

2.) $x_0, \dots, x_n \in [-1, 1]$ paarweise verschieden

$\beta_0, \dots, \beta_n \in \mathbb{R}$ für beliebige Polynome mit
Grad $\leq n$ soll gelten:

$$\int_{-1}^1 p(x) dx = \sum_{i=0}^n \beta_i p(x_i)$$

$$g(x) = \sum_{k=0}^n f(x_k) L_k(x)$$

$$\text{und } \int_a^b g(x) dx = \sum_{k=0}^n f(x_k) \int_a^b L_k(x) dx = \sum_{k=0}^n \beta_k f(x_k)$$

$$\text{mit } \beta_k = \int_a^b L_k(x) dx$$

hier: $p(x) = g(x)$ und $f(x_k) = y_i$ und $a = -1, b = 1$

$$\Rightarrow \int_{-1}^1 p(x) dx = \int_a^b p(x) dx = \sum_{i=0}^n \beta_i f(x_i) = \sum_{i=0}^n \beta_i y_i$$

$$= \sum_{i=0}^n \beta_i p(x_i)$$

$$\text{da } p(x_i) = y_i$$

$$6.) b) T = \frac{b_i - a_i}{2} (f(a_i) + f(b_i))$$

$$\tilde{T} = \frac{b_i - a_i}{4} (f(a_i) + 2f(\frac{a_i + b_i}{2}) + f(b_i))$$

$$\tilde{F} = \frac{1}{3} |\tilde{T} - T|$$

$$\int_1^5 \frac{1}{x} dx$$

$$[1; 5]: T_0 = \frac{5-1}{2} (1 + \frac{1}{5}) = \frac{12}{5}$$

$$\tilde{T}_0 = \frac{5-1}{4} (1 + 2 \cdot \frac{1}{3} + \frac{1}{5}) = \frac{28}{15}$$

$$\Rightarrow \tilde{F} = \frac{1}{3} |\frac{28}{15} - \frac{12}{5}|$$

$$= \frac{1}{3} \cdot \frac{8}{15} = \frac{8}{45} > 0,02$$

$$[1; 3]:$$

$$T_u = 1 \cdot (1 + \frac{1}{3}) = \frac{4}{3}$$

$$\tilde{T}_u = \frac{1}{2} (1 + 1 + \frac{1}{3}) = \frac{7}{6}$$

$$\Rightarrow \tilde{F}_u = \frac{1}{3} |\frac{7}{6} - \frac{4}{3}| = \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$$

$$\Rightarrow \tilde{F}_u > 0,02$$

$$[3; 5]:$$

$$T_v = 1 \cdot (\frac{1}{3} + \frac{1}{5}) = \frac{8}{15}$$

$$\tilde{T}_v = \frac{1}{2} (\frac{1}{3} + \frac{1}{2} + \frac{1}{5}) = \frac{1}{2} \cdot \frac{31}{30} = \frac{31}{60}$$

$$\Rightarrow \tilde{F}_v = \frac{1}{3} |\frac{8}{15} - \frac{31}{60}| = \frac{1}{3} \cdot \frac{1}{60} = \frac{1}{180}$$

$$\Rightarrow \tilde{F}_v < 0,02 \quad \checkmark$$

$$[1; 2]:$$

$$T_w = \frac{1}{2} \cdot (1 + \frac{1}{2}) = \frac{3}{4}$$

$$\begin{aligned} \tilde{T}_w &= \frac{1}{4} \cdot (1 + 2 \cdot \frac{2}{3} + \frac{1}{2}) \\ &= \frac{1}{4} \cdot (\frac{6}{6} + \frac{8}{6} + \frac{3}{6}) = \frac{17}{24} \end{aligned}$$

$$\Rightarrow \tilde{F}_w = \frac{1}{3} |\frac{17}{24} - \frac{3}{4}| = \frac{1}{3} \cdot \frac{1}{24} = \frac{1}{72}$$

$$\Rightarrow \tilde{F}_w < 0,02 \quad \checkmark$$

$$[2; 3]:$$

$$T_z = \frac{1}{2} \cdot (\frac{1}{2} + \frac{1}{3}) = \frac{5}{12}$$

$$\begin{aligned} \tilde{T}_z &= \frac{1}{4} (\frac{1}{2} + 2(\frac{2}{5}) + \frac{1}{3}) \\ &= \frac{1}{4} (\frac{15}{30} + \frac{24}{30} + \frac{10}{30}) = \frac{49}{120} \end{aligned}$$

$$\Rightarrow \tilde{F}_z = \frac{1}{3} |\frac{49}{120} - \frac{5}{12}|$$

$$= \frac{1}{3} \cdot \frac{1}{120} = \frac{1}{360}$$

$$\Rightarrow \tilde{F}_z < 0,02 \quad \checkmark$$

$$\Rightarrow \tilde{T}_{v2} + \tilde{T}_{u1} + \tilde{T}_{z2} = \frac{31}{60} + \frac{17}{24} + \frac{49}{120} = \frac{69}{30} = 1,63$$

$$\text{exakt: } \int_1^5 \frac{1}{x} dx = \ln(5) - \ln(1) \approx 1,609$$