1.)

i	0	1	2	3
x_i	-1	0	1	3
y_i	-2	4	6	22

(a) $L_0(x) = \frac{x-x_1}{x_0-x_1} \cdot \frac{x-x_2}{x_0-x_2} \cdot \frac{x-x_3}{x_0-x_3} = \frac{x-0}{-1} \cdot \frac{x-1}{-2} \cdot \frac{x-3}{-4}$

$= -\frac{1}{8} x \cdot (x-1)(x-3)$

$L_1(x) = \frac{x-x_0}{x_1-x_0} \cdot \frac{x-x_2}{x_1-x_2} \cdot \frac{x-x_3}{x_1-x_3} = \frac{x+1}{1} \cdot \frac{x-1}{-1} \cdot \frac{x-3}{-3}$

$= \frac{1}{3}(x+1)(x-1)(x-3)$

$L_2(x) = \frac{x-x_0}{x_2-x_0} \cdot \frac{x-x_1}{x_2-x_1} \cdot \frac{x-x_3}{x_2-x_3} = \frac{x+1}{2} \cdot \frac{x-0}{1} \cdot \frac{x-3}{-2}$

$= -\frac{1}{4} x (x+1)(x-3)$

$L_3(x) = \frac{x-x_0}{x_3-x_0} \cdot \frac{x-x_1}{x_3-x_1} \cdot \frac{x-x_2}{x_3-x_2} = \frac{x+1}{4} \cdot \frac{x-0}{3} \cdot \frac{x-1}{2}$

$= \frac{1}{24} x (x+1)(x-1)$

$p_3(x) = -2L_0(x) + 4L_1(x) + 6L_2(x) + 22L_3(x)$

$= \frac{1}{4} x(x-1)(x-3) + \frac{4}{3}(x+1)(x-1)(x-3) - \frac{3}{2} x (x+1)(x-3)$

$+ \frac{11}{12} x (x+1)(x-1)$

$= \frac{1}{4} x (x^2 - 4x + 3) + \frac{4}{3}(x^2-1)(x-3) - \frac{3}{2} x (x^2 - 2x - 3)$

$+ \frac{11}{12} x (x^2 - 1)$

$= \frac{1}{4} x^3 - x^2 + \frac{3}{4} x + \frac{4}{3}(x^3 - 3x^2 - x + 3) - \frac{3}{2} x^3 + 3x^2 + \frac{3}{2} x$

$+ \frac{11}{12} x^3 - \frac{11}{12} x$

$= x^3 - 2x^2 + 3x + 4$

1.)

(b) $p_n(x) = d_{00} + d_{10}(x-x_0) + \dots + d_{n0}(x-x_0)\cdot\dots\cdot(x-x_{n-1})$

$d_{00} = -2$

$$d_{10} = \frac{d_{11}-d_{00}}{x_1-x_0} = \frac{4+2}{0+1} = 6$$

$$d_{21} = \frac{d_{22}-d_{11}}{x_2-x_1} = \frac{6-4}{1-0} = 2$$

$$d_{20} = \frac{d_{21}-d_{10}}{x_2-x_0} = \frac{2-6}{1+1} = -2$$

$$d_{32} = \frac{d_{33}-d_{22}}{x_3-x_2} = \frac{22-6}{3-1} = 8$$

$$d_{31} = \frac{d_{32}-d_{21}}{x_3-x_1} = \frac{8-2}{3-0} = 2$$

$$d_{30} = \frac{d_{31}-d_{20}}{x_3-x_0} = \frac{2+2}{3+1} = 1$$

$$\Rightarrow p_3(x) = -2 + 6(x+1) - 2(x+1)(x-0) + (x+1)(x-0)(x-1)$$

$$= -2 + 6x + 6 - 2x^2 - 2x + x^3 - x$$

$$= x^3 - 2x^2 + 3x + 4$$