NUEN 625

Test 1

Due Thursday March 24, 2016

Preliminaries

Consider the the cylinder, shown in Fig. 1, of radius r_0 and height z_0 with center point c and endpoint e. The radial surface of the cylinder is defined by $r = r_0$, and $0 \leq z \leq z_0$. Given any point on the radial surface, the inward-directed normal at that point, $\overrightarrow{n}$, is directed along $-r$, as shown in Fig. 2. All surface source distributions, $Q(\overrightarrow{\Omega})$ are defined in terms of $\mu = \overrightarrow{\Omega} \cdot \overrightarrow{n}$, and are uniform over the axial surface of the cylinder.

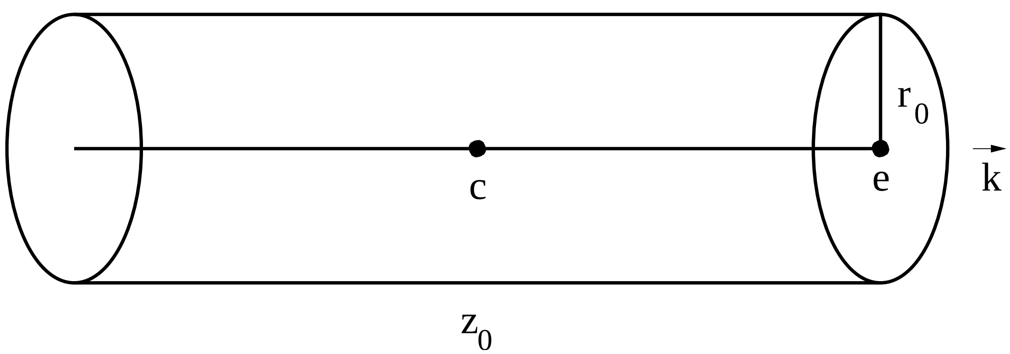

Figure 1: Geometry for cylinder of radius r_0 and height z_0.

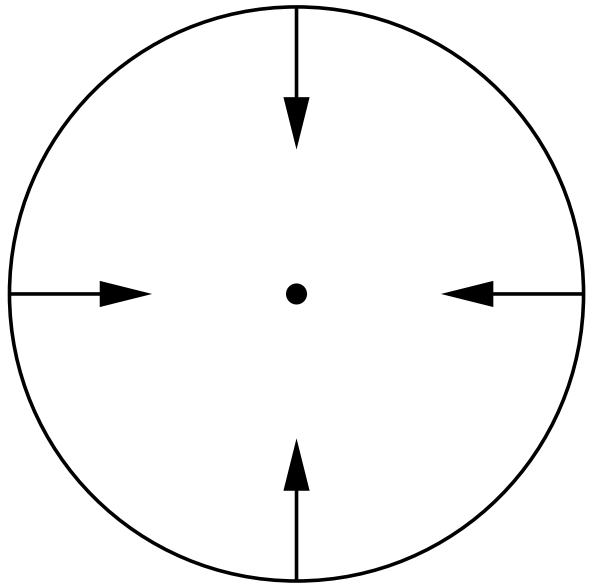

Figure 2: Inward-directed axial surface normals at four points as viewed looking down the axis of the cylinder.

1. Given an isotropic surface source distribution,

$$Q(\overrightarrow{\Omega}) = \frac{Q_0}{4\pi} \quad (p/(cm^2 - sec - steradian),$$

   (a) Calculate the scalar flux, ϕ $(p/cm^2 - sec)$, at point c.

   (b) Calculate the z-component of the current, $(\overrightarrow{J} \cdot \overrightarrow{k})$ $(p/(cm^2 - sec)$, at point c.

   (c) Calculate the scalar flux, ϕ $(p/cm^2 - sec)$, at point e.

   (d) Calculate the z-component of the current, $(\overrightarrow{J} \cdot \overrightarrow{k})$ $(p/(cm^2 - sec)$, at point e.

2. Given an anisotropic surface source distribution,

$$Q(\overrightarrow{\Omega}) = \frac{Q_0 \mu}{4\pi} \quad (p/(cm^2 - sec - steradian),$$

(a) Calculate the scalar flux, ϕ $(p/cm^2 - sec)$, at point c.

(b) Calculate the z-component of the current, $(\overrightarrow{J} \cdot \overrightarrow{k})$ $(p/(cm^2 - sec)$, at point c.

(c) Calculate the scalar flux, ϕ $(p/cm^2 - sec)$, at point e.

(d) Calculate the z-component of the current, $(\overrightarrow{J} \cdot \overrightarrow{k})$ $(p/(cm^2 - sec)$, at point e.

3. Consider the following equation:

$$-\frac{\partial}{\partial x}\frac{1}{3\sigma_t}\frac{\partial \phi}{\partial x} + \sigma_a \phi = 0 \quad \text{where } x \in [0, \infty) \text{ and } j^+(0) = 1.$$

Assuming constant cross sections, solve this equation using a Mark condition.

(a) Evaluate the reflected fraction $j^-(0)/j^+(0)$.

(b) Evaluate the reflected fraction in the limit as $\sigma_a \to 0$.

(c) Evaluate the reflected fraction in the limit as $\sigma_t \to \sigma_a$.

4. Consider the following equation:

$$\frac{df}{dx} + \sigma f = 0\,, \quad \text{for } x \in [x_{i-1/2}, x_{i+1/2}],\ f(x_{i-1/2}) = 1.$$

Solve this equation using the following trial space:

$$
\begin{aligned}
f(x) &= 1\,, \quad \text{for } x = x_{i-1/2}, \\
&= f_L\,, \quad \text{for } x \in (x_{i-1/2}, x_i), \\
&= f_R\,, \quad \text{for } x \in (x_i, x_{i+1/2}], \\
&= (f_L + f_R)/2\,, \quad \text{for } x = x_i,
\end{aligned}
$$

where x_i is located at the midpoint of the cell; and the following weighting space:

$$
\begin{aligned}
W_1(x) &= 1.0\,, \quad \text{for } x \in [x_{i-1/2}, x_i]\,, \\
&= 0, \quad \text{otherwise.}
\end{aligned}
$$

$$
\begin{aligned}
W_2(x) &= 1.0\,, \quad \text{for } x \in [x_i, x_{i+1/2}]\,, \\
&= 0, \quad \text{otherwise.}
\end{aligned}
$$

5. Consider the following system of equations:

$$
\frac{1}{v}\frac{\partial \psi}{\partial t} + \mu \frac{\partial \psi}{\partial x} + \sigma \psi = \frac{\sigma}{4\pi} g\,,
$$

$$
\frac{1}{v}\frac{\partial g}{\partial t} = \sigma \left(\phi - g\right)\,,
$$

where v is a speed, $\psi(t, x, \mu)$ is the usual angular flux, ϕ is the usual scalar flux, $g(t, x)$ is a positive isotropic function, and $\sigma(x)$ is a macroscopic interaction cross section.

Perform an asymptotic expansion for these equations using the following scaling:

$$
\begin{aligned}
v &\rightarrow v/\epsilon\,, \\
\sigma &\rightarrow \sigma/\epsilon\,,
\end{aligned}
$$

and

(a) express ψ as a function of g to leading order,

(b) derive the diffusion equation satisfied by g to leading order.