3

Graphical Design of Planar Linkages

3.1 Introduction

Design engineers are often faced with the task of creating a linkage that generates an irregular motion. Synthesis is the design of a linkage to produce a desired output motion for a given input motion. Chapter 2 acquainted the reader with the position analysis of various planar linkages. This chapter introduces the reader to the graphical design of planar linkages. It follows Chapter 2 because after you have designed a linkage, you need to be able to verify that it moves according to its motion design specifications.

History shows that graphical synthesis of linkages was predominately used before the age of the computer. It is included in this textbook because it helps the reader understand some of the basic concepts necessary for linkage synthesis, even if it is done using a computer. Graphical synthesis will typically get the design close to a possible solution or at least an insight into what might need to be changed to make a linkage design function properly.

Another reason for looking at graphical synthesis is that there are typically more unknown quantities in the design of a planar linkage than there are known physical conditions that must be met. This leads to an infinite number of solutions and an unsolvable solution for a computer program. A graphical synthesis can help the designer narrow down the number of unknowns graphically and through common sense and past experience. Design is typically an exercise in trade-offs between what you want and what you can obtain, while still designing a simple linkage.

A computer analysis results in a large set of numbers, which may be hard to interpolate. Converting some of these numbers to a graphical picture can provide the designer with valuable insight as to what might need to be modified to obtain the desired output motion.

Once a design concept has been determined, a graphical layout of the planar linkage can aid in the dimensional synthesis of the linkage. That is, the assigning of dimensions to the linkage.

Design and Analysis of Mechanisms: A Planar Approach, First Edition. Michael J. Rider.

The desired output of a planar linkage might be for function generation, path generation, or motion generation. **Function generation** is defined as requiring a specific output for each specified input, sort of like a black box with an input and an output. **Path generation** is defined as the control of a specified point such that it follows a specified path in 2D space. The orientation of the point is not considered, just its location. **Motion generation** is defined as the control of a rigid body such that its location and orientation relative to a predefined coordinate system are defined. This concept shows up when it is desired to move a container from point A to B, then tilt it to empty its contents. Both its path and its orientation are important throughout the motion.

3.2 Two-Position Synthesis for a Four-Bar Linkage

First, we will discuss a 2-position synthesis of a 4-bar linkage. In this case, it is desired to have the output link oscillate between two extreme limits when being driven by an input link that rotates through 360° (driven by a motor). The output link will be at its extreme right limit when links 2 and 3 are inline as shown in Figure 3.1a. In this case, the distance from A_o to D is equal to the length of link 3 plus the length of link 2. Its extreme left position will occur when link 2 is folded over on link 3 as shown in Figure 3.1b. In this case, the distance from A_o to D is equal to the length of link 3 minus the length of link 2.

We can use this information to design a 4-bar linkage whose output link, link 4, oscillates between two specified angles while the input link, link 2, rotates through 360°.

Example 3.1 Design a 4-bar linkage with angular motion displacement of 55° for its output link

Problem: Add a dyad (two links) to the output link to create a 4-bar linkage whose input is driven by a motor and whose output oscillates between 53° and 108° with the clockwise motion time being equal to the counterclockwise motion time. The bearings A_o and B_o should not be further apart than 12 in.

Solution: Procedure follows.

1. Pick a location for bearing B_o, draw it, and then specify an x–y reference axis.
2. Draw the output link, link 4, in its two extreme positions, $\theta_{41} = 53°$ and $\theta_{42} = 108°$.
3. Draw a line segment between the ends of link 4 labeled D_1 and D_2, and then extend this line segment in either direction.
4. Bisect the line segment D_1D_2.

