ILP Formulations for Layered Promotion Graph

Layer 0: supplier-promotions (Node NodeIndex(0))

Note: This layer contains 1 promotion variable(s).

Decision Variables

All decision variables are binary.

Presence Variables (Full Price)

- x_1: Item 1 (Coca Cola 500ml) at full price (250)
- x_2: Item 2 (Chicken Sandwich) at full price (400)
- x_3: Item 3 (Water) at full price (100)
- x_4: Item 4 (Crisps) at full price (150)
- x_5: Item 5 (Newspaper) at full price (200)

Promotion Variables (Participation & Discounts)

- y_1: Item 1 with promotion "Supplier: 10% Off Coke" (225)

Objective Function

Minimize:

$$\text{minimize} \quad 250 \cdot x_1 + 400 \cdot x_2 + 100 \cdot x_3$$

$$+150 \cdot x_4 + 200 \cdot x_5 + 225 \cdot y_1$$

Constraints

Exclusivity Constraints

Each item must be purchased exactly once (at full price OR discounted by a single promotion):

$$x_1 + y_1 = 1$$

(Item 1 (Coca Cola 500ml))

$$x_2 = 1$$

(Item 2 (Chicken Sandwich))

$$x_3 = 1$$

(Item 3 (Water))

$$x_4 = 1$$

(Item 4 (Crisps))

$$x_5 = 1$$

(Item 5 (Newspaper))

Full ILP in Standard Form

$$\text{minimize} \quad 250 \cdot x_1 + 400 \cdot x_2 + 100 \cdot x_3$$

$$+150 \cdot x_4 + 200 \cdot x_5 + 225 \cdot y_1$$

$$\text{subject to} \quad x_1 + y_1 = 1$$

$$x_2 = 1$$

$$x_3 = 1$$

$$x_4 = 1$$

$$x_5 = 1$$

$$x_i \in \{0, 1\}$$

Layer 1: base-promotions (Node NodeIndex(2))

Note: This layer contains 8 promotion variable(s).

Decision Variables

All decision variables are binary.

Presence Variables (Full Price)

- x_1: Item 1 (Coca Cola 500ml) at full price (225)
- x_2: Item 2 (Chicken Sandwich) at full price (400)
- x_3: Item 3 (Water) at full price (100)
- x_4: Item 4 (Crisps) at full price (150)
- x_5: Item 5 (Newspaper) at full price (200)

Promotion Variables (Participation & Discounts)

- y_1: Item 1 with promotion "Buy One Get One Free Drinks" (225) [participation]
- y_2: Item 1 with promotion "Buy One Get One Free Drinks" (0) [discount]
- y_3: Item 3 with promotion "Buy One Get One Free Drinks" (100) [participation]
- y_4: Item 3 with promotion "Buy One Get One Free Drinks" (0) [discount]
- y_5: Item 1 with promotion "£5 Meal Deal" (0) [slot]
- y_6: Item 2 with promotion "£5 Meal Deal" (0) [slot]
- y_7: Item 3 with promotion "£5 Meal Deal" (0) [slot]
- y_8: Item 4 with promotion "£5 Meal Deal" (0) [slot]

Auxiliary Variables

- a_1: bundle count for promotion "£5 Meal Deal"
- s_1: DFA state for promotion "Buy One Get One Free Drinks" (pos=0, state=0)
- t_1: DFA take for promotion "Buy One Get One Free Drinks" (pos=0, state=0)
- s_2: DFA state for promotion "Buy One Get One Free Drinks" (pos=0, state=1)
- t_2: DFA take for promotion "Buy One Get One Free Drinks" (pos=0, state=1)
- s_3: DFA state for promotion "Buy One Get One Free Drinks" (pos=1, state=0)
- t_3: DFA take for promotion "Buy One Get One Free Drinks" (pos=1, state=0)
- s_4: DFA state for promotion "Buy One Get One Free Drinks" (pos=1, state=1)
- t_4: DFA take for promotion "Buy One Get One Free Drinks" (pos=1, state=1)
- s_5: DFA state for promotion "Buy One Get One Free Drinks" (pos=2, state=0)
- s_6: DFA state for promotion "Buy One Get One Free Drinks" (pos=2, state=1)

Objective Function

Minimize:

$$\begin{aligned}\text{minimize} \quad & 225 \cdot x_1 + 400 \cdot x_2 + 100 \cdot x_3 \\ & +150 \cdot x_4 + 200 \cdot x_5 + 225 \cdot y_1 \\ & -225 \cdot y_2 + 100 \cdot y_3 - 100 \cdot y_4 \\ & +500 \cdot a_1\end{aligned}$$

Constraints

Exclusivity Constraints

Each item must be purchased exactly once (at full price OR discounted by a single promotion):

$$x_1 + y_1 + y_5 = 1$$

(Item 1 (Coca Cola 500ml))

$$x_2 + y_6 = 1$$

(Item 2 (Chicken Sandwich))

$$x_3 + y_3 + y_7 = 1$$

(Item 3 (Water))

$$x_4 + y_8 = 1$$

(Item 4 (Crisps))

$$x_5 = 1$$

(Item 5 (Newspaper))

