

# Answers to Odd-Numbered Exercises and Tests

## Chapter 1

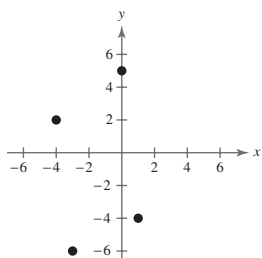
### Section 1.1 (page 9)

#### Vocabulary Check (page 9)

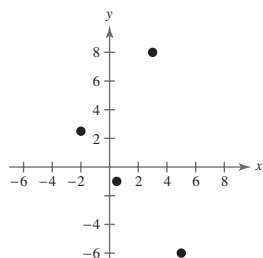
1. (a) v (b) vi (c) i (d) iv (e) iii (f) ii  
 2. Cartesian 3. Distance Formula  
 4. Midpoint Formula

1. A: (2, 6), B: (-6, -2), C: (4, -4), D: (-3, 2)

3.



5.

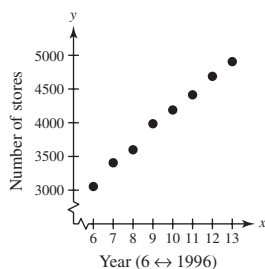


7. (-3, 4) 9. (-5, -5) 11. Quadrant IV

13. Quadrant II 15. Quadrant III or IV

17. Quadrant III 19. Quadrant I or III

21.

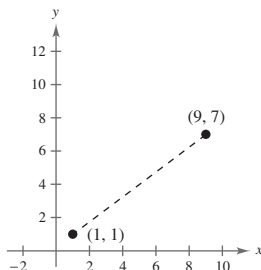


23. 8 25. 5

27. (a) 4, 3, 5 (b)  $4^2 + 3^2 = 5^2$

29. (a) 10, 3,  $\sqrt{109}$  (b)  $10^2 + 3^2 = (\sqrt{109})^2$

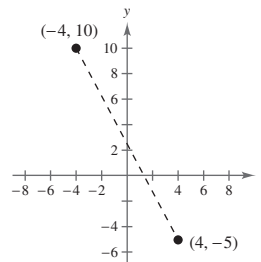
31. (a)



(b) 10

(c) (5, 4)

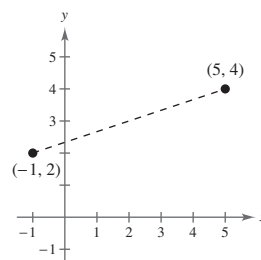
33. (a)



(b) 17

(c)  $(0, \frac{5}{2})$

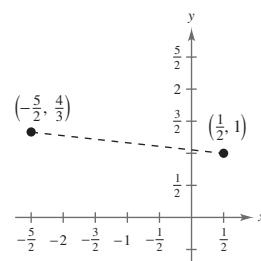
35. (a)



(b)  $2\sqrt{10}$

(c) (2, 3)

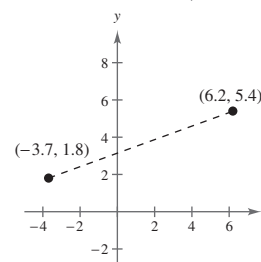
37. (a)



(b)  $\frac{\sqrt{82}}{3}$

(c)  $(-1, \frac{7}{6})$

39. (a)



(b)  $\sqrt{110.97}$

(c) (1.25, 3.6)

41.  $(\sqrt{5})^2 + (\sqrt{45})^2 = (\sqrt{50})^2$

43.  $(2x_m - x_1, 2y_m - y_1)$

45.  $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right), \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right),$   
 $\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$

47.  $2\sqrt{505} \approx 45$  yards 49. \$3803.5 million

51. (0, 1), (4, 2), (1, 4) 53. (-3, 6), (2, 10), (2, 4), (-3, 4)

55. \$3.31 per pound; 2001 57.  $\approx 250\%$

59. (a) The number of artists elected each year seems to be nearly steady except for the first few years. From six to eight artists will be elected in 2008.

(b) The Rock and Roll Hall of Fame was opened in 1986.

61. 1998: \$19,384.5 million; 2000: \$20,223.0 million;  
2002: \$21,061.5 million

63.  $\sqrt[3]{\frac{4.47}{\pi}} \approx 1.12$  inches

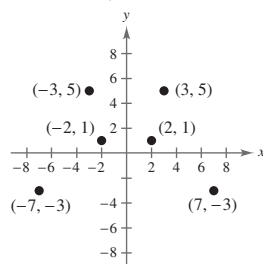
65. Length of side = 43 centimeters;  
area = 800.64 square centimeters

67. (a)  (b)  $l = 1.5w$ ;  $p = 5w$   
(c) 7.5 meters  $\times$  5 meters

69. (a)  (b) 2002

- (c) Answers will vary. Sample answer: Technology now enables us to transport information in many ways other than by mail. The Internet is one example.

71.



- (a) The point is reflected through the  $y$ -axis.  
(b) The point is reflected through the  $x$ -axis.  
(c) The point is reflected through the origin.

73. False. The Midpoint Formula would be used 15 times.

75. No. It depends on the magnitudes of the quantities measured.

77. b    78. c    79. d    80. a    81.  $x = 1$

83.  $x = 2 \pm \sqrt{11}$     85.  $x < \frac{3}{5}$     87.  $14 < x < 22$

## Section 1.2 (page 22)

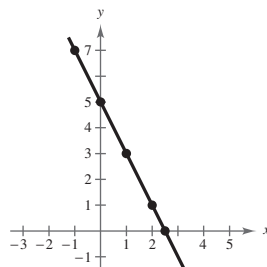
### Vocabulary Check (page 22)

1. solution or solution point    2. graph  
3. intercepts    4.  $y$ -axis    5. circle;  $(h, k)$ ;  $r$   
6. numerical

1. (a) Yes    (b) Yes    3. (a) No    (b) Yes

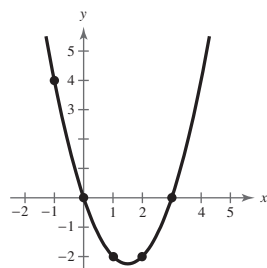
5.

$x$	-1	0	1	2	$\frac{5}{2}$
$y$	7	5	3	1	0
$(x, y)$	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$(\frac{5}{2}, 0)$



7.

$x$	-1	0	1	2	3
$y$	4	0	-2	-2	0
$(x, y)$	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$

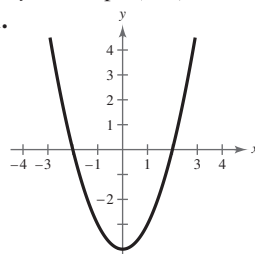


9.  $x$ -intercepts:  $(\pm 2, 0)$   
 $y$ -intercept:  $(0, 16)$

13.  $x$ -intercept:  $(-4, 0)$   
 $y$ -intercept:  $(0, 2)$

17.  $x$ -intercepts:  $(0, 0), (2, 0)$   
 $y$ -intercept:  $(0, 0)$

21.

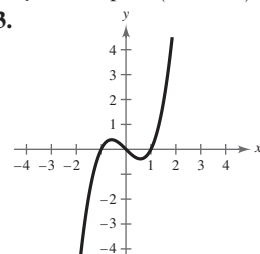


11.  $x$ -intercept:  $(\frac{6}{5}, 0)$   
 $y$ -intercept:  $(0, -6)$

15.  $x$ -intercept:  $(\frac{7}{3}, 0)$   
 $y$ -intercept:  $(0, 7)$

19.  $x$ -intercept:  $(6, 0)$   
 $y$ -intercepts:  $(0, \pm\sqrt{6})$

23.



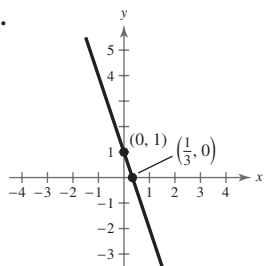
25.  $y$ -axis symmetry

27. Origin symmetry

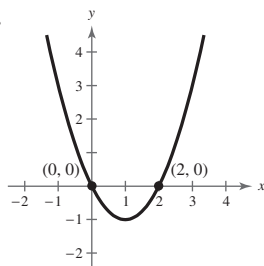
29. Origin symmetry

31.  $x$ -axis symmetry

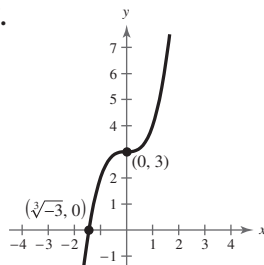
33.



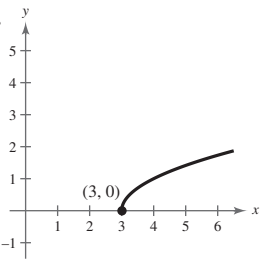
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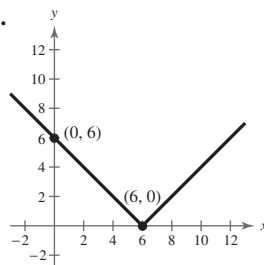
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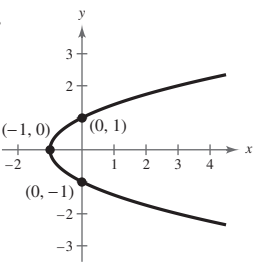
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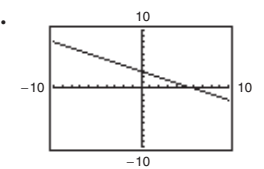
41.



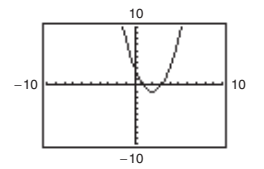
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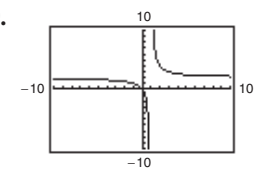
45.


 Intercepts:  $(6, 0)$ ,  $(0, 3)$ 

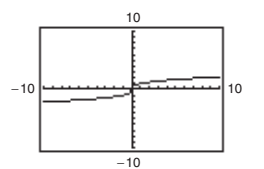
47.


 Intercepts:  $(3, 0)$ ,  $(1, 0)$ ,  $(0, 3)$ 

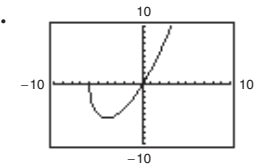
49.


 Intercept:  $(0, 0)$ 

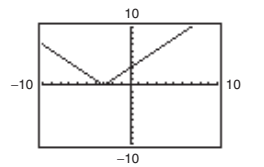
51.


 Intercept:  $(0, 0)$ 

53.


 Intercepts:  $(0, 0)$ ,  $(-6, 0)$ 

55.


 Intercepts:  $(-3, 0)$ ,  $(0, 3)$ 

57.  $x^2 + y^2 = 16$

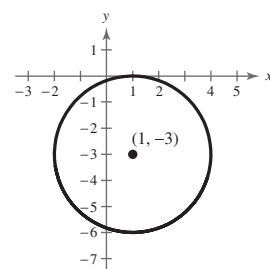
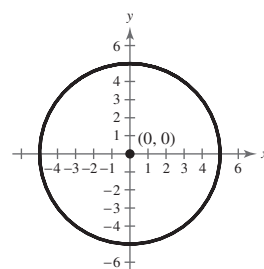
59.  $(x - 2)^2 + (y + 1)^2 = 16$

61.  $(x + 1)^2 + (y - 2)^2 = 5$

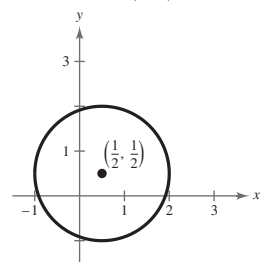
63.  $(x - 3)^2 + (y - 4)^2 = 25$

65. Center:  $(0, 0)$ ; Radius: 5

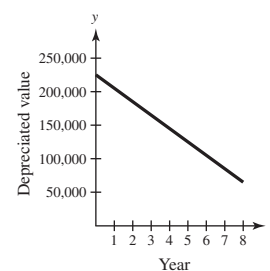
67. Center:  $(1, -3)$ ; Radius: 3



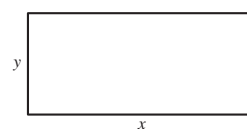
69. Center:  $(\frac{1}{2}, \frac{1}{2})$ ; Radius:  $\frac{3}{2}$



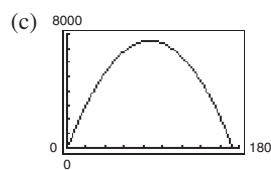
71.



73. (a)



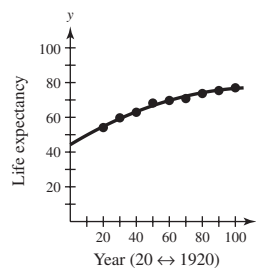
(b) Answers will vary.



(d)  $x = 86\frac{2}{3}$ ,  $y = 86\frac{2}{3}$

 (e) A regulation NFL playing field is 120 yards long and  $53\frac{1}{3}$  yards wide. The actual area is 6400 square yards.

75. (a) and (b)



(c) 66.0 years (d) 2005: 77.0 years; 2010: 77.1 years

(e) Answers will vary.

77. False. A graph is symmetric with respect to the  $x$ -axis if, whenever  $(x, y)$  is on the graph  $(x, -y)$  is also on the graph.

79. The viewing window is incorrect. Change the viewing window. Answers will vary.

81.  $9x^5$ ,  $4x^3$ ,  $-7$     83.  $2\sqrt{2x}$

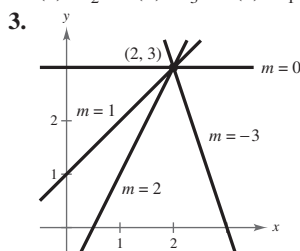
85.  $\frac{10\sqrt{7x}}{x}$     87.  $\sqrt[3]{|t|}$

### Section 1.3 (page 34)

#### Vocabulary Check (page 34)

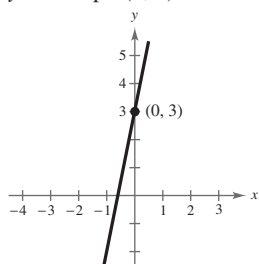
1. linear    2. slope    3. parallel
4. perpendicular    5. rate or rate of change
6. linear extrapolation
7. a. iii    b. i    c. v    d. ii    e. iv

1. (a)  $L_2$     (b)  $L_3$     (c)  $L_1$

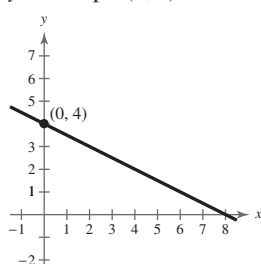


5.  $\frac{3}{2}$     7.  $-4$

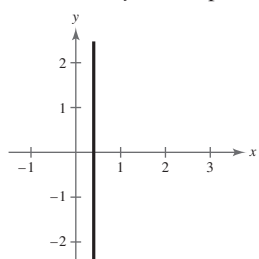
9.  $m = 5$ ;  
y-intercept:  $(0, 3)$



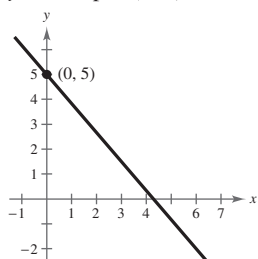
11.  $m = -\frac{1}{2}$ ;  
y-intercept:  $(0, 4)$



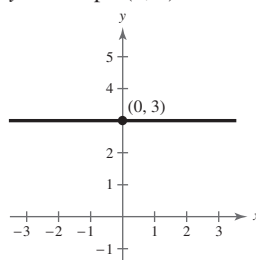
13.  $m$  is undefined.  
There is no y-intercept.



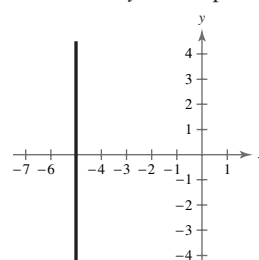
15.  $m = -\frac{7}{6}$ ;  
y-intercept:  $(0, 5)$



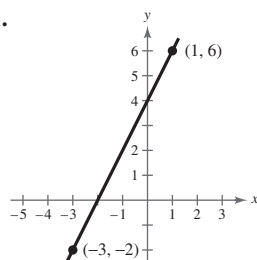
17.  $m = 0$ ;  
y-intercept:  $(0, 3)$



19.  $m$  is undefined.  
There is no y-intercept.

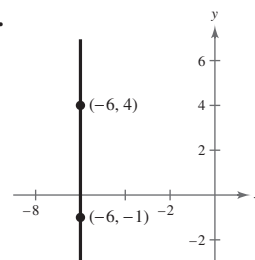


21.



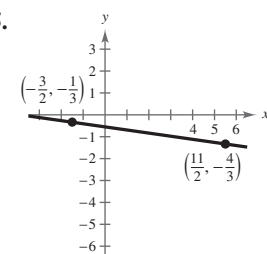
$m = 2$

23.



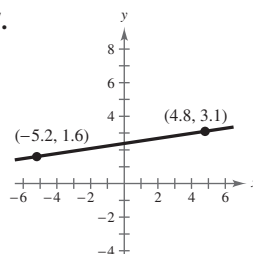
$m$  is undefined.

25.



$m = -\frac{1}{7}$

27.



$m = 0.15$

29.  $(0, 1)$ ,  $(3, 1)$ ,  $(-1, 1)$

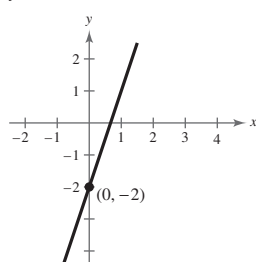
31.  $(6, -5)$ ,  $(7, -4)$ ,  $(8, -3)$

33.  $(-8, 0)$ ,  $(-8, 2)$ ,  $(-8, 3)$

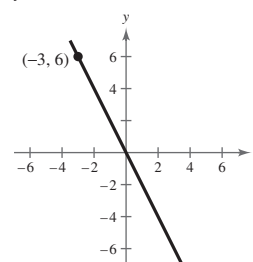
35.  $(-4, 6)$ ,  $(-3, 8)$ ,  $(-2, 10)$

37.  $(9, -1)$ ,  $(11, 0)$ ,  $(13, 1)$

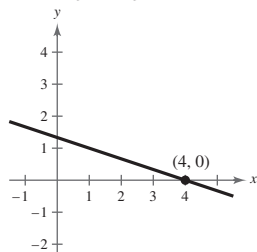
39.  $y = 3x - 2$



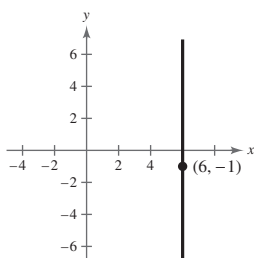
41.  $y = -2x$



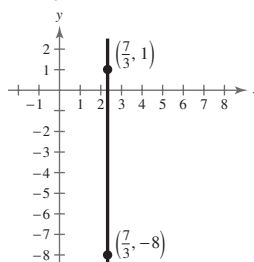
43.  $y = -\frac{1}{3}x + \frac{4}{3}$



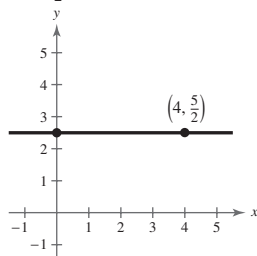
45.  $x = 6$



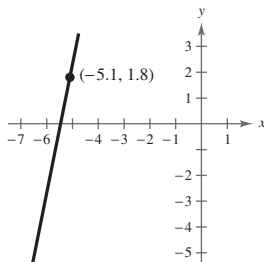
63.  $x = \frac{7}{3}$



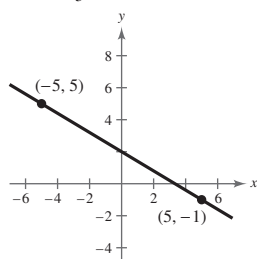
47.  $y = \frac{5}{2}$



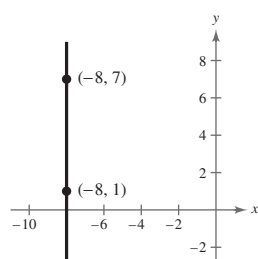
49.  $y = 5x + 27.3$



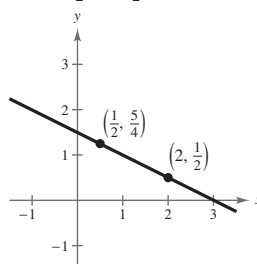
51.  $y = -\frac{3}{5}x + 2$



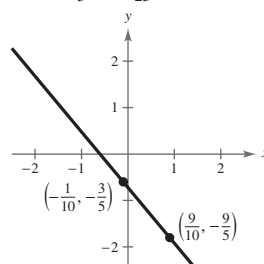
53.  $x = -8$



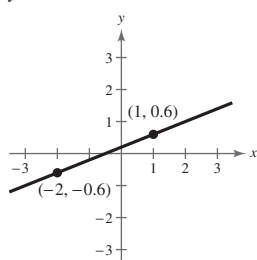
55.  $y = -\frac{1}{2}x + \frac{3}{2}$



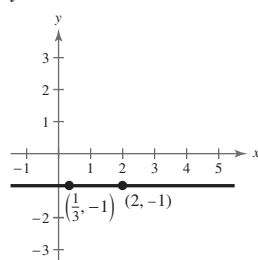
57.  $y = -\frac{6}{5}x - \frac{18}{25}$



59.  $y = 0.4x + 0.2$



61.  $y = -1$



65. Perpendicular

67. Parallel

69. (a)  $y = 2x - 3$  (b)  $y = -\frac{1}{2}x + 2$

71. (a)  $y = -\frac{3}{4}x + \frac{3}{8}$  (b)  $y = \frac{4}{3}x + \frac{127}{72}$

73. (a)  $y = 0$  (b)  $x = -1$

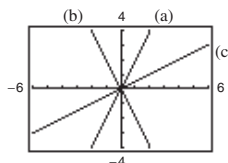
75. (a)  $x = 2$  (b)  $y = 5$

77. (a)  $y = x + 4.3$  (b)  $y = -x + 9.3$

79.  $3x + 2y - 6 = 0$  81.  $12x + 3y + 2 = 0$

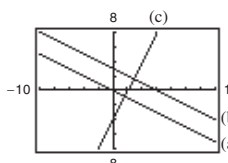
83.  $x + y - 3 = 0$

85. Line (b) is perpendicular to line (c).



87. Line (a) is parallel to line (b).

Line (c) is perpendicular to line (a) and line (b).



89.  $3x - 2y - 1 = 0$  91.  $80x + 12y + 139 = 0$

93. (a) Sales increasing 135 units per year

(b) No change in sales

(c) Sales decreasing 40 units per year

95. (a) Salary increased greatest from 1990 to 1992; Least from 1992 to 1994

(b) Slope of line from 1990 to 2002 is about 2351.83

(c) Salary increased an average of \$2351.83 over the 12 years between 1990 and 2002

97. 12 feet 99.  $V(t) = 3165 - 125t$

101. b; The slope is  $-20$ , which represents the decrease in the amount of the loan each week. The  $y$ -intercept is  $(0, 200)$  which represents the original amount of the loan.

102. c; The slope is  $2$ , which represents the hourly wage per unit produced. The  $y$ -intercept is  $(0, 8.50)$  which represents the initial hourly wage.

103. a; The slope is  $0.32$ , which represents the increase in travel cost for each mile driven. The  $y$ -intercept is  $(0, 30)$  which represents the amount per day for food.

104. d; The slope is  $-100$ , which represents the decrease in the value of the word processor each year. The  $y$ -intercept is  $(0, 750)$  which represents the initial purchase price of the computer.

105.  $y = 0.4825t - 2.2325$ ;  $y(18) \approx \$6.45$ ;  $y(20) \approx \$7.42$

107.  $V = -175t + 875$

109. (a)  $y(t) = 179.5t + 40,571$

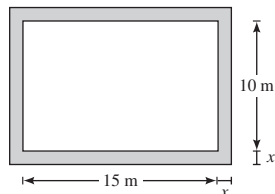
(b)  $y(8) = 42,007$ ;  $y(10) = 42,366$  (c)  $m = 179.5$

111.  $S = 0.85L$

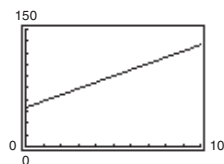
113. (a)  $C = 16.75t + 36,500$  (b)  $R = 27t$

(c)  $P = 10.25t - 36,500$  (d)  $t \approx 3561$  hours

115. (a) (b)  $y = 8x + 50$

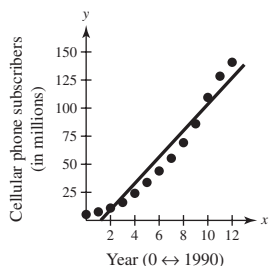


(c) (d)  $m = 8$ , 8 meters



117.  $C = 0.38x + 120$

119. (a) and (b)



(c) Answers will vary. Sample answer:  
 $y = 11.72x - 14.1$

(d) Answers will vary. Sample answer: The  $y$ -intercept indicates that initially there were  $-14.1$  million subscribers which doesn't make sense in the context of this problem. Each year, the number of cellular phone subscribers increases by  $11.72$  million.

(e) The model is accurate.

(f) Answers will vary. Sample answer:  $196.9$  million

121. False. The slope with the greatest magnitude corresponds to the steepest line.

123. Find the distance between each two points and use the Pythagorean Theorem.

125. No. The slope cannot be determined without knowing the scale on the  $y$ -axis. The slopes could be the same.

127.  $V$ -intercept: initial cost; Slope: annual depreciation

129. d 130. c 131. a 132. b

133.  $-1$  135.  $\frac{7}{2}, 7$  137. No solution

139. Answers will vary.

## Section 1.4 (page 48)

### Vocabulary Check (page 48)

- domain; range; function
- verbally; numerically; graphically; algebraically
- independent; dependent
- piecewise-defined
- implied domain
- difference quotient

1. Yes 3. No

5. Yes, each input value has exactly one output value.

7. No, the input values of  $7$  and  $10$  each have two different output values.

9. (a) Function

(b) Not a function, because the element  $1$  in  $A$  corresponds to two elements,  $-2$  and  $1$ , in  $B$ .

(c) Function

(d) Not a function, because not every element in  $A$  is matched with an element in  $B$ .

11. Each is a function. For each year there corresponds one and only one circulation.

13. Not a function 15. Function 17. Function

19. Not a function 21. Function 23. Not a function

25. (a)  $-1$  (b)  $-9$  (c)  $2x - 5$

27. (a)  $36\pi$  (b)  $\frac{9}{2}\pi$  (c)  $\frac{32}{3}\pi r^3$

29. (a)  $1$  (b)  $2.5$  (c)  $3 - 2|x|$

31. (a)  $-\frac{1}{9}$  (b) Undefined (c)  $\frac{1}{y^2 + 6y}$

33. (a)  $1$  (b)  $-1$  (c)  $\frac{|x-1|}{x-1}$

35. (a)  $-1$  (b)  $2$  (c)  $6$

37. (a)  $-7$  (b)  $4$  (c)  $9$

39.

$x$	$-2$	$-1$	$0$	$1$	$2$
$f(x)$	$1$	$-2$	$-3$	$-2$	$1$

41.

$t$	$-5$	$-4$	$-3$	$-2$	$-1$
$h(t)$	$1$	$\frac{1}{2}$	$0$	$\frac{1}{2}$	$1$

43.

$x$	$-2$	$-1$	$0$	$1$	$2$
$f(x)$	$5$	$\frac{9}{2}$	$4$	$1$	$0$

45.  $5$  47.  $\frac{4}{3}$  49.  $\pm 3$  51.  $0, \pm 1$  53.  $2, -1$

55.  $3, 0$  57. All real numbers

59. All real numbers  $t$  except  $t = 0$

61. All real numbers  $y$  such that  $y \geq 0$

63. All real numbers  $x$  such that  $-1 \leq x \leq 1$

65. All real numbers  $x$  except  $x = 0, -2$

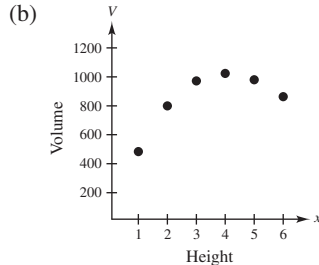
67. All real numbers  $s$  such that  $s \geq 1$  except  $s = 4$   
 69. All real numbers  $x$  such that  $x > 0$   
 71.  $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$   
 73.  $\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$   
 75.  $g(x) = cx^2; c = -2$     76.  $f(x) = cx; c = \frac{1}{4}$

77.  $r(x) = \frac{c}{x}; c = 32$     78.  $h(x) = c\sqrt{|x|}; c = 3$

79.  $3 + h, h \neq 0$     81.  $3x^2 + 3xh + h^2 + 3, h \neq 0$

83.  $-\frac{x+3}{9x^2}, x \neq 3$     85.  $\frac{\sqrt{5x}-5}{x-5}$     87.  $A = \frac{P^2}{16}$

89. (a) The maximum volume is 1024 cubic centimeters.



Yes,  $V$  is a function of  $x$ .

(c)  $V = x(24 - 2x)^2, 0 < x < 12$

91.  $A = \frac{x^2}{2(x-2)}, x > 2$

93. Yes, the ball will be at a height of 6 feet.

95. 1990: \$27,300    97. (a)  $C = 12.30x + 98,000$   
 1991: \$28,052    (b)  $R = 17.98x$   
 1992: \$29,168    (c)  $P = 5.68x - 98,000$   
 1993: \$30,648  
 1994: \$32,492  
 1995: \$34,700  
 1996: \$37,272  
 1997: \$40,208  
 1998: \$41,300  
 1999: \$43,800  
 2000: \$46,300  
 2001: \$48,800  
 2002: \$51,300

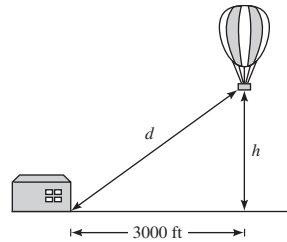
99. (a)  $R = \frac{240n - n^2}{20}, n \geq 80$

(b)

$n$	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

101. (a)



(b)  $h = \sqrt{d^2 - 3000^2}, d \geq 3000$

103. False. The range is  $[-1, \infty)$ .

105. The domain is the set of inputs of the function, and the range is the set of outputs.

107. (a) Yes. The amount you pay in sales tax will increase as the price of the item purchased increases.

- (b) No. The length of time that you study will not necessarily determine how well you do on an exam.

109.  $\frac{15}{8}$     111.  $-\frac{1}{5}$     113.  $2x - 3y - 11 = 0$

115.  $10x + 9y + 15 = 0$

## Section 1.5 (page 61)

### Vocabulary Check (page 61)

1. ordered pairs    2. Vertical Line Test  
 3. zeros    4. decreasing  
 5. maximum    6. average rate of change; secant  
 7. odd    8. even

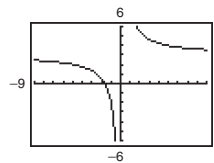
1. Domain:  $(-\infty, -1], [1, \infty)$     3. Domain:  $[-4, 4]$   
 Range:  $[0, \infty)$     Range:  $[0, 4]$

5. (a) 0    (b) -1    (c) 0    (d) -2  
 7. (a) -3    (b) 0    (c) 1    (d) -3    9. Function

11. Not a function    13. Function    15.  $-\frac{5}{2}, 6$

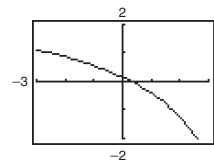
17. 0    19.  $0, \pm\sqrt{2}$     21.  $\pm\frac{1}{2}, 6$     23.  $\frac{1}{2}$

25.



$-\frac{5}{3}$

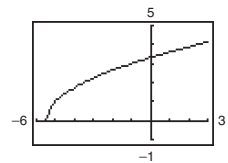
29.



$\frac{1}{3}$

33. Increasing on  $(-\infty, 0)$  and  $(2, \infty)$   
 Decreasing on  $(0, 2)$

27.



$-\frac{11}{2}$

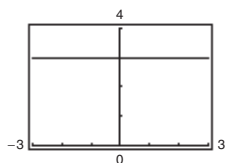
31. Increasing on  $(-\infty, \infty)$

35. Increasing on  $(-\infty, 0)$  and  $(2, \infty)$ ; Constant on  $(0, 2)$

37. Increasing on  $(1, \infty)$ ; Decreasing on  $(-\infty, -1)$

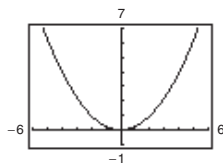
Constant on  $(-1, 1)$

39.



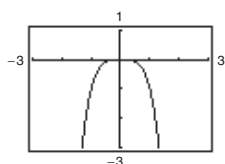
Constant on  $(-\infty, \infty)$

41.



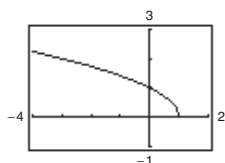
Decreasing on  $(-\infty, 0)$   
Increasing on  $(0, \infty)$

43.



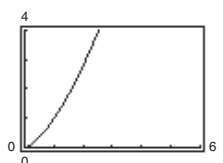
Increasing on  $(-\infty, 0)$   
Decreasing on  $(0, \infty)$

45.



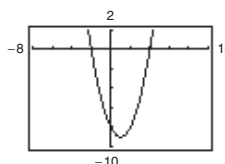
Decreasing on  $(-\infty, 1)$

47.



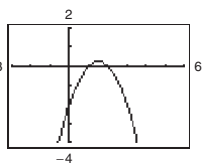
Increasing on  $(0, \infty)$

49.



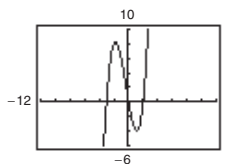
Relative minimum:  
 $(1, -9)$

51.



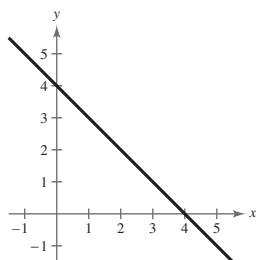
Relative maximum:  
 $(1.5, 0.25)$

53.



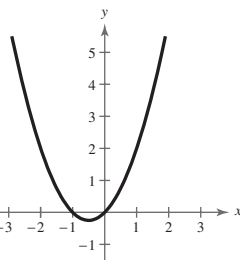
Relative maximum:  $(-1.79, 8.21)$   
Relative minimum:  $(1.12, -4.06)$

55.



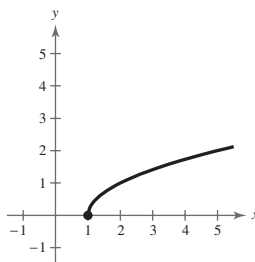
$(-\infty, 4]$

57.



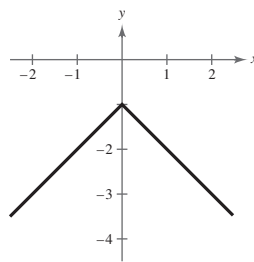
$(-\infty, -1], [0, \infty)$

59.



$[1, \infty)$

61.



$f(x) < 0$  for all  $x$

63. The average rate of change from  $x_1 = 0$  to  $x_2 = 3$  is  $-2$ .

65. The average rate of change from  $x_1 = 1$  to  $x_2 = 5$  is  $18$ .

67. The average rate of change from  $x_1 = 1$  to  $x_2 = 3$  is  $0$ .

69. The average rate of change from  $x_1 = 3$  to  $x_2 = 11$  is  $-\frac{1}{4}$ .

71. Even; y-axis symmetry      73. Odd; origin symmetry

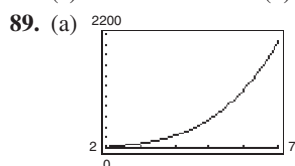
75. Neither even nor odd; no symmetry

77.  $h = -x^2 + 4x - 3$       79.  $h = 2x - x^2$

81.  $L = \frac{1}{2}y^2$       83.  $L = 4 - y^2$

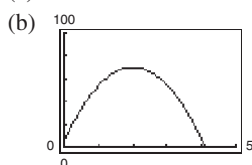
85. (a) (b) 30 watts

87. (a) Ten thousands      (b) Ten millions      (c) Percents



(b) The average rate of change from 2002 to 2007 is 408.56. The estimated revenue is increasing each year at a fast pace.

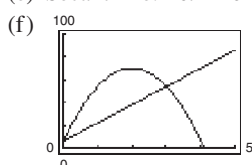
91. (a)  $s = -16t^2 + 64t + 6$



(c) Average rate of change = 16

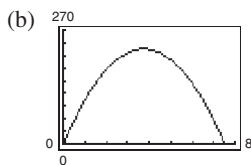
(d) The slope of the secant line is positive.

(e) Secant line:  $16t + 6$





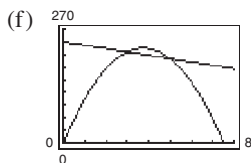
93. (a)  $s = -16t^2 + 120t$



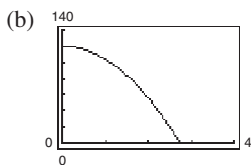
(c) Average rate of change =  $-8$

(d) The slope of the secant line is negative.

(e) Secant line:  $-8t + 240$



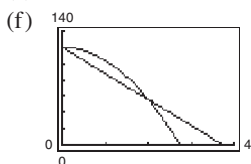
95. (a)  $s = -16t^2 + 120$



(c) Average rate of change =  $-32$

(d) The slope of the secant line is negative.

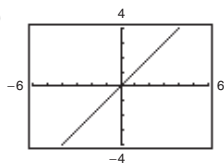
(e) Secant line:  $-32t + 120$

97. False. The function  $f(x) = \sqrt{x^2 + 1}$  has a domain of all real numbers.99. (a) Even. The graph is a reflection in the  $x$ -axis.(b) Even. The graph is a reflection in the  $y$ -axis.(c) Even. The graph is a vertical translation of  $f$ .(d) Neither. The graph is a horizontal translation of  $f$ .

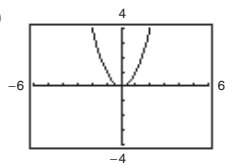
101. (a)  $(\frac{3}{2}, 4)$  (b)  $(\frac{3}{2}, -4)$

103. (a)  $(-4, 9)$  (b)  $(-4, -9)$

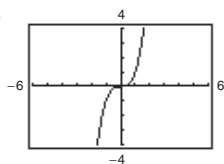
105. (a)



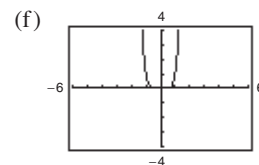
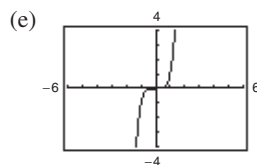
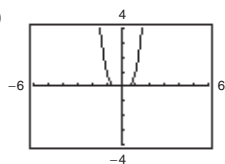
(b)



(c)



(d)



All the graphs pass through the origin. The graphs of the odd powers of  $x$  are symmetric with respect to the origin, and the graphs of the even powers are symmetric with respect to the  $y$ -axis. As the powers increase, the graphs become flatter in the interval  $-1 < x < 1$ .

107. 0, 10    109. 0,  $\pm 1$

111. (a) 37    (b)  $-28$     (c)  $5x - 43$

113. (a)  $-9$     (b)  $2\sqrt{7} - 9$

(c) The given value is not in the domain of the function.

115.  $h + 4$ ,  $h \neq 0$

## Section 1.6 (page 71)

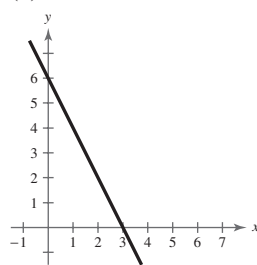
## Vocabulary Check (page 71)

1. g    2. i    3. h    4. a    5. b

6. e    7. f    8. c    9. d

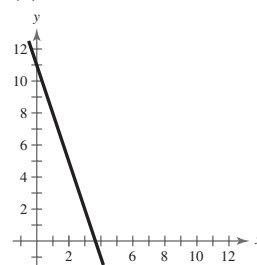
1. (a)  $f(x) = -2x + 6$

(b)



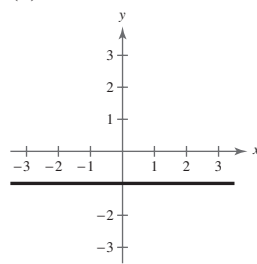
3. (a)  $f(x) = -3x + 11$

(b)



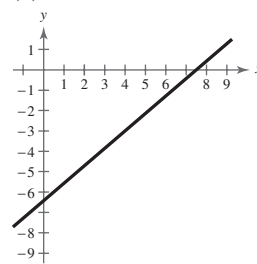
5. (a)  $f(x) = -1$

(b)

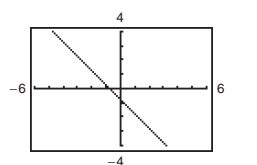


7. (a)  $f(x) = \frac{6}{7}x - \frac{45}{7}$

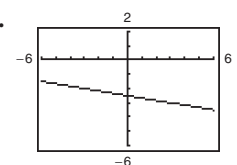
(b)

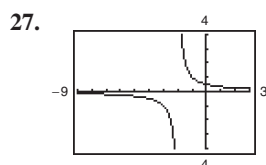
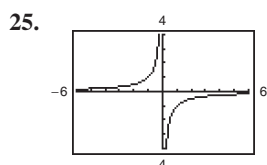
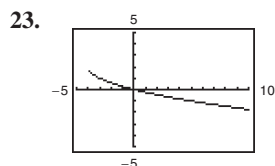
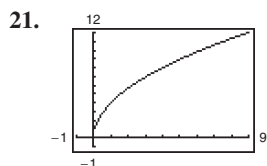
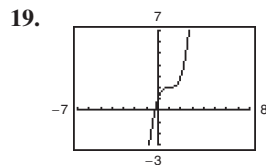
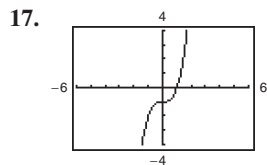
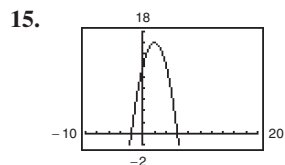
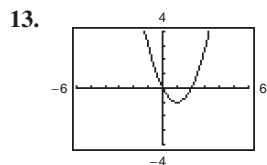


9.



11.



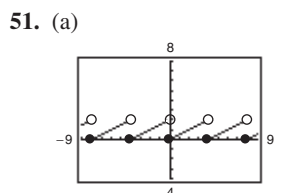
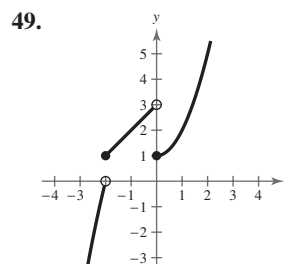
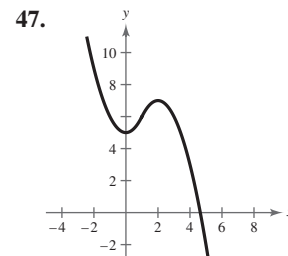
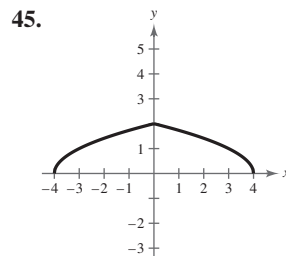
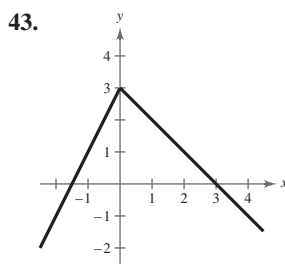
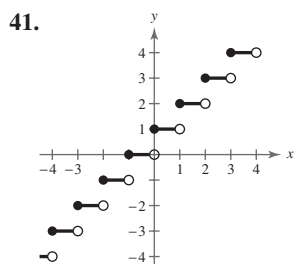
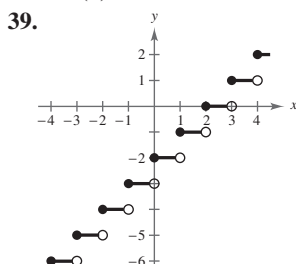
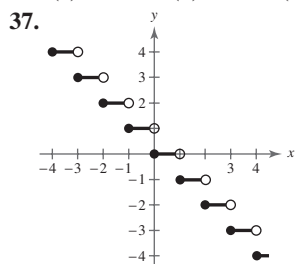


29. (a) 2 (b) 2 (c) -4 (d) 3

31. (a) 1 (b) 3 (c) 7 (d) -19

33. (a) 6 (b) -11 (c) 6 (d) -22

35. (a) -10 (b) -4 (c) -1 (d) 41



(b) Domain:  $(-\infty, \infty)$ ;

Range:  $[0, 2)$

(c) Sawtooth pattern

53. (a)  $f(x) = |x|$  (b)  $g(x) = |x + 2| - 1$

55. (a)  $f(x) = x^3$  (b)  $g(x) = (x - 1)^3 - 2$

57. (a)  $f(x) = 2$  (b)  $g(x) = 2$

59. (a)  $f(x) = x$  (b)  $g(x) = x - 2$

61. (a) (b) \$5.64

63. (a) (b) \$50.25

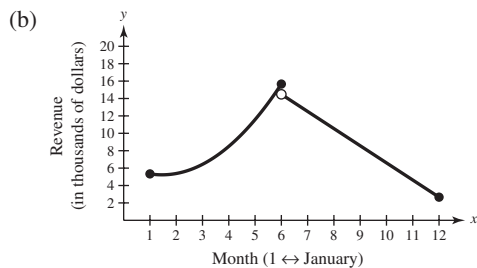
65. (a)  $W(30) = 360$ ;  $W(40) = 480$ ;

$W(45) = 570$ ;  $W(50) = 660$

(b)  $W(h) = \begin{cases} 12h, & 0 < h \leq 45 \\ 18(h - 45) + 540, & h > 45 \end{cases}$

67. (a)  $f(x) = \begin{cases} 0.505x^2 - 1.47x + 6.3, & 1 \leq x \leq 6 \\ -1.97x + 26.3, & 6 < x \leq 12 \end{cases}$

Answers will vary. Sample answer: The domain is determined by inspection of a graph of the data with the two models.



(c)  $f(5) = 11.575, f(11) = 4.63$ ; These values represent the revenue for the months of May and November, respectively.

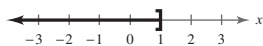
(d) These values are quite close to the actual data values.

69. False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include  $x$ - and  $y$ -intercepts.

71.  $f(x) = \begin{cases} -\frac{4}{3}x + 6, & 0 \leq x \leq 3 \\ -\frac{2}{5}x + \frac{16}{5}, & 3 < x \leq 8 \end{cases}$

73.  $x \leq 1$

75. Neither

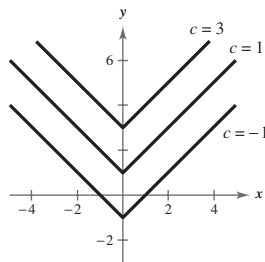


## Section 1.7 (page 79)

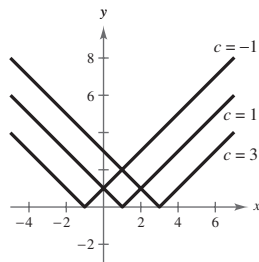
### Vocabulary Check (page 79)

1. rigid    2.  $-f(x); f(-x)$     3. nonrigid
4. horizontal shrink; horizontal stretch
5. vertical stretch; vertical shrink
6. (a) iv    (b) ii    (c) iii    (d) i

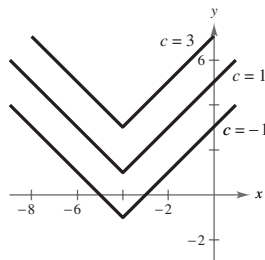
1. (a)



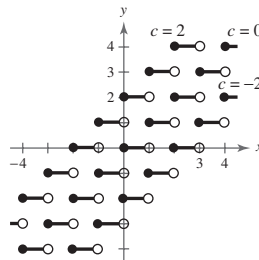
(b)



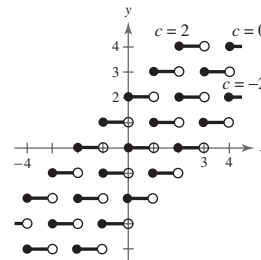
(c)



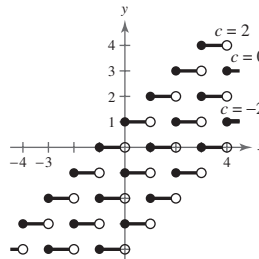
3. (a)



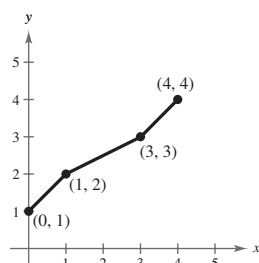
(b)



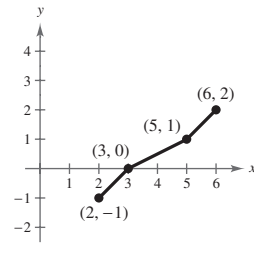
(c)



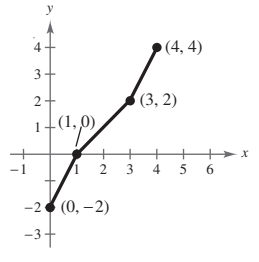
5. (a)



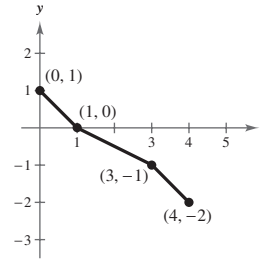
(b)



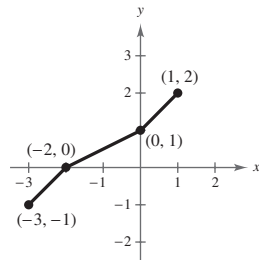
(c)



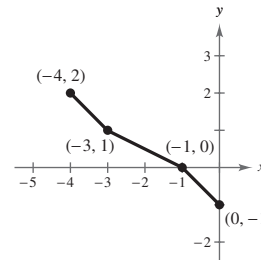
(d)

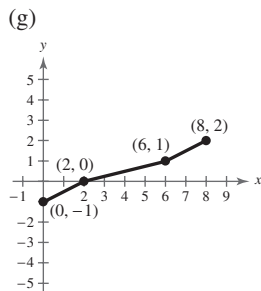


(e)

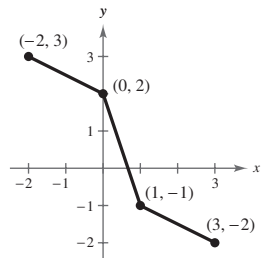


(f)

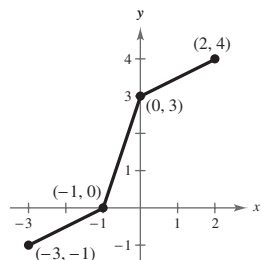




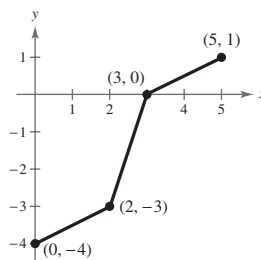
7. (a)



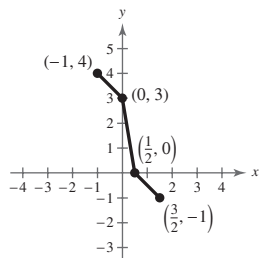
(c)



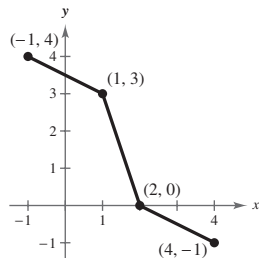
(e)



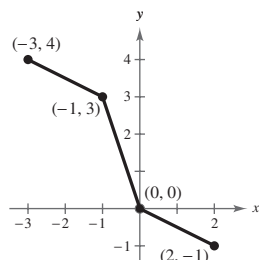
(g)



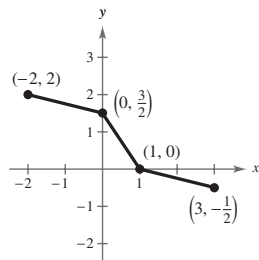
(b)



(d)



(f)



9. (a)  $y = x^2 - 1$  (b)  $y = 1 - (x + 1)^2$   
 (c)  $y = -(x - 2)^2 + 6$  (d)  $y = (x - 5)^2 - 3$

11. (a)  $y = |x| + 5$  (b)  $y = -|x + 3|$   
 (c)  $y = |x - 2| - 4$  (d)  $y = -|x - 6| - 1$

13. Horizontal shift of  $y = x^3$ ;  $y = (x - 2)^3$

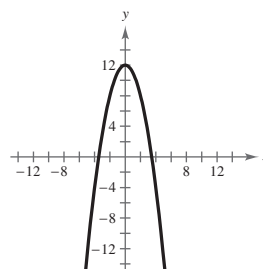
15. Reflection in the  $x$ -axis of  $y = x^2$ ;  $y = -x^2$

17. Reflection in the  $x$ -axis and vertical shift of  $y = \sqrt{x}$ ;  
 $y = 1 - \sqrt{x}$

19. (a)  $f(x) = x^2$

(b) Reflection in the  $x$ -axis, and vertical shift 12 units upward, of  $f(x) = x^2$

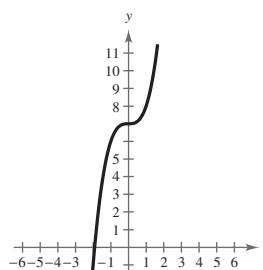
(c) (d)  $g(x) = 12 - f(x)$



21. (a)  $f(x) = x^3$

(b) Vertical shift seven units upward, of  $f(x) = x^3$

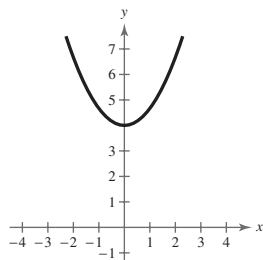
(c) (d)  $g(x) = f(x) + 7$



23. (a)  $f(x) = x^2$

(b) Vertical shrink of two-thirds, and vertical shift four units upward, of  $f(x) = x^2$

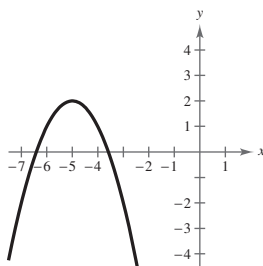
(c) (d)  $g(x) = \frac{2}{3}f(x) + 4$



25. (a)  $f(x) = x^2$

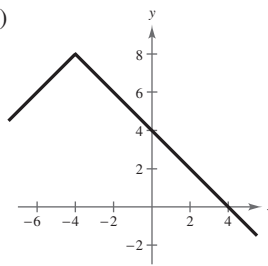
(b) Reflection in the  $x$ -axis, horizontal shift five units to the left, and vertical shift two units upward, of  $f(x) = x^2$

(c)



(d)  $g(x) = 2 - f(x + 5)$

(c)

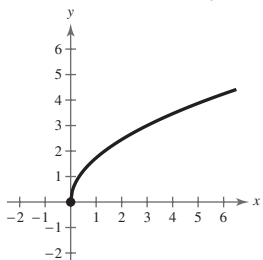


(d)  $g(x) = -f(x + 4) + 8$

27. (a)  $f(x) = \sqrt{x}$

 (b) Horizontal shrink of  $\frac{1}{3}$ , of  $f(x) = \sqrt{x}$ 

(c)

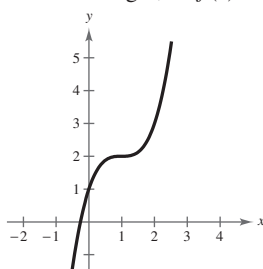


(d)  $g(x) = f(3x)$

29. (a)  $f(x) = x^3$

 (b) Vertical shift two units upward, and horizontal shift one unit to the right, of  $f(x) = x^3$ 

(c)

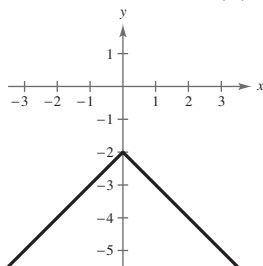


(d)  $g(x) = f(x - 1) + 2$

31. (a)  $f(x) = |x|$

 (b) Reflection in the  $x$ -axis, and vertical shift two units downward, of  $f(x) = |x|$ 

(c)



(d)  $g(x) = -f(x) - 2$

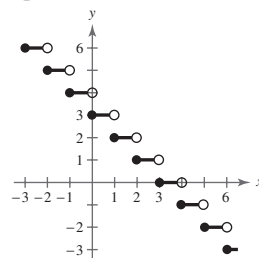
33. (a)  $f(x) = |x|$

 (b) Reflection in the  $x$ -axis, horizontal shift four units to the left, and vertical shift eight units upward, of  $f(x) = |x|$ 

35. (a)  $f(x) = \lfloor x \rfloor$

 (b) Reflection in the  $x$ -axis, and vertical shift three units upward, of  $f(x) = \lfloor x \rfloor$ 

(c)

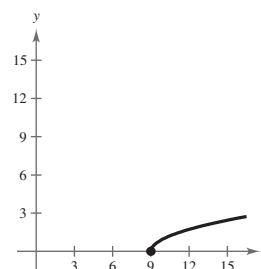


(d)  $g(x) = 3 - f(x)$

37. (a)  $f(x) = \sqrt{x}$

 (b) Horizontal shift of nine units to the right, of  $f(x) = \sqrt{x}$ 

(c)

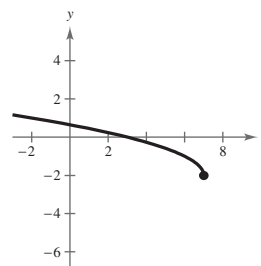


(d)  $g(x) = f(x - 9)$

39. (a)  $f(x) = \sqrt{x}$

 (b) Reflection in the  $y$ -axis, horizontal shift of seven units to the right, and vertical shift two units downward, of  $f(x) = \sqrt{x}$ 

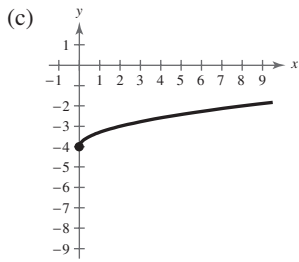
(c)



(d)  $g(x) = f(7 - x) - 2$

41. (a)  $f(x) = \sqrt{x}$

 (b) Horizontal stretch, and vertical shift four units downward, of  $f(x) = \sqrt{x}$



(d)  $g(x) = f\left(\frac{1}{2}x\right) - 4$

43.  $f(x) = (x - 2)^2 - 8$

45.  $f(x) = (x - 13)^3$

47.  $f(x) = -|x| - 10$

49.  $f(x) = -\sqrt{-x + 6}$

51. (a)  $y = -3x^2$  (b)  $y = 4x^2 + 3$

53. (a)  $y = -\frac{1}{2}|x|$  (b)  $y = 3|x| - 3$

55. Vertical stretch of  $y = x^3$ ;  $y = 2x^3$

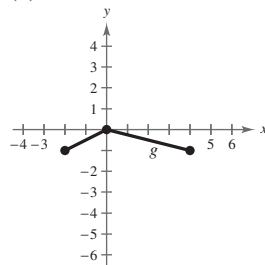
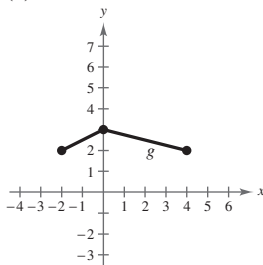
57. Reflection in the  $x$ -axis and vertical shrink of  $y = x^2$ ;  
 $y = -\frac{1}{2}x^2$

59. Reflection in the  $y$ -axis and vertical shrink of  $y = \sqrt{x}$ ;  
 $y = \frac{1}{2}\sqrt{-x}$

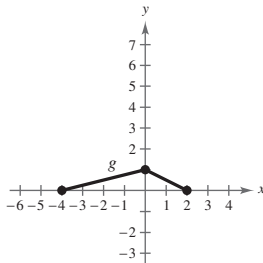
61.  $y = -(x - 2)^3 + 2$

63.  $y = -\sqrt{x} - 3$

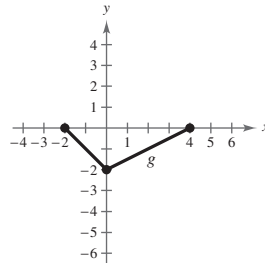
65. (a)



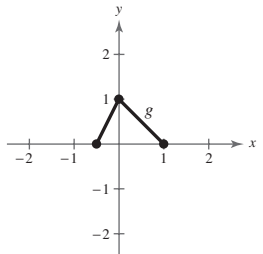
(c)



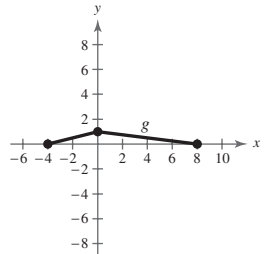
(d)



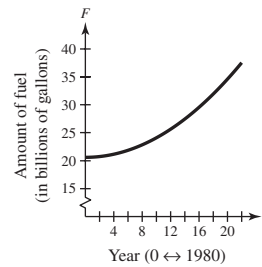
(e)



(f)



67. (a) Horizontal stretch of 0.035 and a vertical shift of 20.6 units upward.



- (b) 0.77-billion-gallon increase in fuel usage by trucks each year

- (c)  $f(t) = 20.6 + 0.035(t + 10)^2$ . The graph is shifted 10 units to the left.

- (d) 52.1 billion gallons. Yes.

69. True.  $|-x| = |x|$

71. (a)  $g(t) = \frac{3}{4}f(t)$  (b)  $g(t) = f(t) + 10,000$

(c)  $g(t) = f(t - 2)$

73.  $(-2, 0), (-1, 1), (0, 2)$

75.  $\frac{4}{x(1-x)}$

77.  $\frac{3x-2}{x(x-1)}$

79.  $\frac{(x-4)\sqrt{x^2-4}}{x^2-4}$

81.  $5(x-3), x \neq -3$

83. (a) 38 (b)  $\frac{57}{4}$  (c)  $x^2 - 12x + 38$

85. All real numbers  $x$  except  $x = 1$

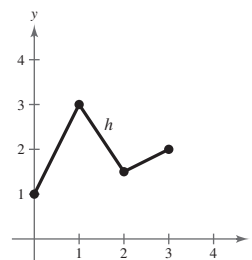
87. All real numbers  $x$  such that  $-9 \leq x \leq 9$

## Section 1.8 (page 89)

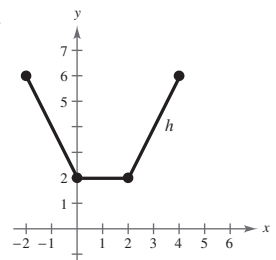
### Vocabulary Check (page 89)

1. addition; subtraction; multiplication; division
2. composition
3.  $g(x)$
4. inner; outer

1.



3.



5. (a)  $2x$  (b) 4 (c)  $x^2 - 4$

(d)  $\frac{x+2}{x-2}$ ; all real numbers  $x$  except  $x = 2$

7. (a)  $x^2 + 4x - 5$  (b)  $x^2 - 4x + 5$  (c)  $4x^3 - 5x^2$

(d)  $\frac{x^2}{4x-5}$ ; all real numbers  $x$  except  $x = \frac{5}{4}$

9. (a)  $x^2 + 6 + \sqrt{1-x}$  (b)  $x^2 + 6 - \sqrt{1-x}$   
 (c)  $(x^2 + 6)\sqrt{1-x}$   
 (d)  $\frac{(x^2 + 6)\sqrt{1-x}}{1-x}$ ; all real numbers  $x$  such that  $x < 1$

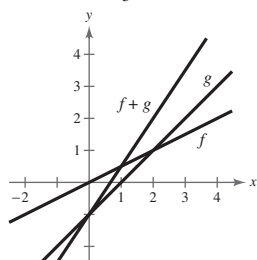
11. (a)  $\frac{x+1}{x^2}$  (b)  $\frac{x-1}{x^2}$  (c)  $\frac{1}{x^3}$

(d)  $x$ ; all real numbers  $x$  except  $x = 0$

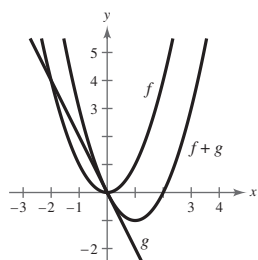
13. 3 15. 5 17.  $9t^2 - 3t + 5$  19. 74

21. 26 23.  $\frac{3}{5}$

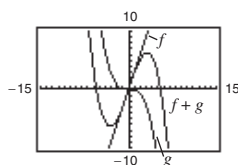
25.



27.



29.



$f(x), g(x)$

31. (a)  $(x-1)^2$  (b)  $x^2 - 1$  (c)  $x^4$

33. (a)  $x$  (b)  $x$  (c)  $\sqrt[3]{\sqrt[3]{x-1}-1}$

35. (a)  $\sqrt{x^2+4}$  (b)  $x+4$

Domains of  $f$  and  $g \circ f$ :  $x \geq -4$

Domains of  $g$  and  $f \circ g$ : all real numbers

37. (a)  $x+1$  (b)  $\sqrt{x^2+1}$

Domains of  $f$  and  $g \circ f$ : all real numbers

Domains of  $g$  and  $f \circ g$ : all real numbers  $x$  such that  $x \geq 0$

39. (a)  $|x+6|$  (b)  $|x|+6$

Domains of  $f, g, f \circ g$ , and  $g \circ f$ : all real numbers

41. (a)  $\frac{1}{x+3}$  (b)  $\frac{1}{x}+3$

Domains of  $f$  and  $g \circ f$ : all real numbers  $x$  except  $x = 0$

Domains of  $g$ : all real numbers

Domains of  $f \circ g$ : all real numbers  $x$  except  $x = -3$

43. (a) 3 (b) 0 45. (a) 0 (b) 4

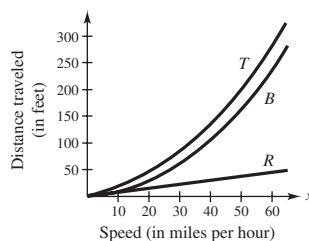
47.  $f(x) = x^2, g(x) = 2x+1$

49.  $f(x) = \sqrt[3]{x}, g(x) = x^2 - 4$

51.  $f(x) = \frac{1}{x}, g(x) = x+2$

53.  $f(x) = \frac{x+3}{4+x}, g(x) = -x^2$

55.  $T = \frac{3}{4}x + \frac{1}{15}x^2$



57. (a)  $c(t) = \frac{p(t) + b(t) - d(t)}{p(t)} \times 100$

(b)  $c(5)$  is the population change in the year 2005.

59. (a)  $(A+N)(t) = 5.31t^2 - 102.0t + 1338$

$$(A+N)(4) = 1014.96$$

$$(A+N)(8) = 861.84$$

$$(A+N)(12) = 878.64$$

- (b)  $(A-N)(t) = 1.41t^2 - 17.6t + 132$

$$(A-N)(4) = 84.16$$

$$(A-N)(8) = 81.44$$

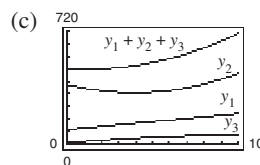
$$(A-N)(12) = 123.84$$

61. (a)  $y_1 = 10.20t + 92.7$

$$y_2 = 3.357t^2 - 26.46t + 379.5$$

$$y_3 = -0.465t^2 + 9.71t + 7.4$$

- (b)  $y_1 + y_2 + y_3 = 2.892t^2 - 6.55t + 479.6$ ; this amount represents the amount spent on health care in the United States.



- (d) In 2008, \$1298.708 billion is estimated to be spent on health services and supplies, and in 2010, \$1505.4 billion is estimated.

63. (a)  $r(x) = \frac{x}{2}$  (b)  $A(r) = \pi r^2$

- (c)  $(A \circ r)(x) = \pi \left(\frac{x}{2}\right)^2$ ;  $(A \circ r)(x)$  represents the area of the circular base of the tank on the square foundation with side length  $x$ .

65. (a)  $N(T(t)) = 30(3t^2 + 2t + 20)$  This represents the number of bacteria in the food as a function of time.

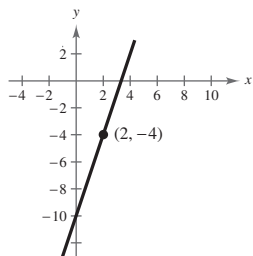
- (b)  $t = 2.846$  hours

67.  $g(f(x))$  represents 3 percent of an amount over \$500,000.

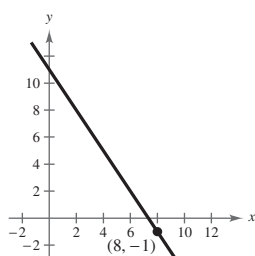
69. False.  $(f \circ g)(x) = 6x + 1$  and  $(g \circ f)(x) = 6x + 6$

71. Answers will vary. 73. 3 75.  $\frac{-4}{x(x+h)}$

77.  $3x - y - 10 = 0$



79.  $3x + 2y - 22 = 0$



## Section 1.9 (page 99)

## Vocabulary Check (page 99)

1. inverse;  $f$ -inverse    2. range; domain  
 3.  $y = x$     4. one-to-one    5. horizontal

1.  $f^{-1}(x) = \frac{1}{6}x$     3.  $f^{-1}(x) = x - 9$

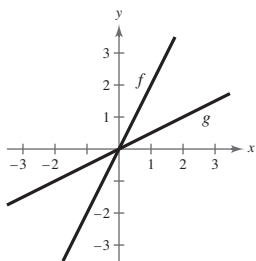
5.  $f^{-1}(x) = \frac{x-1}{3}$     7.  $f^{-1}(x) = x^3$

9. c    10. b    11. a    12. d

13. (a)  $f(g(x)) = f\left(\frac{x}{2}\right) = 2\left(\frac{x}{2}\right) = x$

$$g(f(x)) = g(2x) = \frac{(2x)}{2} = x$$

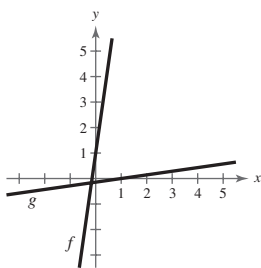
(b)



15. (a)  $f(g(x)) = f\left(\frac{x-1}{7}\right) = 7\left(\frac{x-1}{7}\right) + 1 = x$

$$g(f(x)) = g(7x+1) = \frac{(7x+1)-1}{7} = x$$

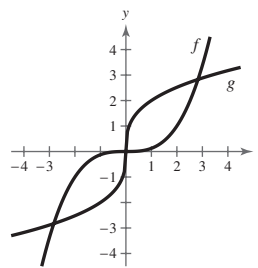
(b)



17. (a)  $f(g(x)) = f(\sqrt[3]{8x}) = \frac{(\sqrt[3]{8x})^3}{8} = x$

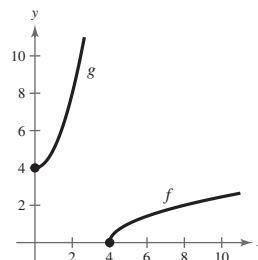
$$g(f(x)) = g\left(\frac{x^3}{8}\right) = \sqrt[3]{8\left(\frac{x^3}{8}\right)} = x$$

(b)



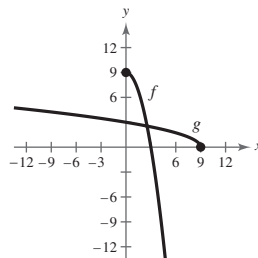
19. (a)  $f(g(x)) = f(x^2 + 4), x \geq 0$   
 $= \sqrt{(x^2 + 4)} - 4 = x$   
 $g(f(x)) = g(\sqrt{x - 4})$   
 $= (\sqrt{x - 4})^2 + 4 = x$

(b)



21. (a)  $f(g(x)) = f(\sqrt{9-x}), x \leq 9$   
 $= 9 - (\sqrt{9-x})^2 = x$   
 $g(f(x)) = g(9-x^2), x \geq 0$   
 $= \sqrt{9 - (9-x^2)} = x$

(b)



23. (a)  $f(g(x)) = f\left(-\frac{5x+1}{x-1}\right) = \frac{-\left(\frac{5x+1}{x-1}\right) - 1}{-\left(\frac{5x+1}{x-1}\right) + 5}$

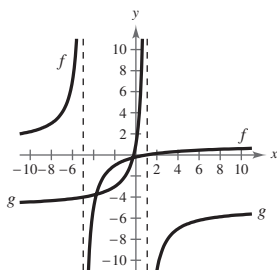
$$= \frac{-5x - 1 - x + 1}{-5x - 1 + 5x - 5} = x$$

$$g(f(x)) = g\left(\frac{x-1}{x+5}\right) = \frac{-5\left(\frac{x-1}{x+5}\right) - 1}{\frac{x-1}{x+5} - 1}$$

$$= \frac{-5x + 5 - x - 5}{x - 1 - x - 5} = x$$



(b)



25. No

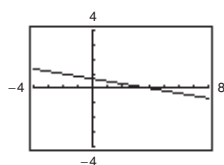
27.

$x$	-2	0	2	4	6	8
$f^{-1}(x)$	-2	-1	0	1	2	3

29. Yes

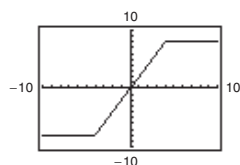
31. No

33.



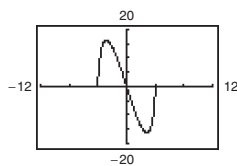
The function has an inverse.

35.



The function does not have an inverse.

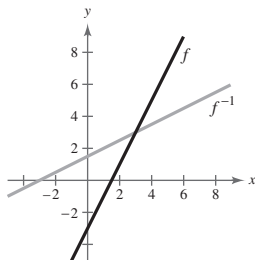
37.



The function does not have an inverse.

39. (a)  $f^{-1}(x) = \frac{x+3}{2}$

(b)

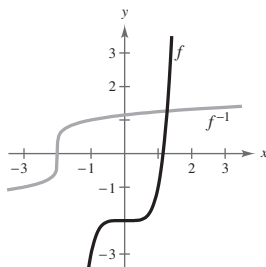


(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .

(d) The domains and ranges of  $f$  and  $f^{-1}$  are all real numbers.

41. (a)  $f^{-1}(x) = \sqrt[5]{x+2}$

(b)

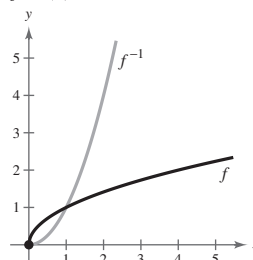


(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .

(d) The domains and ranges of  $f$  and  $f^{-1}$  are all real numbers.

43. (a)  $f^{-1}(x) = x^2, x \geq 0$

(b)

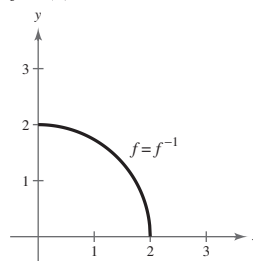


(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .

(d) The domains and ranges of  $f$  and  $f^{-1}$  are all real numbers  $x$  such that  $x \geq 0$ .

45. (a)  $f^{-1}(x) = \sqrt{4-x^2}, 0 \leq x \leq 2$

(b)

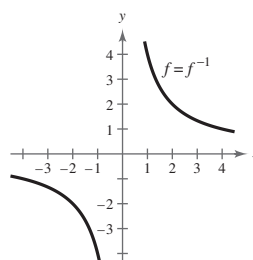


(c) The graph of  $f^{-1}$  is the same as the graph of  $f$ .

(d) The domains and ranges of  $f$  and  $f^{-1}$  are all real numbers  $x$  such that  $0 \leq x \leq 2$ .

47. (a)  $f^{-1}(x) = \frac{4}{x}$

(b)

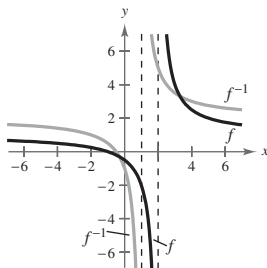


(c) The graph of  $f^{-1}$  is the same as the graph of  $f$ .

(d) The domains and ranges of  $f$  and  $f^{-1}$  are all real numbers  $x$  except  $x = 0$ .

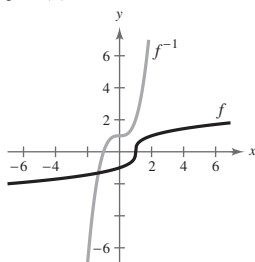
49. (a)  $f^{-1}(x) = \frac{2x+1}{x-1}$

(b)

(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .(d) The domain of  $f$  and the range of  $f^{-1}$  are all real numbers  $x$  except  $x = 2$ . The domain of  $f^{-1}$  and the range of  $f$  are all real numbers  $x$  except  $x = 1$ .

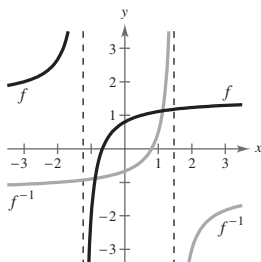
51. (a)  $f^{-1}(x) = x^3 + 1$

(b)

(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .(d) The domains and ranges of  $f$  and  $f^{-1}$  are all real numbers.

53. (a)  $f^{-1}(x) = \frac{5x - 4}{6 - 4x}$

(b)

(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .(d) The domain of  $f$  and the range of  $f^{-1}$  are all real numbers  $x$  except  $x = -\frac{5}{4}$ . The domain of  $f^{-1}$  and the range of  $f$  are all real numbers  $x$  except  $x = \frac{3}{2}$ .55. No inverse 57.  $g^{-1}(x) = 8x$  59. No inverse61.  $f^{-1}(x) = \sqrt{x} - 3$  63. No inverse65. No inverse 67.  $f^{-1}(x) = \frac{x^2 - 3}{2}$ ,  $x \geq 0$ 69. 32 71. 600 73.  $2\sqrt[3]{x+3}$ 75.  $\frac{x+1}{2}$  77.  $\frac{x+1}{2}$ 

79. (a)  $f^{-1}(108,209) = 11$

(b)  $f^{-1}$  represents the year for a given number of households in the United States.

(c)  $y = 1578.68t + 90,183.63$

(d)  $f^{-1} = \frac{t - 90,183.63}{1578.68}$  (e)  $f^{-1}(117,022) = 17$

(f)  $f^{-1}(108,209) = 11.418$ ; the results are similar.

81. (a) Yes

(b)  $f^{-1}$  yields the year for a given number of miles traveled by motor vehicles.

(c)  $f^{-1}(2632) = 8$

(d) No.  $f(t)$  would not pass the Horizontal Line Test.

83. (a)  $y = \sqrt{\frac{x - 245.50}{0.03}}$ ,  $245.5 < x < 545.5$

 $x$  = degrees Fahrenheit;  $y$  = % load

(b)  (c)  $0 < x < 92.11$

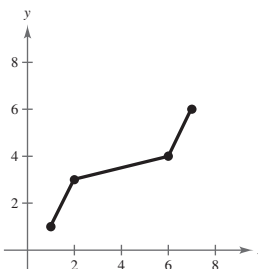
85. False.  $f(x) = x^2$  has no inverse.

87. Answers will vary.

89.

$x$	1	3	4	6
$y$	1	2	6	7

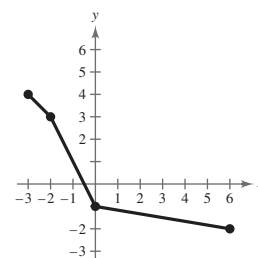
$x$	1	2	6	7
$f^{-1}(x)$	1	3	4	6



91.

$x$	-2	-1	3	4
$y$	6	0	-2	-3

$x$	-3	-2	0	6
$f^{-1}(x)$	4	3	-1	-2



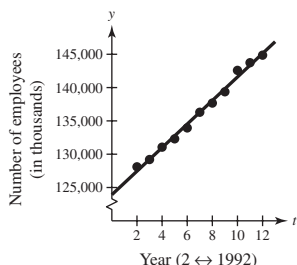
93.  $k = \frac{1}{4}$     95.  $\pm 8$     97.  $\frac{3}{2}$     99.  $3 \pm \sqrt{5}$   
 101.  $5, -\frac{10}{3}$     103. 16, 18

Section 1.10 (page 109)

Vocabulary Check (page 109)

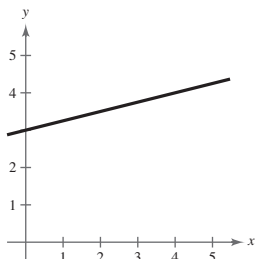
1. variation; regression    2. sum of square differences  
 3. correlation coefficient    4. directly proportional  
 5. constant of variation    6. directly proportional  
 7. inverse    8. combined    9. jointly proportional

1.



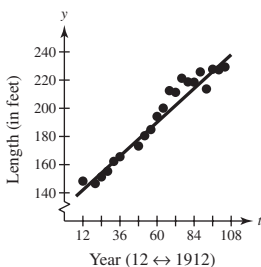
The model is a good fit for the actual data.

