Mathematical Modelling - Assignments week 4:

Numerical Methods for Differential Equations

Assignment 1: Differential equations

Given is the following differential equation (DE) and boundary condition (BC):

$$\frac{dy}{dt} = y - 3t,\ y(0) = 5$$

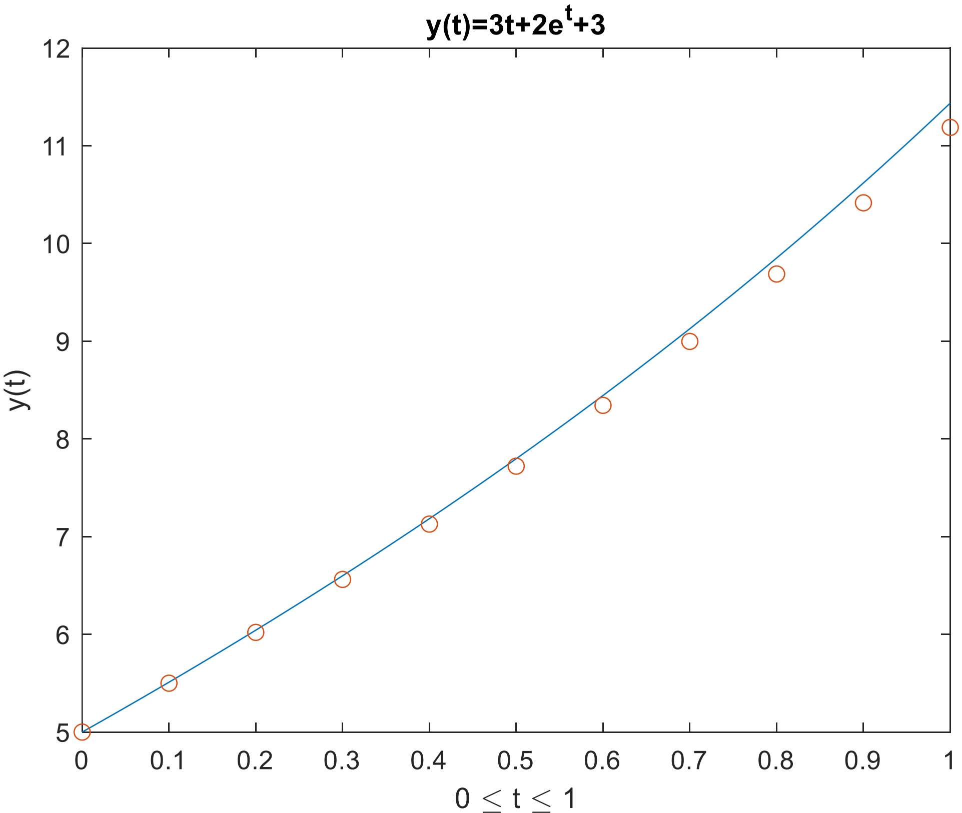
Calculate the analytical solution of this DE and BC.

