The Coarse Scattering Method

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22 October 2018

../graphic/TAMUlogo.png

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In the coarse scattering (CS) method, we make the following substitution to the MG transport equation to reduce the size of the scattering matrices

$$\sum_{g'} \Sigma_{s,\ell,g' \to g} \phi_{\ell,g'} \quad \to \quad S_{\ell,e\to g} \sum_{e'} \Sigma_{s,\ell,e' \to e} \phi_{\ell,e'}$$

where each fine-group g is a subset of a coarse-element e and

$$S_{\ell,e\to g} = \frac{\sum_{g'} \Sigma_{s,\ell,g'\to g} \phi_{\ell,g'}}{\sum_{e'} \Sigma_{s,\ell,e'\to e} \phi_{\ell,e'}}$$

$$\Sigma_{s,\ell,e'\to e} = \sum_{g\in e} \sum_{g'\in e'} \Sigma_{s,\ell,g'\to g}$$

$$\phi_{\ell,e} = \sum_{g\in e} \phi_{\ell,g}$$

The CS method can also be applied to the fission matrix

For fission, we make the following substitution

$$\sum_{g'} \Sigma_{f,g'\to g}\phi_{g'} \quad \to \quad F_{e\to g}\sum_{e'} \Sigma_{f,e'\to e}\phi_{e'}$$

where

$$F_{e\to g} = \frac{\sum_{g'} \Sigma_{f,g'\to g}\phi_{g'}}{\sum_{e'} \Sigma_{f,e'\to e}\phi_{e'}}$$

$$\Sigma_{f,e'\to e} = \sum_{g\in e}\sum_{g'\in e'} \Sigma_{f,g'\to g}$$

$$\phi_e = \sum_{g\in e}\phi_g$$

Example of fine transfer matrix being decomposed into a coarse transfer matrix and mapping operator

../graphic/matrix_decomposition.png

Recomputing the scattering and fission spectra

- Not necessary to recompute both $S_{\ell,e\to g}$ and $F_{e\to g}$ every iteration

$$\left[\frac{1}{v_g}\frac{\partial}{\partial t} + \mu\frac{\partial}{\partial x} + \Sigma_{t,g}(\vec{r},t)\right]\psi_g^{(i+1)}(\vec{r},\mu,t) = q_g(\vec{r},\hat{\Omega},t) + S_{\ell,e\to g}^{(i-s)}\sum_{\ell=0}^{L}\frac{2\ell+1}{2}\sum_{e'}\Sigma_{s,\ell,e'\to e}(\vec{r},t)\phi_{\ell,e'}^{(i)}(\vec{r},t) + \frac{F_{e\to g}^{(i-f)}}{2}\sum_{e'}\Sigma_{f,e'\to e}(\vec{r},t)\phi_{e'}^{(i)m}(\vec{r},t)$$

- Asymptotic computational cost of solving the MG transport equation is order G^2
- Asymptotic computational cost of the CS method is order G in iterations when $S_{\ell,e\to g}$ and $F_{e\to g}$ are not recomputed and order G^2 only in iterations where $S_{\ell,e\to g}$ and $F_{e\to g}$ are recomputed

Particle balance can be maintained

Particle balance is maintained if following residual ρ is less than machine epsilon in the last iteration,

$$\rho = \Big[\sum_{g'} \big(\Sigma_{s,0,g' \to g} + \Sigma_{f,g' \to g} \big) \phi_{g'} \Big] - \Big[S_{0,e \to g} \sum_{e'} \Sigma_{s,0,e' \to e} \phi_{e'} + F_{e \to g} \sum_{e'} \Sigma_{f,e' \to e} \phi_{e'} \Big]$$

Futhermore, it may be possible to perserve particle balance every iteration if the CS method is only applied to Legendre moments greater than 0

Preliminary CS Results for a 1D slab of uranium (20% enriched)

Simulation	MG	CS	CS	CS
Number of Groups	200	200 / 25	200 / 5	200 / 1
Recomputations ($S_{e\to g}$ / $F_{e\to g}$)		2 / 4	2 / 4	2 / 4
k_{eff}	1.15420	1.15420	1.15420	1.15420
RHS Cost	3.26×10^8	1.89×10^8	2.03×10^8	2.34×10^8
Total Cost	3.32×10^8	1.95×10^8	2.11×10^8	2.44×10^8

$$\int_{4\pi} d\Omega \, Y_\ell^m(\hat{\Omega}) Y_{\ell'}^{m'}(\hat{\Omega}) = \frac{4\pi}{2\ell+1}\delta_{\ell\ell'}\delta_{mm'}$$

$$\int_{4\pi} d\Omega \, Y_\ell^m(\hat{\Omega}) Y_0^0(\hat{\Omega}) = 4\pi$$

$$\int_{4\pi} d\Omega \, Y_\ell^m(\hat{\Omega}) = 4\pi$$

Properties of Spherical Harmonics

$$\int_{4\pi} d\Omega \sum_{\ell=0}^{L} \sum_{m=-\ell}^{\ell} \frac{2\ell+1}{4\pi} Y_\ell^m(\hat{\Omega}) \sum_{g'}^{G} \Sigma_{s,\ell,g'\to g} \phi_{\ell,g'}^m = \Sigma_{s,0,g'\to g} \phi_{0,g'}$$

$$\int_{4\pi} d\Omega \Big\{ \sum_{g'}^{G} \Sigma_{s,0,g'\to g} \phi_{0,g'} + \sum_{\ell=1}^{L} \sum_{m=-\ell}^{\ell} \frac{2\ell+1}{4\pi} Y_\ell^m(\hat{\Omega}) \sum_{e'}^{E} \Sigma_{s,\ell,e'\to e} \phi_{\ell,e'}^m \Big\} = \Sigma_{s,0,g'\to g} \phi_{0,g'}$$

Thank you

Questions?