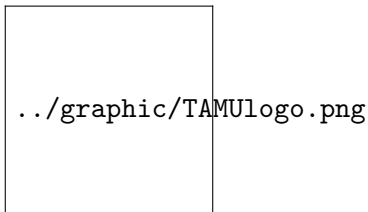


# The Coarse Scattering Method

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## The coarse scattering method

In the coarse scattering (CS) method, we make the following substitution to the MG transport equation to reduce the size of the scattering matrices

$$\sum_{g'} \Sigma_{s,\ell,g' \rightarrow g} \phi_{\ell,g'} \quad \rightarrow \quad S_{\ell,e \rightarrow g} \sum_{e'} \Sigma_{s,\ell,e' \rightarrow e} \phi_{\ell,e'}$$

where each fine-group  $g$  is a subset of a coarse-element  $e$  and

$$S_{\ell,e \rightarrow g} = \frac{\sum_{g'} \Sigma_{s,\ell,g' \rightarrow g} \phi_{\ell,g'}}{\sum_{e'} \Sigma_{s,\ell,e' \rightarrow e} \phi_{\ell,e'}}$$

$$\Sigma_{s,\ell,e' \rightarrow e} = \sum_{g \in e} \sum_{g' \in e'} \Sigma_{s,\ell,g' \rightarrow g}$$

$$\phi_{\ell,e} = \sum_{g \in e} \phi_{\ell,g}$$

## The CS method can also be applied to the fission matrix

For fission, we make the following substitution

$$\sum_{g'} \Sigma_{f,g' \rightarrow g} \phi_{g'} \rightarrow F_{e \rightarrow g} \sum_{e'} \Sigma_{f,e' \rightarrow e} \phi_{e'}$$

where

$$F_{e \rightarrow g} = \frac{\sum_{g'} \Sigma_{f,g' \rightarrow g} \phi_{g'}}{\sum_{e'} \Sigma_{f,e' \rightarrow e} \phi_{e'}}$$

$$\Sigma_{f,e' \rightarrow e} = \sum_{g \in e} \sum_{g' \in e'} \Sigma_{f,g' \rightarrow g}$$

$$\phi_e = \sum_{g \in e} \phi_g$$

## Example of fine transfer matrix being decomposed into a coarse transfer matrix and mapping operator

`../graphic/matrix_decomposition.png`

## Recomputing the scattering and fission spectra

- Not necessary to recompute both  $S_{\ell,e \rightarrow g}$  and  $F_{e \rightarrow g}$  every iteration

$$\left[ \frac{1}{v_g} \frac{\partial}{\partial t} + \mu \frac{\partial}{\partial x} + \Sigma_{t,g}(\vec{r}, t) \right] \psi_g^{(i+1)}(\vec{r}, \mu, t) = q_g(\vec{r}, \hat{\Omega}, t) +$$
$$S_{\ell,e \rightarrow g}^{(i-s)} \sum_{\ell=0}^L \frac{2\ell+1}{2} \sum_{e'} \Sigma_{s,\ell,e' \rightarrow e}(\vec{r}, t) \phi_{\ell,e'}^{(i)}(\vec{r}, t) +$$
$$\frac{F_{e \rightarrow g}^{(i-f)}}{2} \sum_{e'} \Sigma_{f,e' \rightarrow e}(\vec{r}, t) \phi_{e'}^{(i)m}(\vec{r}, t)$$

- Asymptotic computational cost of solving the MG transport equation is order  $G^2$
- Asymptotic computational cost of the CS method is order  $G$  in iterations when  $S_{\ell,e \rightarrow g}$  and  $F_{e \rightarrow g}$  are not recomputed and order  $G^2$  only in iterations where  $S_{\ell,e \rightarrow g}$  and  $F_{e \rightarrow g}$  are recomputed

## Particle balance can be maintained

Particle balance is maintained if following residual  $\rho$  is less than machine epsilon in the last iteration,

$$\rho = \left[ \sum_{g'} (\Sigma_{s,0,g' \rightarrow g} + \Sigma_{f,g' \rightarrow g}) \phi_{g'} \right] - \left[ S_{0,e \rightarrow g} \sum_{e'} \Sigma_{s,0,e' \rightarrow e} \phi_{e'} + F_{e \rightarrow g} \sum_{e'} \Sigma_{f,e' \rightarrow e} \phi_{e'} \right]$$

Futhermore, it may be possible to perserve particle balance every iteration if the CS method is only applied to Legendre moments greater than 0

# Preliminary CS Results for a 1D slab of uranium (20% enriched)

Simulation	MG	CS	CS	CS
Number of Groups	200	200 / 25	200 / 5	200 / 1
Recomputations ( $S_{e \rightarrow g}$ / $F_{e \rightarrow g}$ )		2 / 4	2 / 4	2 / 4
$k_{eff}$	1.15420	1.15420	1.15420	1.15420
RHS Cost	$3.26 \times 10^8$	$1.89 \times 10^8$	$2.03 \times 10^8$	$2.34 \times 10^8$
Total Cost	$3.32 \times 10^8$	$1.95 \times 10^8$	$2.11 \times 10^8$	$2.44 \times 10^8$



# Properties of Spherical Harmonics

$$\int_{4\pi} d\Omega \frac{2\ell+1}{4\pi} Y_{\ell}^m(\hat{\Omega}) Y_{\ell'}^{m'}(\hat{\Omega}) = \delta_{\ell\ell'} \delta_{mm'}$$

$$\int_{4\pi} d\Omega \frac{2\ell+1}{4\pi} Y_{\ell}^m(\hat{\Omega}) Y_0^0(\hat{\Omega}) = \delta_{\ell 0} \delta_{m 0}$$

$$\int_{4\pi} d\Omega \sum_{\ell=0}^L \sum_{m=-\ell}^{\ell} \frac{2\ell+1}{4\pi} Y_{\ell}^m(\hat{\Omega}) Y_0^0(\hat{\Omega}) = 1$$

Particle balanced is maintain if coarse scattering is only used for higher moments

$$\int_{4\pi} d\Omega Y_0^0(\hat{\Omega}) \sum_{\ell=0}^L \sum_{m=-\ell}^{\ell} \frac{2\ell+1}{4\pi} Y_{\ell}^m(\hat{\Omega}) \sum_{g'}^G \Sigma_{s,\ell,g' \rightarrow g} \phi_{\ell,g'}^m = \Sigma_{s,0,g' \rightarrow g} \phi_{0,g'}$$

$$\int_{4\pi} d\Omega Y_0^0(\hat{\Omega}) \left\{ \frac{1}{4\pi} Y_0^0(\hat{\Omega}) \sum_{g'}^G \Sigma_{s,0,g' \rightarrow g} \phi_{0,g'} + \right. \\ \left. \sum_{\ell=1}^L \sum_{m=-\ell}^{\ell} \frac{2\ell+1}{4\pi} Y_{\ell}^m(\hat{\Omega}) S_{e \rightarrow g} \sum_{e'}^E \Sigma_{s,\ell,e' \rightarrow e} \phi_{\ell,e'}^m \right\} = \Sigma_{s,0,g' \rightarrow g} \phi_{0,g'}$$

Thank you

Questions?