University of Applied Sciences

SAXION

Mathematical Modelling
Complex numbers &
Vector and matrix calculations

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Outline today

- **When to use a dot**
- Complex numbers
  - Complex numbers in MATLAB
  - Logarithms of a negative number
- Vectors and Matrices
  - Dot product (inner product)
  - Length, norm & size
  - Cross product (outer product)
  - Matrices
- Fancy plotting
- How to hand in Matlab lab assignment

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When to use a dot (1)

```
>> x = [1 2];
>> y = [3 4];
>> z = x+2
z =
     3     4
>> z = x+y
z =
   4     6
>> z = x.+y
 z = x.+y
       ↑
Error: Invalid use of operator.
```

When to use a dot (2)

```
>> x = [1 2];
>> y = [3 4];
>> z = 2*x
z =
     2     4
>> z = x*y
Error using  *
Incorrect dimensions for matrix multiplication.
>> z = x.*y % elementwise multiplication
z =
     3     8
```

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When to use a dot (3)

```
>> x = [1 2];
>> y = [3 4];
>> z = y/x
z =
     2.2000
```

This 'division' operator on vectors is not commonly used, see `mrdivide` for more info; quite often an elementwise division is intended instead!

```
>> y./x % elementwise division
z =
      3      2
```

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When to use a dot (4)

```
>> M = [1 2;3 4]
M =
     1     2
     3     4
>> R = M.^2 % elementwise exponentiation
R =
     1     4
     9    16
>> R = M^2 % matrix multiplication
R =
     7    10
    15    22
```

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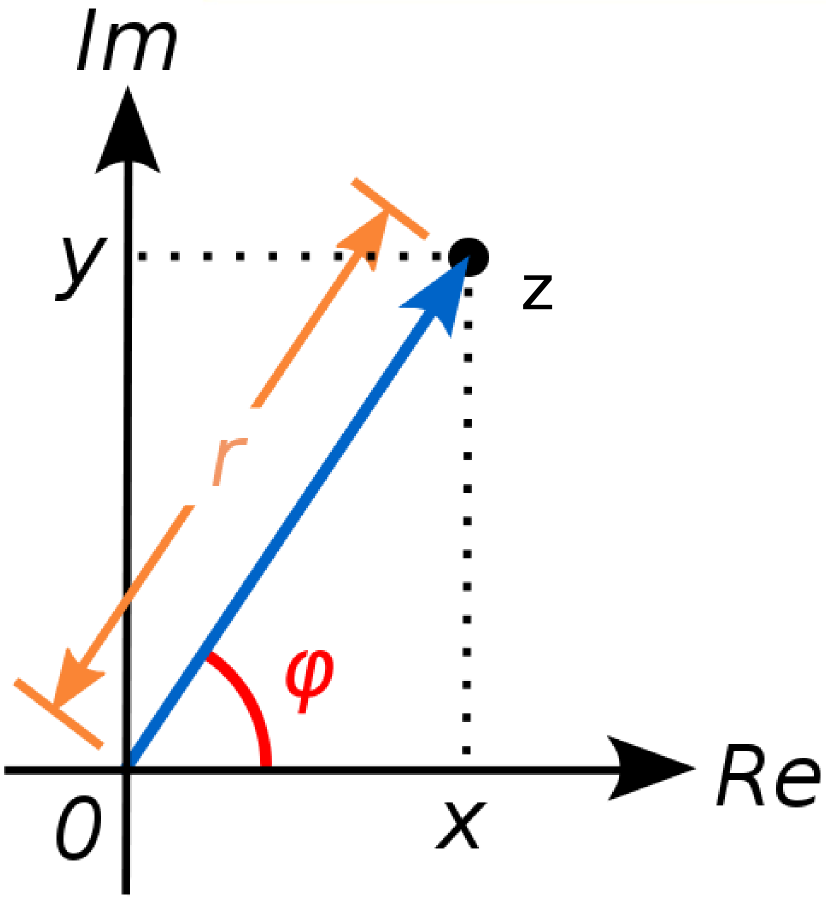
Complex numbers (1)



Source: https://en.wikipedia.org/wiki/Argument_(complex_analysis)

```
% MATLAB
x = real(z);
y = imag(z);
r = abs(z);
phi = angle(z);
```

Complex numbers (2)

For more information about complex numbers, have a look at the first-year course

Trigonometry and complex calculation
Lesson8

Logarithms (1)

```
log(x) % log(x) in MATLAB is ln(x)

>> r = log(0)
r =

  -Inf

>> r = log(-1)
r =

   0.0000 + 3.1416i
```

Logarithms (s)

The input x of $\ln(x)$ is normally defined as a real number, with should be larger than zero.

