## Causal Treatment Effect Aggregation in Boundary Discontinuity Designs Supplemental Appendix

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## Abstract

This supplemental appendix contains theoretical proofs and other technical results.

Keywords: regression discontinuity, treatment effects estimation, causal inference.

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