ETH zürich

Autonomous Mobile Robots

Exercise 3: Line fitting and extraction for robot localization

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Line extraction, EKF, SLAM

Exercise 3

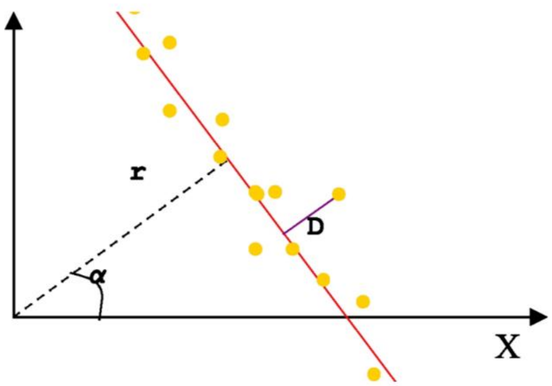
- Line extraction
- Line fitting

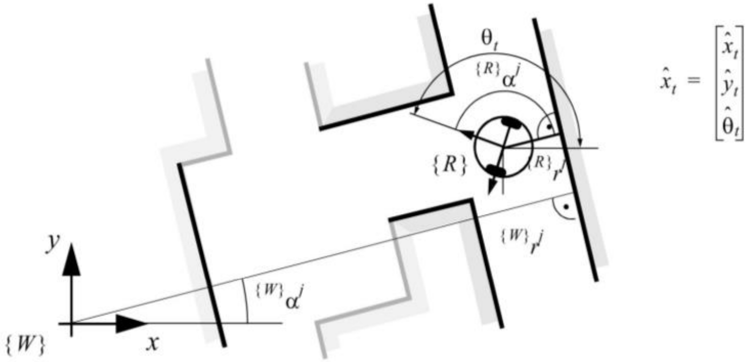
Exercise 4

- EKF
- Localization: Line extraction, given map
- Wheel odometry

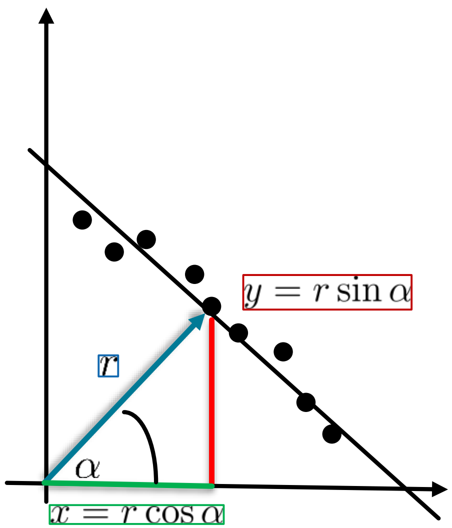
Exercise 5

- Simultaneous Localization and Mapping (SLAM)
- Unknown environment (a-priori)





Line in polar parameters



$$\boxed{x\cos\alpha + y\sin\alpha = r}$$

1. Why switch to polar parameters?
   - Vertical lines not representable in Cartesian coordinates
   - In general simpler representation for e.g. lines or circles
2. Where does the line equation expressed in polar coordinates come from? Easy:

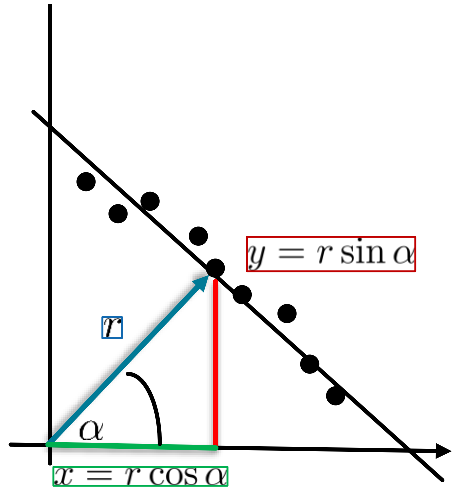
Pythagoras Theorem yields:

$$x^2 + y^2 = r^2 \tag{1}$$

With $x = r\cos\alpha$ and $y = r\sin\alpha$:

$$x \cdot x + y \cdot y = r \cdot r \tag{2}$$
$$x\cos\alpha + y\sin\alpha = r \tag{3}$$

Line fitting / Line regression



$$S(r,\alpha) := \sum_i (r - x^i \cos\alpha - y^i \sin\alpha)^2$$

Solution of (r,α): $\nabla S = 0$, i.e.

$$\frac{\partial S}{\partial \alpha} = 0$$
$$\frac{\partial S}{\partial r} = 0$$

Task:

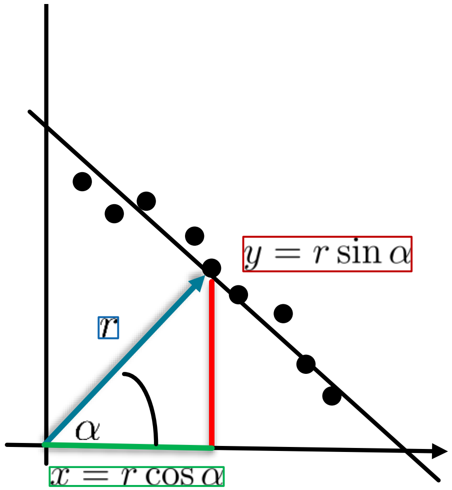
- Derive α. (Hint: Compute r via $\frac{\partial S}{\partial r} = 0$, then plug in result for r into $\frac{\partial S}{\partial \alpha} = 0$ and solve for α.)