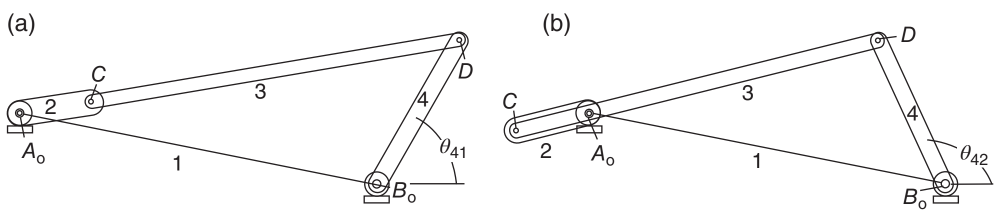

Figure 3.1 Extreme positions of 4-bar. (a) Links 2 and 3 extended and (b) links 2 and 3 overlap

5. Select a location on this line such that bearing A_o is less than the specified distance from bearing B_o as defined in the problem statement. Link 3 should be 3–4 times the length of link 2. If you make (A_o to D_2) = (D_2 to D_1), then L_3 is three times L_2.
6. Set your compass at a radius equal to half of line segment D_1D_2. Place your compass at bearing A_o and draw a circle. This will be the path of point C at the end of link 2. This distance from A_o to C is the length of link 2.
7. Label the right intersection point of the line and the circle as C_1. Label the left intersection point as C_2.
8. The distance between C_1 and D_1 is the same as the distance between C_2 and D_2. This distance is the length of link 3, the coupler link.
9. Measure the distances between A_o and C_1, C_1 and D_1, D_1 and B_o, and A_o and B_o. These four distances represent the lengths for link 2, link 3, link 4, and link 1 (see Figure 3.2).
10. Check the Grashof condition to be sure that the input link, link 2, can rotate through 360° without taking the linkage apart, then putting it back together again in its second configuration. Need: $L_{\text{shortest}} + L_{\text{longest}} < L_{\text{other}} + L_{\text{last}}$. If this is not true, select a new location for bearing A_o further from B_o and try again. If this is not possible, shorten link 4 and start again.
11. Use the information learned in Chapter 2 to analyze the linkage in its two extreme positions to verify your design.

Answers: $\boxed{L_1 = 10.0'' \text{ at } 138° \text{ from } B_o \text{ to } A_o}$, $\boxed{L_2 = 2.77''}$, $\boxed{L_3 = 8.47''}$, and $\boxed{L_4 = 6.0''}$.

Since C_2, A_o, and C_1 lie along the same line, the angle of rotation from C_1 to C_2 is 180° and from C_2 to C_1 is 180°. This guarantees that the clockwise motion time for link 4 will be equal to its counterclockwise motion time.

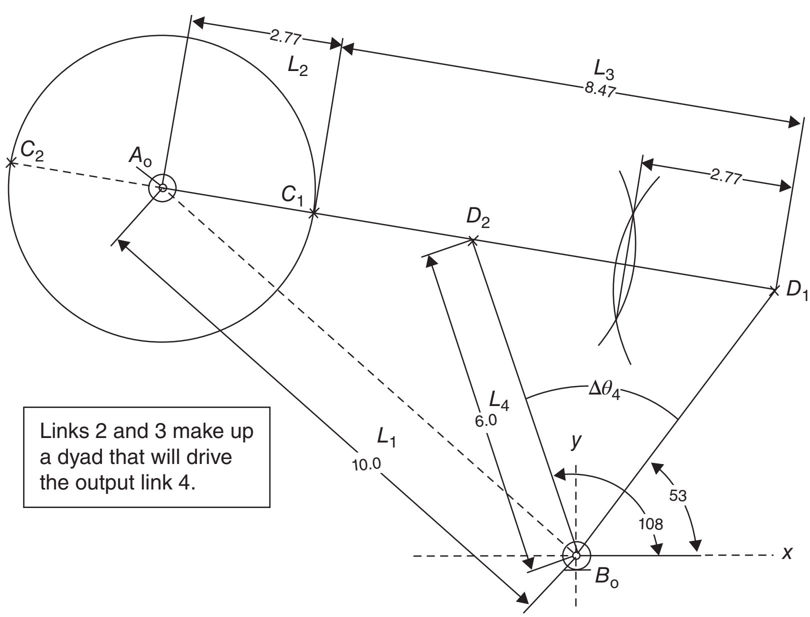

Figure 3.2 Graphical 2-angular positions

3.3 Two-Position Synthesis for a Quick Return 4-Bar Linkage

If you want to design a quick-return 4-bar linkage where the clockwise motion is longer than the counterclockwise motion, then the design procedure above needs to be modified slightly. Before we do this, let us look at a 4-bar linkage with an unequal timing ratio for its output oscillation (consider Figure 3.3). With the input link, link 2, rotating counterclockwise, the output link, link 4, will move from D_1 to D_2 after link 2 has rotated through 196.7°. Link 4 will move from D_2 back to D_1 after link 2 has rotated through 163.3°. If the input link is rotating at a constant speed, for example, driven by a motor, then link 4's timing clockwise will be different from its timing counterclockwise.