3.



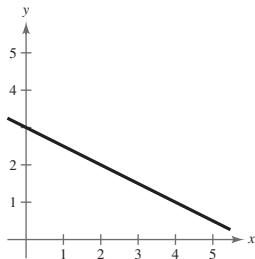
$$y = \frac{1}{4}x + 3$$

7. (a) and (b)



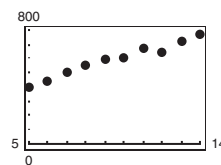
- (c)  $y = 1.03t + 130.27$     (d) The models are similar.  
 (e) Part (b): 238 feet; Part (c): 241.51 feet  
 (f) Answers will vary.

5.



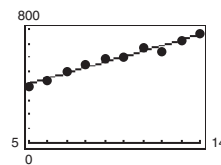
$$y = -\frac{1}{2}x + 3$$

9. (a)



(b)  $S = 38.4t + 224$

(c)



The model is a good fit.

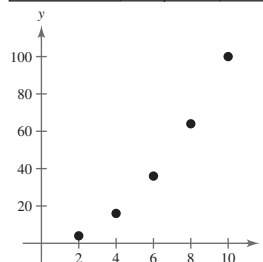
(d) 2005: \$800 million; 2007: \$876.8 million

(e) Each year the annual gross ticket sales for Broadway shows in New York City increase by \$38.4 million.

11. Inversely

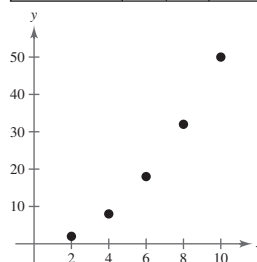
13.

$x$	2	4	6	8	10
$y = kx^2$	4	16	36	64	100



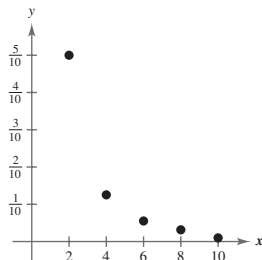
15.

$x$	2	4	6	8	10
$y = kx^2$	2	8	18	32	50



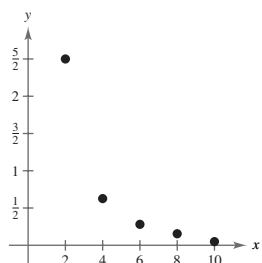
17.

$x$	2	4	6	8	10
$y = k/x^2$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{18}$	$\frac{1}{32}$	$\frac{1}{50}$



19.

$x$	2	4	6	8	10
$y = k/x^2$	$\frac{5}{2}$	$\frac{5}{8}$	$\frac{5}{18}$	$\frac{5}{32}$	$\frac{1}{10}$



21.  $y = \frac{5}{x}$     23.  $y = -\frac{7}{10}x$     25.  $y = \frac{12}{5}x$

27.  $y = 205x$     29.  $I = 0.035P$

31. Model:  $y = \frac{33}{13}x$ ; 25.4 centimeters, 50.8 centimeters

33.  $y = 0.0368x$ ; \$7360

35. (a) 0.05 meter    (b)  $176\frac{2}{3}$  newtons

37. 39.47 pounds    39.  $A = kr^2$     41.  $y = \frac{k}{x^2}$

43.  $F = \frac{kg}{r^2}$     45.  $P = \frac{k}{V}$     47.  $F = \frac{km_1m_2}{r^2}$

49. The area of a triangle is jointly proportional to its base and height.

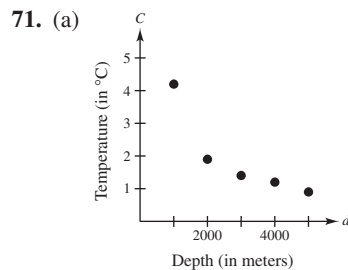
51. The volume of a sphere varies directly as the cube of its radius.

53. Average speed is directly proportional to the distance and inversely proportional to the time.

55.  $A = \pi r^2$     57.  $y = \frac{28}{x}$     59.  $F = 14rs^3$

61.  $z = \frac{2x^2}{3y}$     63.  $\approx 0.61$  mile per hour    65. 506 feet

67. 1470 joules    69. The velocity is increased by one-third.



(b) Yes.  $k_1 = 4200$ ,  $k_2 = 3800$ ,  $k_3 = 4200$ ,  $k_4 = 4800$ ,  $k_5 = 4500$

(c)  $C = \frac{4300}{d}$

(d)  (e)  $\approx 1433$  meters

73. (a)  (b) 0.2857 microwatt per square centimeter

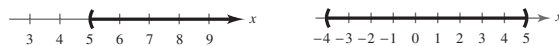
75. False.  $y$  will increase if  $k$  is positive and  $y$  will decrease if  $k$  is negative.

77. True. The closer the value of  $|r|$  is to 1, the better the fit.

79. The accuracy is questionable when based on such limited data.

81.  $x > 5$

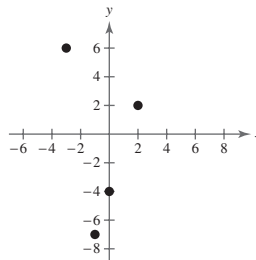
83.  $-4 < x < 5$



85. (a)  $-\frac{5}{3}$     (b)  $-\frac{7}{3}$     (c) 21    87. Answers will vary.

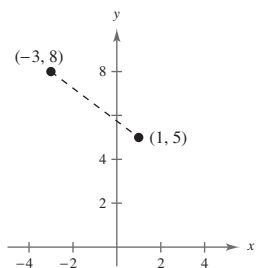
## Review Exercises (page 117)

1.



3. Quadrant IV

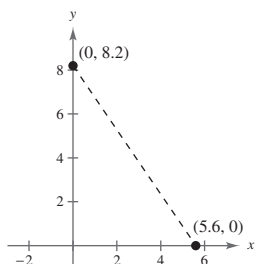
5. (a)



(b) 5

(c)  $(-1, \frac{13}{2})$

7. (a)



(b) 9.9

(c)  $(2.8, 4.1)$

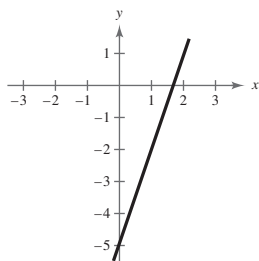
9.  $(2, 5)$ ,  $(4, 5)$ ,  $(2, 0)$ ,  $(4, 0)$

11. \$656.45 million

13. Radius  $\approx 22.5$  centimeters

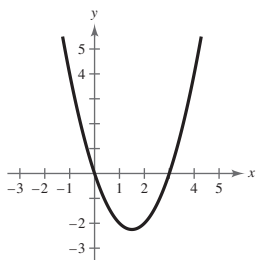
15.

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1

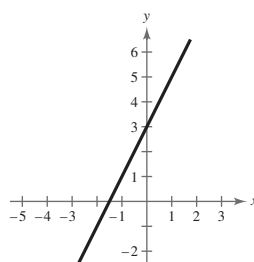


17.

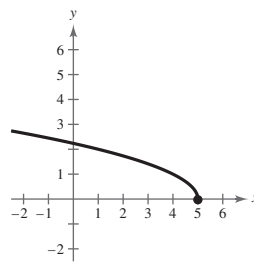
x	-1	0	1	2	3	4
y	4	0	-2	-2	0	4



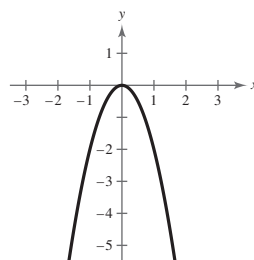
19.



21.



23.

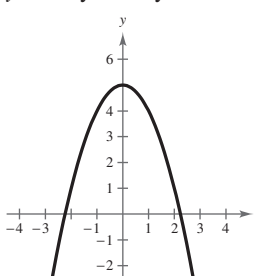
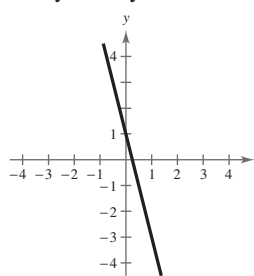


25. x-intercept:  $(-\frac{7}{2}, 0)$   
y-intercept:  $(0, 7)$

27. x-intercepts:  $(1, 0)$ ,  $(5, 0)$   
y-intercept:  $(0, 5)$

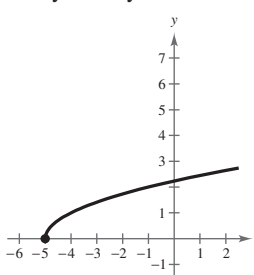
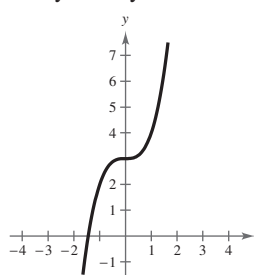
29. No symmetry

31. y-axis symmetry



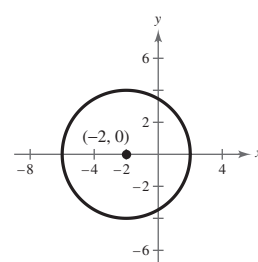
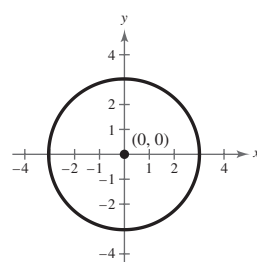
33. No symmetry

35. No symmetry

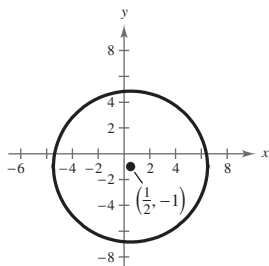


37. Center:  $(0, 0)$ ;  
Radius: 3

39. Center:  $(-2, 0)$ ;  
Radius: 4



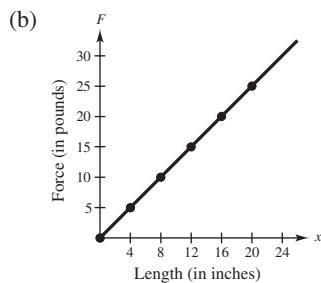
41. Center:
- $(\frac{1}{2}, -1)$
- ; Radius: 6



- 43.
- $(x - 2)^2 + (y + 3)^2 = 13$

45. (a)

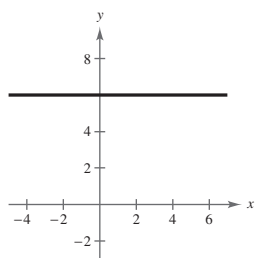
$x$	0	4	8	12	16	20
$F$	0	5	10	15	20	25



(c) 12.5 pounds

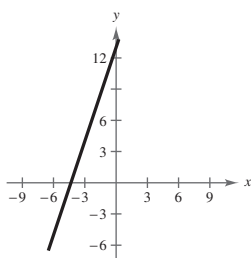
47. slope: 0

y-intercept: 6

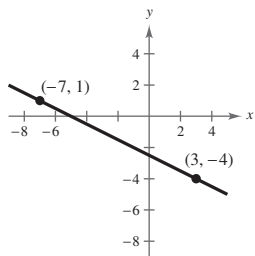


49. slope: 3

y-intercept: 13

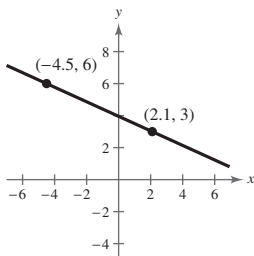


- 51.



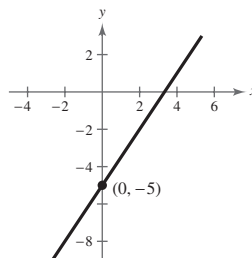
$$m = -\frac{1}{2}$$

- 53.

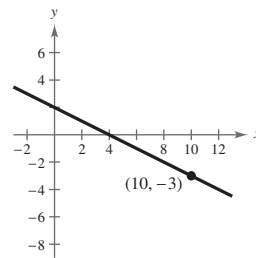


$$m = -\frac{5}{11}$$

- 55.
- $y = \frac{3}{2}x - 5$



- 57.
- $y = -\frac{1}{2}x + 2$



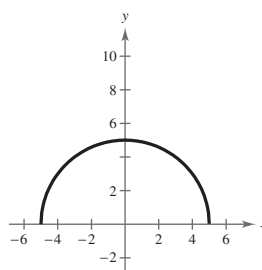
- 59.
- $x = 0$
- 61.
- $y = -\frac{4}{3}x + \frac{8}{3}$

63. (a)
- $y = \frac{5}{4}x - \frac{23}{4}$
- (b)
- $y = -\frac{4}{5}x + \frac{2}{5}$

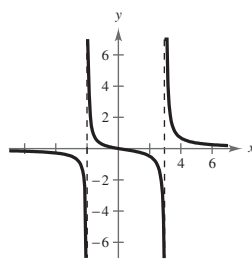
- 65.
- $V = 850t + 7400$
- ,
- $6 \leq t \leq 11$
67. No 69. Yes

71. (a) 5 (b) 17 (c)
- $t^4 + 1$
- (d)
- $t^2 + 2t + 2$

73. All real numbers
- $x$
- such that
- $-5 \leq x \leq 5$



75. All real numbers
- $x$
- except
- $x = 3, -2$



77. (a) 16 feet per second (b) 1.5 seconds

(c) -16 feet per second

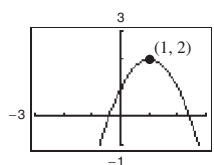
- 79.
- $4x + 2h + 3$
- ,
- $h \neq 0$
81. Function

83. Not a function 85.
- $\frac{7}{3}, 3$
- 87.
- $-\frac{3}{8}$

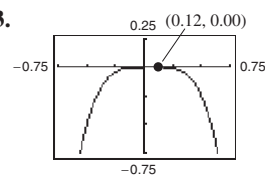
89. Increasing on
- $(0, \infty)$

Decreasing on  $(-\infty, -1)$ Constant on  $(-1, 0)$ 

- 91.



- 93.

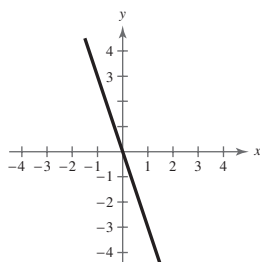


95. 4    97.  $\frac{1 - \sqrt{2}}{2}$

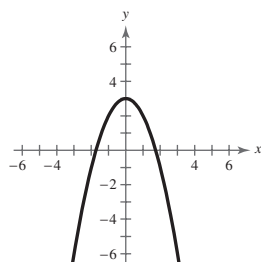
99. Neither even nor odd

101. Odd

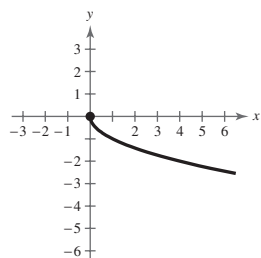
103.  $f(x) = -3x$



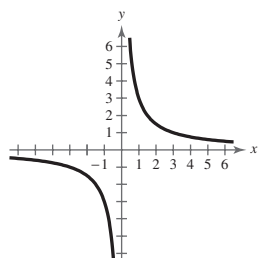
105.



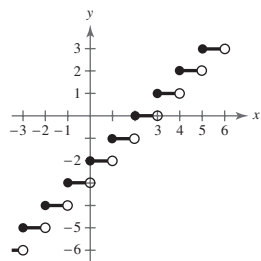
107.



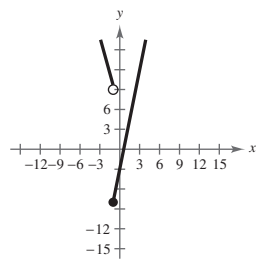
109.



111.



113.

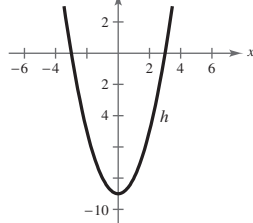


115.  $y = x^3$

117. (a)  $f(x) = x^2$

(b) Vertical shift of nine units downward

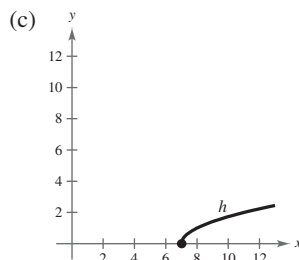
(c)



(d)  $h(x) = f(x) - 9$

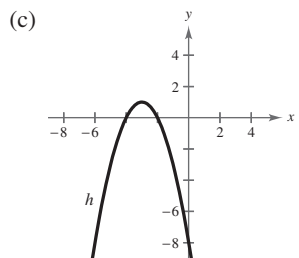
119. (a)  $f(x) = \sqrt{x}$

(b) Horizontal shift of seven units to the right



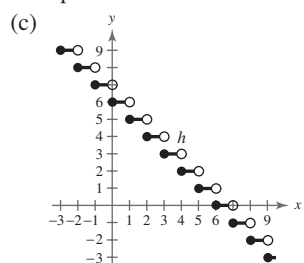
(d)  $h(x) = f(x - 7)$

121. (a)  $f(x) = x^2$

 (b) Reflection in the  $x$ -axis, horizontal shift of three units to the left, and vertical shift of one unit upward


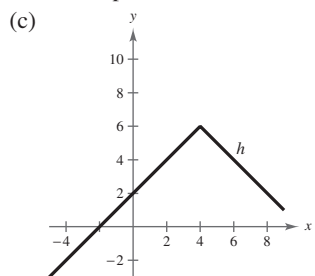
(d)  $h(x) = -f(x + 3) + 1$

123. (a)  $f(x) = \lfloor x \rfloor$

 (b) Reflection in the  $x$ -axis and vertical shift of six units upward


(d)  $h(x) = -f(x) + 6$

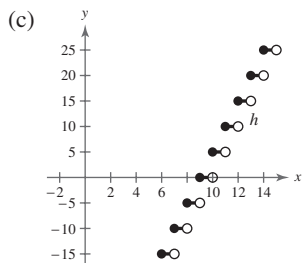
125. (a)  $f(x) = |x|$

 (b) Reflections in the  $x$ -axis and the  $y$ -axis, horizontal shift of four units to the right, and vertical shift of six units upward


(d)  $h(x) = -f(-x + 4) + 6$

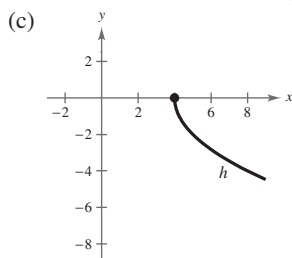
127. (a)  $f(x) = \llbracket x \rrbracket$

(b) Horizontal shift of nine units to the right and vertical stretch



(d)  $h(x) = 5f(x - 9)$

129. (a)  $f(x) = \sqrt{x}$

(b) Reflection in the  $x$ -axis, vertical stretch, and horizontal shift of four units to the right

(d)  $h(x) = -2f(x - 4)$

131. (a)  $x^2 + 2x + 2$  (b)  $x^2 - 2x + 4$

(c)  $2x^3 - x^2 + 6x - 3$

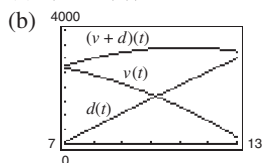
(d)  $\frac{x^2 + 3}{2x - 1}$ ; all real numbers  $x$  except  $x = \frac{1}{2}$

133. (a)  $x - \frac{8}{3}$  (b)  $x - 8$

Domains of  $f$ ,  $g$ ,  $f \circ g$ , and  $g \circ f$ : all real numbers

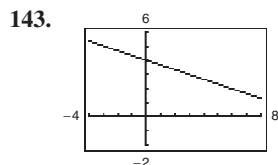
135.  $f(x) = x^3$ ,  $g(x) = 6x - 5$

137. (a)  $(v + d)(t) = -36.04t^2 + 804.6t - 1112$

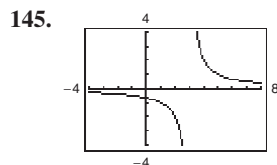


(c)  $(v + d)(10) = 3330$

139.  $f^{-1}(x) = x + 7$  141. The function has an inverse.

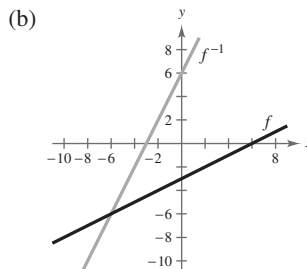


The function has an inverse.

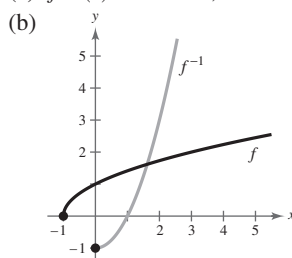


The function has an inverse.

147. (a)  $f^{-1}(x) = 2x + 6$

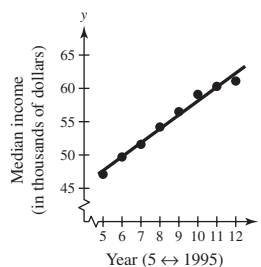
(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .(d) Both  $f$  and  $f^{-1}$  have domains and ranges that are all real numbers.

149. (a)  $f^{-1}(x) = x^2 - 1$ ,  $x \geq 0$

(c) The graph of  $f^{-1}$  is the reflection of the graph of  $f$  in the line  $y = x$ .(d)  $f$  has a domain of  $[-1, \infty)$  and a range of  $[0, \infty)$ ;  $f^{-1}$  has a domain of  $[0, \infty)$  and a range of  $[-1, \infty)$ .

151.  $x \geq 4$ ;  $f^{-1}(x) = \sqrt{\frac{x}{2}} + 4$

153. (a)



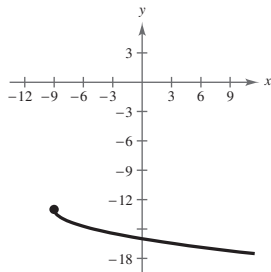
(b) The model is a good fit for the actual data.

155. Model:  $m = \frac{8}{5}k$ ; 3.2 kilometers, 16 kilometers

157. A factor of 4 159.  $\approx 2$  hours, 26 minutes



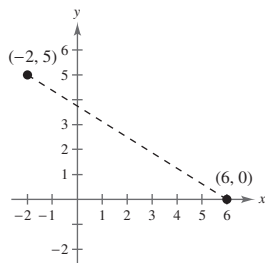
161. False. The graph is reflected in the  $x$ -axis, shifted nine units to the left, and then shifted 13 units downward.



163. True. If  $y$  is directly proportional to  $x$ , then  $y = kx$ , so  $x = (1/k)y$ . Therefore,  $x$  is directly proportional to  $y$ .
165. A function from a set  $A$  to a set  $B$  is a relation that assigns to each element  $x$  in the set  $A$  exactly one element  $y$  in the set  $B$ .

### Chapter Test (page 123)

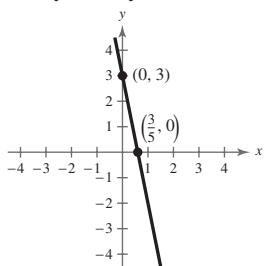
1.



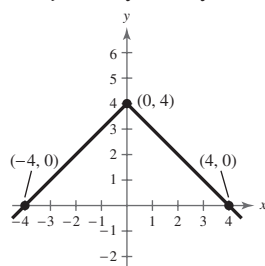
Midpoint:  $(2, \frac{5}{2})$ ; Distance:  $\sqrt{89}$

2.  $\approx 11.937$  centimeters

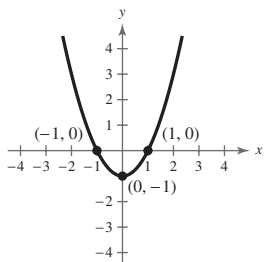
3. No symmetry



4.  $y$ -axis symmetry



5.  $y$ -axis symmetry



6.  $(x - 1)^2 + (y - 3)^2 = 16$       7.  $2x + y - 1 = 0$

8.  $17x + 10y - 59 = 0$

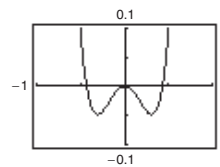
9. (a)  $4x - 7y + 44 = 0$       (b)  $7x + 4y - 53 = 0$

10. (a)  $-\frac{1}{8}$       (b)  $-\frac{1}{28}$       (c)  $\frac{\sqrt{x}}{x^2 - 18x}$

11.  $-10 \leq x \leq 10$

12. (a)  $0, \pm 0.4314$

(b)



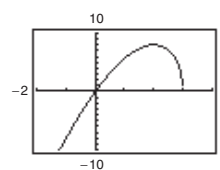
(c) Increasing on  $(-0.31, 0)$ ,  $(0.31, \infty)$

Decreasing on  $(-\infty, -0.31)$ ,  $(0, 0.31)$

(d) Even

13. (a) 0, 3

(b)



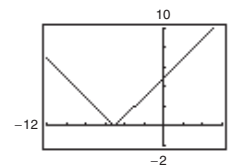
(c) Increasing on  $(-\infty, 2)$

Decreasing on  $(2, 3)$

(d) Neither even nor odd

14. (a)  $-5$

(b)

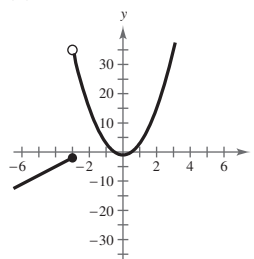


(c) Increasing on  $(-5, \infty)$

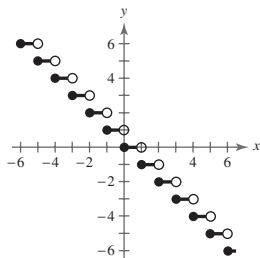
Decreasing on  $(-\infty, -5)$

(d) Neither even nor odd

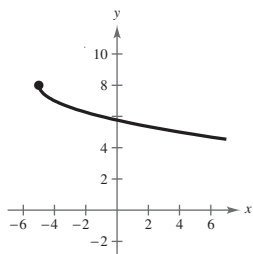
15.



16. Reflection in the
- $x$
- axis of
- $y = \lfloor x \rfloor$



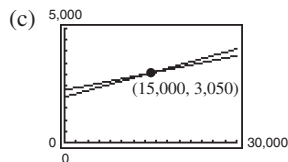
17. Reflection in the
- $x$
- axis, horizontal shift, and vertical shift of
- $y = \sqrt{x}$



18. (a)  $2x^2 - 4x - 2$  (b)  $4x^2 + 4x - 12$   
 (c)  $-3x^4 - 12x^3 + 22x^2 + 28x - 35$   
 (d)  $\frac{3x^2 - 7}{-x^2 - 4x + 5}, x \neq -5, 1$   
 (e)  $3x^4 + 24x^3 + 18x^2 - 120x + 68$   
 (f)  $-9x^4 + 30x^2 - 16$
19. (a)  $\frac{1 + 2x^{3/2}}{x}, x > 0$  (b)  $\frac{1 - 2x^{3/2}}{x}, x > 0$   
 (c)  $\frac{2\sqrt{x}}{x}, x > 0$  (d)  $\frac{1}{2x^{3/2}}, x > 0$   
 (e)  $\frac{\sqrt{x}}{2x}, x > 0$  (f)  $\frac{2\sqrt{x}}{x}, x > 0$
20.  $f^{-1}(x) = \sqrt[3]{x - 8}$  21. No inverse  
 22.  $f^{-1}(x) = (\frac{1}{3}x)^{2/3}, x \geq 0$  23.  $v = 6\sqrt{s}$   
 24.  $A = \frac{25}{6}xy$  25.  $b = \frac{48}{a}$

**Problem Solving** (page 125)

1. (a)
- $W_1 = 2000 + 0.07S$
- (b)
- $W_2 = 2300 + 0.05S$



Both jobs pay the same monthly salary if sales equal \$15,000.

- (d) No. Job 1 would pay \$3400 and job 2 would pay \$3300.

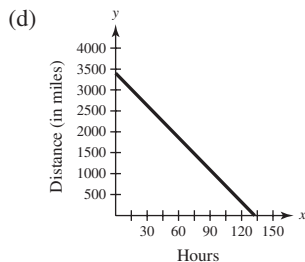
3. (a) The function will be even.  
 (b) The function will be odd.  
 (c) The function will be neither even nor odd.
5.  $f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$   
 $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$   
 $= f(x)$

7. (a)
- $81\frac{2}{3}$
- hours (b)
- $25\frac{5}{7}$
- miles per hour

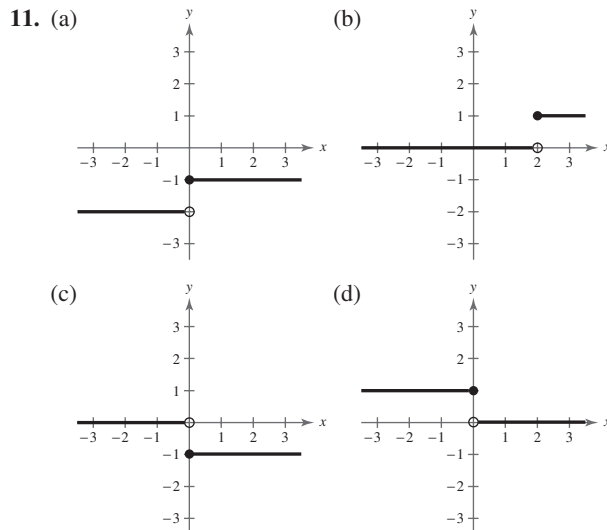
(c)  $y = \frac{-180}{7}x + 3400$

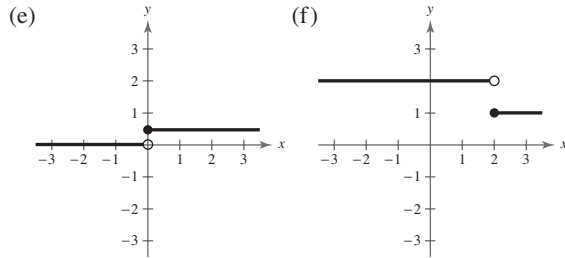
Domain:  $0 \leq x \leq \frac{1190}{9}$

Range:  $0 \leq y \leq 3400$



9. (a)  $(f \circ g)(x) = 4x + 24$  (b)  $(f \circ g)^{-1}(x) = \frac{1}{4}x - 6$   
 (c)  $f^{-1}(x) = \frac{1}{4}x; g^{-1}(x) = x - 6$   
 (d)  $(g^{-1} \circ f^{-1})(x) = \frac{1}{4}x - 6$   
 (e)  $(f \circ g)(x) = 8x^3 + 1; (f \circ g)^{-1}(x) = \frac{1}{2}\sqrt[3]{x - 1};$   
 $f^{-1}(x) = \sqrt[3]{x - 1}; g^{-1}(x) = \frac{1}{2}x;$   
 $(g^{-1} \circ f^{-1})(x) = \frac{1}{2}\sqrt[3]{x - 1}$   
 (f) Answers will vary. (g)  $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$





13. Proof

15. (a)

$x$	-4	-2	0	4
$f(f^{-1}(x))$	-4	-2	0	4

(b)

$x$	-3	-2	0	1
$(f + f^{-1})(x)$	5	1	-3	-5

(c)

$x$	-3	-2	0	1
$(f \circ f^{-1})(x)$	4	0	2	6

(d)

$x$	-4	-3	0	4
$ f^{-1}(x) $	2	1	1	3

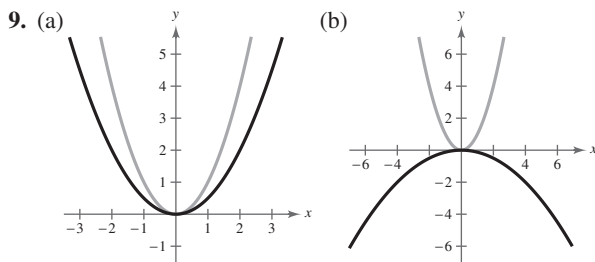
## Chapter 2

### Section 2.1 (page 134)

#### Vocabulary Check (page 134)

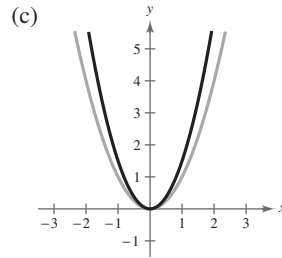
1. nonnegative integer; real
2. quadratic; parabola
3. axis
4. positive; minimum
5. negative; maximum

1. g
2. c
3. b
4. h
5. f
6. a
7. e
8. d

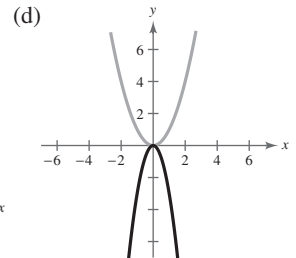


Vertical shrink

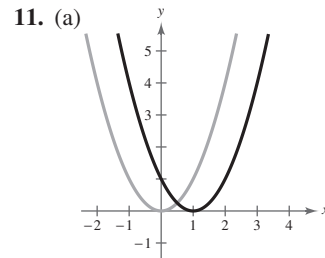
Vertical shrink and reflection in the  $x$ -axis



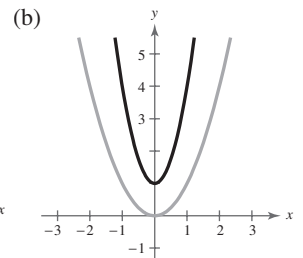
Vertical stretch



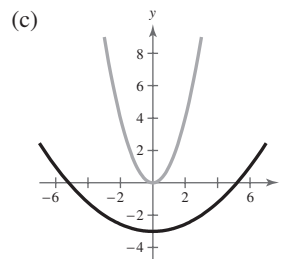
Vertical stretch and reflection in the  $x$ -axis



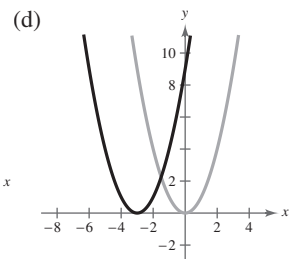
Horizontal shift



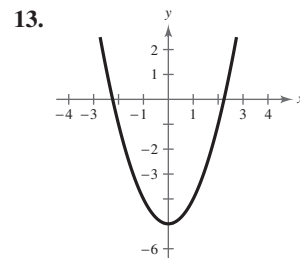
Horizontal shrink and vertical shift



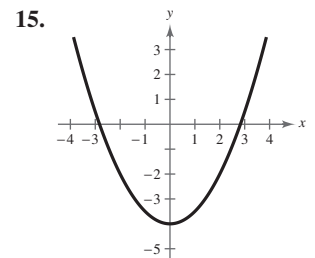
Horizontal stretch and vertical shift



Horizontal shift

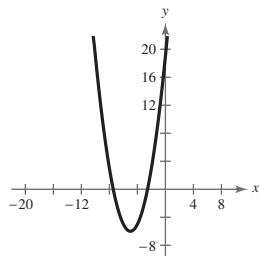


Vertex:  $(0, -5)$   
Axis of symmetry:  $y$ -axis  
 $x$ -intercepts:  $(\pm\sqrt{5}, 0)$



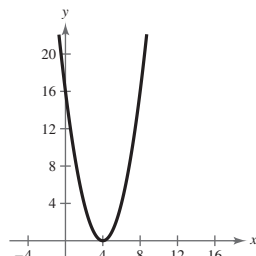
Vertex:  $(0, -4)$   
Axis of symmetry:  $y$ -axis  
 $x$ -intercepts:  $(\pm 2\sqrt{2}, 0)$

17.



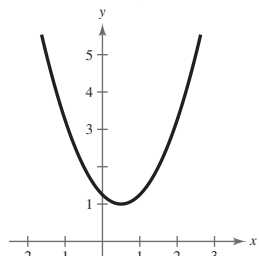
Vertex:  $(-5, -6)$   
 Axis of symmetry:  $x = -5$   
 x-intercepts:  $(-5 \pm \sqrt{6}, 0)$

19.



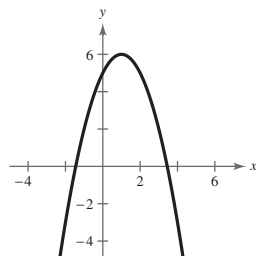
Vertex:  $(4, 0)$   
 Axis of symmetry:  $x = 4$   
 x-intercept:  $(4, 0)$

21.



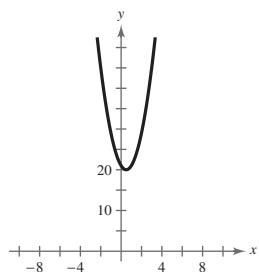
Vertex:  $(\frac{1}{2}, 1)$   
 Axis of symmetry:  $x = \frac{1}{2}$   
 No x-intercept

23.



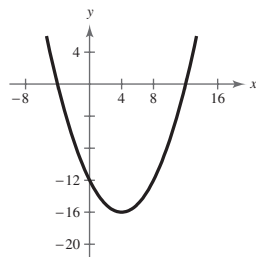
Vertex:  $(1, 6)$   
 Axis of symmetry:  $x = 1$   
 x-intercepts:  $(1 \pm \sqrt{6}, 0)$

25.



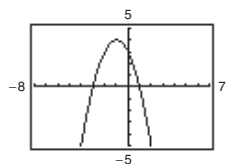
Vertex:  $(\frac{1}{2}, 20)$   
 Axis of symmetry:  $x = \frac{1}{2}$   
 No x-intercept

27.



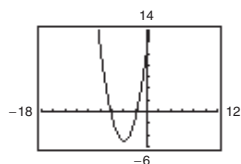
Vertex:  $(4, -16)$   
 Axis of symmetry:  $x = 4$   
 x-intercepts:  $(-4, 0), (12, 0)$

29.



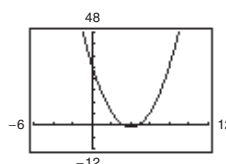
Vertex:  $(-1, 4)$   
 Axis of symmetry:  $x = -1$   
 x-intercepts:  $(1, 0), (-3, 0)$

31.



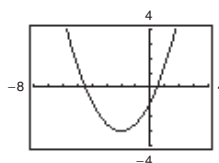
Vertex:  $(-4, -5)$   
 Axis of symmetry:  $x = -4$   
 x-intercepts:  $(-4 \pm \sqrt{5}, 0)$

33.



Vertex:  $(4, -1)$   
 Axis of symmetry:  $x = 4$   
 x-intercepts:  $(4 \pm \frac{1}{2}\sqrt{2}, 0)$

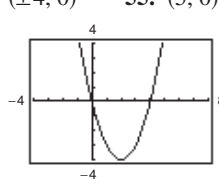
35.



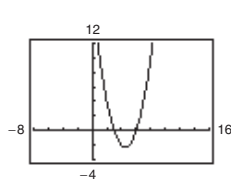
Vertex:  $(-2, -3)$   
 Axis of symmetry:  $x = -2$   
 x-intercepts:  $(-2 \pm \sqrt{6}, 0)$

37.  $y = (x - 1)^2$ 39.  $y = -(x + 1)^2 + 4$ 41.  $y = -2(x + 2)^2 + 2$ 43.  $f(x) = (x + 2)^2 + 5$ 45.  $f(x) = -\frac{1}{2}(x - 3)^2 + 4$ 47.  $f(x) = \frac{3}{4}(x - 5)^2 + 12$ 49.  $f(x) = -\frac{24}{49}(x + \frac{1}{4})^2 + \frac{3}{2}$ 51.  $f(x) = -\frac{16}{3}(x + \frac{5}{2})^2$ 53.  $(\pm 4, 0)$ 55.  $(5, 0), (-1, 0)$ 

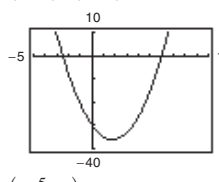
57.

 $(0, 0), (4, 0)$ 

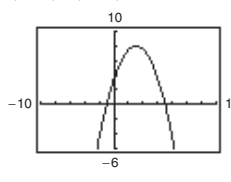
59.

 $(3, 0), (6, 0)$ 

61.

 $(-\frac{5}{2}, 0), (6, 0)$ 

63.

 $(7, 0), (-1, 0)$ 65.  $f(x) = x^2 - 2x - 3$ 67.  $f(x) = x^2 - 10x$  $g(x) = -x^2 + 2x + 3$  $g(x) = -x^2 + 10x$ 69.  $f(x) = 2x^2 + 7x + 3$  $g(x) = -2x^2 - 7x - 3$ 

71. 55, 55

73. 12, 6

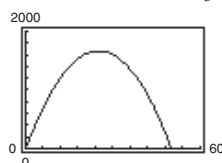
75. (a)  $A = \frac{8x(50 - x)}{3}$ 

(b)

$x$	5	10	15	20	25	30
$A$	600	1067	1400	1600	1667	1600

 $x = 25$  feet,  $y = 33\frac{1}{3}$  feet

(c)

 $x = 25$  feet,  $y = 33\frac{1}{3}$  feet(d)  $A = -\frac{8}{3}(x - 25)^2 + \frac{5000}{3}$ 

(e) They are identical.

77. 16 feet

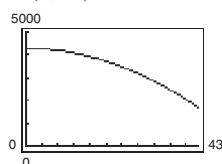
79. 20 fixtures

81. 350,000 units

83. (a) \$14,000,000; \$14,375,000; \$13,500,000

(b) 24; \$14,400

85. (a)



(b) 4299; answers will vary.

(c) 8879; 24

87. (a)  (b) 69.6 miles per hour

89. True. The equation has no real solutions, so the graph has no  $x$ -intercepts.

91.  $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

93. Yes. A graph of a quadratic equation whose vertex is on the  $x$ -axis has only one  $x$ -intercept.

95.  $y = -\frac{1}{3}x + \frac{5}{3}$     97.  $y = \frac{5}{4}x + 3$     99. 27

101.  $-\frac{1408}{49}$     103. 109    105. Answers will vary.

## Section 2.2 (page 148)

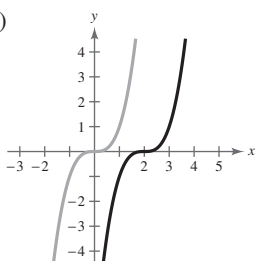
### Vocabulary Check (page 148)

1. continuous
2. Leading Coefficient Test
3.  $n$ ;  $n - 1$
4. (a) solution; (b)  $(x - a)$ ; (c)  $x$ -intercept
5. touches; crosses
6. standard
7. Intermediate Value

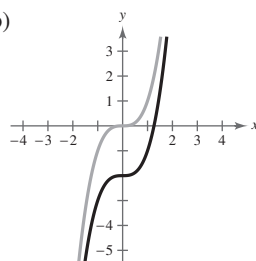
1. c    2. g    3. h    4. f

5. a    6. e    7. d    8. b

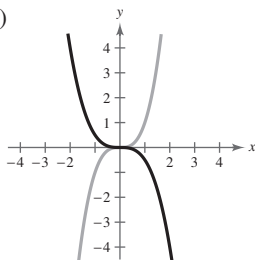
9. (a)



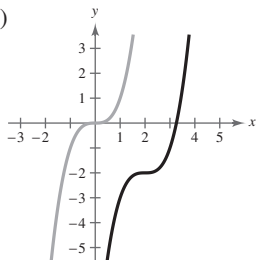
(b)



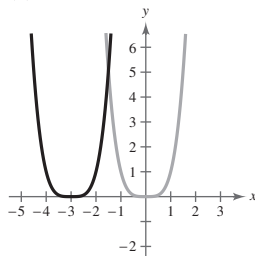
(c)



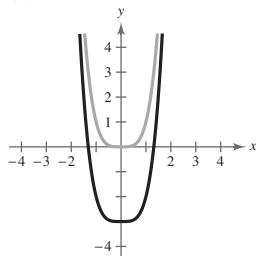
(d)



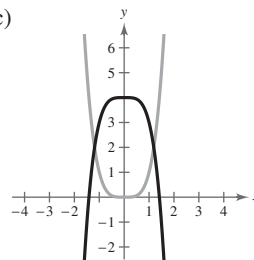
11. (a)



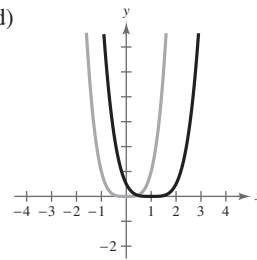
(b)



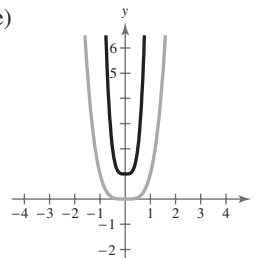
(c)



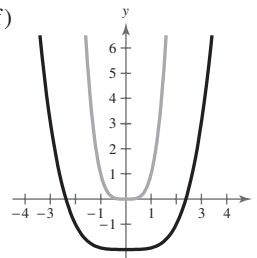
(d)



(e)



(f)



13. Falls to the left, rises to the right

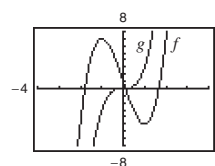
15. Falls to the left, falls to the right

17. Rises to the left, falls to the right

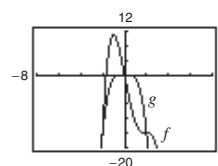
19. Rises to the left, falls to the right

21. Falls to the left, falls to the right

23.



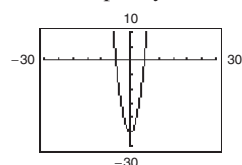
25.



27. (a)  $\pm 5$

(b) odd multiplicity; number of turning points: 1

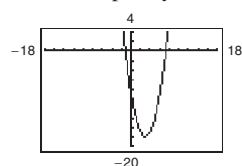
(c)



29. (a) 3

(b) even multiplicity; number of turning points: 1

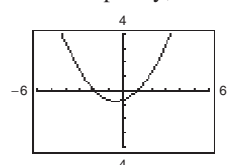
(c)



31. (a) -2, 1

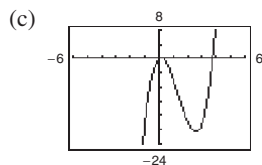
(b) odd multiplicity; number of turning points: 1

(c)



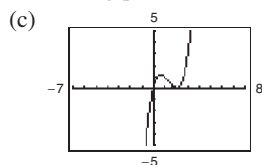
33. (a)  $0, 2 \pm \sqrt{3}$

(b) odd multiplicity; number of turning points: 2

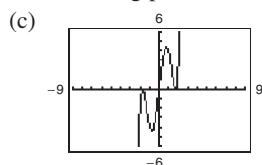


35. (a) 0, 2

(b) 0, odd multiplicity; 2, even multiplicity; number of turning points: 2

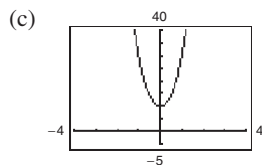


37. (a)  $0, \pm\sqrt{3}$

(b) 0, odd multiplicity;  $\pm\sqrt{3}$ , even multiplicity; number of turning points: 4

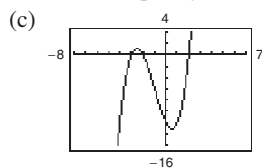
39. (a) No real zeros

(b) number of turning points: 1

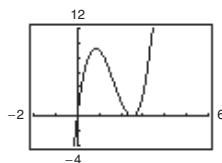


41. (a)  $\pm 2, -3$

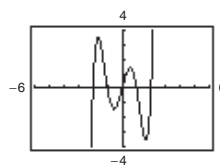
(b) odd multiplicity; number of turning points: 2



43. (a)

(b)  $x$ -intercepts:  $(0, 0), (\frac{5}{2}, 0)$  (c)  $x = 0, \frac{5}{2}$ (d) The answers in part (c) match the  $x$ -intercepts.

45. (a)

(b)  $x$ -intercepts:  $(0, 0), (\pm 1, 0), (\pm 2, 0)$ (c)  $x = 0, 1, -1, 2, -2$ (d) The answers in part (c) match the  $x$ -intercepts.

47.  $f(x) = x^2 - 10x$  49.  $f(x) = x^2 + 4x - 12$

51.  $f(x) = x^3 + 5x^2 + 6x$

53.  $f(x) = x^4 - 4x^3 - 9x^2 + 36x$

55.  $f(x) = x^2 - 2x - 2$  57.  $f(x) = x^2 + 4x + 4$

59.  $f(x) = x^3 + 2x^2 - 3x$  61.  $f(x) = x^3 - 3x$

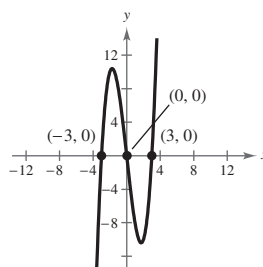
63.  $f(x) = x^4 + x^3 - 15x^2 + 23x - 10$

65.  $f(x) = x^5 + 16x^4 + 96x^3 + 256x^2 + 256x$

67. (a) Falls to the left, rises to the right

(b)  $0, \pm 3$  (c) Answers will vary.

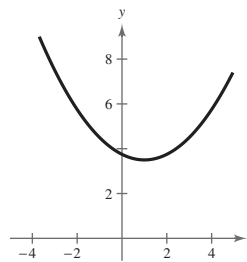
(d)



69. (a) Rises to the left, rises to the right

(b) No zeros (c) Answers will vary.

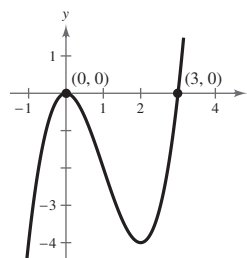
(d)



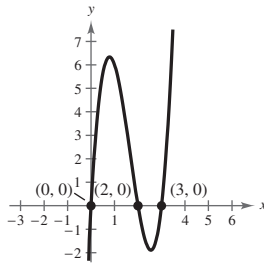
71. (a) Falls to the left, rises to the right

(b) 0, 3 (c) Answers will vary.

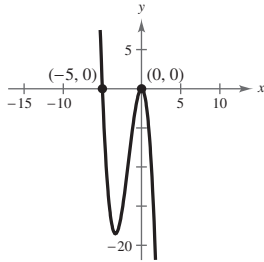
(d)



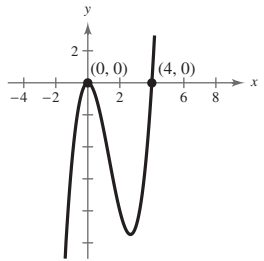
73. (a) Falls to the left, rises to the right  
(b) 0, 2, 3 (c) Answers will vary.  
(d)



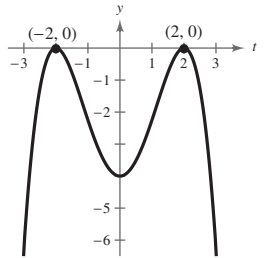
75. (a) Rises to the left, falls to the right  
(b) -5, 0 (c) Answers will vary.  
(d)



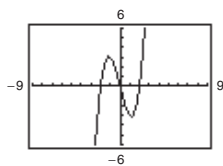
77. (a) Falls to the left, rises to the right  
(b) 0, 4 (c) Answers will vary.  
(d)



79. (a) Falls to the left, falls to the right  
(b)  $\pm 2$  (c) Answers will vary.  
(d)

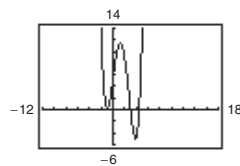


81.



Zeros: 0,  $\pm 2$ ,  
odd multiplicity

83.



Zeros: -1,  
even multiplicity;  
 $3, \frac{9}{2}$ , odd multiplicity

85.  $[-1, 0], [1, 2], [2, 3]; \approx -0.879, 1.347, 2.532$

87.  $[-2, -1], [0, 1]; \approx -1.585, 0.779$

89. (a)  $V = l \times w \times h$   
 $= (36 - 2x)(36 - 2x)x$   
 $= x(36 - 2x)^2$

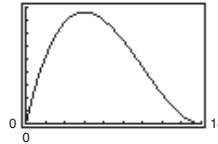
- (b) Domain:  $0 < x < 18$

(c)

$x$	1	2	3	4	5	6	7
$V$	1156	2048	2700	3136	3380	3456	3388

6 inches  $\times$  24 inches  $\times$  24 inches

(d) 3600



$x = 6$

91. (a)  $A = -2x^2 + 12x$  (b)  $V = -384x^2 + 2304x$

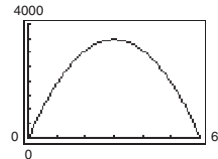
- (c)  $0 \text{ inches} < x < 6 \text{ inches}$

(d)

$x$	$V_1$
0	0
1	4320
2	8640
3	12960
4	17280
5	21600
6	25920
7	30240
8	34560
9	38880
10	43200
11	47520
12	51840
13	56160
14	60480
15	64800
16	69120
17	73440
18	77760
19	82080
20	86400
21	90720
22	95040
23	99360
24	103680
25	108000
26	112320
27	116640
28	120960
29	125280
30	129600
31	133920
32	138240
33	142560
34	146880
35	151200
36	155520
37	159840
38	164160
39	168480
40	172800
41	177120
42	181440
43	185760
44	190080
45	194400
46	198720
47	203040
48	207360
49	211680
50	216000
51	220320
52	224640
53	228960
54	233280
55	237600
56	241920
57	246240
58	250560
59	254880
60	259200
61	263520
62	267840
63	272160
64	276480
65	280800
66	285120
67	289440
68	293760
69	298080
70	302400
71	306720
72	311040
73	315360
74	319680
75	324000
76	328320
77	332640
78	336960
79	341280
80	345600
81	349920
82	354240
83	358560
84	362880
85	367200
86	371520
87	375840
88	380160
89	384480
90	388800
91	393120
92	397440
93	401760
94	406080
95	410400
96	414720
97	419040
98	423360
99	427680
100	432000

When  $x = 3$ , the volume is maximum at  $V = 3456$ ;  
dimensions of gutter are 3 inches  $\times$  6 inches  $\times$  3 inches.

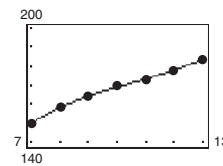
(e)



The maximum value is the same.

- (f) No. Answers will vary.

93.



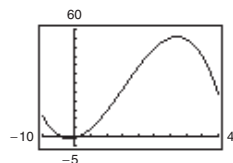
The model is a good fit.

95. Region 1: 259,370

Region 2: 223,470

Answers will vary.

97. (a)

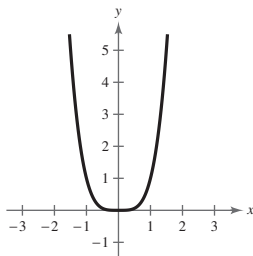


- (b)  $t \approx 15$  (c) Vertex: (15.22, 2.54)

- (d) The results are approximately equal.

99. False. A fifth-degree polynomial can have at most four turning points.
101. True. The degree of the function is odd and its leading coefficient is negative, so the graph rises to the left and falls to the right.

103.



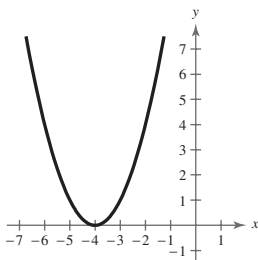
- (a) Vertical shift of two units; Even  
 (b) Horizontal shift of two units; Neither even nor odd  
 (c) Reflection in the  $y$ -axis; Even  
 (d) Reflection in the  $x$ -axis; Even  
 (e) Horizontal stretch; Even  
 (f) Vertical shrink; Even  
 (g)  $g(x) = x^3$ ; Neither odd nor even  
 (h)  $g(x) = x^{16}$ ; Even

105.  $(5x - 8)(x + 3)$     107.  $x^2(4x + 5)(x - 3)$

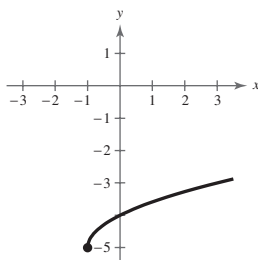
109.  $-\frac{7}{2}, 4$     111.  $-\frac{5}{4}, \frac{1}{3}$     113.  $1 \pm \sqrt{22}$

115.  $\frac{-5 \pm \sqrt{185}}{4}$

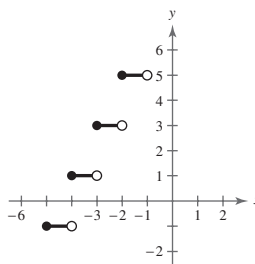
117. Horizontal translation four units to the left of
- $y = x^2$



119. Horizontal translation one unit left and vertical translation five units down of
- $y = \sqrt{x}$



121. Vertical stretch by a factor of 2 and vertical translation nine units up of
- $y = \lfloor x \rfloor$



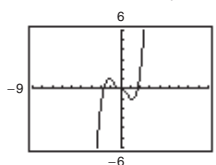
## Section 2.3 (page 159)

## Vocabulary Check (page 159)

1. dividend; divisor; quotient; remainder  
 2. improper; proper    3. synthetic division  
 4. factor    5. remainder

1. Answers will vary.

3.    5.
- $2x + 4$



7.  $x^2 - 3x + 1$     9.  $x^3 + 3x^2 - 1$     11.  $7 - \frac{11}{x + 2}$

13.  $3x + 5 - \frac{2x - 3}{2x^2 + 1}$     15.  $x^2 + 2x + 4 + \frac{2x - 11}{x^2 - 2x + 3}$

17.  $x + 3 + \frac{6x^2 - 8x + 3}{(x - 1)^3}$     19.  $3x^2 - 2x + 5$

21.  $4x^2 - 9$     23.  $-x^2 + 10x - 25$

25.  $5x^2 + 14x + 56 + \frac{232}{x - 4}$

27.  $10x^3 + 10x^2 + 60x + 360 + \frac{1360}{x - 6}$

29.  $x^2 - 8x + 64$

31.  $-3x^3 - 6x^2 - 12x - 24 - \frac{48}{x - 2}$

33.  $-x^3 - 6x^2 - 36x - 36 - \frac{216}{x - 6}$

35.  $4x^2 + 14x - 30$

37.  $f(x) = (x - 4)(x^2 + 3x - 2) + 3$ ,  $f(4) = 3$

39.  $f(x) = (x + \frac{2}{3})(15x^3 - 6x + 4) + \frac{34}{3}$ ,  $f(-\frac{2}{3}) = \frac{34}{3}$

41.  $f(x) = (x - \sqrt{2})[x^2 + (3 + \sqrt{2})x + 3\sqrt{2}] - 8$ ,  
 $f(\sqrt{2}) = -8$

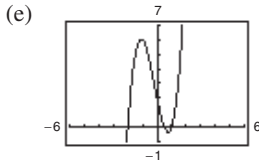
43.  $f(x) = (x - 1 + \sqrt{3})[-4x^2 + (2 + 4\sqrt{3})x + (2 + 2\sqrt{3})]$ ,  
 $f(1 - \sqrt{3}) = 0$

45. (a) 1    (b) 4    (c) 4    (d) 1954

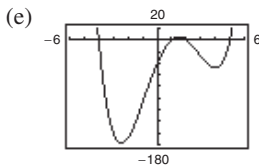
47. (a) 97    (b)  $-\frac{5}{3}$     (c) 17    (d) -199



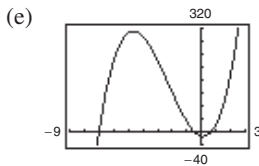
49.  $(x - 2)(x + 3)(x - 1)$ ; Zeros: 2, -3, 1  
 51.  $(2x - 1)(x - 5)(x - 2)$ ; Zeros:  $\frac{1}{2}$ , 5, 2  
 53.  $(x + \sqrt{3})(x - \sqrt{3})(x + 2)$ ; Zeros:  $-\sqrt{3}$ ,  $\sqrt{3}$ , -2  
 55.  $(x - 1)(x - 1 - \sqrt{3})(x - 1 + \sqrt{3})$ ;  
 Zeros: 1,  $1 + \sqrt{3}$ ,  $1 - \sqrt{3}$   
 57. (a) Answers will vary. (b)  $2x - 1$   
 (c)  $f(x) = (2x - 1)(x + 2)(x - 1)$  (d)  $\frac{1}{2}$ , -2, 1



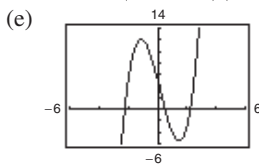
59. (a) Answers will vary. (b)  $(x - 1)$ ,  $(x - 2)$   
 (c)  $f(x) = (x - 1)(x - 2)(x - 5)(x + 4)$   
 (d) 1, 2, 5, -4



61. (a) Answers will vary. (b)  $x + 7$   
 (c)  $f(x) = (x + 7)(2x + 1)(3x - 2)$   
 (d)  $-7$ ,  $-\frac{1}{2}$ ,  $\frac{2}{3}$



63. (a) Answers will vary. (b)  $(x - \sqrt{5})$   
 (c)  $f(x) = (x - \sqrt{5})(x + \sqrt{5})(2x - 1)$  (d)  $\pm\sqrt{5}$ ,  $\frac{1}{2}$

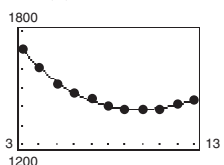


65. (a) Zeros are 2 and  $\approx \pm 2.236$ .  
 (b)  $x = 2$  (c)  $f(x) = (x - 2)(x - \sqrt{5})(x + \sqrt{5})$   
 67. (a) Zeros are -2,  $\approx 0.268$ , and  $\approx 3.732$ .  
 (b)  $x = -2$

(c)  $h(t) = (t + 2)[t - (2 + \sqrt{3})][t - (2 - \sqrt{3})]$

69.  $2x^2 - x - 1$ ,  $x \neq \frac{3}{2}$  71.  $x^2 + 3x$ ,  $x \neq -2, -1$

73. (a) and (b)



$M = -0.242x^3 + 12.43x^2 - 173.4x + 2118$

- (c)

$t$	3	4	5	6	7	8
$M(t)$	1703	1608	1531	1473	1430	1402

$t$	9	10	11	12	13
$M(t)$	1388	1385	1392	1409	1433

Answers will vary.

- (d) 1614 thousand. No, because the model will approach negative infinity quickly.

75. False.  $-\frac{4}{7}$  is a zero of  $f$ .  
 77. True. The degree of the numerator is greater than the degree of the denominator.  
 79.  $x^{2n} + 6x^n + 9$  81. The remainder is 0.  
 83.  $c = -210$  85. 0;  $x + 3$  is a factor of  $f$ .  
 87.  $\pm\frac{5}{3}$  89.  $-\frac{7}{5}, 2$  91.  $\frac{-3 \pm \sqrt{3}}{2}$   
 93.  $f(x) = x^3 - 7x^2 + 12x$   
 95.  $f(x) = x^3 + x^2 - 7x - 3$

## Section 2.4 (page 167)

### Vocabulary Check (page 167)

1. (a) iii (b) i (c) ii 2.  $\sqrt{-1}$ ; -1  
 3. complex numbers;  $a + bi$  4. principal square  
 5. complex conjugates

1.  $a = -10$ ,  $b = 6$  3.  $a = 6$ ,  $b = 5$  5.  $4 + 3i$   
 7.  $2 - 3\sqrt{3}i$  9.  $5\sqrt{3}i$  11. 8  
 13.  $-1 - 6i$  15.  $0.3i$  17.  $11 - i$  19. 4  
 21.  $3 - 3\sqrt{2}i$  23.  $-14 + 20i$  25.  $\frac{1}{6} + \frac{7}{6}i$   
 27.  $5 + i$  29.  $12 + 30i$  31. 24 33.  $-9 + 40i$   
 35. -10 37.  $6 - 3i$ , 45 39.  $-1 + \sqrt{5}i$ , 6  
 41.  $-2\sqrt{5}i$ , 20 43.  $\sqrt{8}$ , 8 45.  $-5i$   
 47.  $\frac{8}{41} + \frac{10}{41}i$  49.  $\frac{4}{5} + \frac{3}{5}i$  51.  $-5 - 6i$   
 53.  $-\frac{120}{1681} - \frac{27}{1681}i$  55.  $-\frac{1}{2} - \frac{5}{2}i$  57.  $\frac{62}{949} + \frac{297}{949}i$   
 59.  $-2\sqrt{3}$  61. -10  
 63.  $(21 + 5\sqrt{2}) + (7\sqrt{5} - 3\sqrt{10})i$  65.  $1 \pm i$   
 67.  $-2 \pm \frac{1}{2}i$  69.  $-\frac{5}{2}, -\frac{3}{2}$  71.  $2 \pm \sqrt{2}i$   
 73.  $\frac{5}{7} \pm \frac{5\sqrt{15}}{7}$  75.  $-1 + 6i$   
 77.  $-5i$  79.  $-375\sqrt{3}i$  81.  $i$   
 83. (a)  $z_1 = 9 + 16i$ ,  $z_2 = 20 - 10i$   
 (b)  $z = \frac{11,240}{877} + \frac{4630}{877}i$   
 85. (a) 16 (b) 16 (c) 16 (d) 16  
 87. False. If the complex number is real, the number equals its conjugate.

89. False.

$$i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = 1 - 1 + 1 - i + i = 1$$

91. Proof    93.  $-x^2 - 3x + 12$     95.  $3x^2 + \frac{23}{2}x - 2$ 97.  $-31$     99.  $\frac{27}{2}$     101.  $a = \frac{\sqrt{3V\pi b}}{2\pi b}$     103. 1 liter

## Section 2.5 (page 179)

## Vocabulary Check (page 179)

1. Fundamental Theorem of Algebra
2. Linear Factorization Theorem    3. Rational Zero
4. conjugate    5. irreducible over the reals
6. Descartes' Rule of Signs    7. lower; upper

1. 0, 6    3. 2, -4    5. -6,  $\pm i$     7.  $\pm 1, \pm 3$   
 9.  $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2}, \pm \frac{45}{2}$   
 11. 1, 2, 3    13. 1, -1, 4    15. -1, -10    17.  $\frac{1}{2}, -1$   
 19. -2, 3,  $\pm \frac{2}{3}$     21. -1, 2    23.  $-6, \frac{1}{2}, 1$   
 25. (a)  $\pm 1, \pm 2, \pm 4$

(b)  (c) -2, -1, 2

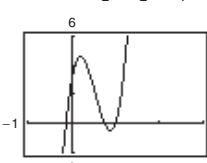
27. (a)
- $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

(b)  (c)  $-\frac{1}{4}, 1, 3$ 

29. (a)
- $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

(b)  (c)  $-\frac{1}{2}, 1, 2, 4$ 

31. (a)
- $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{1}{8}, \pm \frac{3}{8}, \pm \frac{1}{16}, \pm \frac{3}{16}, \pm \frac{1}{32}, \pm \frac{3}{32}$

(b)  (c)  $1, \frac{3}{4}, -\frac{1}{8}$ 

33. (a)
- $\pm 1, \approx \pm 1.414$

(b)  $f(x) = (x+1)(x-1)(x+\sqrt{2})(x-\sqrt{2})$

35. (a) 0, 3, 4,
- $\approx \pm 1.414$

(b)  $h(x) = x(x-3)(x-4)(x+\sqrt{2})(x-\sqrt{2})$

- 37.
- $x^3 - x^2 + 25x - 25$
- 39.
- $x^3 + 4x^2 - 31x - 174$

- 41.
- $3x^4 - 17x^3 + 25x^2 + 23x - 22$

43. (a)
- $(x^2 + 9)(x^2 - 3)$

(b)  $(x^2 + 9)(x + \sqrt{3})(x - \sqrt{3})$

(c)  $(x + 3i)(x - 3i)(x + \sqrt{3})(x - \sqrt{3})$

45. (a)
- $(x^2 - 2x - 2)(x^2 - 2x + 3)$

(b)  $(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})(x^2 - 2x + 3)$

(c)  $(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})(x - 1 + \sqrt{2}i)(x - 1 - \sqrt{2}i)$

- 47.
- $-\frac{3}{2}, \pm 5i$
- 49.
- $\pm 2i, 1, -\frac{1}{2}$
- 51.
- $-3 \pm i, \frac{1}{4}$

- 53.
- $2, -3 \pm \sqrt{2}i, 1$
- 55.
- $\pm 5i; (x+5i)(x-5i)$

- 57.
- $2 \pm \sqrt{3}; (x-2-\sqrt{3})(x-2+\sqrt{3})$

- 59.
- $\pm 3, \pm 3i; (x+3)(x-3)(x+3i)(x-3i)$

- 61.
- $1 \pm i; (z-1+i)(z-1-i)$

- 63.
- $2, 2 \pm i; (x-2)(x-2+i)(x-2-i)$

- 65.
- $-2, 1 \pm \sqrt{2}i; (x+2)(x-1+\sqrt{2}i)(x-1-\sqrt{2}i)$

- 67.
- $-\frac{1}{5}, 1 \pm \sqrt{5}i; (5x+1)(x-1+\sqrt{5}i)(x-1-\sqrt{5}i)$

- 69.
- $2, \pm 2i; (x-2)^2(x+2i)(x-2i)$

- 71.
- $\pm i, \pm 3i; (x+i)(x-i)(x+3i)(x-3i)$

- 73.
- $-10, -7 \pm 5i$
- 75.
- $-\frac{3}{4}, 1 \pm \frac{1}{2}i$

- 77.
- $-2, -\frac{1}{2}, \pm i$
79. No real zeros

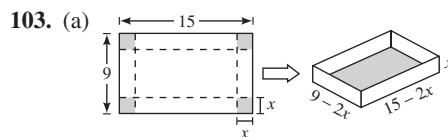
81. No real zeros    83. One positive zero

85. One or three positive zeros

87. Answers will vary.    89. Answers will vary.

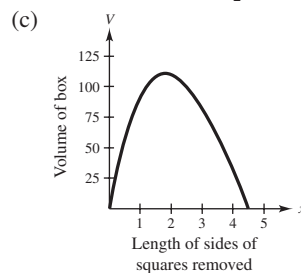
- 91.
- $1, -\frac{1}{2}$
- 93.
- $-\frac{3}{4}$
- 95.
- $\pm 2, \pm \frac{3}{2}$
- 97.
- $\pm 1, \frac{1}{4}$

99. d    100. a    101. b    102. c



(b)  $V = x(9-2x)(15-2x)$

Domain:  $0 < x < \frac{9}{2}$

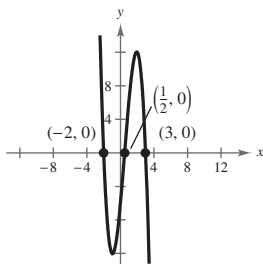


1.82 centimeters  $\times$  5.36 centimeters  $\times$  11.36 centimeters

(d)  $\frac{1}{2}, \frac{7}{2}, 8$ ; 8 is not in the domain of  $V$ .

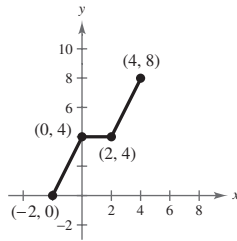
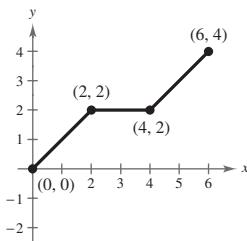
- 105.
- $x \approx 38.4$
- , or \$384,000

107. (a)  $V = x^3 + 9x^2 + 26x + 24 = 120$   
 (b) 4 feet by 5 feet by 6 feet
109.  $x \approx 40$ , or 4000 units
111. No. Setting  $p = 9,000,000$  and solving the resulting equation yields imaginary roots.
113. False. The most complex zeros it can have is two, and the Linear Factorization Theorem guarantees that there are three linear factors, so one zero must be real.
115.  $r_1, r_2, r_3$     117.  $5 + r_1, 5 + r_2, 5 + r_3$
119. The zeros cannot be determined.
121. (a)  $0 < k < 4$     (b)  $k = 4$     (c)  $k < 0$     (d)  $k > 4$
123. Answers will vary. There are infinitely many possible functions for  $f$ . Sample equation and graph:  
 $f(x) = -2x^3 + 3x^2 + 11x - 6$

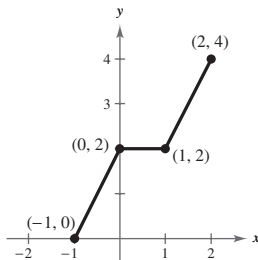


125. Answers will vary.
127. (a)  $x^2 + b$     (b)  $x^2 - 2ax + a^2 + b^2$
129.  $-11 + 9i$     131.  $20 + 40i$

133.    135.



137.



## Section 2.6 (page 193)

### Vocabulary Check (page 193)

1. rational functions    2. vertical asymptote  
 3. horizontal asymptote    4. slant asymptote

$x$	$f(x)$	$x$	$f(x)$	$x$	$f(x)$
0.5	-2	1.5	2	5	0.25
0.9	-10	1.1	10	10	$0.\overline{1}$
0.99	-100	1.01	100	100	$0.0\overline{1}$
0.999	-1000	1.001	1000	1000	$0.00\overline{1}$

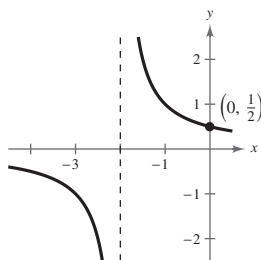
- (b) Vertical asymptote:  $x = 1$   
 Horizontal asymptote:  $y = 0$
- (c) Domain: all real numbers  $x$  except  $x = 1$

3. (a)

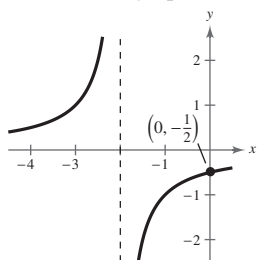
$x$	$f(x)$	$x$	$f(x)$	$x$	$f(x)$
0.5	-1	1.5	5.4	5	3.125
0.9	-12.79	1.1	17.29	10	$3.\overline{03}$
0.99	-147.8	1.01	152.3	100	$3.000\overline{03}$
0.999	-1498	1.001	1502	1000	3

- (b) Vertical asymptotes:  $x = \pm 1$   
 Horizontal asymptote:  $y = 3$
- (c) Domain: all real numbers  $x$  except  $x = \pm 1$
5. Domain: all real numbers  $x$  except  $x = 0$   
 Vertical asymptote:  $x = 0$   
 Horizontal asymptote:  $y = 0$
7. Domain: all real numbers  $x$  except  $x = 2$   
 Vertical asymptote:  $x = 2$   
 Horizontal asymptote:  $y = -1$
9. Domain: all real numbers  $x$  except  $x = \pm 1$   
 Vertical asymptotes:  $x = \pm 1$
11. Domain: all real numbers  $x$   
 Horizontal asymptote:  $y = 3$
13. d    14. a    15. c    16. b
17. 1    19. 6
21. Domain: all real numbers  $x$  except  $x = \pm 4$ ;  
 Vertical asymptote:  $x = -4$ ; horizontal asymptote:  $y = 0$
23. Domain: all real numbers  $x$  except  $x = -1, 3$ ;  
 Vertical asymptote:  $x = 3$ ; horizontal asymptote:  $y = 1$
25. Domain: all real numbers  $x$  except  $x = -1, \frac{1}{2}$ ;  
 Vertical asymptote:  $x = \frac{1}{2}$ ; horizontal asymptote:  $y = \frac{1}{2}$
27. (a) Domain: all real numbers  $x$  except  $x = -2$   
 (b) y-intercept:  $(0, \frac{1}{2})$   
 (c) Vertical asymptote:  $x = -2$   
 Horizontal asymptote:  $y = 0$

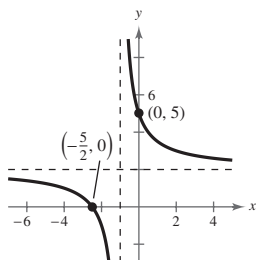
(d)

29. (a) Domain: all real numbers  $x$  except  $x = -2$ (b)  $y$ -intercept:  $(0, -\frac{1}{2})$ (c) Vertical asymptote:  $x = -2$ Horizontal asymptote:  $y = 0$ 

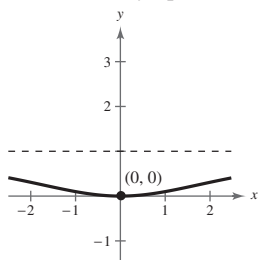
(d)

31. (a) Domain: all real numbers  $x$  except  $x = -1$ (b)  $x$ -intercept:  $(-\frac{5}{2}, 0)$  $y$ -intercept:  $(0, 5)$ (c) Vertical asymptote:  $x = -1$ Horizontal asymptote:  $y = 2$ 

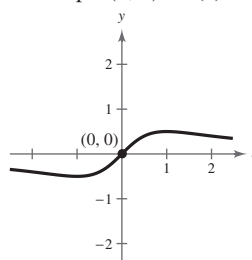
(d)

33. (a) Domain: all real numbers  $x$ (b) Intercept:  $(0, 0)$ (c) Horizontal asymptote:  $y = 1$ 

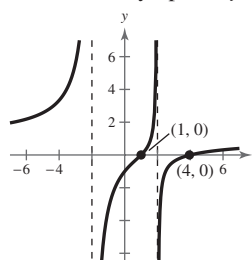
(d)

35. (a) Domain: all real numbers  $s$ (b) Intercept:  $(0, 0)$  (c) Horizontal asymptote:  $y = 0$ 

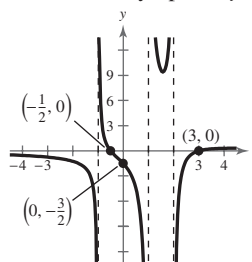
(d)

37. (a) Domain: all real numbers  $x$  except  $x = \pm 2$ (b)  $x$ -intercepts:  $(1, 0)$  and  $(4, 0)$  $y$ -intercept:  $(0, -1)$ (c) Vertical asymptotes:  $x = \pm 2$ Horizontal asymptote:  $y = 1$ 

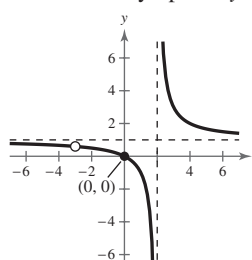
(d)

39. (a) Domain: all real numbers  $x$  except  $x = \pm 1, 2$ (b)  $x$ -intercept:  $(3, 0)$ ,  $(-\frac{1}{2}, 0)$  $y$ -intercept:  $(0, -\frac{3}{2})$ (c) Vertical asymptotes:  $x = 2$ ,  $x = \pm 1$ Horizontal asymptote:  $y = 0$ 

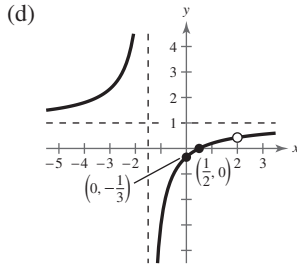
(d)

41. (a) Domain: all real numbers  $x$  except  $x = 2, -3$ (b) Intercept:  $(0, 0)$ (c) Vertical asymptote:  $x = 2$ Horizontal asymptote:  $y = 1$ 

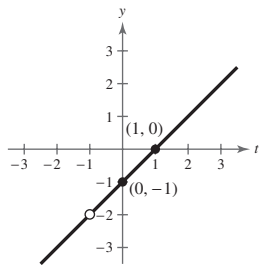
(d)



43. (a) Domain: all real numbers  $x$  except  $x = -\frac{3}{2}, 2$   
 (b)  $x$ -intercept:  $(\frac{1}{2}, 0)$   
 $y$ -intercept:  $(0, \frac{1}{3})$   
 (c) Vertical asymptote:  $x = -\frac{3}{2}$   
 Horizontal asymptote:  $y = 1$

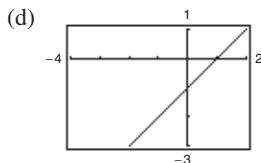


45. (a) Domain: all real numbers  $t$  except  $t = -1$   
 (b)  $t$ -intercept:  $(1, 0)$   
 $y$ -intercept:  $(0, -1)$   
 (c) Vertical asymptote: None  
 Horizontal asymptote: None



47. (a) Domain of  $f$ : all real numbers  $x$  except  $x = -1$   
 Domain of  $g$ : all real numbers  $x$   
 (b)  $x - 1$ ; Vertical asymptotes: none  
 (c)

$x$	-3	-2	-1.5	-1	-0.5	0	1
$f(x)$	-4	-3	-2.5	Undef.	-1.5	-1	0
$g(x)$	-4	-3	-2.5	-2	-1.5	-1	0



- (e) Because there are only a finite number of pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

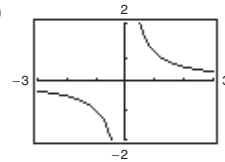
49. (a) Domain of  $f$ : all real numbers  $x$  except  $x = 0, 2$   
 Domain of  $g$ : all real numbers  $x$  except  $x = 0$

- (b)  $\frac{1}{x}$ ; Vertical asymptote:  $x = 0$

(c)

$x$	-0.5	0	0.5	1	1.5	2	3
$f(x)$	-2	Undef.	2	1	$\frac{2}{3}$	Undef.	$\frac{1}{3}$
$g(x)$	-2	Undef.	2	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$

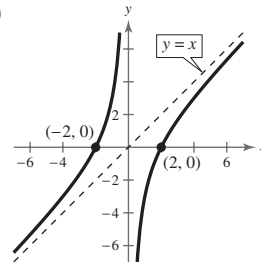
(d)



- (e) Because there are only a finite number of pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

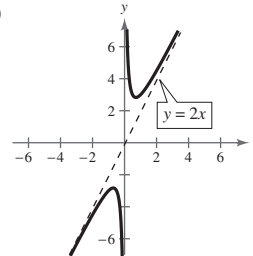
51. (a) Domain: all real numbers  $x$  except  $x = 0$   
 (b)  $x$ -intercepts:  $(2, 0), (-2, 0)$   
 (c) Vertical asymptote:  $x = 0$   
 Slant asymptote:  $y = x$

(d)



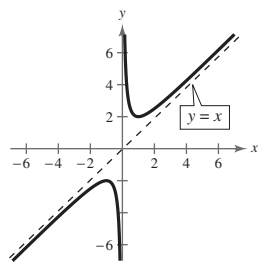
53. (a) Domain: all real numbers  $x$  except  $x = 0$   
 (b) No intercepts  
 (c) Vertical asymptote:  $x = 0$   
 Slant asymptote:  $y = 2x$

(d)

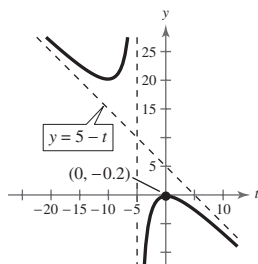


55. (a) Domain: all real numbers  $x$  except  $x = 0$   
 (b) No intercepts  
 (c) Vertical asymptote:  $x = 0$   
 Slant asymptote:  $y = x$

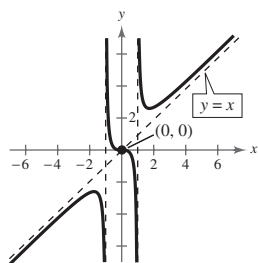
(d)

57. (a) Domain: all real numbers  $t$  except  $t = -5$ (b)  $y$ -intercept:  $(0, -0.2)$ (c) Vertical asymptote:  $t = -5$ Slant asymptote:  $y = -t + 5$ 

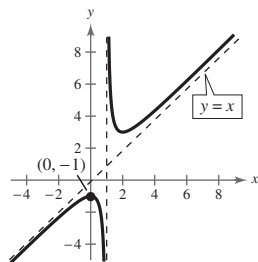
(d)

59. (a) Domain: all real numbers  $x$  except  $x = \pm 1$ (b) Intercept:  $(0, 0)$ (c) Vertical asymptotes:  $x = \pm 1$ Slant asymptote:  $y = x$ 

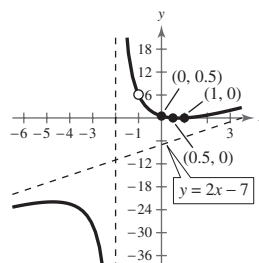
(d)

61. (a) Domain: all real numbers  $x$  except  $x = 1$ (b)  $y$ -intercept:  $(0, -1)$ (c) Vertical asymptote:  $x = 1$ Slant asymptote:  $y = x$ 

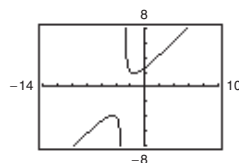
(d)

63. (a) Domain: all real numbers  $x$  except  $x = -1, -2$ (b)  $y$ -intercept:  $(0, 0.5)$  $x$ -intercepts:  $(0.5, 0), (1, 0)$ (c) Vertical asymptote:  $x = -2$ Slant asymptote:  $y = 2x - 7$ 

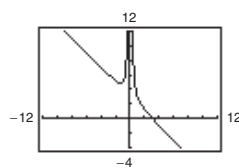
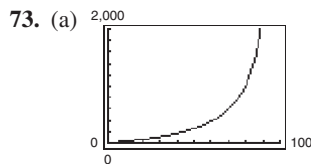
(d)



65.