You will need the following identities:

$$2\sin(\alpha)\cos(\alpha) = \sin(2\alpha) \quad (1)$$
$$\cos^2(\alpha) - \sin^2(\alpha) = \cos(2\alpha) \quad (2)$$

Line fitting / Line regression

$$S(r,\alpha) := \sum_i (r - x^i \cos\alpha - y^i \sin\alpha)^2 \qquad (1)$$



$$\frac{\partial S(r,\alpha)}{\partial r} = 2\sum_i (r - x^i \cos\alpha - y^i \sin\alpha) = 0 \qquad (1)$$

$$Nr - \cos\alpha \sum_i x^i - \sin\alpha \sum_i y^i = 0 \qquad (2)$$

$$\boxed{\begin{aligned} r &= \cos\alpha \frac{\sum_i x^i}{N} + \sin\alpha \frac{\sum_i y^i}{N} && (3) \\ &= x_c \cos\alpha + y_c \sin\alpha && (4) \end{aligned}}$$

That is, the line passes through the centroid (for a squared cost function).

Line fitting / Line regression

$$S(r,\alpha) := \sum_i (r - x^i \cos\alpha - y^i \sin\alpha)^2 \quad (1)$$

$$2\sin(\alpha)\cos(\alpha) = \sin(2\alpha) \quad (2)$$

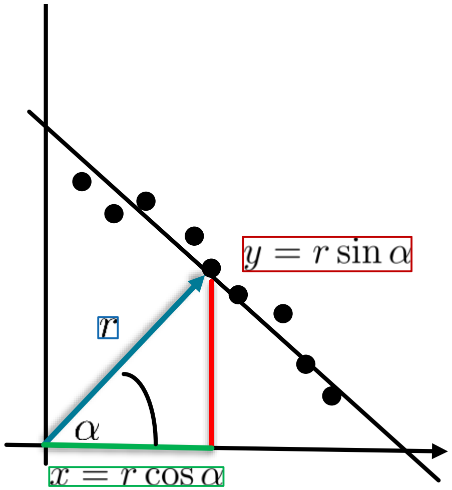
$$\cos^2(\alpha) - \sin^2(\alpha) = \cos(2\alpha) \quad (3)$$

$$r = x_c \cos\alpha + y_c \sin\alpha \quad (4)$$

$$\frac{\partial S(r,\alpha)}{\partial \alpha} = \frac{\partial}{\partial\alpha}\sum_i (x_c\cos\alpha + y_c\sin\alpha - x^i\cos\alpha - y^i\sin\alpha)^2 \quad (1)$$

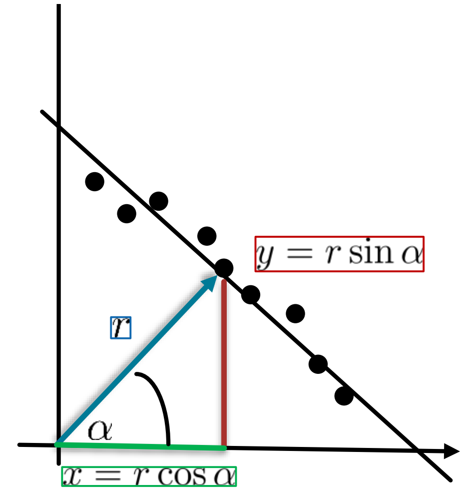
$$= \frac{\partial}{\partial\alpha}\sum_i (\cos\alpha(x_c - x^i) + \sin\alpha(y_c - y^i))^2 \quad (2)$$

$$= 2\sum_i (\tilde{x}\cos\alpha + \tilde{y}\sin\alpha)(-\tilde{x}\sin\alpha + \tilde{y}\cos\alpha) \quad (3)$$