If the input is rotating at 1 revolution per second, then it will take $\frac{196.7}{360}(1\ \mathrm{s}) = 0.546\ \mathrm{s}$ to move from D_1 to D_2. It will take 0.454 s to return from D_2 to D_1, thus the quick return. Note that 196.7° minus 163.3° equals 33.4° or two times 16.7°. We can use this information when designing a quick-return 4-bar linkage. In addition, the timing ratio is $Q = \frac{196.7}{163.3} = \frac{0.5464}{0.4536} \approx 1.205$. The time over is approximately 20% higher than the return time. If you know the timing ratio, Q, then you can calculate the two input angles between the extreme positions and the angle, δ, between link 3 in its two extreme positions. For example, given a timing ratio of $Q = 1.2$, the construction angle delta can be calculated as follows.

$$\delta = 180 \cdot \left(\frac{Q-1}{Q+1}\right)$$

$$\delta = 180 \cdot \left(\frac{1.2-1}{1.2+1}\right) = 16.4^\circ$$

$$\theta_{\mathrm{long}} = 180^\circ + \delta = 180^\circ + 16.4^\circ = 196.4^\circ$$

$$\theta_{\mathrm{short}} = 180^\circ - \delta = 180^\circ - 16.4^\circ = 163.6^\circ$$

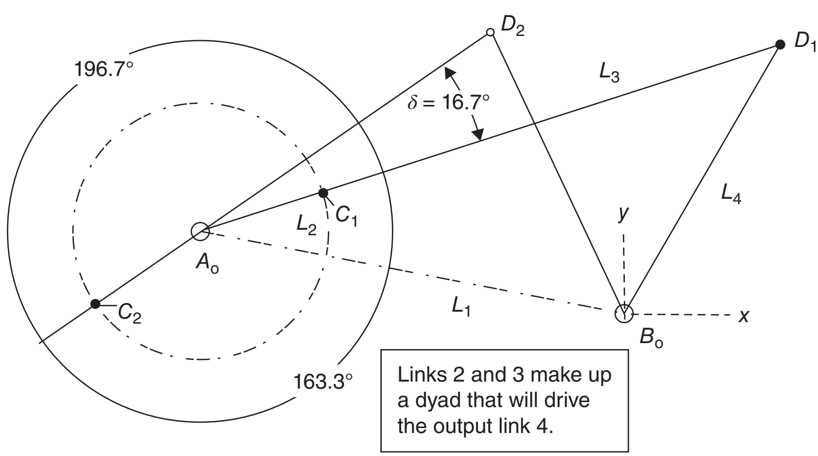

Figure 3.3 Quick-return 4-bar

These values come very close to the values shown in Figure 3.3. The difference lies in the fact that the timing ratio was slightly greater than 1.2 and we used 1.2 for these calculations.

Example 3.2 Design a quick-return 4-bar linkage with an angular motion displacement of 58° for its output link

Problem: Design a quick-return 4-bar linkage whose input is driven by a constant speed motor and whose output oscillates between 58° with the clockwise motion time being 1.25 times slower than the counterclockwise motion time. The bearings A_o and B_o should not be further apart than 12 in.

Solution: Procedure follows.

1. Pick a location for bearing B_o, draw it, and then specify an x–y reference axis.
2. Pick an initial length and angle for the output link, link 4. Draw link 4 in its two extreme positions, (I picked) $\theta_{41} = 52°$ and $\theta_{42} = 110°$. Label the two ends of link 4, D_1 and D_2.
3. Draw a line segment at a 45° angle from link 4, an unspecified length.
4. Using the timing ratio, Q, calculate the construction angle, δ.

$$\delta = 180° \cdot \left(\frac{Q-1}{Q+1}\right) = 180° \cdot \left(\frac{1.25-1}{1.25+1}\right) = 20°$$