Domain: all real numbers  $x$  except  $x = -3$ Vertical asymptote:  $x = -3$ Slant asymptote:  $y = x + 2$  $y = x + 2$ 

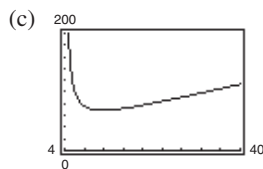
67.

Domain: all real numbers  $x$  except  $x = 0$ Vertical asymptote:  $x = 0$ Slant asymptote:  $y = -x + 3$  $y = -x + 3$ 69. (a)  $(-1, 0)$  (b)  $-1$ 71. (a)  $(1, 0), (-1, 0)$  (b)  $\pm 1$ 

(b) \$28.33 million; \$170 million; \$765 million

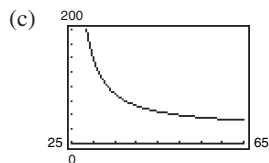
(c) No. The function is undefined at  $p = 100$ .

75. (a) 333 deer, 500 deer, 800 deer (b) 1500 deer

77. (a) Answers will vary. (b)  $(4, \infty)$ 11.75 inches  $\times$  5.87 inches

79. (a) Answers will vary.

 (b) Vertical asymptote:  $x = 25$ 

 Horizontal asymptote:  $y = 25$ 


(d)

$x$	30	35	40	45	50	55	60
$y$	150	87.5	66.7	56.3	50	45.8	42.9

(e) Yes. You would expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

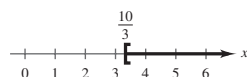
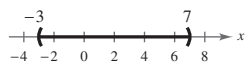
(f) No. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

81. False. Polynomials do not have vertical asymptotes.

 83. Answers will vary. Sample answer:  $f(x) = \frac{2x^2}{x^2 + 1}$ 

 85.  $(x - 7)(x - 8)$ 

 87.  $(x - 5)(x + 2i)(x - 2i)$ 

 89.  $x \geq \frac{10}{3}$ 

 91.  $-3 < x < 7$ 


93. Answers will vary.

## Section 2.7 (page 204)

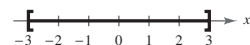
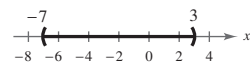
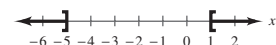
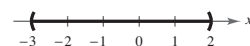
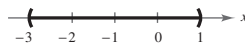
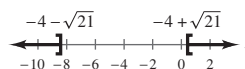
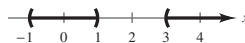
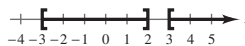
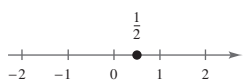
### Vocabulary Check (page 204)

1. critical; test intervals      2. zeros; undefined values  
3.  $P = R - C$

1. (a) No      (b) Yes      (c) Yes      (d) No

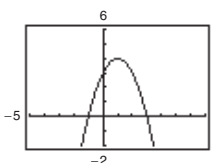
3. (a) Yes      (b) No      (c) No      (d) Yes

 5.  $2, -\frac{3}{2}$       7.  $\frac{7}{2}, 5$ 

 9.  $[-3, 3]$ 

 11.  $(-7, 3)$ 

 13.  $(-\infty, -5] \cup [1, \infty)$ 

 15.  $(-3, 2)$ 

 17.  $(-3, 1)$ 

 19.  $(-\infty, -4 - \sqrt{21}] \cup [-4 + \sqrt{21}, \infty)$ 

 21.  $(-1, 1) \cup (3, \infty)$ 

 23.  $[-3, 2] \cup [3, \infty)$ 

 25.  $x = \frac{1}{2}$ 

 27.  $(-\infty, 0) \cup (0, \frac{3}{2})$ 

 29.  $[-2, 0] \cup [2, \infty)$ 

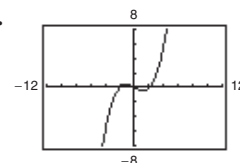
33.


 (a)  $x \leq -1, x \geq 3$ 

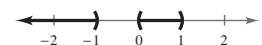
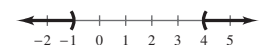
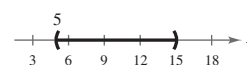
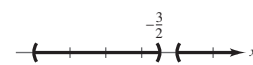
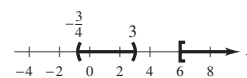
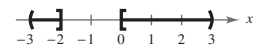
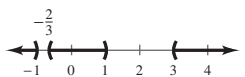
 (b)  $0 \leq x \leq 2$ 

 31.  $[-2, \infty)$ 

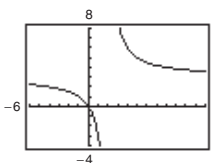
35.


 (a)  $-2 \leq x \leq 0,$ 
 $2 \leq x < \infty$ 

 (b)  $x \leq 4$ 

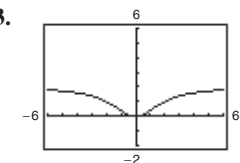
 37.  $(-\infty, -1) \cup (0, 1)$ 

 39.  $(-\infty, -1) \cup (4, \infty)$ 

 41.  $(5, 15)$ 

 43.  $(-5, -\frac{3}{2}) \cup (-1, \infty)$ 

 45.  $(-\frac{3}{4}, 3) \cup [6, \infty)$ 

 47.  $(-3, -2] \cup [0, 3)$ 

 49.  $(-\infty, -1) \cup (-\frac{2}{3}, 1) \cup (3, \infty)$ 


51.


 (a)  $0 \leq x < 2$ 

 (b)  $2 < x \leq 4$ 

53.


 (a)  $|x| \geq 2$ 

 (b)  $-\infty < x < \infty$

55.  $[-2, 2]$  57.  $(-\infty, 3] \cup [4, \infty)$

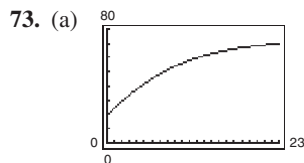
59.  $(-5, 0] \cup (7, \infty)$  61.  $(-3.51, 3.51)$

63.  $(-0.13, 25.13)$  65.  $(2.26, 2.39)$

67. (a)  $t = 10$  seconds (b)  $4 \text{ seconds} < t < 6 \text{ seconds}$

69.  $13.8 \text{ meters} \leq L \leq 36.2 \text{ meters}$

71.  $40,000 \leq x \leq 50,000$ ;  $50.00 \leq p \leq 55.00$



(b)	$t$	24	26	28	30	32	34
	$C$	70.5	71.6	72.9	74.6	76.8	79.6

2011

(c)  $t \approx 31$

(d)	$t$	36	37	38	39
	$C$	83.2	85.4	87.8	90.5

$t$	40	41	42	43
$C$	93.5	96.8	100.4	104.4

2016 to 2021

(e)  $37 \leq t \leq 41$  (f) Answers will vary.

75.  $R_1 \geq 2$  ohms

77. True. The test intervals are  $(-\infty, -3)$ ,  $(-3, 1)$ ,  $(1, 4)$ , and  $(4, \infty)$ .

79.  $(-\infty, -4] \cup [4, \infty)$  81.  $(-\infty, -2\sqrt{30}] \cup [2\sqrt{30}, \infty)$

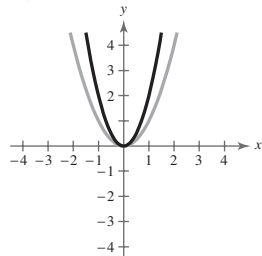
83. (a) If  $a > 0$  and  $c \leq 0$ ,  $b$  can be any real number. If  $a > 0$  and  $c > 0$ ,  $b < -2\sqrt{ac}$  or  $b > 2\sqrt{ac}$ .

(b) 0

85.  $(2x + 5)^2$  87.  $(x + 3)(x + 2)(x - 2)$  89.  $2x^2 + x$

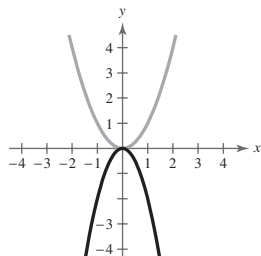
**Review Exercises** (page 208)

1. (a)

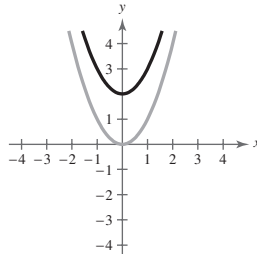


Vertical stretch

(b)

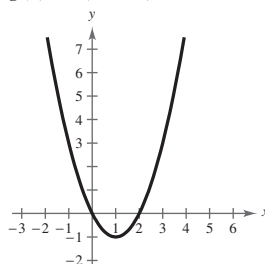
Vertical stretch and reflection in the  $x$ -axis

(c)

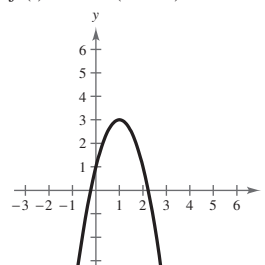


Vertical shift

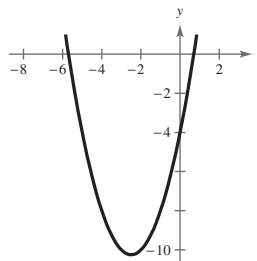
3.  $g(x) = (x - 1)^2 - 1$

Vertex:  $(1, -1)$ Axis of symmetry:  $x = 1$  $x$ -intercepts:  $(0, 0)$ ,  $(2, 0)$ 

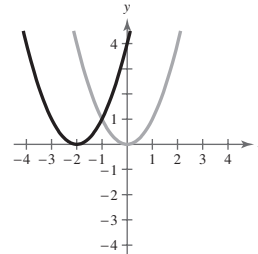
7.  $f(t) = -2(t - 1)^2 + 3$

Vertex:  $(1, 3)$ Axis of symmetry:  $t = 1$  $t$ -intercepts:  $\left(1 \pm \frac{\sqrt{6}}{2}, 0\right)$ 

11.  $h(x) = \left(x + \frac{5}{2}\right)^2 - \frac{41}{4}$

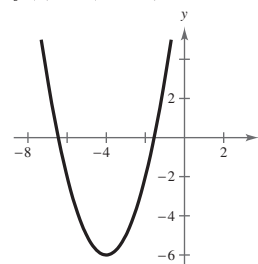
Vertex:  $\left(-\frac{5}{2}, -\frac{41}{4}\right)$ Axis of symmetry:  $x = -\frac{5}{2}$  $x$ -intercepts:  $\left(\frac{\pm\sqrt{41}-5}{2}, 0\right)$ 

(d)

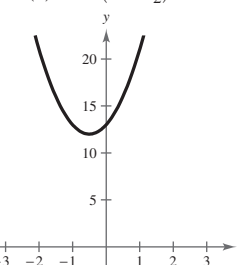


Horizontal shift

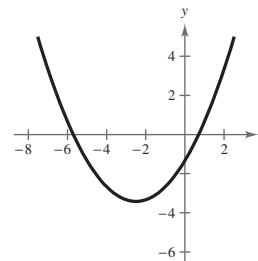
5.  $f(x) = (x + 4)^2 - 6$

Vertex:  $(-4, -6)$ Axis of symmetry:  $x = -4$  $x$ -intercepts:  $(-4 \pm \sqrt{6}, 0)$ 

9.  $h(x) = 4\left(x + \frac{1}{2}\right)^2 + 12$

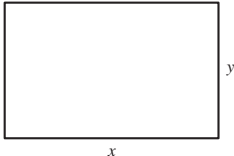
Vertex:  $\left(-\frac{1}{2}, 12\right)$ Axis of symmetry:  $x = -\frac{1}{2}$ No  $x$ -intercept

13.  $f(x) = \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12}$

Vertex:  $\left(-\frac{5}{2}, -\frac{41}{12}\right)$ Axis of symmetry:  $x = -\frac{5}{2}$  $x$ -intercepts:  $\left(\frac{\pm\sqrt{41}-5}{2}, 0\right)$

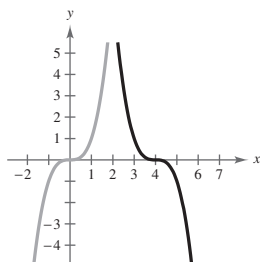


15.  $f(x) = -\frac{1}{2}(x-4)^2 + 1$  17.  $f(x) = (x-1)^2 - 4$

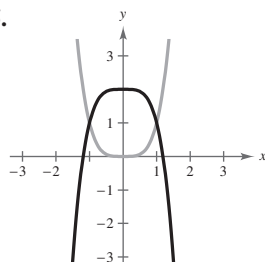
19. (a)  (b)  $y = 100 - x$   
 $A = 100x - x^2$   
 (c)  $x = 50, y = 50$

21. 1091 units

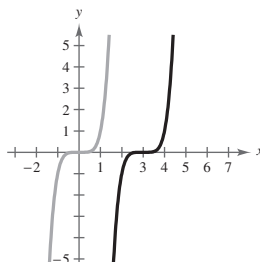
23.



25.



27.



29. Falls to the left, falls to the right

31. Rises to the left, rises to the right

33.  $-7, \frac{3}{2}$ , odd multiplicity; turning point: 1

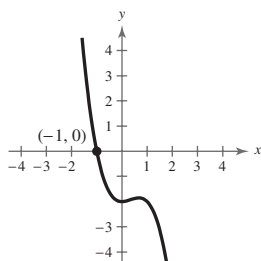
35.  $0, \pm\sqrt{3}$ , odd multiplicity; turning points: 2

37. 0, even multiplicity;  $\frac{5}{3}$ , odd multiplicity; turning points: 2

39. (a) Rises to the left, falls to the right (b) -1

(c) Answers will vary.

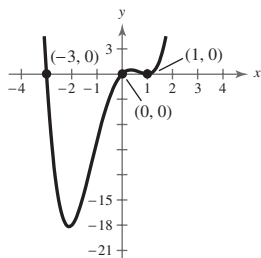
(d)



41. (a) Rises to the right, rises to the left (b) -3, 0, 1

(c) Answers will vary.

(d)



43. (a)  $[-1, 0]$  (b)  $\approx -0.900$

45. (a)  $[-1, 0], [1, 2]$  (b)  $\approx -0.200, \approx 1.772$

47.  $8x + 5 + \frac{2}{3x-2}$  49.  $5x + 2$

51.  $x^2 - 3x + 2 - \frac{1}{x^2 + 2}$

53.  $6x^3 + 8x^2 - 11x - 4 - \frac{8}{x-2}$

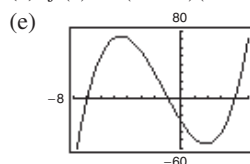
55.  $2x^2 - 11x - 6$

57. (a) Yes (b) Yes (c) Yes (d) No

59. (a) -421 (b) -9

61. (a) Answers will vary. (b)  $(x+7), (x+1)$

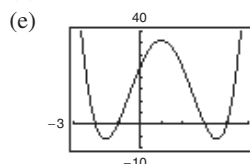
(c)  $f(x) = (x+7)(x+1)(x-4)$  (d) -7, -1, 4



63. (a) Answers will vary. (b)  $(x+1), (x-4)$

(c)  $f(x) = (x+1)(x-4)(x+2)(x-3)$

(d) -2, -1, 3, 4



65.  $6 + 2i$  67.  $-1 + 3i$  69.  $3 + 7i$

71.  $40 + 65i$  73.  $-4 - 46i$  75.  $\frac{23}{17} + \frac{10}{17}i$

77.  $\frac{21}{13} - \frac{1}{13}i$  79.  $\pm\frac{\sqrt{3}}{3}i$  81.  $1 \pm 3i$

83. 0, 2 85. 8, 1 87.  $-4, 6, \pm 2i$

89.  $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{5}{4}, \pm \frac{15}{4}$

91. -1, -3, 6 93. 1, 8 95. -4, 3

97.  $3x^4 - 14x^3 + 17x^2 - 42x + 24$

99.  $4, \pm i$  101.  $-3, \frac{1}{2}, 2 \pm i$

103. 0, 1, -5;  $f(x) = x(x-1)(x+5)$

105.  $-4, 2 \pm 3i$ ;  $g(x) = (x+4)^2(x-2-3i)(x-2+3i)$

107. Two or no positive zeros, one negative zero

109. Answers will vary.

111. Domain: all real numbers  $x$  except  $x = -12$

113. Domain: all real numbers  $x$  except  $x = 6, 4$

115. Vertical asymptote:  $x = -3$

Horizontal asymptote:  $y = 0$

117. Vertical asymptote:  $x = -3$

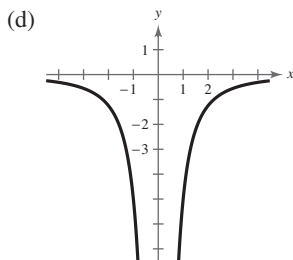
Horizontal asymptote:  $y = 0$

119. (a) Domain: all real numbers  $x$  except  $x = 0$

(b) No intercepts

(c) Vertical asymptote:  $x = 0$

Horizontal asymptote:  $y = 0$



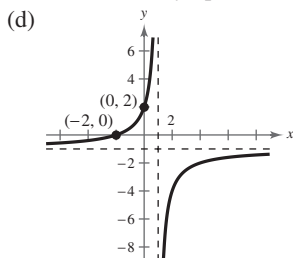
121. (a) Domain: all real numbers  $x$  except  $x = 1$

(b)  $x$ -intercept:  $(-2, 0)$

$y$ -intercept:  $(0, 2)$

(c) Vertical asymptote:  $x = 1$

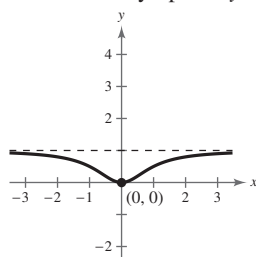
Horizontal asymptote:  $y = -1$



123. (a) Domain: all real numbers  $x$  (b) Intercept:  $(0, 0)$

(c) Horizontal asymptote:  $y = 1$

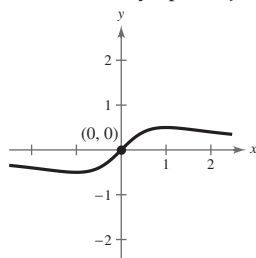
(d)



125. (a) Domain: all real numbers  $x$  (b) Intercept:  $(0, 0)$

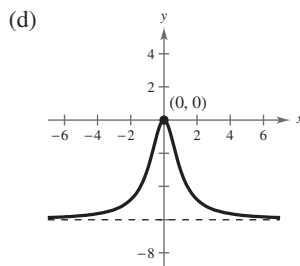
(c) Horizontal asymptote:  $y = 0$

(d)



127. (a) Domain: all real numbers  $x$  (b) Intercept:  $(0, 0)$

(c) Horizontal asymptote:  $y = -6$



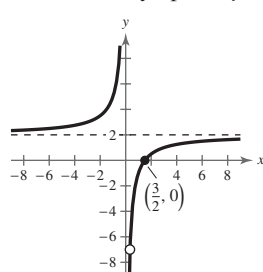
129. (a) Domain: all real numbers  $x$  except  $x = 0, \frac{1}{3}$

(b)  $x$ -intercept:  $(1.5, 0)$

(c) Vertical asymptote:  $x = 0$

Horizontal asymptote:  $y = 2$

(d)

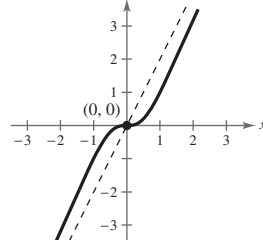


131. (a) Domain: all real numbers  $x$

(b) Intercept:  $(0, 0)$

(c) Slant asymptote:  $y = 2x$

(d)



133. (a) Domain: all real numbers  $x$  except  $x = \frac{4}{3}$

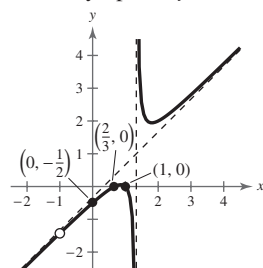
(b)  $y$ -intercept:  $(0, -0.5)$

$x$ -intercepts:  $(\frac{2}{3}, 0), (1, 0)$

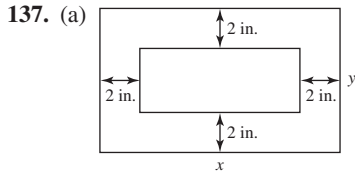
(c) Vertical asymptote:  $x = \frac{4}{3}$

Slant asymptote:  $y = x - \frac{1}{3}$

(d)



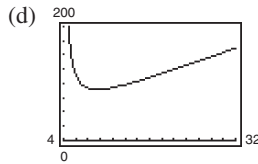
135. \$0.50 is the horizontal asymptote of the function.



(b)  $(x - 4)(y - 4) = 30$   
 $y = \frac{4x + 14}{x - 4}$

Area =  $x \left( \frac{4x + 14}{x - 4} \right)$   
 $= \frac{2x(2x + 7)}{x - 4}$

(c)  $4 < x < \infty$



9.48 inches  $\times$  9.48 inches

139.  $(-\frac{4}{3}, \frac{1}{2})$  141.  $[-4, 0] \cup [4, \infty)$

143.  $[-5, -1) \cup (1, \infty)$  145.  $[-4, -3] \cup (0, \infty)$

147. 4.9%

149. False. A fourth-degree polynomial can have at most four zeros, and complex zeros occur in conjugate pairs.

151. Find the vertex of the quadratic function and write the function in standard form. If the leading coefficient is positive, the vertex is a minimum. If the leading coefficient is negative, the vertex is a maximum.

153. An asymptote of a graph is a line to which the graph becomes arbitrarily close as  $x$  increases or decreases without bound.

### Chapter Test (page 212)

1. (a) Reflection in the  $x$ -axis followed by a vertical translation

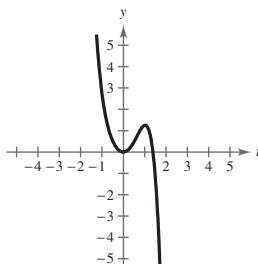
(b) Horizontal translation

2.  $y = (x - 3)^2 - 6$

3. (a) 50 feet

(b) 5. Yes, changing the constant term results in a vertical translation of the graph and therefore changes the maximum height.

4. Rises to the left, falls to the right



5.  $3x + \frac{x-1}{x^2+1}$  6.  $2x^3 + 4x^2 + 3x + 6 + \frac{9}{x-2}$

7.  $(4x - 1)(x - \sqrt{3})(x + \sqrt{3})$ ;

Solutions:  $\frac{1}{4}, \pm\sqrt{3}$

8. (a)  $-3 + 5i$  (b) 7 9.  $2 - i$

10.  $f(x) = x^4 - 9x^3 + 28x^2 - 30x$

11.  $f(x) = x^4 - 6x^3 + 16x^2 - 24x + 16$

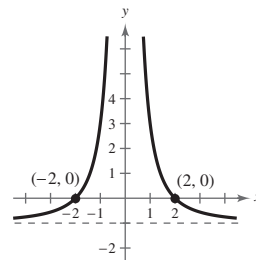
12.  $-2, \pm\sqrt{5}i$  13.  $-2, 4, -1 \pm \sqrt{2}i$

14.  $x$ -intercepts:  $(-2, 0), (2, 0)$

No  $y$ -intercept

Vertical asymptote:  $x = 0$

Horizontal asymptote:  $y = -1$

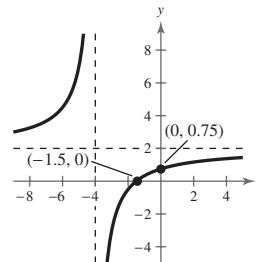


15.  $x$ -intercept:  $(-1.5, 0)$

$y$ -intercept:  $(0, 0.75)$

Vertical asymptote:  $x = -4$

Horizontal asymptote:  $y = 2$

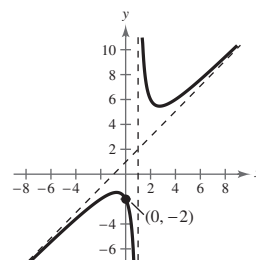


16. No  $x$ -intercept

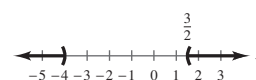
$y$ -intercept:  $(0, -2)$

Vertical asymptote:  $x = 1$

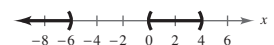
Slant asymptote:  $y = x + 1$



17.  $x < -4$  or  $x > \frac{3}{2}$



18.  $x < -6$  or  $0 < x < 4$



**Problem Solving** (page 215)

1. Answers will vary.
3. 2 inches  $\times$  2 inches  $\times$  5 inches
5. (a) and (b)  $y = -x^2 + 5x - 4$
7. (a)  $f(x) = (x - 2)x^2 + 5 = x^3 - 2x^2 + 5$   
 (b)  $f(x) = -(x + 3)x^2 + 1 = -x^3 - 3x^2 + 1$
9.  $(a + bi)(a - bi) = a^2 + abi - abi - b^2i^2$   
 $= a^2 + b^2$
11. (a) As  $|a|$  increases, the graph stretches vertically. For  $a < 0$ , the graph is reflected in the  $x$ -axis.  
 (b) As  $|b|$  increases, the vertical asymptote is translated. For  $b > 0$ , the graph is translated to the right. For  $b < 0$ , the graph is reflected in the  $x$ -axis and is translated to the left.

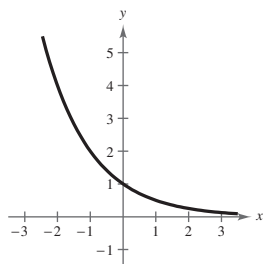
**Chapter 3****Section 3.1** (page 226)**Vocabulary Check** (page 226)

1. algebraic      2. transcendental
3. natural exponential; natural      4.  $A = P\left(1 + \frac{r}{n}\right)^{nt}$
5.  $A = Pe^{rt}$

1. 946.852      3. 0.006      5. 1767.767
7. d      8. c      9. a      10. b

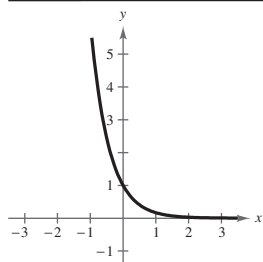
11.

$x$	-2	-1	0	1	2
$f(x)$	4	2	1	0.5	0.25



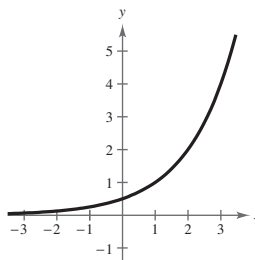
13.

$x$	-2	-1	0	1	2
$f(x)$	36	6	1	0.167	0.028

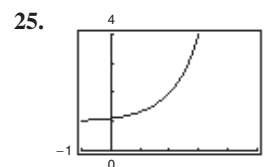
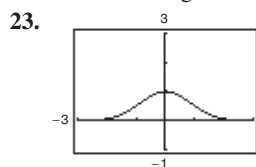


15.

$x$	-2	-1	0	1	2
$f(x)$	0.125	0.25	0.5	1	2



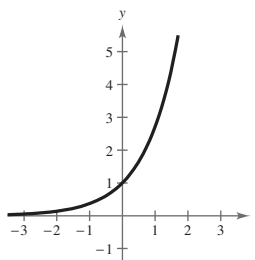
17. Shift the graph of  $f$  four units to the right.
19. Shift the graph of  $f$  five units upward.
21. Reflect the graph of  $f$  in the  $x$ -axis and  $y$ -axis and shift six units to the right.



27. 0.472      29.  $3.857 \times 10^{-22}$       31. 7166.647

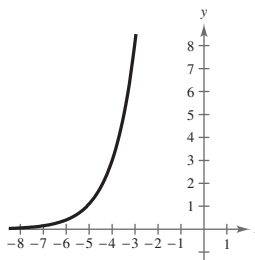
33.

$x$	-2	-1	0	1	2
$f(x)$	0.135	0.368	1	2.718	7.389



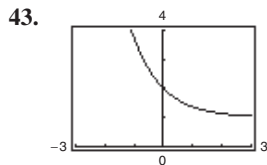
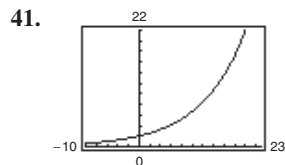
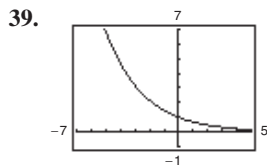
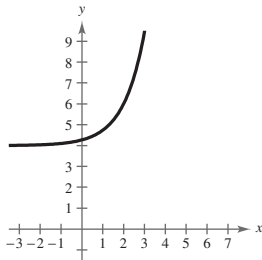
35.

$x$	-8	-7	-6	-5	-4
$f(x)$	0.055	0.149	0.406	1.104	3



37.

$x$	-2	-1	0	1	2
$f(x)$	4.037	4.100	4.271	4.736	6



45.  $x = 2$     47.  $x = -3$     49.  $x = \frac{1}{3}$     51.  $x = 3, -1$

53.

$n$	1	2	4
$A$	\$3200.21	\$3205.09	\$3207.57

$n$	12	365	Continuous
$A$	\$3209.23	\$3210.06	\$3210.06

55.

$n$	1	2	4
$A$	\$4515.28	\$4535.05	\$4545.11

$n$	12	365	Continuous
$A$	\$4551.89	\$4555.18	\$4555.30

57.

$t$	10	20	30
$A$	\$17,901.90	\$26,706.49	\$39,841.40

$t$	40	50
$A$	\$59,436.39	\$88,668.67

59.

$t$	10	20	30
$A$	\$22,986.49	\$44,031.56	\$84,344.25

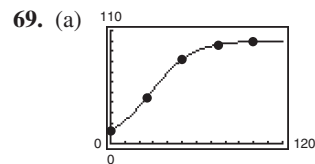
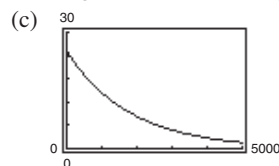
$t$	40	50
$A$	\$161,564.86	\$309,484.08

61. \$222,822.57    63. \$35.45

65. (a)  $V(1) = 10,000.298$     (b)  $V(1.5) = 100,004.47$

(c)  $V(2) = 1,000,059.6$

67. (a) 25 grams    (b) 16.21 grams



(b)

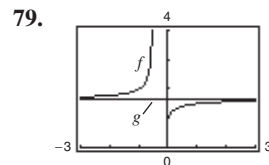
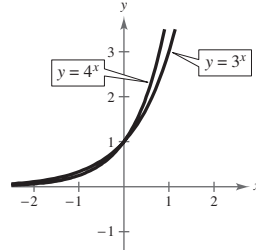
$x$	0	25	50	75	100
Model	12.5	44.5	81.82	96.19	99.3
Actual	12	44	81	96	99

(c) 63.14%    (d) 38 masses

71. True. As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -2$  but never reaches  $-2$ .

73.  $f(x) = h(x)$     75.  $f(x) = g(x) = h(x)$

77. (a)  $x < 0$     (b)  $x > 0$

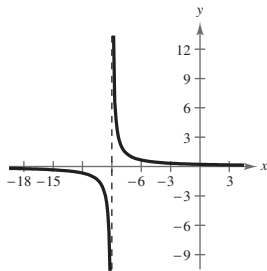


As  $x \rightarrow \infty$ ,  $f(x) \rightarrow g(x)$ .

As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow g(x)$ .

81.  $y = \pm \sqrt{25 - x^2}$

83.



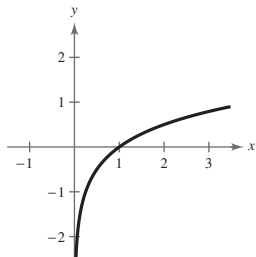
85. Answers will vary.

**Section 3.2** (page 236)**Vocabulary Check** (page 236)

1. logarithmic    2. 10    3. natural;  $e$   
 4.  $a^{\log_a x} = x$     5.  $x = y$

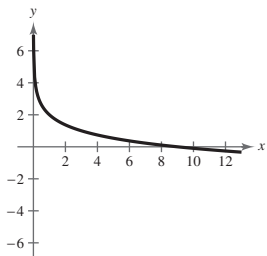
1.  $4^3 = 64$     3.  $7^{-2} = \frac{1}{49}$     5.  $32^{2/5} = 4$   
 7.  $36^{1/2} = 6$     9.  $\log_5 125 = 3$     11.  $\log_{81} 3 = \frac{1}{4}$   
 13.  $\log_6 \frac{1}{36} = -2$     15.  $\log_7 1 = 0$     17. 4    19. 0  
 21. 2    23. -0.0972    25. 1.097    27. 4    29. 1

31.



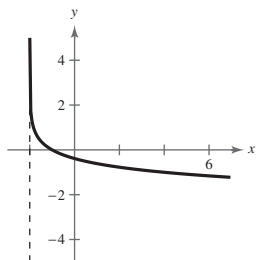
Domain:  $(0, \infty)$   
 $x$ -intercept:  $(1, 0)$   
 Vertical asymptote:  $x = 0$

33.



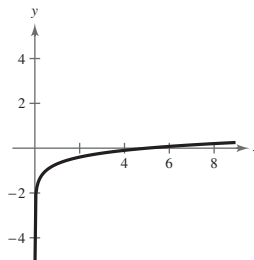
Domain:  $(0, \infty)$   
 $x$ -intercept:  $(9, 0)$   
 Vertical asymptote:  $x = 0$

35.



Domain:  $(-2, \infty)$   
 $x$ -intercept:  $(-1, 0)$   
 Vertical asymptote:  $x = -2$

37.

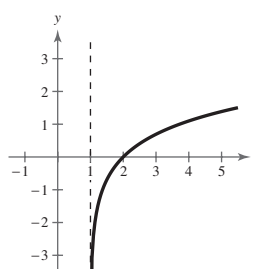


Domain:  $(0, \infty)$   
 $x$ -intercept:  $(5, 0)$   
 Vertical asymptote:  $x = 0$

39. c    40. f    41. d    42. e    43. b    44. a

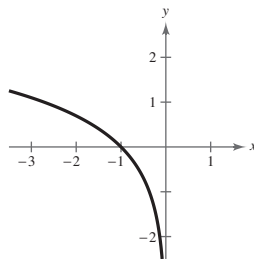
45.  $e^{-0.693 \dots} = \frac{1}{2}$     47.  $e^{1.386 \dots} = 4$ 49.  $e^{5.521 \dots} = 250$     51.  $e^0 = 1$ 53.  $\ln 20.0855 \dots = 3$     55.  $\ln 1.6487 \dots = \frac{1}{2}$ 57.  $\ln 0.6065 \dots = -0.5$     59.  $\ln 4 = x$     61. 2.91363. -0.575    65. 3    67.  $-\frac{2}{3}$ 

69.



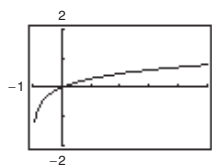
Domain:  $(1, \infty)$   
 $x$ -intercept:  $(2, 0)$   
 Vertical asymptote:  $x = 1$

71.

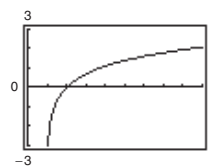


Domain:  $(-\infty, 0)$   
 $x$ -intercept:  $(-1, 0)$   
 Vertical asymptote:  $x = 0$

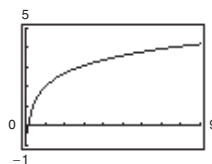
73.



75.



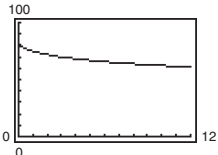
77.

79.  $x = 3$     81.  $x = 7$     83.  $x = 4$     85.  $x = -5, 5$ 

87. (a) 30 years; 20 years    (b) \$396,234; \$301,123.20

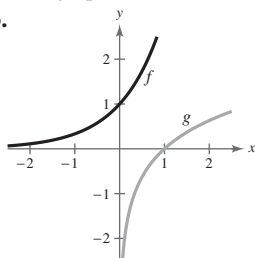
(c) \$246,234; \$151,123.20

(d)  $x = 1000$ ; The monthly payment must be greater than \$1000.

89. (a)  (b) 80  
(c) 68.1  
(d) 62.3

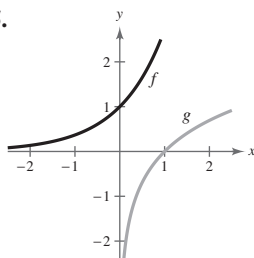
91. False. Reflecting  $g(x)$  about the line  $y = x$  will determine the graph of  $f(x)$ .

93.



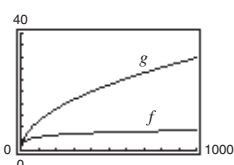
The functions  $f$  and  $g$  are inverses.

95.



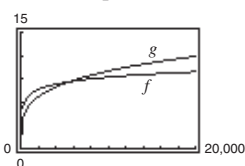
The functions  $f$  and  $g$  are inverses.

97. (a)



$g(x)$ ; The natural log function grows at a slower rate than the square root function.

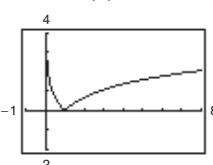
- (b)



$g(x)$ ; The natural log function grows at a slower rate than the fourth root function.

99. (a) False (b) True (c) True (d) False

101. (a)



- (c) Relative minimum:  $(1, 0)$

103. 15 105. 4300 107. 1028

### Section 3.3 (page 243)

#### Vocabulary Check (page 243)

1. change-of-base 2.  $\frac{\log x}{\log a} = \frac{\ln x}{\ln a}$   
3. c 4. a 5. b

1. (a)  $\frac{\log x}{\log 5}$  (b)  $\frac{\ln x}{\ln 5}$  3. (a)  $\frac{\log x}{\log \frac{1}{5}}$  (b)  $\frac{\ln x}{\ln \frac{1}{5}}$

5. (a)  $\frac{\log \frac{3}{10}}{\log x}$  (b)  $\frac{\ln \frac{3}{10}}{\ln x}$  7. (a)  $\frac{\log x}{\log 2.6}$  (b)  $\frac{\ln x}{\ln 2.6}$   
9. 1.771 11. -2.000 13. -0.417 15. 2.633  
17.  $\frac{3}{2}$  19.  $-3 - \log_5 2$  21.  $6 + \ln 5$  23. 2  
25.  $\frac{3}{4}$  27. 2.4 29. -9 is not in the domain of  $\log_3 x$ .  
31. 4.5 33.  $-\frac{1}{2}$  35. 7 37. 2  
39.  $\log_4 5 + \log_4 x$  41.  $4 \log_8 x$  43.  $1 - \log_5 x$   
45.  $\frac{1}{2} \ln z$  47.  $\ln x + \ln y + 2 \ln z$   
49.  $\ln z + 2 \ln(z-1)$  51.  $\frac{1}{2} \log_2(a-1) - 2 \log_2 3$   
53.  $\frac{1}{3} \ln x - \frac{1}{3} \ln y$  55.  $4 \ln x + \frac{1}{2} \ln y - 5 \ln z$   
57.  $2 \log_5 x - 2 \log_5 y - 3 \log_5 z$   
59.  $\frac{3}{4} \ln x + \frac{1}{4} \ln(x^2 + 3)$  61.  $\ln 3x$  63.  $\log_4 \frac{z}{y}$

65.  $\log_2(x+4)^2$  67.  $\log_3 \sqrt[4]{5x}$  69.  $\ln \frac{x}{(x+1)^3}$

71.  $\log \frac{xz^3}{y^2}$  73.  $\ln \frac{x}{(x^2-4)^4}$  75.  $\ln \sqrt[3]{\frac{x(x+3)^2}{x^2-1}}$

77.  $\log_8 \frac{\sqrt[3]{y(y+4)^2}}{y-1}$

79.  $\log_2 \frac{32}{4} = \log_2 32 - \log_2 4$ ; Property 2

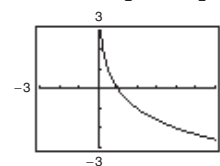
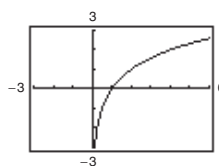
81.  $\beta = 10(\log I + 12)$ ; 60 dB 83.  $\approx 3$

85.  $y = 256.24 - 20.8 \ln x$

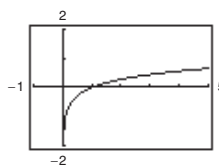
87. False.  $\ln 1 = 0$  89. False.  $\ln(x-2) \neq \ln x - \ln 2$

91. False.  $u = v^2$  93. Answers will vary.

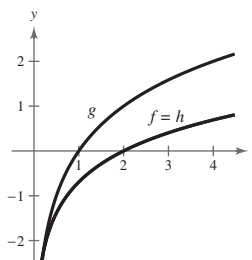
95.  $f(x) = \frac{\log x}{\log 2} = \frac{\ln x}{\ln 2}$  97.  $f(x) = \frac{\log x}{\log \frac{1}{2}} = \frac{\ln x}{\ln \frac{1}{2}}$



99.  $f(x) = \frac{\log x}{\log 11.8} = \frac{\ln x}{\ln 11.8}$



101.  $f(x) = h(x)$ ; Property 2



103.  $\frac{3x^4}{2y^3}, x \neq 0$     105.  $1, x \neq 0, y \neq 0$

107.  $-1, \frac{1}{3}$     109.  $\frac{-1 \pm \sqrt{97}}{6}$

### Section 3.4 (page 253)

#### Vocabulary Check (page 253)

- solve
- (a)  $x = y$     (b)  $x = y$     (c)  $x$     (d)  $x$
- extraneous

1. (a) Yes    (b) No

3. (a) No    (b) Yes    (c) Yes, approximate

5. (a) Yes, approximate    (b) No    (c) Yes

7. (a) No    (b) Yes    (c) Yes, approximate

9. 2    11. -5    13. 2    15.  $\ln 2 \approx 0.693$

17.  $e^{-1} \approx 0.368$     19. 64    21. (3, 8)    23. (9, 2)

25. 2, -1    27.  $\approx 1.618, \approx -0.618$

29.  $\frac{\ln 5}{\ln 3} \approx 1.465$     31.  $\ln 5 \approx 1.609$     33.  $\ln 28 \approx 3.332$

35.  $\frac{\ln 80}{2 \ln 3} \approx 1.994$     37. 2    39. 4

41.  $3 - \frac{\ln 565}{\ln 2} \approx -6.142$     43.  $\frac{1}{3} \log\left(\frac{3}{2}\right) \approx 0.059$

45.  $1 + \frac{\ln 7}{\ln 5} \approx 2.209$     47.  $\frac{\ln 12}{3} \approx 0.828$

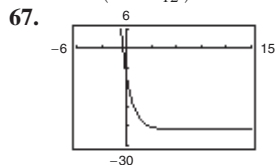
49.  $-\ln \frac{3}{5} \approx 0.511$     51. 0    53.  $\frac{\ln \frac{8}{3}}{3 \ln 2} + \frac{1}{3} \approx 0.805$

55.  $\ln 5 \approx 1.609$     57.  $\ln 4 \approx 1.386$

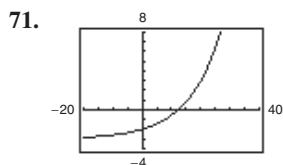
59.  $2 \ln 75 \approx 8.635$     61.  $\frac{1}{2} \ln 1498 \approx 3.656$

63.  $\frac{\ln 4}{365 \ln\left(1 + \frac{0.065}{365}\right)} \approx 21.330$

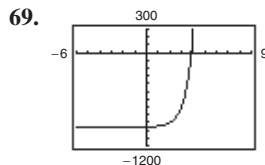
65.  $\frac{\ln 2}{12 \ln\left(1 + \frac{0.10}{12}\right)} \approx 6.960$



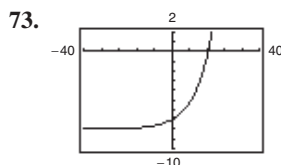
-0.427



12.207



3.847



16.636

75.  $e^{-3} \approx 0.050$     77.  $\frac{e^{2.4}}{2} \approx 5.512$     79. 1,000,000

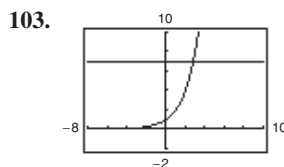
81.  $\frac{e^{10/3}}{5} \approx 5.606$     83.  $e^2 - 2 \approx 5.389$

85.  $e^{-2/3} \approx 0.513$     87.  $2(3^{11/6}) \approx 14.988$

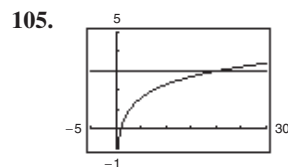
89. No solution    91.  $1 + \sqrt{1 + e} \approx 2.928$

93. No solution    95. 7    97.  $\frac{-1 + \sqrt{17}}{2} \approx 1.562$

99. 2    101.  $\frac{725 + 125\sqrt{33}}{8} \approx 180.384$



2.807



20.086

107. (a) 8.2 years    (b) 12.9 years

109. (a) 1426 units    (b) 1498 units

111. (a) (b)  $V = 6.7$ ; The yield will approach 6.7 million cubic feet per acre.  
(c) 29.3 years

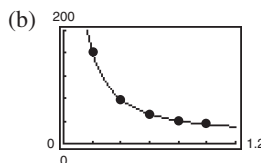
113. 2001

115. (a)  $y = 100$  and  $y = 0$ ; The range falls between 0% and 100%.

- (b) Males: 69.71 inches    Females: 64.51 inches

117. (a)

$x$	0.2	0.4	0.6	0.8	1.0
$y$	162.6	78.5	52.5	40.5	33.9



The model appears to fit the data well.

- (c) 1.2 meters

- (d) No. According to the model, when the number of  $g$ 's is less than 23,  $x$  is between 2.276 meters and 4.404 meters, which isn't realistic in most vehicles.

119.  $\log_b uv = \log_b u + \log_b v$

True by Property 1 in Section 5.3.

121.  $\log_b(u - v) = \log_b u - \log_b v$

False.

$1.95 \approx \log(100 - 10) \neq \log 100 - \log 10 = 1$

123. Yes. See Exercise 93.

125. Yes. Time to double:  $t = \frac{\ln 2}{r}$ ;

Time to quadruple:  $t = \frac{\ln 4}{r} = 2\left(\frac{\ln 2}{r}\right)$



127.  $4|x|y^2\sqrt{3y}$       129.  $5\sqrt[3]{3}$

131.       133. 

135. 1.226      137. -5.595

### Section 3.5 (page 264)

#### Vocabulary Check (page 264)

- $y = ae^{bx}$ ;  $y = ae^{-bx}$
- $y = a + b \ln x$ ;  $y = a + b \log x$
- normally distributed      4. bell; average value
- sigmoidal

	1. c	2. e	3. b	4. a	5. d	6. f
<i>Initial Investment</i>						
<i>Annual % Rate</i>						
<i>Time to Double</i>						
<i>Amount After 10 years</i>						

7. \$1000      3.5%      19.8 yr      \$1419.07  
 9. \$750      8.9438%      7.75 yr      \$1834.36  
 11. \$500      11.0%      6.3 yr      \$1505.00  
 13. \$6376.28      4.5%      15.4 yr      \$10,000.00

15. \$112,087.09

17. (a) 6.642 years      (b) 6.330 years  
 (c) 6.302 years      (d) 6.301 years

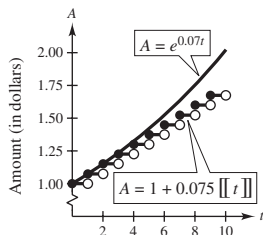
19. 

$r$	2%	4%	6%	8%	10%	12%
$t$	54.93	27.47	18.31	13.73	10.99	9.16

21. 

$r$	2%	4%	6%	8%	10%	12%
$t$	55.48	28.01	18.85	14.27	11.53	9.69

#### 23. Continuous compounding



	Half-life (years)	Initial Quantity	Amount After 1000 Years
25.	1599	10 g	6.48 g
27.	5715	2.26 g	2 g
29.	24,100	2.16 g	2.1 g

31.  $y = e^{0.7675x}$       33.  $y = 5e^{-0.4024x}$

35. (a) Decreasing due to the negative exponent.

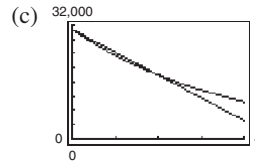
- (b) 2000: population of 2430 thousand  
 2003: population of 2408.95 thousand

(c) 2018

37.  $k = 0.2988$ ;  $\approx 5,309,734$  hits      39. 3.15 hours

41. (a)  $\approx 12,180$  years old      (b)  $\approx 4797$  years old

43. (a)  $V = -6394t + 30,788$       (b)  $V = 30,788e^{-0.268t}$



The exponential model depreciates faster.

(d) 

$t$	1	3
$V = -6394t + 30,788$	24,394	11,606
$V = 30,788e^{-0.268t}$	23,550	13,779

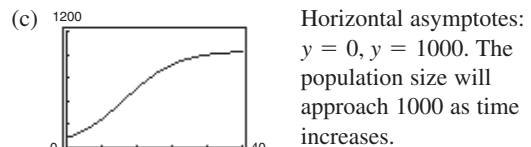
(e) Answers will vary.

45. (a)  $S(t) = 100(1 - e^{-0.1625t})$

(b)       (c) 55,625

47. (a)       (b) 100

49. (a) 203 animals      (b) 13 years



51. (a)  $10^{7.9} \approx 79,432,823$       (b)  $10^{8.3} \approx 199,526,231$

(c)  $10^{4.2} \approx 15,849$

53. (a) 20 decibels      (b) 70 decibels

(c) 40 decibels      (d) 120 decibels

55. 95%      57. 4.64      59.  $1.58 \times 10^{-6}$  moles per liter

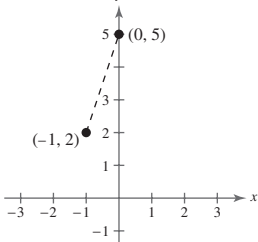
61.  $10^{5.1}$       63. 3:00 A.M.

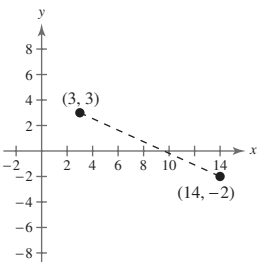
65. (a)       (b)  $\approx 21$  years; Yes

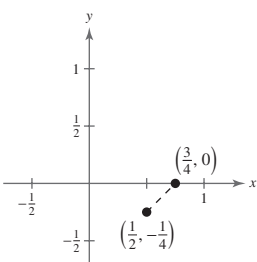
67. False. The domain can be the set of real numbers for a logistic growth function.

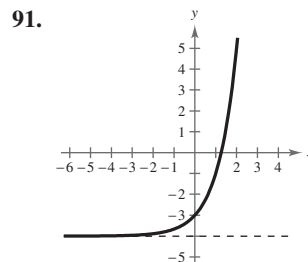
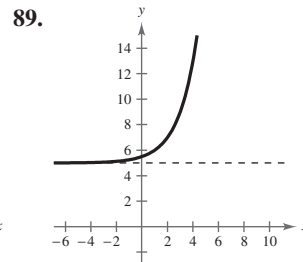
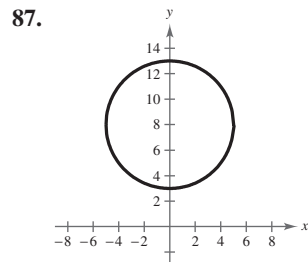
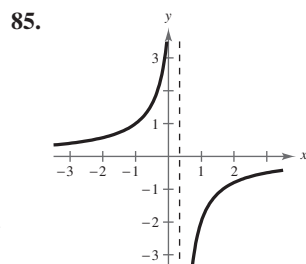
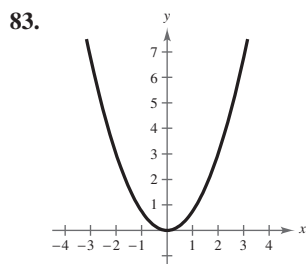
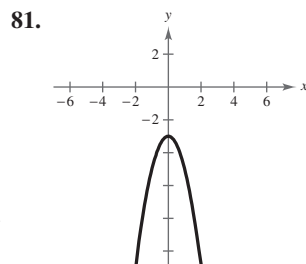
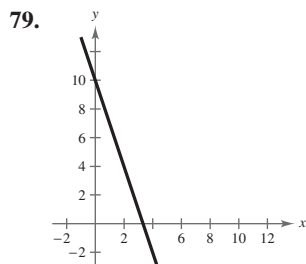
69. False. The graph of  $f(x)$  is the graph of  $g(x)$  shifted upward five units.

71. (a) Logarithmic (b) Logistic (c) Exponential  
(d) Linear (e) None of the above (f) Exponential

73. (a)  (b)  $\sqrt{10}$   
(c)  $(-\frac{1}{2}, \frac{7}{2})$   
(d) 3

75. (a)  (b)  $\sqrt{146}$   
(c)  $(\frac{17}{2}, \frac{1}{2})$   
(d)  $-\frac{5}{11}$

77. (a)  (b)  $\sqrt{\frac{1}{8}}$   
(c)  $(\frac{5}{8}, -\frac{1}{8})$   
(d) 1



93. Answers will vary.

### Review Exercises (page 271)

1. 76.699 3. 0.337 5. 1456.529

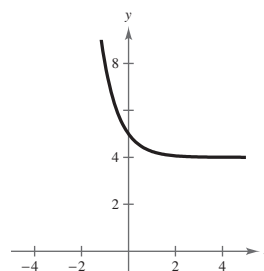
7. c 8. d 9. a 10. b

11. Shift the graph of  $f$  one unit to the right.

13. Reflect  $f$  in the  $x$ -axis and shift two units to the left.

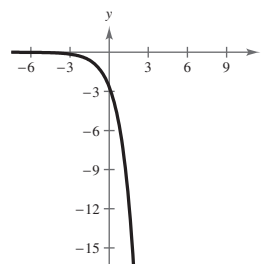
15.

$x$	-1	0	1	2	3
$f(x)$	8	5	4.25	4.063	4.016



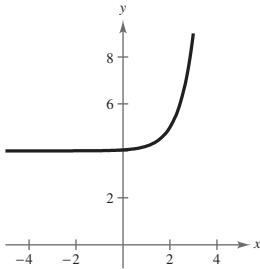
17.

$x$	-2	-1	0	1	2
$f(x)$	-0.377	-1	-2.65	-7.023	-18.61



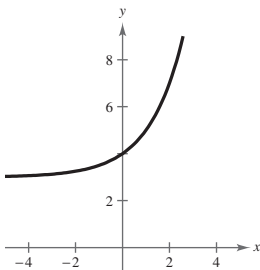
19.

$x$	-1	0	1	2	3
$f(x)$	4.008	4.04	4.2	5	9



21.

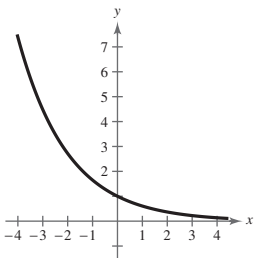
$x$	-2	-1	0	1	2
$f(x)$	3.25	3.5	4	5	7



23.  $x = -4$     25.  $x = \frac{22}{5}$     27. 2980.958    29. 0.183

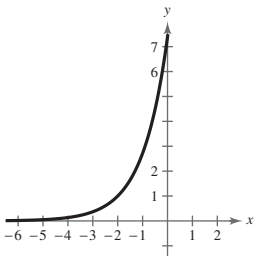
31.

$x$	-2	-1	0	1	2
$h(x)$	2.72	1.65	1	0.61	0.37



33.

$x$	-3	-2	-1	0	1
$f(x)$	0.37	1	2.72	7.39	20.09



35.

$n$	1	2	4	12
$A$	\$6569.98	\$6635.43	\$6669.46	\$6692.64

$n$	365	Continuous
$A$	\$6704.00	\$6704.39

37. (a) 0.154    (b) 0.487    (c) 0.811

39. (a) \$1,069,047.14    (b) 7.9 years

41.  $\log_4 64 = 3$     43.  $\ln 2.2255 \dots = 0.8$

45. 3    47. -3    49.  $x = 7$     51.  $x = -5$

53. Domain:  $(0, \infty)$

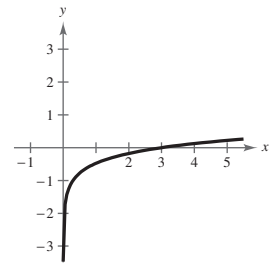
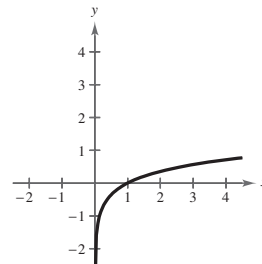
$x$ -intercept:  $(1, 0)$

Vertical asymptote:  $x = 0$

55. Domain:  $(0, \infty)$

$x$ -intercept:  $(3, 0)$

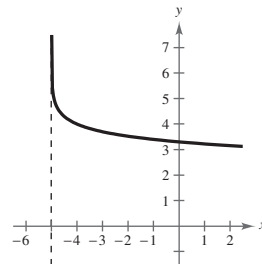
Vertical asymptote:  $x = 0$



57. Domain:  $(-5, \infty)$

$x$ -intercept:  $(9995, 0)$

Vertical asymptote:  $x = -5$



59. 3.118    61. -12    63. 2.034

65. Domain:  $(0, \infty)$

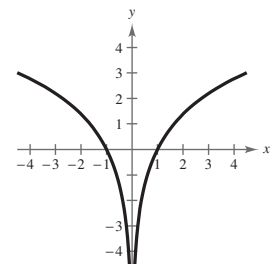
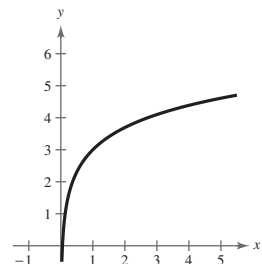
$x$ -intercept:  $(e^{-3}, 0)$

Vertical asymptote:  $x = 0$

67. Domain:  $(-\infty, 0), (0, \infty)$

$x$ -intercept:  $(\pm 1, 0)$

Vertical asymptote:  $x = 0$



69. 53.4 inches    71. 1.585

73. -2.322

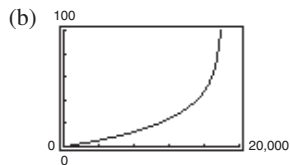
75.  $\log 2 + 2 \log 3 \approx 1.255$

77.  $2 \ln 2 + \ln 5 \approx 2.996$

79.  $1 + 2 \log_5 x$     81.  $1 + \log_3 2 - \frac{1}{3} \log_3 x$   
 83.  $2 \ln x + 2 \ln y + \ln z$     85.  $\ln(x+3) - \ln x - \ln y$   
 87.  $\log_2 5x$     89.  $\ln \frac{x}{\sqrt[4]{y}}$     91.  $\log_8 y^7 \sqrt[3]{x+4}$

93.  $\ln \frac{\sqrt{2x-1}}{(x+1)^2}$

95. (a)  $0 \leq h < 18,000$



Vertical asymptote:  
 $h = 18,000$

(c) The plane is climbing at a slower rate, so the time required increases.

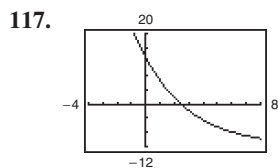
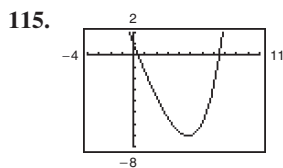
(d) 5.46 minutes

97. 3    99.  $\ln 3 \approx 1.099$     101. 16

103.  $e^4 \approx 54.598$     105.  $\ln 12 \approx 2.485$     107.  $x = 1, 3$

109.  $\frac{\ln 22}{\ln 2} \approx 4.459$     111.  $\frac{\ln 17}{\ln 5} \approx 1.760$

113.  $\ln 2 \approx 0.693$ ,  $\ln 5 \approx 1.609$



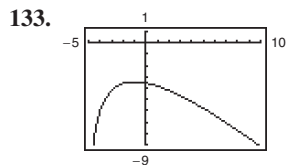
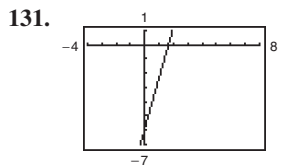
7.480; 0.392

2.447

119.  $\frac{1}{3}e^{8.2} \approx 1213.650$     121.  $\frac{1}{4}e^{7.5} \approx 452.011$

123.  $3e^2 \approx 22.167$     125.  $e^4 - 1 \approx 53.598$

127. No solution    129. 0.900



1.643

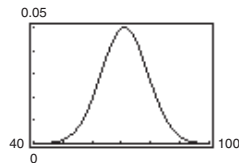
No solution

135. 15.2 years    137. e    138. b    139. f

140. d    141. a    142. c    143.  $y = 2e^{0.1014x}$

145. 2008    147. (a) 13.8629%    (b) \$11,486.98

149. (a)    (b) 71



151.  $10^{-3.5}$  watt per square centimeter

153. True by the inverse properties

155.  $b$  and  $d$  are negative.

$a$  and  $c$  are positive.

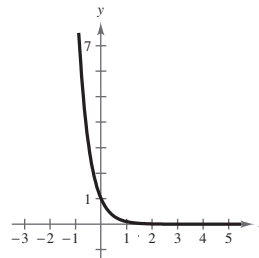
Answers will vary.

## Chapter Test (page 275)

1. 1123.690    2. 687.291    3. 0.497    4. 22.198

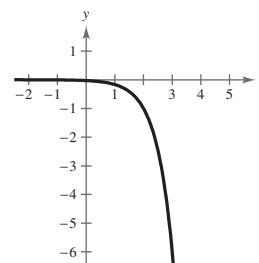
5.

$x$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	10	3.162	1	0.316	0.1



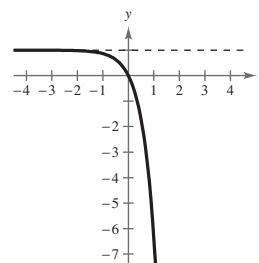
6.

$x$	-1	0	1	2	3
$f(x)$	-0.005	-0.028	-0.167	-1	-6



7.

$x$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	0.865	0.632	0	-1.718	-6.389

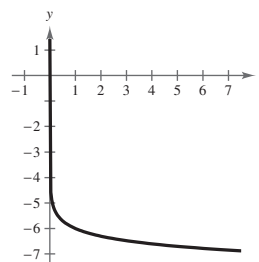


8. (a) -0.89    (b) 9.2

9.

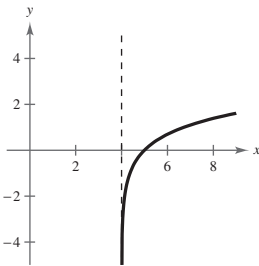
$x$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	4
$f(x)$	-5.699	-6	-6.176	-6.301	-6.602

Vertical asymptote:  $x = 0$



10.

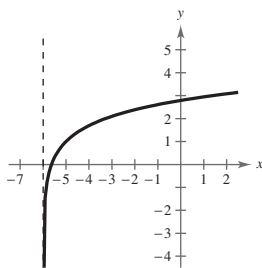
$x$	5	7	9	11	13
$f(x)$	0	1.099	1.609	1.946	2.197



Vertical asymptote:  $x = 4$

11.

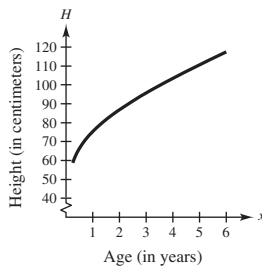
$x$	-5	-3	-1	0	1
$f(x)$	1	2.099	2.609	2.792	2.946



Vertical asymptote:  $x = -6$

12. 1.945      13. 0.115      14. 1.328  
 15.  $\log_2 3 + 4 \log_2 |a|$       16.  $\ln 5 + \frac{1}{2} \ln x - \ln 6$   
 17.  $(\log 7 + 2 \log x) - (\log y + 3 \log z)$   
 18.  $\log_3 13y$       19.  $\ln \frac{x^4}{y^4}$       20.  $\ln \frac{x^2(x-5)}{y^3}$   
 21.  $x = -2$       22.  $x = \frac{\ln 44}{-5} \approx -0.757$   
 23.  $\frac{\ln 197}{4} \approx 1.321$       24.  $e^{1/2} \approx 1.649$   
 25.  $e^{-11/4} \approx 0.0639$       26.  $\frac{800}{501} \approx 1.597$   
 27.  $y = 2745e^{0.1570x}$       28. 55%  
 29. (a)

$x$	$\frac{1}{4}$	1	2	4	5	6
$H$	58.720	75.332	86.828	103.43	110.59	117.38

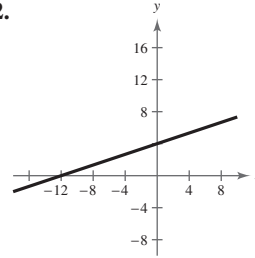


(b) 103 centimeters; 103.43 centimeters

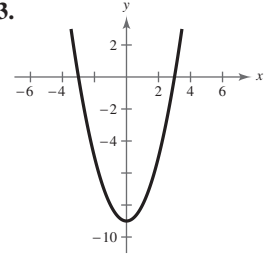
### Cumulative Test for Chapters 1–3 (page 276)

1. (a) Midpoint:  $(1, \frac{3}{2})$ ; Distance:  $\sqrt{41}$

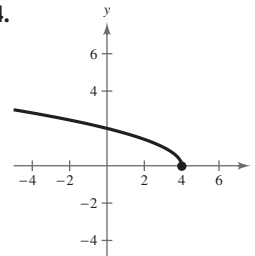
2.



3.

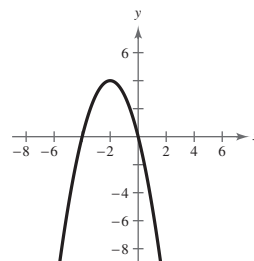


4.

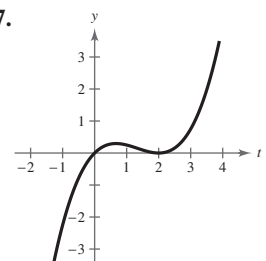


5.  $y = 2x + 2$

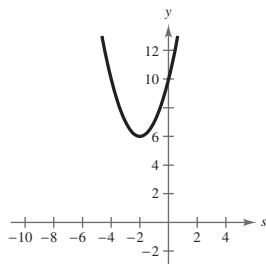
6. For some values of  $x$  there correspond two values of  $y$ .  
 7. (a)  $\frac{3}{2}$       (b) Division by 0 is undefined.      (c)  $\frac{s+2}{s}$   
 8. (a) Vertical shrink by  $\frac{1}{2}$   
 (b) Vertical shift of two units upward  
 (c) Horizontal shift of two units to the left  
 9. (a)  $5x - 2$       (b)  $-3x - 4$       (c)  $4x^2 - 11x - 3$   
 (d)  $\frac{x-3}{4x+1}$ ; Domain: all real numbers  $x$  except  $x = -\frac{1}{4}$   
 10. (a)  $\sqrt{x-1} + x^2 + 1$       (b)  $\sqrt{x-1} - x^2 - 1$   
 (c)  $x^2\sqrt{x-1} + \sqrt{x-1}$   
 (d)  $\frac{\sqrt{x-1}}{x^2+1}$ ; Domain: all real numbers  $x$  such that  $x \geq 1$   
 11. (a)  $2x + 12$       (b)  $\sqrt{2x^2 + 6}$   
 Domain of  $f \circ g$ : all real numbers  $x$  such that  $x \geq -6$   
 Domain of  $g \circ f$ : all real numbers  
 12. (a)  $|x| - 2$       (b)  $|x - 2|$   
 Domain of  $f \circ g$  and  $g \circ f$ : all real numbers  
 13. Yes;  $h^{-1}(x) = \frac{1}{5}(x + 2)$       14. 2438.65 kilowatts  
 15.  $y = -\frac{3}{4}(x + 8)^2 + 5$   
 16.



17.



18.



19.  $-2, \pm 2i; (x+2)(x+2i)(x-2i)$

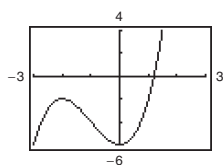
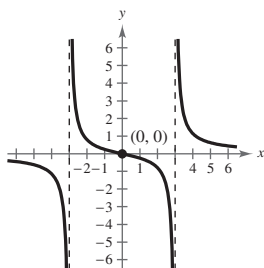
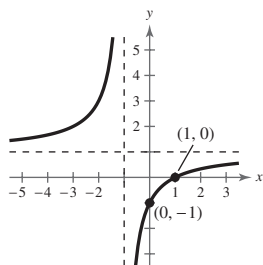
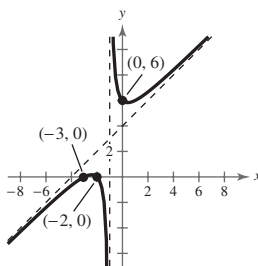
20.  $-7, 0, 3; x(x)(x-3)(x+7)$

21.  $4, -\frac{1}{2}, 1 \pm 3i; (x-4)(2x+1)(x-1+3i)(x-1-3i)$

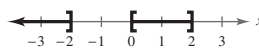
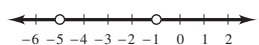
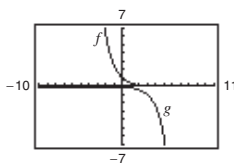
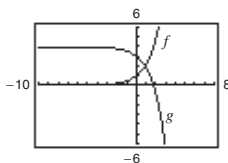
22.  $3x - 2 - \frac{3x - 2}{2x^2 + 1}$

23.  $2x^3 - x^2 + 2x - 10 + \frac{25}{x+2}$

24.

Interval:  $[1, 2]; 1.20$ 25. Intercept:  $(0, 0)$ Vertical asymptotes:  $x = \pm 3$ Horizontal asymptote:  $y = 0$ 26. y-intercept:  $(0, -1)$ x-intercept:  $(1, 0)$ Horizontal asymptote:  $y = 1$ Vertical asymptote:  $x = -1$ 27. y-intercept:  $(0, 6)$ x-intercepts:  $(-2, 0), (-3, 0)$ Slant asymptote:  $y = x + 4$ Vertical asymptote:  $x = -1$ 

28.  $x \leq -2$  or  $0 \leq x \leq 2$

29. All real numbers  $x$  such that  $x < -5$  or  $x > -1$ 30. Reflect  $f$  in the  $x$ -axis and  $y$ -axis, and shift three units to the right.31. Reflect  $f$  in the  $x$ -axis, and shift four units upward.32. 1.991    33.  $-0.067$     34. 1.717    35. 0.281

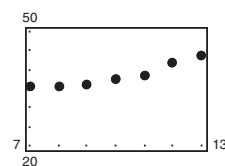
36.  $\ln(x+4) + \ln(x-4) - 4 \ln x, x > 4$

37.  $\ln \frac{x^2}{\sqrt{x+5}}, x > 0$     38.  $x = \frac{\ln 12}{2} \approx 1.242$

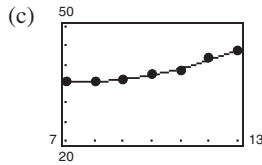
39.  $\ln 3 \approx 1.099$  or  $3 \ln 2 \approx 2.079$

40.  $e^6 - 2 \approx 401.429$

41. (a)



(b)  $S = 0.274t^2 - 4.08t + 50.6$

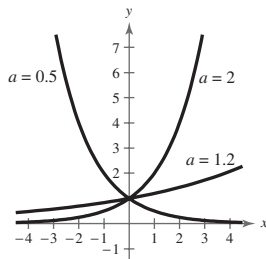


The model is a good fit for the data.

(d) 65.9 Yes, this is a reasonable answer.

### Problem Solving (page 279)

1.

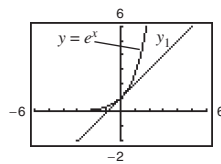


$$y = 0.5^x \text{ and } y = 1.2^x \\ 0 \leq a \leq 1.44$$

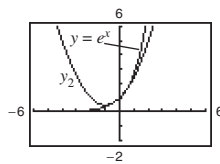
3. As  $x \rightarrow \infty$ , the graph of  $e^x$  increases at a greater rate than the graph of  $x^n$ .

5. Answers will vary.

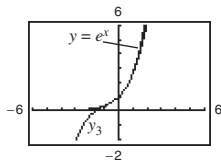
7. (a)



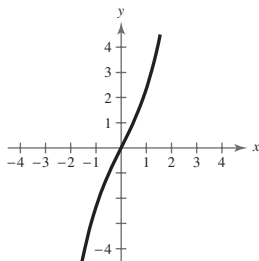
(b)



(c)



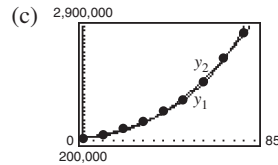
9.



$$f^{-1}(x) = \ln\left(\frac{x + \sqrt{x^2 + 4}}{2}\right)$$

11. c     13.  $t = \frac{\ln c_1 - \ln c_2}{\left(\frac{1}{k_2} - \frac{1}{k_1}\right) \ln \frac{1}{2}}$

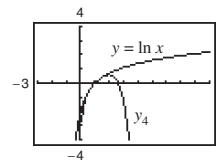
15. (a)  $y_1 = 252,606(1.0310)^t$   
(b)  $y_2 = 400.88t^2 - 1464.6t + 291,782$



(d) The exponential model is a better fit. No, because the model is rapidly approaching infinity.

17.  $1, e^2$

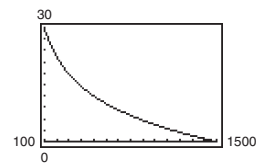
19.  $y_4 = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4$



The pattern implies that

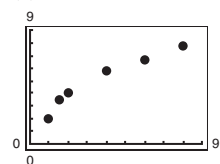
$$\ln x = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \dots$$

21.



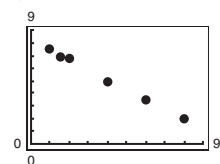
17.7 cubic feet per minute

23. (a)



(b)–(e)  
Answers will vary.

25. (a)



(b)–(e)  
Answers will vary.

## Chapter 4

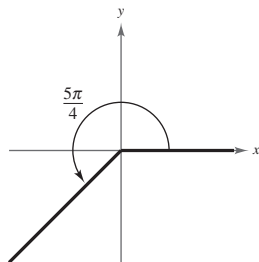
### Section 4.1 (page 290)

#### Vocabulary Check (page 290)

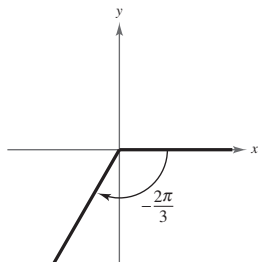
1. Trigonometry    2. angle    3. coterminal
4. radian    5. acute; obtuse
6. complementary; supplementary    7. degree
8. linear    9. angular    10.  $A = \frac{1}{2}r^2\theta$

1. 2 radians    3.  $-3$  radians    5. 1 radian
7. (a) Quadrant I    (b) Quadrant III
9. (a) Quadrant IV    (b) Quadrant III
11. (a) Quadrant III    (b) Quadrant II

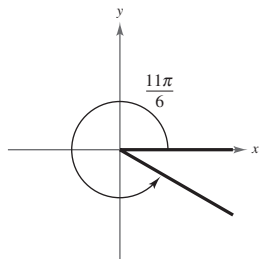
13. (a)



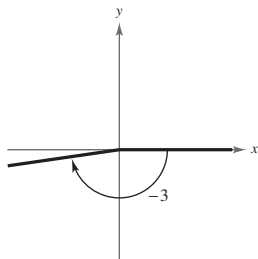
(b)



15. (a)



(b)



17. (a)  $\frac{13\pi}{6}, -\frac{11\pi}{6}$  (b)  $\frac{17\pi}{6}, -\frac{7\pi}{6}$

19. (a)  $\frac{8\pi}{3}, -\frac{4\pi}{3}$  (b)  $\frac{25\pi}{12}, -\frac{23\pi}{12}$

21. (a) Complement:  $\frac{\pi}{6}$ ; Supplement:  $\frac{2\pi}{3}$

(b) Complement: none; Supplement:  $\frac{\pi}{4}$

23. (a) Complement:  $\frac{\pi}{2} - 1 \approx 0.57$ ;

Supplement:  $\pi - 1 \approx 2.14$

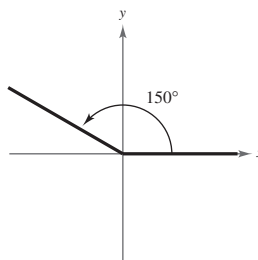
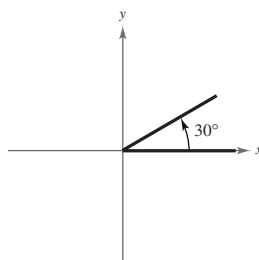
(b) Complement: none; Supplement:  $\pi - 2 \approx 1.14$

25.  $210^\circ$  27.  $-60^\circ$  29.  $165^\circ$

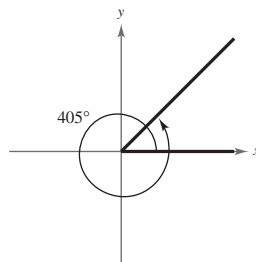
31. (a) Quadrant II (b) Quadrant IV

33. (a) Quadrant III (b) Quadrant I

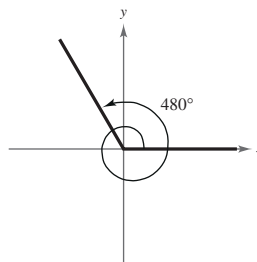
35. (a) (b)



37. (a)



(b)



39. (a)  $405^\circ, -315^\circ$  (b)  $324^\circ, -396^\circ$

41. (a)  $600^\circ, -120^\circ$  (b)  $180^\circ, -540^\circ$

43. (a) Complement:  $72^\circ$ ; Supplement:  $162^\circ$

(b) Complement: none; Supplement:  $65^\circ$

45. (a) Complement:  $11^\circ$ ; Supplement:  $101^\circ$

(b) Complement: none; Supplement:  $30^\circ$

47. (a)  $\frac{\pi}{6}$  (b)  $\frac{5\pi}{6}$  49. (a)  $-\frac{\pi}{9}$  (b)  $-\frac{4\pi}{3}$

51. (a)  $270^\circ$  (b)  $210^\circ$  53. (a)  $420^\circ$  (b)  $-66^\circ$

55. 2.007 57.  $-3.776$  59. 9.285 61.  $-0.014$

63.  $25.714^\circ$  65.  $337.500^\circ$  67.  $-756.000^\circ$

69.  $-114.592^\circ$  71. (a)  $54.75^\circ$  (b)  $-128.5^\circ$

73. (a)  $85.308^\circ$  (b)  $330.007^\circ$

75. (a)  $240^\circ 36'$  (b)  $-145^\circ 48'$

77. (a)  $2^\circ 30'$  (b)  $-3^\circ 34' 48''$

79.  $\frac{6}{5}$  radians 81.  $\frac{32}{7}$  radians 83.  $\frac{2}{9}$  radian

85.  $\frac{50}{29}$  radians 87.  $15\pi$  inches  $\approx 47.12$  inches

89. 3 meters 91.  $\frac{8\pi}{3}$  square inches  $\approx 8.38$  square inches

93. 12.27 square feet 95. 591.3 miles

97.  $0.071$  radian  $\approx 4.04^\circ$  99.  $\frac{5}{12}$  radian

101. (a) 728.3 revolutions per minute

(b) 4576 radians per minute

103. (a)  $10,400\pi$  radians per minute

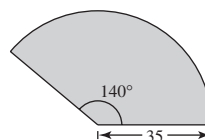
$\approx 32,672.56$  radians per minute

(b)  $9425\pi/3$  feet per minute  $\approx 9869.84$  feet per minute

105. (a)  $[400\pi, 1000\pi]$  radians per minute

(b)  $[2400\pi, 6000\pi]$  centimeters per minute

107.