Promotion Constraints

$$y_6 - a_1 \geq 0$$

(slot min for promotion “£5 Meal Deal”)

$$y_6 - a_1 \leq 0$$

(slot max for promotion “£5 Meal Deal”)

$$y_5 + y_7 - a_1 \geq 0$$

(slot min for promotion “£5 Meal Deal”)

$$y_5 + y_7 - a_1 \leq 0$$

(slot max for promotion “£5 Meal Deal”)

$$y_8 - a_1 \geq 0$$

(slot min for promotion “£5 Meal Deal”)

$$y_8 - a_1 \leq 0$$

(slot max for promotion “£5 Meal Deal”)

$$s_1 + s_2 = 1$$

(DFA state uniqueness for promotion “Buy One Get One Free Drinks”)

$$s_3 + s_4 = 1$$

(DFA state uniqueness for promotion “Buy One Get One Free Drinks”)

$$s_5 + s_6 = 1$$

(DFA state uniqueness for promotion “Buy One Get One Free Drinks”)

$$s_1 = 1$$

(DFA initial state for promotion “Buy One Get One Free Drinks”)

$$s_5 = 1$$

(DFA final state for promotion “Buy One Get One Free Drinks”)

$$-s_1 + s_3 + t_1 - t_2 = 0$$

(DFA state transition for promotion “Buy One Get One Free Drinks”)

$$-s_2 + s_4 - t_1 + t_2 = 0$$

(DFA state transition for promotion “Buy One Get One Free Drinks”)

$$-s_3 + s_5 + t_3 - t_4 = 0$$

(DFA state transition for promotion “Buy One Get One Free Drinks”)

$$-s_4 + s_6 - t_3 + t_4 = 0$$

(DFA state transition for promotion “Buy One Get One Free Drinks”)

$$y_1 - t_1 - t_2 = 0$$

(DFA link participation for promotion “Buy One Get One Free Drinks”)

$$y_3 - t_3 - t_4 = 0$$

(DFA link participation for promotion “Buy One Get One Free Drinks”)

$$y_2 - t_2 = 0$$

(DFA link discount for promotion “Buy One Get One Free Drinks”)

$$y_4 - t_4 = 0$$

(DFA link discount for promotion “Buy One Get One Free Drinks”)

$$-s_1 + t_1 \leq 0$$

(DFA restrict transitions for promotion “Buy One Get One Free Drinks”)

$$-s_2 + t_2 \leq 0$$

(DFA restrict transitions for promotion “Buy One Get One Free Drinks”)

$$-s_3 + t_3 \leq 0$$

(DFA restrict transitions for promotion “Buy One Get One Free Drinks”)

$$-s_4 + t_4 \leq 0$$

(DFA restrict transitions for promotion “Buy One Get One Free Drinks”)

Full ILP in Standard Form

$$\begin{aligned}
\text{minimize} \quad & 225 \cdot x_1 + 400 \cdot x_2 + 100 \cdot x_3 \\
& +150 \cdot x_4 + 200 \cdot x_5 + 225 \cdot y_1 \\
& -225 \cdot y_2 + 100 \cdot y_3 - 100 \cdot y_4 \\
& +500 \cdot a_1 \\
\text{subject to} \quad & x_1 + y_1 + y_5 = 1 \\
& x_2 + y_6 = 1
\end{aligned}$$

$$x_3 + y_3 + y_7 = 1$$

$$x_4 + y_8 = 1$$

$$x_5 = 1$$

$$y_6 - a_1 \geq 0$$

$$y_6 - a_1 \leq 0$$

$$y_5 + y_7 - a_1 \geq 0$$

$$y_5 + y_7 - a_1 \leq 0$$

$$y_8 - a_1 \geq 0$$

$$y_8 - a_1 \leq 0$$

$$s_1 + s_2 = 1$$

$$s_3 + s_4 = 1$$

$$s_5 + s_6 = 1$$

$$s_1 = 1$$

$$s_5 = 1$$

$$-s_1 + s_3 + t_1 - t_2 = 0$$

$$-s_2 + s_4 - t_1 + t_2 = 0$$

$$-s_3 + s_5 + t_3 - t_4 = 0$$

$$-s_4 + s_6 - t_3 + t_4 = 0$$

$$y_1 - t_1 - t_2 = 0$$

$$y_3 - t_3 - t_4 = 0$$

$$y_2 - t_2 = 0$$

$$y_4 - t_4 = 0$$

$$-s_1 + t_1 \leq 0$$

$$-s_2 + t_2 \leq 0$$

$$-s_3 + t_3 \leq 0$$

$$-s_4 + t_4 \leq 0$$

$$x_i \in \{0, 1\}$$

Layer 2: staff-discount (Node NodeIndex(1))

Note: This layer contains 2 promotion variable(s).

Decision Variables

All decision variables are binary.

Presence Variables (Full Price)

- x_1: Item 1 (Coca Cola 500ml) at full price (100)
- x_2: Item 2 (Chicken Sandwich) at full price (200)

Promotion Variables (Participation & Discounts)

- y_1: Item 1 with promotion "5% Staff discount" (95)
- y_2: Item 2 with promotion "5% Staff discount" (190)

Objective Function

Minimize:

$$\text{minimize} \quad 100 \cdot x_1 + 200 \cdot x_2 + 95 \cdot y_1$$

$$+190 \cdot y_2$$

Constraints

Exclusivity Constraints

Each item must be purchased exactly once (at full price OR discounted by a single promotion):

$$x_1 + y_1 = 1$$

(Item 1 (Coca Cola 500ml))

$$x_2 + y_2 = 1$$

(Item 2 (Chicken Sandwich))

Full ILP in Standard Form

$$\text{minimize} \quad 100 \cdot x_1 + 200 \cdot x_2 + 95 \cdot y_1$$

$$+190 \cdot y_2$$

$$\text{subject to} \quad x_1 + y_1 = 1$$

$$x_2 + y_2 = 1$$

$$x_i \in \{0, 1\}$$