5. Calculate the orientation of a second line so that it is oriented δ away from the first line and passes through D_2 (see Figure 3.4). $(\theta_{41} = 52°) - 45° + (\delta = 20°) = 27°$, so draw a line at 27° that passes through D_2.
6. The intersection of these two line segments is the location for bearing A_o. Be sure the location of A_o is less than the specified distance from bearing B_o as defined in the problem statement if appropriate.
7. The distance from A_o to D_1 is the sum of links 3 and 2. The distance from A_o to D_2 is the difference between links 3 and 2. Measure these two distances, and then calculate

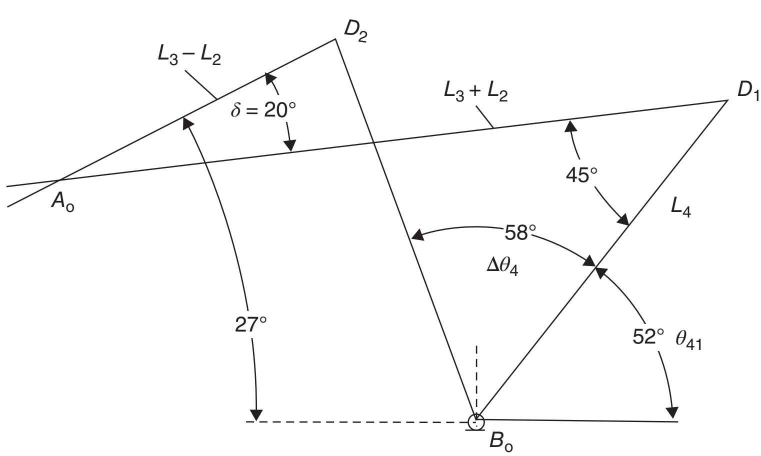

Figure 3.4 Construct second line

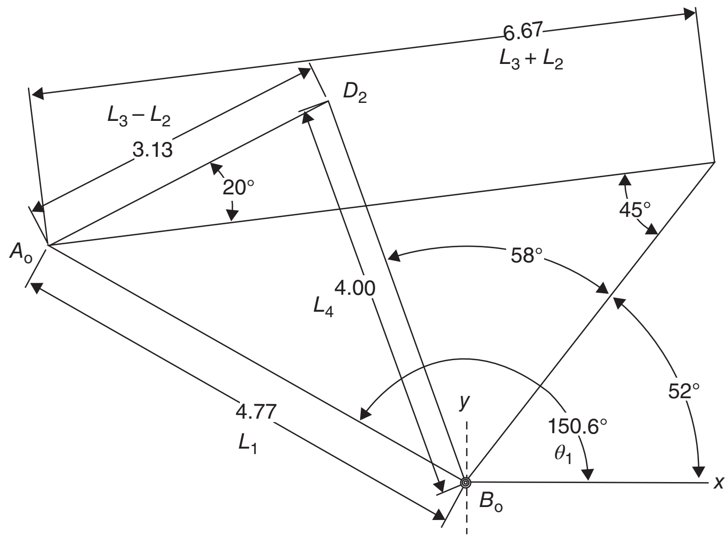

Figure 3.5 Quick-return 2-angular positions

the lengths of links 2 and 3 by solving two linear equations for two unknowns (see Figure 3.5).

$$L_3 + L_2 = 6.67$$

$$\underline{L_3 - L_2 = 3.13}$$

$$2L_3 = 9.80$$

$$L_3 = 4.90 \text{ in.}$$

$$L_2 = 4.90 - 3.13 = 1.77 \text{ in.}$$

8. Set your compass at a radius equal to the length of link 2. Place your compass at bearing A_o and draw a circle. This will be the path of point C at the end of link 2.
9. Label the right intersection point of the line and the circle between A_o and D_1 as C_1.
10. Extend line D_2 to A_o so it crosses the left edge of the circle. Label this left intersection point as C_2.
11. Measure the distances between D_1 and B_o, and A_o and B_o. These two distances represent the lengths for link 4 and link 1.
12. Check the Grashof condition to be sure that the input link, link 2, can rotate through 360° without taking the linkage apart, then putting it back together again in its second configuration. Need: $L_{\text{shortest}} + L_{\text{longest}} < L_{\text{other}} + L_{\text{last}}$. If this is not true, select an angle different from 45° for the first line segment and try again.
13. Use the information learned in Chapter 2 to analyze the linkage in its two extreme positions to verify your design. Also, verify that the timing is correct per the design statement. Note that the input, link 2, must rotate clockwise for the clockwise motion of the output link to be the longer time.