$A = 476.39\pi$  square meters  $\approx 1496.62$  square meters

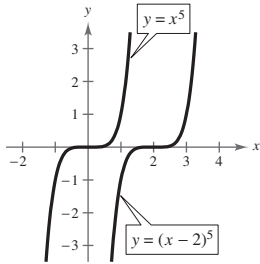
109. False. A measurement of  $4\pi$  radians corresponds to two complete revolutions from the initial to the terminal side of an angle.



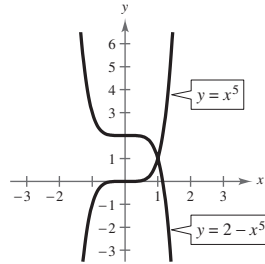
111. False. The terminal side of the angle lies on the  $x$ -axis.  
 113. Increases. The linear velocity is proportional to the radius.  
 115. The arc length is increasing. If  $\theta$  is constant, the length of the arc is proportional to the radius ( $s = r\theta$ ).

117.  $\frac{\sqrt{2}}{2}$       119.  $2\sqrt{10}$

121.



123.



## Section 4.2 (page 299)

### Vocabulary Check (page 299)

1. unit circle      2. periodic  
 3. period      4. odd; even

1.  $\sin \theta = \frac{15}{17}$        $\csc \theta = \frac{17}{15}$   
 $\cos \theta = -\frac{8}{17}$        $\sec \theta = -\frac{17}{8}$   
 $\tan \theta = -\frac{15}{8}$        $\cot \theta = -\frac{8}{15}$   
 3.  $\sin \theta = -\frac{5}{13}$        $\csc \theta = -\frac{13}{5}$   
 $\cos \theta = \frac{12}{13}$        $\sec \theta = \frac{13}{12}$   
 $\tan \theta = -\frac{5}{12}$        $\cot \theta = -\frac{12}{5}$   
 5.  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$       7.  $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$       9.  $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$   
 11.  $(0, -1)$   
 13.  $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$       15.  $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$   
 $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$        $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$   
 $\tan \frac{\pi}{4} = 1$        $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$   
 17.  $\sin\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$       19.  $\sin \frac{11\pi}{6} = -\frac{1}{2}$   
 $\cos\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$        $\cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$   
 $\tan\left(-\frac{7\pi}{4}\right) = 1$        $\tan \frac{11\pi}{6} = -\frac{\sqrt{3}}{3}$   
 21.  $\sin\left(-\frac{3\pi}{2}\right) = 1$   
 $\cos\left(-\frac{3\pi}{2}\right) = 0$   
 $\tan\left(-\frac{3\pi}{2}\right)$  is undefined.

23.  $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$        $\csc \frac{3\pi}{4} = \sqrt{2}$   
 $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$        $\sec \frac{3\pi}{4} = -\sqrt{2}$   
 $\tan \frac{3\pi}{4} = -1$        $\cot \frac{3\pi}{4} = -1$   
 25.  $\sin\left(-\frac{\pi}{2}\right) = -1$        $\csc\left(-\frac{\pi}{2}\right) = -1$   
 $\cos\left(-\frac{\pi}{2}\right) = 0$        $\sec\left(-\frac{\pi}{2}\right)$  is undefined.  
 $\tan\left(-\frac{\pi}{2}\right)$  is undefined.       $\cot\left(-\frac{\pi}{2}\right) = 0$   
 27.  $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$        $\csc\left(\frac{4\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$   
 $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$        $\sec\left(\frac{4\pi}{3}\right) = -2$   
 $\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$        $\cot\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{3}$

29.  $\sin 5\pi = \sin \pi = 0$       31.  $\cos \frac{8\pi}{3} = \cos \frac{2\pi}{3} = -\frac{1}{2}$

33.  $\cos\left(-\frac{15\pi}{2}\right) = \cos \frac{\pi}{2} = 0$

35.  $\sin\left(-\frac{9\pi}{4}\right) = \sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$

37. (a)  $-\frac{1}{3}$       (b)  $-3$       39. (a)  $-\frac{1}{5}$       (b)  $-5$

41. (a)  $\frac{4}{5}$       (b)  $-\frac{4}{5}$       43. 0.7071      45. 1.0378

47.  $-0.1288$       49. 1.3940      51.  $-1.4486$

53. (a)  $-1$       (b)  $-0.4$

55. (a) 0.25 or 2.89      (b) 1.82 or 4.46

57. (a)

$t$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$y$	0.25	0.0138	$-0.1501$	$-0.0249$	0.0883

(b)  $t \approx 5.5$       (c) The displacement decreases.

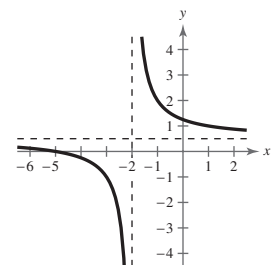
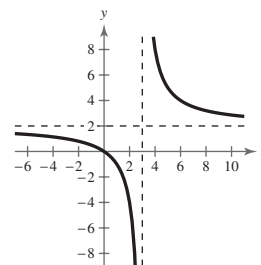
59. False.  $\sin(-t) = -\sin t$  means that the function is odd, not that the sine of a negative angle is a negative number.

61. (a)  $y$ -axis symmetry      (b)  $\sin t_1 = \sin(\pi - t_1)$

(c)  $\cos(\pi - t_1) = -\cos t_1$

63.  $f^{-1}(x) = \frac{2}{3}(x + 1)$       65.  $f^{-1}(x) = \sqrt{x^2 + 4}$ ,  $x \geq 0$

67.



## Section 4.3 (page 308)

**Vocabulary Check** (page 308)

1. (a) v (b) iv (c) vi (d) iii (e) i (f) ii  
 2. opposite; adjacent; hypotenuse  
 3. elevation; depression

$$\begin{aligned} 1. \sin \theta &= \frac{3}{5} & \csc \theta &= \frac{5}{3} \\ \cos \theta &= \frac{4}{5} & \sec \theta &= \frac{5}{4} \\ \tan \theta &= \frac{3}{4} & \cot \theta &= \frac{4}{3} \\ 3. \sin \theta &= \frac{9}{41} & \csc \theta &= \frac{41}{9} \\ \cos \theta &= \frac{40}{41} & \sec \theta &= \frac{41}{40} \\ \tan \theta &= \frac{9}{40} & \cot \theta &= \frac{40}{9} \\ 5. \sin \theta &= \frac{1}{3} & \csc \theta &= 3 \\ \cos \theta &= \frac{2\sqrt{2}}{3} & \sec \theta &= \frac{3\sqrt{2}}{4} \\ \tan \theta &= \frac{\sqrt{2}}{4} & \cot \theta &= 2\sqrt{2} \end{aligned}$$

The triangles are similar, and corresponding sides are proportional.

$$\begin{aligned} 7. \sin \theta &= \frac{3}{5} & \csc \theta &= \frac{5}{3} \\ \cos \theta &= \frac{4}{5} & \sec \theta &= \frac{5}{4} \\ \tan \theta &= \frac{3}{4} & \cot \theta &= \frac{4}{3} \end{aligned}$$

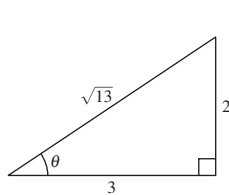
The triangles are similar, and corresponding sides are proportional.

$$\begin{aligned} 9. \quad & \cos \theta = \frac{\sqrt{7}}{4} & \sec \theta &= \frac{4\sqrt{7}}{7} \\ & \tan \theta = \frac{3\sqrt{7}}{7} & \cot \theta &= \frac{\sqrt{7}}{3} \\ & \csc \theta = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} 11. \quad & \sin \theta = \frac{\sqrt{3}}{2} & \csc \theta &= \frac{2\sqrt{3}}{3} \\ & \cos \theta = \frac{1}{2} & \cot \theta &= \frac{\sqrt{3}}{3} \\ & \tan \theta = \sqrt{3} \end{aligned}$$

$$\begin{aligned} 13. \quad & \sin \theta = \frac{3\sqrt{10}}{10} & \sec \theta &= \sqrt{10} \\ & \cos \theta = \frac{\sqrt{10}}{10} & \cot \theta &= \frac{1}{3} \\ & \csc \theta = \frac{\sqrt{10}}{3} \end{aligned}$$

15.



$$\begin{aligned} \sin \theta &= \frac{2\sqrt{13}}{13} & \csc \theta &= \frac{\sqrt{13}}{2} \\ \cos \theta &= \frac{3\sqrt{13}}{13} & \sec \theta &= \frac{\sqrt{13}}{3} \\ \tan \theta &= \frac{2}{3} \end{aligned}$$

$$17. \frac{\pi}{6}; \frac{1}{2} \quad 19. 60^\circ; \sqrt{3} \quad 21. 60^\circ; \frac{\pi}{3} \quad 23. 30^\circ; \frac{\sqrt{3}}{2}$$

$$25. 45^\circ; \frac{\pi}{4} \quad 27. (a) \sqrt{3} \quad (b) \frac{1}{2} \quad (c) \frac{\sqrt{3}}{2} \quad (d) \frac{\sqrt{3}}{3}$$

$$29. (a) \frac{2\sqrt{13}}{13} \quad (b) \frac{3\sqrt{13}}{13} \quad (c) \frac{2}{3} \quad (d) \frac{\sqrt{13}}{2}$$

$$31. (a) 3 \quad (b) \frac{2\sqrt{2}}{3} \quad (c) \frac{\sqrt{2}}{4} \quad (d) \frac{1}{3}$$

33–41. Answers will vary. 43. (a) 0.1736 (b) 0.1736

45. (a) 0.2815 (b) 3.5523

47. (a) 1.3499 (b) 1.3432

49. (a) 5.0273 (b) 0.1989

51. (a) 1.8527 (b) 0.9817

$$53. (a) 30^\circ = \frac{\pi}{6} \quad (b) 30^\circ = \frac{\pi}{6}$$

$$55. (a) 60^\circ = \frac{\pi}{3} \quad (b) 45^\circ = \frac{\pi}{4}$$

$$57. (a) 60^\circ = \frac{\pi}{3} \quad (b) 45^\circ = \frac{\pi}{4}$$

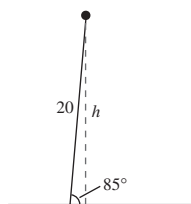
$$59. 30\sqrt{3} \quad 61. \frac{32\sqrt{3}}{3}$$

$$63. 443.2 \text{ meters}; 323.3 \text{ meters} \quad 65. 30^\circ = \frac{\pi}{6}$$

67. (a) 371.1 feet (b) 341.6 feet  
 (c) Moving down line at 61.8 feet per second  
 Dropping vertically at 24.2 feet per second

$$69. (x_1, y_1) = (28\sqrt{3}, 28) \\ (x_2, y_2) = (28, 28\sqrt{3})$$

$$71. (a) \quad \sin 85^\circ = \frac{h}{20} \\ (c) 19.9 \text{ meters}$$

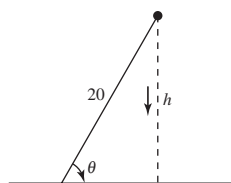


(d) The side of the triangle labeled  $h$  will become shorter.

Angle, $\theta$	$80^\circ$	$70^\circ$	$60^\circ$	$50^\circ$
Height	19.7	18.8	17.3	15.3

Angle, $\theta$	$40^\circ$	$30^\circ$	$20^\circ$	$10^\circ$
Height	12.9	10.0	6.8	3.5

(f) As  $\theta \rightarrow 0^\circ$ ,  $h \rightarrow 0$ .


73. True,  $\csc x = \frac{1}{\sin x}$ . 75. False,  $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \neq 1$ .

 77. False,  $1.7321 \neq 0.0349$ .

79. Corresponding sides of similar triangles are proportional.

81. (a)

$\theta$	0.1	0.2	0.3	0.4	0.5
$\sin \theta$	0.0998	0.1987	0.2955	0.3894	0.4794

 (b)  $\theta$  (c) As  $\theta$  approaches 0,  $\sin \theta$  approaches 0.

83.  $\frac{x}{x-2}$ ,  $x \neq \pm 2$  85.  $\frac{2(x^2 - 5x - 10)}{(x-2)(x+2)^2}$

# Section 4.4 (page 318)

## Vocabulary Check (page 318)

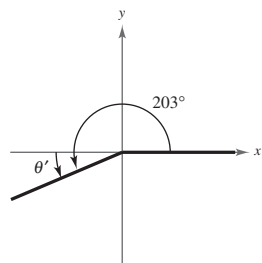
1.  $\frac{y}{r}$
2.  $\csc \theta$
3.  $\frac{y}{x}$
4.  $\frac{r}{x}$
5.  $\cos \theta$
6.  $\cot \theta$
7. reference

1. (a)  $\sin \theta = \frac{3}{5}$   
 $\cos \theta = \frac{4}{5}$   
 $\tan \theta = \frac{3}{4}$   
 $\csc \theta = \frac{5}{3}$   
 $\sec \theta = \frac{5}{4}$   
 $\cot \theta = \frac{4}{3}$
- (b)  $\sin \theta = -\frac{15}{17}$   
 $\cos \theta = \frac{8}{17}$   
 $\tan \theta = -\frac{15}{8}$   
 $\csc \theta = -\frac{17}{15}$   
 $\sec \theta = \frac{17}{8}$   
 $\cot \theta = -\frac{8}{15}$
3. (a)  $\sin \theta = -\frac{1}{2}$   
 $\cos \theta = -\frac{\sqrt{3}}{2}$   
 $\tan \theta = \frac{\sqrt{3}}{3}$   
 $\csc \theta = -2$   
 $\sec \theta = -\frac{2\sqrt{3}}{3}$   
 $\cot \theta = \sqrt{3}$
- (b)  $\sin \theta = \frac{\sqrt{17}}{17}$   
 $\cos \theta = -\frac{4\sqrt{17}}{17}$   
 $\tan \theta = -\frac{1}{4}$   
 $\csc \theta = \sqrt{17}$   
 $\sec \theta = -\frac{\sqrt{17}}{4}$   
 $\cot \theta = -4$
5.  $\sin \theta = \frac{24}{25}$   
 $\cos \theta = \frac{7}{25}$   
 $\tan \theta = \frac{24}{7}$
- $\csc \theta = \frac{25}{24}$   
 $\sec \theta = \frac{25}{7}$   
 $\cot \theta = \frac{7}{24}$

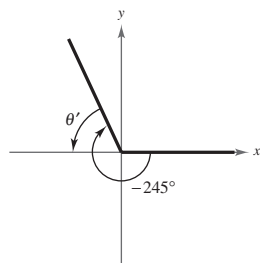
7.  $\sin \theta = \frac{5\sqrt{29}}{29}$   
 $\cos \theta = -\frac{2\sqrt{29}}{29}$   
 $\tan \theta = -\frac{5}{2}$
- $\csc \theta = \frac{\sqrt{29}}{5}$   
 $\sec \theta = -\frac{\sqrt{29}}{2}$   
 $\cot \theta = -\frac{2}{5}$
9.  $\sin \theta = \frac{68\sqrt{5849}}{5849}$   
 $\cos \theta = -\frac{35\sqrt{5849}}{5849}$   
 $\tan \theta = -\frac{68}{35}$
- $\csc \theta = \frac{\sqrt{5849}}{68}$   
 $\sec \theta = -\frac{\sqrt{5849}}{35}$   
 $\cot \theta = -\frac{35}{68}$
11. Quadrant III
13. Quadrant II
15.  $\sin \theta = \frac{5}{3}$   
 $\cos \theta = -\frac{4}{5}$   
 $\tan \theta = -\frac{3}{4}$
- $\csc \theta = \frac{5}{3}$   
 $\sec \theta = -\frac{5}{4}$   
 $\cot \theta = -\frac{4}{3}$
17.  $\sin \theta = -\frac{15}{17}$   
 $\cos \theta = \frac{8}{17}$   
 $\tan \theta = -\frac{15}{8}$
- $\csc \theta = -\frac{17}{15}$   
 $\sec \theta = \frac{17}{8}$   
 $\cot \theta = -\frac{8}{15}$
19.  $\sin \theta = -\frac{\sqrt{10}}{10}$   
 $\cos \theta = \frac{3\sqrt{10}}{10}$   
 $\tan \theta = -\frac{1}{3}$
- $\csc \theta = -\sqrt{10}$   
 $\sec \theta = \frac{\sqrt{10}}{3}$   
 $\cot \theta = -3$
21.  $\sin \theta = \frac{\sqrt{3}}{2}$   
 $\cos \theta = -\frac{1}{2}$   
 $\tan \theta = -\sqrt{3}$
- $\csc \theta = \frac{2\sqrt{3}}{3}$   
 $\sec \theta = -2$   
 $\cot \theta = -\frac{\sqrt{3}}{3}$
23.  $\sin \theta = 0$   
 $\cos \theta = -1$   
 $\tan \theta = 0$
- $\csc \theta$  is undefined.  
 $\sec \theta = -1$   
 $\cot \theta$  is undefined.
25.  $\sin \theta = \frac{\sqrt{2}}{2}$   
 $\cos \theta = -\frac{\sqrt{2}}{2}$   
 $\tan \theta = -1$
- $\csc \theta = \sqrt{2}$   
 $\sec \theta = -\sqrt{2}$   
 $\cot \theta = -1$
27.  $\sin \theta = -\frac{2\sqrt{5}}{5}$   
 $\cos \theta = -\frac{\sqrt{5}}{5}$   
 $\tan \theta = 2$
- $\csc \theta = -\frac{\sqrt{5}}{2}$   
 $\sec \theta = -\sqrt{5}$   
 $\cot \theta = \frac{1}{2}$

29. 0    31. Undefined    33. 1    35. Undefined

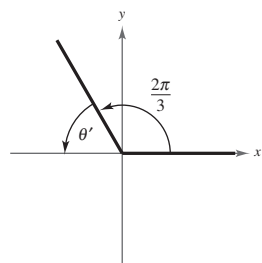
37.  $\theta' = 23^\circ$



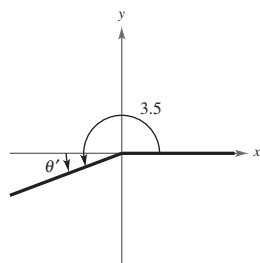
39.  $\theta' = 65^\circ$



41.  $\theta' = \frac{\pi}{3}$



43.  $\theta' = 3.5 - \pi$



45.  $\sin 225^\circ = -\frac{\sqrt{2}}{2}$

$\cos 225^\circ = -\frac{\sqrt{2}}{2}$

$\tan 225^\circ = 1$

49.  $\sin(-150^\circ) = -\frac{1}{2}$

$\cos(-150^\circ) = -\frac{\sqrt{3}}{2}$

$\tan(-150^\circ) = \frac{\sqrt{3}}{3}$

53.  $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

57.  $\sin\left(-\frac{3\pi}{2}\right) = 1$

$\cos\left(-\frac{3\pi}{2}\right) = 0$

$\tan\left(-\frac{3\pi}{2}\right)$  is undefined.

59.  $\frac{4}{5}$     61.  $-\frac{\sqrt{13}}{2}$     63.  $\frac{8}{5}$     65. 0.1736

67. -0.3420    69. -1.4826    71. 3.2361

73. 4.6373    75. 0.3640    77. -0.6052

79. -0.4142

81. (a)  $30^\circ = \frac{\pi}{6}$ ,  $150^\circ = \frac{5\pi}{6}$     (b)  $210^\circ = \frac{7\pi}{6}$ ,  $330^\circ = \frac{11\pi}{6}$

83. (a)  $60^\circ = \frac{\pi}{3}$ ,  $120^\circ = \frac{2\pi}{3}$     (b)  $135^\circ = \frac{3\pi}{4}$ ,  $315^\circ = \frac{7\pi}{4}$

85. (a)  $45^\circ = \frac{\pi}{4}$ ,  $225^\circ = \frac{5\pi}{4}$     (b)  $150^\circ = \frac{5\pi}{6}$ ,  $330^\circ = \frac{11\pi}{6}$

87. (a)  $N = 22.099 \sin(0.522t - 2.219) + 55.008$

$F = 36.641 \sin(0.502t - 1.831) + 25.610$

(b) February:  $N = 34.6^\circ$ ,  $F = -1.4^\circ$

March:  $N = 41.6^\circ$ ,  $F = 13.9^\circ$

May:  $N = 63.4^\circ$ ,  $F = 48.6^\circ$

June:  $N = 72.5^\circ$ ,  $F = 59.5^\circ$

August:  $N = 75.5^\circ$ ,  $F = 55.6^\circ$

September:  $N = 68.6^\circ$ ,  $F = 41.7^\circ$

November:  $N = 46.8^\circ$ ,  $F = 6.5^\circ$

(c) Answers will vary.

89. (a) 2 centimeters    (b) 0.14 centimeter

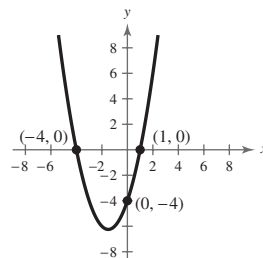
(c) -1.98 centimeters

91. 0.79 ampere

93. False. In each of the four quadrants, the signs of the secant function and cosine function will be the same, because these functions are reciprocals of each other.

95. As  $\theta$  increases from  $0^\circ$  to  $90^\circ$ ,  $x$  decreases from 12 cm to 0 cm and  $y$  increases from 0 cm to 12 cm. Therefore,  $\sin \theta = y/12$  increases from 0 to 1 and  $\cos \theta = x/12$  decreases from 1 to 0. Thus,  $\tan \theta = y/x$  and increases without bound. When  $\theta = 90^\circ$ , the tangent is undefined.

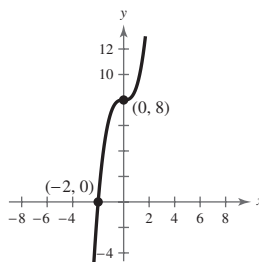
97.

 $x$ -intercepts:

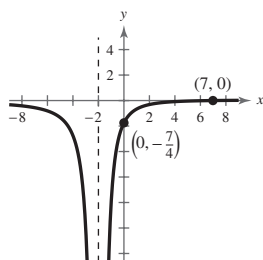
(1, 0), (-4, 0)

 $y$ -intercept: (0, -4)Domain: all real numbers  $x$ 

99.

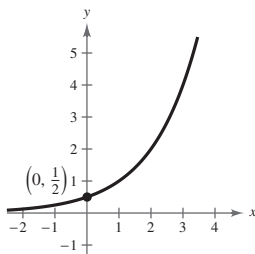
 $x$ -intercept: (-2, 0) $y$ -intercept: (0, 8)Domain: all real numbers  $x$

101.



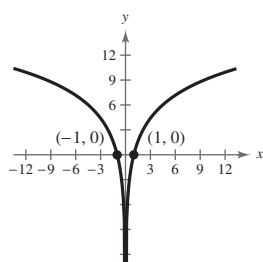
x-intercept:  $(7, 0)$   
 y-intercept:  $(0, -\frac{7}{4})$   
 Vertical asymptote:  
 $x = -2$   
 Horizontal asymptote:  
 $y = 0$   
 Domain: all real numbers  $x$   
 except  $x = -2$

103.



y-intercept:  $(0, \frac{1}{2})$   
 Horizontal asymptote:  
 $y = 0$   
 Domain: all real numbers  $x$

105.



x-intercepts:  $(\pm 1, 0)$   
 Vertical asymptote:  $x = 0$   
 Domain: all real numbers  $x$   
 except  $x = 0$

# Section 4.5 (page 328)

## Vocabulary Check (page 328)

1. cycle
2. amplitude
3.  $\frac{2\pi}{b}$
4. phase shift
5. vertical shift

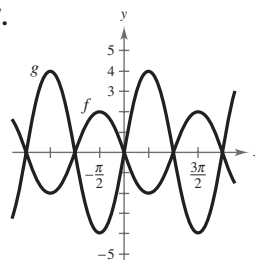
1. Period:  $\pi$   
Amplitude: 3
3. Period:  $4\pi$   
Amplitude:  $\frac{5}{2}$
5. Period: 6  
Amplitude:  $\frac{1}{2}$

7. Period:  $2\pi$   
Amplitude: 3
9. Period:  $\frac{\pi}{5}$   
Amplitude: 3

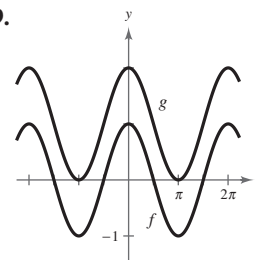
11. Period:  $3\pi$   
Amplitude:  $\frac{1}{2}$
13. Period: 1  
Amplitude:  $\frac{1}{4}$

15.  $g$  is a shift of  $f$   $\pi$  units to the right.
17.  $g$  is a reflection of  $f$  in the  $x$ -axis.
19. The period of  $f$  is twice the period of  $g$ .
21.  $g$  is a shift of  $f$  three units upward.
23. The graph of  $g$  has twice the amplitude of the graph of  $f$ .
25. The graph of  $g$  is a horizontal shift of the graph of  $f$   $\pi$  units to the right.

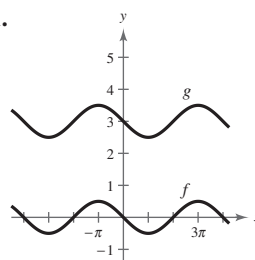
27.



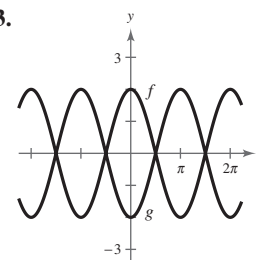
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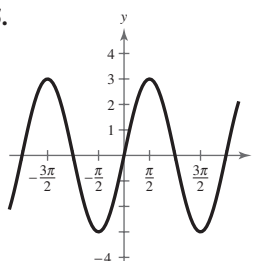
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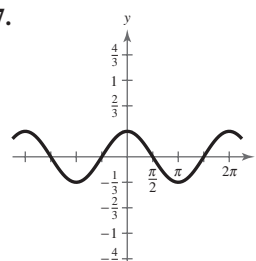
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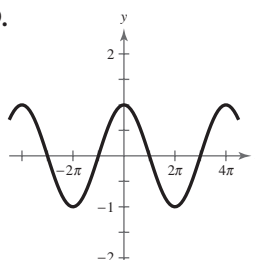
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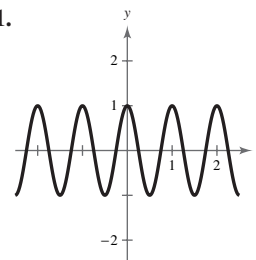
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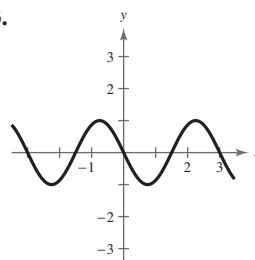
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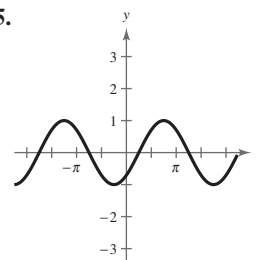
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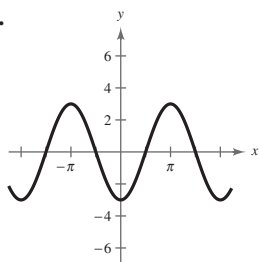
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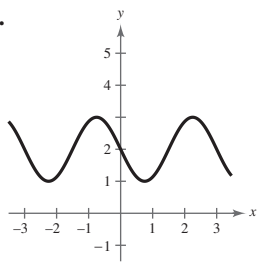
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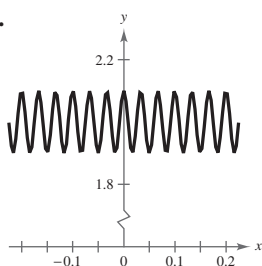
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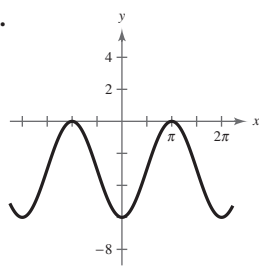
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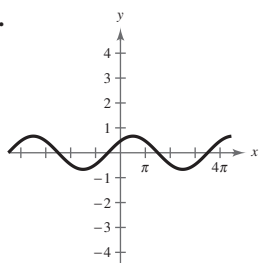
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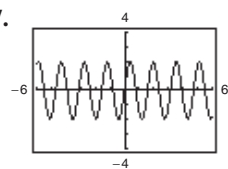
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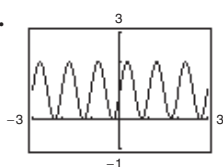
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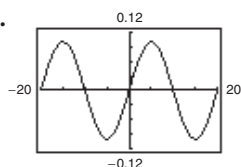
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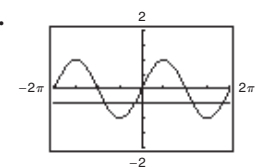
59.



61.

63.  $a = 2, d = 1$     65.  $a = -4, d = 4$ 67.  $a = -3, b = 2, c = 0$     69.  $a = 2, b = 1, c = -\frac{\pi}{4}$ 

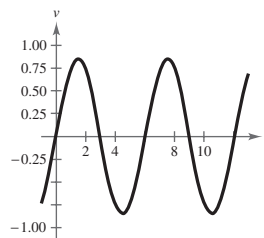
71.



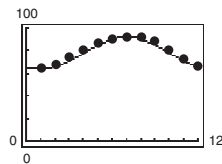
$$x = -\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

73. (a) 6 seconds    (b) 10 cycles per minute

(c)

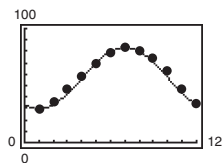
75. (a)  $C(t) = 56.55 + 26.95 \cos\left(\frac{\pi}{6}t - 3.67\right)$ 

(b)



The model is a good fit.

(c)



The model is a good fit.

(d) Tallahassee:  $77.90^\circ$ ; Chicago:  $56.55^\circ$ 

The constant term gives the annual average temperature.

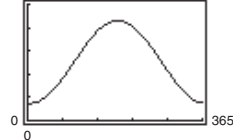
(e) 12; yes; one full period is one year.

(f) Chicago; amplitude; the greater the amplitude, the greater the variability in temperature.

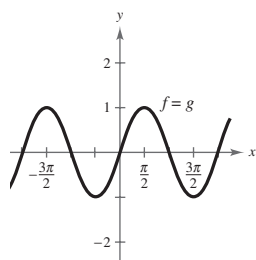
77. (a)  $\frac{1}{440}$  second    (b) 440 cycles per second

79. (a) 365; answers will vary.

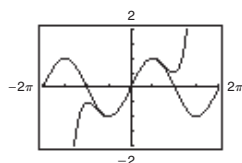
(b) 30.3 gallons; the constant term

(c)  $124 < t < 252$ 81. False. The graph of  $f(x) = \sin(x + 2\pi)$  translates the graph of  $f(x) = \sin x$  exactly one period to the left so that the two graphs look identical.83. True. Because  $\cos x = \sin\left(x + \frac{\pi}{2}\right)$ ,  $y = -\cos x$  is a reflection in the  $x$ -axis of  $y = \sin\left(x + \frac{\pi}{2}\right)$ .

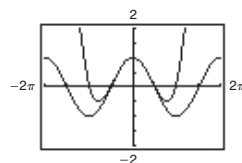
85.

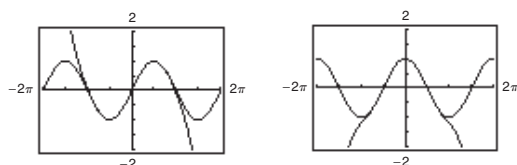

 Conjecture:  $\sin x = \cos\left(x - \frac{\pi}{2}\right)$ 

87. (a)


 The graphs appear to coincide from  $-\frac{\pi}{2}$  to  $\frac{\pi}{2}$ .

(b)


 The graphs appear to coincide from  $-\frac{\pi}{2}$  to  $\frac{\pi}{2}$ .

 (c)  $-\frac{x^7}{7!}, -\frac{x^6}{6!}$ 


The interval of accuracy increased.

 89.  $\frac{1}{2} \log_{10}(x - 2)$     91.  $3 \ln t - \ln(t - 1)$ 

 93.  $\log_{10} \sqrt{xy}$     95.  $\ln \frac{3x}{y^4}$     97. Answers will vary.

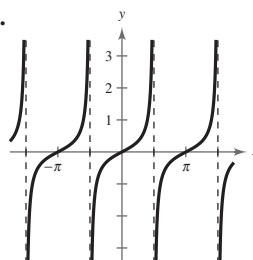
### Section 4.6 (page 339)

#### Vocabulary Check (page 339)

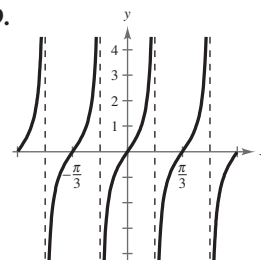
1. vertical    2. reciprocal    3. damping
4.  $\pi$     5.  $x \neq n\pi$     6.  $(-\infty, -1] \cup [1, \infty)$
7.  $2\pi$

1. e,  $\pi$     2. c,  $2\pi$     3. a, 1    4. d,  $2\pi$
5. f, 4    6. b, 4

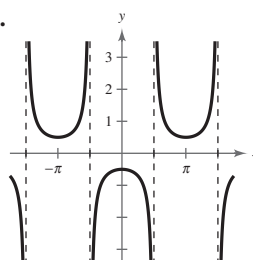
7.



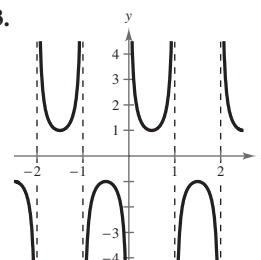
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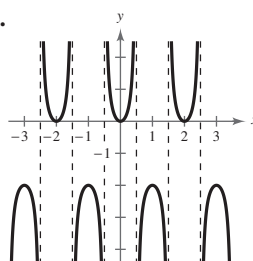
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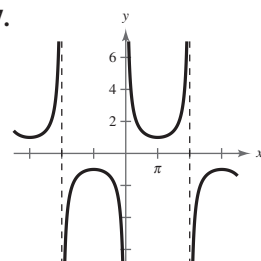
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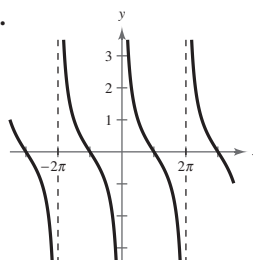
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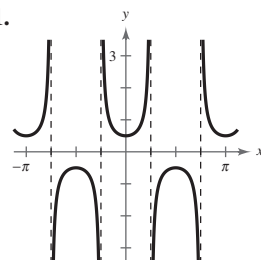
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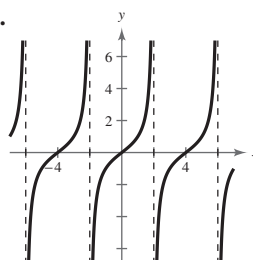
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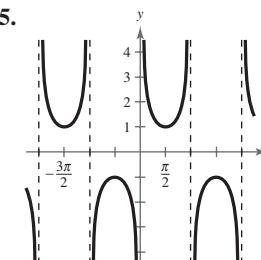
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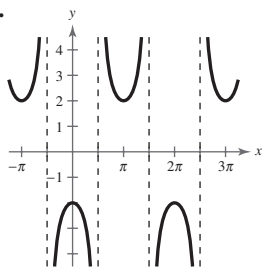
23.



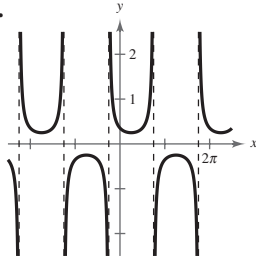
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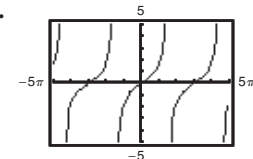
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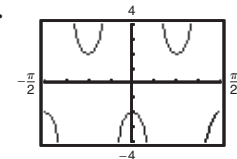
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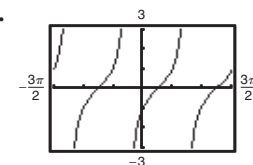
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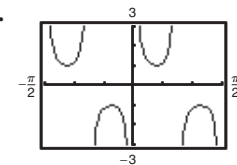
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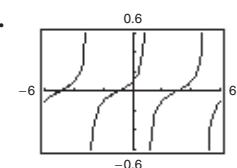
35.



37.



39.



$$41. -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$

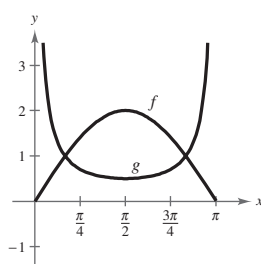
$$43. -\frac{4\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$$

$$45. -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$47. -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

49. Even

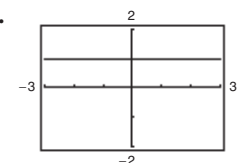
51. (a)



$$(b) \frac{\pi}{6} < x < \frac{5\pi}{6}$$

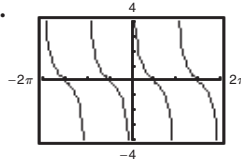
(c)  $f$  approaches 0 and  $g$  approaches  $+\infty$  because the cosecant is the reciprocal of the sine.

53.



The expressions are equivalent except that when  $\sin x = 0$ ,  $y_1$  is undefined.

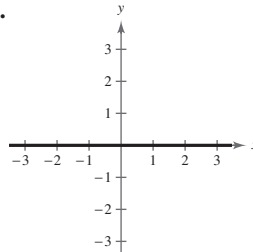
55.



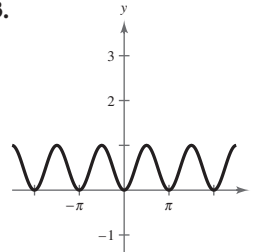
The expressions are equivalent.

57. d,  $f \rightarrow 0$  as  $x \rightarrow 0$ .58. a,  $f \rightarrow 0$  as  $x \rightarrow 0$ .59. b,  $g \rightarrow 0$  as  $x \rightarrow 0$ .60. c,  $g \rightarrow 0$  as  $x \rightarrow 0$ .

61.



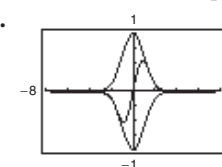
63.



The functions are equal.

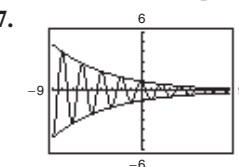
The functions are equal.

65.



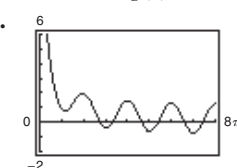
As  $x \rightarrow \infty$ ,  $g(x) \rightarrow 0$ .

67.



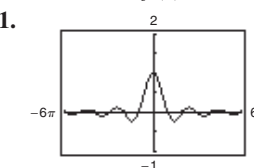
As  $x \rightarrow \infty$ ,  $f(x) \rightarrow 0$ .

69.



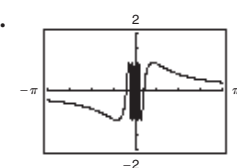
As  $x \rightarrow 0$ ,  $y \rightarrow \infty$ .

71.

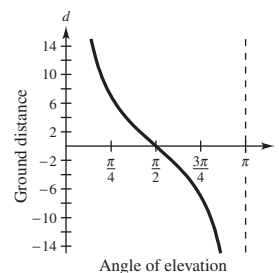


As  $x \rightarrow 0$ ,  $g(x) \rightarrow 1$ .

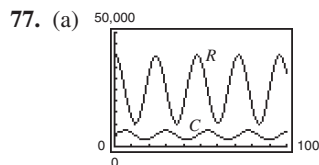
73.



As  $x \rightarrow 0$ ,  $f(x)$  oscillates between 1 and  $-1$ .

75.  $d = 7 \cot x$ 





- (b) As the predator population increases, the number of prey decreases. When the number of prey is small, the number of predators decreases.

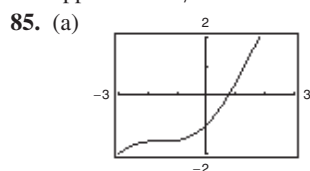
(c) C: 24 months; R: 24 months

79. (a) H: 12 months; L: 12 months

(b) Summer; winter (c) 1 month

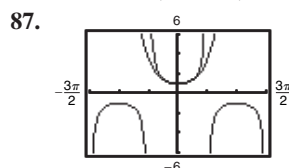
81. True. For a given value of  $x$ , the  $y$ -coordinate of  $\csc x$  is the reciprocal of the  $y$ -coordinate of  $\sin x$ .

83. As  $x$  approaches  $\pi/2$  from the left,  $f$  approaches  $\infty$ . As  $x$  approaches  $\pi/2$  from the right,  $f$  approaches  $-\infty$ .



0.7391

(b) 1, 0.5403, 0.8576, 0.6543, 0.7935, 0.7014, 0.7640, 0.7221, 0.7504, 0.7314, . . . ; 0.7391



The graphs appear to coincide on the interval  $-1.1 \leq x \leq 1.1$ .

89.  $\frac{\ln 54}{2} \approx 1.994$  91.  $-\ln 2 \approx -0.693$

93.  $\frac{2 + e^{73}}{3} \approx 1.684 \times 10^{31}$

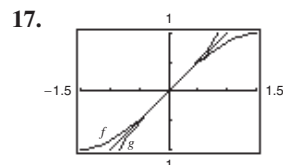
95.  $\pm \sqrt{e^{3.2} - 1} \approx \pm 4.851$  97. 2

## Section 4.7 (page 349)

### Vocabulary Check (page 349)

- $y = \sin^{-1} x$ ;  $-1 \leq x \leq 1$
- $y = \arccos x$ ;  $0 \leq y \leq \pi$
- $y = \tan^{-1} x$ ;  $-\infty < x < \infty$ ;  $-\frac{\pi}{2} < y < \frac{\pi}{2}$

- $\frac{\pi}{6}$  3.  $\frac{\pi}{3}$  5.  $\frac{\pi}{6}$  7.  $\frac{5\pi}{6}$  9.  $-\frac{\pi}{3}$
- $\frac{2\pi}{3}$  13.  $\frac{\pi}{3}$  15. 0



19. 1.29 21. -0.85 23. -1.25 25. 0.32

27. 1.99 29. 0.74 31. 0.85 33. 1.29

35.  $-\frac{\pi}{3}, -\frac{\sqrt{3}}{3}, 1$  37.  $\theta = \arctan \frac{x}{4}$

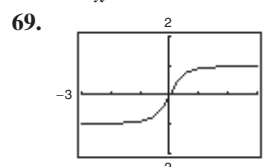
39.  $\theta = \arcsin \frac{x+2}{5}$  41.  $\theta = \arccos \frac{x+3}{2x}$

43. 0.3 45. -0.1 47. 0 49.  $\frac{3}{5}$  51.  $\frac{\sqrt{5}}{5}$

53.  $\frac{12}{13}$  55.  $\frac{\sqrt{34}}{5}$  57.  $\frac{\sqrt{5}}{3}$  59.  $\frac{1}{x}$

61.  $\sqrt{1-4x^2}$  63.  $\sqrt{1-x^2}$  65.  $\frac{\sqrt{9-x^2}}{x}$

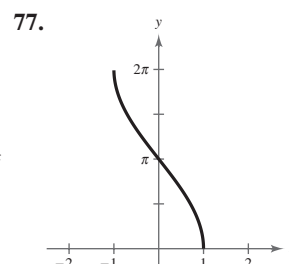
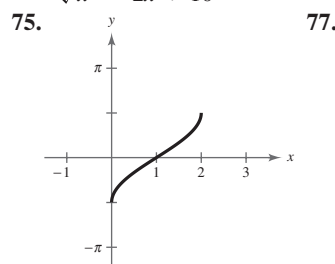
67.  $\frac{\sqrt{x^2+2}}{x}$



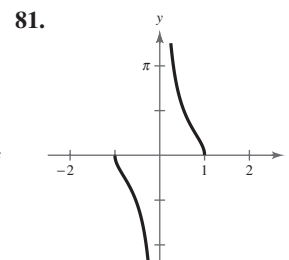
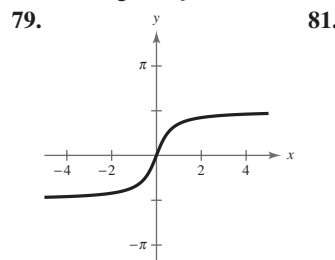
Asymptotes:  $y = \pm 1$

71.  $\frac{9}{\sqrt{x^2+81}}, x > 0$ ;  $\frac{-9}{\sqrt{x^2+81}}, x < 0$

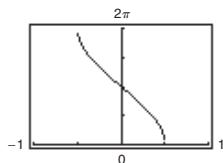
73.  $\frac{|x-1|}{\sqrt{x^2-2x+10}}$



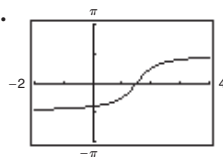
The graph of  $g$  is a horizontal shift one unit to the right of  $f$ .



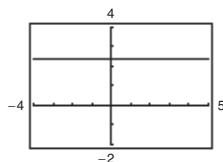
83.



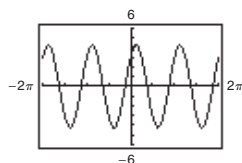
85.



87.



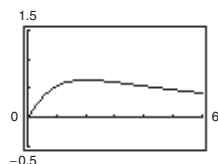
89.  $3\sqrt{2} \sin\left(2t + \frac{\pi}{4}\right)$



The graph implies that the identity is true.

91. (a)  $\theta = \arcsin \frac{5}{s}$  (b) 0.13, 0.25

93. (a)

(b) 2 feet (c)  $\beta = 0$ ; As  $x$  increases,  $\beta$  approaches 0.

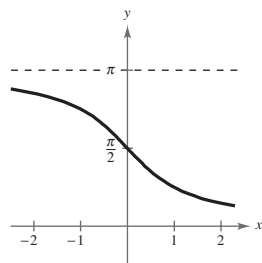
95. (a)  $\theta \approx 26.0^\circ$  (b) 24.4 feet

97. (a)  $\theta = \arctan \frac{x}{20}$  (b)  $14.0^\circ, 31.0^\circ$

99. False.  $\frac{5\pi}{4}$  is not in the range of the arctangent.

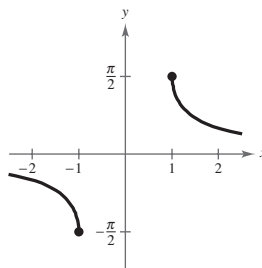
101. Domain:  $(-\infty, \infty)$

Range:  $(0, \pi)$



103. Domain:  $(-\infty, -1] \cup [1, \infty)$

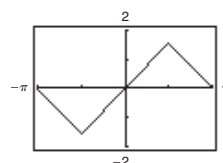
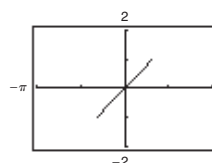
Range:  $[-\pi/2, 0) \cup (0, \pi/2]$



105. (a)  $\frac{\pi}{4}$  (b)  $\frac{\pi}{2}$  (c) 1.25 (d) 2.03

107. (a)  $f \circ f^{-1}$

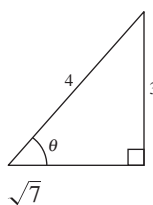
$f^{-1} \circ f$

(b) The domains and ranges of the functions are restricted. The graphs of  $f \circ f^{-1}$  and  $f^{-1} \circ f$  differ because of the domains and ranges of  $f$  and  $f^{-1}$ .

109. 1279.284

111. 117.391

113.

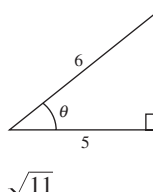


$$\cos \theta = \frac{\sqrt{7}}{4} \quad \sec \theta = \frac{4\sqrt{7}}{7}$$

$$\tan \theta = \frac{3\sqrt{7}}{7} \quad \cot \theta = \frac{\sqrt{7}}{3}$$

$$\csc \theta = \frac{4}{3}$$

115.



$$\sin \theta = \frac{\sqrt{11}}{6} \quad \sec \theta = \frac{6}{5}$$

$$\tan \theta = \frac{\sqrt{11}}{5} \quad \cot \theta = \frac{5\sqrt{11}}{11}$$

$$\csc \theta = \frac{6\sqrt{11}}{11}$$

117. Eight people

119. (a) \$21,253.63 (b) \$21,275.17

(c) \$21,285.66 (d) \$21,286.01

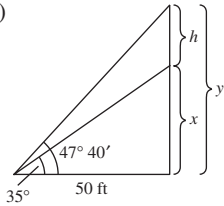
## Section 4.8 (page 359)

## Vocabulary Check (page 359)

1. elevation; depression
2. bearing
3. harmonic motion

1.  $a \approx 3.64$       3.  $a \approx 8.26$       5.  $c \approx 11.66$   
 $c \approx 10.64$        $c \approx 25.38$        $A \approx 30.96^\circ$   
 $B = 70^\circ$        $A = 19^\circ$        $B \approx 59.04^\circ$   
 7.  $a \approx 49.48$       9.  $a \approx 91.34$       11. 2.56 inches  
 $A \approx 72.08^\circ$        $b \approx 420.70$   
 $B \approx 17.92^\circ$        $B = 77^\circ 45'$   
 13. 19.99 inches      15. 107.2 feet      17. 19.7 feet

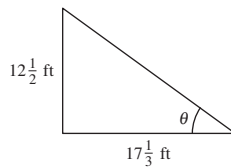
19. (a)



(b)  $h = 50(\tan 47^\circ 40' - \tan 35^\circ)$       (c) 19.9 feet

21. 2236.8 feet

23. (a)



(b)  $\tan \theta = \frac{12\frac{1}{2}}{17\frac{1}{3}}$       (c)  $35.8^\circ$

 25.  $2.06^\circ$       27. 0.73 mile

29. 554 miles north; 709 miles east

 31. (a) 58.18 nautical miles west;  
104.95 nautical miles south

 (b) S  $36.7^\circ$  W; distance = 130.9 nautical miles

 33. (a) N  $58^\circ$  E      (b) 68.82 meters

 35. N  $56.31^\circ$  W      37. 1933.3 feet

 39.  $\approx 3.23$  miles or  $\approx 17,054$  feet

 41.  $78.7^\circ$       43.  $35.3^\circ$       45. 29.4 inches

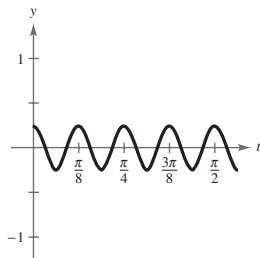
 47.  $y = \sqrt{3}r$       49.  $a \approx 12.2$ ,  $b \approx 7$ 

 51.  $d = 4 \sin(\pi t)$       53.  $d = 3 \cos\left(\frac{4\pi t}{3}\right)$ 

 55. (a) 4      (b) 4      (c) 4      (d)  $\frac{1}{16}$ 

 57. (a)  $\frac{1}{16}$       (b) 60      (c) 0      (d)  $\frac{1}{120}$       59.  $\omega = 528\pi$ 

61. (a)


 (b)  $\frac{\pi}{8}$       (c)  $\frac{\pi}{32}$ 

63. (a)

Base 1	Base 2	Altitude	Area
8	$8 + 16 \cos 30^\circ$	$8 \sin 30^\circ$	59.7
8	$8 + 16 \cos 40^\circ$	$8 \sin 40^\circ$	72.7
8	$8 + 16 \cos 50^\circ$	$8 \sin 50^\circ$	80.5
8	$8 + 16 \cos 60^\circ$	$8 \sin 60^\circ$	83.1
8	$8 + 16 \cos 70^\circ$	$8 \sin 70^\circ$	80.7
8	$8 + 16 \cos 80^\circ$	$8 \sin 80^\circ$	74.0
8	$8 + 16 \cos 90^\circ$	$8 \sin 90^\circ$	64.0

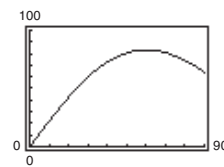
(b)

Base 1	Base 2	Altitude	Area
8	$8 + 16 \cos 56^\circ$	$8 \sin 56^\circ$	82.73
8	$8 + 16 \cos 58^\circ$	$8 \sin 58^\circ$	83.04
8	$8 + 16 \cos 59^\circ$	$8 \sin 59^\circ$	83.11
8	$8 + 16 \cos 60^\circ$	$8 \sin 60^\circ$	83.14
8	$8 + 16 \cos 61^\circ$	$8 \sin 61^\circ$	83.11
8	$8 + 16 \cos 62^\circ$	$8 \sin 62^\circ$	83.04

83.14 square feet

 (c)  $A = 64(1 + \cos \theta)(\sin \theta)$ 

(d)

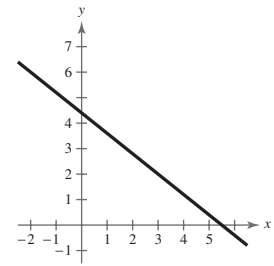
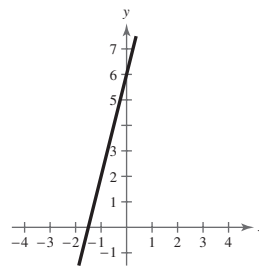

 $\approx 83.1$  square feet when  $\theta = 60^\circ$ 

The answers are the same.

65. False. The tower is leaning, so it is not perfectly vertical and does not form a right angle with the ground.

 67. No. N  $24^\circ$  E means 24 degrees east of north.

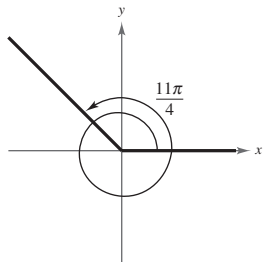
 69.  $y = 4x + 6$ 

 71.  $y = -\frac{4}{5}x + \frac{22}{5}$ 


## Review Exercises (page 365)

1. 0.5 radian

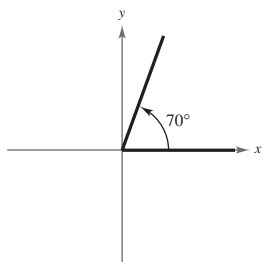
3. (a)



(b) Quadrant II

(c)  $\frac{3\pi}{4}, -\frac{5\pi}{4}$

7. (a)



(b) Quadrant I

(c)  $430^\circ, -290^\circ$

11. 8.378    13. -0.589    15. 128.571°

17.  $-200.535^\circ$     19. 478.17 inches

21. (a)  $66\frac{2}{3}\pi$  radians per minute

(b)  $400\pi$  inches per minute

23. Area = 339.28 square inches

25.  $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$     27.  $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

29.  $\sin \frac{7\pi}{6} = -\frac{1}{2}$      $\csc \frac{7\pi}{6} = -2$

$\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$      $\sec \frac{7\pi}{6} = -\frac{2\sqrt{3}}{3}$

$\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$      $\cot \frac{7\pi}{6} = \sqrt{3}$

31.  $\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$      $\csc\left(-\frac{2\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$

$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$      $\sec\left(-\frac{2\pi}{3}\right) = -2$

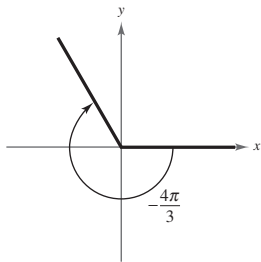
$\tan\left(-\frac{2\pi}{3}\right) = \sqrt{3}$      $\cot\left(-\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{3}$

33.  $\sin \frac{11\pi}{4} = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

35.  $\sin\left(-\frac{17\pi}{6}\right) = \sin \frac{7\pi}{6} = -\frac{1}{2}$

37. -75.3130    39. 3.2361

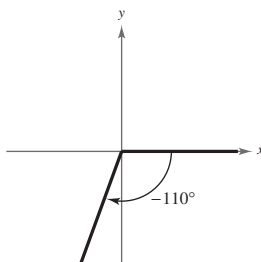
5. (a)



(b) Quadrant II

(c)  $\frac{2\pi}{3}, -\frac{10\pi}{3}$

9. (a)



(b) Quadrant III

(c)  $250^\circ, -470^\circ$

41.  $\sin \theta = \frac{4\sqrt{41}}{41}$

$\cos \theta = \frac{5\sqrt{41}}{41}$

$\tan \theta = \frac{4}{5}$

$\csc \theta = \frac{\sqrt{41}}{4}$

$\sec \theta = \frac{\sqrt{41}}{5}$

$\cot \theta = \frac{5}{4}$

43.  $\sin \theta = \frac{\sqrt{3}}{2}$

$\cos \theta = \frac{1}{2}$

$\tan \theta = \sqrt{3}$

$\csc \theta = \frac{2\sqrt{3}}{3}$

$\sec \theta = 2$

$\cot \theta = \frac{\sqrt{3}}{3}$

45. (a) 3    (b)  $\frac{2\sqrt{2}}{3}$     (c)  $\frac{3\sqrt{2}}{4}$     (d)  $\frac{\sqrt{2}}{4}$

47. (a)  $\frac{1}{4}$     (b)  $\frac{\sqrt{15}}{4}$     (c)  $\frac{4\sqrt{15}}{15}$     (d)  $\frac{\sqrt{15}}{15}$

49. 0.6494    51. 0.5621    53. 3.6722    55. 71.3 meters

57.  $\sin \theta = \frac{4}{5}$      $\csc \theta = \frac{5}{4}$

$\cos \theta = \frac{3}{5}$      $\sec \theta = \frac{5}{3}$

$\tan \theta = \frac{4}{3}$      $\cot \theta = \frac{3}{4}$

59.  $\sin \theta = \frac{15\sqrt{241}}{241}$      $\csc \theta = \frac{\sqrt{241}}{15}$

$\cos \theta = \frac{4\sqrt{241}}{241}$      $\sec \theta = \frac{\sqrt{241}}{4}$

$\tan \theta = \frac{15}{4}$      $\cot \theta = \frac{4}{15}$

61.  $\sin \theta = \frac{9\sqrt{82}}{82}$      $\csc \theta = \frac{\sqrt{82}}{9}$

$\cos \theta = \frac{-\sqrt{82}}{82}$      $\sec \theta = -\sqrt{82}$

$\tan \theta = -9$      $\cot \theta = -\frac{1}{9}$

63.  $\sin \theta = \frac{4\sqrt{17}}{17}$      $\csc \theta = \frac{\sqrt{17}}{4}$

$\cos \theta = \frac{\sqrt{17}}{17}$      $\sec \theta = \sqrt{17}$

$\tan \theta = 4$      $\cot \theta = \frac{1}{4}$

65.  $\sin \theta = -\frac{\sqrt{11}}{6}$     67.  $\cos \theta = -\frac{\sqrt{55}}{8}$

$\cos \theta = \frac{5}{6}$      $\tan \theta = -\frac{3\sqrt{55}}{55}$

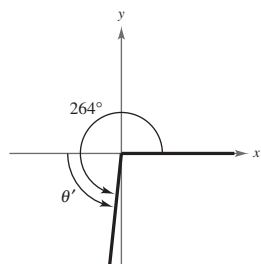
$\tan \theta = -\frac{\sqrt{11}}{5}$      $\csc \theta = \frac{8}{3}$

$\csc \theta = -\frac{6\sqrt{11}}{11}$      $\sec \theta = -\frac{8\sqrt{55}}{55}$

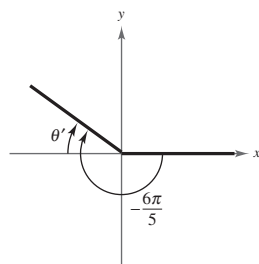
$\cot \theta = -\frac{5\sqrt{11}}{11}$      $\cot \theta = -\frac{\sqrt{55}}{3}$

$$\begin{aligned}
 69. \sin \theta &= \frac{\sqrt{21}}{5} \\
 \tan \theta &= -\frac{\sqrt{21}}{2} \\
 \csc \theta &= \frac{5\sqrt{21}}{21} \\
 \sec \theta &= -\frac{5}{2} \\
 \cot \theta &= -\frac{2\sqrt{21}}{21}
 \end{aligned}$$

$$71. \theta' = 84^\circ$$



$$73. \theta' = \frac{\pi}{5}$$



$$75. \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}; \cos \frac{\pi}{3} = \frac{1}{2}; \tan \frac{\pi}{3} = \sqrt{3}$$

$$77. \sin\left(-\frac{7\pi}{3}\right) = -\frac{\sqrt{3}}{2}; \cos\left(-\frac{7\pi}{3}\right) = \frac{1}{2}; \tan\left(-\frac{7\pi}{3}\right) = -\sqrt{3}$$

$$79. \sin 495^\circ = \frac{\sqrt{2}}{2}; \cos 495^\circ = -\frac{\sqrt{2}}{2}; \tan 495^\circ = -1$$

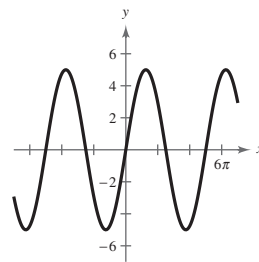
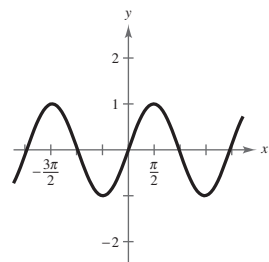
$$81. \sin(-240^\circ) = \frac{\sqrt{3}}{2}; \cos(-240^\circ) = -\frac{1}{2}; \tan(-240^\circ) = -\sqrt{3}$$

$$83. -0.7568 \quad 85. 0.0584$$

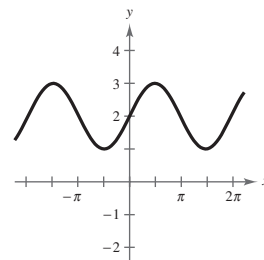
$$87. 3.2361$$

$$89.$$

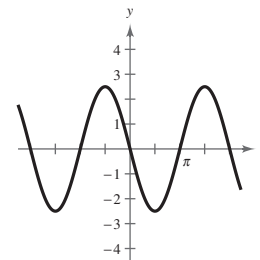
$$91.$$



$$93.$$



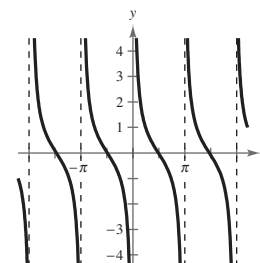
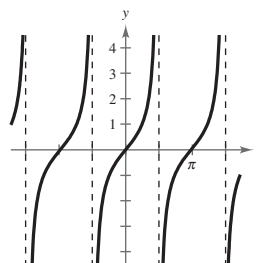
$$95.$$



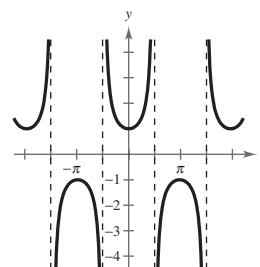
$$97. (a) y = 2 \sin 528\pi x \quad (b) 264 \text{ cycles per second}$$

$$99.$$

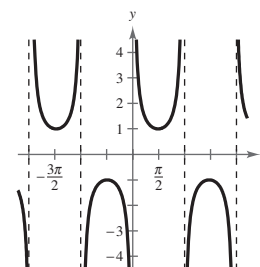
$$101.$$



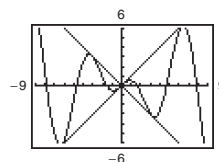
$$103.$$



$$105.$$



$$107.$$

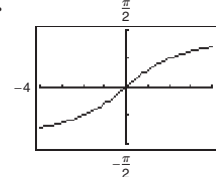
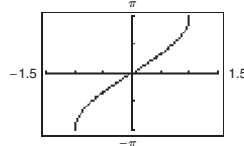


$$\text{As } x \rightarrow +\infty, f(x) \rightarrow +\infty$$

$$109. -\frac{\pi}{6} \quad 111. 0.41 \quad 113. -0.46 \quad 115. \frac{\pi}{6}$$

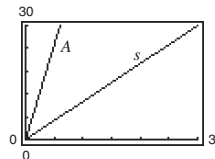
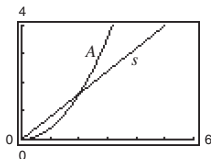
$$117. \pi \quad 119. 1.24 \quad 121. -0.98$$

$$123. \quad 125.$$



$$127. \frac{4}{5} \quad 129. \frac{13}{5} \quad 131. \frac{\sqrt{4-x^2}}{x} \quad 133. 66.8^\circ$$

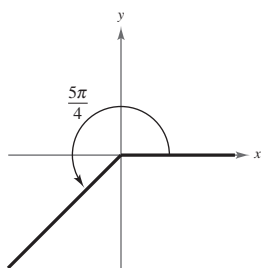
135. 1221 miles,  $85.6^\circ$   
 137. False. The sine or cosine function is often useful for modeling simple harmonic motion.  
 139. False. For each  $\theta$  there corresponds exactly one value of  $y$ .  
 141. d; The period is  $2\pi$  and the amplitude is 3.  
 143. b; The period is 2 and the amplitude is 2.  
 145. The function is undefined because  $\sec \theta = 1/\cos \theta$ .  
 147. The ranges of the other four trigonometric functions are  $(-\infty, \infty)$  or  $(-\infty, -1] \cup [1, \infty)$ .  
 149. (a)  $A = 0.4r^2, r > 0$ ;  $s = 0.8r, r > 0$   
 (b)  $A = 50\theta, \theta > 0$ ;  $s = 10\theta, \theta > 0$



The area function increases more rapidly.

### Chapter Test (page 369)

1. (a)



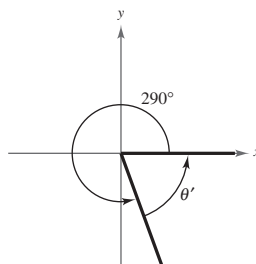
- (b)  $\frac{13\pi}{4}, -\frac{3\pi}{4}$   
 (c)  $225^\circ$

2. 3000 radians per minute      3.  $\approx 709.04$  square feet

4.  $\sin \theta = \frac{3\sqrt{10}}{10}$        $\csc \theta = \frac{\sqrt{10}}{3}$   
 $\cos \theta = -\frac{\sqrt{10}}{10}$        $\sec \theta = -\sqrt{10}$   
 $\tan \theta = -3$        $\cot \theta = -\frac{1}{3}$

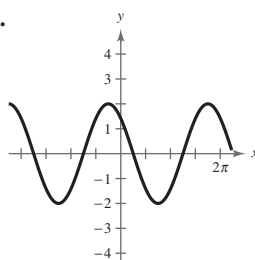
5. For  $0 \leq \theta < \frac{\pi}{2}$ :      For  $\pi \leq \theta < \frac{3\pi}{2}$ :  
 $\sin \theta = \frac{3\sqrt{13}}{13}$        $\sin \theta = -\frac{3\sqrt{13}}{13}$   
 $\cos \theta = \frac{2\sqrt{13}}{13}$        $\cos \theta = -\frac{2\sqrt{13}}{13}$   
 $\csc \theta = \frac{\sqrt{13}}{3}$        $\csc \theta = -\frac{\sqrt{13}}{3}$   
 $\sec \theta = \frac{\sqrt{13}}{2}$        $\sec \theta = -\frac{\sqrt{13}}{2}$   
 $\cot \theta = \frac{2}{3}$        $\cot \theta = \frac{2}{3}$

6.  $\theta' = 70^\circ$

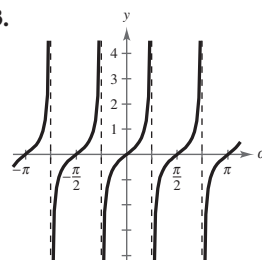


7. Quadrant III      8.  $150^\circ, 210^\circ$       9. 1.33, 1.81  
 10.  $\sin \theta = -\frac{4}{5}$       11.  $\sin \theta = \frac{15}{17}$   
 $\tan \theta = -\frac{4}{3}$        $\cos \theta = -\frac{8}{17}$   
 $\csc \theta = -\frac{5}{4}$        $\tan \theta = -\frac{15}{8}$   
 $\sec \theta = \frac{5}{3}$        $\csc \theta = \frac{17}{15}$   
 $\cot \theta = -\frac{3}{4}$        $\cot \theta = -\frac{8}{15}$

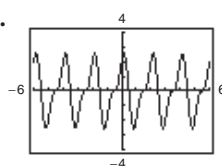
12.



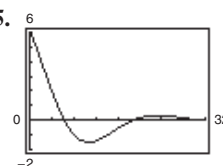
13.



14.



15.



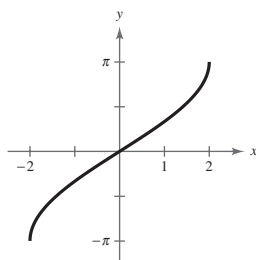
Period: 2

Not periodic

16.  $a = -2, b = \frac{1}{2}, c = -\frac{\pi}{4}$

17.  $\frac{\sqrt{5}}{2}$

18.



19.  $310.1^\circ$       20.  $d = -6 \cos \pi t$

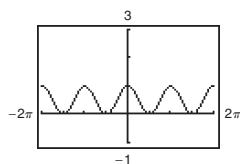
### Problem Solving (page 371)

1. (a)  $\frac{11\pi}{2}$  radians or  $990^\circ$       (b)  $\approx 816.42$  feet

3. (a) 4767 feet (b) 3705 feet  
(c)  $w = 2183$  feet,

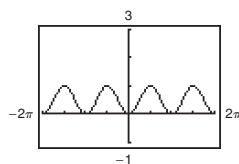
$$\tan 63^\circ = \frac{w + 3705}{3000}$$

5. (a)



Even

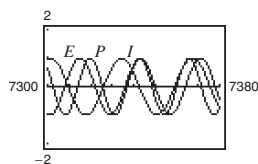
(b)



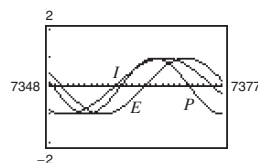
Even

7.  $h = 51 - 50 \sin\left(8\pi t + \frac{\pi}{2}\right)$

9. (a)



(b)



- (c)  $P(7369) = 0.631$   
 $E(7369) = 0.901$   
 $I(7369) = 0.945$

11. (a) 3.35, 7.35 (b)  $-0.65$   
(c) Yes. There is a difference of nine periods between the values.  
13. (a)  $40.5^\circ$  (b)  $x \approx 1.71$  feet;  $y \approx 3.46$  feet  
(c)  $\approx 1.75$  feet  
(d) As you move closer to the rock,  $d$  must get smaller and smaller. The angles  $\theta_1$  and  $\theta_2$  will decrease along with the distance  $y$ , so  $d$  will decrease.

## Chapter 5

### Section 5.1 (page 379)

#### Vocabulary Check (page 379)

1.  $\tan u$  2.  $\cos u$  3.  $\cot u$  4.  $\csc u$   
5.  $\cot^2 u$  6.  $\sec^2 u$  7.  $\cos u$  8.  $\csc u$   
9.  $\cos u$  10.  $-\tan u$

1.  $\sin x = \frac{\sqrt{3}}{2}$

$$\cos x = -\frac{1}{2}$$

$$\tan x = -\sqrt{3}$$

$$\csc x = \frac{2\sqrt{3}}{3}$$

$$\sec x = -2$$

$$\cot x = -\frac{\sqrt{3}}{3}$$

5.  $\sin x = -\frac{5}{13}$

$$\cos x = -\frac{12}{13}$$

$$\tan x = \frac{5}{12}$$

$$\sec x = -\frac{13}{12}$$

$$\csc x = -\frac{13}{5}$$

$$\cot x = \frac{12}{5}$$

9.  $\sin x = \frac{1}{3}$

$$\cos x = -\frac{2\sqrt{2}}{3}$$

$$\tan x = -\frac{\sqrt{2}}{4}$$

$$\csc x = 3$$

$$\sec x = -\frac{3\sqrt{2}}{4}$$

$$\cot x = -2\sqrt{2}$$

13.  $\sin \theta = -1$

$$\cos \theta = 0$$

$$\tan \theta \text{ is undefined.}$$

$$\cot \theta = 0$$

$$\csc \theta = -1$$

$$\sec \theta \text{ is undefined.}$$

3.  $\sin \theta = -\frac{\sqrt{2}}{2}$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$

$$\sec \theta = \sqrt{2}$$

$$\csc \theta = -\sqrt{2}$$

$$\cot \theta = -1$$

7.  $\sin \phi = -\frac{\sqrt{5}}{3}$

$$\cos \phi = \frac{2}{3}$$

$$\tan \phi = -\frac{\sqrt{5}}{2}$$

$$\sec \phi = \frac{3}{2}$$

$$\csc \phi = -\frac{3\sqrt{5}}{5}$$

$$\cot \phi = -\frac{2\sqrt{5}}{5}$$

11.  $\sin \theta = -\frac{2\sqrt{5}}{5}$

$$\cos \theta = -\frac{\sqrt{5}}{5}$$

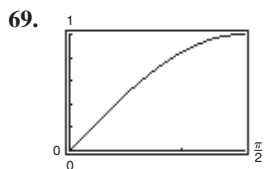
$$\tan \theta = 2$$

$$\csc \theta = -\frac{\sqrt{5}}{2}$$

$$\sec \theta = -\sqrt{5}$$

$$\cot \theta = \frac{1}{2}$$

15. d 16. a 17. b 18. f 19. e 20. c  
21. b 22. c 23. f 24. a 25. e 26. d  
27.  $\csc \theta$  29.  $\cos^2 \phi$  31.  $\cos x$  33.  $\sin^2 x$   
35. 1 37.  $\tan x$  39.  $1 + \sin y$  41.  $\sec \beta$   
43.  $\cos u + \sin u$  45.  $\sin^2 x$  47.  $\sin^2 x \tan^2 x$   
49.  $\sec x + 1$  51.  $\sec^4 x$  53.  $\sin^2 x - \cos^2 x$   
55.  $\cot^2 x (\csc x - 1)$  57.  $1 + 2 \sin x \cos x$   
59.  $4 \cot^2 x$  61.  $2 \csc^2 x$  63.  $2 \sec x$   
65.  $1 + \cos y$  67.  $3(\sec x + \tan x)$



$x$	0.2	0.4	0.6	0.8	1.0
$y_1$	0.1987	0.3894	0.5646	0.7174	0.8415
$y_2$	0.1987	0.3894	0.5646	0.7174	0.8415

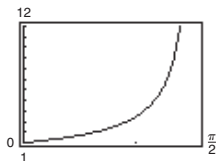
$x$	1.2	1.4
$y_1$	0.9320	0.9854
$y_2$	0.9320	0.9854

$$y_1 = y_2$$

71. 

$x$	0.2	0.4	0.6	0.8	1.0
$y_1$	1.2230	1.5085	1.8958	2.4650	3.4082
$y_2$	1.2230	1.5085	1.8958	2.4650	3.4082

$x$	1.2	1.4
$y_1$	5.3319	11.6814
$y_2$	5.3319	11.6814



$$y_1 = y_2$$

73.  $\csc x$     75.  $\tan x$     77.  $3 \sin \theta$     79.  $3 \tan \theta$

81.  $5 \sec \theta$     83.  $3 \cos \theta = 3$ ;  $\sin \theta = 0$ ;  $\cos \theta = 1$

85.  $4 \sin \theta = 2\sqrt{2}$ ;  $\sin \theta = \frac{\sqrt{2}}{2}$ ;  $\cos \theta = \frac{\sqrt{2}}{2}$

87.  $0 \leq \theta \leq \pi$     89.  $0 \leq \theta < \frac{\pi}{2}$ ,  $\frac{3\pi}{2} < \theta < 2\pi$

91.  $\ln|\cot x|$     93.  $\ln|\csc t \sec t|$

95. (a)  $\csc^2 132^\circ - \cot^2 132^\circ \approx 1.8107 - 0.8107 = 1$

(b)  $\csc^2 \frac{2\pi}{7} - \cot^2 \frac{2\pi}{7} \approx 1.6360 - 0.6360 = 1$

97. (a)  $\cos(90^\circ - 80^\circ) = \sin 80^\circ \approx 0.9848$

(b)  $\cos\left(\frac{\pi}{2} - 0.8\right) = \sin 0.8 \approx 0.7174$

99.  $\mu = \tan \theta$

101. True. For example,  $\sin(-x) = -\sin x$ .

103. 1, 1    105.  $\infty$ , 0

107. Not an identity because  $\cos \theta = \pm \sqrt{1 - \sin^2 \theta}$

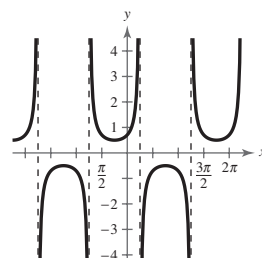
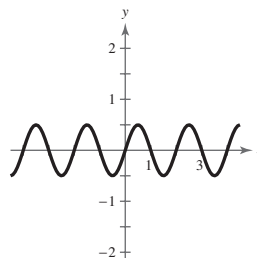
109. Not an identity because  $\frac{\sin k\theta}{\cos k\theta} = \tan k\theta$

111. An identity because  $\sin \theta \cdot \frac{1}{\sin \theta} = 1$

113. Answers will vary.    115.  $x - 25$

117.  $\frac{x^2 + 6x - 8}{(x + 5)(x - 8)}$     119.  $\frac{-5x^2 + 8x + 28}{(x^2 - 4)(x + 4)}$

121.    123.



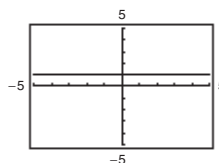
## Section 5.2 (page 387)

### Vocabulary Check (page 387)

- identity
- conditional equation
- $\tan u$
- $\cot u$
- $\cos^2 u$
- $\sin u$
- $-\csc u$
- $\sec u$

1–37. Answers will vary.

39. (a)



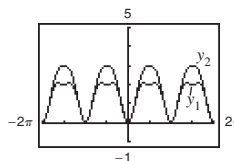
(b)

$x$	$y_1$	$y_2$
0	1	1
1	1	1
2	1	1
3	1	1
4	1	1
5	1	1
$x = -3$	1	1

Identity

(c) Answers will vary.

