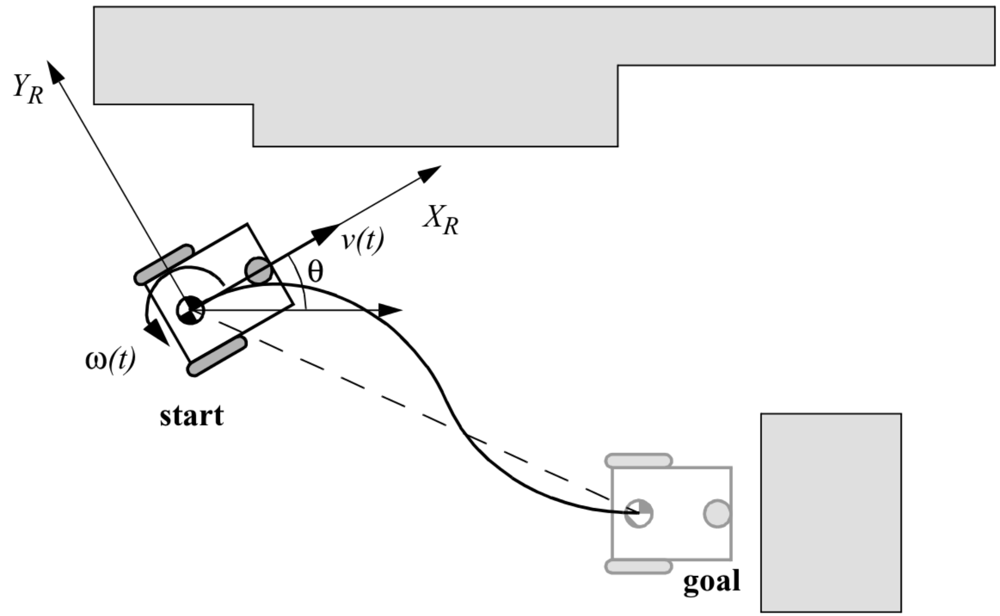
# Mobile Robot Kinematics

- Differential drive kinematic model

$$v = \frac{r\dot{\phi}_r}{2} + \frac{r\dot{\phi}_l}{2}$$
$$\omega = \frac{r\dot{\phi}_r}{2l} - \frac{r\dot{\phi}_l}{2l}$$

- Robot description: *wheel speeds +intrinsics ↔ robot speed*

# Controllability

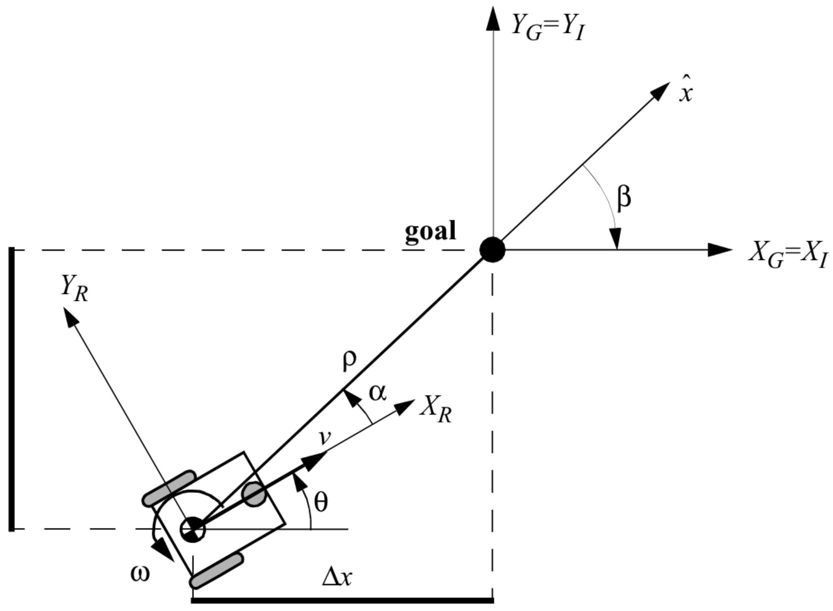



- How can the robot reach a goal position from its current state?

# Control

- Holonomic / Non-holonomic
- Open-loop / Feedback control
- Open-loop:
  - Trajectory synthesis → follow set-points
  - Can't adapt to dynamic changes in environments
  - Imperfect execution of trajectories (*boundaries, lines...*)
- Feedback:
  - Always knows current state <-> execute next step

# Control




To simplify the calculation, the goal reference frame was aligned with θ_g, such that the goal heading direction is simply the Xg axis. This differs from the exercise, where you have to find the polar coordinates with respect to a variable θ_g goal heading direction.

- **Kinematic model**: Wheel speeds needed for achieving particular robot motion
- **Control**: What robot motion (speeds) will be needed to get to the goal state

# Control

- Error :- $e = {}^{R}[x, y, \theta]^{T}$

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \, {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$ **=>** $$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \quad \text{with } k_{ij} = k(t, e)$$

- Proportional control: Robot not actuated when at the goal position
- Find K such that $\lim_{t \to \infty} e(t) = 0$

# Control

- Motion of the robot in inertial frame
- Kinematic of differential-drive robot in Inertial frame:

$$^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- Cartesian → Polar
  - Easier description for the given problem
- What does the controller need?
  - Position → decide speed
  - Heading → for alignment to goal location
  - Orientation → for aligning with goal orientation

# Control

- Inertial → Polar transformation

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$

- Kinematics model:-

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \dfrac{\sin\alpha}{\rho} & -1 \\ -\dfrac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

**NOTE**:
heading ↔ speed relation

# Control Law

- Simple linear control law:

$$v = k_\rho \rho$$

$$\omega = k_\alpha \alpha + k_\beta \beta$$

- Closed-loop system description:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

# Local Stability

- Linearizing the system around equilibrium (*cosx = 1; sinx = x*)

$$\begin{bmatrix}\dot{\rho}\\ \dot{\alpha}\\ \dot{\beta}\end{bmatrix} = \overset{A}{\begin{bmatrix}-k_\rho & 0 & 0\\ 0 & -(k_\alpha - k_\rho) & -k_\beta\\ 0 & -k_\rho & 0\end{bmatrix}}\begin{bmatrix}\rho\\ \alpha\\ \beta\end{bmatrix}$$

- Locally exponentially stable if the eigenvalues of the matrix *A* have -*ve* real part.
- Characteristic polynomial of A ( det(A-λ*I) ):

$$(\lambda + k_\rho)(\lambda^2 + \lambda(k_\alpha - k_\rho) - k_\rho k_\beta)$$

# Local Stability

- Negative real parts for all the roots of characteristic polynomial:

$$k_\rho > 0 \;\; ; \quad -k_\beta > 0 \;\; ; \quad k_\alpha - k_\rho > 0$$

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**NOTE**:
heading ↔ speed
relation