Answers: $\boxed{L_1 = 4.77'' \text{ at } 150.6^\circ \text{ from } B_o \text{ to } A_o}$, $\boxed{L_2 = 1.77''}$, $\boxed{L_3 = 4.90''}$, and $\boxed{L_4 = 4.00''}$.

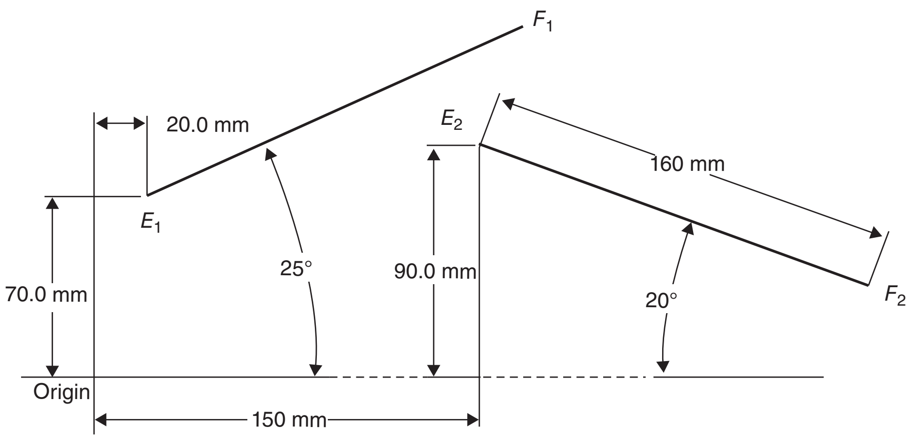

Figure 3.6 Output link in two positions

3.4 Two-Positions for Coupler Link

Next, we will look at the case where the output link contains a line segment that must move between two locations and orientations and be driven by a constant speed motor (see Figure 3.6).

Since this is part of link 4, the first step is to determine the pivot point for link 4 so that the defined segment will move and rotate as desired. This can be done by drawing a perpendicular bisector between E_1 and E_2 and a perpendicular bisector between F_1 and F_2. The intersection of these two bisectors is the proper location for the bearing, B_o, of link 4. Although the line B_oE_2 looks perpendicular to E_2F_2, it may not be. This angle on Figure 3.7 is 83.8°. Note that the angle $B_oE_1F_1$ is equal to the angle $B_oE_2F_2$.

Add a dyad to drive link 4 between these two extreme positions. The rest of the 4-bar linkage design is exactly like the previous two examples since the extreme limits of link 4 are now defined. Follow Example 3.1 if the timing ratio is one. Follow Example 3.2 if a quick-return 4-bar linkage is desired.

3.5 Three Positions of the Coupler Link

In the case where the coupler link (see Figure 3.8) must be in three different locations and at three different orientations, the following method can be used. Since C_1, C_2, and C_3 are equal distance from bearing A_o, drawing a circle through these three points will locate bearing A_o. In addition, D_1, D_2, and D_3 are equal distance from bearing B_o, and thus drawing a circle through these three points will locate bearing B_o.