41. (a)



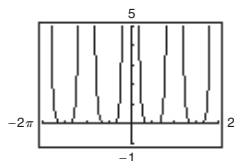
(b)

$x$	$y_1$	$y_2$
0	2	3
1	2	3
2	2	3
3	2	3
4	2	3
5	2	3
6	2	3
$x = -4.71238898038$	2	3

Not an identity

(c) Answers will vary.

43. (a)



(b)

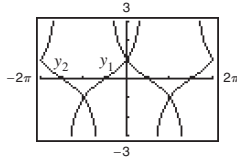
$x$	$y_1$	$y_2$
0	2.422	2.422
1	2.422	2.422
2	2.422	2.422
3	2.422	2.422
4	2.422	2.422
5	2.422	2.422
6	2.422	2.422
$x = -3$	2.422	2.422

Identity

(c) Answers will vary.



45. (a)



(b)

X	Y <sub>1</sub>	Y <sub>2</sub>
-3	-.8676	-1.153
-2	-.218	-4.588
-1	-.29341	3.4082
0	3.4082	-.218
1	-4.588	-.8676
2	-1.153	-.29341
3	-.218	-.218

Not an identity

(c) Answers will vary.

47 and 49. Answers will vary. 51. 1 53. 2

55. Answers will vary.

57. False. An identity is an equation that is true for all real values of  $\theta$ .

59. The equation is not an identity because  $\sin \theta = \pm\sqrt{1 - \cos^2 \theta}$ .

Possible answer:  $\frac{7\pi}{4}$

61.  $2 + (3 - \sqrt{26})i$  63.  $-8 + 4i$

65.  $-3 \pm \sqrt{21}$  67.  $1 \pm \sqrt{5}$

### Section 5.3 (page 396)

#### Vocabulary Check (page 396)

1. general 2. quadratic 3. extraneous

1–5. Answers will vary. 7.  $\frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

9.  $\frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$  11.  $\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

13.  $n\pi, \frac{3\pi}{2} + 2n\pi$  15.  $\frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

17.  $\frac{\pi}{8} + \frac{n\pi}{2}, \frac{3\pi}{8} + \frac{n\pi}{2}$  19.  $\frac{n\pi}{3}, \frac{\pi}{4} + n\pi$

21.  $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$  23.  $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

25.  $\frac{\pi}{3}, \frac{5\pi}{3}, \pi$  27. No solution 29.  $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

31.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$  33.  $\frac{\pi}{2}$  35.  $\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

37.  $\frac{\pi}{12} + \frac{n\pi}{3}$  39.  $\frac{\pi}{2} + 4n\pi, \frac{7\pi}{2} + 4n\pi$  41.  $-1 + 4n$

43.  $-2 + 6n, 2 + 6n$  45. 2.678, 5.820

47. 1.047, 5.236 49. 0.860, 3.426

51. 0, 2.678, 3.142, 5.820

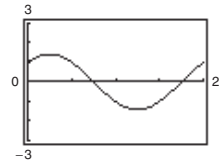
53. 0.983, 1.768, 4.124, 4.910

55. 0.3398, 0.8481, 2.2935, 2.8018

57. 1.9357, 2.7767, 5.0773, 5.9183

59.  $\frac{\pi}{4}, \frac{5\pi}{4}, \arctan 5, \arctan 5 + \pi$  61.  $\frac{\pi}{3}, \frac{5\pi}{3}$

63. (a)



(b)  $\frac{\pi}{4} \approx 0.7854$

$\frac{5\pi}{4} \approx 3.9270$

Maximum: (0.7854, 1.4142)

Minimum: (3.9270, -1.4142)

65. 1

67. (a) All real numbers  $x$  except  $x = 0$

(b)  $y$ -axis symmetry; Horizontal asymptote:  $y = 1$

(c) Oscillates (d) Infinitely many solutions

(e) Yes, 0.6366

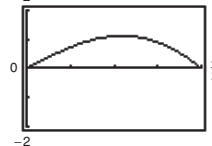
69. 0.04 second, 0.43 second, 0.83 second

71. February, March, and April 73.  $36.9^\circ, 53.1^\circ$

75. (a) Between  $t = 8$  seconds and  $t = 24$  seconds

(b) 5 times:  $t = 16, 48, 80, 112, 144$  seconds

77. (a) 2 (b)  $0.6 < x < 1.1$



$A \approx 1.12$

79. True. The first equation has a smaller period than the second equation, so it will have more solutions in the interval  $[0, 2\pi)$ .

81. 1 83.  $C = 24^\circ$

$a \approx 54.8$

$b \approx 50.1$

85.  $\sin 390^\circ = \frac{1}{2}$  87.  $\sin(-1845^\circ) = -\frac{\sqrt{2}}{2}$

$\cos 390^\circ = \frac{\sqrt{3}}{2}$   $\cos(-1845^\circ) = \frac{\sqrt{2}}{2}$

$\tan 390^\circ = \frac{\sqrt{3}}{3}$   $\tan(-1845^\circ) = -1$

89.  $1.36^\circ$  91. Answers will vary.

### Section 5.4 (page 404)

#### Vocabulary Check (page 404)

1.  $\sin u \cos v - \cos u \sin v$

2.  $\cos u \cos v - \sin u \sin v$  3.  $\frac{\tan u + \tan v}{1 - \tan u \tan v}$

4.  $\sin u \cos v + \cos u \sin v$

5.  $\cos u \cos v + \sin u \sin v$  6.  $\frac{\tan u - \tan v}{1 + \tan u \tan v}$

1. (a)  $\frac{-\sqrt{2} - \sqrt{6}}{4}$  (b)  $\frac{-1 + \sqrt{2}}{2}$

3. (a)  $\frac{\sqrt{2} - \sqrt{6}}{4}$  (b)  $\frac{\sqrt{2} + 1}{2}$

5. (a)  $\frac{1}{2}$  (b)  $\frac{-\sqrt{3} - 1}{2}$

7.  $\sin 105^\circ = \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\cos 105^\circ = \frac{\sqrt{2}}{4}(1 - \sqrt{3})$

$\tan 105^\circ = -2 - \sqrt{3}$

9.  $\sin 195^\circ = \frac{\sqrt{2}}{4}(1 - \sqrt{3})$

$\cos 195^\circ = -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\tan 195^\circ = 2 - \sqrt{3}$

11.  $\sin \frac{11\pi}{12} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

$\cos \frac{11\pi}{12} = -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\tan \frac{11\pi}{12} = -2 + \sqrt{3}$

13.  $\sin \frac{17\pi}{12} = -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\cos \frac{17\pi}{12} = \frac{\sqrt{2}}{4}(1 - \sqrt{3})$

$\tan \frac{17\pi}{12} = 2 + \sqrt{3}$

15.  $\sin 285^\circ = -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\cos 285^\circ = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

$\tan 285^\circ = -(2 + \sqrt{3})$

17.  $\sin(-165^\circ) = -\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

$\cos(-165^\circ) = -\frac{\sqrt{2}}{4}(1 + \sqrt{3})$

$\tan(-165^\circ) = 2 - \sqrt{3}$

19.  $\sin \frac{13\pi}{12} = \frac{\sqrt{2}}{4}(1 - \sqrt{3})$

$\cos \frac{13\pi}{12} = -\frac{\sqrt{2}}{4}(1 + \sqrt{3})$

$\tan \frac{13\pi}{12} = 2 - \sqrt{3}$

21.  $\sin\left(-\frac{13\pi}{12}\right) = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

$\cos\left(-\frac{13\pi}{12}\right) = -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\tan\left(-\frac{13\pi}{12}\right) = -2 + \sqrt{3}$

23.  $\cos 40^\circ$  25.  $\tan 239^\circ$  27.  $\sin 1.8$  29.  $\tan 3x$

31.  $-\frac{\sqrt{3}}{2}$  33.  $\frac{\sqrt{3}}{2}$  35.  $-1$  37.  $-\frac{63}{65}$

39.  $\frac{16}{65}$  41.  $-\frac{63}{16}$  43.  $\frac{65}{56}$  45.  $\frac{3}{5}$  47.  $-\frac{44}{117}$

49.  $\frac{5}{3}$  51.  $1$  53.  $0$  55–63. Answers will vary.

65.  $-\sin x$  67.  $-\cos \theta$  69.  $\frac{\pi}{2}$  71.  $\frac{5\pi}{4}, \frac{7\pi}{4}$

73.  $\frac{\pi}{4}, \frac{7\pi}{4}$

75. (a)  $y = \frac{5}{12} \sin(2t + 0.6435)$

(b)  $\frac{5}{12}$  feet (c)  $\frac{1}{\pi}$  cycle per second

77. False.  $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

79. False.

$\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2} = \sin x$

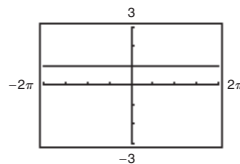
81–83. Answers will vary.

85. (a)  $\sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right)$  (b)  $\sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right)$

87. (a)  $13 \sin(3\theta + 0.3948)$  (b)  $13 \cos(3\theta - 1.1760)$

89.  $2 \cos \theta$  91. Proof 93.  $15^\circ$

95.



$\sin^2\left(\theta + \frac{\pi}{4}\right) + \sin^2\left(\theta - \frac{\pi}{4}\right) = 1$

97.  $f^{-1}(x) = \frac{x + 15}{5}$

99. Because  $f$  is not one-to-one,  $f^{-1}$  does not exist.

101.  $4x - 3$  103.  $6x - 3$

## Section 5.5 (page 415)

### Vocabulary Check (page 415)

1.  $2 \sin u \cos u$  2.  $\cos^2 u$

3.  $\cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$

4.  $\tan^2 u$  5.  $\pm \sqrt{\frac{1 - \cos u}{2}}$

6.  $\frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$

7.  $\frac{1}{2}[\cos(u - v) + \cos(u + v)]$

8.  $\frac{1}{2}[\sin(u + v) + \sin(u - v)]$

9.  $2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$

10.  $-2 \sin\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$

1.  $\frac{\sqrt{17}}{17}$     3.  $\frac{15}{17}$     5.  $\frac{8}{15}$     7.  $\frac{17}{8}$     9.  $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

11.  $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$     13.  $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

15.  $\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$     17.  $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

19.  $3 \sin 2x$     21.  $4 \cos 2x$

23.  $\sin 2u = \frac{24}{25}$   
 $\cos 2u = -\frac{7}{25}$   
 $\tan 2u = -\frac{24}{7}$

25.  $\sin 2u = \frac{24}{25}$   
 $\cos 2u = \frac{7}{25}$   
 $\tan 2u = \frac{24}{7}$

27.  $\sin 2u = -\frac{4\sqrt{21}}{25}$     29.  $\frac{1}{8}(3 + 4 \cos 2x + \cos 4x)$

$\cos 2u = -\frac{17}{25}$

$\tan 2u = \frac{4\sqrt{21}}{17}$

31.  $\frac{1}{8}(1 - \cos 4x)$

33.  $\frac{1}{16}(1 + \cos 2x - \cos 4x - \cos 2x \cos 4x)$

35.  $\frac{4\sqrt{17}}{17}$     37.  $\frac{1}{4}$     39.  $\sqrt{17}$

41.  $\sin 75^\circ = \frac{1}{2}\sqrt{2 + \sqrt{3}}$   
 $\cos 75^\circ = \frac{1}{2}\sqrt{2 - \sqrt{3}}$   
 $\tan 75^\circ = 2 + \sqrt{3}$

43.  $\sin 112^\circ 30' = \frac{1}{2}\sqrt{2 + \sqrt{2}}$   
 $\cos 112^\circ 30' = -\frac{1}{2}\sqrt{2 - \sqrt{2}}$   
 $\tan 112^\circ 30' = -1 - \sqrt{2}$

45.  $\sin \frac{\pi}{8} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$     47.  $\sin \frac{3\pi}{8} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$\cos \frac{\pi}{8} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$\cos \frac{3\pi}{8} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$

$\tan \frac{\pi}{8} = \sqrt{2} - 1$

$\tan \frac{3\pi}{8} = \sqrt{2} + 1$

49.  $\sin \frac{u}{2} = \frac{5\sqrt{26}}{26}$

$\cos \frac{u}{2} = \frac{\sqrt{26}}{26}$

$\tan \frac{u}{2} = 5$

51.  $\sin \frac{u}{2} = \sqrt{\frac{89 - 8\sqrt{89}}{178}}$   
 $\cos \frac{u}{2} = -\sqrt{\frac{89 + 8\sqrt{89}}{178}}$   
 $\tan \frac{u}{2} = \frac{8 - \sqrt{89}}{5}$

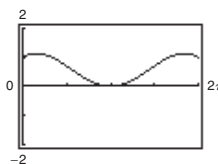
53.  $\sin \frac{u}{2} = \frac{3\sqrt{10}}{10}$

$\cos \frac{u}{2} = -\frac{\sqrt{10}}{10}$

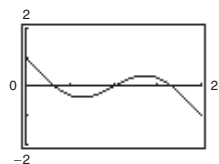
$\tan \frac{u}{2} = -3$

55.  $|\sin 3x|$     57.  $-\tan 4x$

59.  $\pi$



61.  $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$



63.  $3\left(\sin \frac{\pi}{2} + \sin 0\right)$

65.  $5(\cos 60^\circ + \cos 90^\circ)$

67.  $\frac{1}{2}(\sin 10\theta + \sin 2\theta)$

69.  $\frac{5}{2}(\cos 8\beta + \cos 2\beta)$

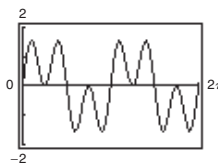
71.  $\frac{1}{2}(\cos 2y - \cos 2x)$     73.  $\frac{1}{2}(\sin 2\theta + \sin 2\pi)$

75.  $2 \cos 4\theta \sin \theta$     77.  $2 \cos 4x \cos 2x$

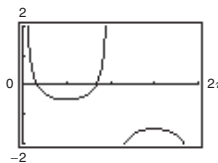
79.  $2 \cos \alpha \sin \beta$     81.  $-2 \sin \theta \sin \frac{\pi}{2}$

83.  $\frac{\sqrt{3} + 1}{2}$     85.  $-\sqrt{2}$

87.  $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

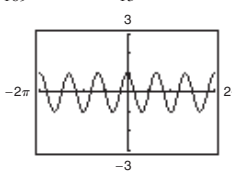


89.  $\frac{\pi}{6}, \frac{5\pi}{6}$

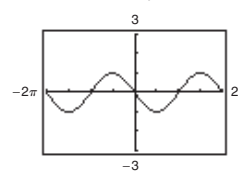


91.  $\frac{25}{169}$     93.  $\frac{4}{13}$     95–109. Answers will vary.

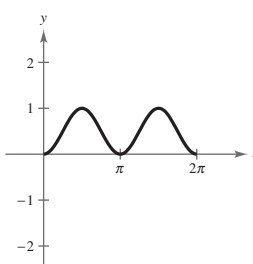
111.



113.



115.



117.  $2x\sqrt{1-x^2}$  119.  $23.85^\circ$

121. (a)  $\pi$  (b) 0.4482

(c) 760 miles per hour; 3420 miles per hour

(d)  $\theta = 2 \sin^{-1}\left(\frac{1}{M}\right)$

123. False. For  $u < 0$ ,

$$\begin{aligned}\sin 2u &= -\sin(-2u) \\ &= -2 \sin(-u) \cos(-u) \\ &= -2(-\sin u) \cos u \\ &= 2 \sin u \cos u.\end{aligned}$$

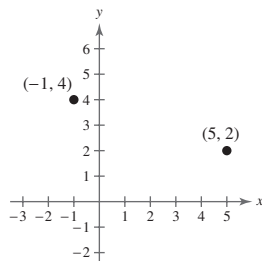
125. (a)  (b)  $\pi$ Maximum:  $(\pi, 3)$ 

127. (a)  $\frac{1}{4}(3 + \cos 4x)$  (b)  $2 \cos^4 x - 2 \cos^2 x + 1$

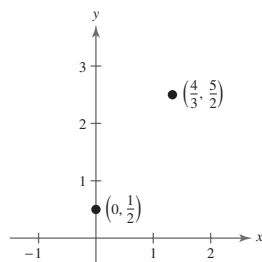
(c)  $1 - 2 \sin^2 x \cos^2 x$  (d)  $1 - \frac{1}{2} \sin^2 2x$

(e) No. There is often more than one way to rewrite a trigonometric expression.

129. (a)

(b) Distance =  $2\sqrt{10}$  (c) Midpoint:  $(2, 3)$ 

131. (a)

(b) Distance =  $\frac{2}{3}\sqrt{13}$  (c) Midpoint:  $(\frac{2}{3}, \frac{3}{2})$ 133. (a) Complement:  $35^\circ$ ; supplement:  $125^\circ$ (b) No complement; supplement:  $18^\circ$ 

135. (a) Complement:  $\frac{4\pi}{9}$ ; supplement:  $\frac{17\pi}{18}$

(b) Complement:  $\frac{\pi}{20}$ ; supplement:  $\frac{11\pi}{20}$

137. September: \$235,000; October: \$272,600

139.  $\approx 127$  feet

## Review Exercises (page 420)

1.  $\sec x$  3.  $\cos x$  5.  $\cot x$

7.  $\tan x = \frac{3}{4}$  9.  $\cos x = \frac{\sqrt{2}}{2}$

$\csc x = \frac{5}{3}$   $\tan x = -1$

$\sec x = \frac{5}{4}$   $\csc x = -\sqrt{2}$

$\cot x = \frac{4}{3}$   $\sec x = \sqrt{2}$

11.  $\sin^2 x$  13. 1 15.  $\cot \theta$  17.  $\cot^2 x$

19.  $\sec x + 2 \sin x$  21.  $-2 \tan^2 \theta$

23–31. Answers will vary.

33.  $\frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$  35.  $\frac{\pi}{6} + n\pi$

37.  $\frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$  39.  $0, \frac{2\pi}{3}, \frac{4\pi}{3}$  41.  $0, \frac{\pi}{2}, \pi$

43.  $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

45.  $0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$  47.  $0, \pi$

49.  $\arctan(-4) + \pi, \arctan(-4) + 2\pi, \arctan 3, \pi + \arctan 3$

51.  $\sin 285^\circ = -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\cos 285^\circ = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

$\tan 285^\circ = -2 - \sqrt{3}$

53.  $\sin \frac{25\pi}{12} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

$\cos \frac{25\pi}{12} = \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

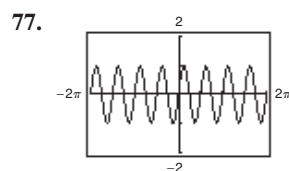
$\tan \frac{25\pi}{12} = 2 - \sqrt{3}$

55.  $\sin 15^\circ$  57.  $\tan 35^\circ$  59.  $-\frac{3}{52}(5 + 4\sqrt{7})$

61.  $\frac{1}{52}(5\sqrt{7} + 36)$  63.  $\frac{1}{52}(5\sqrt{7} - 36)$

65–69. Answers will vary. 71.  $\frac{\pi}{4}, \frac{7\pi}{4}$  73.  $\frac{\pi}{6}, \frac{11\pi}{6}$

75.  $\sin 2u = \frac{24}{25}$   
 $\cos 2u = -\frac{7}{25}$   
 $\tan 2u = -\frac{24}{7}$



79.  $\frac{1 - \cos 4x}{1 + \cos 4x}$  81.  $\frac{3 - 4 \cos 2x + \cos 4x}{4(1 + \cos 2x)}$

83.  $\sin(-75^\circ) = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\cos(-75^\circ) = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\tan(-75^\circ) = -2 - \sqrt{3}$

$$85. \sin \frac{19\pi}{12} = -\frac{1}{2}\sqrt{2+\sqrt{3}} \quad 87. \sin \frac{u}{2} = \frac{\sqrt{10}}{10}$$

$$\cos \frac{19\pi}{12} = \frac{1}{2}\sqrt{2-\sqrt{3}} \quad \cos \frac{u}{2} = \frac{3\sqrt{10}}{10}$$

$$\tan \frac{19\pi}{12} = -2 - \sqrt{3} \quad \tan \frac{u}{2} = \frac{1}{3}$$

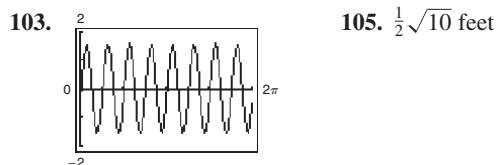
$$89. \sin \frac{u}{2} = \frac{3\sqrt{14}}{14} \quad 91. -|\cos 5x|$$

$$\cos \frac{u}{2} = \frac{\sqrt{70}}{14}$$

$$\tan \frac{u}{2} = \frac{3\sqrt{5}}{5}$$

$$93. \frac{1}{2} \sin \frac{\pi}{3} \quad 95. \frac{1}{2}(\cos 2\theta + \cos 8\theta) \quad 97. 2 \cos 3\theta \sin \theta$$

$$99. -2 \sin x \sin \frac{\pi}{6} \quad 101. \theta = 15^\circ \text{ or } \frac{\pi}{12}$$



107. False. If  $(\pi/2) < \theta < \pi$ , then  $\cos(\theta/2) > 0$ . The sign of  $\cos(\theta/2)$  depends on the quadrant in which  $\theta/2$  lies.

$$\begin{aligned} 109. \text{ True. } 4 \sin(-x) \cos(-x) &= 4(-\sin x) \cos x \\ &= -4 \sin x \cos x \\ &= -2(2 \sin x \cos x) \\ &= -2 \sin 2x \end{aligned}$$

111. Reciprocal identities:

$$\sin \theta = \frac{1}{\csc \theta}, \cos \theta = \frac{1}{\sec \theta}, \tan \theta = \frac{1}{\cot \theta},$$

$$\csc \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{1}{\tan \theta}$$

$$\text{Quotient identities: } \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\text{Pythagorean identities: } \sin^2 \theta + \cos^2 \theta = 1,$$

$$1 + \tan^2 \theta = \sec^2 \theta, 1 + \cot^2 \theta = \csc^2 \theta$$

$$113. -1 \leq \sin x \leq 1 \text{ for all } x \quad 115. y_1 = y_2 + 1$$

$$117. -1.8431, 2.1758, 3.9903, 8.8935, 9.8820$$

### Chapter Test (page 423)

$$1. \sin \theta = -\frac{3\sqrt{13}}{13} \quad 2. 1 \quad 3. 1 \quad 4. \csc \theta \sec \theta$$

$$\cos \theta = -\frac{2\sqrt{13}}{13}$$

$$\csc \theta = -\frac{\sqrt{13}}{3}$$

$$\sec \theta = -\frac{\sqrt{13}}{2}$$

$$\cot \theta = \frac{2}{3}$$

$$5. \theta = 0, \frac{\pi}{2} < \theta \leq \pi, \frac{3\pi}{2} < \theta < 2\pi$$

6.  7-12. Answers will vary.

$$13. \frac{1}{16} \left( \frac{10 - 15 \cos 2x + 6 \cos 4x - \cos 6x}{1 + \cos 2x} \right) \quad 14. \tan 2\theta$$

$$15. 2(\sin 6\theta + \sin 2\theta) \quad 16. -2 \cos \frac{7\theta}{2} \sin \frac{\theta}{2}$$

$$17. 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4} \quad 18. \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$19. \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \quad 20. \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$21. -2.938, -2.663, 1.170 \quad 22. \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$23. \sin 2u = \frac{4}{5}, \tan 2u = -\frac{4}{3}, \cos 2u = -\frac{3}{5}$$

24. Day 123 to day 223

25.  $t = 0.26$  minute

0.58 minute

0.89 minute

1.20 minutes

1.52 minutes

1.83 minutes

### Problem Solving (page 427)

$$1. (a) \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

$$\tan \theta = \pm \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$$

$$\cot \theta = \pm \frac{\sqrt{1 - \sin^2 \theta}}{\sin \theta}$$

$$\sec \theta = \pm \frac{1}{\sqrt{1 - \sin^2 \theta}}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$(b) \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

$$\tan \theta = \pm \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$$

$$\csc \theta = \pm \frac{1}{\sqrt{1 - \cos^2 \theta}}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

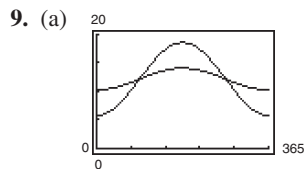
$$\cot \theta = \pm \frac{\cos \theta}{\sqrt{1 - \cos^2 \theta}}$$

3. Answers will vary. 5.  $u + v = w$

$$7. \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$$



- (b)  $t = 91, t = 274$ ; Spring Equinox and Fall Equinox  
 (c) Seward; The amplitudes: 6.4 and 1.9  
 (d) 365.2 days

11. (a)  $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$  (b)  $\frac{2\pi}{3} \leq x \leq \frac{4\pi}{3}$

(c)  $\frac{\pi}{2} < x < \pi, \frac{3\pi}{2} < x < 2\pi$

(d)  $0 \leq x \leq \frac{\pi}{4}, \frac{5\pi}{4} \leq x \leq 2\pi$

13. (a)  $\sin(u + v + w)$   
 $= \sin u \cos v \cos w - \sin u \sin v \sin w$   
 $+ \cos u \sin v \cos w + \cos u \cos v \sin w$

(b)  $\tan(u + v + w)$   
 $= \frac{\tan u + \tan v + \tan w - \tan u \tan v \tan w}{1 - \tan u \tan v - \tan u \tan w - \tan v \tan w}$

15. (a) (b) 233.3 times per second

## Chapter 6

### Section 6.1 (page 436)

#### Vocabulary Check (page 436)

1. oblique    2.  $\frac{b}{\sin B}$     3.  $\frac{1}{2}ac \sin B$

1.  $C = 105^\circ, b \approx 28.28, c \approx 38.64$   
 3.  $C = 120^\circ, b \approx 4.75, c \approx 7.17$   
 5.  $B \approx 21.55^\circ, C \approx 122.45^\circ, c \approx 11.49$   
 7.  $B = 60.9^\circ, b \approx 19.32, c \approx 6.36$   
 9.  $B = 42^\circ 4', a \approx 22.05, b \approx 14.88$   
 11.  $A \approx 10^\circ 11', C \approx 154^\circ 19', c \approx 11.03$   
 13.  $A \approx 25.57^\circ, B \approx 9.43^\circ, a \approx 10.53$   
 15.  $B \approx 18^\circ 13', C \approx 51^\circ 32', c \approx 40.06$   
 17.  $C = 83^\circ, a \approx 0.62, b \approx 0.51$

19.  $B \approx 48.74^\circ, C \approx 21.26^\circ, c \approx 48.23$

21. No solution

23. Two solutions:

$B \approx 72.21^\circ, C \approx 49.79^\circ, c \approx 10.27$

$B \approx 107.79^\circ, C \approx 14.21^\circ, c \approx 3.30$

25. (a)  $b \leq 5, b = \frac{5}{\sin 36^\circ}$  (b)  $5 < b < \frac{5}{\sin 36^\circ}$

(c)  $b > \frac{5}{\sin 36^\circ}$

27. (a)  $b \leq 10.8, b = \frac{10.8}{\sin 10^\circ}$  (b)  $10.8 < b < \frac{10.8}{\sin 10^\circ}$

(c)  $b > \frac{10.8}{\sin 10^\circ}$

29. 10.4    31. 1675.2    33. 3204.5    35. 15.3 meters

37.  $16.1^\circ$     39. 77 meters

41. (a) (b) 22.6 miles  
 (c) 21.4 miles  
 (d) 7.3 miles

43. 3.2 miles

45. True. If an angle of a triangle is obtuse (greater than  $90^\circ$ ), then the other two angles must be acute and therefore less than  $90^\circ$ . The triangle is oblique.

47. (a)  $\alpha = \arcsin(0.5 \sin \beta)$

(b) Domain:  $0 < \beta < \pi$   
 Range:  $0 < \alpha < \frac{\pi}{6}$

(c)  $c = \frac{18 \sin[\pi - \beta - \arcsin(0.5 \sin \beta)]}{\sin \beta}$

(d) Domain:  $0 < \beta < \pi$   
 Range:  $9 < c < 27$

(e)

$\beta$	0.4	0.8	1.2	1.6
$\alpha$	0.1960	0.3669	0.4848	0.5234
$c$	25.95	23.07	19.19	15.33

$\beta$	2.0	2.4	2.8
$\alpha$	0.4720	0.3445	0.1683
$c$	12.29	10.31	9.27

As  $\beta$  increases from 0 to  $\pi$ ,  $\alpha$  increases and then decreases, and  $c$  decreases from 27 to 9.

49.  $\cos x$     51.  $\sin^2 x$

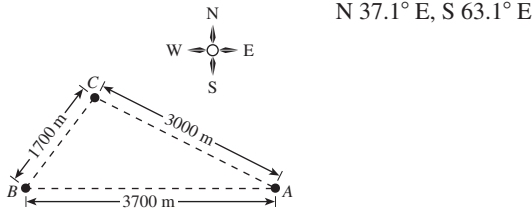
**Section 6.2** (page 443)

**Vocabulary Check** (page 443)

1. Cosines    2.  $b^2 = a^2 + c^2 - 2ac \cos B$   
3. Heron's Area Formula

1.  $A \approx 23.07^\circ, B \approx 34.05^\circ, C \approx 122.88^\circ$   
3.  $B \approx 23.79^\circ, C \approx 126.21^\circ, a \approx 18.59$   
5.  $A \approx 31.99^\circ, B \approx 42.39^\circ, C \approx 105.63^\circ$   
7.  $A \approx 92.94^\circ, B \approx 43.53^\circ, C \approx 43.53^\circ$   
9.  $B \approx 13.45^\circ, C \approx 31.55^\circ, a \approx 12.16$   
11.  $A \approx 141^\circ 45', C \approx 27^\circ 40', b \approx 11.87$   
13.  $A = 27^\circ 10', C = 27^\circ 10', b \approx 56.94$   
15.  $A \approx 33.80^\circ, B \approx 103.20^\circ, c \approx 0.54$

	$a$	$b$	$c$	$d$	$\theta$	$\phi$
17.	5	8	12.07	5.69	$45^\circ$	$135.1^\circ$
19.	10	14	20	13.86	$68.2^\circ$	$111.8^\circ$
21.	15	16.96	25	20	$77.2^\circ$	$102.8^\circ$
23.	16.25	25.	10.4	27.	52.11	
29.						



31. 373.3 meters    33.  $72.3^\circ$     35. 43.3 miles  
37. (a) N  $58.4^\circ$  W    (b) S  $81.5^\circ$  W    39. 63.7 feet  
41. 24.2 miles  
43.  $\overline{PQ} \approx 9.4, \overline{QS} = 5, \overline{RS} \approx 12.8$

45.	$d$ (inches)	9	10	12	13	14
	$\theta$ (degrees)	$60.9^\circ$	$69.5^\circ$	$88.0^\circ$	$98.2^\circ$	$109.6^\circ$
	$s$ (inches)	20.88	20.28	18.99	18.28	17.48

$d$ (inches)	15	16
$\theta$ (degrees)	$122.9^\circ$	$139.8^\circ$
$s$ (inches)	16.55	15.37

47. 46,837.5 square feet    49. \$83,336.37  
51. False. For  $s$  to be the average of the lengths of the three sides of the triangle,  $s$  would be equal to  $(a + b + c)/3$ .  
53. False. The three side lengths do not form a triangle.  
55. (a) 570.60    (b) 5910    (c) 177  
57. Answers will vary.

59.  $-\frac{\pi}{2}$     61.  $\frac{\pi}{3}$     63.  $-\frac{\pi}{3}$

65.  $\frac{1}{\sqrt{1-4x^2}}$     67.  $\frac{1}{x-2}$

69.  $\cos \theta = 1$   
 $\sec \theta = 1$   
 $\csc \theta$  is undefined.

71.  $\tan \theta = -\frac{\sqrt{3}}{3}$

$\sec \theta = \frac{2\sqrt{3}}{3}$

$\csc \theta = -2$

73.  $-2 \sin \frac{7\pi}{12} \sin \frac{\pi}{4}$

**Section 6.3** (page 456)

**Vocabulary Check** (page 456)

1. directed line segment    2. initial; terminal  
3. magnitude    4. vector  
5. standard position    6. unit vector  
7. multiplication; addition    8. resultant  
9. linear combination; horizontal; vertical

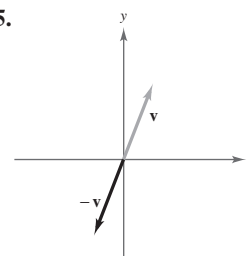
1.  $\|\mathbf{u}\| = \|\mathbf{v}\| = \sqrt{17}$ ,  $\text{slope}_{\mathbf{u}} = \text{slope}_{\mathbf{v}} = \frac{1}{4}$   
 $\mathbf{u}$  and  $\mathbf{v}$  have the same magnitude and direction, so they are equal.

3.  $\mathbf{v} = \langle 3, 2 \rangle$ ;  $\|\mathbf{v}\| = \sqrt{13}$     5.  $\mathbf{v} = \langle -3, 2 \rangle$ ;  $\|\mathbf{v}\| = \sqrt{13}$

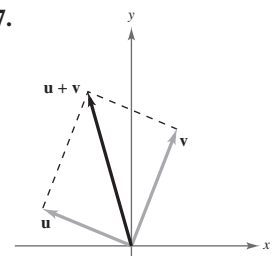
7.  $\mathbf{v} = \langle 0, 5 \rangle$ ;  $\|\mathbf{v}\| = 5$     9.  $\mathbf{v} = \langle 16, 7 \rangle$ ;  $\|\mathbf{v}\| = \sqrt{305}$

11.  $\mathbf{v} = \langle 8, 6 \rangle$ ;  $\|\mathbf{v}\| = 10$     13.  $\mathbf{v} = \langle -9, -12 \rangle$ ;  $\|\mathbf{v}\| = 15$

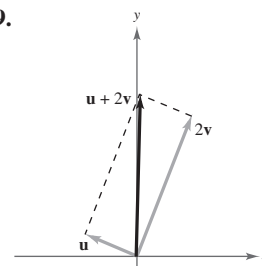
15.

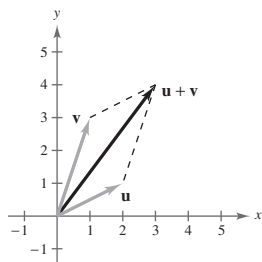
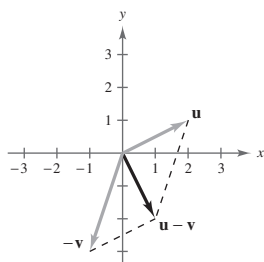
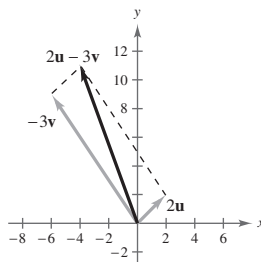
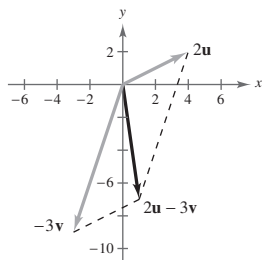
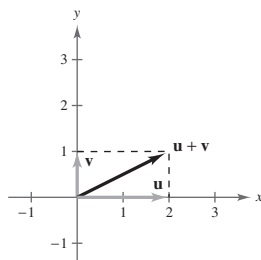
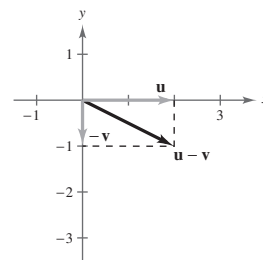
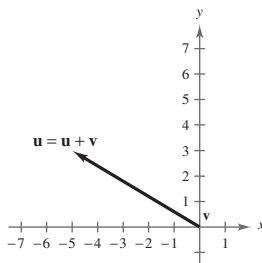
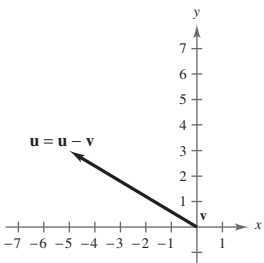
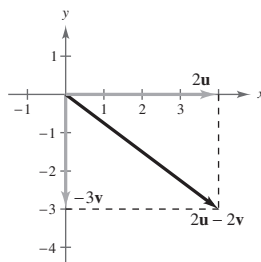
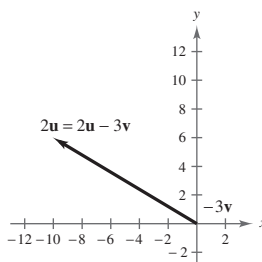
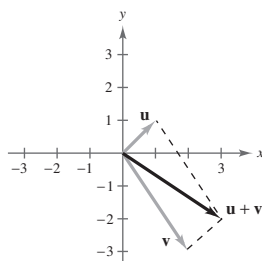
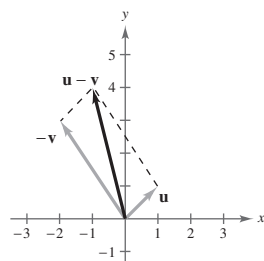
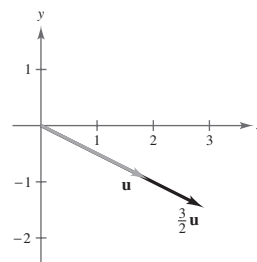


17.



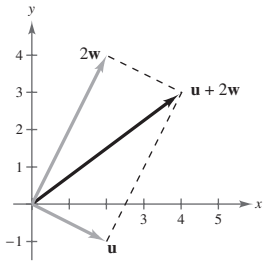
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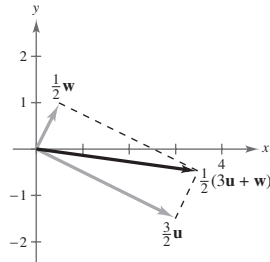
21. (a)  $\langle 3, 4 \rangle$ (b)  $\langle 1, -2 \rangle$ (c)  $-4\mathbf{i} + 11\mathbf{j}$ (c)  $\langle 1, -7 \rangle$ 27. (a)  $2\mathbf{i} + \mathbf{j}$ (b)  $2\mathbf{i} - \mathbf{j}$ 23. (a)  $\langle -5, 3 \rangle$ (b)  $\langle -5, 3 \rangle$ (c)  $4\mathbf{i} - 3\mathbf{j}$ (c)  $\langle -10, 6 \rangle$ 25. (a)  $3\mathbf{i} - 2\mathbf{j}$ (b)  $-\mathbf{i} + 4\mathbf{j}$ 29.  $\langle 1, 0 \rangle$  31.  $\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$  33.  $\frac{3\sqrt{10}}{10}\mathbf{i} - \frac{\sqrt{10}}{10}\mathbf{j}$ 35.  $\mathbf{j}$  37.  $\frac{\sqrt{5}}{5}\mathbf{i} - \frac{2\sqrt{5}}{5}\mathbf{j}$  39.  $\left\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$ 41.  $\left\langle \frac{18\sqrt{29}}{29}, \frac{45\sqrt{29}}{29} \right\rangle$  43.  $7\mathbf{i} + 4\mathbf{j}$  45.  $3\mathbf{i} + 8\mathbf{j}$ 47.  $\mathbf{v} = \left\langle 3, -\frac{3}{2} \right\rangle$ 



49.  $\mathbf{v} = \langle 4, 3 \rangle$



51.  $\mathbf{v} = \langle \frac{7}{2}, -\frac{1}{2} \rangle$

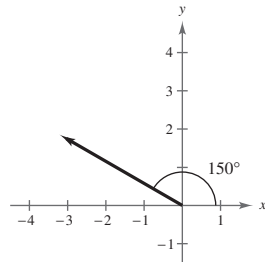
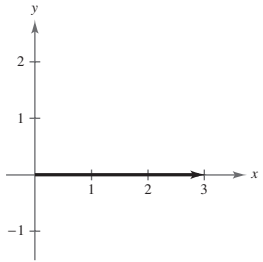


53.  $\|\mathbf{v}\| = 3; \theta = 60^\circ$

55.  $\|\mathbf{v}\| = 6\sqrt{2}; \theta = 315^\circ$

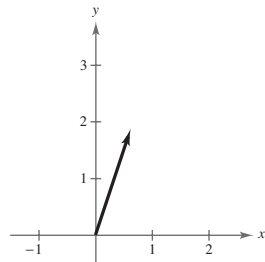
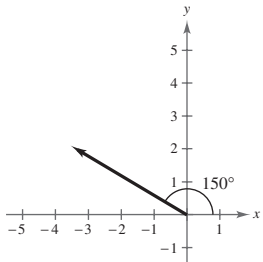
57.  $\mathbf{v} = \langle 3, 0 \rangle$

59.  $\mathbf{v} = \langle -\frac{7\sqrt{3}}{4}, \frac{7}{4} \rangle$



61.  $\mathbf{v} = \langle -\frac{3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2} \rangle$

63.  $\mathbf{v} = \langle \frac{\sqrt{10}}{5}, \frac{3\sqrt{10}}{5} \rangle$



65.  $\langle 5, 5 \rangle$

67.  $\langle 10\sqrt{2} - 50, 10\sqrt{2} \rangle$

69.  $90^\circ$

71.  $62.7^\circ$

73.  $12.8^\circ; 398.32$  newtons

75.  $71.3^\circ; 228.5$  pounds

77. Vertical component:  $70 \sin 35^\circ \approx 40.15$  feet per second

Horizontal component:  $70 \cos 35^\circ \approx 57.34$  feet per second

79.  $T_{AC} \approx 1758.8$  pounds

81.  $3154.4$  pounds

$T_{BC} \approx 1305.4$  pounds

83. N  $21.4^\circ$  E;  $138.7$  kilometers per hour

85.  $1928.4$  foot-pounds

87. True. See Example 1.

89. (a)  $0^\circ$  (b)  $180^\circ$

(c) No. The magnitude is at most equal to the sum when the angle between the vectors is  $0^\circ$ .

91. Answers will vary.

93.  $\langle 1, 3 \rangle$  or  $\langle -1, -3 \rangle$

95.  $8 \tan \theta$

97.  $6 \sec \theta$

99.  $\frac{\pi}{2} + n\pi, \pi + 2n\pi$

101.  $n\pi, \frac{\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$

## Section 6.4 (page 467)

## Vocabulary Check (page 467)

1. dot product

2.  $\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$

3. orthogonal

4.  $\left( \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v}$

5.  $\|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \|\overrightarrow{PQ}\|; \mathbf{F} \cdot \overrightarrow{PQ}$

1.  $-9$

3.  $-11$

5.  $6$

7.  $-12$

9.  $8$ ; scalar

11.  $\langle -6, 8 \rangle$ ; vector

13.  $\langle -66, -66 \rangle$ ; vector

15.  $\sqrt{5} - 1$ ; scalar

17.  $4$ ; scalar

19.  $13$

21.  $5\sqrt{41}$

23.  $6$

25.  $90^\circ$

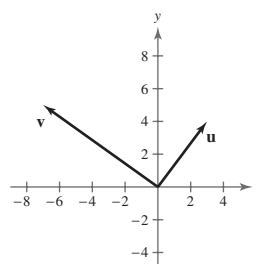
27.  $143.13^\circ$

29.  $60.26^\circ$

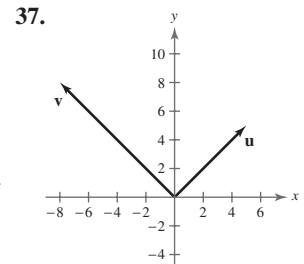
31.  $90^\circ$

33.  $\frac{5\pi}{12}$

35.



$\approx 91.33^\circ$



$90^\circ$

39.  $26.57^\circ, 63.43^\circ, 90^\circ$

41.  $41.63^\circ, 53.13^\circ, 85.24^\circ$

43.  $-20$

45.  $-229.1$

47. Parallel

49. Neither

51. Orthogonal

53.  $\frac{1}{37}\langle 84, 14 \rangle, \frac{1}{37}\langle -10, 60 \rangle$

55.  $\frac{45}{229}\langle 2, 15 \rangle, \frac{6}{229}\langle -15, 2 \rangle$

57.  $0$

59.  $\langle -5, 3 \rangle, \langle 5, -3 \rangle$

61.  $\frac{2}{3}\mathbf{i} + \frac{1}{2}\mathbf{j}, -\frac{2}{3}\mathbf{i} - \frac{1}{2}\mathbf{j}$

63.  $32$

65. (a)  $\$58,762.50$ ; This value gives the total revenue that can be earned by selling all of the units.

(b)  $1.05\mathbf{v}$

67. (a) Force =  $30,000 \sin d$

(b)

$d$	$0^\circ$	$1^\circ$	$2^\circ$	$3^\circ$	$4^\circ$	$5^\circ$
Force	0	523.6	1047.0	1570.1	2092.7	2614.7

$d$	$6^\circ$	$7^\circ$	$8^\circ$	$9^\circ$	$10^\circ$
Force	3135.9	3656.1	4175.2	4693.0	5209.4

(c)  $29,885.8$  pounds

69.  $735$  newton-meters

71.  $779.4$  foot-pounds

73.  $21,650.64$  foot-pounds

75. False. Work is represented by a scalar.

77. (a)  $\theta = \frac{\pi}{2}$  (b)  $0 \leq \theta < \frac{\pi}{2}$  (c)  $\frac{\pi}{2} < \theta \leq \pi$

79. Answers will vary.

81.  $12\sqrt{7}$

83.  $-2\sqrt{6}$

85.  $0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$

87.  $0, \pi$

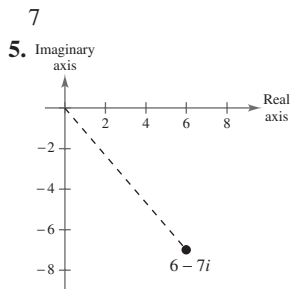
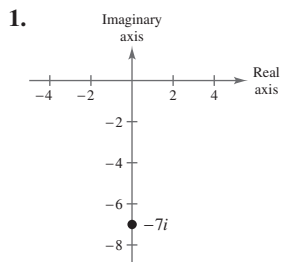
89.  $-\frac{253}{325}$

91.  $\frac{204}{325}$

## Section 6.5 (page 478)

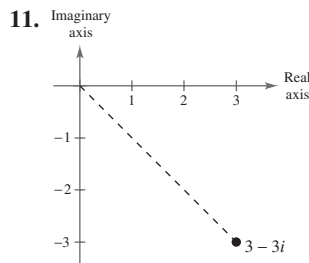
**Vocabulary Check** (page 478)

1. absolute value
2. trigonometric form; modulus; argument
3. DeMoivre's      4.  $n$ th root

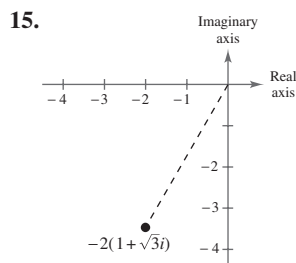


$\sqrt{85}$

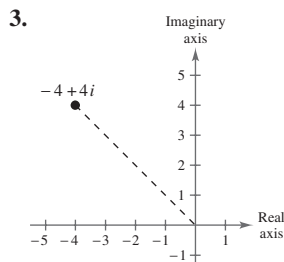
7.  $3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$



$3\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$

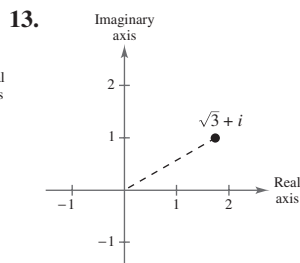


$4\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$

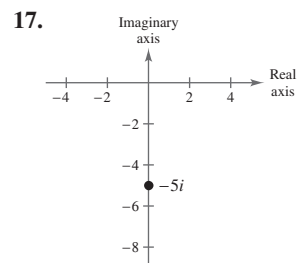


$4\sqrt{2}$

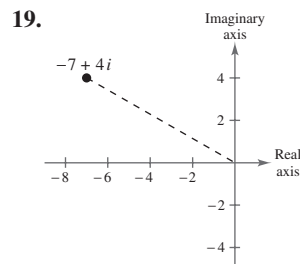
9.  $\sqrt{10}(\cos 5.96 + i \sin 5.96)$



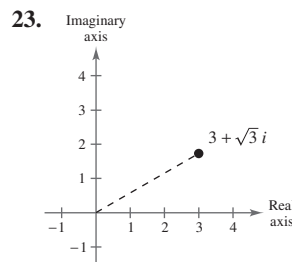
$2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$



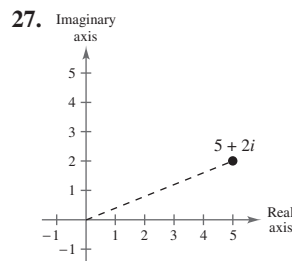
$5\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$



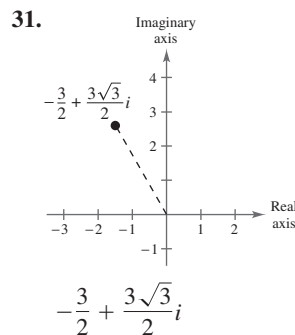
$\sqrt{65}(\cos 2.62 + i \sin 2.62)$



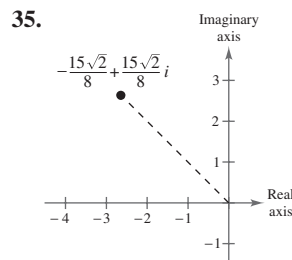
$2\sqrt{3}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$



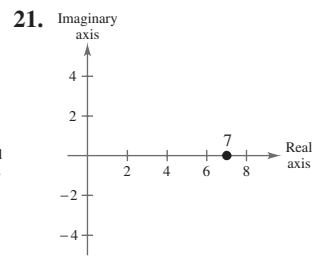
$\sqrt{29}(\cos 0.38 + i \sin 0.38)$



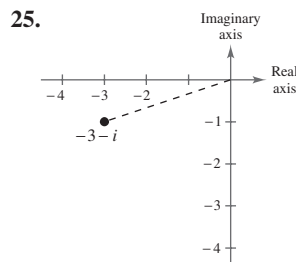
$-\frac{3}{2} + \frac{3\sqrt{3}}{2}i$



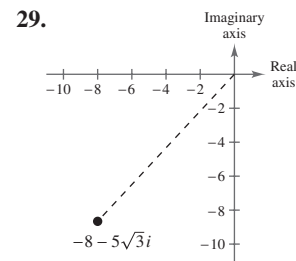
$-\frac{15\sqrt{2}}{8} + \frac{15\sqrt{2}}{8}i$



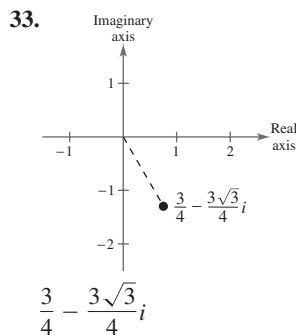
$7(\cos 0 + i \sin 0)$



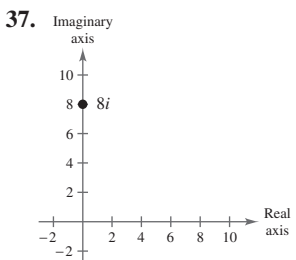
$\sqrt{10}(\cos 3.46 + i \sin 3.46)$



$\sqrt{139}(\cos 3.97 + i \sin 3.97)$

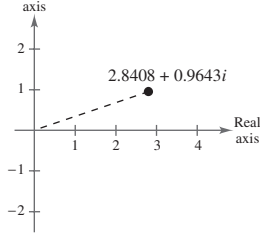


$\frac{3}{4} - \frac{3\sqrt{3}}{4}i$



$8i$

39. Imaginary axis

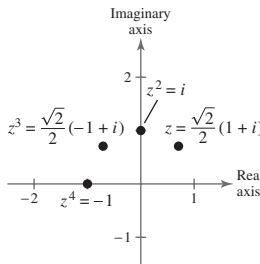


$$2.8408 + 0.9643i$$

41.  $4.6985 + 1.7101i$

43.  $-2.9044 + 0.7511i$

45.



The absolute value of each is 1.

47.  $12\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

49.  $\frac{10}{9}(\cos 200^\circ + i \sin 200^\circ)$

51.  $0.27(\cos 150^\circ + i \sin 150^\circ)$

53.  $\cos 30^\circ + i \sin 30^\circ$

55.  $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$

57.  $4(\cos 302^\circ + i \sin 302^\circ)$

59. (a)  $\left[2\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]\left[\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)\right]$

(b)  $4(\cos 0 + i \sin 0) = 4$

(c) 4

61. (a)  $\left[2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)\right]\left[\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]$

(b)  $2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) = 2 - 2i$

(c)  $-2i - 2i^2 = -2i + 2 = 2 - 2i$

63. (a)  $[5(\cos 0.93 + i \sin 0.93)] \div \left[2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)\right]$

(b)  $\frac{5}{2}(\cos 1.97 + i \sin 1.97) \approx -0.982 + 2.299i$

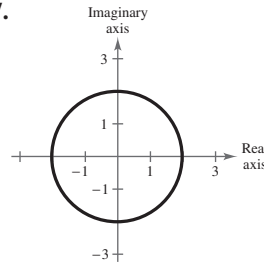
(c)  $\approx -0.982 + 2.299i$

65. (a)  $[5(\cos 0 + i \sin 0)] \div [\sqrt{13}(\cos 0.98 + i \sin 0.98)]$

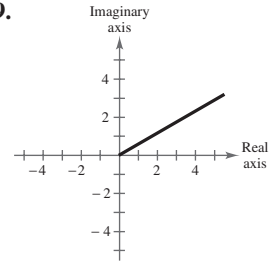
(b)  $\frac{5}{\sqrt{13}}(\cos 5.30 + i \sin 5.30) \approx 0.769 - 1.154i$

(c)  $\frac{10}{13} - \frac{15}{13}i \approx 0.769 - 1.154i$

67.



69.



71.  $-4 - 4i$

73.  $-32i$

75.  $-128\sqrt{3} - 128i$

77.  $\frac{125}{2} + \frac{125\sqrt{3}}{2}i$

79.  $-1$

81.  $608.0 + 144.7i$

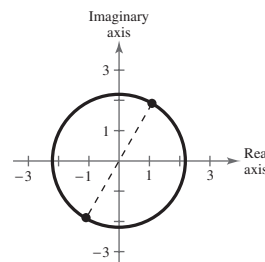
83.  $-597 - 122i$

85.  $\frac{81}{2} + \frac{81\sqrt{3}}{2}i$

87.  $32i$

89. (a)  $\sqrt{5}(\cos 60^\circ + i \sin 60^\circ)$   
 $\sqrt{5}(\cos 240^\circ + i \sin 240^\circ)$

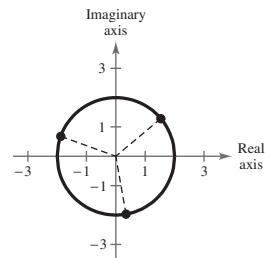
(b)



(c)  $\frac{\sqrt{5}}{2} + \frac{\sqrt{15}}{2}i, -\frac{\sqrt{5}}{2} - \frac{\sqrt{15}}{2}i$

91. (a)  $2\left(\cos \frac{2\pi}{9} + i \sin \frac{2\pi}{9}\right)$   
 $2\left(\cos \frac{8\pi}{9} + i \sin \frac{8\pi}{9}\right)$   
 $2\left(\cos \frac{14\pi}{9} + i \sin \frac{14\pi}{9}\right)$

(b)



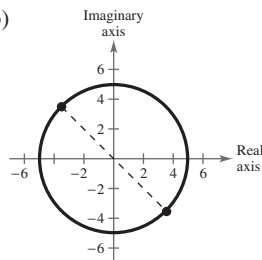
(c)  $1.5321 + 1.2856i, -1.8794 + 0.6840i,$   
 $0.3473 - 1.9696i$

93. (a)  $5\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$  (b)

$$5\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

(c)  $-\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$

$$\frac{5\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}i$$

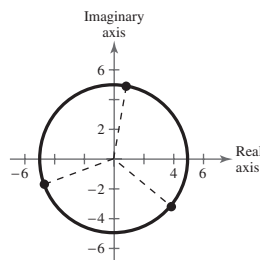


95. (a)  $5\left(\cos \frac{4\pi}{9} + i \sin \frac{4\pi}{9}\right)$

$$5\left(\cos \frac{10\pi}{9} + i \sin \frac{10\pi}{9}\right)$$

$$5\left(\cos \frac{16\pi}{9} + i \sin \frac{16\pi}{9}\right)$$

(b)



(c)  $0.8682 + 4.9240i$ ,  
 $-4.6985 - 1.7101i$ ,  
 $3.8302 - 3.2140i$

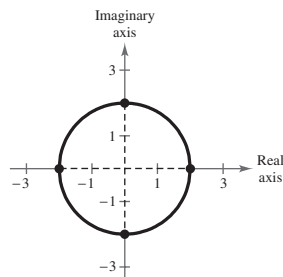
97. (a)  $2(\cos 0 + i \sin 0)$

$$2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

$$2(\cos \pi + i \sin \pi)$$

$$2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$$

(b)



(c)  $2, 2i, -2, -2i$

99. (a)  $\cos 0 + i \sin 0$

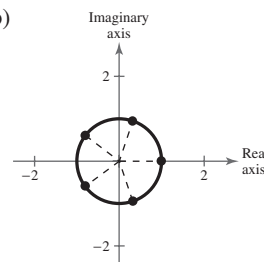
$$\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$$

(b)

$$\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$$

$$\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$$

$$\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}$$



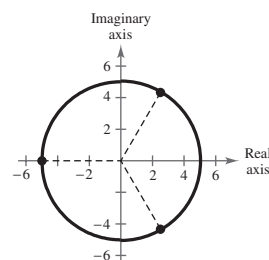
(c)  $1, 0.3090 + 0.9511i$ ,  
 $-0.8090 + 0.5878i, -0.8090 - 0.5878i$ ,  
 $0.3090 - 0.9511i$

101. (a)  $5\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

$$5(\cos \pi + i \sin \pi)$$

$$5\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$$

(b)



(c)  $\frac{5}{2} + \frac{5\sqrt{3}}{2}i, -5, \frac{5}{2} - \frac{5\sqrt{3}}{2}i$

103. (a)  $2\sqrt{2}\left(\cos \frac{3\pi}{20} + i \sin \frac{3\pi}{20}\right)$

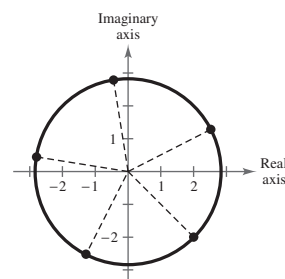
$$2\sqrt{2}\left(\cos \frac{11\pi}{20} + i \sin \frac{11\pi}{20}\right)$$

$$2\sqrt{2}\left(\cos \frac{19\pi}{20} + i \sin \frac{19\pi}{20}\right)$$

$$2\sqrt{2}\left(\cos \frac{27\pi}{20} + i \sin \frac{27\pi}{20}\right)$$

$$2\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

(b)



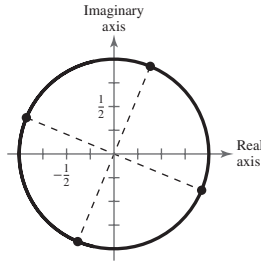
(c)  $2.5201 + 1.2841i, -0.4425 + 2.7936i$ ,  
 $-2.7936 + 0.4425i, -1.2841 - 2.5201i, 2 - 2i$

$$105. \cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}$$

$$\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}$$

$$\cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8}$$

$$\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8}$$



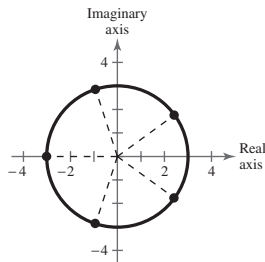
$$107. 3\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$$

$$3\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$$

$$3(\cos \pi + i \sin \pi)$$

$$3\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$$

$$3\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$$

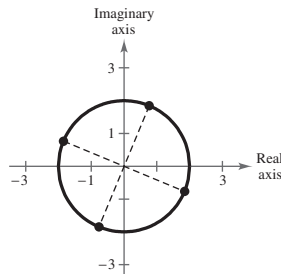


$$109. 2\left(\cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}\right)$$

$$2\left(\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}\right)$$

$$2\left(\cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8}\right)$$

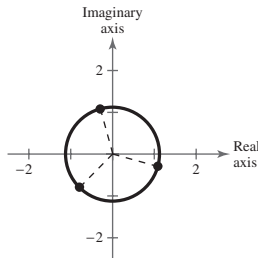
$$2\left(\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8}\right)$$



$$111. \sqrt[6]{2}\left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}\right)$$

$$\sqrt[6]{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$$

$$\sqrt[6]{2}\left(\cos \frac{23\pi}{12} + i \sin \frac{23\pi}{12}\right)$$



113. True, by the definition of the absolute value of a complex number.

115. True.  $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)] = 0$  if and only if  $r_1 = 0$  and/or  $r_2 = 0$ .

117. Answers will vary.

119. (a)  $r^2$  (b)  $\cos 2\theta + i \sin 2\theta$

121. Answers will vary.

123. (a)  $2(\cos 30^\circ + i \sin 30^\circ)$  (b)  $8i$   
 $2(\cos 150^\circ + i \sin 150^\circ)$   
 $2(\cos 270^\circ + i \sin 270^\circ)$

125.  $B = 68^\circ, b \approx 19.80, c \approx 21.36$

127.  $B = 60^\circ, a \approx 65.01, c \approx 130.02$

129.  $B = 47^\circ 45', a \approx 7.53, b \approx 8.29$

131. 16; 2    133.  $\frac{1}{16}; \frac{4}{5}$     135.  $3(\sin 11\theta + \sin 5\theta)$

## Review Exercises (page 482)

1.  $C = 74^\circ, b \approx 13.19, c \approx 13.41$

3.  $A = 26^\circ, a \approx 24.89, c \approx 56.23$

5.  $C = 66^\circ, a \approx 2.53, b \approx 9.11$

7.  $B = 108^\circ, a \approx 11.76, c \approx 21.49$

9.  $A \approx 20.41^\circ, C \approx 9.59^\circ, a \approx 20.92$

11.  $B \approx 39.48^\circ, C \approx 65.52^\circ, c \approx 48.24$

13. 7.9    15. 33.5

17. 31.1 meters    19. 31.01 feet

21.  $A \approx 29.69^\circ, B \approx 52.41^\circ, C \approx 97.90^\circ$

23.  $A \approx 29.92^\circ, B \approx 86.18^\circ, C \approx 63.90^\circ$

25.  $A = 35^\circ, C = 35^\circ, b \approx 6.55$

27.  $A \approx 45.76^\circ, B \approx 91.24^\circ, c \approx 21.42$

29.  $\approx 4.3$  feet,  $\approx 12.6$  feet

31. 615.1 meters    33. 9.80    35. 8.36

37.  $\|\mathbf{u}\| = \|\mathbf{v}\| = \sqrt{61}$ ,  $\text{slope}_{\mathbf{u}} = \text{slope}_{\mathbf{v}} = \frac{5}{6}$

39.  $\langle 7, -5 \rangle$     41.  $\langle 7, -7 \rangle$     43.  $\langle -4, 4\sqrt{3} \rangle$

45. (a)  $\langle -4, 3 \rangle$  (b)  $\langle 2, -9 \rangle$  (c)  $\langle -3, -9 \rangle$

(d)  $\langle -11, -3 \rangle$

47. (a)  $\langle -1, 6 \rangle$  (b)  $\langle -9, -2 \rangle$  (c)  $\langle -15, 6 \rangle$

(d)  $\langle -17, 18 \rangle$

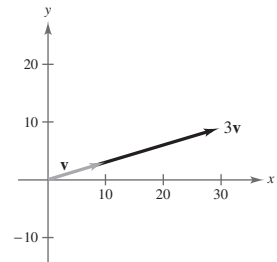
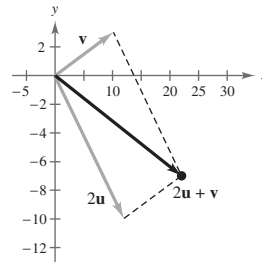
49. (a)  $7\mathbf{i} + 2\mathbf{j}$  (b)  $-3\mathbf{i} - 4\mathbf{j}$  (c)  $6\mathbf{i} - 3\mathbf{j}$

(d)  $20\mathbf{i} + \mathbf{j}$

51. (a)  $3\mathbf{i} + 6\mathbf{j}$  (b)  $5\mathbf{i} - 6\mathbf{j}$  (c)  $12\mathbf{i}$  (d)  $18\mathbf{i} + 12\mathbf{j}$

53.  $\langle 22, -7 \rangle$

55.  $\langle 30, 9 \rangle$



57.  $-3\mathbf{i} + 4\mathbf{j}$     59.  $6\mathbf{i} + 4\mathbf{j}$

61.  $10\sqrt{2}(\cos 135^\circ \mathbf{i} + \sin 135^\circ \mathbf{j})$

63.  $\|\mathbf{v}\| = 7; \theta = 60^\circ$     65.  $\|\mathbf{v}\| = \sqrt{41}; \theta = 38.7^\circ$

67.  $\|\mathbf{v}\| = 3\sqrt{2}; \theta = 225^\circ$

69. The resultant force is 133.92 pounds and  $5.6^\circ$  from the 85-pound force.

71. 422.30 miles per hour;  $130.4^\circ$     73. 45

75. -2    77. 50; scalar    79.  $\langle 6, -8 \rangle$ ; vector

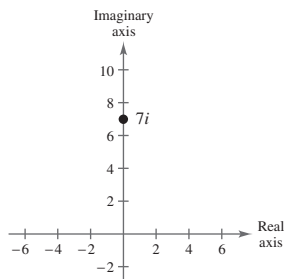
81.  $\frac{11\pi}{12}$     83.  $160.5^\circ$

85. Orthogonal    87. Neither

89.  $-\frac{13}{17}\langle 4, 1 \rangle, \frac{16}{17}\langle -1, 4 \rangle$     91.  $\frac{5}{2}\langle -1, 1 \rangle, \frac{9}{2}\langle 1, 1 \rangle$

93. 48    95. 72,000 foot-pounds

97.



7

$$101. 5\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$$

$$103. 6\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$$

$$105. (a) z_1 = 4\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$$

$$z_2 = 10\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$$

$$(b) z_1 z_2 = 40\left(\cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3}\right)$$

$$\frac{z_1}{z_2} = \frac{2}{5}\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

$$107. \frac{625}{2} + \frac{625\sqrt{3}}{2}i \quad 109. 2035 - 828i$$

$$111. (a) 3\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$3\left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}\right)$$

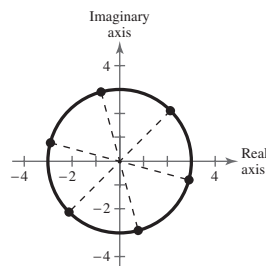
$$3\left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}\right)$$

$$3\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$$

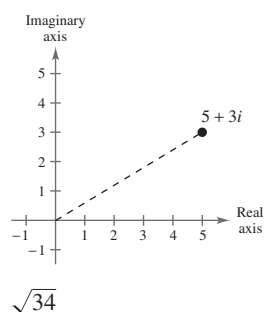
$$3\left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12}\right)$$

$$3\left(\cos \frac{23\pi}{12} + i \sin \frac{23\pi}{12}\right)$$

(b)



99.



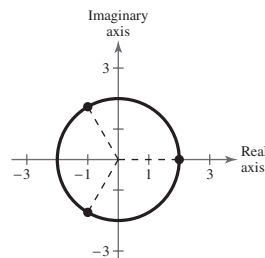
$$(c) \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i, -0.7765 + 2.898i, \\ -2.898 + 0.7765i, -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i, \\ 0.7765 - 2.898i, 2.898 - 0.7765i$$

$$113. (a) 2(\cos 0 + i \sin 0)$$

$$2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$$

$$2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$$

(b)



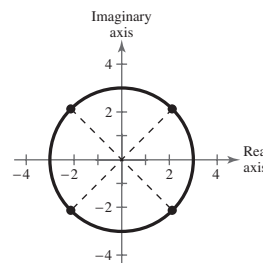
$$(c) 2, -1 + \sqrt{3}i, -1 - \sqrt{3}i$$

$$115. 3\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$$

$$3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$$

$$3\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$

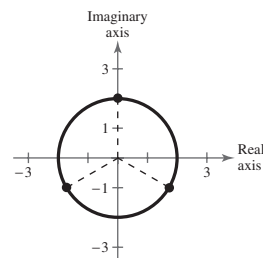
$$3\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$



$$117. 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 2i$$

$$2\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) = -\sqrt{3} - i$$

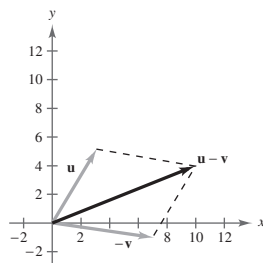
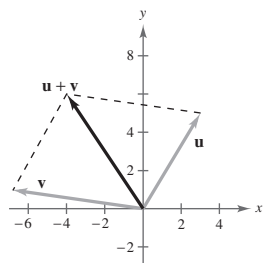
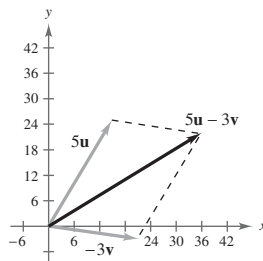
$$2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right) = \sqrt{3} - i$$



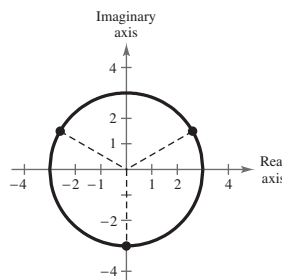
119. True.  $\sin 90^\circ$  is defined in the Law of Sines.
121. True. By definition,  $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$ , so  $\mathbf{v} = \|\mathbf{v}\|\mathbf{u}$ .
123. False. The solutions to  $x^2 - 8i = 0$  are  $x = 2 + 2i$  and  $x = -2 - 2i$ .
125.  $a^2 = b^2 + c^2 - 2bc \cos A$ ,  $b^2 = a^2 + c^2 - 2ac \cos B$ ,  
 $c^2 = a^2 + b^2 - 2ab \cos C$
127. **A and C**
129. If  $k > 0$ , the direction is the same and the magnitude is  $k$  times as great.  
 If  $k < 0$ , the result is a vector in the opposite direction and the magnitude is  $|k|$  times as great.
131. (a)  $4(\cos 60^\circ + i \sin 60^\circ)$  (b)  $-64$   
 $4(\cos 180^\circ + i \sin 180^\circ)$   
 $4(\cos 300^\circ + i \sin 300^\circ)$
133.  $z_1 z_2 = -4$ ;  $\frac{z_1}{z_2} = \cos(2\theta - \pi) + i \sin(2\theta - \pi)$   
 $= -\cos 2\theta - i \sin 2\theta$

### Chapter Test (page 486)

- $C = 88^\circ$ ,  $b \approx 27.81$ ,  $c \approx 29.98$
- $A = 43^\circ$ ,  $b \approx 25.75$ ,  $c \approx 14.45$
- Two solutions:  
 $B \approx 29.12^\circ$ ,  $C \approx 126.88^\circ$ ,  $c \approx 22.03$   
 $B \approx 150.88^\circ$ ,  $C \approx 5.12^\circ$ ,  $c \approx 2.46$
- No solution
- $A \approx 39.96^\circ$ ,  $C \approx 40.04^\circ$ ,  $c \approx 15.02$
- $A \approx 23.43^\circ$ ,  $B \approx 33.57^\circ$ ,  $c \approx 86.46$
- 2052.5 square meters
- 606.3 miles;  $29.1^\circ$
- $\langle 14, -23 \rangle$
- $\left\langle \frac{18\sqrt{34}}{17}, -\frac{30\sqrt{34}}{17} \right\rangle$
- $\langle -4, 6 \rangle$
- $\langle 10, 4 \rangle$

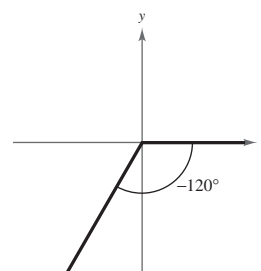

 13.  $\langle 36, 22 \rangle$ 

 14.  $\left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$ 

- $14.9^\circ$ ; 250.15 pounds
- $135^\circ$
- No
- $\frac{37}{26}\langle 5, 1 \rangle$ ;  $\frac{29}{26}\langle -1, 5 \rangle$
- $\approx 104$  pounds
- $5\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$
- $-3 + 3\sqrt{3}i$
- $-\frac{6561}{2} - \frac{6561\sqrt{3}}{2}i$
- $5832i$
- $4\sqrt[4]{2}\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$   
 $4\sqrt[4]{2}\left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}\right)$   
 $4\sqrt[4]{2}\left(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12}\right)$   
 $4\sqrt[4]{2}\left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12}\right)$
- $3\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$   
 $3\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$   
 $3\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$



### Cumulative Test for Chapters 4–6 (page 487)

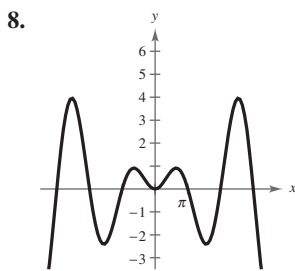
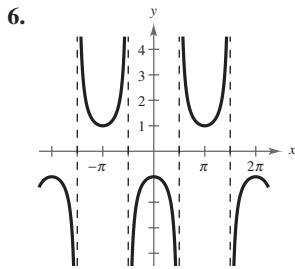
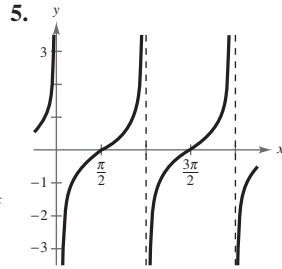
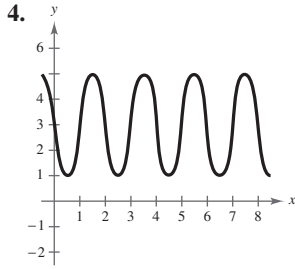
1. (a)



- $240^\circ$
- $-\frac{2\pi}{3}$
- $60^\circ$

$$\begin{aligned} \text{(e)} \quad \sin(-120^\circ) &= -\frac{\sqrt{3}}{2} & \csc(-120^\circ) &= -\frac{2\sqrt{3}}{3} \\ \cos(-120^\circ) &= -\frac{1}{2} & \sec(-120^\circ) &= -2 \\ \tan(-120^\circ) &= \sqrt{3} & \cot(-120^\circ) &= \frac{\sqrt{3}}{3} \end{aligned}$$

 2.  $134.6^\circ$     3.  $\frac{3}{5}$



7.  $a = -3, b = \pi, c = 0$

9. 6.7    10.  $\frac{3}{4}$

11.  $\sqrt{1-4x^2}$     12. 1    13.  $2 \tan \theta$

14–16. Answers will vary.    17.  $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

18.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$     19.  $\frac{3\pi}{2}$     20.  $\frac{16}{63}$     21.  $\frac{4}{3}$

22.  $\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}$     23.  $\frac{5}{2}(\sin \frac{5\pi}{2} - \sin \pi)$

24.  $2 \cos 6x \cos 2x$

25.  $B \approx 26.39^\circ, C \approx 123.61^\circ, c \approx 15.0$

26.  $B \approx 52.48^\circ, C \approx 97.52^\circ, a \approx 5.04$

27.  $B = 60^\circ, a \approx 5.77, c \approx 11.55$

28.  $A = 26.38^\circ, B \approx 62.72^\circ, C \approx 90.90^\circ$

29. 36.4 square inches    30. 85.2 square inches

31.  $3\mathbf{i} + 5\mathbf{j}$     32.  $\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$     33. -5

34.  $-\frac{1}{13}\langle 1, 5 \rangle; \frac{21}{13}\langle 5, -1 \rangle$

35.  $2\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$     36.  $-12\sqrt{3} + 12i$

37.  $\cos 0 + i \sin 0 = 1$

$$\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

38.  $3\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$   
 $3\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$   
 $3(\cos \pi + i \sin \pi)$   
 $3\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$   
 $3\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$

39.  $\approx 395.8$  radians per minute;  $\approx 8312.6$  inches per minute

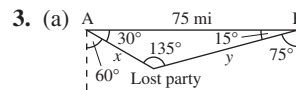
40. Area = 63.67 square yards    41. 5 feet    42.  $22.6^\circ$

43.  $d = 4 \cos \frac{\pi}{4}t$     44.  $32.6^\circ$ ; 543.9 kilometers per hour

45. 425 foot-pounds

### Problem Solving (page 493)

1. 2.01 feet

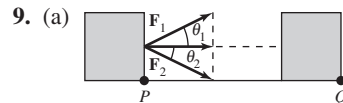


(b) Station A: 27.45 miles; Station B: 53.03 miles

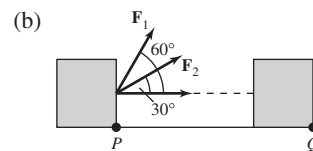
(c) 11.03 miles; S  $21.7^\circ$  E

5. (a) (i)  $\sqrt{2}$     (ii)  $\sqrt{5}$     (iii) 1  
 (iv) 1    (v) 1    (vi) 1  
 (b) (i) 1    (ii)  $3\sqrt{2}$     (iii)  $\sqrt{13}$   
 (iv) 1    (v) 1    (vi) 1  
 (c) (i)  $\frac{\sqrt{5}}{2}$     (ii)  $\sqrt{13}$     (iii)  $\frac{\sqrt{85}}{2}$   
 (iv) 1    (v) 1    (vi) 1  
 (d) (i)  $2\sqrt{5}$     (ii)  $5\sqrt{2}$     (iii)  $5\sqrt{2}$   
 (iv) 1    (v) 1    (vi) 1r

7.  $\mathbf{w} = \frac{1}{2}(\mathbf{u} + \mathbf{v})$ ;  $\mathbf{w} = \frac{1}{2}(\mathbf{v} - \mathbf{u})$



The amount of work done by  $\mathbf{F}_1$  is equal to the amount of work done by  $\mathbf{F}_2$ .



The amount of work done by  $\mathbf{F}_2$  is  $\sqrt{3}$  times as great as the amount of work done by  $\mathbf{F}_1$ .



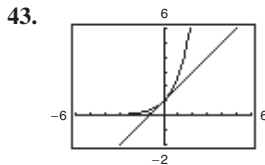
# Chapter 7

## Section 7.1 (page 503)

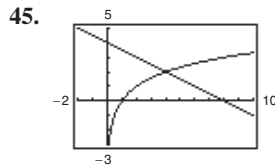
### Vocabulary Check (page 503)

1. system of equations      2. solution
3. solving      4. substitution
5. point of intersection      6. break-even

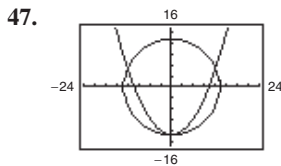
1. (a) No    (b) No    (c) No    (d) Yes
3. (a) No    (b) Yes    (c) No    (d) No
5. (2, 2)      7. (2, 6), (-1, 3)
9. (0, -5), (4, 3)      11. (0, 0), (2, -4)
13. (0, 1), (1, -1), (3, 1)      15. (5, 5)      17.  $(\frac{1}{2}, 3)$
19. (1, 1)      21.  $(\frac{20}{3}, \frac{40}{3})$       23. No solution
25. (-2, 4), (0, 0)      27. No solution      29. (4, 3)
31.  $(\frac{5}{2}, \frac{3}{2})$       33. (2, 2), (4, 0)      35. (1, 4), (4, 7)
37.  $(4, -\frac{1}{2})$       39. No solution      41. (4, 3), (-4, 3)



(0, 1)



(4, 2)



(0, -13),  $(\pm 12, 5)$

49. (1, 2)      51. (-2, 0),  $(\frac{29}{10}, \frac{21}{10})$       53. No solution
55. (0.287, 1.751)      57. (-1, 0), (0, 1), (1, 0)
59.  $(\frac{1}{2}, 2)$ ,  $(-4, -\frac{1}{4})$       61. 192 units
63. (a) 781 units    (b) 3708 units
65. (a) 8 weeks

(b)

	1	2	3	4
$360 - 24x$	336	312	228	264
$24 + 18x$	42	60	78	96

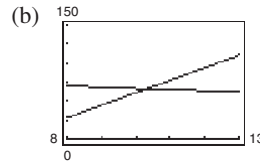
	5	6	7	8
$360 - 24x$	240	216	192	168
$24 + 18x$	114	132	150	168

67. More than \$11,666.67

69. (a) 
$$\begin{cases} x + y = 25,000 \\ 0.06x + 0.085y = 2,000 \end{cases}$$
- (b)

Decreases; Interest is fixed.