Analytical solution = y(t)= $3t+2e^t+3$

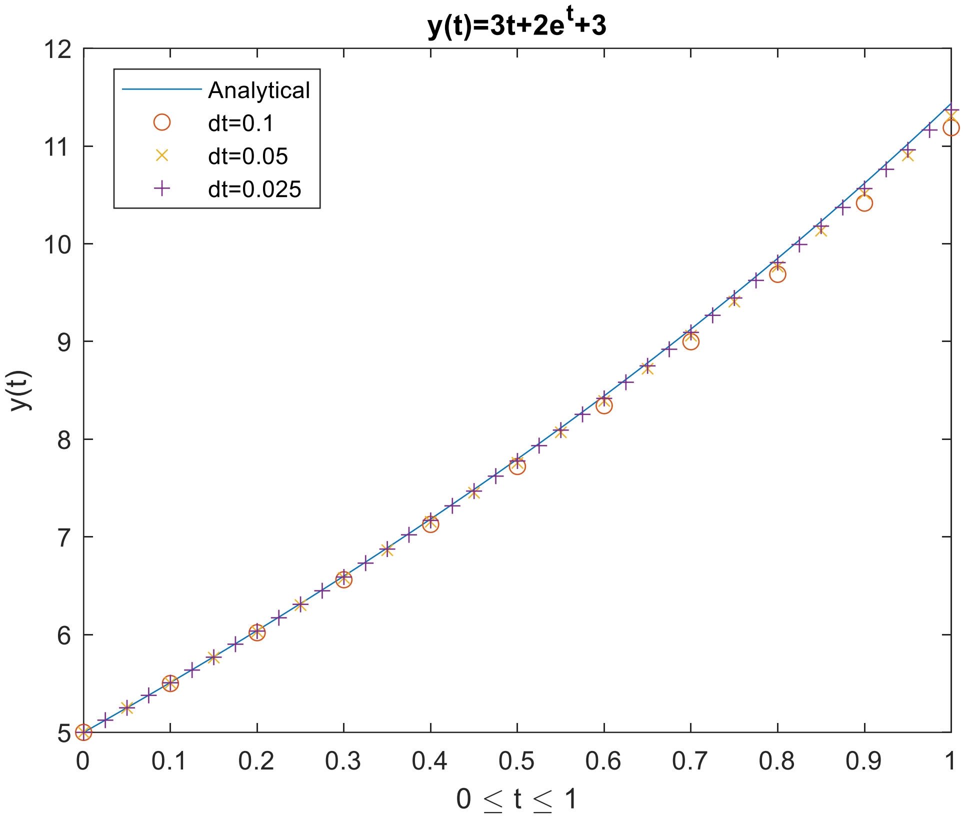
Assignment 2: Euler with Matlab

(a) Write a Matlab function `righthandside` to implement the righthand side of the DE.
(b) Write the Euler function itself in the matlab file `Euler.m`.
(c) Extend the Matlab script file `assignments_week4.m` with code for solving the DE for the end time t = 1.

- In this script you should start time t = 0 and step size dt = 0.1.
- Plot the solution using markers (e.g. circles). Don't draw a connection line through the points of the solution.
- The analytical solution has been given in the previous assignment. Evaluate this solution at the same time values as the Euler solution.
- Plot the analytical solution in the same figure, use this time a line and no markers.



- Determine now the Euler solution for the step sizes 0.05 and 0.025, and plot these in the same figure.



- Calculate the error ratios $\frac{E_{dt=0.05}}{E_{dt=0.1}}$ and $\frac{E_{dt=0.025}}{E_{dt=0.05}}$ for t=1 by means of matlab code.

  Don't use a semicolon in the final result, so that the output will be printed

$$\frac{E_{dt=0.05}}{E_{dt=0.1}} = 0.5218$$

$$\frac{E_{dt=0.025}}{E_{dt=0.05}} = 0.5112$$

Assignment 3: Heun with Matlab

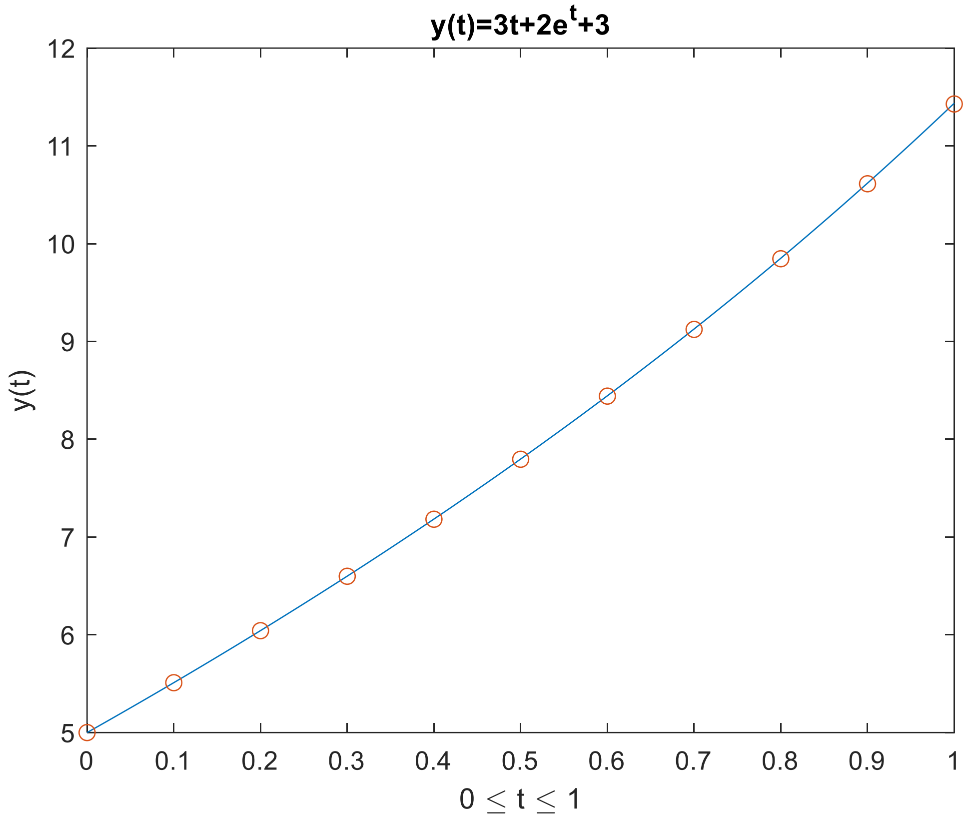
Answer the same questions as in assignment 2, but now with the Heun method (so you should use the step sizes 0.1, 0.05 and 0.025. Of course you can reuse the Matlab function `righthandside`.