However, we can also extend the input domain to the complex numbers (this is what MATLAB does):

Let c be the complex number $a + jb$,

then c can also be written as $r.e^{j\varphi}$

So, $\ln(c) = \ln(r.e^{j\varphi}) = \ln(r) + \ln(e^{j\varphi}) = \ln(r) + j\varphi$

Example:

$\ln(-1) = \ln(1.e^{j\pi}) = j\pi$

$\ln(-2) = \ln(2.e^{j\pi}) = \ln(2) + j\pi$

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Dot product (inner product)

```
>> a=[1 2];
>> b=[3 4];
>> d=dot(a,b); %
```
$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + \cdots + a_n \cdot b_n$

```
d =
    11
```

or as vector multiplication:

```
d = a * b';
```

```
>> a=[2 0];
>> b=[0 5];
>> d=dot(a,b)
d =
     0
```

Length, norm & size

```
>> v=[3 4];
>> l=length(v)
l =
     2

>> n=norm(v)
```
% $\|\vec{v}\| = \sqrt{{v_1}^2 + {v_2}^2 + \cdots + {v_n}^2}$

```
n =
     5

>> s=size(v)
s =
     1     2
```

Cross product (outer product)

```
>> a=[1 2 3];
>> b=[4 5 6];
>> v=cross(a,b)
v =
    -3      6     -3
>> w=dot(a,v)
w =
     0
```

a. $\vec{v} \perp \vec{a}$ and $\vec{v} \perp \vec{b}$

b. $||\vec{v}||$ is equal to area of parallelogram formed by $\vec{a}$ and $\vec{b}$

c. $\vec{v}$ has a direction according to right-hand screw rule, a.k.a. corkscrew rule

Matrices (1)

```
>> A=[1 2;3 4]
A =
     1     2
     3     4

>> A_transpose=A'
A_transpose =
     1     3
     2     4

Note: be careful with complex vectors
      transpose is also complex conjugate!
```

Matrices (2)

```
>> A=[1+2j 2-3j;3 4]
A =
   1.0000 + 2.0000i   2.0000 - 3.0000i
   3.0000 + 0.0000i   4.0000 + 0.0000i

>> A_transpose=A'
A_transpose =
   1.0000 - 2.0000i   3.0000 + 0.0000i
   2.0000 + 3.0000i   4.0000 + 0.0000i
```

```
Note: be careful with complex vectors
      transpose is also complex conjugate!
```

Matrices (3)

Solving the equation $A \cdot x = b$

$$3 \cdot x_1 - 2 \cdot x_2 = 12$$

$$2 \cdot x_1 + 5 \cdot x_2 = 17$$

```
>> A = [3 -2; 2 5];
>> b = [12; 17];
```

We want to find the value of x_1 and x_2

Matrices (2)

Solving the equation $A \cdot x = b$

```
>> A = [3 -2; 2 5];
>> b = [12; 17]
```

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$$
$$\mathbf{A}^{-1} \cdot \mathbf{A} \cdot \mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b}$$
$$\mathbf{I} \cdot \mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b}$$
$$\mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b}$$

```
>> x=inv(A)*b
x =
    4.9474
    1.4211
```

Solving the equation $A \cdot x = b$

```
>> A = [3 -2; 2 5];
>> b = [12; 17];
```

Same result but computationally more efficient than using `x=inv(A)*b`:

```
>> x = A\b; % Matrix left division a.k.a. backslash
x =
    4.9474
    1.4211
```

Special Matrices

```
>> Z = zeros(2,4)
Z = 0 0 0 0
    0 0 0 0
>> O = ones(1,3);
>> I = eye(3)
I = Identity Matrix
   1 0 0
   0 1 0
   0 0 1
>> Runif= rand(2,5);
>> Rnorm= randn(3,3);
>> M = magic(4);
```

```
>> D = diag([1 2 3])
D =
     1     0     0
     0     2     0
     0     0     3

>> diag( D )
ans =
     1
     2
     3
```

Selecting rows and columns from a matrix

```
row1 = M(1,:);
col3 = M(:,3);
col3and4 = M(:,3:4);
col3and4fromrow2 = M(2,3:4);
col2and4fromrow3 = M(3,[2,4]);
```

Vector- and matrix calculation

For more information about vector- and matrix calculation, see the first-year course Trigonometry and complex calculation

```
>> a = [ 1 2 3 ];   b = [ 4 5 6 ];
>> a  * b        Incorrect dimensions for matrix multiplication.
>> a' * b'       Incorrect dimensions for matrix multiplication.
>> a .* b        ans =    4   10   18
>> a  * b'       ans =   32
>> a' * b        ans =    4    5    6
                          8   10   12
                         12   15   18
```