- (c) \$5000
71. (a) Solar:  $0.1429t^2 - 4.46t + 96.8$   
Wind:  $16.371t - 102.7$



- (c) Point of intersection: (10.3, 66.01). Consumption of solar and wind energy are equal at this point in time in the year 2000.
- (d)  $t = 10.3, 135.47$
- (e) The results are the same, but due to the given parameters,  $t = 135.47$  is not of significance.
- (f) Answers will vary.
73. 6 meters  $\times$  9 meters      75. 9 inches  $\times$  12 inches
77. 8 kilometers  $\times$  12 kilometers
79. False. To solve a system of equations by substitution, you can solve for either variable in one of the two equations and then back-substitute.
81. 1. *Solve* one of the equations for one variable in terms of the other.  
2. *Substitute* the expression found in Step 1 into the other equation to obtain an equation in one variable.  
3. *Solve* the equation obtained in Step 2.  
4. *Back-substitute* the value obtained in Step 3 into the expression obtained in Step 1 to find the value of the other variable.  
5. *Check* that the solution satisfies *each* of the original equations.

83. (a)  $y = 2x$     (b)  $y = 0$     (c)  $y = x - 2$

85.  $2x + 7y - 45 = 0$       87.  $y - 3 = 0$

89.  $30x - 17y - 18 = 0$

91. Domain: All real numbers  $x$  except  $x = 6$

Horizontal asymptote:  $y = 0$

Vertical asymptote:  $x = 6$

93. Domain: All real numbers  $x$  except  $x = \pm 4$

Horizontal asymptote:  $y = 1$

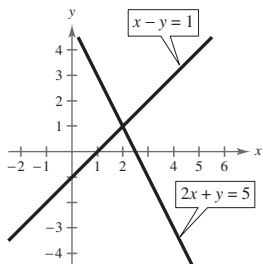
Vertical asymptotes:  $x = \pm 4$

## Section 7.2 (page 515)

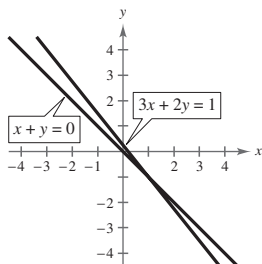
## Vocabulary Check (page 515)

1. elimination    2. equivalent  
3. consistent; inconsistent    4. equilibrium point

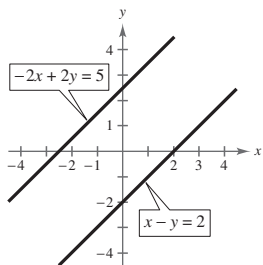
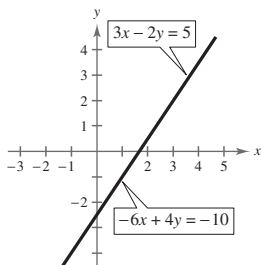
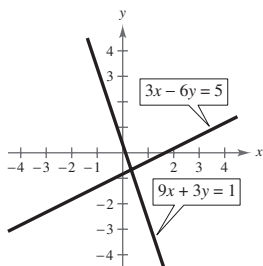
1. (2, 1)



3. (1, -1)



5. No solution

7.  $(a, \frac{3}{2}a - \frac{5}{2})$ 9.  $(\frac{1}{3}, -\frac{2}{3})$ 11.  $(\frac{5}{2}, \frac{3}{4})$     13. (3, 4)    15. (4, -1)    17.  $(\frac{12}{7}, \frac{18}{7})$ 19. No solution    21.  $(\frac{18}{5}, \frac{3}{5})$ 23. Infinitely many solutions:  $(a, -\frac{1}{2} + \frac{5}{6}a)$ 25.  $(\frac{90}{31}, -\frac{67}{31})$     27.  $(-\frac{6}{35}, \frac{43}{35})$     29. (5, -2)

31. b; one solution; consistent

32. a; infinitely many solutions; consistent

33. c; one solution; consistent

34. d; no solutions; inconsistent

35. (4, 1)    37. (2, -1)    39. (6, -3)    41.  $(\frac{43}{6}, \frac{25}{6})$ 

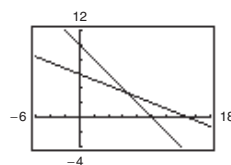
43. 550 miles per hour, 50 miles per hour

45. (80, 10)    47. (2,000,000, 100)

49. Cheeseburger: 310 calories; fries: 230 calories

$$51. (a) \begin{cases} x + y = 10 \\ 0.2x + 0.5y = 3 \end{cases}$$

(b)



Decreases

(c) 20% solution:  $6\frac{2}{3}$  liters  
50% solution:  $3\frac{1}{3}$  liters

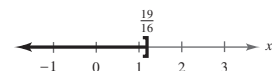
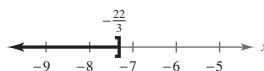
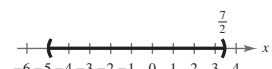
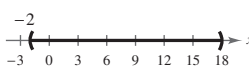
53. \$6000    55. 400 adult, 1035 student

57.  $y = 0.97x + 2.1$     59.  $y = 0.32x + 4.1$ 61.  $y = -2x + 4$ 63. (a)  $y = 14x + 19$     (b) 41.4 bushels per acre

65. False. Two lines that coincide have infinitely many points of intersection.

67. No. Two lines will intersect only once or will coincide, and if they coincide the system will have infinitely many solutions.

69. (39,600, 398). It is necessary to change the scale on the axes to see the point of intersection.

71.  $k = -4$ 73.  $x \leq -\frac{22}{3}$ 75.  $x \leq \frac{19}{16}$ 77.  $-2 < x < 18$ 79.  $-5 < x < \frac{7}{2}$ 81.  $\ln 6x$     83.  $\log_9 \frac{12}{x}$     85. No solution

87. Answers will vary.

## Section 7.3 (page 527)

## Vocabulary Check (page 527)

1. row-echelon    2. ordered triple  
3. Gaussian    4. row operation  
5. nonsquare    6. position

1. (a) No    (b) No    (c) No    (d) Yes

3. (a) No    (b) No    (c) Yes    (d) No

5. (1, -2, 4)    7. (3, 10, 2)    9.  $(\frac{1}{2}, -2, 2)$ 

$$11. \begin{cases} x - 2y + 3z = 5 \\ y - 2z = 9 \\ 2x - 3z = 0 \end{cases}$$

First step in putting the system in row-echelon form

13. (1, 2, 3)    15. (-4, 8, 5)    17. (5, -2, 0)

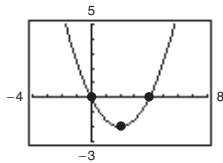
19. No solution    21.  $(-\frac{1}{2}, 1, \frac{3}{2})$ 23.  $(-3a + 10, 5a - 7, a)$     25.  $(-a + 3, a + 1, a)$ 27.  $(2a, 21a - 2, 8a)$     29.  $(-\frac{3}{2}a + \frac{1}{2}, -\frac{2}{3}a + 1, a)$

31. (1, 1, 1)    33. No solution    35. (0, 0, 0)

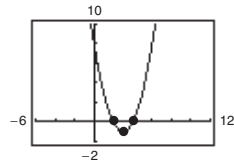
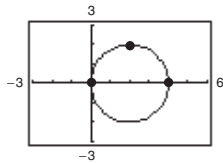
37.  $(9a, -35a, 67a)$     39.  $s = -16t^2 + 144$

41.  $s = -16t^2 - 32t + 500$

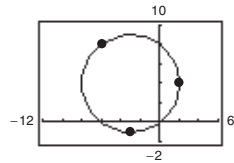
43.  $y = \frac{1}{2}x^2 - 2x$     45.  $y = x^2 - 6x + 8$



47.  $x^2 + y^2 - 4x = 0$



49.  $x^2 + y^2 + 6x - 8y = 0$



51. 6 touchdowns, 6 extra-point kicks, and 1 field goal

53. \$300,000 at 8%

\$400,000 at 9%

\$75,000 at 10%

55.  $250,000 - \frac{1}{2}s$  in certificates of deposit

$125,000 + \frac{1}{2}s$  in municipal bonds

$125,000 - s$  in blue-chip stocks

$s$  in growth stocks

57. Brand X = 4 lb

Brand Y = 9 lb

Brand Z = 9 lb

59. Vanilla = 2 lb

Hazelnut = 4 lb

French Roast = 4 lb

61. Television = 30 ads

Radio = 10 ads

Newspaper = 20 ads

63. (a) Not possible

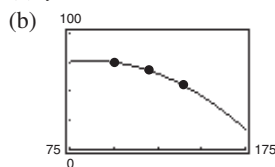
(b) No gallons of 10%, 6 gallons of 15%, 6 gallons of 25%

(c) 4 gallons of 10%, No gallons of 15%, 8 gallons of 25%

65.  $I_1 = 1, I_2 = 2, I_3 = 1$

67.  $y = x^2 - x$     69.  $y = -\frac{5}{24}x^2 - \frac{3}{10}x + \frac{41}{6}$

71. (a)  $y = -0.0075x^2 + 1.3x + 20$



(c)

$x$	100	120	140
$y$	75	68	55

The values are the same.

(d) 24.25%    (e) 156 females

73. Touchdowns = 8; Field goals = 2;

Two-point conversions = 1; Extra-point kicks = 5

75.  $x = 5$     77.  $x = \pm\sqrt{2}/2$  or  $x = 0$

$y = 5$      $y = \frac{1}{2}$      $y = 0$

$\lambda = -5$      $\lambda = 1$      $\lambda = 0$

79. False. Equation 2 does not have a leading coefficient of 1.

81. No. Answers will vary.

83. 
$$\begin{cases} 3x + y - z = 9 \\ x + 2y - z = 0 \\ -x + y + 3z = 1 \end{cases} \quad \begin{cases} x + y + z = 5 \\ x - 2z = 0 \\ 2y + z = 0 \end{cases}$$

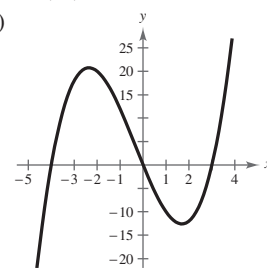
85. 
$$\begin{cases} x + 2y - 4z = -5 \\ -x - 4y + 8z = 13 \\ x + 6y + 4z = 7 \end{cases} \quad \begin{cases} x + 2y + 4z = 9 \\ y + 2z = 3 \\ x - 4z = -4 \end{cases}$$

87. 6.375    89. 80,000    91.  $11 + i$

93.  $22 + 3i$     95.  $\frac{7}{2} + \frac{7}{2}i$

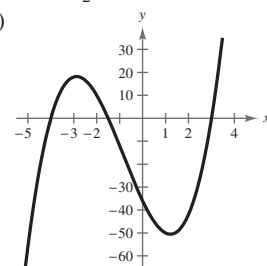
97. (a) -4, 0, 3

(b)



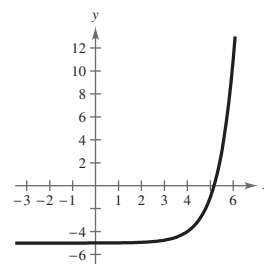
99. (a) -4,  $-\frac{3}{2}$ , 3

(b)



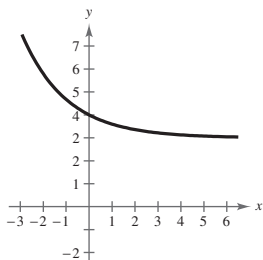
101.

$x$	-2	0	2	4	5
$y$	-5	-4.996	-4.938	-4	-1



103.

$x$	-2	-1	0	1	2
$y$	5.793	4.671	4	3.598	3.358



105. (40, 40)

107. Answers will vary.

## Section 7.4 (page 539)

## Vocabulary Check (page 539)

1. partial fraction decomposition    2. improper  
3. linear; quadratic; irreducible    4. basic equation

1. b    2. c    3. d    4. a

$$5. \frac{A}{x} + \frac{B}{x-14} \quad 7. \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-10}$$

$$9. \frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} \quad 11. \frac{A}{x} + \frac{Bx+C}{x^2+10}$$

$$13. \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \quad 15. \frac{1}{2} \left( \frac{1}{x-1} - \frac{1}{x+1} \right)$$

$$17. \frac{1}{x} - \frac{1}{x+1} \quad 19. \frac{1}{x} - \frac{2}{2x+1}$$

$$21. \frac{1}{x-1} - \frac{1}{x+2} \quad 23. -\frac{3}{x} - \frac{1}{x+2} + \frac{5}{x-2}$$

$$25. \frac{3}{x} - \frac{1}{x^2} + \frac{1}{x+1} \quad 27. \frac{3}{x-3} + \frac{9}{(x-3)^2}$$

$$29. -\frac{1}{x} + \frac{2x}{x^2+1} \quad 31. -\frac{1}{x-1} + \frac{x+2}{x^2-2}$$

$$33. \frac{1}{6} \left( \frac{2}{x^2+2} - \frac{1}{x+2} + \frac{1}{x-2} \right)$$

$$35. \frac{1}{8} \left( \frac{1}{2x+1} + \frac{1}{2x-1} - \frac{4x}{4x^2+1} \right)$$

$$37. \frac{1}{x+1} + \frac{2}{x^2-2x+3} \quad 39. 1 - \frac{2x+1}{x^2+x+1}$$

$$41. 2x - 7 + \frac{17}{x+2} + \frac{1}{x+1}$$

$$43. x + 3 + \frac{6}{x-1} + \frac{4}{(x-1)^2} + \frac{1}{(x-1)^3}$$

$$45. \frac{3}{2x-1} - \frac{2}{x+1} \quad 47. \frac{2}{x} - \frac{1}{x^2} - \frac{2}{x+1}$$

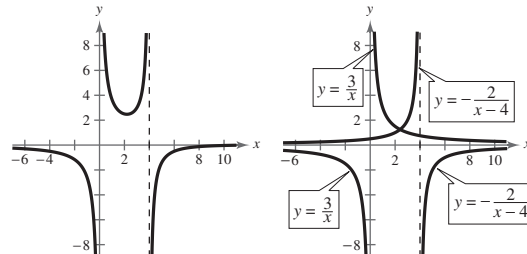
$$49. \frac{1}{x^2+2} + \frac{x}{(x^2+2)^2} \quad 51. 2x + \frac{1}{2} \left( \frac{3}{x-4} - \frac{1}{x+2} \right)$$

$$53. (a) \frac{3}{x} - \frac{2}{x-4}$$

$$y = \frac{x-12}{x(x-4)}$$

$$y = \frac{3}{x}, y = -\frac{2}{x-4}$$

(b)



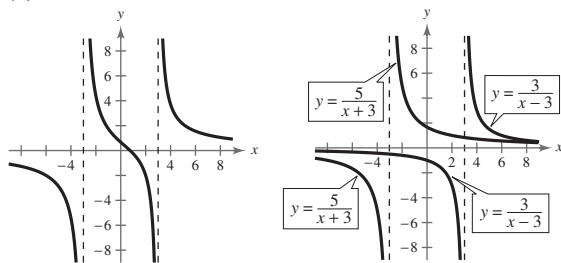
(c) The vertical asymptotes are the same.

$$55. (a) \frac{3}{x-3} + \frac{5}{x+3}$$

$$y = \frac{2(4x-3)}{x^2-9}$$

$$y = \frac{3}{x-3}, y = \frac{5}{x+3}$$

(b)



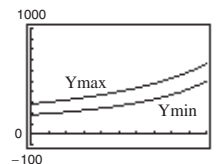
(c) The vertical asymptotes are the same.

$$57. (a) \frac{2000}{7-4x} - \frac{2000}{11-7x}, \quad 0 < x \leq 1$$

$$(b) Y_{\max} = \left| \frac{2000}{7-4x} \right|$$

$$Y_{\min} = \left| \frac{-2000}{11-7x} \right|$$

(c)

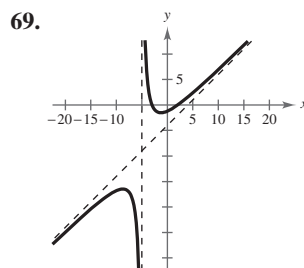
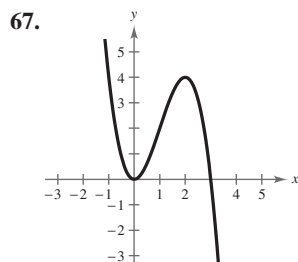
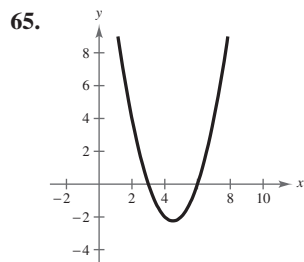


(d) Maximum: 400°F  
Minimum: 266.7°F

59. False. The partial fraction decomposition is

$$\frac{A}{x+10} + \frac{B}{x-10} + \frac{C}{(x-10)^2}$$

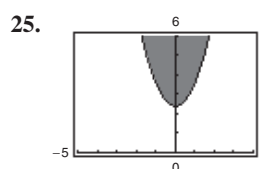
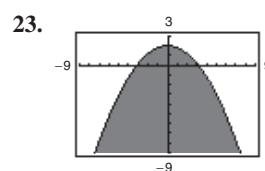
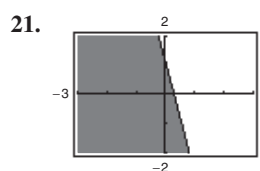
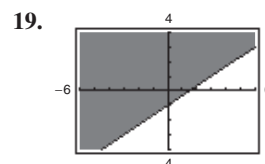
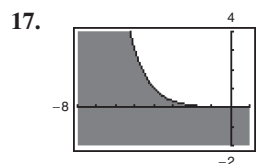
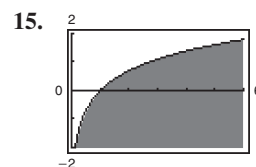
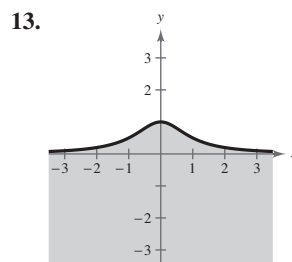
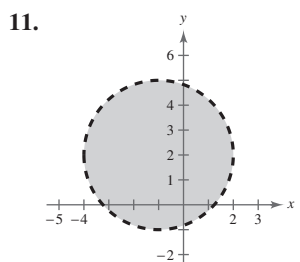
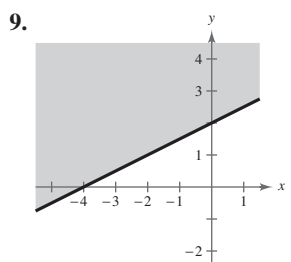
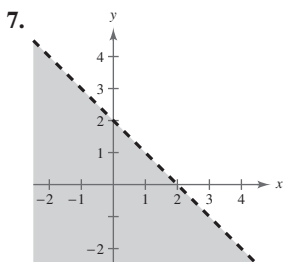
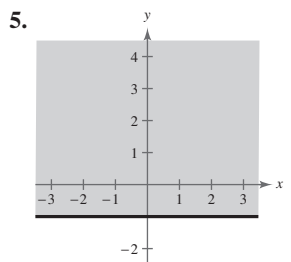
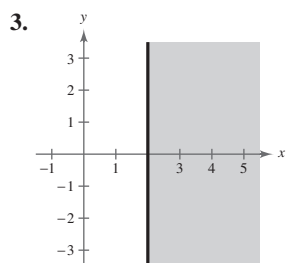
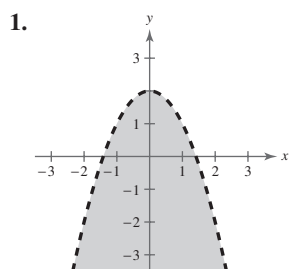
$$61. \frac{1}{2a} \left( \frac{1}{a+x} + \frac{1}{a-x} \right) \quad 63. \frac{1}{a} \left( \frac{1}{y} + \frac{1}{a-y} \right)$$



Section 7.5 (page 548)

Vocabulary Check (page 548)

1. solution
2. graph
3. linear
4. solution
5. consumer surplus



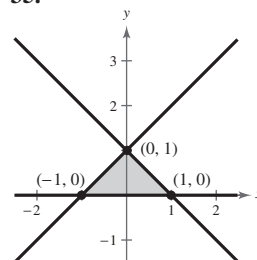
27.  $y \leq \frac{1}{2}x + 2$

29.  $y \geq -\frac{2}{3}x + 2$

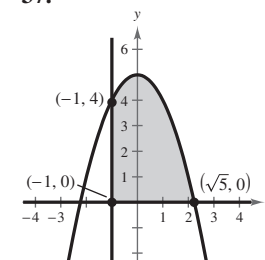
31. (a) No (b) No (c) Yes (d) Yes

33. (a) Yes (b) No (c) Yes (d) Yes

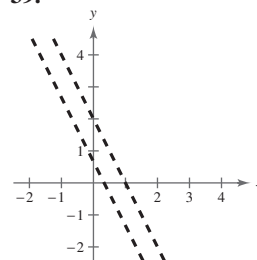
35.



37.

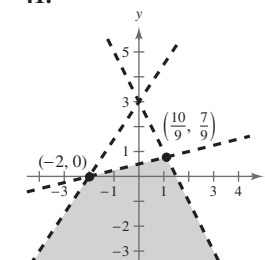


39.

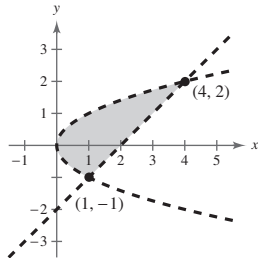


No solution

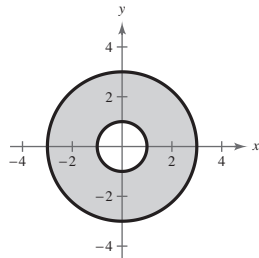
41.



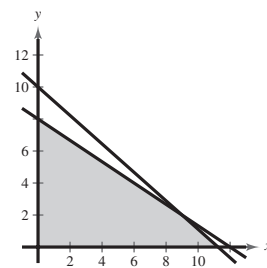
43.



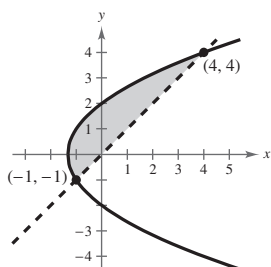
45.



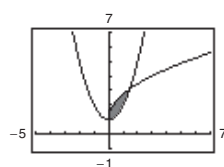
$$69. \begin{cases} x + \frac{3}{2}y \leq 12 \\ \frac{4}{3}x + \frac{3}{2}y \leq 15 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



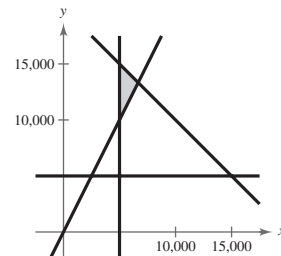
47.



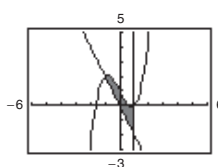
49.



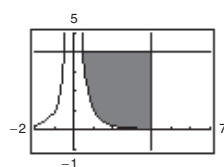
$$71. \begin{cases} x + y \leq 20,000 \\ y \geq 2x \\ x \geq 5,000 \\ y \geq 5,000 \end{cases}$$



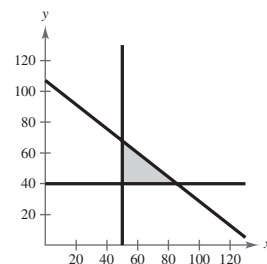
51.



53.



$$73. \begin{cases} 55x + 70y \leq 7500 \\ x \geq 50 \\ y \geq 40 \end{cases}$$



$$55. \begin{cases} y \leq 4 - x \\ x \geq 0 \\ y \geq 0 \end{cases}$$

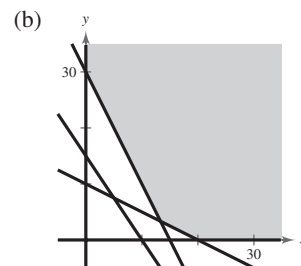
$$57. \begin{cases} y \geq 4 - x \\ y \geq 2 - \frac{1}{4}x \\ x \geq 0, y \geq 0 \end{cases}$$

$$59. \begin{cases} x^2 + y^2 \leq 16 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$$61. \begin{cases} 2 \leq x \leq 5 \\ 1 \leq y \leq 7 \end{cases}$$

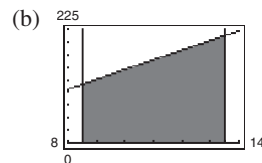
$$63. \begin{cases} y \leq \frac{3}{2}x \\ y \leq -x + 5 \\ y \geq 0 \end{cases}$$

$$75. (a) \begin{cases} 20x + 10y \geq 300 \\ 15x + 10y \geq 150 \\ 10x + 20y \geq 200 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



(c) Answers will vary.

$$77. (a) y = 19.17t - 46.61$$

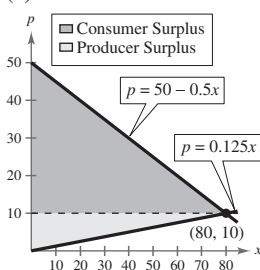


$$(c) \text{ Total retail sales} = \frac{h}{2}(a + b) = \$821.3 \text{ billion}$$

79. True. The figure is a rectangle with a length of 9 units and a width of 11 units.

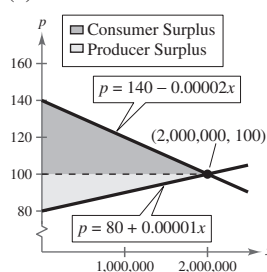
81. The graph is a half-line on the real number line; on the rectangular coordinate system, the graph is a half-plane.

65. (a)

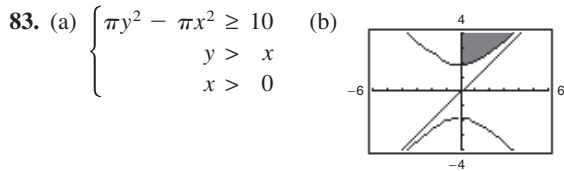


(b) Consumer surplus: \$1600  
Producer surplus: \$400

67. (a)



(b) Consumer surplus: \$40,000,000  
Producer surplus: \$20,000,000



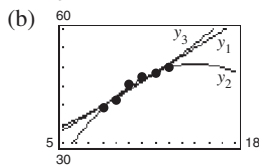
(c) The line is an asymptote to the boundary. The larger the circles, the closer the radii can be and the constraint will still be satisfied.

85. d    86. b    87. c    88. a

89.  $5x + 3y - 8 = 0$     91.  $28x + 17y + 13 = 0$

93.  $x + y + 1.8 = 0$

95. (a)  $y_1 = 2.17t + 22.5$   
 $y_2 = -0.241t^2 + 7.23t - 3.4$   
 $y_3 = 27(1.05^t)$



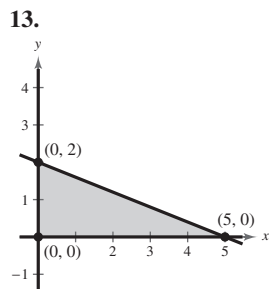
(c) The quadratic model is the best fit for the data.  
 (d) \$48.66

## Section 7.6 (page 558)

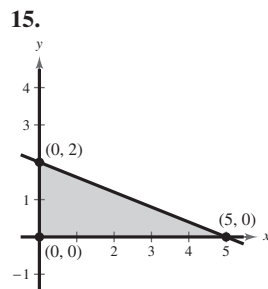
### Vocabulary Check (page 558)

1. optimization    2. linear programming
3. objective    4. constraints; feasible solutions
5. vertex

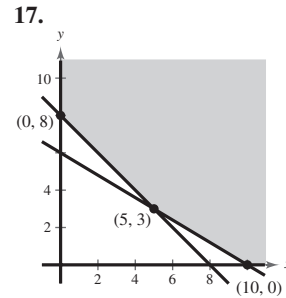
1. Minimum at (0, 0): 0    3. Minimum at (0, 0): 0  
 Maximum at (5, 0): 20    Maximum at (0, 5): 40
5. Minimum at (0, 0): 0    7. Minimum at (0, 0): 0  
 Maximum at (3, 4): 17    Maximum at (4, 0): 20
9. Minimum at (0, 0): 0  
 Maximum at (60, 20): 740
11. Minimum at (0, 0): 0  
 Maximum at any point on the line segment connecting (60, 20) and (30, 45): 2100



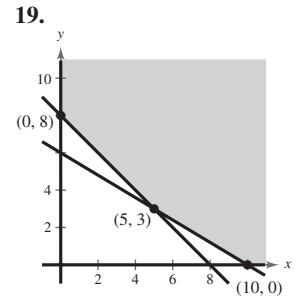
Minimum at (0, 0): 0  
 Maximum at (5, 0): 30



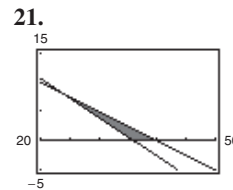
Minimum at (0, 0): 0  
 Maximum at (0, 2): 48



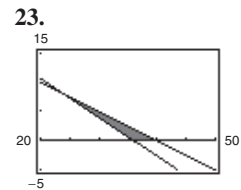
Minimum at (5, 3): 35  
 No maximum



Minimum at (10, 0): 20  
 No maximum



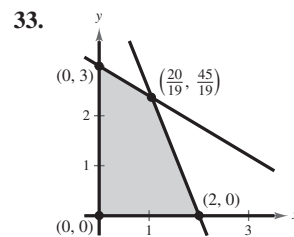
Minimum at (24, 8): 104  
 Maximum at (40, 0): 160



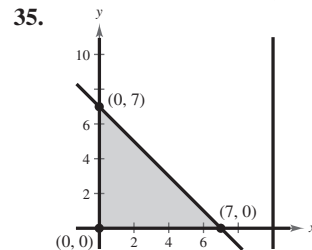
Minimum at (36, 0): 36  
 Maximum at (24, 8): 56

25. Maximum at (3, 6): 12  
 29. Maximum at (0, 5): 25

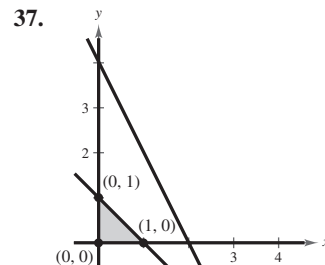
27. Maximum at (0, 10): 10  
 31. Maximum at  $(\frac{22}{3}, \frac{19}{6})$ :  $\frac{271}{6}$



The maximum, 5, occurs at any point on the line segment connecting (2, 0) and  $(\frac{20}{19}, \frac{45}{19})$ .



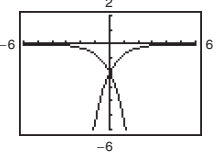
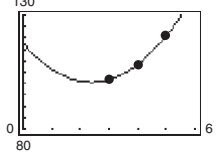
The constraint  $x \leq 10$  is extraneous. Maximum at (0, 7): 14



The constraint  $2x + y \leq 4$  is extraneous.  
 Maximum at (0, 1): 4

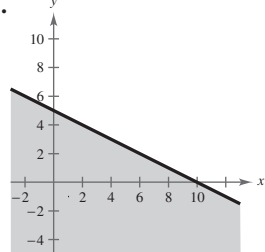
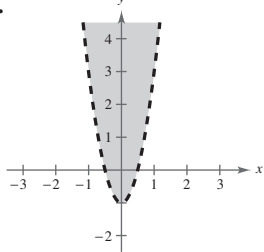
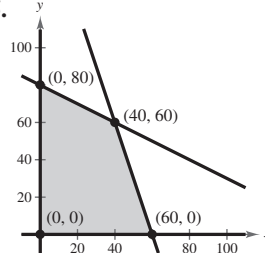
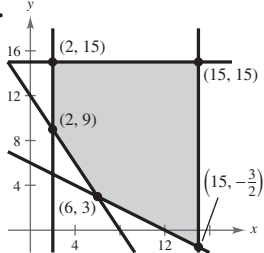
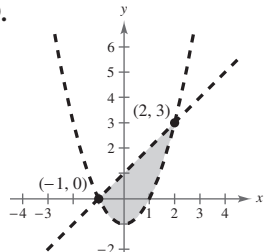
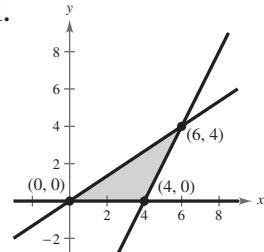
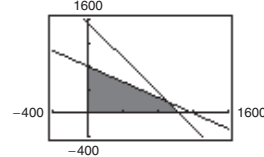
39. 750 units of model A  
1000 units of model B  
Optimal profit: \$83,750
41. 216 units of \$300 model  
0 units of \$250 model  
Optimal profit: \$8640
43. Three bags of brand X  
Six bags of brand Y  
Optimal cost: \$195
45. 0 tax returns  
12 audits  
Optimal revenue: \$30,000
47. \$62,500 to type A  
\$187,500 to type B  
Optimal return: \$23,750
49. True. The objective function has a maximum value at any point on the line segment connecting the two vertices.
51. (a)  $t \geq 9$  (b)  $\frac{3}{4} \leq t \leq 9$
53.  $z = x + 5y$  55.  $z = 4x + y$  57.  $\frac{9}{2(x+3)}, x \neq 0$
59.  $\frac{x^2 + 2x - 13}{x(x-2)}, x \neq \pm 3$  61.  $\ln 3 \approx 1.099$
63.  $4 \ln 38 \approx 14.550$  65.  $\frac{1}{3}e^{12/7} \approx 1.851$
67.  $(-4, 3, -7)$

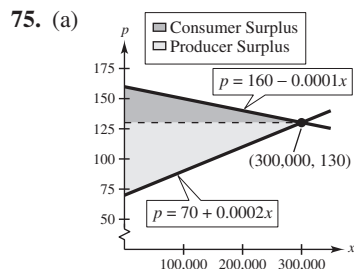
### Review Exercises (page 563)

1.  $(1, 1)$  3.  $(0.25, 0.625)$  5.  $(5, 4)$
7.  $(0, 0), (2, 8), (-2, 8)$  9.  $(4, -2)$
11.  $(1.41, -0.66), (-1.41, 10.66)$
13. 
- $(0, -2)$
15. 3847 units 17. 96 meters  $\times$  144 meters 19.  $(\frac{5}{2}, 3)$
21.  $(-0.5, 0.8)$  23.  $(0, 0)$  25.  $(\frac{8}{5}a + \frac{14}{5}, a)$
27. d, one solution, consistent
28. c, infinite solutions, consistent
29. b, no solution, inconsistent
30. a, one solution, consistent
31.  $(\frac{500,000}{7}, \frac{159}{7})$  33.  $(2, -4, -5)$
35.  $(\frac{24}{5}, \frac{22}{5}, -\frac{8}{5})$  37.  $(3a + 4, 2a + 5, a)$
39.  $(a - 4, a - 3, a)$  41.  $y = 2x^2 + x - 5$
43.  $x^2 + y^2 - 4x + 4y - 1 = 0$
45. (a)  $y = 3x^2 - 14.3x + 117.6$
- (b) 
- (c) 195.2; yes.

The model is a good fit.

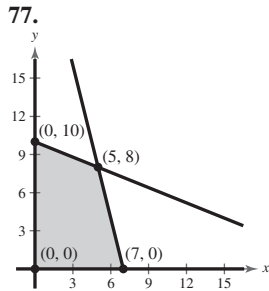
47. \$16,000 at 7%  
\$13,000 at 9%  
\$11,000 at 11%

49.  $\frac{A}{x} + \frac{B}{x+20}$  51.  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-5}$
53.  $\frac{3}{x+2} - \frac{4}{x+4}$  55.  $1 - \frac{25}{8(x+5)} + \frac{9}{8(x-3)}$
57.  $\frac{1}{2}(\frac{3}{x-1} - \frac{x-3}{x^2+1})$  59.  $\frac{3x}{x^2+1} + \frac{x}{(x^2+1)^2}$
61. 
63. 
65. 
67. 
69. 
71. 
73. 
$$\begin{cases} 20x + 30y \leq 24,000 \\ 12x + 8y \leq 12,400 \\ x \geq 0 \\ y \geq 0 \end{cases}$$
- 

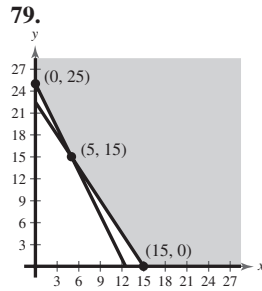


- (b) Consumer surplus: \$4,500,000  
Producer surplus: \$9,000,000

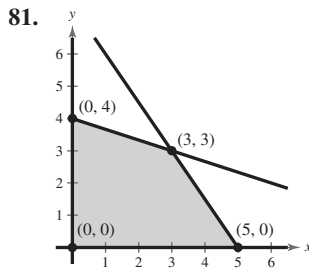




Minimum at  $(0, 0)$ : 0  
Maximum at  $(5, 8)$ : 47



Minimum at  $(15, 0)$ : 26.25  
No maximum



Minimum at  $(0, 0)$ : 0  
Maximum at  $(3, 3)$ : 48

83. 72 haircuts  
0 permanents  
Optimal revenue: \$1800

85. Three bags of brand X  
Two bags of brand Y  
Optimal cost: \$105

87. False. To represent a region covered by an isosceles trapezoid, the last two inequality signs should be  $\leq$ .

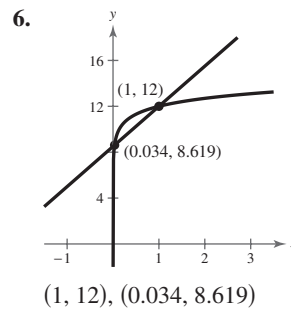
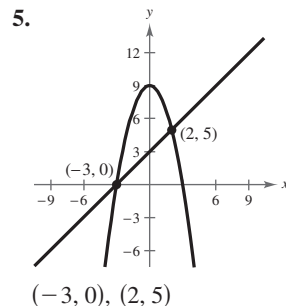
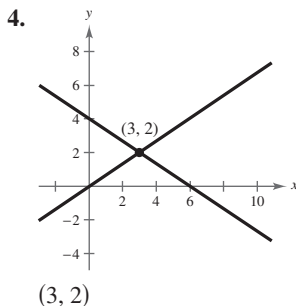
89.  $\begin{cases} x + y = 2 \\ x - y = -14 \end{cases}$       91.  $\begin{cases} 3x + y = 7 \\ -6x + 3y = 1 \end{cases}$

93.  $\begin{cases} x + y + z = 6 \\ x + y - z = 0 \\ x - y - z = 2 \end{cases}$       95.  $\begin{cases} 2x + 2y - 3z = 7 \\ x - 2y + z = 4 \\ -x + 4y - z = -1 \end{cases}$

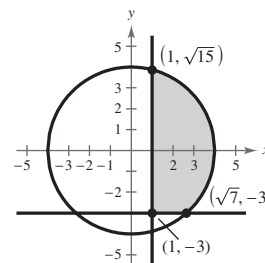
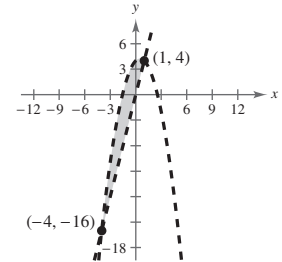
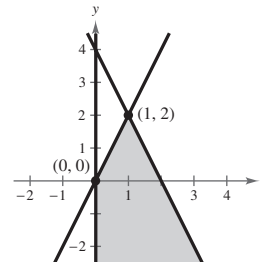
97. An inconsistent system of linear equations has no solution.  
99. Answers will vary.

### Chapter Test (page 567)

1.  $(-3, 4)$       2.  $(0, -1), (1, 0), (2, 1)$   
3.  $(8, 4), (2, -2)$

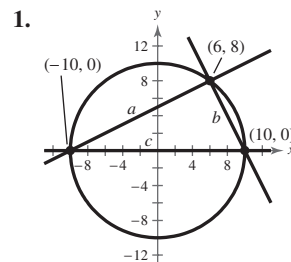


7.  $(1, 5)$       8.  $(2, -1)$       9.  $(2, -3, 1)$       10. No solution  
11.  $-\frac{1}{x+1} + \frac{3}{x-2}$       12.  $\frac{2}{x^2} + \frac{3}{2-x}$   
13.  $-\frac{5}{x} + \frac{3}{x+1} + \frac{3}{x-1}$       14.  $-\frac{2}{x} + \frac{3x}{x^2+2}$   
15.      16.



19. 8%: \$20,000      20.  $y = -\frac{1}{2}x^2 + x + 6$   
8.5%: \$30,000  
21. 0 units of model I, 5300 units of model II  
Optimal profit: \$212,000

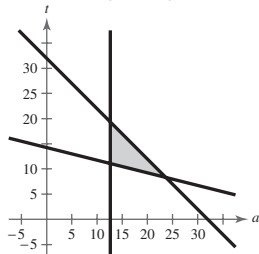
### Problem Solving (page 569)



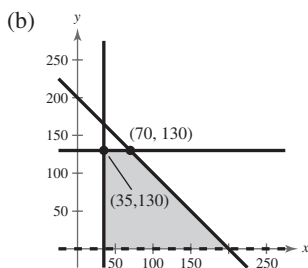
$a = 8\sqrt{5}, b = 4\sqrt{5}, c = 20$   
 $(8\sqrt{5})^2 + (4\sqrt{5})^2 = 20^2$   
Therefore, the triangle is a right triangle.

3.  $ad \neq bc$  5. (a) One (b) Two (c) Four7. 10.1 feet high;  $\approx 252.7$  feet long 9. \$12.0011. (a)  $(3, -4)$  (b)  $\left(\frac{2}{-a+5}, \frac{1}{4a-1}, \frac{1}{a}\right)$ 13. (a)  $\left(\frac{-5a+16}{6}, \frac{5a-16}{6}, a\right)$ (b)  $\left(\frac{-11a+36}{14}, \frac{13a-40}{14}, a\right)$ (c)  $(-a+3, a-3, a)$  (d) Infinitely many

$$15. \begin{cases} a + t \leq 32 \\ 0.15a \geq 1.9 \\ 193a + 772t \geq 11,000 \end{cases}$$



$$17. (a) \begin{cases} x + y \leq 200 \\ x \geq 35 \\ 0 < y \leq 130 \end{cases}$$



(c) No, because the total cholesterol is greater than 200 milligrams per deciliter.

(d) LDL: 140 milligrams per deciliter

HDL: 50 milligrams per deciliter

Total: 190 milligrams per deciliter

(e)  $(50, 120)$ ;  $\frac{170}{50} = 3.4 < 4$ ; answers will vary.

## Chapter 8

### Section 8.1 (page 582)

#### Vocabulary Check (page 582)

1. matrix 2. square 3. main diagonal
4. row; column 5. augmented 6. coefficient
7. row-equivalent 8. reduced row-echelon form
9. Gauss-Jordan elimination

1.  $1 \times 2$  3.  $3 \times 1$  5.  $2 \times 2$ 

$$7. \begin{bmatrix} 4 & -3 & : & -5 \\ -1 & 3 & : & 12 \end{bmatrix} \quad 9. \begin{bmatrix} 1 & 10 & -2 & : & 2 \\ 5 & -3 & 4 & : & 0 \\ 2 & 1 & 0 & : & 6 \end{bmatrix}$$

$$11. \begin{bmatrix} 7 & -5 & 1 & : & 13 \\ 19 & 0 & -8 & : & 10 \end{bmatrix} \quad 13. \begin{cases} x + 2y = 7 \\ 2x - 3y = 4 \end{cases}$$

$$15. \begin{cases} 2x + 5z = -12 \\ y - 2z = 7 \\ 6x + 3y = 2 \end{cases}$$

$$17. \begin{cases} 9x + 12y + 3z = 0 \\ -2x + 18y + 5z + 2w = 10 \\ x + 7y - 8z = -4 \\ 3x + 2z = -10 \end{cases}$$

$$19. \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

$$21. \begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 5 & -2 & 6 \\ 0 & 3 & 20 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 1 & -\frac{2}{5} & \frac{6}{5} \\ 0 & 3 & 20 & 4 \end{bmatrix}$$

23. Add 5 times Row 2 to Row 1.

25. Interchange Row 1 and Row 2.

Add 4 times new Row 1 to Row 3.

$$27. (a) \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 3 & 1 & -1 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -5 & -10 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 0 \end{bmatrix} \quad (d) \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(e) \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix is in reduced row-echelon form.

29. Reduced row-echelon form

31. Not in row-echelon form

$$33. \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad 35. \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$37. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 39. \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$41. \begin{bmatrix} 1 & 0 & 3 & 16 \\ 0 & 1 & 2 & 12 \end{bmatrix} \quad 43. \begin{cases} x - 2y = 4 \\ y = -3 \end{cases} \quad (-2, -3)$$

$$45. \begin{cases} x - y + 2z = 4 \\ y - z = 2 \\ z = -2 \end{cases} \quad (8, 0, -2)$$

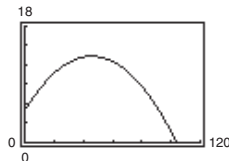
47.  $(3, -4)$  49.  $(-4, -10, 4)$  51.  $(3, 2)$ 53.  $(-5, 6)$  55.  $(-1, -4)$  57. Inconsistent59.  $(4, -3, 2)$  61.  $(7, -3, 4)$  63.  $(-4, -3, 6)$ 65.  $(2a + 1, 3a + 2, a)$ 67.  $(4 + 5b + 4a, 2 - 3b - 3a, b, a)$  69. Inconsistent71.  $(0, 2 - 4a, a)$  73.  $(1, 0, 4, -2)$ 75.  $(-2a, a, a, 0)$  77. Yes;  $(-1, 1, -3)$  79. No

$$81. \begin{bmatrix} 1 & 3 & \frac{3}{2} & : & 4 \\ 0 & 1 & \frac{7}{4} & : & -\frac{3}{2} \\ 0 & 0 & 1 & : & 2 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 1 & : & 3 \\ 0 & 1 & 2 & : & -1 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

83.  $\frac{4x^2}{(x+1)^2(x-1)} = \frac{1}{x-1} + \frac{3}{x+1} - \frac{2}{(x+1)^2}$

85. \$150,000 at 7%      87.  $y = x^2 + 2x + 5$   
 \$750,000 at 8%  
 \$600,000 at 10%

89. (a)  $y = -0.004x^2 + 0.367x + 5$   
 (b)



- (c) 13 feet, 104 feet      (d) 13.418 feet, 103.793 feet  
 (e) The results are similar.

91. (a)  $x_1 = s, x_2 = t, x_3 = 600 - s, x_4 = s - t,$   
 $x_5 = 500 - t, x_6 = s, x_7 = t$   
 (b)  $x_1 = 0, x_2 = 0, x_3 = 600, x_4 = 0, x_5 = 500,$   
 $x_6 = 0, x_7 = 0$   
 (c)  $x_1 = 0, x_2 = -500, x_3 = 600, x_4 = 500,$   
 $x_5 = 1000, x_6 = 0, x_7 = -500$

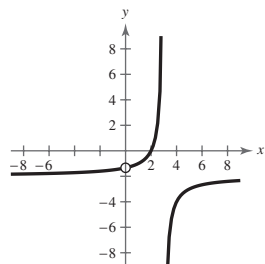
93. False. It is a  $2 \times 4$  matrix.

95. False. Gaussian elimination reduces a matrix until a row-echelon form is obtained; Gauss-Jordan elimination reduces a matrix until a reduced row-echelon form is obtained.

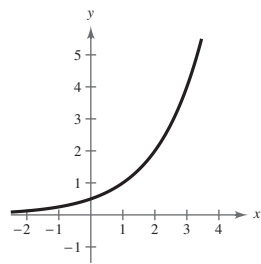
97. (a) There exists a row with all zeros except for the entry in the last column.  
 (b) There are fewer rows with nonzero entries than there are variables and no rows as in (a).

99. They are the same.

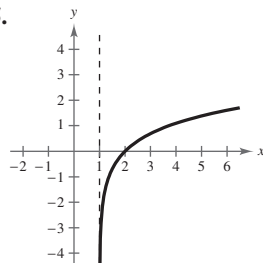
101.



103.



105.



## Section 8.2 (page 597)

### Vocabulary Check (page 597)

1. equal      2. scalars      3. zero;  $O$       4. identity  
 5. (a) iii      (b) iv      (c) i      (d) v      (e) ii  
 6. (a) ii      (b) iv      (c) i      (d) iii

1.  $x = -4, y = 22$

3.  $x = 2, y = 3$

5. (a)  $\begin{bmatrix} 3 & -2 \\ 1 & 7 \end{bmatrix}$       (b)  $\begin{bmatrix} -1 & 0 \\ 3 & -9 \end{bmatrix}$       (c)  $\begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix}$

(d)  $\begin{bmatrix} -1 & -1 \\ 8 & -19 \end{bmatrix}$

7. (a)  $\begin{bmatrix} 7 & 3 \\ 1 & 9 \\ -2 & 15 \end{bmatrix}$       (b)  $\begin{bmatrix} 5 & -5 \\ 3 & -1 \\ -4 & -5 \end{bmatrix}$       (c)  $\begin{bmatrix} 18 & -3 \\ 6 & 12 \\ -9 & 15 \end{bmatrix}$

(d)  $\begin{bmatrix} 16 & -11 \\ 8 & 2 \\ -11 & -5 \end{bmatrix}$

9. (a)  $\begin{bmatrix} 3 & 3 & -2 & 1 & 1 \\ -2 & 5 & 7 & -6 & -8 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 1 & 0 & -1 & 1 \\ 4 & -3 & -11 & 6 & 6 \end{bmatrix}$

(c)  $\begin{bmatrix} 6 & 6 & -3 & 0 & 3 \\ 3 & 3 & -6 & 0 & -3 \end{bmatrix}$

(d)  $\begin{bmatrix} 4 & 4 & -1 & -2 & 3 \\ 9 & -5 & -24 & 12 & 11 \end{bmatrix}$

11. (a), (b), and (d) not possible

(c)  $\begin{bmatrix} 18 & 0 & 9 \\ -3 & -12 & 0 \end{bmatrix}$

13.  $\begin{bmatrix} -8 & -7 \\ 15 & -1 \end{bmatrix}$

15.  $\begin{bmatrix} -24 & -4 & 12 \\ -12 & 32 & 12 \end{bmatrix}$

17.  $\begin{bmatrix} 10 & 8 \\ -59 & 9 \end{bmatrix}$

19.  $\begin{bmatrix} -17.143 & 2.143 \\ 11.571 & 10.286 \end{bmatrix}$

21.  $\begin{bmatrix} -1.581 & -3.739 \\ -4.252 & -13.249 \\ 9.713 & -0.362 \end{bmatrix}$

23.  $\begin{bmatrix} -6 & -9 \\ -1 & 0 \\ 17 & -10 \end{bmatrix}$

25.  $\begin{bmatrix} 3 & 3 \\ -\frac{1}{2} & 0 \\ -\frac{13}{2} & \frac{11}{2} \end{bmatrix}$

27. Not possible

29.  $\begin{bmatrix} 3 & -4 \\ 10 & 16 \\ 26 & 46 \end{bmatrix}$

31.  $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -10 \end{bmatrix}$

Order:  $3 \times 2$

Order:  $3 \times 3$

$$33. \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 35. \begin{bmatrix} 41 & 7 & 7 \\ 42 & 5 & 25 \\ -10 & -25 & 45 \end{bmatrix}$$

Order:  $3 \times 3$ 

$$37. \begin{bmatrix} 151 & 25 & 48 \\ 516 & 279 & 387 \\ 47 & -20 & 87 \end{bmatrix} \quad 39. \text{Not possible}$$

$$41. (a) \begin{bmatrix} 0 & 15 \\ 6 & 12 \end{bmatrix} \quad (b) \begin{bmatrix} -2 & 2 \\ 31 & 14 \end{bmatrix} \quad (c) \begin{bmatrix} 9 & 6 \\ 12 & 12 \end{bmatrix}$$

$$43. (a) \begin{bmatrix} 0 & -10 \\ 10 & 0 \end{bmatrix} \quad (b) \begin{bmatrix} 0 & -10 \\ 10 & 0 \end{bmatrix} \quad (c) \begin{bmatrix} 8 & -6 \\ 6 & 8 \end{bmatrix}$$

$$45. (a) \begin{bmatrix} 7 & 7 & 14 \\ 8 & 8 & 16 \\ -1 & -1 & -2 \end{bmatrix} \quad (b) [13] \quad (c) \text{Not possible}$$

$$47. \begin{bmatrix} 5 & 8 \\ -4 & -16 \end{bmatrix} \quad 49. \begin{bmatrix} -4 & 10 \\ 3 & 14 \end{bmatrix}$$

$$51. (a) \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \quad (b) \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$53. (a) \begin{bmatrix} -2 & -3 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -36 \end{bmatrix} \quad (b) \begin{bmatrix} -7 \\ 6 \end{bmatrix}$$

$$55. (a) \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -1 \\ 2 & -5 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 17 \end{bmatrix} \quad (b) \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$57. (a) \begin{bmatrix} 1 & -5 & 2 \\ -3 & 1 & -1 \\ 0 & -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -20 \\ 8 \\ -16 \end{bmatrix} \quad (b) \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

$$59. \begin{bmatrix} 84 & 60 & 30 \\ 42 & 120 & 84 \end{bmatrix}$$

$$61. (a) A = \begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix}$$

The entries represent the numbers of bushels of each crop that are shipped to each outlet.

$$(b) B = [\$3.50 \quad \$6.00]$$

The entries represent the profits per bushel of each crop.

$$(c) BA = [\$1037.50 \quad \$1400 \quad \$1012.50]$$

The entries represent the profits from both crops at each of the three outlets.

$$63. \begin{bmatrix} \$15,770 & \$18,300 \\ \$26,500 & \$29,250 \\ \$21,260 & \$24,150 \end{bmatrix}$$

The entries represent the wholesale and retail values of the inventories at the three outlets.

$$65. P^3 = \begin{bmatrix} 0.300 & 0.175 & 0.175 \\ 0.308 & 0.433 & 0.217 \\ 0.392 & 0.392 & 0.608 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.250 & 0.188 & 0.188 \\ 0.315 & 0.377 & 0.248 \\ 0.435 & 0.435 & 0.565 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.225 & 0.194 & 0.194 \\ 0.314 & 0.345 & 0.267 \\ 0.461 & 0.461 & 0.539 \end{bmatrix}$$

$$P^6 = \begin{bmatrix} 0.213 & 0.197 & 0.197 \\ 0.311 & 0.326 & 0.280 \\ 0.477 & 0.477 & 0.523 \end{bmatrix}$$

$$P^7 = \begin{bmatrix} 0.206 & 0.198 & 0.198 \\ 0.308 & 0.316 & 0.288 \\ 0.486 & 0.486 & 0.514 \end{bmatrix}$$

$$P^8 = \begin{bmatrix} 0.203 & 0.199 & 0.199 \\ 0.305 & 0.309 & 0.292 \\ 0.492 & 0.492 & 0.508 \end{bmatrix}$$

Approaches the matrix

$$\begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$67. (a) \begin{matrix} \text{Sales \$} & \text{Profit} \\ 447 & 115 \\ 624.5 & 161 \\ 731.2 & 188 \end{matrix} \quad (b) \$464$$

$$\begin{bmatrix} 447 & 115 \\ 624.5 & 161 \\ 731.2 & 188 \end{bmatrix}$$

The entries represent the total sales and profits for each type of milk.

$$69. (a) [2 \quad 0.5 \quad 3]$$

$$(b) 120 \text{ lb} \quad 150 \text{ lb}$$

$$[473.5 \quad 588.5]$$

The entries represent the total calories burned.

71. True. The sum of two matrices of different orders is undefined.

73. Not possible      75. Not possible      77.  $2 \times 2$

$$79. 2 \times 3 \quad 81. AC = BC = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

83.  $AB$  is a diagonal matrix whose entries are the products of the corresponding entries of  $A$  and  $B$ .

$$85. -8, \frac{4}{3} \quad 87. 0, \frac{-5 \pm \sqrt{37}}{4} \quad 89. 4, \pm \frac{\sqrt{15}}{3}i$$

$$91. (7, -\frac{1}{2}) \quad 93. (3, -1)$$

### Section 8.3 (page 608)

#### Vocabulary Check (page 608)

1. square      2. inverse

3. nonsingular; singular      4.  $A^{-1}B$

1–9.  $AB = I$  and  $BA = I$ 

11.  $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$  13.  $\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$  15.  $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

17. Does not exist 19. Does not exist

21.  $\begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}$  23.  $\begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & \frac{1}{4} & 0 \\ \frac{7}{20} & -\frac{1}{4} & \frac{1}{5} \end{bmatrix}$

25.  $\begin{bmatrix} -\frac{1}{8} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -\frac{1}{5} \end{bmatrix}$  27.  $\begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$

29.  $\begin{bmatrix} -1.5 & 1.5 & 1 \\ 4.5 & -3.5 & -3 \\ -1 & 1 & 1 \end{bmatrix}$  31.  $\begin{bmatrix} -12 & -5 & -9 \\ -4 & -2 & -4 \\ -8 & -4 & -6 \end{bmatrix}$

33.  $\begin{bmatrix} 0 & -1.8\overline{1} & 0.9\overline{0} \\ -10 & 5 & 5 \\ 10 & -2.7\overline{2} & -3.6\overline{3} \end{bmatrix}$  35. Does not exist

37.  $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$  39.  $\begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ -\frac{2}{19} & \frac{5}{19} \end{bmatrix}$

41. Does not exist 43.  $\begin{bmatrix} \frac{16}{59} & \frac{15}{59} \\ -\frac{4}{59} & \frac{70}{59} \end{bmatrix}$

45. (5, 0) 47. (-8, -6) 49. (3, 8, -11)

51. (2, 1, 0, 0) 53. (2, -2) 55. No solution

57. (-4, -8) 59. (-1, 3, 2)

61.  $(\frac{5}{16}a + \frac{13}{16}, \frac{19}{16}a + \frac{11}{16}, a)$  63. (-7, 3, -2)

65. (5, 0, -2, 3)

67. \$7000 in AAA-rated bonds

\$1000 in A-rated bonds

\$2000 in B-rated bonds

69. \$9000 in AAA-rated bonds

\$1000 in A-rated bonds

\$2000 in B-rated bonds

71. (a)  $I_1 = -3$  amperes (b)  $I_1 = 2$  amperes

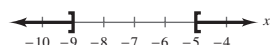
$I_2 = 8$  amperes  $I_2 = 3$  amperes

$I_3 = 5$  amperes  $I_3 = 5$  amperes

73. True. If  $B$  is the inverse of  $A$ , then  $AB = I = BA$ .

75. Answers will vary.

77.  $x \geq -5$  or  $x \leq -9$



79.  $\frac{2 \ln 315}{\ln 3} \approx 10.472$  81.  $2^{6.5} \approx 90.510$

83. Answers will vary.

## Section 8.4 (page 616)

## Vocabulary Check (page 616)

1. determinant 2. minor  
3. cofactor 4. expanding by cofactors

1. 5 3. 5 5. 27 7. 0 9. 6 11. -9  
13. 72 15.  $\frac{11}{6}$  17. -0.002 19. -4.842 21. 0  
23. (a)  $M_{11} = -5, M_{12} = 2, M_{21} = 4, M_{22} = 3$   
(b)  $C_{11} = -5, C_{12} = -2, C_{21} = -4, C_{22} = 3$   
25. (a)  $M_{11} = -4, M_{12} = -2, M_{21} = 1, M_{22} = 3$   
(b)  $C_{11} = -4, C_{12} = 2, C_{21} = -1, C_{22} = 3$   
27. (a)  $M_{11} = 3, M_{12} = -4, M_{13} = 1, M_{21} = 2, M_{22} = 2,$   
 $M_{23} = -4, M_{31} = -4, M_{32} = 10, M_{33} = 8$   
(b)  $C_{11} = 3, C_{12} = 4, C_{13} = 1, C_{21} = -2, C_{22} = 2,$   
 $C_{23} = 4, C_{31} = -4, C_{32} = -10, C_{33} = 8$   
29. (a)  $M_{11} = 30, M_{12} = 12, M_{13} = 11, M_{21} = -36,$   
 $M_{22} = 26, M_{23} = 7, M_{31} = -4, M_{32} = -42, M_{33} = 12$   
(b)  $C_{11} = 30, C_{12} = -12, C_{13} = 11, C_{21} = 36, C_{22} = 26,$   
 $C_{23} = -7, C_{31} = -4, C_{32} = 42, C_{33} = 12$   
31. (a) -75 (b) -75 33. (a) 96 (b) 96  
35. (a) 170 (b) 170 37. 0 39. 0 41. -9  
43. -58 45. -30 47. -168 49. 0  
51. 412 53. -126 55. 0 57. -336 59. 410  
61. (a) -3 (b) -2 (c)  $\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$  (d) 6  
63. (a) -8 (b) 0 (c)  $\begin{bmatrix} -4 & 4 \\ 1 & -1 \end{bmatrix}$  (d) 0  
65. (a) -21 (b) -19 (c)  $\begin{bmatrix} 7 & 1 & 4 \\ -8 & 9 & -3 \\ 7 & -3 & 9 \end{bmatrix}$  (d) 399  
67. (a) 2 (b) -6 (c)  $\begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{bmatrix}$  (d) -12  
69–73. Answers will vary. 75. -1, 4 77. -1, -4  
79.  $8uv - 1$  81.  $e^{5x}$  83.  $1 - \ln x$   
85. True. If an entire row is zero, then each cofactor in the expansion is multiplied by zero.  
87. Answers will vary.  
89. A square matrix is a square array of numbers. The determinant of a square matrix is a real number.  
91. (a) Columns 2 and 3 of  $A$  were interchanged.  
 $|A| = -115 = -|B|$   
(b) Rows 1 and 3 of  $A$  were interchanged.  
 $|A| = -40 = -|B|$   
93. (a) Multiply Row 1 by 5.  
(b) Multiply Column 2 by 4 and Column 3 by 3.  
95. All real numbers  $x$

97. All real numbers  $x$  such that  $-4 \leq x \leq 4$ 99. All real numbers  $t$  such that  $t > 1$ 

101.  103.  $\begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ 2 & 1 \end{bmatrix}$

105. Does not exist

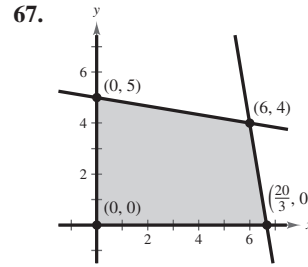
**Section 8.5** (page 628)**Vocabulary Check** (page 628)

1. Cramer's Rule 2. collinear

3.  $A = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$  4. cryptogram

5. uncoded; coded

1. (2, -2) 3. Not possible 5.  $(\frac{32}{7}, \frac{30}{7})$   
 7. (-1, 3, 2) 9. (-2, 1, -1) 11.  $(0, -\frac{1}{2}, \frac{1}{2})$   
 13. (1, 2, 1) 15. 7 17. 14 19.  $\frac{33}{8}$  21.  $\frac{5}{2}$   
 23. 28 25.  $y = \frac{16}{5}$  or  $y = 0$   
 27.  $y = -3$  or  $y = -11$  29. 250 square miles  
 31. Collinear 33. Not collinear 35. Collinear  
 37.  $y = -3$  39.  $3x - 5y = 0$  41.  $x + 3y - 5 = 0$   
 43.  $2x + 3y - 8 = 0$   
 45. Uncoded: [20 18 15], [21 2 12], [5 0 9], [14 0 18],  
                   [9 22 5], [18 0 3], [9 20 25]  
       Encoded: -52 10 27 -49 3 34 -49 13 27  
                   -94 22 54 1 1 -7 0 -12 9  
                   -121 41 55  
 47. -6 -35 -69 11 20 17 6 -16 -58 46 79 67  
 49. -5 -41 -87 91 207 257 11 -5 -41 40 80  
       84 76 177 227  
 51. HAPPY NEW YEAR  
 53. CLASS IS CANCELED  
 55. SEND PLANES 57. MEET ME TONIGHT RON  
 59. False. The denominator is the determinant of the coefficient matrix.  
 61. False. If the determinant of the coefficient matrix is zero, the system has either no solution or infinitely many solutions.  
 63. (-6, 4) 65. (-1, 0, -3)



Minimum at (0, 0): 0

Maximum at (6, 4): 52

**Review Exercises** (page 632)

1.  $3 \times 1$  3.  $1 \times 1$  5.  $\begin{bmatrix} 3 & -10 & : & 15 \\ 5 & 4 & : & 22 \end{bmatrix}$   
 7.  $\begin{cases} 5x + y + 7z = -9 \\ 4x + 2y = 10 \\ 9x + 4y + 2z = 3 \end{cases}$  9.  $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$   
 11.  $\begin{cases} x + 2y + 3z = 9 \\ y - 2z = 2 \\ z = 0 \end{cases}$  13.  $\begin{cases} x - 5y + 4z = 1 \\ y + 2z = 3 \\ z = 4 \end{cases}$   
                   (5, 2, 0)                   (-40, -5, 4)  
 15. (10, -12) 17.  $(-\frac{1}{5}, \frac{7}{10})$  19. (5, 2, -6)  
 21.  $(-2a + \frac{3}{2}, 2a + 1, a)$  23. (1, 0, 4, 3)  
 25. (2, -3, 3) 27. (2, 3, -1) 29. (2, 6, -10, -3)  
 31.  $x = 12, y = -7$  33.  $x = 1, y = 11$   
 35. (a)  $\begin{bmatrix} -1 & 8 \\ 15 & 13 \end{bmatrix}$  (b)  $\begin{bmatrix} 5 & -12 \\ -9 & -3 \end{bmatrix}$   
      (c)  $\begin{bmatrix} 8 & -8 \\ 12 & 20 \end{bmatrix}$  (d)  $\begin{bmatrix} -7 & 28 \\ 39 & 29 \end{bmatrix}$   
 37. (a)  $\begin{bmatrix} 5 & 7 \\ -3 & 14 \\ 31 & 42 \end{bmatrix}$  (b)  $\begin{bmatrix} 5 & 1 \\ -11 & -10 \\ -9 & -38 \end{bmatrix}$   
      (c)  $\begin{bmatrix} 20 & 16 \\ -28 & 8 \\ 44 & 8 \end{bmatrix}$  (d)  $\begin{bmatrix} 5 & 13 \\ 5 & 38 \\ 71 & 122 \end{bmatrix}$   
 39.  $\begin{bmatrix} 17 & -17 \\ 13 & 2 \end{bmatrix}$  41.  $\begin{bmatrix} 54 & 4 \\ -2 & 24 \\ -4 & 32 \end{bmatrix}$   
 43.  $\begin{bmatrix} 48 & -18 & -3 \\ 15 & 51 & 33 \end{bmatrix}$  45.  $\begin{bmatrix} -14 & -4 \\ 7 & -17 \\ -17 & -2 \end{bmatrix}$   
 47.  $\begin{bmatrix} 3 & \frac{2}{3} \\ -\frac{4}{3} & \frac{11}{3} \\ \frac{10}{3} & 0 \end{bmatrix}$  49.  $\begin{bmatrix} -30 & 4 \\ 51 & 70 \end{bmatrix}$   
 51.  $\begin{bmatrix} 100 & 220 \\ 12 & -4 \\ 84 & 212 \end{bmatrix}$  53.  $\begin{bmatrix} 14 & -2 & 8 \\ 14 & -10 & 40 \\ 36 & -12 & 48 \end{bmatrix}$

55.  $\begin{bmatrix} 44 & 4 \\ 20 & 8 \end{bmatrix}$  57.  $\begin{bmatrix} 24 & -8 \\ 36 & -12 \end{bmatrix}$  59.  $\begin{bmatrix} 1 & 17 \\ 12 & 36 \end{bmatrix}$
61.  $\begin{bmatrix} 14 & -22 & 22 \\ 19 & -41 & 80 \\ 42 & -66 & 66 \end{bmatrix}$  63.  $\begin{bmatrix} 76 & 114 & 133 \\ 38 & 95 & 76 \end{bmatrix}$
65. [\$274,150 \$303,150]  
The merchandise shipped to warehouse 1 is worth \$274,150 and the merchandise shipped to warehouse 2 is worth \$303,150.
- 67–69.  $AB = I$  and  $BA = I$
71.  $\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$  73.  $\begin{bmatrix} 13 & 6 & -4 \\ -12 & -5 & 3 \\ 5 & 2 & -1 \end{bmatrix}$
75.  $\begin{bmatrix} \frac{1}{2} & -1 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{2}{3} & -\frac{5}{6} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$  77.  $\begin{bmatrix} -3 & 6 & -5.5 & 3.5 \\ 1 & -2 & 2 & -1 \\ 7 & -15 & 14.5 & -9.5 \\ -1 & 2.5 & -2.5 & 1.5 \end{bmatrix}$
79.  $\begin{bmatrix} 1 & -1 \\ 4 & -\frac{7}{2} \end{bmatrix}$  81.  $\begin{bmatrix} 2 & \frac{20}{3} \\ \frac{1}{10} & \frac{1}{6} \end{bmatrix}$  83. (36, 11)
85.  $(-6, -1)$  87.  $(2, -1, -2)$  89.  $(6, 1, -1)$
91.  $(-3, 1)$  93.  $(1, 1, -2)$  95.  $-42$  97. 550
99. (a)  $M_{11} = 4, M_{12} = 7, M_{21} = -1, M_{22} = 2$   
(b)  $C_{11} = 4, C_{12} = -7, C_{21} = 1, C_{22} = 2$
101. (a)  $M_{11} = 30, M_{12} = -12, M_{13} = -21,$   
 $M_{21} = 20, M_{22} = 19, M_{23} = 22, M_{31} = 5,$   
 $M_{32} = -2, M_{33} = 19$   
(b)  $C_{11} = 30, C_{12} = 12, C_{13} = -21,$   
 $C_{21} = -20, C_{22} = 19, C_{23} = -22,$   
 $C_{31} = 5, C_{32} = 2, C_{33} = 19$
103. 130 105. 279 107. (4, 7) 109.  $(-1, 4, 5)$
111. 16 113. 10 115. Collinear
117.  $x - 2y + 4 = 0$  119.  $2x + 6y - 13 = 0$
121. Uncoded:  $[12 \ 15 \ 15], [11 \ 0 \ 15], [21 \ 20 \ 0],$   
 $[2 \ 5 \ 12], [15 \ 23 \ 0]$   
Encoded:  $-21 \ 6 \ 0 \ -68 \ 8 \ 45 \ 102 \ -42 \ -60 \ -53$   
 $20 \ 21 \ 99 \ -30 \ -69$
123. SEE YOU FRIDAY
125. False. The matrix must be square.
127. The matrix must be square and its determinant nonzero.
129. No. The first two matrices describe a system of equations with one solution. The third matrix describes a system with infinitely many solutions.
131.  $\lambda = \pm 2\sqrt{10} - 3$

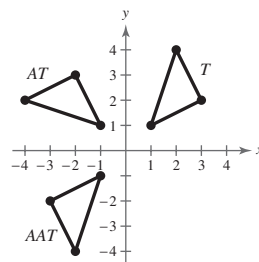
### Chapter Test (page 637)

1.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.  $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
3.  $\begin{bmatrix} 4 & 3 & -2 & : & 14 \\ -1 & -1 & 2 & : & -5 \\ 3 & 1 & -4 & : & 8 \end{bmatrix}, (1, 3, -\frac{1}{2})$
4. (a)  $\begin{bmatrix} 1 & 5 \\ 0 & -4 \end{bmatrix}$   
(b)  $\begin{bmatrix} 15 & 12 \\ -12 & -12 \end{bmatrix}$   
(c)  $\begin{bmatrix} 7 & 14 \\ -4 & -12 \end{bmatrix}$   
(d)  $\begin{bmatrix} 4 & -5 \\ 0 & 4 \end{bmatrix}$
5.  $\begin{bmatrix} \frac{1}{2} & \frac{2}{5} \\ 1 & \frac{3}{5} \end{bmatrix}$
6.  $\begin{bmatrix} -\frac{5}{2} & 4 & -3 \\ 5 & -7 & 6 \\ 4 & -6 & 5 \end{bmatrix}$
7. (13, 22) 8.  $-196$  9. 29 10. 43
11.  $(-3, 5)$  12.  $(-2, 4, 6)$  13. 7
14. Uncoded:  $[11 \ 14 \ 15], [3 \ 11 \ 0], [15 \ 14 \ 0], [23 \ 15 \ 15],$   
 $[4 \ 0 \ 0]$   
Encoded:  $115 \ -41 \ -59 \ 14 \ -3 \ -11 \ 29 \ -15$   
 $-14 \ 128 \ -53 \ -60 \ 4 \ -4 \ 0$
15. 75 liters of 60% solution  
25 liters of 20% solution

### Problem Solving (page 639)

1. (a)  $AT = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix}$   
 $AAT = \begin{bmatrix} -1 & -2 & -3 \\ -1 & -4 & -2 \end{bmatrix}$



A represents a counterclockwise rotation.

- (b) AAT is rotated clockwise  $90^\circ$  to obtain AT. AT is then rotated clockwise  $90^\circ$  to obtain T.

3. (a) Yes (b) No (c) No (d) No
5. (a) Gold Cable Company: 28,750 subscribers  
Galaxy Cable Company: 35,750 subscribers  
Nonsubscribers: 35,500  
Answers will vary.
- (b) Gold Cable Company: 30,813 subscribers  
Galaxy Cable Company: 39,675 subscribers  
Nonsubscribers: 29,513  
Answers will vary.
- (c) Gold Cable Company: 31,947 subscribers  
Galaxy Cable Company: 42,329 subscribers  
Nonsubscribers: 25,724  
Answers will vary.
- (d) Cable companies are increasing the number of subscribers, while the nonsubscribers are decreasing.
7.  $x = 6$  9–11. Answers will vary.
13. Sulfur: 32 atomic mass units  
Nitrogen: 14 atomic mass units  
Fluorine: 19 atomic mass units
15.  $A^T = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}$   $B^T = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$   
 $(AB)^T = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix} = B^T A^T$
17. (a)  $A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$   
(b) JOHN RETURN TO BASE
19.  $|A| = 0$

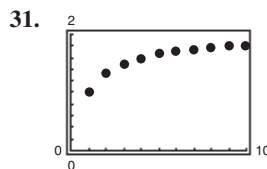
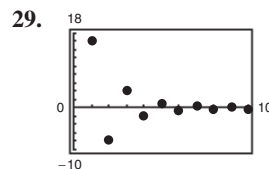
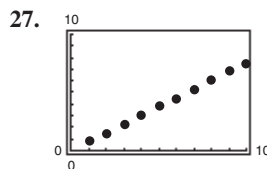
## Chapter 9

### Section 9.1 (page 649)

#### Vocabulary Check (page 649)

1. infinite sequence 2. terms 3. finite  
4. recursively 5. factorial  
6. summation notation 7. index; upper; lower  
8. series 9.  $n$ th partial sum

1. 4, 7, 10, 13, 16 3. 2, 4, 8, 16, 32  
5.  $-2, 4, -8, 16, -32$  7.  $3, 2, \frac{5}{3}, \frac{3}{2}, \frac{7}{5}$   
9.  $3, \frac{12}{11}, \frac{9}{13}, \frac{24}{47}, \frac{15}{37}$  11.  $0, 1, 0, \frac{1}{2}, 0$  13.  $\frac{5}{3}, \frac{17}{9}, \frac{53}{27}, \frac{161}{81}, \frac{485}{243}$   
15.  $1, \frac{1}{2^{3/2}}, \frac{1}{3^{3/2}}, \frac{1}{8}, \frac{1}{5^{3/2}}$  17.  $-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}$   
19.  $\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}$  21.  $0, 0, 6, 24, 60$  23.  $-73$  25.  $\frac{44}{239}$



33. c 34. b 35. d 36. a 37.  $a_n = 3n - 2$
39.  $a_n = n^2 - 1$  41.  $a_n = \frac{(-1)^n(n+1)}{n+2}$
43.  $a_n = \frac{n+1}{2n-1}$  45.  $a_n = \frac{1}{n^2}$  47.  $a_n = (-1)^{n+1}$
49.  $a_n = 1 + \frac{1}{n}$  51. 28, 24, 20, 16, 12
53. 3, 4, 6, 10, 18 55. 6, 8, 10, 12, 14  
 $a_n = 2n + 4$
57. 81, 27, 9, 3, 1 59.  $1, 3, \frac{9}{2}, \frac{9}{2}, \frac{27}{8}$   
 $a_n = \frac{243}{3^n}$
61.  $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}$  63.  $1, \frac{1}{2}, \frac{1}{24}, \frac{1}{720}, \frac{1}{40,320}$
65.  $\frac{1}{30}$  67. 90 69.  $n + 1$
71.  $\frac{1}{2n(2n+1)}$  73. 35 75. 40 77. 30
79.  $\frac{9}{5}$  81. 88 83. 30 85. 81 87.  $\frac{47}{60}$
89.  $\sum_{i=1}^9 \frac{1}{3i}$  91.  $\sum_{i=1}^8 \left[ 2\left(\frac{i}{8}\right) + 3 \right]$  93.  $\sum_{i=1}^6 (-1)^{i+1} 3^i$
95.  $\sum_{i=1}^{20} \frac{(-1)^{i+1}}{i^2}$  97.  $\sum_{i=1}^5 \frac{2^i - 1}{2^{i+1}}$  99.  $\frac{75}{16}$  101.  $-\frac{3}{2}$
103.  $\frac{2}{3}$  105.  $\frac{7}{9}$

107. (a)  $A_1 = \$5100.00$ ,  $A_2 = \$5202.00$ ,  $A_3 = \$5306.04$ ,  
 $A_4 = \$5412.16$ ,  $A_5 = \$5520.40$ ,  $A_6 = \$5630.81$ ,  
 $A_7 = \$5743.43$ ,  $A_8 = \$5858.30$

(b)  $A_{40} = \$11,040.20$

109. (a)  $b_n = 60.57n - 182$

(b)  $c_n = 1.61n^2 + 26.8n - 9.5$

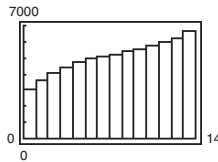
$n$	8	9	10	11	12	13
$a_n$	311	357	419	481	548	608
$b_n$	303	363	424	484	545	605
$c_n$	308	362	420	480	544	611

The quadratic model is a better fit.

- (d) The quadratic model; 995



111. (a)  $a_0 = \$3102.9$ ,  $a_1 = \$3644.3$ ,  $a_2 = \$4079.6$ ,  
 $a_3 = \$4425.3$ ,  $a_4 = \$4698.2$ ,  $a_5 = \$4914.8$ ,  
 $a_6 = \$5091.8$ ,  $a_7 = \$5245.7$ ,  $a_8 = \$5393.2$ ,  
 $a_9 = \$5550.9$ ,  $a_{10} = \$5735.5$ ,  $a_{11} = \$5963.5$ ,  
 $a_{12} = \$6251.5$ ,  $a_{13} = \$6616.3$



(b) The federal debt is increasing.

113. True by the Properties of Sums

115. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

$$1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}$$

117. \$500.95      119. Answers will vary.