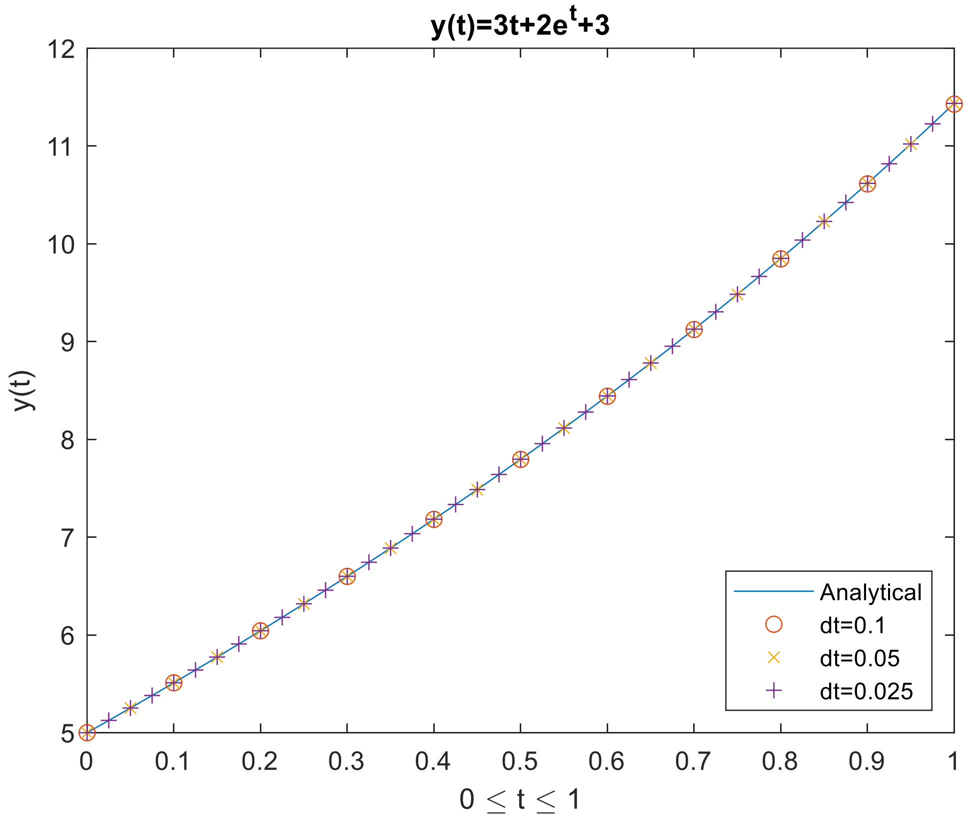
So you have to implement a new function Heun in the file `Heun.m`, which has the same parameters as the Euler function, but of course a different implementation.

To compare the Heun method with the Euler method, we will solve again the DE of the first assignment

Plot the solutions (with different markers) in the same figure, together with the analytical solution.

Don't forget to calculate the error ratios too.





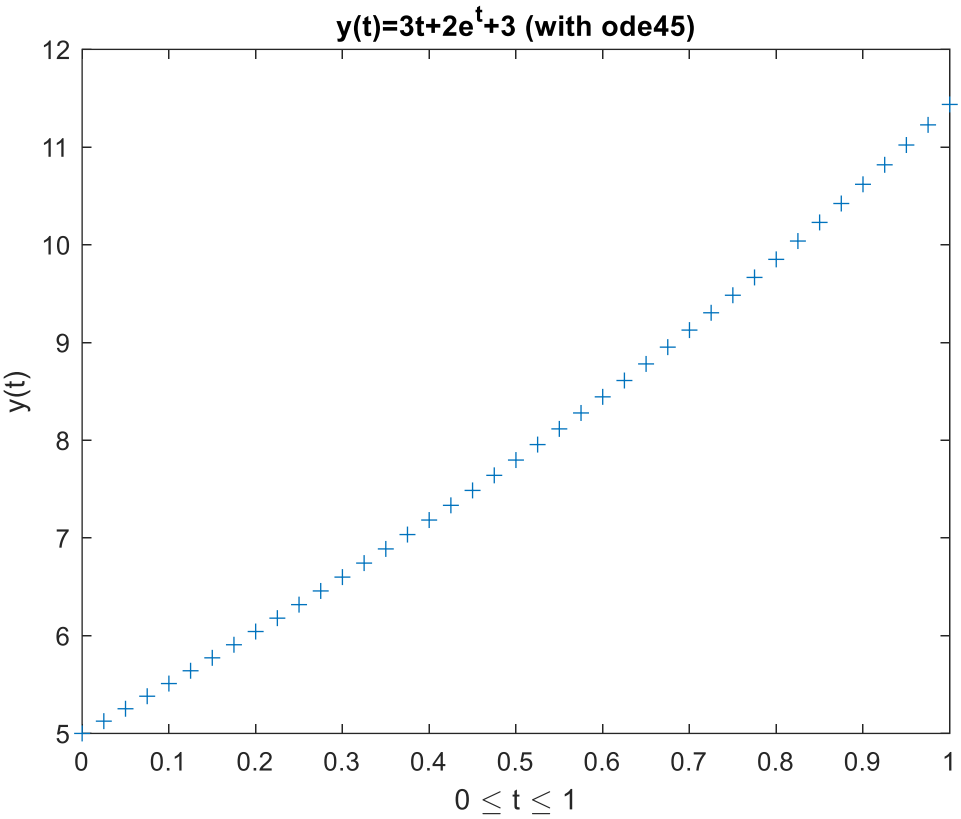
$$\frac{E_{dt=0.05}}{E_{dt=0.1}} = 0.2596$$