Line fitting / Line regression

$$
\begin{aligned}
\frac{\partial S(r,\alpha)}{\partial \alpha} &= \frac{\partial}{\partial \alpha}\sum_i (x_c \cos\alpha + y_c \sin\alpha - x^i \cos\alpha - y^i \sin\alpha)^2 && (1)\\
&= \frac{\partial}{\partial \alpha}\sum_i (\cos\alpha(x_c - x^i) + \sin\alpha(y_c - y^i))^2 && (2)\\
&= 2\sum_i (\tilde{x}\cos\alpha + \tilde{y}\sin\alpha)(-\tilde{x}\sin\alpha + \tilde{y}\cos\alpha) && (3)
\end{aligned}
$$



$$-\cos\alpha\sin\alpha\sum\tilde{x}^2 + \cos^2\alpha\sum\tilde{x}\tilde{y} - sin^2\alpha\sum\tilde{x}\tilde{y} + \sin\alpha\cos\alpha\sum\tilde{y}^2 = 0$$

$$\underline{\sin\alpha\cos\alpha}\sum(\tilde{y}^2 - \tilde{x}^2) + \underline{(\cos^2\alpha - \sin^2\alpha)}\sum\tilde{x}\tilde{y} = 0 \quad \Big|\cdot 2$$

$$\sin(2\alpha)\sum(\tilde{y}^2 - \tilde{x}^2) + 2\cos(2\alpha)\sum\tilde{x}\tilde{y} = 0$$

$$\frac{\sin(2\alpha)}{\cos(2\alpha)} = -\frac{2\sum\tilde{x}\tilde{y}}{\sum(\tilde{y}^2 - \tilde{x}^2)}$$

$$\boxed{\alpha = \frac{1}{2}\tan^{-1}\left(\frac{-2\sum\tilde{x}\tilde{y}}{\sum(\tilde{y}^2 - \tilde{x}^2)}\right)}$$

$$\boxed{\begin{aligned} 2\sin(\alpha)\cos(\alpha) &= \sin(2\alpha)\\ \cos^2(\alpha) - \sin^2(\alpha) &= \cos(2\alpha)\end{aligned}}$$

Trigonometric identities

Matlab implementation

Editor - /tmp/ethzasl_amr_exercise3 (copy)/code/Ex3_LineExtraction/fitLine.m

fitLine.m

```matlab
%--------------------------------------------------------------------
% This function computes the parameters (r, alpha) of a line passing
% through input points that minimize the total-least-square error.
%
% Input:   XY - [2,N] : Input points
%
% Output:  alpha, r: paramters of the fitted line

function [alpha, r] = fitLine(XY)
% Compute the centroid of the point set (xmw, ymw) considering that
% the centroid of a finite set of points can be computed as
% the arithmetic mean of each coordinate of the points.

% XY(1,:) contains x position of the points
% XY(2,:) contains y position of the points


    xc = TODO
    yc = TODO

    % compute parameter alpha (see exercise pages)
    num   = TODO
    denom = TODO
    alpha = TODO

    % compute parameter r (see exercise pages)
    r = TODO


% Eliminate negative radii
if r < 0,
    alpha = alpha + pi;
    if alpha > pi, alpha = alpha - 2 * pi; end
    r = -r;
end

end
```

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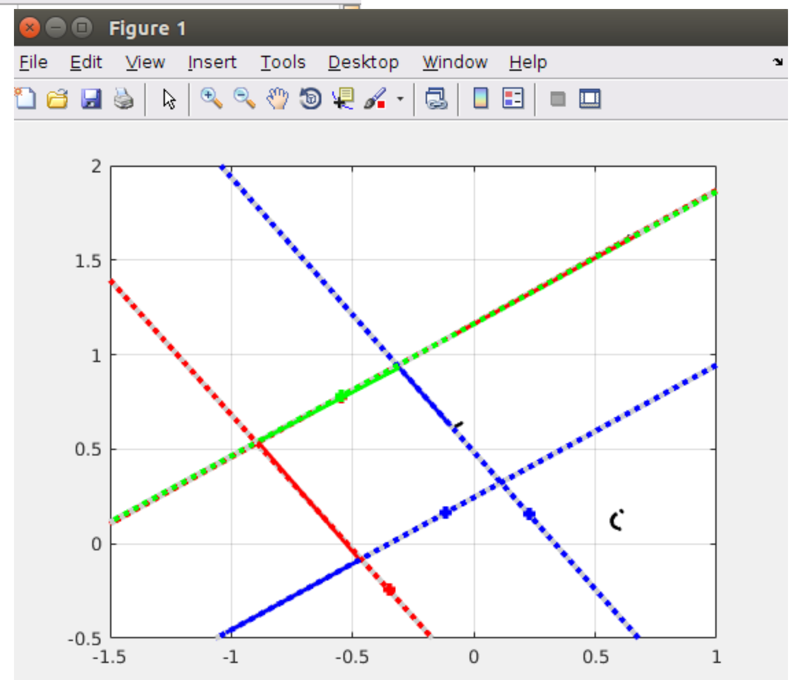


```
>> testLineExtraction
Testing laser scan 1: OK
Testing laser scan 2: OK
Testing laser scan 3: OK
Testing laser scan 4: OK
Testing laser scan 5: OK
Testing laser scan 6: OK
```