121.  $x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}$

123.  $-\frac{x^2}{2}, \frac{x^4}{24}, -\frac{x^6}{720}, \frac{x^8}{40,320}, -\frac{x^{10}}{3,628,800}$

125.  $f^{-1}(x) = \frac{x+3}{4}$       127.  $h^{-1}(x) = \frac{x^2-1}{5}, x \geq 0$

129. (a)  $\begin{bmatrix} 8 & 1 \\ -2 & 6 \end{bmatrix}$       (b)  $\begin{bmatrix} -26 & 1 \\ 12 & -21 \end{bmatrix}$

- (c)  $\begin{bmatrix} 18 & 9 \\ 10 & 7 \end{bmatrix}$       (d)  $\begin{bmatrix} 4 & 2 \\ 24 & 21 \end{bmatrix}$

131. (a)  $\begin{bmatrix} -3 & -7 & 4 \\ 4 & 4 & 1 \\ 1 & 4 & 3 \end{bmatrix}$       (b)  $\begin{bmatrix} 10 & 25 & -10 \\ -12 & -11 & 3 \\ -3 & -9 & -8 \end{bmatrix}$

- (c)  $\begin{bmatrix} -2 & 7 & -16 \\ 4 & 42 & 45 \\ 1 & 23 & 48 \end{bmatrix}$       (d)  $\begin{bmatrix} 16 & 31 & 42 \\ 10 & 47 & 31 \\ 13 & 22 & 25 \end{bmatrix}$

133. 26      135. -194

## Section 9.2 (page 659)

### Vocabulary Check (page 659)

1. arithmetic; common      2.  $a_n = dn + c$   
 3. sum of a finite arithmetic sequence

1. Arithmetic sequence,  $d = -2$   
 3. Not an arithmetic sequence  
 5. Arithmetic sequence,  $d = -\frac{1}{4}$   
 7. Not an arithmetic sequence  
 9. Not an arithmetic sequence  
 11. 8, 11, 14, 17, 20  
 Arithmetic sequence,  $d = 3$   
 13. 7, 3, -1, -5, -9  
 Arithmetic sequence,  $d = -4$

15. -1, 1, -1, 1, -1

Not an arithmetic sequence

17.  $-3, \frac{3}{2}, -1, \frac{3}{4}, -\frac{3}{5}$

Not an arithmetic sequence

19.  $a_n = 3n - 2$       21.  $a_n = -8n + 108$

23.  $a_n = 2xn - x$       25.  $a_n = -\frac{5}{2}n + \frac{13}{2}$

27.  $a_n = \frac{10}{3}n + \frac{5}{3}$       29.  $a_n = -3n + 103$

31. 5, 11, 17, 23, 29      33. -2.6, -3.0, -3.4, -3.8, -4.2

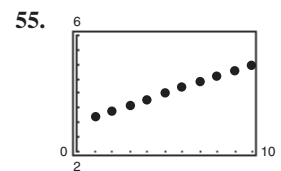
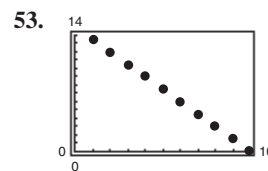
35. 2, 6, 10, 14, 18      37. -2, 2, 6, 10, 14

39. 15, 19, 23, 27, 31;  $d = 4$ ;  $a_n = 4n + 11$

41. 200, 190, 180, 170, 160;  $d = -10$ ;  $a_n = -10n + 210$

43.  $\frac{5}{8}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{1}{8}$ ;  $d = -\frac{1}{8}$ ;  $a_n = -\frac{1}{8}n + \frac{3}{4}$

45. 59      47. 18.6      49. b      50. d      51. c      52. a



57. 620      59. 17.4      61. 265      63. 4000

65. 10,000      67. 1275      69. 30,030      71. 355

73. 160,000      75. 520      77. 2725      79. 10,120

81. (a) \$40,000      (b) \$217,500      83. 2340 seats

85. 405 bricks      87. 490 meters

89. (a)  $a_n = -25n + 225$       (b) \$900

91. \$70,500; answers will vary.

93. (a)

Month	1	2	3	4	5	6
Monthly payment	\$220	\$218	\$216	\$214	\$212	\$210
Unpaid balance	\$1800	\$1600	\$1400	\$1200	\$1000	\$800

- (b) \$110

95. (a)  $a_n = 1098n + 17,588$

- (b)  $a_n = 1114.9n + 17,795$ ; the models are similar.

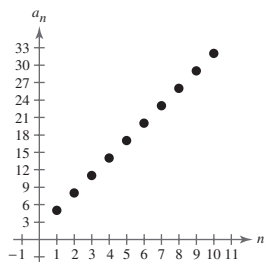
- (c)      (d) 2004: \$32,960  
 2005: \$34,058

- (e) Answers will vary.

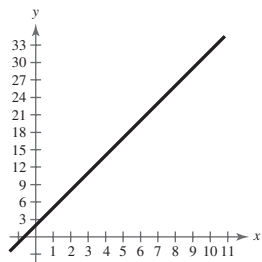
97. True. Given  $a_1$  and  $a_2$ ,  $d = a_2 - a_1$  and  
 $a_n = a_1 + (n - 1)d$ .

99. Answers will vary.

101. (a)



(b)

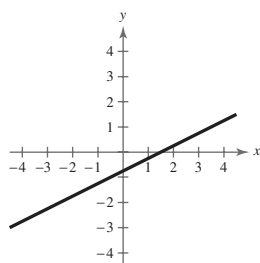


(c) The graph of  $y = 3x + 2$  contains all points on the line. The graph of  $a_n = 2 + 3n$  contains only points at the positive integers.

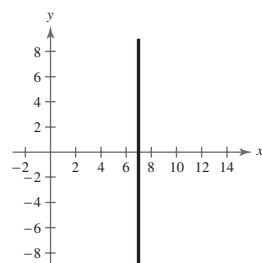
(d) The slope of the line and the common difference of the arithmetic sequence are equal.

103. 4

105. Slope:  $\frac{1}{2}$ ;  
y-intercept:  $(0, -\frac{3}{4})$



107. Slope: undefined;  
No y-intercept

109.  $x = 1, y = 5, z = -1$ 

111. Answers will vary.

## Section 9.3 (page 669)

## Vocabulary Check (page 669)

1. geometric; common 2.  $a_n = a_1 r^{n-1}$ 3.  $S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right)$  4. geometric series5.  $S = \frac{a_1}{1 - r}$ 1. Geometric sequence,  $r = 3$ 

3. Not a geometric sequence

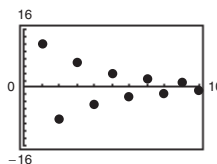
5. Geometric sequence,  $r = -\frac{1}{2}$ 7. Geometric sequence,  $r = 2$ 

9. Not a geometric sequence

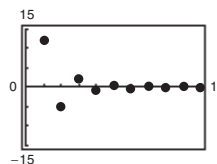
11. 2, 6, 18, 54, 162 13.  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ 15.  $5, -\frac{1}{2}, \frac{1}{20}, -\frac{1}{200}, \frac{1}{2000}$  17.  $1, e, e^2, e^3, e^4$ 19.  $2, \frac{x}{2}, \frac{x^2}{8}, \frac{x^3}{32}, \frac{x^4}{128}$ 21. 64, 32, 16, 8, 4;  $r = \frac{1}{2}$ ;  $a_n = 128\left(\frac{1}{2}\right)^n$ 23. 7, 14, 28, 56, 112;  $r = 2$ ;  $a_n = \frac{7}{2}(2)^n$ 25.  $6, -9, \frac{27}{2}, -\frac{81}{4}, \frac{243}{8}$ ;  $r = -\frac{3}{2}$ ;  $a_n = -4\left(-\frac{3}{2}\right)^n$ 27.  $a_n = 4\left(\frac{1}{2}\right)^{n-1}$ ;  $\frac{1}{128}$  29.  $a_n = 6\left(-\frac{1}{3}\right)^{n-1}$ ;  $-\frac{2}{3^{10}}$ 31.  $a_n = 100e^{x(n-1)}$ ;  $100e^{8x}$ 33.  $a_n = 500(1.02)^{n-1}$ ;  $\approx 1082.372$  35. 45,92737. 50,388,480 39.  $a_3 = 9$  41.  $a_6 = -2$ 

43. a 44. c 45. b 46. d

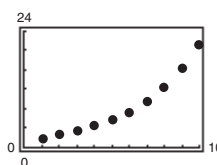
47.



49.



51.



53. 511

55. 171

57. 43

59.  $\frac{1365}{32}$ 

61. 29,921.311

63. 592.647

65. 2092.596

67.  $\frac{8}{5}$ 

69. 6.400

71. 3.750

73.  $\sum_{n=1}^7 5(3)^{n-1}$ 75.  $\sum_{n=1}^7 2\left(-\frac{1}{4}\right)^{n-1}$ 77.  $\sum_{n=1}^6 0.1(4)^{n-1}$ 

79. 2

81.  $\frac{2}{3}$ 83.  $\frac{16}{3}$ 85.  $\frac{5}{3}$ 

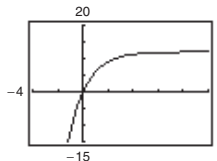
87. -30

89. 32

91. Undefined

93.  $\frac{4}{11}$ 95.  $\frac{7}{22}$ 

97.

Horizontal asymptote:  $y = 12$ 

Corresponds to the sum of the series

99. (a)  $a_n = 1190.88(1.006)^n$ 

(b) The population is growing at a rate of 0.6% per year.

(c) 1342.2 million. This value is close to the prediction.

(d) 2007

101. (a) \$3714.87 (b) \$3722.16 (c) \$3725.85

(d) \$3728.32 (e) \$3729.52

103. \$7011.89 105. Answers will vary.

107. (a) \$26,198.27 (b) \$26,263.88

109. (a) \$118,590.12 (b) \$118,788.73

111. Answers will vary. 113. \$1600

115.  $\approx \$2181.82$  117. 126 square inches

119. \$3,623,993.23

121. False. A sequence is geometric if the ratios of consecutive terms are the same.

123. Given a real number  $r$  between  $-1$  and  $1$ , as the exponent  $n$  increases,  $r^n$  approaches zero.

125.  $x^2 + 2x$     127.  $3x^2 + 6x + 1$   
 129.  $x(3x + 8)(3x - 8)$     131.  $(3x + 1)(2x - 5)$   
 133.  $\frac{3x}{x - 3}$ ,  $x \neq -3$     135.  $\frac{2x + 1}{3}$ ,  $x \neq 0, -\frac{1}{2}$   
 137.  $\frac{5x^2 + 9x - 30}{(x + 2)(x - 2)}$     139. Answers will vary.

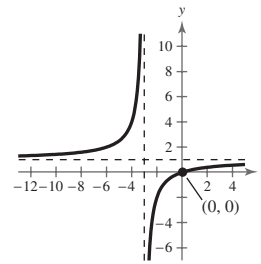
**Section 9.4** (page 681)

**Vocabulary Check** (page 681)

1. mathematical induction    2. first  
 3. arithmetic    4. second

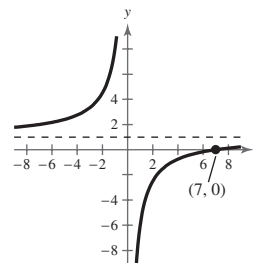
1.  $\frac{5}{(k + 1)(k + 2)}$     3.  $\frac{(k + 1)^2(k + 2)^2}{4}$   
 5–33. Answers will vary.    35.  $S_n = n(2n - 1)$   
 37.  $S_n = 10 - 10\left(\frac{9}{10}\right)^n$     39.  $S_n = \frac{n}{2(n + 1)}$   
 41. 120    43. 91    45. 979    47. 70    49. -3402  
 51. 0, 3, 6, 9, 12, 15  
 First differences: 3, 3, 3, 3, 3  
 Second differences: 0, 0, 0, 0  
 Linear  
 53. 3, 1, -2, -6, -11, -17  
 First differences: -2, -3, -4, -5, -6  
 Second differences: -1, -1, -1, -1  
 Quadratic  
 55. 2, 4, 16, 256, 65,536, 4,294,967,296  
 First differences: 2, 12, 240, 65,280, 4,294,901,760  
 Second differences: 10, 228, 65,040, 4,294,836,480  
 Neither  
 57.  $a_n = n^2 - n + 3$     59.  $a_n = \frac{1}{2}n^2 + n - 3$   
 61. (a) 2.2, 2.4, 2.2, 2.3, 0.9  
 (b) A linear model can be used.  
 $a_n = 2.2n + 102.7$   
 (c)  $a_n = 2.08n + 103.9$   
 (d) Part b:  $a_n = 142.3$ ; Part c:  $a_n = 141.34$   
 These are very similar.  
 63. True.  $P_7$  may be false.  
 65. True. If the second differences are all zero, then the first differences are all the same and the sequence is arithmetic.  
 67.  $4x^4 - 4x^2 + 1$     69.  $-64x^3 + 240x^2 - 300x + 125$   
 71. (a) Domain: all real numbers  $x$  except  $x = -3$   
 (b) Intercept: (0, 0)  
 (c) Vertical asymptote:  $x = -3$   
 Horizontal asymptote:  $y = 1$

(d)



73. (a) Domain: all real numbers  $t$  except  $t = 0$   
 (b)  $t$ -intercept: (7, 0)  
 (c) Vertical asymptote:  $t = 0$   
 Horizontal asymptote:  $y = 1$

(d)



**Section 9.5** (page 688)

**Vocabulary Check** (page 688)

1. binomial coefficients  
 2. Binomial Theorem; Pascal's Triangle  
 3.  $\binom{n}{r}$ ;  ${}_nC_r$     4. expanding a binomial

1. 10    3. 1    5. 15,504    7. 210    9. 4950  
 11. 56    13. 35    15.  $x^4 + 4x^3 + 6x^2 + 4x + 1$   
 17.  $a^4 + 24a^3 + 216a^2 + 864a + 1296$   
 19.  $y^3 - 12y^2 + 48y - 64$   
 21.  $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$   
 23.  $r^6 + 18r^5s + 135r^4s^2 + 540r^3s^3 + 1215r^2s^4 + 1458rs^5 + 729s^6$   
 25.  $243a^5 - 1620a^4b + 4320a^3b^2 - 5760a^2b^3 + 3840ab^4 - 1024b^5$   
 27.  $8x^3 + 12x^2y + 6xy^2 + y^3$   
 29.  $x^8 + 4x^6y^2 + 6x^4y^4 + 4x^2y^6 + y^8$   
 31.  $\frac{1}{x^5} + \frac{5y}{x^4} + \frac{10y^2}{x^3} + \frac{10y^3}{x^2} + \frac{5y^4}{x} + y^5$   
 33.  $2x^4 - 24x^3 + 113x^2 - 246x + 207$   
 35.  $32t^5 - 80t^4s + 80t^3s^2 - 40t^2s^3 + 10ts^4 - s^5$

37.  $x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

39.  $120x^7y^3$  41.  $360x^3y^2$  43.  $1,259,712x^2y^7$

45.  $32,476,950,000x^4y^8$  47.  $1,732,104$

49. 180 51.  $-326,592$  53. 210

55.  $x^2 + 12x^{3/2} + 54x + 108x^{1/2} + 81$

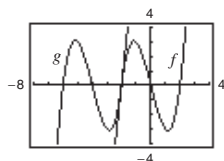
57.  $x^2 - 3x^{4/3}y^{1/3} + 3x^{2/3}y^{2/3} - y$

59.  $3x^2 + 3xh + h^2, h \neq 0$  61.  $\frac{1}{\sqrt{x+h} + \sqrt{x}}, h \neq 0$

63.  $-4$  65.  $2035 + 828i$  67. 1

69. 1.172 71. 510,568.785

73.

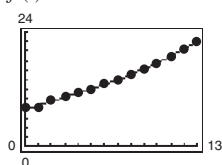
 $g$  is shifted four units to the left of  $f$ .

$g(x) = x^3 + 12x^2 + 44x + 48$

75. 0.273 77. 0.171

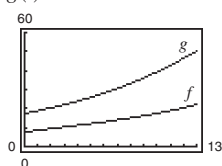
79. (a)  $f(t) = 0.0025t^3 - 0.015t^2 + 0.88t + 7.7$

(b)



(c)  $g(t) = 0.0025t^3 + 0.06t^2 + 1.33t + 17.5$

(d)



(e)  $f(t)$ : 33.26 gallons;  $g(t)$ : 33.26 gallons; yes

(f) The trend is for the per capita consumption of bottled water to increase. This may be due to the increasing concern with contaminants in tap water.

81. True. The coefficients from the Binomial Theorem can be used to find the numbers in Pascal's Triangle.

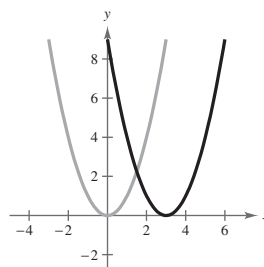
83. False. The coefficient of the  $x^{10}$ -term is 1,732,104 and the coefficient of the  $x^{14}$ -term is 192,456.

$$\begin{array}{cccccccccccccccc}
 85. & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \\
 & 1 & 9 & 36 & 84 & 126 & 126 & 84 & 36 & 9 & 1 \\
 & 1 & 10 & 45 & 120 & 210 & 252 & 210 & 120 & 45 & 10 & 1
 \end{array}$$

87. The signs of the terms in the expansion of  $(x - y)^n$  alternate between positive and negative.

89–91. Answers will vary.

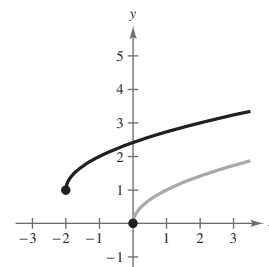
93.



$g(x) = (x - 3)^2$

97.  $\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

95.



$g(x) = \sqrt{x + 2} + 1$

## Section 9.6 (page 698)

## Vocabulary Check (page 698)

1. Fundamental Counting Principle 2. permutation

3.  ${}_nP_r = \frac{n!}{(n-r)!}$  4. distinguishable permutations

5. combinations

1. 6 3. 5 5. 3 7. 8 9. 30 11. 30

13. 64 15. 175,760,000

17. (a) 900 (b) 648 (c) 180 (d) 600

19. 64,000 21. (a) 40,320 (b) 384 23. 24

25. 336 27. 120 29.  $n = 5$  or  $n = 6$

31. 1,860,480 33. 970,200 35. 15,504 37. 120

39. 11,880 41. 420 43. 2520

45. ABCD, ABDC, ACBD, ACDB, ADCB, ADCB, BADC, BADC, CABD, CADB, DABC, DACB, BCAD, BDAC, CBAD, CDAB, DBAC, DCAB, BCDA, BDCA, CBDA, CDBA, DBCA, DCBA

47. 1,816,214,400 49. 5,586,853,480

51. AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF

53. 324,632 55. (a) 35 (b) 63 (c) 203

57. (a) 3744 (b) 24 59. 292,600

61. 5 63. 20

65. (a) 146,107,962

(b) If the jackpot is won, there is only one winning number.

(c) There are 28,989,675 possible winning numbers in the state lottery, which is considerably less than the possible number of winning Powerball numbers.

67. False. It is an example of a combination.

69. They are equal.

71–73. Proof

75. No. For some calculators the number is too great.

77. (a) 35 (b) 8 (c) 83

79. (a)  $-4$  (b) 0 (c) 0 81. 8.30 83. 35

Section 9.7 (page 709)

**Vocabulary Check** (page 709)

1. experiment; outcomes      2. sample space
3. probability      4. impossible; certain
5. mutually exclusive      6. independent
7. complement      8. (a) iii (b) i (c) iv (d) ii

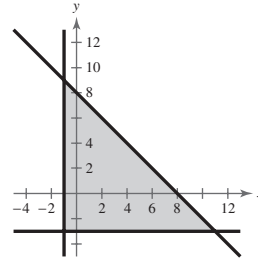
1.  $\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$
3.  $\{ABC, ACB, BAC, BCA, CAB, CBA\}$
5.  $\{AB, AC, AD, AE, BC, BD, BE, CD, CE, DE\}$
7.  $\frac{3}{8}$       9.  $\frac{7}{8}$       11.  $\frac{3}{13}$       13.  $\frac{3}{26}$       15.  $\frac{1}{12}$       17.  $\frac{11}{12}$
19.  $\frac{1}{3}$       21.  $\frac{1}{5}$       23.  $\frac{2}{5}$       25. 0.3      27.  $\frac{3}{4}$       29. 0.86
31.  $\frac{18}{35}$       33. (a) 58% (b) 95.6% (c) 0.4%
35. (a) 243 (b)  $\frac{1}{50}$  (c)  $\frac{16}{25}$
37. (a)  $\frac{112}{209}$  (b)  $\frac{97}{209}$  (c)  $\frac{274}{627}$
39.  $P(\{\text{Taylor wins}\}) = \frac{1}{2}$   
 $P(\{\text{Moore wins}\}) = P(\{\text{Jenkins wins}\}) = \frac{1}{4}$
41. (a)  $\frac{21}{1292}$  (b)  $\frac{225}{646}$  (c)  $\frac{49}{323}$
43. (a)  $\frac{1}{120}$  (b)  $\frac{1}{24}$       45. (a)  $\frac{5}{13}$  (b)  $\frac{1}{2}$  (c)  $\frac{4}{13}$
47. (a)  $\frac{14}{55}$  (b)  $\frac{12}{55}$  (c)  $\frac{54}{55}$       49. 0.4746
51. (a) 0.9702 (b) 0.9998 (c) 0.0002
53. (a)  $\frac{1}{16}$  (b)  $\frac{1}{8}$  (c)  $\frac{15}{16}$
55. (a)  $\frac{1}{38}$  (b)  $\frac{9}{19}$  (c)  $\frac{10}{19}$  (d)  $\frac{1}{1444}$  (e)  $\frac{729}{6859}$
- (f) The probabilities are slightly better in European roulette.
57. True. Two events are independent if the occurrence of one has no effect on the occurrence of the other.
59. (a) As you consider successive people with distinct birthdays, the probabilities must decrease to take into account the birth dates already used. Because the birth dates of people are independent events, multiply the respective probabilities of distinct birthdays.  
 (b)  $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365}$  (c) Answers will vary.  
 (d)  $Q_n$  is the probability that the birthdays are *not* distinct, which is equivalent to at least two people having the same birthday.

(e)

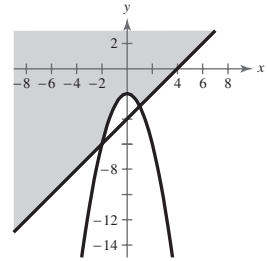
$n$	10	15	20	23	30	40	50
$P_n$	0.88	0.75	0.59	0.49	0.29	0.11	0.03
$Q_n$	0.12	0.25	0.41	0.51	0.71	0.89	0.97

- (f) 23
61. No real solution      63.  $0, \frac{1 \pm \sqrt{13}}{2}$       65. -4
67.  $\frac{11}{2}$       69. -10

71.



73.



**Review Exercises** (page 715)

1. 8, 5, 4,  $\frac{7}{2}$ ,  $\frac{16}{5}$       3. 72, 36, 12, 3,  $\frac{3}{5}$       5.  $a_n = 2(-1)^n$
7.  $a_n = \frac{4}{n}$       9. 120      11. 1      13. 30      15.  $\frac{205}{24}$
17. 6050      19.  $\sum_{k=1}^{20} \frac{1}{2k}$       21.  $\frac{5}{9}$       23.  $\frac{2}{99}$
25. (a)  $A_1 = \$10,067$ ,  $A_2 = \$10,134$ ,  $A_3 = \$10,201$ ,  
 $A_4 = \$10,269$ ,  $A_5 = \$10,338$ ,  $A_6 = \$10,407$ ,  
 $A_7 = \$10,476$ ,  $A_8 = \$10,546$ ,  $A_9 = \$10,616$ ,  
 $A_{10} = \$10,687$   
 (b)  $A_{120} = \$22,196.40$
27. Arithmetic sequence,  $d = -2$
29. Arithmetic sequence,  $d = \frac{1}{2}$       31. 4, 7, 10, 13, 16
33. 25, 28, 31, 34, 37      35.  $a_n = 12n - 5$
37.  $a_n = 3ny - 2y$       39.  $a_n = -7n + 107$
41. 80      43. 88      45. 25,250
47. (a) \$43,000 (b) \$192,500
49. Geometric sequence,  $r = 2$
51. Geometric sequence,  $r = -2$       53. 4, -1,  $\frac{1}{4}$ ,  $-\frac{1}{16}$ ,  $\frac{1}{64}$
55. 9, 6, 4,  $\frac{8}{3}$ ,  $\frac{16}{9}$  or 9, -6, 4,  $-\frac{8}{3}$ ,  $\frac{16}{9}$
57.  $a_n = 16(-\frac{1}{2})^{n-1}$ ;  $\approx -3.052 \times 10^{-5}$
59.  $a_n = 100(1.05)^{n-1}$ ;  $\approx 252.695$
61. 127      63.  $\frac{15}{16}$       65. 31      67. 24.85
69. 5486.45      71. 8      73.  $\frac{10}{9}$       75. 12
77. (a)  $a_t = 120,000(0.7)^t$  (b) \$20,168.40
- 79–81. Answers will vary.      83.  $S_n = n(2n + 7)$
85.  $S_n = \frac{5}{2}[1 - (\frac{3}{5})^n]$       87. 465      89. 4648
91. 5, 10, 15, 20, 25  
 First differences: 5, 5, 5, 5  
 Second differences: 0, 0, 0  
 Linear
93. 16, 15, 14, 13, 12  
 First differences: -1, -1, -1, -1  
 Second differences: 0, 0, 0  
 Linear
95. 15      97. 56      99. 35      101. 28
103.  $x^4 + 16x^3 + 96x^2 + 256x + 256$
105.  $a^5 - 15a^4b + 90a^3b^2 - 270a^2b^3 + 405ab^4 - 243b^5$
107. 41 + 840i      109. 11      111. 10,000      113. 720
115. 56      117.  $\frac{1}{9}$       119. (a) 43% (b) 82%

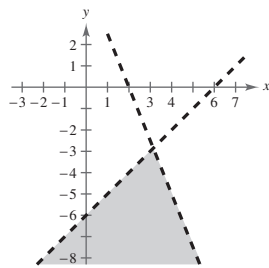
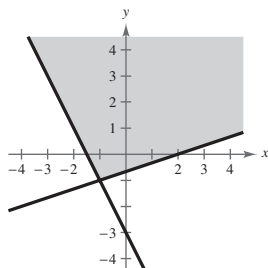
121.  $\frac{1}{216}$  123.  $\frac{3}{4}$   
 125. True.  $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)n!}{n!} = (n+2)(n+1)$   
 127. True by Properties of Sums  
 129. False. When  $r$  equals 0 or 1, then the results are the same.  
 131. In the sequence in part (a), the odd-numbered terms are negative, whereas in the sequence in part (b), the even-numbered terms are negative.  
 133. Each term of the sequence is defined in terms of preceding terms.  
 135. d 136. a 137. b 138. c  
 139. 240, 440, 810, 1490, 2740

### Chapter Test (page 719)

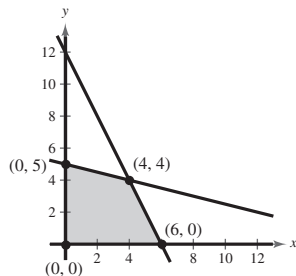
1.  $-\frac{1}{5}, \frac{1}{8}, -\frac{1}{11}, \frac{1}{14}, -\frac{1}{17}$  2.  $a_n = \frac{n+2}{n!}$   
 3. 50, 61, 72; 140 4.  $a_n = 0.8n + 1.4$   
 5. 5, 10, 20, 40, 80 6. 86, 100 7. 189  
 8. 4 9. Answers will vary.  
 10.  $x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$  11. -108,864  
 12. (a) 72 (b) 328,440 13. (a) 330 (b) 720,720  
 14. 26,000 15. 720 16.  $\frac{1}{15}$  17.  $3.908 \times 10^{-10}$   
 18. 25%

### Cumulative Test for Chapters 7–9 (page 720)

1.  $(1, 2), (-\frac{3}{2}, \frac{3}{4})$  2.  $(2, -1)$   
 3.  $(4, 2, -3)$  4.  $(1, -2, 1)$   
 5.



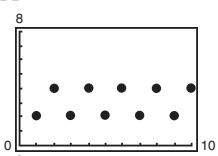
7.

Maximum at  $(4, 4)$ :  $z = 20$ Minimum at  $(0, 0)$ :  $z = 0$ 

8. \$0.75 mixture: 120 pounds; \$1.25 mixture: 80 pounds  
 9.  $y = \frac{1}{3}x^2 - 2x + 4$

10.  $\begin{bmatrix} -1 & 2 & -1 & \vdots & 9 \\ 2 & -1 & 2 & \vdots & -9 \\ 3 & 3 & -4 & \vdots & 7 \end{bmatrix}$  11.  $(-2, 3, -1)$   
 12.  $\begin{bmatrix} 3 & 3 \\ 0 & 2 \end{bmatrix}$  13.  $\begin{bmatrix} 2 & -6 \\ -2 & 0 \end{bmatrix}$  14.  $\begin{bmatrix} 6 & -6 \\ -3 & 2 \end{bmatrix}$   
 15.  $\begin{bmatrix} -4 & 12 \\ 3 & -3 \end{bmatrix}$  16. 84 17.  $\begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$   
 18. Gym shoes: \$198.36 million  
 Jogging shoes: \$358.48 million  
 Walking shoes: \$167.17 million  
 19.  $(-5, 4)$  20.  $(-3, 4, 2)$  21. 9  
 22.  $\frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, -\frac{1}{11}, \frac{1}{13}$  23.  $a_n = \frac{(n+1)!}{n+3}$   
 24. 920 25. (a) 65.4 (b)  $a_n = 3.2n + 1.4$   
 26. 3, 6, 12, 24, 48 27.  $\frac{13}{9}$  28. Answers will vary.  
 29.  $z^4 - 12z^3 + 54z^2 - 108z + 81$  30. 210 31. 600  
 32. 70 33. 120 34. 453,600 35. 151,200  
 36. 720 37.  $\frac{1}{4}$

### Problem Solving (page 725)

1. 1, 1.5, 1.41 $\bar{6}$ , 1.414215686, 1.414213562, 1.414213562, . . .  
 $x_n$  approaches  $\sqrt{2}$ .  
 3. (a)  (b) If  $n$  is odd,  $a_n = 2$ , and if  $n$  is even,  $a_n = 4$ .  
 (c) 

$n$	1	10	101	1000	10,001
$a_n$	2	4	2	4	2

  
 (d) It is not possible to find the value of  $a_n$  as  $n$  approaches infinity.  
 5. (a) 3, 5, 7, 9, 11, 13, 15, 17;  $a_n = 2n + 1$   
 (b) To obtain the arithmetic sequence, find the differences of consecutive terms of the sequence of perfect cubes. Then find the differences of consecutive terms of this sequence.  
 (c) 12, 18, 24, 30, 36, 42, 48;  $a_n = 6n + 6$   
 (d) To obtain the arithmetic sequence, find the third sequence obtained by taking differences of consecutive terms in consecutive sequences.  
 (e) 60, 84, 108, 132, 156, 180;  $a_n = 24n + 36$   
 7.  $s_n = \left(\frac{1}{2}\right)^{n-1}$   
 $a_n = \frac{\sqrt{3}}{4}s_n^2$   
 9. Answers will vary.

11. (a) Answers will vary. (b) 17,710  
 13.  $\frac{1}{3}$  15. (a) -\$0.71 (b) 2.53, 24 turns

# Chapter 10

## Section 10.1 (page 732)

### Vocabulary Check (page 732)

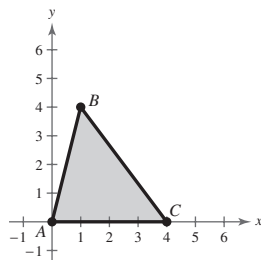
1. inclination
2.  $\tan \theta$
3.  $\frac{|m_2 - m_1|}{1 + m_1 m_2}$
4.  $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

1.  $\frac{\sqrt{3}}{3}$
3.  $-1$
5.  $\sqrt{3}$
7. 3.2236
9.  $\frac{3\pi}{4}$  radians,  $135^\circ$
11.  $\frac{\pi}{4}$  radian,  $45^\circ$
13. 0.6435 radian,  $36.9^\circ$
15. 1.0517 radians,  $60.3^\circ$
17. 2.1112 radians,  $121.0^\circ$
19. 1.2490 radians,  $71.6^\circ$
21. 2.1112 radians,  $121.0^\circ$
23. 1.1071 radians,  $63.4^\circ$
25. 0.1974 radian,  $11.3^\circ$
27. 1.4289 radians,  $81.9^\circ$
29. 0.9273 radian,  $53.1^\circ$
31. 0.8187 radian,  $46.9^\circ$
33.  $(2, 1) \leftrightarrow (4, 4)$ : slope =  $\frac{3}{2}$   
 $(4, 4) \leftrightarrow (6, 2)$ : slope =  $-1$   
 $(6, 2) \leftrightarrow (2, 1)$ : slope =  $\frac{1}{4}$   
 $(2, 1)$ :  $42.3^\circ$ ;  $(4, 4)$ :  $78.7^\circ$ ;  $(6, 2)$ :  $59.0^\circ$
35.  $(-4, -1) \leftrightarrow (3, 2)$ : slope =  $\frac{3}{7}$   
 $(3, 2) \leftrightarrow (1, 0)$ : slope =  $1$   
 $(1, 0) \leftrightarrow (-4, -1)$ : slope =  $\frac{1}{5}$   
 $(-4, -1)$ :  $11.9^\circ$ ;  $(3, 2)$ :  $21.8^\circ$ ;  $(1, 0)$ :  $146.3^\circ$

37. 0
39.  $\frac{7}{5}$
41. 7
43.  $\frac{8\sqrt{37}}{37} \approx 1.3152$

45.

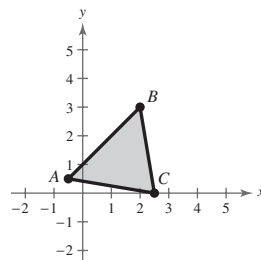
(a)



- (b) 4 (c) 8

47.

(a)



- (b)  $\frac{35\sqrt{37}}{74}$  (c)  $\frac{35}{8}$

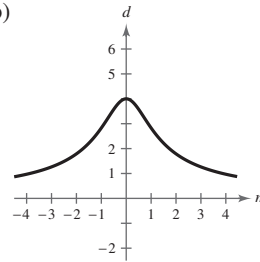
49.  $2\sqrt{2}$
51. 0.1003, 1054 feet
53.  $31.0^\circ$

55.  $\alpha \approx 33.69^\circ$ ;  $\beta \approx 56.31^\circ$

57. True. The inclination of a line is related to its slope by  $m = \tan \theta$ . If the angle is greater than  $\pi/2$  but less than  $\pi$ , then the angle is in the second quadrant, where the tangent function is negative.

59. (a)  $d = \frac{4}{\sqrt{m^2 + 1}}$

(b)

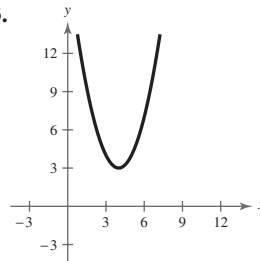


(c)  $m = 0$

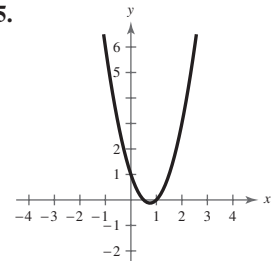
- (d) The graph has a horizontal asymptote at  $d = 0$ . As the slope becomes larger, the distance between the origin and the line  $y = mx + 4$ , becomes smaller and approaches 0.

61. x-intercept:  $(7, 0)$   
y-intercept:  $(0, 49)$
63. x-intercepts:  $(5 \pm \sqrt{5}, 0)$   
y-intercept:  $(0, 20)$
65. x-intercepts:  $\left(\frac{7 \pm \sqrt{53}}{2}, 0\right)$   
y-intercept:  $(0, -1)$
67.  $f(x) = 3\left(x + \frac{1}{3}\right)^2 - \frac{49}{3}$   
Vertex:  $\left(-\frac{1}{3}, -\frac{49}{3}\right)$
69.  $f(x) = 5\left(x + \frac{17}{5}\right)^2 - \frac{324}{5}$   
Vertex:  $\left(-\frac{17}{5}, -\frac{324}{5}\right)$
71.  $f(x) = 6\left(x - \frac{1}{12}\right)^2 - \frac{289}{24}$   
Vertex:  $\left(\frac{1}{12}, -\frac{289}{24}\right)$

73.



75.

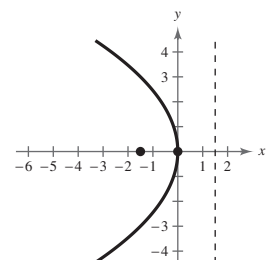
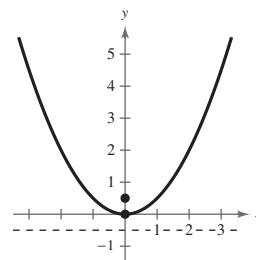


## Section 10.2 (page 740)

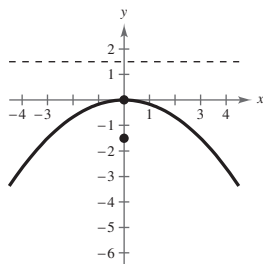
### Vocabulary Check (page 740)

1. conic
2. locus
3. parabola; directrix; focus
4. axis
5. vertex
6. focal chord
7. tangent

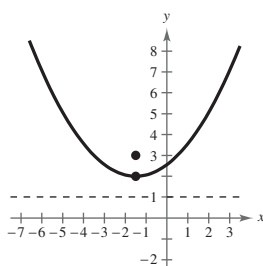
1. A circle is formed when a plane intersects the top or bottom half of a double-napped cone and is perpendicular to the axis of the cone.
3. A parabola is formed when a plane intersects the top or bottom half of a double-napped cone, is parallel to the side of the cone, and does not intersect the vertex.
5. e
6. b
7. d
8. f
9. a
10. c
11. Vertex:  $(0, 0)$   
Focus:  $\left(0, \frac{1}{2}\right)$   
Directrix:  $y = -\frac{1}{2}$
13. Vertex:  $(0, 0)$   
Focus:  $\left(-\frac{3}{2}, 0\right)$   
Directrix:  $x = \frac{3}{2}$



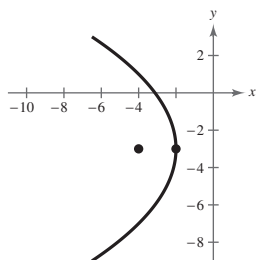
15. Vertex:  $(0, 0)$   
Focus:  $(0, -\frac{3}{2})$   
Directrix:  $y = \frac{3}{2}$



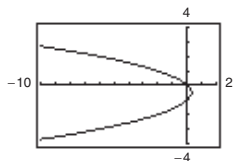
19. Vertex:  $(-\frac{3}{2}, 2)$   
Focus:  $(-\frac{3}{2}, 3)$   
Directrix:  $y = 1$



23. Vertex:  $(-2, -3)$   
Focus:  $(-4, -3)$   
Directrix:  $x = 0$

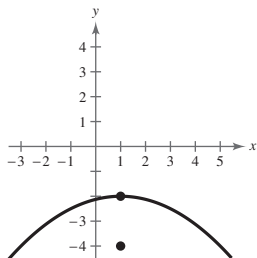


27. Vertex:  $(\frac{1}{4}, -\frac{1}{2})$   
Focus:  $(0, -\frac{1}{2})$   
Directrix:  $x = \frac{1}{2}$

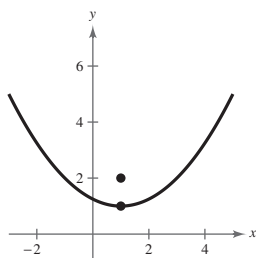


29.  $x^2 = \frac{3}{2}y$     31.  $x^2 = -6y$     33.  $y^2 = -8x$   
35.  $x^2 = 4y$     37.  $y^2 = -8x$     39.  $y^2 = 9x$   
41.  $(x - 3)^2 = -(y - 1)$     43.  $y^2 = 4(x + 4)$

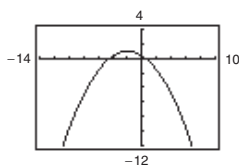
17. Vertex:  $(1, -2)$   
Focus:  $(1, -4)$   
Directrix:  $y = 0$



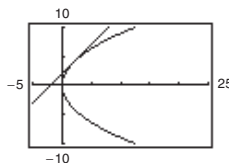
21. Vertex:  $(1, 1)$   
Focus:  $(1, 2)$   
Directrix:  $y = 0$



25. Vertex:  $(-2, 1)$   
Focus:  $(-2, -\frac{1}{2})$   
Directrix:  $x = -2$



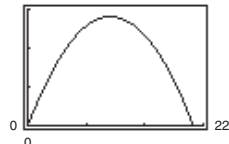
45.  $(y - 2)^2 = -8(x - 5)$     47.  $x^2 = 8(y - 4)$   
49.  $(y - 2)^2 = 8x$     51.  $y = \sqrt{6(x + 1)} + 3$   
53.



$(2, 4)$

55.  $4x - y - 8 = 0$ ;  $(2, 0)$

59.  $15,000$



$x = 106$  units

63. (a)  $y = -\frac{1}{640}x^2$     (b) 8 feet

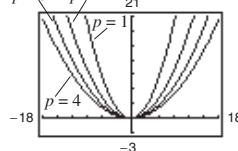
65. (a)  $17,500\sqrt{2}$  miles per hour  $\approx 24,750$  miles per hour

- (b)  $x^2 = -16,400(y - 4100)$

67. (a)  $x^2 = -64(y - 75)$     (b) 69.3 feet

69. False. If the graph crossed the directrix, there would exist points closer to the directrix than the focus.

71. (a)  $p = 3$      $p = 2$      $p = 1$      $p = 4$



As  $p$  increases, the graph becomes wider.

- (b)  $(0, 1), (0, 2), (0, 3), (0, 4)$     (c) 4, 8, 12, 16;  $4|p|$

- (d) Easy way to determine two additional points on the graph

73.  $m = \frac{x_1}{2p}$     75.  $\pm 1, \pm 2, \pm 4$

77.  $\pm \frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

79.  $f(x) = x^3 - 7x^2 + 17x - 15$     81.  $\frac{1}{2}, -\frac{5}{3}, \pm 2$

83.  $B \approx 23.67^\circ, C \approx 121.33^\circ, c \approx 14.89$

85.  $C = 89^\circ, a \approx 1.93, b \approx 2.33$

87.  $A \approx 16.39^\circ, B \approx 23.77^\circ, C \approx 139.84^\circ$

89.  $B \approx 24.62^\circ, C \approx 90.38^\circ, a \approx 10.88$

### Section 10.3 (page 750)

#### Vocabulary Check (page 750)

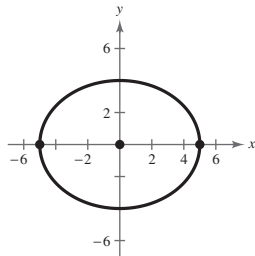
1. ellipse; foci    2. major axis; center  
3. minor axis    4. eccentricity



1. b    2. c    3. d    4. f    5. a    6. e

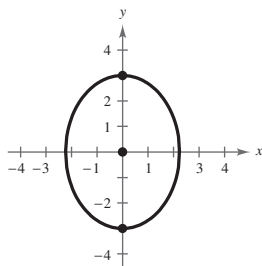
7. Ellipse

Center:  $(0, 0)$   
 Vertices:  $(\pm 5, 0)$   
 Foci:  $(\pm 3, 0)$   
 Eccentricity:  $\frac{3}{5}$



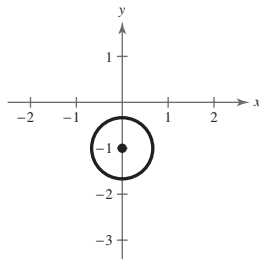
11. Ellipse

Center:  $(0, 0)$   
 Vertices:  $(0, \pm 3)$   
 Foci:  $(0, \pm 2)$   
 Eccentricity:  $\frac{2}{3}$



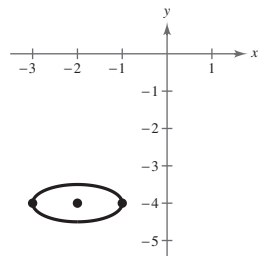
15. Circle

Center:  $(0, -1)$   
 Radius:  $\frac{2}{3}$



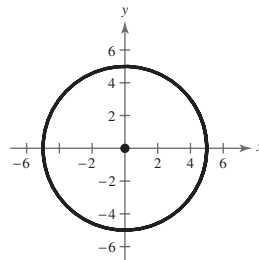
17. Ellipse

Center:  $(-2, -4)$   
 Vertices:  $(-3, -4), (-1, -4)$   
 Foci:  $\left(\frac{-4 \pm \sqrt{3}}{2}, -4\right)$   
 Eccentricity:  $\frac{\sqrt{3}}{2}$



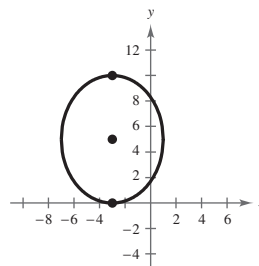
9. Circle

Center:  $(0, 0)$   
 Radius: 5



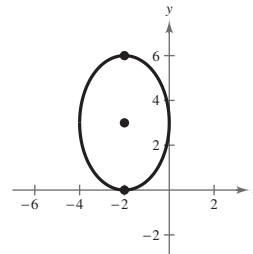
13. Ellipse

Center:  $(-3, 5)$   
 Vertices:  $(-3, 10), (-3, 0)$   
 Foci:  $(-3, 8), (-3, 2)$   
 Eccentricity:  $\frac{3}{5}$



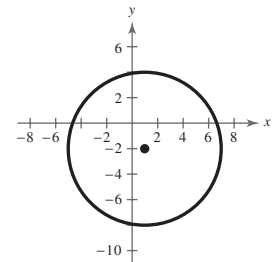
19. Ellipse

Center:  $(-2, 3)$   
 Vertices:  $(-2, 6), (-2, 0)$   
 Foci:  $(-2, 3 \pm \sqrt{5})$   
 Eccentricity:  $\frac{\sqrt{5}}{3}$



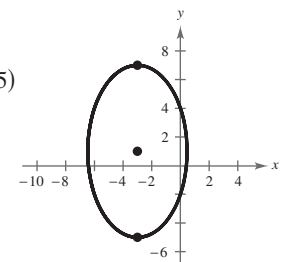
21. Circle

Center:  $(1, -2)$   
 Radius: 6



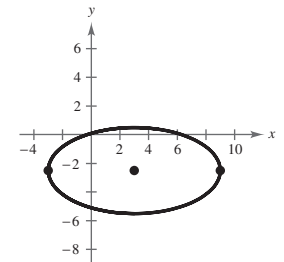
23. Ellipse

Center:  $(-3, 1)$   
 Vertices:  $(-3, 7), (-3, -5)$   
 Foci:  $(-3, 1 \pm 2\sqrt{6})$   
 Eccentricity:  $\frac{\sqrt{6}}{3}$



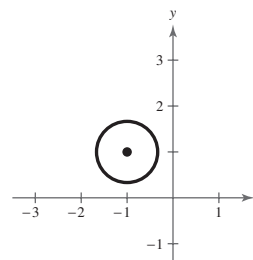
25. Ellipse

Center:  $\left(3, -\frac{5}{2}\right)$   
 Vertices:  $\left(9, -\frac{5}{2}\right), \left(-3, -\frac{5}{2}\right)$   
 Foci:  $\left(3 \pm 3\sqrt{3}, -\frac{5}{2}\right)$   
 Eccentricity:  $\frac{\sqrt{3}}{2}$



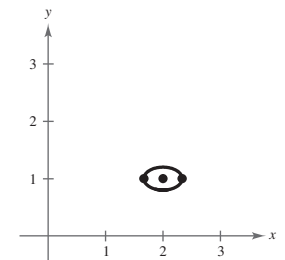
27. Circle

Center:  $(-1, 1)$   
 Radius:  $\frac{2}{3}$

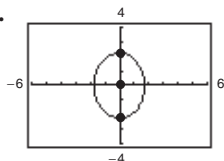


29. Ellipse

Center:  $(2, 1)$   
 Vertices:  $\left(\frac{7}{3}, 1\right), \left(\frac{5}{3}, 1\right)$   
 Foci:  $\left(\frac{34}{15}, 1\right), \left(\frac{26}{15}, 1\right)$   
 Eccentricity:  $\frac{4}{5}$

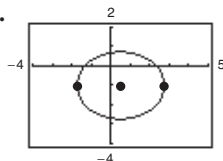


31.



Center:  $(0, 0)$   
 Vertices:  $(0, \pm\sqrt{5})$   
 Foci:  $(0, \pm\sqrt{2})$

33.



Center:  $(\frac{1}{2}, -1)$   
 Vertices:  $(\frac{1}{2} \pm \sqrt{5}, -1)$   
 Foci:  $(\frac{1}{2} \pm \sqrt{2}, -1)$

35.  $\frac{x^2}{4} + \frac{y^2}{16} = 1$     37.  $\frac{x^2}{36} + \frac{y^2}{32} = 1$     39.  $\frac{x^2}{36} + \frac{y^2}{11} = 1$

41.  $\frac{21x^2}{400} + \frac{y^2}{25} = 1$     43.  $\frac{(x-2)^2}{1} + \frac{(y-3)^2}{9} = 1$

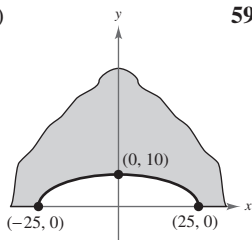
45.  $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{9} = 1$

47.  $\frac{(x-2)^2}{4} + \frac{(y-4)^2}{1} = 1$     49.  $\frac{x^2}{48} + \frac{(y-4)^2}{64} = 1$

51.  $\frac{x^2}{16} + \frac{(y-4)^2}{12} = 1$     53.  $\frac{(x-2)^2}{4} + \frac{(y-2)^2}{1} = 1$

55.  $\frac{x^2}{25} + \frac{y^2}{16} = 1$

57. (a)

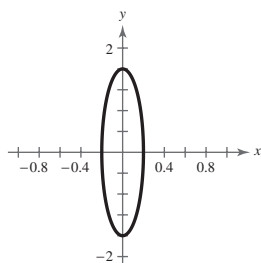


(b)  $\frac{x^2}{625} + \frac{y^2}{100} = 1$

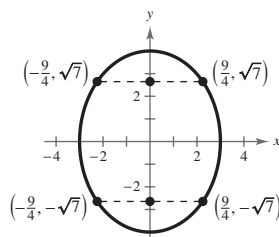
(c) Yes

61. (a)  $\frac{x^2}{0.04} + \frac{y^2}{2.56} = 1$

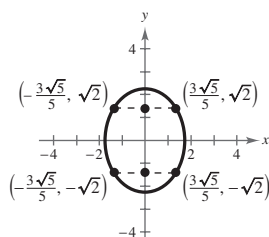
(b)



63.



65.

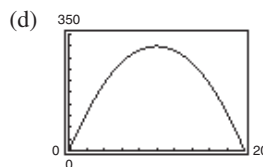


67. False. The graph of  $x^2/4 + y^4 = 1$  is not an ellipse. The degree of  $y$  is 4, not 2.

69. (a)  $A = \pi a(20 - a)$     (b)  $\frac{x^2}{196} + \frac{y^2}{36} = 1$

$a$	8	9	10	11	12	13
$A$	301.6	311.0	314.2	311.0	301.6	285.9

$a = 10$ , circle



The shape of an ellipse with a maximum area is a circle. The maximum area is found when  $a = 10$  (verified in part c) and therefore  $b = 10$ , so the equation produces a circle.

71. Geometric    73. Arithmetic    75. 547    77. 340.15

## Section 10.4 (page 760)

### Vocabulary Check (page 760)

- hyperbola; foci
- branches
- transverse axis; center
- asymptotes
- $Ax^2 + Cy^2 + Dx + Ey + F = 0$

1. b    2. c    3. a    4. d

5. Center:  $(0, 0)$

Vertices:  $(\pm 1, 0)$

Foci:  $(\pm\sqrt{2}, 0)$

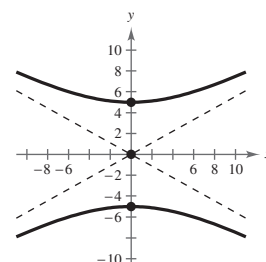
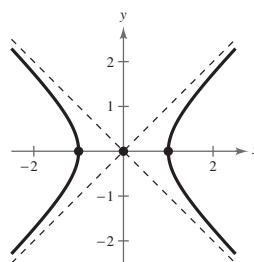
Asymptotes:  $y = \pm x$

7. Center:  $(0, 0)$

Vertices:  $(0, \pm 5)$

Foci:  $(0, \pm\sqrt{106})$

Asymptotes:  $y = \pm\frac{5}{9}x$

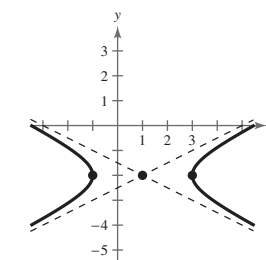


9. Center:  $(1, -2)$

Vertices:  $(3, -2), (-1, -2)$

Foci:  $(1 \pm \sqrt{5}, -2)$

Asymptotes:  
 $y = -2 \pm \frac{1}{2}(x - 1)$



11. Center:  $(2, -6)$

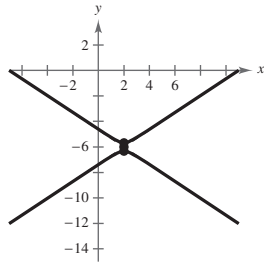
Vertices:

$$\left(2, -\frac{17}{3}\right), \left(2, -\frac{19}{3}\right)$$

Foci:  $\left(2, -6 \pm \frac{\sqrt{13}}{6}\right)$

Asymptotes:

$$y = -6 \pm \frac{2}{3}(x - 2)$$



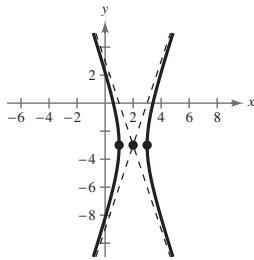
13. Center:  $(2, -3)$

Vertices:  $(3, -3), (1, -3)$

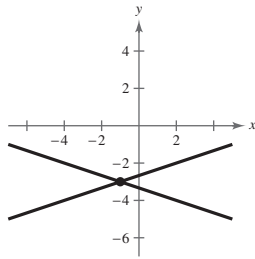
Foci:  $(2 \pm \sqrt{10}, -3)$

Asymptotes:

$$y = -3 \pm 3(x - 2)$$



15. The graph of this equation is two lines intersecting at  $(-1, -3)$ .

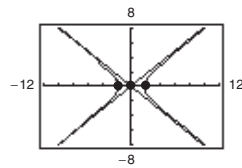


17. Center:  $(0, 0)$

Vertices:  $(\pm\sqrt{3}, 0)$

Foci:  $(\pm\sqrt{5}, 0)$

Asymptotes:  $y = \pm\frac{\sqrt{6}}{3}x$



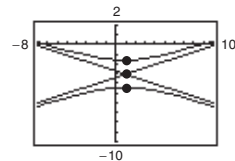
19. Center:  $(1, -3)$

Vertices:  $(1, -3 \pm \sqrt{2})$

Foci:  $(1, -3 \pm 2\sqrt{5})$

Asymptotes:

$$y = -3 \pm \frac{1}{3}(x - 1)$$



21.  $\frac{y^2}{4} - \frac{x^2}{12} = 1$       23.  $\frac{x^2}{1} - \frac{y^2}{25} = 1$

25.  $\frac{17y^2}{1024} - \frac{17x^2}{64} = 1$       27.  $\frac{(x-4)^2}{4} - \frac{y^2}{12} = 1$

29.  $\frac{(y-5)^2}{16} - \frac{(x-4)^2}{9} = 1$       31.  $\frac{y^2}{9} - \frac{4(x-2)^2}{9} = 1$

33.  $\frac{(y-2)^2}{4} - \frac{x^2}{4} = 1$       35.  $\frac{(x-2)^2}{1} - \frac{(y-2)^2}{1} = 1$

37.  $\frac{(x-3)^2}{9} - \frac{(y-2)^2}{4} = 1$

39. (a)  $\frac{x^2}{1} - \frac{y^2}{169/3} = 1$       (b)  $\approx 2.403$  feet

41.  $(3300, -2750)$       43.  $(12(\sqrt{5} - 1), 0) \approx (14.83, 0)$

45. Circle      47. Hyperbola      49. Hyperbola

51. Parabola      53. Ellipse      55. Parabola

57. Ellipse      59. Circle

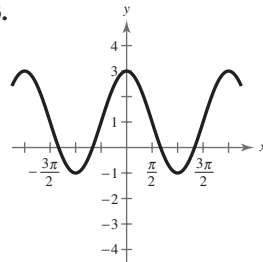
61. True. For a hyperbola,  $c^2 = a^2 + b^2$ . The larger the ratio of  $b$  to  $a$ , the larger the eccentricity of the hyperbola,  $e = c/a$ .

63. Answers will vary.

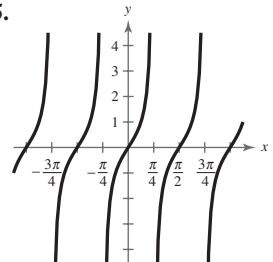
65.  $y = 1 - 3\sqrt{\frac{(x-3)^2}{4} - 1}$       67.  $x(x+4)(x-4)$

69.  $2x(x-6)^2$       71.  $2(2x+3)(4x^2-6x+9)$

- 73.



- 75.



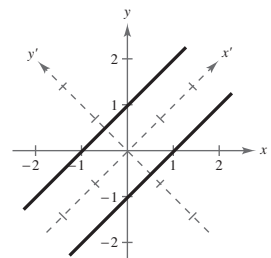
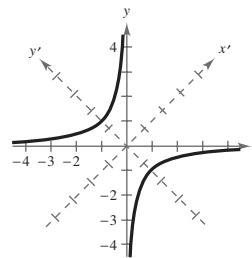
## Section 10.5 (page 769)

### Vocabulary Check (page 769)

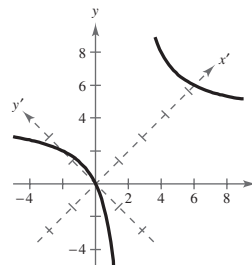
- rotation of axes
- $A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$
- invariant under rotation
- discriminant

1.  $(3, 0)$       3.  $\left(\frac{3 + \sqrt{3}}{2}, \frac{3\sqrt{3} - 1}{2}\right)$       5.  $\left(\frac{3\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

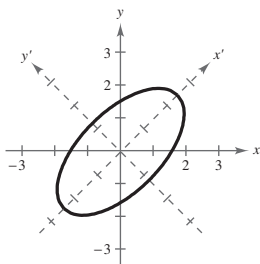
7.  $\frac{(y')^2}{2} - \frac{(x')^2}{2} = 1$       9.  $y' = \pm\frac{\sqrt{2}}{2}$



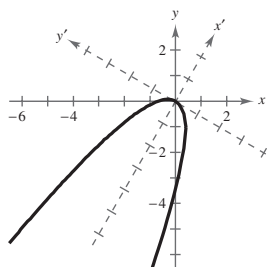
11.  $\frac{(x' - 3\sqrt{2})^2}{16} - \frac{(y' - \sqrt{2})^2}{16} = 1$



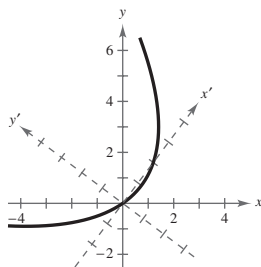
13.  $\frac{(x')^2}{6} + \frac{(y')^2}{\frac{3}{2}} = 1$



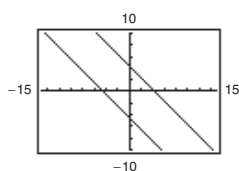
15.  $(y')^2 = -x'$



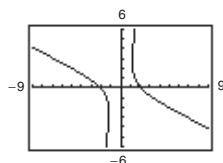
17.  $(x' - 1)^2 = 6(y' + \frac{1}{6})$



19.

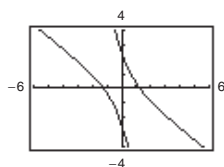


21.



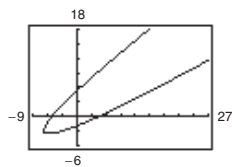
$\theta = 45^\circ$

23.



$\theta \approx 26.57^\circ$

25.



$\theta \approx 31.72^\circ$

27. e

28. f

29. b

30. a

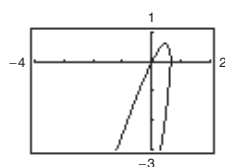
31. d

32. c

33. (a) Parabola

(b)  $y = \frac{(8x - 5) \pm \sqrt{(8x - 5)^2 - 4(16x^2 - 10x)}}{2}$

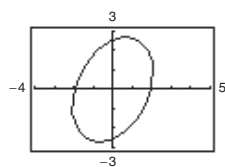
(c)



35. (a) Ellipse

(b)  $y = \frac{6x \pm \sqrt{36x^2 - 28(12x^2 - 45)}}{14}$

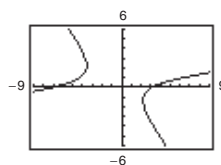
(c)



37. (a) Hyperbola

(b)  $y = \frac{6x \pm \sqrt{36x^2 + 20(x^2 + 4x - 22)}}{-10}$

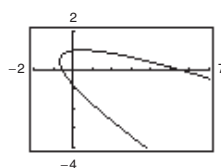
(c)



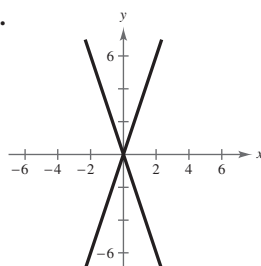
39. (a) Parabola

(b)  $y = \frac{-(4x - 1) \pm \sqrt{(4x - 1)^2 - 16(x^2 - 5x - 3)}}{8}$

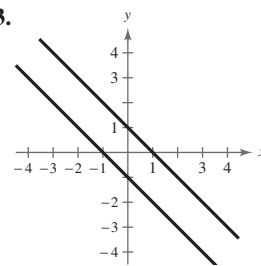
(c)



41.



43.



45. (2, 2), (2, 4)

47. (-8, 12)

49. (0, 8), (12, 8)

51. (0, 4)

53.  $(1, \sqrt{3}), (1, -\sqrt{3})$

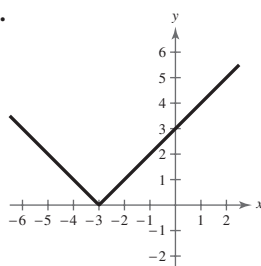
55. No solution

57.  $(0, \frac{3}{2}), (-3, 0)$

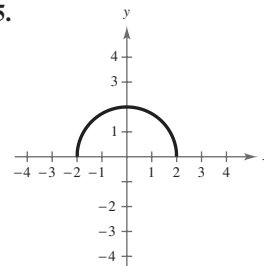
59. True. The graph of the equation can be classified by finding the discriminant. For a graph to be a hyperbola, the discriminant must be greater than zero. If  $k \geq \frac{1}{4}$ , then the discriminant would be less than or equal to zero.

61. Answers will vary.

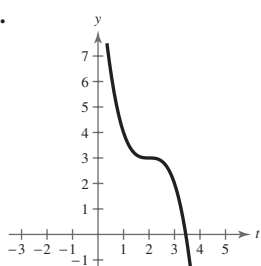
63.



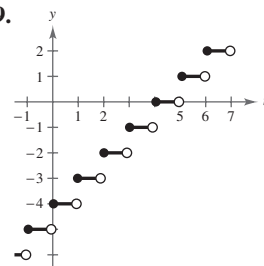
65.



67.



69.



71. Area = 45.11 square units

73. Area = 48.60 square units

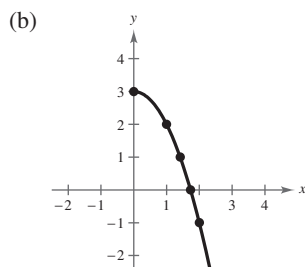
## Section 10.6 (page 776)

**Vocabulary Check** (page 776)

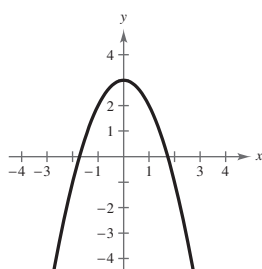
1. plane curve; parametric; parameter
2. orientation
3. eliminating the parameter

1. (a)

$t$	0	1	2	3	4
$x$	0	1	$\sqrt{2}$	$\sqrt{3}$	2
$y$	3	2	1	0	-1



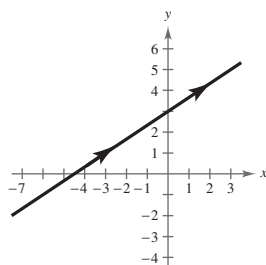
(c)  $y = 3 - x^2$



The graph of the rectangular equation shows the entire parabola rather than just the right half.

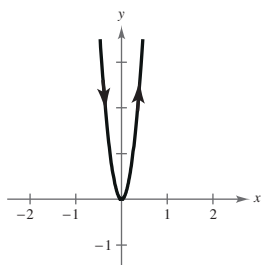
The graph of the rectangular equation continues the graph into the second and third quadrants.

3. (a)



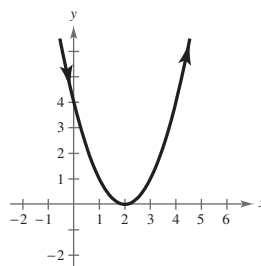
(b)  $y = \frac{2}{3}x + 3$

5. (a)



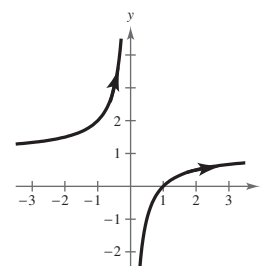
(b)  $y = 16x^2$

7. (a)



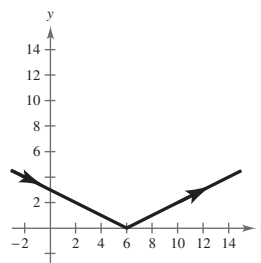
(b)  $y = x^2 - 4x + 4$

9. (a)



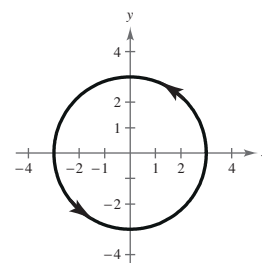
(b)  $y = \frac{(x-1)}{x}$

11. (a)



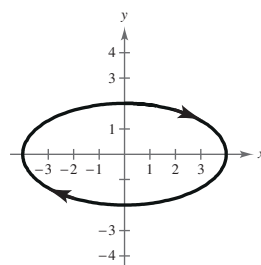
(b)  $y = \left| \frac{x}{2} - 3 \right|$

13. (a)



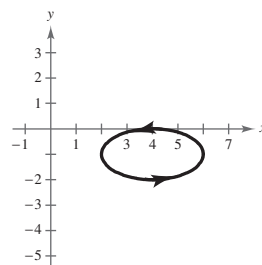
(b)  $\frac{y^2}{9} + \frac{x^2}{9} = 1$

15. (a)



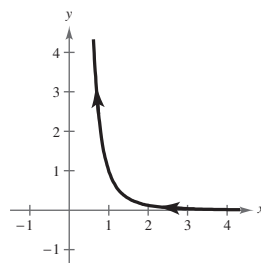
(b)  $\frac{x^2}{16} + \frac{y^2}{4} = 1$

17. (a)



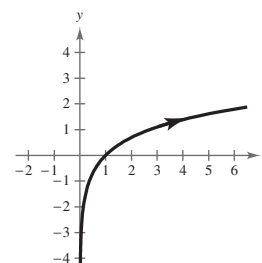
(b)  $\frac{(x-4)^2}{4} + (y+1)^2 = 1$

19. (a)



(b)  $y = \frac{1}{x^3}$

21. (a)



(b)  $y = \ln x$

23. Each curve represents a portion of the line  $y = 2x + 1$ .

Domain	Orientation
(a) $(-\infty, \infty)$	Left to right
(b) $[-1, 1]$	Depends on $\theta$
(c) $(0, \infty)$	Right to left
(d) $(0, \infty)$	Left to right

25.  $y - y_1 = m(x - x_1)$

27.  $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

29.  $x = 6t$

$y = -3t$

33.  $x = 4 \cos \theta$

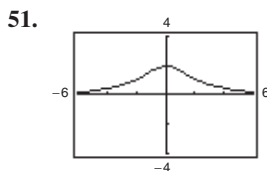
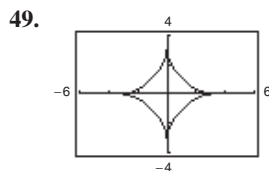
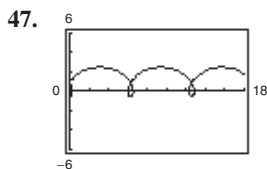
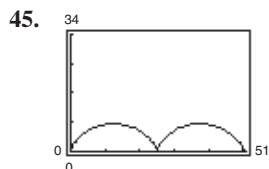
$y = \sqrt{7} \sin \theta$

37. (a)  $x = t, y = 3t - 2$  (b)  $x = -t + 2, y = -3t + 4$

39. (a)  $x = t, y = t^2$  (b)  $x = -t + 2, y = t^2 - 4t + 4$

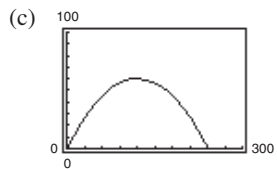
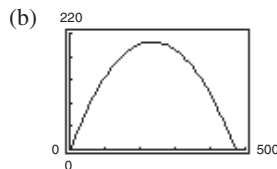
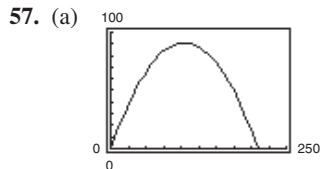
41. (a)  $x = t, y = t^2 + 1$  (b)  $x = -t + 2, y = t^2 - 4t + 5$

43. (a)  $x = t, y = \frac{1}{t}$  (b)  $x = -t + 2, y = -\frac{1}{t - 2}$

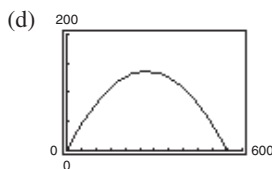


53. b  
Domain:  $[-2, 2]$   
Range:  $[-1, 1]$

55. d  
Domain:  $(-\infty, \infty)$   
Range:  $(-\infty, \infty)$   
Maximum height: 90.7 feet  
Range: 209.6 feet



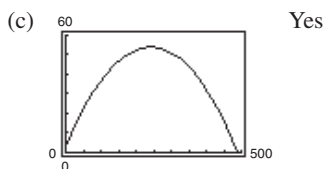
- Maximum height: 60.5 feet  
Range: 242.0 feet



- Maximum height: 136.1 feet  
Range: 544.5 feet

59. (a)  $x = (146.67 \cos \theta)t$   
 $y = 3 + (146.67 \sin \theta)t - 16t^2$

- (b) No



- (d)  $19.3^\circ$

61. Answers will vary.

63.  $x = a\theta - b \sin \theta$   
 $y = a - b \cos \theta$

65. True

$x = t$

$y = t^2 + 1 \Rightarrow y = x^2 + 1$

$x = 3t$

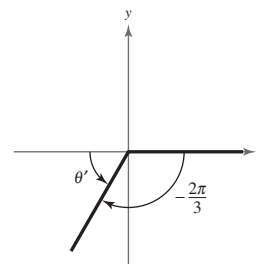
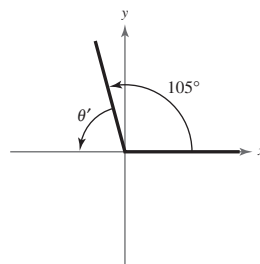
$y = 9t^2 + 1 \Rightarrow y = x^2 + 1$

67. Parametric equations are useful when graphing two functions simultaneously on the same coordinate system. For example, they are useful when tracking the path of an object so that the position and the time associated with that position can be determined.

69.  $(5, 2)$  71.  $(1, -2, 1)$

73.  $\theta' = 75^\circ$

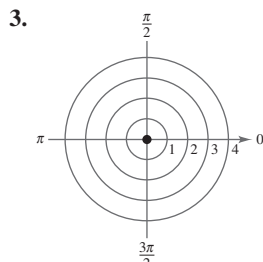
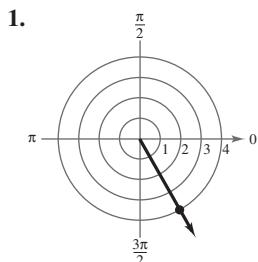
75.  $\theta' = \frac{\pi}{3}$



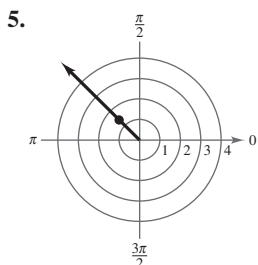
## Section 10.7 (page 783)

## Vocabulary Check (page 783)

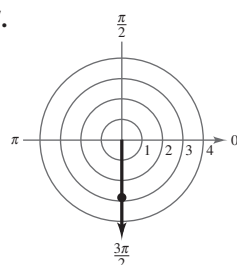
1. pole      2. directed distance; directed angle  
 3. polar      4.  $x = r \cos \theta$        $\tan \theta = \frac{y}{x}$   
 $y = r \sin \theta$        $r^2 = x^2 + y^2$



5.  $\left(4, \frac{5\pi}{3}\right), \left(-4, -\frac{4\pi}{3}\right)$



7.  $\left(0, \frac{5\pi}{6}\right), \left(0, -\frac{13\pi}{6}\right)$



$(\sqrt{2}, 8.64), (-\sqrt{2}, -0.78) \quad (2\sqrt{2}, 10.99), (-2\sqrt{2}, 7.85)$

9.  $(0, 3)$       11.  $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$       13.  $(-\sqrt{2}, \sqrt{2})$

15.  $(-1.1340, -2.2280)$       17.  $\left(\sqrt{2}, \frac{\pi}{4}\right)$       19.  $(6, \pi)$

21.  $(5, 2.2143)$       23.  $\left(\sqrt{6}, \frac{5\pi}{4}\right)$       25.  $(3\sqrt{13}, 0.9828)$

27.  $(\sqrt{13}, 5.6952)$       29.  $(\sqrt{7}, 0.8571)$

31.  $\left(\frac{17}{6}, 0.4900\right)$       33.  $r = 3$       35.  $r = 4 \csc \theta$

37.  $r = 10 \sec \theta$       39.  $r = \frac{-2}{3 \cos \theta - \sin \theta}$

41.  $r^2 = 16 \sec \theta \csc \theta = 32 \csc 2\theta$

43.  $r = \frac{4}{1 - \cos \theta}$  or  $-\frac{4}{1 + \cos \theta}$

45.  $r = a$       47.  $r = 2a \cos \theta$       49.  $x^2 + y^2 - 4y = 0$

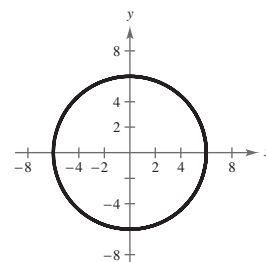
51.  $\sqrt{3}x + y = 0$       53.  $x^2 + y^2 = 16$

55.  $y = 4$       57.  $x^2 + y^2 - x^{2/3} = 0$

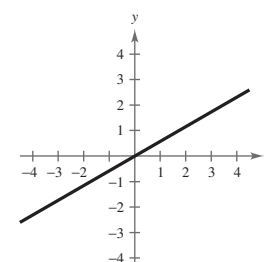
59.  $(x^2 + y^2)^2 = 6x^2y - 2y^3$       61.  $x^2 + 4y - 4 = 0$

63.  $4x^2 - 5y^2 - 36y - 36 = 0$

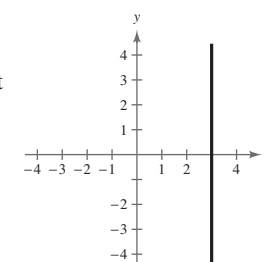
65. The graph of the polar equation consists of all points that are six units from the pole.  
 $x^2 + y^2 = 36$



67. The graph of the polar equation consists of all points on the line that make an angle of  $\pi/6$  with the positive polar axis.  
 $-\sqrt{3}x + 3y = 0$



69. The graph of the polar equation is not evident by simple inspection, so convert to rectangular form.  
 $x - 3 = 0$



71. True. Because  $r$  is a directed distance, the point  $(r, \theta)$  can be represented as  $(r, \theta \pm 2\pi n)$ .

73.  $(x - h)^2 + (y - k)^2 = h^2 + k^2$

Radius:  $\sqrt{h^2 + k^2}$

Center:  $(h, k)$

75. (a) Answers will vary.

- (b)  $(r_1, \theta_1), (r_2, \theta_2)$  and the pole are collinear.

$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2} = |r_1 - r_2|$

This represents the distance between two points on the line  $\theta = \theta_1 = \theta_2$ .

(c)  $d = \sqrt{r_1^2 + r_2^2}$

This is the result of the Pythagorean Theorem.

- (d) Answers will vary. For example:

Points:  $(3, \pi/6), (4, \pi/3)$

Distance: 2.053

Points:  $(-3, 7\pi/6), (-4, 4\pi/3)$

Distance: 2.053

77.  $2 \log_6 x + \log_6 z - \log_6 3 - \log_6 y$

79.  $\ln x + 2 \ln(x + 4)$       81.  $\log_7 \frac{x}{3y}$       83.  $\ln \sqrt{x}(x - 2)$

85.  $(2, 3)$       87.  $\left(\frac{8}{7}, \frac{88}{35}, \frac{8}{5}\right)$       89.  $(2, -3, 3)$

91. Not collinear      93. Collinear

## Section 10.8 (page 791)

## Vocabulary Check (page 791)

1.  $\theta = \frac{\pi}{2}$     2. polar axis    3. convex limaçon  
4. circle    5. lemniscate    6. cardioid

1. Rose curve with 4 petals    3. Limaçon with inner loop  
5. Rose curve with 4 petals    7. Polar axis

9.  $\theta = \frac{\pi}{2}$     11.  $\theta = \frac{\pi}{2}$ , polar axis, pole

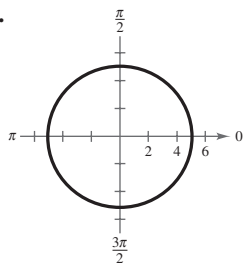
13. Maximum:  $|r| = 20$  when  $\theta = \frac{3\pi}{2}$

Zero:  $r = 0$  when  $\theta = \frac{\pi}{2}$

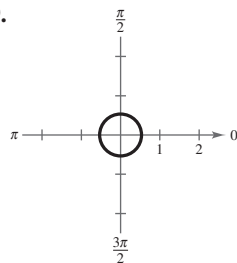
15. Maximum:  $|r| = 4$  when  $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$

Zero:  $r = 0$  when  $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

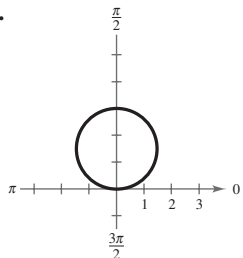
17.



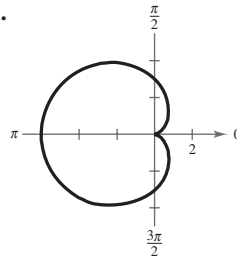
19.



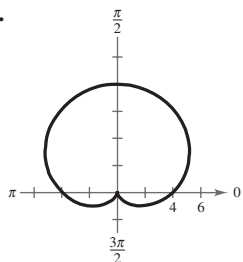
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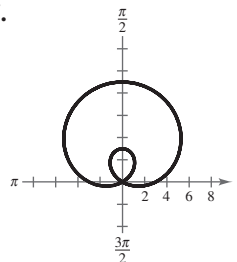
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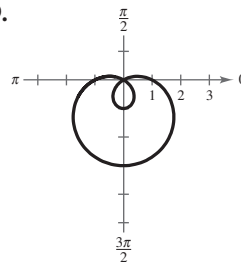
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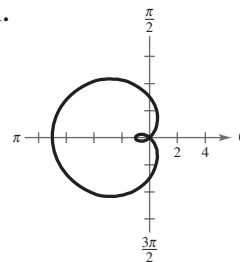
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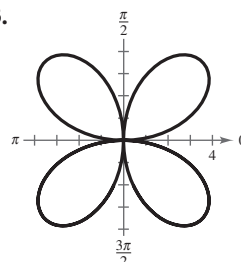
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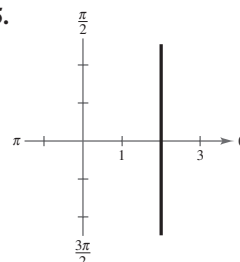
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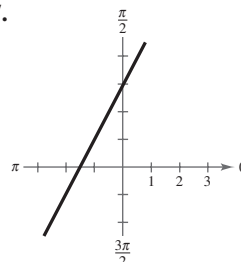
33.



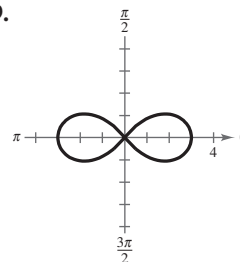
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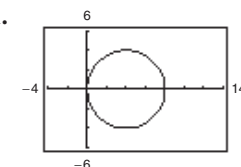
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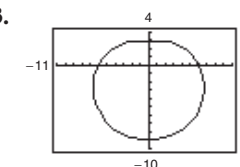
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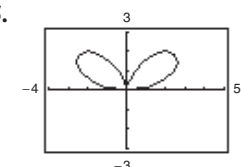
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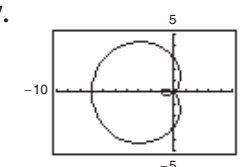
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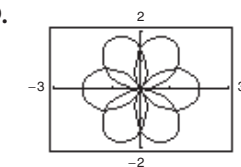
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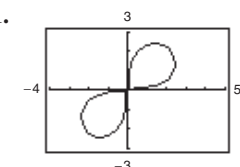
47.



49.



51.

