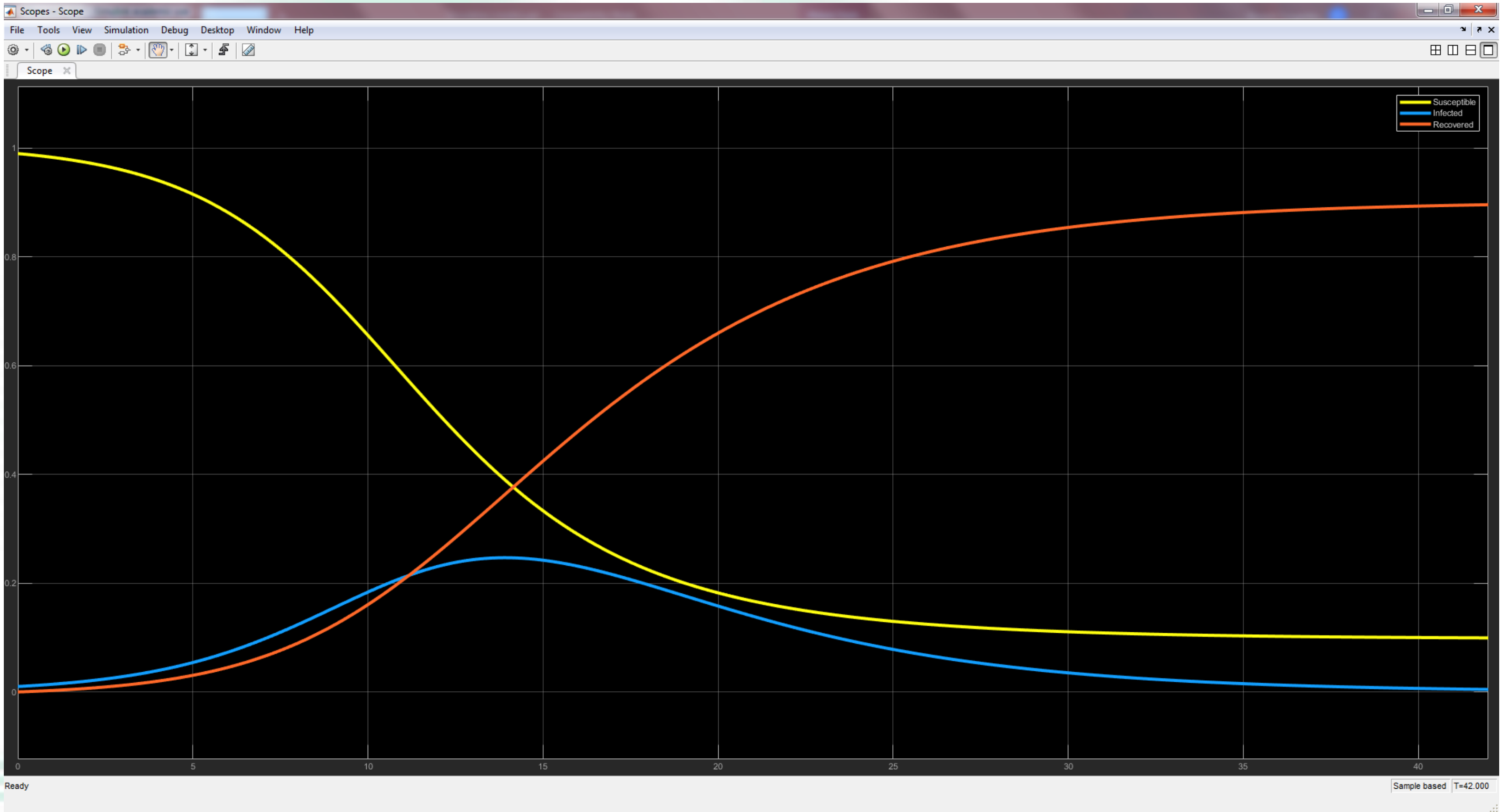
The number of infected people and the derivative (i.e. the change of the number infected people) in one single figure (note the inflection point and the maximum)

SIR epidemy model (Vb)



Transmission rate 1.82 gives maximum number of infections 61.3 % after 4.22 days (simulation time 10 days)

SIR epidemy model (Vc)

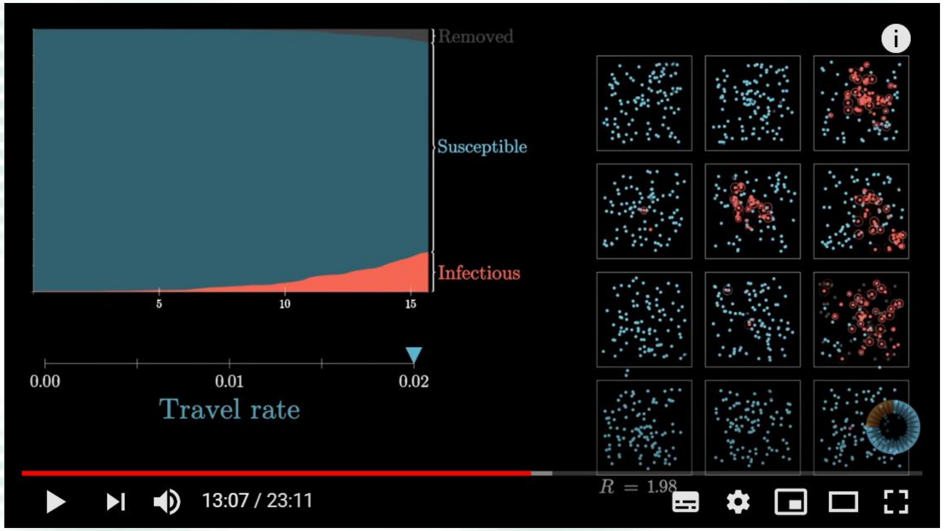


Transmission rate 0.59 gives maximum number of infections 24.6 % after 13.92 days (simulation time 42 days)

Remarks

The SIR model is a so called macro scale model, in which the population as a whole is modelled.

An other time of model is a micro scale model, in which the individuals are modelled separately; an example of these types of models is agent based modelling.



References

Sparks, Ben, (2020, March 25). *The Coronavirus Curve – Numberphile*, retrieved March 27, 2020 from https://www.youtube.com/watch?v=k6nLfCbAzgo

(terminology updated to SIR terminology, and data adapted to data used in reference mentioned above)

Sanderson, Grant, (2020, March 27). *Simulating an epidemic – 3Blue1Brown*, retrieved May 17, 2020 from https://www.youtube.com/watch?v=gxAaO2rsdIs