Final Exam

Due Monday, May 9, 2016

1. Consider the following problem in spherical geometry: $r \in [0, 10]$ cm, $\sigma_t = 5$ cm^{-1} and $\sigma_a = .001$ cm^{-1}, $\psi(10, \mu) = a$ for all $\mu_m < 0$ where a is a constant chosen to obtain a unit incident partial current, S_{16} quadrature, and 500 spatial cells.

   (a) Perform a calculation for this problem and plot your scalar flux solution.

   (b) Perform the calculation again with $\psi(10, \mu_1) = a$ and $\psi(10, \mu_m) = 0$ for all other $\mu_m < 0$ where a is a constant chosen to obtain a unit incident partial current. Plot your scalar flux solution. What is the rapid variation of the solution near the outer boundary called?

   (c) Evaluate the analytic slab-geometry asymptotic Dirichlet boundary condition assuming an incident distribution equal to a delta-function at $\mu = \mu_1$ normalized to give a unit partial current. How does this value compare with your computed spherical-geometry value?

   (d) Perform the calculation with the isotropically incident flux again with 10 cells and plot your scalar flux solution together with the 500 cell solution.

(e) Perform the calculation with the anisotropically incident flux again with 10 cells and plot your scalar flux solution together with the 500 cell solution. What is the main source of error in the 10-cell solution?

2. Consider the following problem in spherical geometry: $r \in [0, 1]\ cm$, $\sigma_t = 3\ cm^{-1}$ and $\sigma_a = 1\ cm^{-1}$, $\psi(1, \mu) = a$ for all $\mu_m < 0$ where a is a constant chosen to obtain a unit incident partial current, S_{16} quadrature, and 50 spatial cells.

(a) Compute the following response using the forward method:

$$R = \oint \int_{\mu>0} \psi(1, \mu)\mu_m \, d\mu \, dA \,, \tag{1}$$

(b) Compute the same response using the adjoint method. How do the forward and adjoint responses compare? Explain your result.

(c) Use first-order pertubation theory to compute the derivative of this response with respect to an additive perturbation of the absorption cross section, $\frac{\delta R}{\delta \sigma_a}$, i.e., compute $\frac{\partial R}{\partial \sigma_a}$.

(d) Assume $\delta\sigma_a = 0.05\ cm^{-1}$ and compute the corresponding perturbation in the response using both the response derivative and a forward calculation for the perturbed system. How do the two perturbed responses compare? Explain your result.