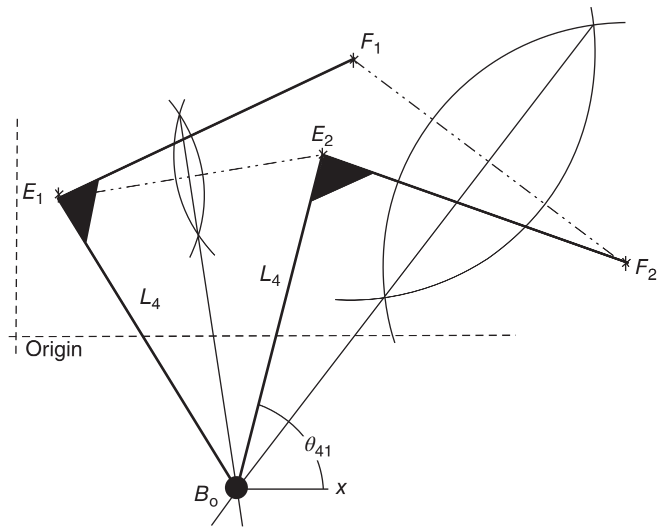

Figure 3.7 Locate bearing B_o

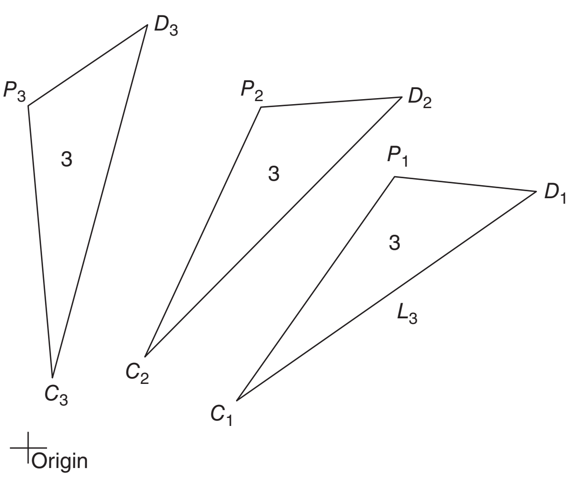

Figure 3.8 Coupler link in three positions

Example 3.3 Design a 4-bar linkage whose coupler link goes through three specific locations

Problem: Design a 4-bar linkage whose 5.00 in. long coupler link goes through the three positions shown in Figure 3.9, and then determine if the 4-bar's link 2 can rotate through 360°. Position 1: end point at (0.50, 3.38) inches and link 3 horizontal. Position 2: end point at (3.00, 3.84) inches and link 3 is 17° clockwise from horizontal. Position 3: end point at (3.75, 2.44) inches and link 3 is 26 clockwise from horizontal.

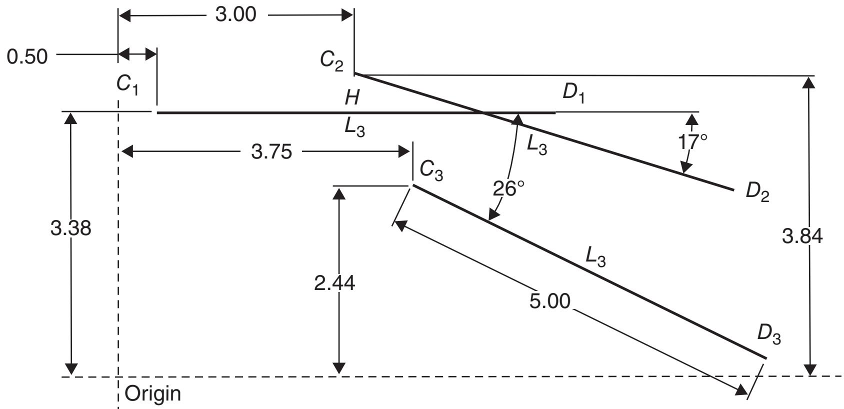

Figure 3.9 Three positions for coupler link

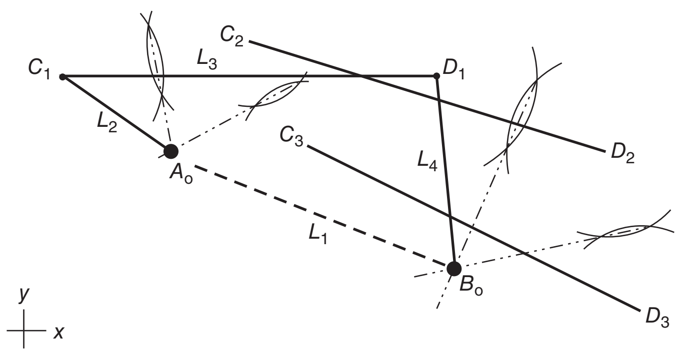

Figure 3.10 4-Bar designed

Solution: Procedure follows.