BSXfun, or implicite usage if BSX functionality

The binary singleton expansion function performs broadcasting, that is, it applies a binary function f element-by-element to two array arguments A and B, and expands singleton dimensions in either input argument, if necessary

>> a = ones(5, 5);	b = 4;	a .* b
[5x5]	[1x1]→[5x5]	[5x5]
>> a = ones(5, 5);	b = 1:5;	a .* b
[5x5]	[1x5]→[5x5]	[5x5]
>> a = ones(5, 5);	b = (1:5) ';	a .* b
[5x5]	[5x1]→[5x5]	[5x5]

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Fancy plotting

Create clear plots:

- Label your axes
  - `xlabel('Time (s)')`
  - `ylabel('Voltage (V)')`
- Limit your axes
  - X-axis only: `xlim([xmin xmax])`
  - Y-axis only: `ylim([ymin ymax])`
  - Both: `axis([xmin xmax ymin ymax])`
- Multiple plots: `figure` or `subplot`
- Add legend: `legend('Volt1', 'Volt2')`
- Add title: `title('Graph name')`
- Bring a plot to the front: `shg`
- Set grid and zoom on: `grid on; zoom on;`

Fancy plotting

Link multiple plots (axes): Try this at home:

```
>> hFig = figure;   hFig.Color = 'w';
>> hSp1 = subplot(211);  grid on;   zoom on;
>> plot( 10 + rand(1,100), 'ro' );
>> hSp2 = subplot(212);  grid on;   zoom on;
>> plot( randn(1,100), 'b+-' );
>> linkaxes([ hSp1 hSp2 ], 'x' );% x-axis only
Or:
>> linkaxes([ hSp1 hSp2 ]);       % both axes
```

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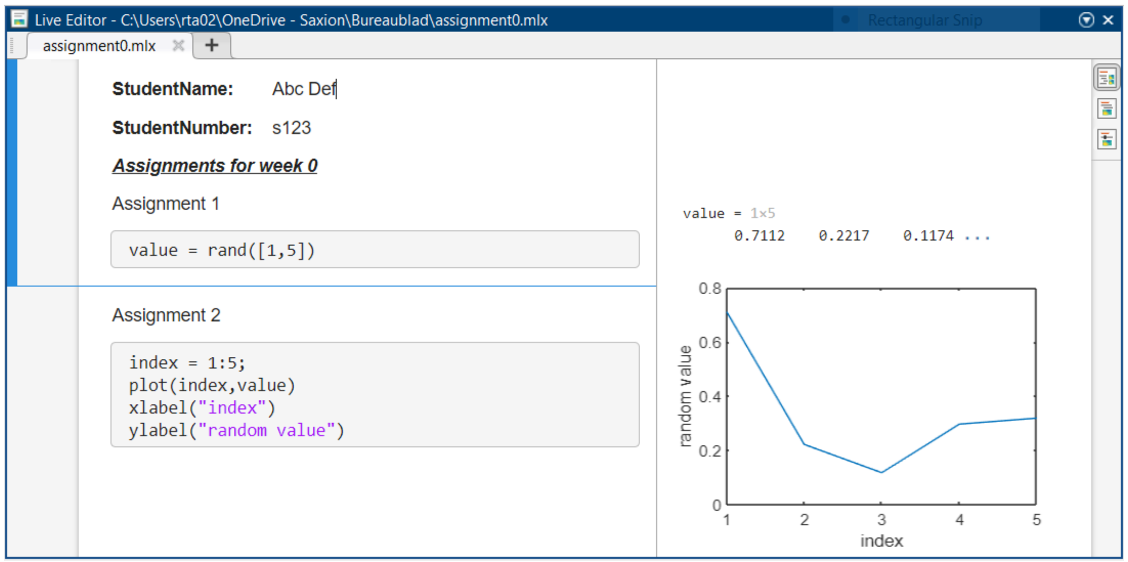
Practical lab assignments

- Submit the practical assignments on BrightSpace both as MATLAB live script (*.mxl) and exported pdf file



- Please note the use of comment (name, student number, assignment) and sections
- A more advanced MATLAB live script can be found at BrightSpace

Practical assignments

- If you are requested to plot, always use a title, xlabel and ylabel and a legend (if applicable). Take care of good titles and don’t forget to clearly indicate the units used at the x- and y-axes.

Questions