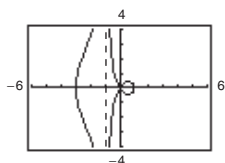


$$0 \leq \theta < 4\pi$$

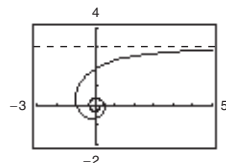
$$0 \leq \theta < \pi$$



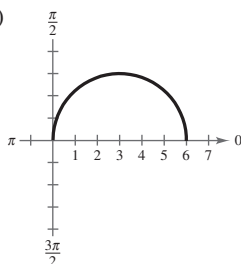
53.



55.

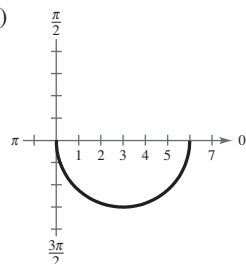

 57. True. For a graph to have polar axis symmetry, replace  $(r, \theta)$  by  $(r, -\theta)$  or  $(-r, \pi - \theta)$ .

59. (a)



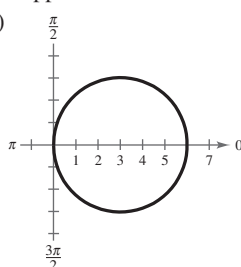
Upper half of circle

(b)



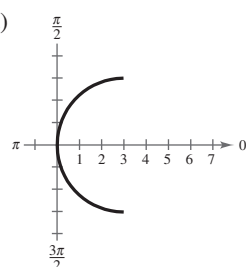
Lower half of circle

(c)



Full circle

(d)



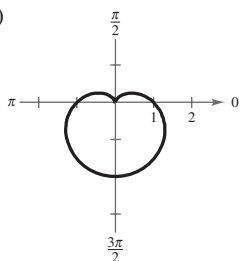
Left half of circle

61. Answers will vary.

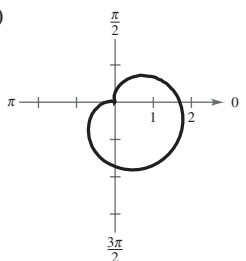
 63. (a)  $r = 2 - \frac{\sqrt{2}}{2}(\sin \theta - \cos \theta)$  (b)  $r = 2 + \cos \theta$ 

 (c)  $r = 2 + \sin \theta$  (d)  $r = 2 - \cos \theta$ 

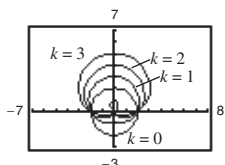
65. (a)



(b)



67.


 $k = 0$ , circle

 $k = 1$ , convex limaçon

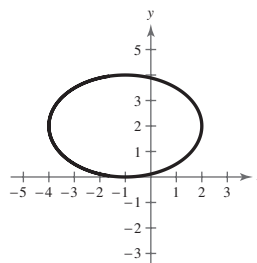
 $k = 2$ , cardioid

 $k = 3$ , limaçon with inner loop

 69.  $\pm 3$ 

 71.  $\frac{13}{5}$ 

73.  $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$



## Section 10.9 (page 797)

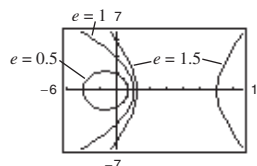
## Vocabulary Check (page 797)

1. conic    2. eccentricity;
- $e$
3. vertical; right
- 
4. (a) iii    (b) i    (c) ii

1.  $e = 1$ :  $r = \frac{4}{1 + \cos \theta}$ , parabola

$e = 0.5$ :  $r = \frac{2}{1 + 0.5 \cos \theta}$ , ellipse

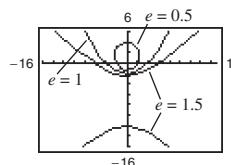
$e = 1.5$ :  $r = \frac{6}{1 + 1.5 \cos \theta}$ , hyperbola



3.  $e = 1$ :  $r = \frac{4}{1 - \sin \theta}$ , parabola

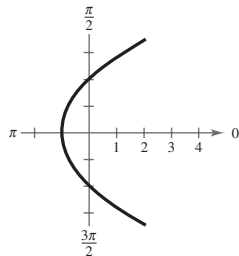
$e = 0.5$ :  $r = \frac{2}{1 - 0.5 \sin \theta}$ , ellipse

$e = 1.5$ :  $r = \frac{6}{1 - 1.5 \sin \theta}$ , hyperbola

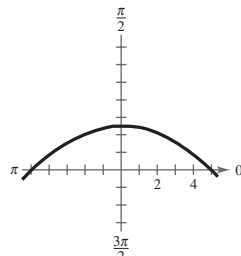


5. f    6. c    7. d    8. e    9. a    10. b

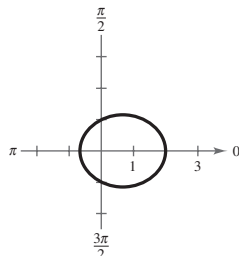
11. Parabola



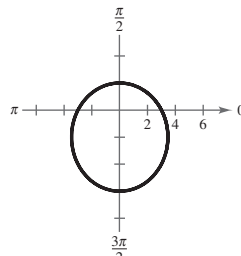
13. Parabola



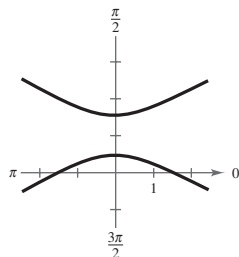
15. Ellipse



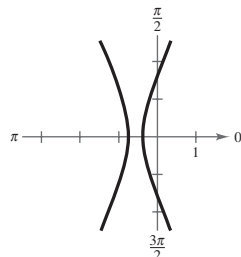
17. Ellipse



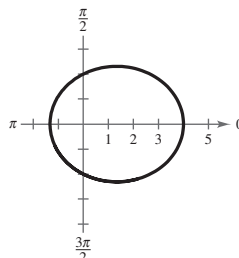
19. Hyperbola



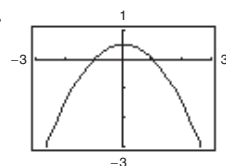
21. Hyperbola



23. Ellipse

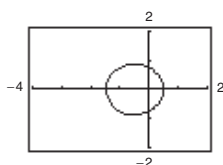


25.



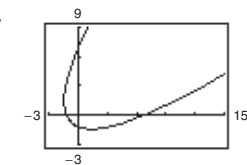
Parabola

27.

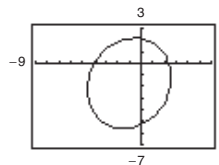


Ellipse

29.



31.



33.  $r = \frac{1}{1 - \cos \theta}$

35.  $r = \frac{1}{2 + \sin \theta}$

37.  $r = \frac{2}{1 + 2 \cos \theta}$

39.  $r = \frac{2}{1 - \sin \theta}$

41.  $r = \frac{10}{1 - \cos \theta}$

43.  $r = \frac{10}{3 + 2 \cos \theta}$

45.  $r = \frac{20}{3 - 2 \cos \theta}$

47.  $r = \frac{9}{4 - 5 \sin \theta}$

49. Answers will vary.

51.  $r = \frac{9.5929 \times 10^7}{1 - 0.0167 \cos \theta}$

Perihelion:  $9.4354 \times 10^7$  milesAphelion:  $9.7558 \times 10^7$  miles

53.  $r = \frac{1.0820 \times 10^8}{1 - 0.0068 \cos \theta}$

Perihelion:  $1.0747 \times 10^8$  kilometersAphelion:  $1.0894 \times 10^8$  kilometers

55.  $r = \frac{1.4039 \times 10^8}{1 - 0.0934 \cos \theta}$

Perihelion:  $1.2840 \times 10^8$  milesAphelion:  $1.5486 \times 10^8$  miles

57.  $r = \frac{0.624}{1 + 0.847 \sin \pi/2}$ ;  $r = 0.338$  astronomical unit

59. True. The graphs represent the same hyperbola.

61. True. The conic is an ellipse because the eccentricity is less than 1.

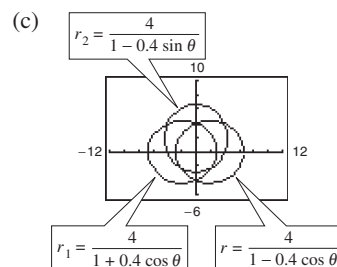
63. Answers will vary. 65.  $r^2 = \frac{24,336}{169 - 25 \cos^2 \theta}$

67.  $r^2 = \frac{144}{25 \cos^2 \theta - 9}$

69.  $r^2 = \frac{144}{25 \sin^2 \theta - 16}$

71. (a) Ellipse

(b) The given polar equation,  $r$ , has a vertical directrix to the left of the pole. The equation,  $r_1$ , has a vertical directrix to the right of the pole, and the equation,  $r_2$ , has a horizontal directrix below the pole.



73.  $\frac{\pi}{6} + n\pi$     75.  $\frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

77.  $\frac{\pi}{2} + n\pi$     79.  $\frac{\sqrt{2}}{10}$     81.  $\frac{7\sqrt{2}}{10}$

83.  $\sin 2u = -\frac{24}{25}$

$$\cos 2u = -\frac{7}{25}$$

$$\tan 2u = \frac{24}{7}$$

85.  $a_n = -\frac{1}{4}n + \frac{1}{4}$     87.  $a_n = 9n$     89. 220    91. 720

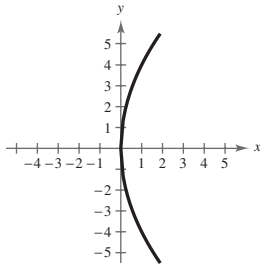
**Review Exercises (page 801)**

1.  $\frac{\pi}{4}$  radian,  $45^\circ$     3. 1.1071 radians,  $63.43^\circ$

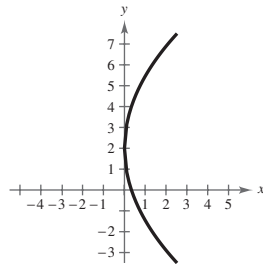
5. 0.4424 radian,  $25.35^\circ$     7. 0.6588 radian,  $37.75^\circ$

9.  $2\sqrt{2}$     11. Hyperbola

13.  $y^2 = 16x$



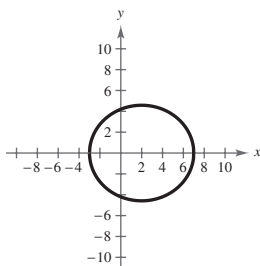
15.  $(y - 2)^2 = 12x$



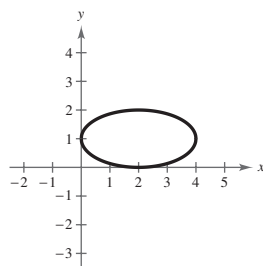
17.  $y = -2x + 2$ ; (1, 0)

19.  $8\sqrt{6}$  meters

21.  $\frac{(x-2)^2}{25} + \frac{y^2}{21} = 1$



23.  $\frac{(x-2)^2}{4} + (y-1)^2 = 1$



25. The foci occur 3 feet from the center of the arch on a line connecting the tops of the pillars.

27. Center:  $(-2, 1)$

Vertices:  
 $(-2, 11), (-2, -9)$

Foci:  $(-2, 1 \pm \sqrt{19})$

Eccentricity:  $\frac{\sqrt{19}}{10}$

29. Center:  $(1, -4)$

Vertices:  $(1, 0), (1, -8)$   
 Foci:  $(1, -4 \pm \sqrt{7})$

Eccentricity:  $\frac{\sqrt{7}}{4}$

31.  $y^2 - \frac{x^2}{8} = 1$     33.  $\frac{5(x-4)^2}{16} - \frac{5y^2}{64} = 1$

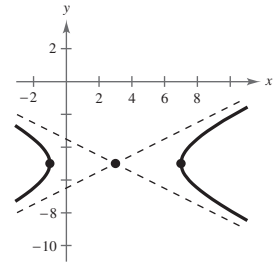
35. Center:  $(3, -5)$

Vertices:  $(7, -5), (-1, -5)$

Foci:  $(3 \pm 2\sqrt{5}, -5)$

Asymptotes:

$y = -5 \pm \frac{1}{2}(x - 3)$



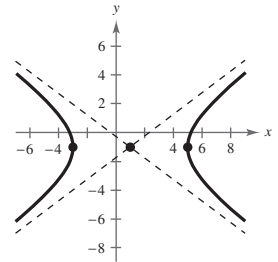
37. Center:  $(1, -1)$

Vertices:  $(5, -1), (-3, -1)$

Foci:  $(6, -1), (-4, -1)$

Asymptotes:

$y = -1 \pm \frac{3}{4}(x - 1)$

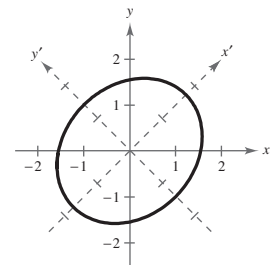
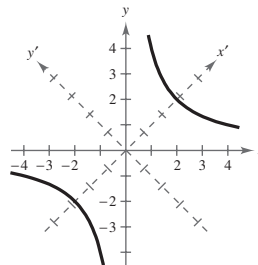


39. 72 miles    41. Hyperbola

43. Ellipse

45.  $\frac{(x')^2}{8} - \frac{(y')^2}{8} = 1$

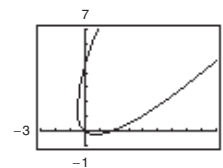
47.  $\frac{(x')^2}{3} + \frac{(y')^2}{2} = 1$



49. (a) Parabola

(b)  $y = \frac{24x + 40 \pm \sqrt{(24x + 40)^2 - 36(16x^2 - 30x)}}{18}$

(c)

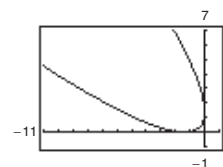


51. (a) Parabola

(b)

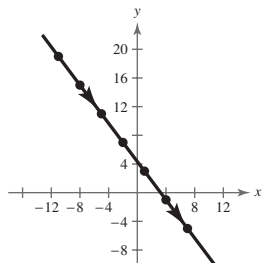
$$y = \frac{-(2x - 2\sqrt{2}) \pm \sqrt{(2x - 2\sqrt{2})^2 - 4(x^2 + 2\sqrt{2}x + 2)}}{2}$$

(c)

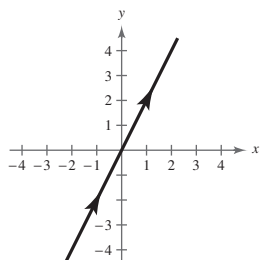


53.

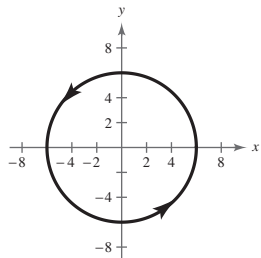
$t$	-3	-2	-1	0	1	2	3
$x$	-11	-8	-5	-2	1	4	7
$y$	19	15	11	7	3	-1	-5



55. (a)

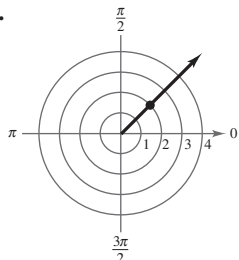
(b)  $y = 2x$ 

59. (a)



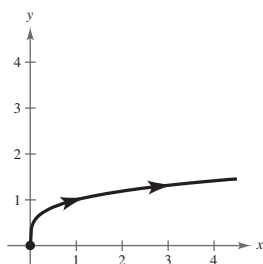
61.  $x = 5 + 6 \cos \theta$   
 $y = 4 + 6 \sin \theta$

65.



$$\left(2, \frac{9\pi}{4}\right), \left(-2, \frac{5\pi}{4}\right)$$

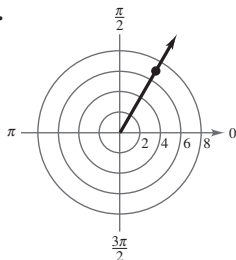
57. (a)



(b)  $y = \sqrt[4]{x}$   
 (b)  $x^2 + y^2 = 36$

63.  $x = 3 \tan \theta$   
 $y = 4 \sec \theta$

67.



$$(7, 1.05), (-7, 10.47)$$

69.  $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  71.  $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$  73.  $\left(2, \frac{\pi}{2}\right)$

75.  $(2\sqrt{13}, 0.9828)$  77.  $r = 7$  79.  $r = 6 \sin \theta$

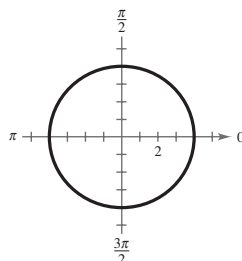
81.  $r^2 = 10 \csc 2\theta$  83.  $x^2 + y^2 = 25$

85.  $x^2 + y^2 = 3x$  87.  $x^2 + y^2 = y^{2/3}$

89. Symmetry:  $\theta = \frac{\pi}{2}$ , polar axis, pole

Maximum value of  $|r|$ :  $|r| = 4$  for all values of  $\theta$

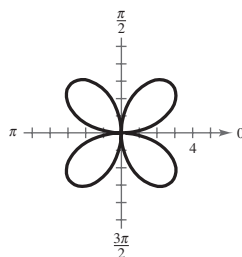
No zeros of  $r$



91. Symmetry:  $\theta = \frac{\pi}{2}$ , polar axis, pole

Maximum value of  $|r|$ :  $|r| = 4$  when  $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

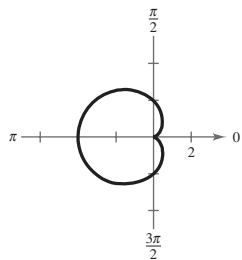
Zeros of  $r$ :  $r = 0$  when  $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$



93. Symmetry: polar axis

Maximum value of  $|r|$ :  $|r| = 4$  when  $\theta = 0$

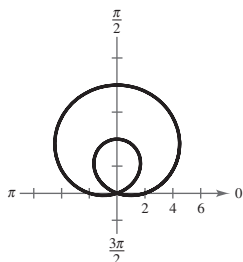
Zeros of  $r$ :  $r = 0$  when  $\theta = \pi$



95. Symmetry:  $\theta = \frac{\pi}{2}$

Maximum value of  $|r|$ :  $|r| = 8$  when  $\theta = \frac{\pi}{2}$

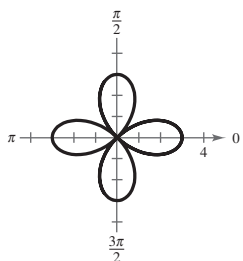
Zeros of  $r$ :  $r = 0$  when  $\theta = 3.4814, 5.9433$



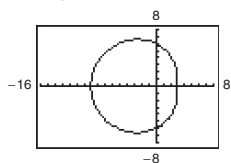
97. Symmetry:  $\theta = \frac{\pi}{2}$ , polar axis, pole

Maximum value of  $|r|$ :  $|r| = 3$  when  $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

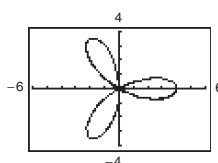
Zeros of  $r$ :  $r = 0$  when  $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



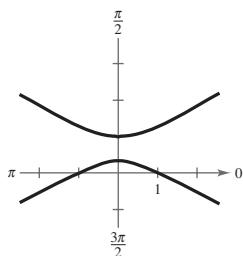
99. Limaçon



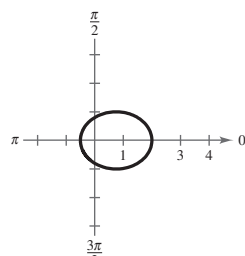
101. Rose curve



103. Hyperbola



105. Ellipse



107.  $r = \frac{4}{1 - \cos \theta}$       109.  $r = \frac{5}{3 - 2 \cos \theta}$

111.  $r = \frac{7978.81}{1 - 0.937 \cos \theta}$ ; 11,011.87 miles

113. False. When classifying an equation of the form  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ , its graph can be determined by its discriminant. For a graph to be a parabola, its discriminant,  $B^2 - 4AC$ , must equal zero. So, if  $B = 0$ , then  $A$  or  $C$  equals 0.

115. False. The following are two sets of parametric equations for the line.

$$x = t, y = 3 - 2t$$

$$x = 3t, y = 3 - 6t$$

117. 5. The ellipse becomes more circular and approaches a circle of radius 5.

119. (a) The speed would double.

- (b) The elliptical orbit would be flatter; the length of the major axis would be greater.

121. (a) The graphs are the same.

- (b) The graphs are the same.

## Chapter Test (page 805)

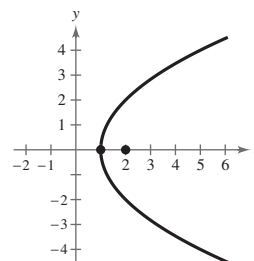
1. 0.2783 radian,  $15.9^\circ$       2. 0.8330 radian,  $47.7^\circ$

3.  $\frac{7\sqrt{2}}{2}$

4. Parabola:  $y^2 = 4(x - 1)$

Vertex: (1, 0)

Focus: (2, 0)



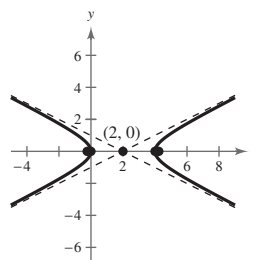
5. Hyperbola:  $\frac{(x - 2)^2}{4} - y^2 = 1$

Center: (2, 0)

Vertices: (0, 0), (4, 0)

Foci:  $(2 \pm \sqrt{5}, 0)$

Asymptotes:  $y = \pm \frac{1}{2}(x - 2)$

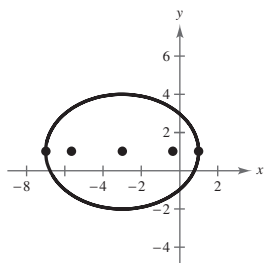


6. Ellipse:  $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{9} = 1$

Center:  $(-3, 1)$

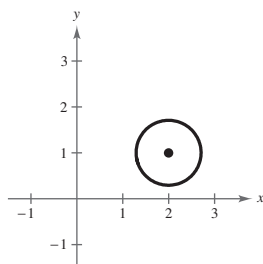
Vertices:  $(1, 1), (-7, 1)$

Foci:  $(-3 \pm \sqrt{7}, 1)$



7. Circle:  $(x-2)^2 + (y-1)^2 = \frac{1}{2}$

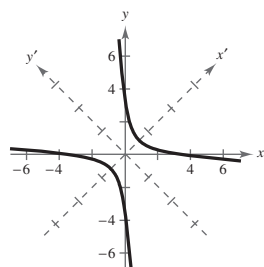
Center:  $(2, 1)$



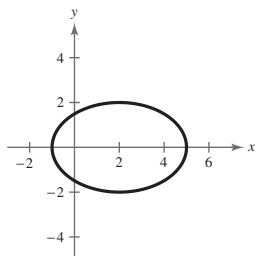
8.  $(x-3)^2 = \frac{3}{2}(y+2)$       9.  $\frac{5(y-2)^2}{4} - \frac{5x^2}{16} = 1$

10. (a)  $45^\circ$

(b)



11.



12.  $x = 6 + 4t$   
 $y = 4 + 7t$

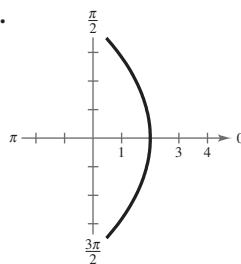
$\frac{(x-2)^2}{9} + \frac{y^2}{4} = 1$

13.  $(\sqrt{3}, -1)$

14.  $(2\sqrt{2}, \frac{7\pi}{4}), (-2\sqrt{2}, \frac{3\pi}{4}), (2\sqrt{2}, -\frac{\pi}{4})$

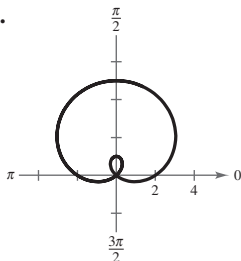
15.  $r = 4 \sin \theta$

16.



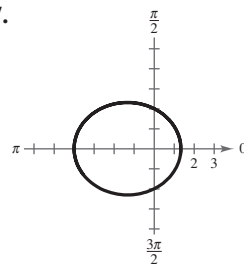
Parabola

18.



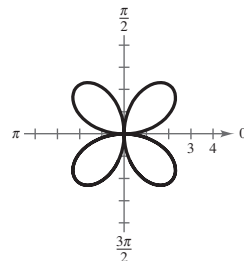
Limaçon with inner loop

17.



Ellipse

19.



Rose curve

20. Answers will vary. For example:  $r = \frac{1}{1 + 0.25 \sin \theta}$

21. Slope: 0.1511; Change in elevation: 789 feet

22. No; Yes

### Problem Solving (page 809)

1. (a) 1.2016 radians      (b) 2420 feet, 5971 feet

3.  $y^2 = 4p(x + p)$

5. (a) Since  $d_1 + d_2 \leq 20$ , by definition, the outer bound that the boat can travel is an ellipse. The islands are the foci.

(b) Island 1:  $(-6, 0)$ ;

Island 2:  $(6, 0)$

(c) 20 miles; Vertex:  $(10, 0)$

(d)  $\frac{x^2}{100} + \frac{y^2}{64} = 1$

7. Answers will vary.

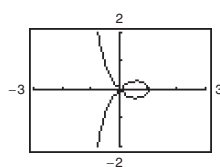
9. Answers will vary. For example:

$x = \cos(-t)$

$y = 2 \sin(-t)$

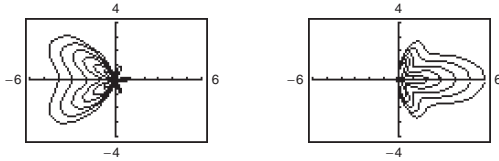
11. (a)  $y^2 = x^2 \left( \frac{1-x}{1+x} \right)$       (b)  $r = \cos 2\theta \sec \theta$

(c)



13. Circle

15.



For  $n \geq 1$ , a bell is produced.

For  $n \leq -1$ , a heart is produced.

For  $n = 0$ , a rose curve is produced.

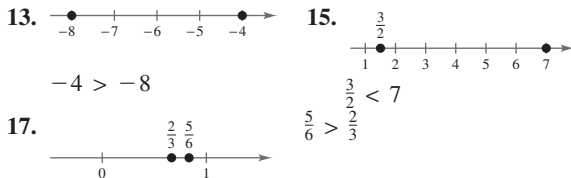
## Appendix A

### Appendix A.1 (page A8)

#### Vocabulary Check (page A8)

1. rational
2. irrational
3. absolute value
4. composite
5. prime
6. variables; constants
7. terms
8. coefficient
9. Zero-Factor Property

1. (a) 5, 1, 2 (b) 0, 5, 1, 2  
(c) -9, 5, 0, 1, -4, 2, -11  
(d)  $-\frac{7}{2}, \frac{2}{3}, -9, 5, 0, 1, -4, 2, -11$  (e)  $\sqrt{2}$
3. (a) 1 (b) 1 (c) -13, 1, -6  
(d) 2.01, -13, 1, -6, 0.666...  
(e) 0.010110111...
5. (a)  $\frac{6}{3}, 8$  (b)  $\frac{6}{3}, 8$  (c)  $\frac{6}{3}, -1, 8, -22$   
(d)  $-\frac{1}{3}, \frac{6}{3}, -7.5, -1, 8, -22$  (e)  $-\pi, \frac{1}{2}\sqrt{2}$
7. 0.625 9.  $0.\overline{123}$  11.  $-1 < 2.5$



13.  $-4 > -8$
15.  $\frac{3}{2} < 7$
17.  $\frac{5}{6} > \frac{2}{3}$
19. (a)  $x \leq 5$  denotes the set of all real numbers less than or equal to 5.  
(b) (c) Unbounded
21. (a)  $x < 0$  denotes the set of all real numbers less than 0.  
(b) (c) Unbounded
23. (a)  $[4, \infty)$  denotes the set of all real numbers greater than or equal to 4.  
(b) (c) Unbounded
25. (a)  $-2 < x < 2$  denotes the set of all real numbers greater than -2 and less than 2.  
(b) (c) Bounded
27. (a)  $-1 \leq x < 0$  denotes the set of all real numbers greater than or equal to -1 and less than 0.  
(b) (c) Bounded

29. (a)  $[-2, 5)$  denotes the set of all real numbers greater than or equal to -2 and less than 5.

- (b) (c) Bounded

31.  $-2 < x \leq 4$  33.  $y \geq 0$  35.  $10 \leq t \leq 22$
37.  $W > 65$  39. 10 41. 5 43. -1 45. -1
47. -1 49.  $|-3| > -|-3|$  51.  $-5 = -|5|$
53.  $-|-2| = -|2|$  55. 51 57.  $\frac{5}{2}$  59.  $\frac{128}{75}$
61.  $|\$113,356 - \$112,700| = \$656 > \$500$   
 $0.05(\$112,700) = \$5635$

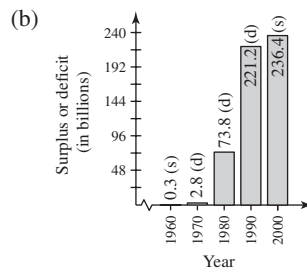
Because the actual expenses differ from the budget by more than \$500, there is failure to meet the "budget variance test."

63.  $|\$37,335 - \$37,640| = \$305 < \$500$   
 $0.05(\$37,640) = \$1882$

Because the difference between the actual expenses and the budget is less than \$500 and less than 5% of the budgeted amount, there is compliance with the "budget variance test."

65. (a)

Year	Expenditures (in billions)	Surplus or deficit (in billions)
1960	\$92.2	\$0.3 (s)
1970	\$195.6	\$2.8 (d)
1980	\$590.9	\$73.8 (d)
1990	\$1253.2	\$221.2 (d)
2000	\$1788.8	\$236.4 (s)



67.  $|x - 5| \leq 3$  69.  $|y| \geq 6$
71.  $|326 - 351| = 25$  miles
73.  $7x$  and 4 are the terms; 7 is the coefficient.
75.  $\sqrt{3}x^2$ ,  $-8x$ , and  $-11$  are the terms;  $\sqrt{3}$  and  $-8$  are the coefficients.
77.  $4x^3$ ,  $x/2$ , and  $-5$  are the terms; 4 and  $\frac{1}{2}$  are the coefficients.
79. (a) -10 (b) -6 81. (a) 14 (b) 2
83. (a) Division by 0 is undefined. (b) 0
85. Commutative Property of Addition
87. Multiplicative Inverse Property
89. Distributive Property
91. Multiplicative Identity Property

**93.** Associative Property of Addition

**95.** Distributive Property

**97.**  $\frac{1}{2}$       **99.**  $\frac{3}{8}$       **101.** 48      **103.**  $\frac{5x}{12}$

**105.** (a)

$n$	1	0.5	0.01	0.0001	0.000001
$5/n$	5	10	500	50,000	5,000,000

(b) The value of  $5/n$  approaches infinity as  $n$  approaches 0.

**107.** False. If  $a < b$ , then  $\frac{1}{a} > \frac{1}{b}$ , where  $a \neq b \neq 0$ .

**109.** (a) No. If one variable is negative and the other is positive, the expressions are unequal.

(b)  $|u + v| \leq |u| + |v|$

The expressions are equal when  $u$  and  $v$  have the same sign. If  $u$  and  $v$  differ in sign,  $|u + v|$  is less than  $|u| + |v|$ .

**111.** The only even prime number is 2, because its only factors are itself and 1.

**113.** (a) Negative      (b) Negative

**115.** Yes.  $|a| = -a$  if  $a < 0$ .

## Appendix A.2 (page A20)

### Vocabulary Check (page A20)

- exponent; base
- scientific notation
- square root
- principle  $n$ th root
- index; radicand
- simplest form
- conjugates
- rationalizing
- power; index

**1.**  $8 \times 8 \times 8 \times 8 \times 8$

**3.**  $4.9^6$       **5.** (a) 27      (b) 81

**7.** (a) 1      (b)  $-9$       **9.** (a)  $\frac{243}{64}$       (b)  $-1$

**11.** (a)  $\frac{5}{6}$       (b) 4      **13.**  $-1600$       **15.** 2.125

**17.**  $-24$       **19.** 6      **21.**  $-54$       **23.** 1

**25.** (a)  $-125z^3$       (b)  $5x^6$       **27.**  $24y^2$       (b)  $3x^2$

**29.** (a)  $\frac{7}{x}$       (b)  $\frac{4}{3}(x + y)^2$       **31.** (a) 1      (b)  $\frac{1}{4x^4}$

**33.** (a)  $-2x^3$       (b)  $\frac{10}{x}$       **35.** (a)  $3^{3n}$       (b)  $\frac{b^5}{a^5}$

**37.**  $5.73 \times 10^7$  square miles

**39.**  $8.99 \times 10^{-5}$  gram per cubic centimeter

**41.** 4,568,000,000 ounces

**43.** 0.00000000000000000016022 coulomb

**45.** (a) 50,000      (b) 200,000

**47.** (a) 954.448      (b)  $3.077 \times 10^{10}$

**49.** (a) 67,082.039      (b) 39.791

**51.** (a) 3      (b)  $\frac{3}{2}$       **53.** (a)  $\frac{1}{8}$       (b)  $\frac{27}{8}$

**55.** (a)  $-4$       (b) 2      **57.** (a) 7.550      (b)  $-7.225$

**59.** (a)  $-0.011$       (b) 0.005      **61.** (a) 4      (b)  $2\sqrt[3]{3}x$

**63.** (a)  $2\sqrt{2}$       (b)  $3\sqrt[3]{2}$       **65.** (a)  $6x\sqrt{2x}$       (b)  $\frac{18\sqrt{z}}{z^2}$

**67.** (a)  $2x\sqrt[3]{2x^2}$       (b)  $\frac{5|x|\sqrt{3}}{y^2}$

**69.** (a)  $34\sqrt{2}$       (b)  $22\sqrt{2}$       **71.** (a)  $2\sqrt{x}$       (b)  $4\sqrt{y}$

**73.** (a)  $13\sqrt{x+1}$       (b)  $18\sqrt{5x}$

**75.**  $\sqrt{5} + \sqrt{3} > \sqrt{5+3}$       **77.**  $5 > \sqrt{3^2 + 2^2}$

**79.**  $\frac{\sqrt{3}}{3}$       **81.**  $\frac{5 + \sqrt{3}}{11}$       **83.**  $\frac{2}{\sqrt{2}}$

**85.**  $\frac{2}{3(\sqrt{5} - \sqrt{3})}$       **87.**  $9^{1/2}$       **89.**  $\sqrt[5]{32}$

**91.**  $(-216)^{1/3}$       **93.**  $81^{3/4}$       **95.**  $\frac{2}{x}$       **97.**  $\frac{1}{x^3}$ ,  $x > 0$

**99.** (a)  $\sqrt{3}$       (b)  $\sqrt[3]{(x+1)^2}$

**101.** (a)  $2\sqrt[4]{2}$       (b)  $\sqrt[8]{2x}$       **103.**  $\frac{\pi}{2} \approx 1.57$  seconds

**105.** (a)

$h$	0	1	2	3	4	5	6
$t$	0	2.93	5.48	7.67	9.53	11.08	12.32

$h$	7	8	9	10	11	12
$t$	13.29	14.00	14.50	14.80	14.93	14.96

(b)  $t \rightarrow 8.64\sqrt{3} \approx 14.96$

**107.** True. When dividing variables, you subtract exponents.

**109.**  $a^0 = 1$ ,  $a \neq 0$ , using the property  $\frac{a^m}{a^n} = a^{m-n}$ :  
 $\frac{a^m}{a^m} = a^{m-m} = a^0 = 1$ .

**111.** When any positive integer is squared, the units digit is 0, 1, 4, 5, 6, or 9. Therefore,  $\sqrt{5233}$  is not an integer.

## Appendix A.3 (page A31)

### Vocabulary Check (page A31)

- $n$ ;  $a_n$ ;  $a_0$
- descending
- monomial; binomial; trinomial
- like terms
- First terms; Outer terms; Inner terms; Last terms
- factoring
- completely factored

**1.** d      **2.** e      **3.** b      **4.** a      **5.** f      **6.** c

**7.**  $-2x^3 + 4x^2 - 3x + 20$       **9.**  $-15x^4 + 1$

**11.** (a)  $-\frac{1}{2}x^5 + 14x$

(b) Degree: 5; Leading coefficient:  $-\frac{1}{2}$

(c) Binomial



13. (a)  $-3x^4 + 2x^2 - 5$   
 (b) Degree: 4; Leading coefficient:  $-3$   
 (c) Trinomial
15. (a)  $x^5 - 1$   
 (b) Degree: 5; Leading coefficient: 1  
 (c) Binomial
17. (a) 3  
 (b) Degree: 0; Leading coefficient: 3  
 (c) Monomial
19. (a)  $-4x^5 + 6x^4 + 1$   
 (b) Degree: 5; Leading coefficient:  $-4$   
 (c) Trinomial
21. (a)  $4x^3y$   
 (b) Degree: 3; Leading coefficient: 4  
 (c) Monomial
23. Polynomial:  $-3x^3 + 2x + 8$
25. Not a polynomial because it includes a term with a negative exponent
27. Polynomial:  $-y^4 + y^3 + y^2$     29.  $-2x - 10$
31.  $3x^3 - 2x + 2$     33.  $8.3x^3 + 29.7x^2 + 11$
35.  $12z + 8$     37.  $3x^3 - 6x^2 + 3x$     39.  $-15z^2 + 5z$
41.  $-4x^4 + 4x$     43.  $7.5x^3 + 9x$     45.  $-\frac{1}{2}x^2 - 12x$
47.  $x^2 + 7x + 12$     49.  $6x^2 - 7x - 5$
51.  $x^4 + x^2 + 1$     53.  $x^2 - 100$     55.  $x^2 - 4y^2$
57.  $4x^2 + 12x + 9$     59.  $4x^2 - 20xy + 25y^2$
61.  $x^3 + 3x^2 + 3x + 1$     63.  $8x^3 - 12x^2y + 6xy^2 - y^3$
65.  $16x^6 - 24x^3 + 9$     67.  $m^2 - n^2 - 6m + 9$
69.  $x^2 + 2xy + y^2 - 6x - 6y + 9$     71.  $4r^4 - 25$
73.  $\frac{1}{4}x^2 - 3x + 9$     75.  $\frac{1}{9}x^2 - 4$
77.  $1.44x^2 + 7.2x + 9$     79.  $2.25x^2 - 16$
81.  $2x^2 + 2x$     83.  $u^4 - 16$     85.  $x - y$
87.  $x^2 - 2\sqrt{5}x + 5$     89.  $3(x + 2)$
91.  $2x(x^2 - 3)$     93.  $(x - 1)(x + 6)$
95.  $(x + 3)(x - 1)$     97.  $\frac{1}{2}(x + 8)$
99.  $\frac{1}{2}x(x^2 + 4x - 10)$     101.  $\frac{2}{3}(x - 6)(x - 3)$
103.  $(x + 9)(x - 9)$     105.  $2(4y - 3)(4y + 3)$
107.  $(4x + \frac{1}{3})(4x - \frac{1}{3})$     109.  $(x + 1)(x - 3)$
111.  $(3u + 2v)(3u - 2v)$     113.  $(x - 2)^2$
115.  $(2t + 1)^2$     117.  $(5y - 1)^2$     119.  $(3u + 4v)^2$
121.  $(x - \frac{2}{3})^2$     123.  $(x - 2)(x^2 + 2x + 4)$
125.  $(y + 4)(y^2 - 4y + 16)$
127.  $(2t - 1)(4t^2 + 2t + 1)$
129.  $(u + 3v)(u^2 - 3uv + 9v^2)$     131.  $(x + 2)(x - 1)$
133.  $(s - 3)(s - 2)$     135.  $-(y + 5)(y - 4)$
137.  $(x - 20)(x - 10)$     139.  $(3x - 2)(x - 1)$
141.  $(5x + 1)(x + 5)$     143.  $-(3z - 2)(3z + 1)$
145.  $(x - 1)(x^2 + 2)$     147.  $(2x - 1)(x^2 - 3)$
149.  $(3 + x)(2 - x^3)$     151.  $(3x^2 - 1)(2x + 1)$
153.  $(x + 2)(3x + 4)$     155.  $(2x - 1)(3x + 2)$
157.  $(3x - 1)(5x - 2)$     159.  $6(x + 3)(x - 3)$

161.  $x^2(x - 4)$     163.  $(x - 1)^2$     165.  $(1 - 2x)^2$
167.  $-2x(x + 1)(x - 2)$     169.  $(9x + 1)(x + 1)$
171.  $\frac{1}{81}(x + 36)(x - 18)$     173.  $(3x + 1)(x^2 + 5)$
175.  $x(x - 4)(x^2 + 1)$     177.  $\frac{1}{4}(x^2 + 3)(x + 12)$
179.  $(t + 6)(t - 8)$     181.  $(x + 2)(x + 4)(x - 2)(x - 4)$
183.  $5(x + 2)(x^2 - 2x + 4)$     185.  $(3 - 4x)(23 - 60x)$
187.  $5(1 - x)^2(3x + 2)(4x + 3)$
189.  $(x - 2)^2(x + 1)^3(7x - 5)$
191.  $3(x^6 + 1)^4(3x + 2)^2(33x^6 + 20x^5 + 3)$
193.  $-14, 14, -2, 2$     195.  $-11, 11, -4, 4, -1, 1$
197. Two possible answers: 2,  $-12$
199. Two possible answers:  $-2, -4$
201. (a)  $P = 22x - 25,000$     (b) \$85,000
203. (a)  $500r^2 + 1000r + 500$

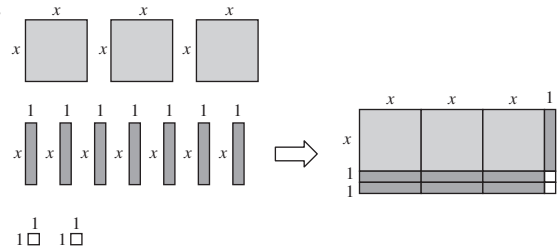
$r$	$2\frac{1}{2}\%$	3%	4%
$500(1 + r)^2$	\$525.31	\$530.45	\$540.80

$r$	$4\frac{1}{2}\%$	5%
$500(1 + r)^2$	\$546.01	\$551.25

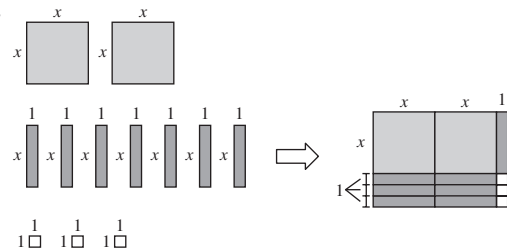
- (c) The amount increases with increasing  $r$ .
205. (a)  $V = 4x^3 - 88x^2 + 468x$

$x$ (cm)	1	2	3
$V$ (cm <sup>3</sup> )	384	616	720

207.  $44x + 308$     209. (a)  $3x^2 + 8x$     (b)  $30x^2$
- 211.



213.



215.  $4\pi(r + 1)$     217.  $4(6 - x)(6 + x)$
219. (a)  $\pi h(R - r)(R + r)$     (b)  $V = 2\pi \left[ \left( \frac{R + r}{2} \right) (R - r) \right] h$
221. False.  $(4x^2 + 1)(3x + 1) = 12x^3 + 4x^2 + 3x + 1$

223. True.  $a^2 - b^2 = (a + b)(a - b)$       225.  $m + n$   
 227.  $-x^3 + 8x^2 + 2x + 7$       229.  $(x^n + y^n)(x^n - y^n)$   
 231.  $x^{3n} - y^{2n}$  is completely factored.  
 233. Answers will vary. Sample answer:  $x^2 - 3$

## Appendix A.4 (page A42)

### Vocabulary Check (page A42)

1. domain      2. rational expression      3. complex  
 4. smaller      5. equivalent      6. difference quotient

1. All real numbers      3. All nonnegative real numbers  
 5. All real numbers  $x$  such that  $x \neq 2$   
 7. All real numbers  $x$  such that  $x \geq -1$       9.  $3x$ ,  $x \neq 0$   
 11.  $\frac{3x}{2}$ ,  $x \neq 0$       13.  $\frac{3y}{y+1}$ ,  $x \neq 0$       15.  $\frac{-4y}{5}$ ,  $y \neq \frac{1}{2}$   
 17.  $-\frac{1}{2}$ ,  $x \neq 5$       19.  $y - 4$ ,  $y \neq -4$   
 21.  $\frac{x(x+3)}{x-2}$ ,  $x \neq -2$       23.  $\frac{y-4}{y+6}$ ,  $y \neq 3$   
 25.  $\frac{-(x^2+1)}{(x+2)}$ ,  $x \neq 2$       27.  $z - 2$

29.	$x$	0	1	2	3	4	5	6
	$\frac{x^2 - 2x - 3}{x - 3}$	1	2	3	Undef.	5	6	7
	$x + 1$	1	2	3	4	5	6	7

The expressions are equivalent except at  $x = 3$ .

31. The expression cannot be simplified.  
 33.  $\frac{\pi}{4}$ ,  $r \neq 0$       35.  $\frac{1}{5(x-2)}$ ,  $x \neq 1$   
 37.  $\frac{r+1}{r}$ ,  $r \neq 1$       39.  $\frac{t-3}{(t+3)(t-2)}$ ,  $t \neq -2$   
 41.  $\frac{(x+6)(x+1)}{x^2}$ ,  $x \neq 6$       43.  $\frac{x+5}{x-1}$       45.  $\frac{6x+13}{x+3}$   
 47.  $-\frac{2}{x-2}$       49.  $-\frac{x^2+3}{(x+1)(x-2)(x-3)}$   
 51.  $\frac{2-x}{x^2+1}$ ,  $x \neq 0$   
 53. The error was incorrect subtraction in the numerator.  
 55.  $\frac{1}{2}$ ,  $x \neq 2$       57.  $x(x+1)$ ,  $x \neq -1, 0$   
 59.  $\frac{2x-1}{2x}$ ,  $x > 0$       61.  $\frac{x^7-2}{x^2}$       63.  $\frac{-1}{(x^2+1)^5}$   
 65.  $\frac{2x^3-2x^2-5}{(x-1)^{1/2}}$       67.  $\frac{3x-1}{3}$ ,  $x \neq 0$   
 69.  $\frac{-1}{x(x+h)}$ ,  $h \neq 0$

71.  $\frac{-1}{(x-4)(x+h-4)}$ ,  $h \neq 0$       73.  $\frac{1}{\sqrt{x+2} + \sqrt{x}}$   
 75.  $\frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$ ,  $h \neq 0$   
 77.  $\frac{x}{2(2x+1)}$ ,  $x \neq 0$   
 79. (a)  $\frac{1}{16}$  minute      (b)  $\frac{x}{16}$  minute(s)      (c)  $\frac{60}{16} = \frac{15}{4}$  minutes  
 81. (a) 9.09%      (b)  $\frac{288(MN-P)}{N(MN+12P)}$ ; 9.09%

83. (a)

$t$	0	2	4	6	8	10
$T$	75	55.9	48.3	45	43.3	42.3

$t$	12	14	16	18	20	22
$T$	41.7	41.3	41.1	40.9	40.7	40.6

- (b) The model is approaching a  $T$ -value of 40.  
 85. False. In order for the simplified expression to be equivalent to the original expression, the domain of the simplified expression needs to be restricted. If  $n$  is even,  $x \neq -1, 1$ . If  $n$  is odd,  $x \neq 1$ .  
 87. Completely factor each polynomial in the numerator and in the denominator. Then conclude that there are no common factors.

## Appendix A.5 (page A56)

### Vocabulary Check (page A56)

1. equation      2. solve      3. identities; conditional  
 4.  $ax + b = 0$       5. extraneous  
 6. quadratic equation  
 7. factoring; extracting square roots; completing the square; Quadratic Formula

1. Identity      3. Conditional equation      5. Identity  
 7. Identity      9. Conditional equation      11. 4  
 13. -9      15. 5      17. 9      19. No solution  
 21. -4      23.  $-\frac{6}{5}$       25. 9  
 27. No solution. The  $x$ -terms sum to zero.      29. 10  
 31. 4      33. 3      35. 0  
 37. No solution. The variable is divided out.  
 39. No solution. The solution is extraneous.  
 41. 2      43. No solution. The solution is extraneous.  
 45. 0      47. All real numbers  $x$   
 49.  $2x^2 + 8x - 3 = 0$       51.  $x^2 - 6x + 6 = 0$   
 53.  $3x^2 - 90x - 10 = 0$       55. 0,  $-\frac{1}{2}$       57. 4, -2  
 59. -5      61. 3,  $-\frac{1}{2}$       63. 2, -6      65.  $-\frac{20}{3}$ , -4  
 67. -a      69.  $\pm 7$       71.  $\pm \sqrt{11}$       73.  $\pm 3\sqrt{3}$

75. 8, 16    77.  $-2 \pm \sqrt{14}$     79.  $\frac{1 \pm 3\sqrt{2}}{2}$
81. 2    83. 4, -8    85.  $\sqrt{11} - 6, -\sqrt{11} - 6$
87.  $1 \pm \frac{\sqrt{6}}{3}$     89.  $2 \pm 2\sqrt{3}$     91.  $\frac{-5 \pm \sqrt{89}}{4}$
93.  $\frac{1}{2}, -1$     95.  $\frac{1}{4}, -\frac{3}{4}$     97.  $1 \pm \sqrt{3}$
99.  $-7 \pm \sqrt{5}$     101.  $-4 \pm 2\sqrt{5}$     103.  $\frac{2}{3} \pm \frac{\sqrt{7}}{3}$
105.  $-\frac{4}{3}$     107.  $-\frac{1}{2} \pm \sqrt{2}$     109.  $\frac{2}{7}$     111.  $2 \pm \frac{\sqrt{6}}{2}$
113.  $6 \pm \sqrt{11}$     115.  $-\frac{3}{8} \pm \frac{\sqrt{265}}{8}$     117. 0.976, -0.643
119. 1.355, -14.071    121. 1.687, -0.488
123. -0.290, -2.200    125.  $1 \pm \sqrt{2}$     127. 6, -12
129.  $\frac{1}{2} \pm \sqrt{3}$     131.  $-\frac{1}{2}$     133.  $\frac{3}{4} \pm \frac{\sqrt{97}}{4}$
135.  $0, \pm \frac{3\sqrt{2}}{2}$     137.  $\pm 3$     139. -6    141. -3, 0
143. 3, 1, -1    145.  $\pm 1$     147.  $\pm \sqrt{3}, \pm 1$
149.  $\pm \frac{1}{2}, \pm 4$     151. 1, -2    153. 50    155. 26
157. -16    159. 2, -5    161. 0    163. 9
165.  $-3 \pm 16\sqrt{2}$     167.  $\pm \sqrt{14}$     169. 1    171.  $2, -\frac{3}{2}$
173.  $\frac{-3 \pm \sqrt{21}}{6}$     175. 4, -5    177.  $\frac{1 \pm \sqrt{31}}{3}$
179. 3, -2    181.  $\sqrt{3}, -3$     183.  $3, \frac{-1 - \sqrt{17}}{2}$
185. (a) 61.2 inches  
 (b) Yes. The estimated height of a male with a 19-inch femur is 69.4 inches.  
 (c)
- | Height, $x$ | Female femur length | Male femur length |
|-------------|---------------------|-------------------|
| 60          | 15.48               | 14.79             |
| 70          | 19.80               | 19.28             |
| 80          | 24.12               | 23.77             |
| 90          | 28.44               | 28.26             |
| 100         | 32.76               | 32.75             |
| 110         | 37.08               | 37.24             |
- 100 inches  
 (d)  $x \approx 100.59$ ; There would not be a problem because it is not likely for either a male or a female to be 100 inches tall (which is 8 feet 4 inches tall).
187.  $y = -0.25t + 8$ ; after about 28 hours
189. 6 inches  $\times$  6 inches  $\times$  2 inches
191.  $\frac{20\sqrt{3}}{3} \approx 11.55$  inches
193. (a) 1998    (b) During 2007

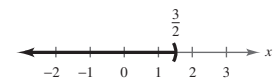
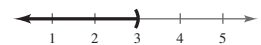
195. 500 units
197. False.  $x(3 - x) = 10$   
 $3x - x^2 = 10$   
 The equation cannot be written in the form  $ax + b = 0$ .
199. False. See Example 14 on page A55.
201. Equivalent equations have the same solution set, and one is derived from the other by steps for generating equivalent equations.  
 $2x = 5, 2x + 3 = 8$
203. Yes. The student should have subtracted  $15x$  from both sides to make the right side of the equation equal to zero. Factoring out an  $x$  shows that there are two solutions,  $x = 0$  and  $x = 6$ .
205.  $x^2 - 3x - 18 = 0$     207.  $x^2 - 22x + 112 = 0$
209.  $x^2 - 2x - 1 = 0$     211.  $a = 9, b = 9$
213. (a)  $x = 0, -\frac{b}{a}$     (b)  $x = 0, 1$

## Appendix A.6 (page A66)

### Vocabulary Check (page A66)

1. solution set    2. graph    3. negative  
 4. solution set    5. double    6. union

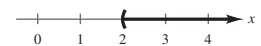
1.  $-1 \leq x \leq 5$ . Bounded    3.  $x > 11$ . Unbounded  
 5.  $x < -2$ . Unbounded  
 7. b    8. f    9. d    10. c    11. e    12. a  
 13. (a) Yes    (b) No    (c) Yes    (d) No  
 15. (a) Yes    (b) No    (c) No    (d) Yes  
 17. (a) Yes    (b) Yes    (c) Yes    (d) No  
 19.  $x < 3$     21.  $x < \frac{3}{2}$



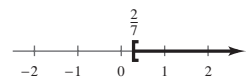
23.  $x \geq 12$



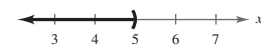
25.  $x > 2$



27.  $x \geq \frac{2}{7}$



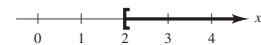
29.  $x < 5$



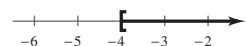
31.  $x \geq 4$



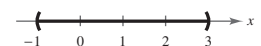
33.  $x \geq 2$



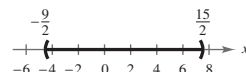
35.  $x \geq -4$



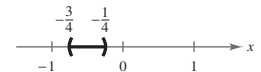
37.  $-1 < x < 3$



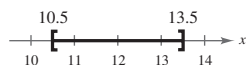
39.  $-\frac{9}{2} < x < \frac{15}{2}$



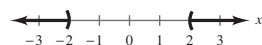
41.  $-\frac{3}{4} < x < -\frac{1}{4}$



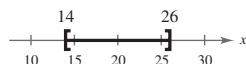
43.  $10.5 \leq x \leq 13.5$



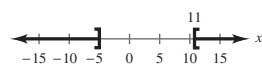
47.  $x < -2, x > 2$



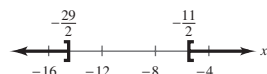
51.  $14 \leq x \leq 26$



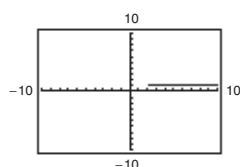
55.  $x \leq -5, x \geq 11$



59.  $x \leq -\frac{29}{2}, x \geq -\frac{11}{2}$

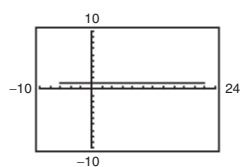


61.



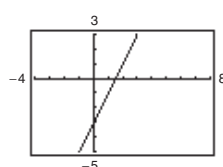
$x > 2$

65.



$-6 \leq x \leq 22$

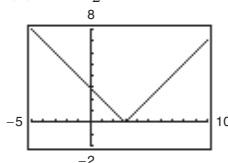
69.



(a)  $x \geq 2$

(b)  $x \leq \frac{3}{2}$

73.



(a)  $1 \leq x \leq 5$

(b)  $x \leq -1, x \geq 7$

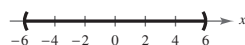
75.  $[5, \infty)$  77.  $[-3, \infty)$  79.  $(-\infty, \frac{7}{2}]$

81. All real numbers within eight units of 10

83.  $|x| \leq 3$

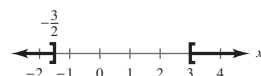
85.  $|x - 7| \geq 3$  87.  $|x - 12| < 10$

45.  $-6 < x < 6$

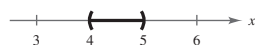


49. No solution

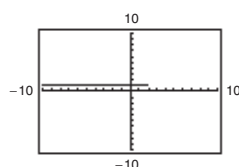
53.  $x \leq -\frac{3}{2}, x \geq 3$



57.  $4 < x < 5$

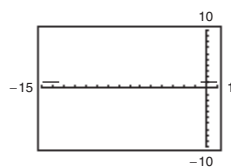


63.



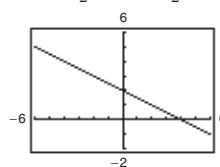
$x \leq 2$

67.



$x \leq -\frac{27}{2}, x \geq -\frac{1}{2}$

71.



(a)  $-2 \leq x \leq 4$

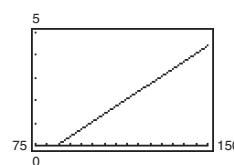
(b)  $x \leq 4$

89.  $|x + 3| > 4$  91.  $x > 6$

93.  $r > 3.125\%$  95.  $x \geq 36$

97.  $134 \leq x \leq 234$

99. (a)



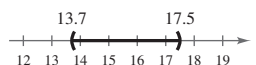
(b)  $x \geq 129$

101. (a)  $1 \leq t \leq 10$  (b)  $t > 16$

103.  $106.864 \text{ square inches} \leq \text{area} \leq 109.464 \text{ square inches}$

105. Might be undercharged or overcharged by \$0.19.

107.  $13.7 < t < 17.5$



109.  $20 \leq h \leq 80$

111. False.  $c$  has to be greater than zero.

113. b

## Appendix A.7 (page A75)

## Vocabulary Check (page A75)

1. numerator 2. reciprocal

1. Change all signs when distributing the minus sign.

$$2x - (3y + 4) = 2x - 3y - 4$$

3. Change all signs when distributing the minus sign.

$$\frac{4}{16x - (2x + 1)} = \frac{4}{14x - 1}$$

- 5.
- $z$
- occurs twice as a factor.

$$(5z)(6z) = 30z^2$$

7. The fraction as a whole is multiplied by
- $a$
- , not the numerator and denominator separately.

$$a\left(\frac{x}{y}\right) = \frac{ax}{y}$$

- 9.
- $\sqrt{x + 9}$
- cannot be simplified.

11. Divide out common factors, not common terms.

$$\frac{2x^2 + 1}{5x} \text{ cannot be simplified.}$$

13. To get rid of negative exponents:

$$\frac{1}{a^{-1} + b^{-1}} = \frac{1}{a^{-1} + b^{-1}} \cdot \frac{ab}{ab} = \frac{ab}{b + a}$$

15. Factor within grouping symbols before applying exponent to each factor.

$$(x^2 + 5x)^{1/2} = [x(x + 5)]^{1/2} = x^{1/2}(x + 5)^{1/2}$$

17. To add fractions, first find a common denominator.

$$\frac{3}{x} + \frac{4}{y} = \frac{3y + 4x}{xy}$$

19.  $3x + 2$     21.  $2x^2 + x + 15$     23.  $\frac{1}{3}$     25. 2

27.  $\frac{1}{2x^2}$     29.  $\frac{25}{9}, \frac{49}{16}$     31. 1, 2    33.  $1 - 5x$

35.  $1 - 7x$     37.  $3x - 1$     39.  $3x^2(2x - 1)^{-3}$

41.  $\frac{4}{3}x^{-1} + 4x^{-4} - 7x(2x)^{-1/3}$     43.  $\frac{16}{x} - 5 - x$

45.  $4x^{8/3} - 7x^{5/3} + \frac{1}{x^{1/3}}$     47.  $\frac{3}{x^{1/2}} - 5x^{3/2} - x^{7/2}$

49.  $\frac{-7x^2 - 4x + 9}{(x^2 - 3)^3(x + 1)^4}$     51.  $\frac{27x^2 - 24x + 2}{(6x + 1)^4}$

53.  $\frac{-1}{(x + 3)^{2/3}(x + 2)^{7/4}}$     55.  $\frac{4x - 3}{(3x - 1)^{4/3}}$     57.  $\frac{x}{x^2 + 4}$

59.  $\frac{(3x - 2)^{1/2}(15x^2 - 4x + 45)}{2(x^2 + 5)^{1/2}}$

61. (a)

$x$	0.5	1.0	1.5	2.0
$t$	1.70	1.72	1.78	1.89

$x$	2.5	3.0	3.5	4.0
$t$	2.02	2.18	2.36	2.57

(b)  $x = 0.5$  mile

(c)  $\frac{3x\sqrt{x^2 - 8x + 20} + (x - 4)\sqrt{x^2 + 4}}{6\sqrt{x^2 + 4}\sqrt{x^2 - 8x + 20}}$

63. True.  $x^{-1} + y^{-2} = \frac{1}{x} + \frac{1}{y^2} = \frac{y^2 + x}{xy^2}$

65. True.  $\frac{1}{\sqrt{x} + 4} = \frac{1}{\sqrt{x} + 4} \cdot \frac{\sqrt{x} - 4}{\sqrt{x} - 4} = \frac{\sqrt{x} - 4}{x - 16}$

67. Add exponents when multiplying powers with like bases.

$$x^n \cdot x^{3n} = x^{4n}$$

69. When a binomial is squared, there is also a middle term.

$$(x^n + y^n)^2 = x^{2n} + 2x^ny^n + y^{2n} \neq x^{2n} + y^{2n}$$

71. The two answers are equivalent and can be obtained by factoring.

$$\frac{1}{10}(2x - 1)^{5/2} + \frac{1}{6}(2x - 1)^{3/2}$$

$$= \frac{1}{60}(2x - 1)^{3/2}[6(2x - 1) + 10]$$

$$= \frac{1}{60}(2x - 1)^{3/2}(12x + 4)$$

$$= \frac{4}{60}(2x - 1)^{3/2}(3x + 1)$$

$$= \frac{1}{15}(2x - 1)^{3/2}(3x + 1)$$

(a)  $\frac{2}{5}(2x - 3)^{3/2}(x + 1)$     (b)  $\frac{8}{15}(4 + x)^{3/2}(x - 1)$



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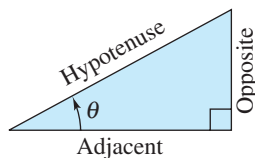
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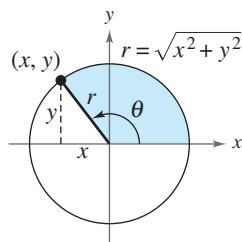
## Definition of the Six Trigonometric Functions

Right triangle definitions, where  $0 < \theta < \pi/2$



$$\begin{aligned}\sin \theta &= \frac{\text{opp.}}{\text{hyp.}} & \csc \theta &= \frac{\text{hyp.}}{\text{opp.}} \\ \cos \theta &= \frac{\text{adj.}}{\text{hyp.}} & \sec \theta &= \frac{\text{hyp.}}{\text{adj.}} \\ \tan \theta &= \frac{\text{opp.}}{\text{adj.}} & \cot \theta &= \frac{\text{adj.}}{\text{opp.}}\end{aligned}$$

Circular function definitions, where  $\theta$  is any angle



$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

## Reciprocal Identities

$$\begin{aligned}\sin u &= \frac{1}{\csc u} & \cos u &= \frac{1}{\sec u} & \tan u &= \frac{1}{\cot u} \\ \csc u &= \frac{1}{\sin u} & \sec u &= \frac{1}{\cos u} & \cot u &= \frac{1}{\tan u}\end{aligned}$$

## Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

## Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

## Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \quad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \quad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

## Even/Odd Identities

$$\sin(-u) = -\sin u \quad \cot(-u) = -\cot u$$

$$\cos(-u) = \cos u \quad \sec(-u) = \sec u$$

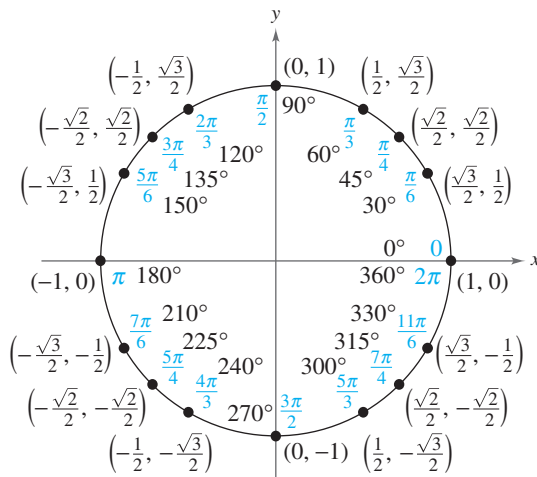
$$\tan(-u) = -\tan u \quad \csc(-u) = -\csc u$$

## Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$



## Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

## Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

## Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

## Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

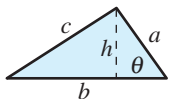
## FORMULAS FROM GEOMETRY

### Triangle:

$$h = a \sin \theta$$

$$\text{Area} = \frac{1}{2}bh$$

$$c^2 = a^2 + b^2 - 2ab \cos \theta \text{ (Law of Cosines)}$$



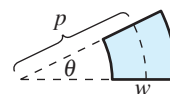
### Sector of Circular Ring:

$$\text{Area} = \theta pw$$

$p$  = average radius,

$w$  = width of ring,

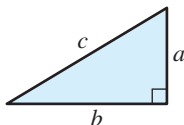
$\theta$  in radians



### Right Triangle:

Pythagorean Theorem

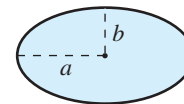
$$c^2 = a^2 + b^2$$



### Ellipse:

$$\text{Area} = \pi ab$$

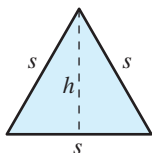
$$\text{Circumference} \approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$$



### Equilateral Triangle:

$$h = \frac{\sqrt{3}s}{2}$$

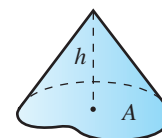
$$\text{Area} = \frac{\sqrt{3}s^2}{4}$$



### Cone:

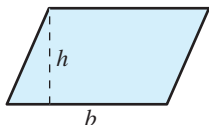
$$\text{Volume} = \frac{Ah}{3}$$

$A$  = area of base



### Parallelogram:

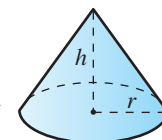
$$\text{Area} = bh$$



### Right Circular Cone:

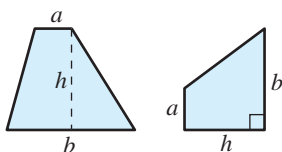
$$\text{Volume} = \frac{\pi r^2 h}{3}$$

$$\text{Lateral Surface Area} = \pi r \sqrt{r^2 + h^2}$$



### Trapezoid:

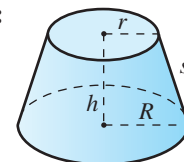
$$\text{Area} = \frac{h}{2}(a + b)$$



### Frustum of Right Circular Cone:

$$\text{Volume} = \frac{\pi(r^2 + rR + R^2)h}{3}$$

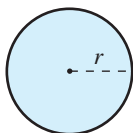
$$\text{Lateral Surface Area} = \pi s(R + r)$$



### Circle:

$$\text{Area} = \pi r^2$$

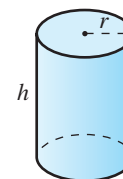
$$\text{Circumference} = 2\pi r$$



### Right Circular Cylinder:

$$\text{Volume} = \pi r^2 h$$

$$\text{Lateral Surface Area} = 2\pi r h$$

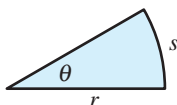


### Sector of Circle:

$$\text{Area} = \frac{\theta r^2}{2}$$

$$s = r\theta$$

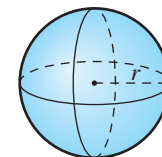
$\theta$  in radians



### Sphere:

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface Area} = 4\pi r^2$$



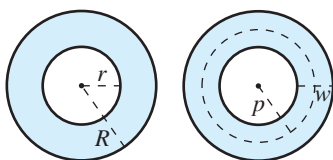
### Circular Ring:

$$\text{Area} = \pi(R^2 - r^2)$$

$$= 2\pi pw$$

$p$  = average radius,

$w$  = width of ring

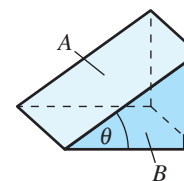


### Wedge:

$$A = B \sec \theta$$

$A$  = area of upper face,

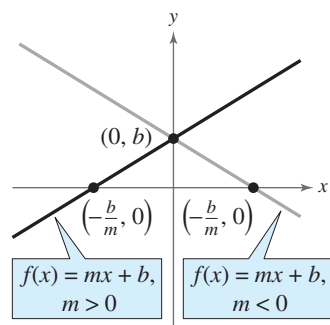
$B$  = area of base



## GRAPHS OF PARENT FUNCTIONS

### Linear Function

$$f(x) = mx + b$$



Domain:  $(-\infty, \infty)$

Range:  $(-\infty, \infty)$

x-intercept:  $(-b/m, 0)$

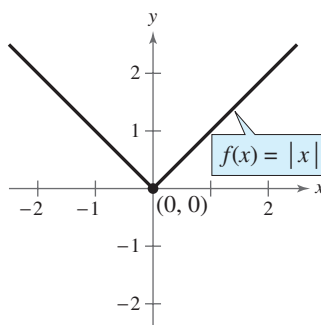
y-intercept:  $(0, b)$

Increasing when  $m > 0$

Decreasing when  $m < 0$

### Absolute Value Function

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Domain:  $(-\infty, \infty)$

Range:  $[0, \infty)$

Intercept:  $(0, 0)$

Decreasing on  $(-\infty, 0)$

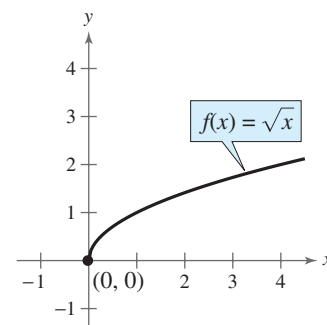
Increasing on  $(0, \infty)$

Even function

y-axis symmetry

### Square Root Function

$$f(x) = \sqrt{x}$$



Domain:  $[0, \infty)$

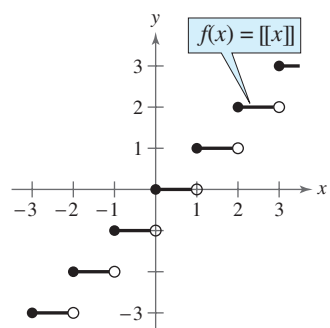
Range:  $[0, \infty)$

Intercept:  $(0, 0)$

Increasing on  $(0, \infty)$

### Greatest Integer Function

$$f(x) = \llbracket x \rrbracket$$



Domain:  $(-\infty, \infty)$

Range: the set of integers

x-intercepts: in the interval  $[0, 1)$

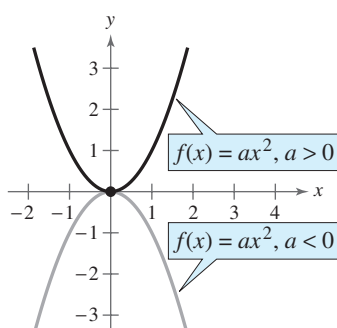
y-intercept:  $(0, 0)$

Constant between each pair of consecutive integers

Jumps vertically one unit at each integer value

### Quadratic (Squaring) Function

$$f(x) = ax^2$$



Domain:  $(-\infty, \infty)$

Range ( $a > 0$ ):  $[0, \infty)$

Range ( $a < 0$ ):  $(-\infty, 0]$

Intercept:  $(0, 0)$

Decreasing on  $(-\infty, 0)$  for  $a > 0$

Increasing on  $(0, \infty)$  for  $a > 0$

Increasing on  $(-\infty, 0)$  for  $a < 0$

Decreasing on  $(0, \infty)$  for  $a < 0$

Even function

y-axis symmetry

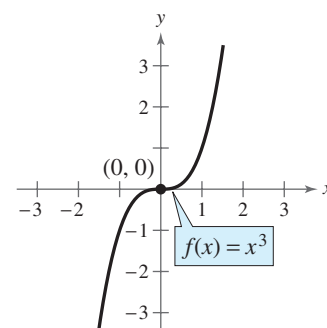
Relative minimum ( $a > 0$ ),

relative maximum ( $a < 0$ ),

or vertex:  $(0, 0)$

### Cubic Function

$$f(x) = x^3$$



Domain:  $(-\infty, \infty)$

Range:  $(-\infty, \infty)$

Intercept:  $(0, 0)$

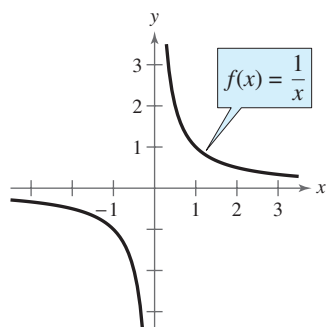
Increasing on  $(-\infty, \infty)$

Odd function

Origin symmetry

## Rational (Reciprocal) Function

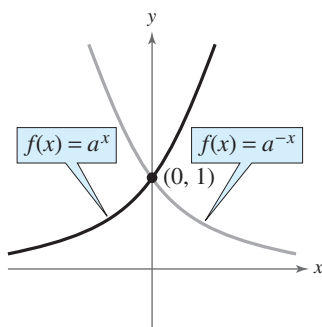
$$f(x) = \frac{1}{x}$$



Domain:  $(-\infty, 0) \cup (0, \infty)$   
 Range:  $(-\infty, 0) \cup (0, \infty)$   
 No intercepts  
 Decreasing on  $(-\infty, 0)$  and  $(0, \infty)$   
 Odd function  
 Origin symmetry  
 Vertical asymptote: y-axis  
 Horizontal asymptote: x-axis

## Exponential Function

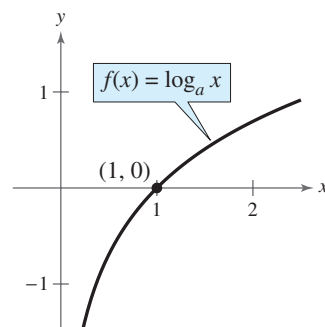
$$f(x) = a^x, a > 0, a \neq 1$$



Domain:  $(-\infty, \infty)$   
 Range:  $(0, \infty)$   
 Intercept:  $(0, 1)$   
 Increasing on  $(-\infty, \infty)$   
 for  $f(x) = a^x$   
 Decreasing on  $(-\infty, \infty)$   
 for  $f(x) = a^{-x}$   
 Horizontal asymptote: x-axis  
 Continuous

## Logarithmic Function

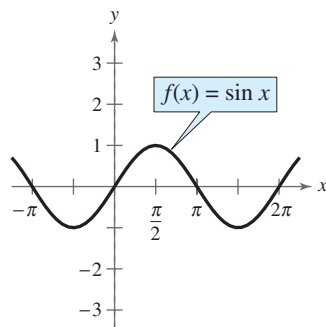
$$f(x) = \log_a x, a > 0, a \neq 1$$



Domain:  $(0, \infty)$   
 Range:  $(-\infty, \infty)$   
 Intercept:  $(1, 0)$   
 Increasing on  $(0, \infty)$   
 Vertical asymptote: y-axis  
 Continuous  
 Reflection of graph of  $f(x) = a^x$   
 in the line  $y = x$

## Sine Function

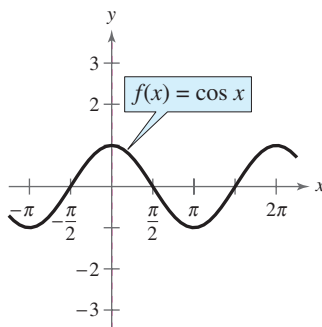
$$f(x) = \sin x$$



Domain:  $(-\infty, \infty)$   
 Range:  $[-1, 1]$   
 Period:  $2\pi$   
 x-intercepts:  $(n\pi, 0)$   
 y-intercept:  $(0, 0)$   
 Odd function  
 Origin symmetry

## Cosine Function

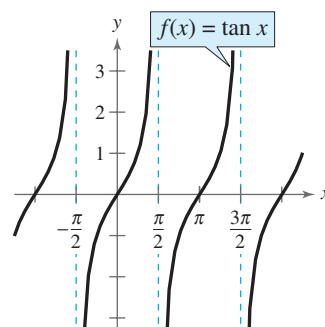
$$f(x) = \cos x$$



Domain:  $(-\infty, \infty)$   
 Range:  $[-1, 1]$   
 Period:  $2\pi$   
 x-intercepts:  $(\frac{\pi}{2} + n\pi, 0)$   
 y-intercept:  $(0, 1)$   
 Even function  
 y-axis symmetry

## Tangent Function

$$f(x) = \tan x$$

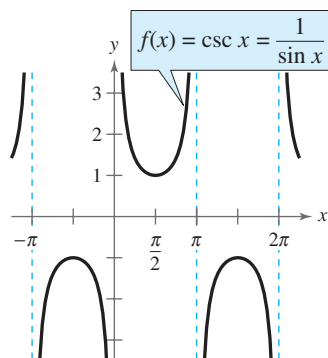


Domain: all  $x \neq \frac{\pi}{2} + n\pi$   
 Range:  $(-\infty, \infty)$   
 Period:  $\pi$   
 x-intercepts:  $(n\pi, 0)$   
 y-intercept:  $(0, 0)$   
 Vertical asymptotes:  

$$x = \frac{\pi}{2} + n\pi$$
  
 Odd function  
 Origin symmetry

## Cosecant Function

$$f(x) = \csc x$$



Domain: all  $x \neq n\pi$

Range:  $(-\infty, -1] \cup [1, \infty)$

Period:  $2\pi$

No intercepts

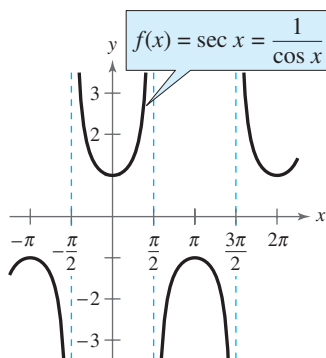
Vertical asymptotes:  $x = n\pi$

Odd function

Origin symmetry

## Secant Function

$$f(x) = \sec x$$



Domain: all  $x \neq \frac{\pi}{2} + n\pi$

Range:  $(-\infty, -1] \cup [1, \infty)$

Period:  $2\pi$

y-intercept:  $(0, 1)$

Vertical asymptotes:

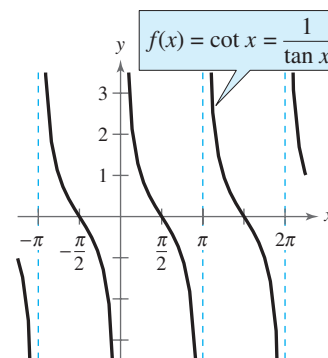
$$x = \frac{\pi}{2} + n\pi$$

Even function

y-axis symmetry

## Cotangent Function

$$f(x) = \cot x$$



Domain: all  $x \neq n\pi$

Range:  $(-\infty, \infty)$

Period:  $\pi$

x-intercepts:  $\left(\frac{\pi}{2} + n\pi, 0\right)$

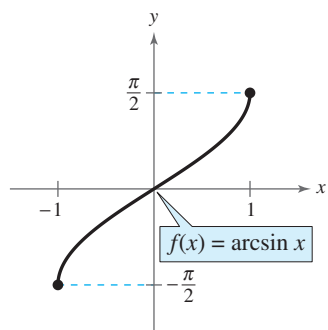
Vertical asymptotes:  $x = n\pi$

Odd function

Origin symmetry

## Inverse Sine Function

$$f(x) = \arcsin x$$



Domain:  $[-1, 1]$

Range:  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

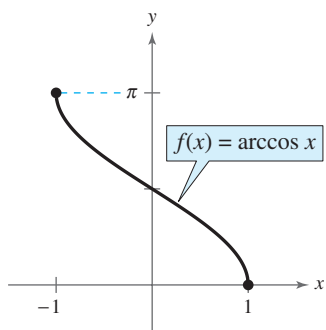
Intercept:  $(0, 0)$

Odd function

Origin symmetry

## Inverse Cosine Function

$$f(x) = \arccos x$$



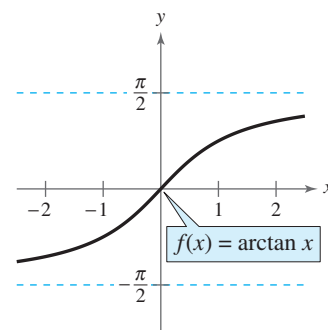
Domain:  $[-1, 1]$

Range:  $[0, \pi]$

y-intercept:  $\left(0, \frac{\pi}{2}\right)$

## Inverse Tangent Function

$$f(x) = \arctan x$$



Domain:  $(-\infty, \infty)$

Range:  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Intercept:  $(0, 0)$

Horizontal asymptotes:

$$y = \pm \frac{\pi}{2}$$

Odd function

Origin symmetry