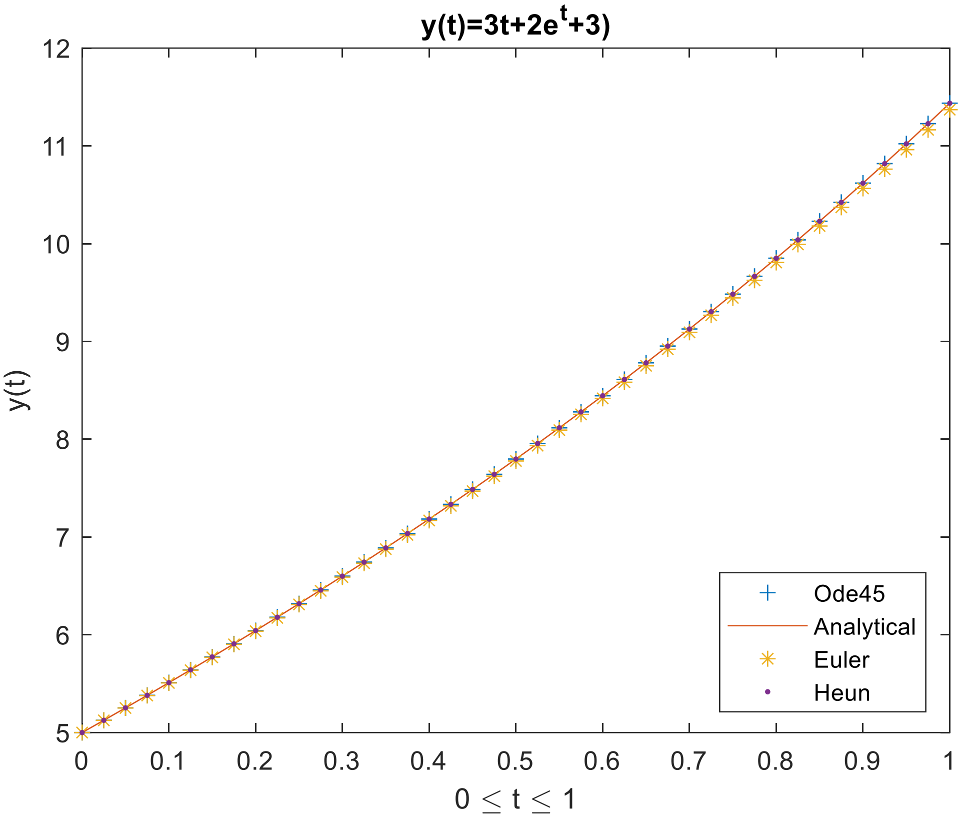
$$\frac{E_{dt=0.025}}{E_{dt=0.05}} = 0.2548$$

Assignment 4: The Matlab command ode45

In this assignment we are going to use this built-in Matlab function.
Add a line in your Matlab script file in which you solve the DE of the first assignment, now using the ode45 method. Take again as end time 1. Use as time domain = [t0 t_end].
Plot the solution (using markers) in the same figure as where the Euler and Heun solution have been plotted.

You should submit the matlab script file as well as Matlab files for the the functions you wrote at the blackboard submit point.




y(t)=3t+2e^t+3)
y(t)
0 ≤ t ≤ 1
12
11
10
9
8
7
6
5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Ode45
Analytical
Euler
Heun