1. Draw the coupler link, link 3, in its three specified locations.
2. Mark one end of the coupler link as C_1, C_2, and C_3.
3. Mark the other end on link 3 as D_1, D_2, and D_3.
4. Draw a perpendicular bisector between points C_1 and C_2.
5. Draw a perpendicular bisector between points C_2 and C_3.
6. The intersection of these two perpendicular bisectors is the center for a circle that will go through points C_1, C_2, and C_3. This is the location for bearing A_o.
7. Draw a line between A_o and C_1. This will be link 2 (see Figure 3.10).
8. Draw a perpendicular bisector between points D_1 and D_2.
9. Draw a perpendicular bisector between points D_2 and D_3.
10. The intersection of these two perpendicular bisectors is the center for a circle that will go through points D_1, D_2, and D_3. This is the location for bearing B_o (see Figure 3.10).

11. Draw a line between B_o and D_1. This will be link 4.
12. Measure the length of Links 2 and 4. Locate the bearings relative to the origin.
13. Check to see if link 2 can rotate through 360° using Grashof condition.

Grashof Condition for this linkage: $(L_2 + L_3) < (L_1 + L_4)$?

Is $(1.78 + 5.00) < (4.08 + 2.57) \rightarrow$ NO, Link 2 will not rotate through 360°. This is a triple-rocker linkage. No link will rotate through 360°. If we want this linkage to move through the three positions defined, then we will have to add a dyad (two more links with one of these links rotating through 360°) to our design solution. We can select either link 2 or link 4, determine its range of motion, and then add links 5 and 6 with link 6 being driven by a motor (see Example 3.1).

Answer: $\boxed{L_2 = 1.78''}$, $\boxed{L_4 = 2.57''}$, $\boxed{L_1 = 4.08''}$, $\boxed{A_o = (1.97'', 2.39'')}$, $\boxed{B_o = (5.74'', 0.82'')}$, and $\boxed{L_3 = 5.00''}$.

3.6 Coupler Point Goes Through Three Points

If we need a coupler point to go through three different points and the orientation of the coupler link is not important, then a different design method is used (see Figure 3.11).

Procedure:

1. Locate the three prescribed positions (P_1, P_2, and P_3).
2. Select a location for the fixed pivot point, bearing A_o.
3. Choose a length for link 2 (A_oC) and draw the path of point C (circle) centered at A_o. Make sure the length of link 2 is greater than half the distance between the any two specified "P" points.
4. Pick a point for C_3 on the newly drawn circle, relative to P_3. Picking the point farthest from bearing A_o guarantees that link 3 should reach a possible location of link 2.
5. With length C_3P_3 determined and fixed, find C_2 from P_2 and C_1 from P_1. If the length C_2P_2 or C_1P_1 does not intersect the "C" circle drawn, then begin again, choosing a different location for C_3, thus a different length.

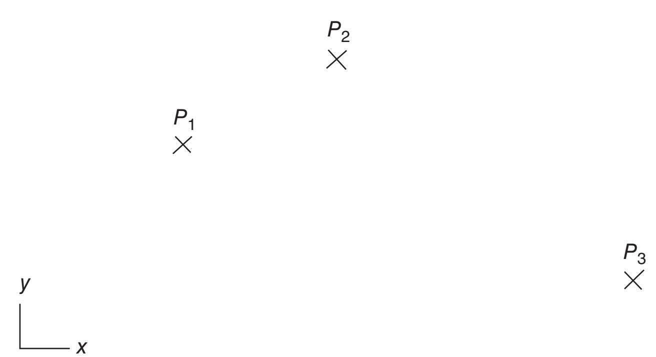

Figure 3.11 Three coupler points

6. Draw line C_1D_1 at some length and some angle away from C_1P_1. Draw line P_1D_1 to complete link 3.
7. Locate D_2 using the sizes from step 6 above. Link 3 is the same size in each position; that is, $C_1D_1 = C_2D_2 = C_3D_3$ and $P_1D_1 = P_2D_2 = P_3D_3$.
8. Locate D_3 using the sizes from step 6 above.
9. Construct perpendicular bisectors for D_1D_2, and D_2D_3 to locate the second bearing B_o as before. If you do not like the location of bearing Bo or the size of link 4, go back to step 2, 3, or 4 and begin again.
10. Measure the length of link 2, link 3, and link 4. Locate the position of A_o and B_o. The 4-bar linkage consists of:

$$
\begin{aligned}
L_1 &= A_oB_o \\
L_2 &= A_oC_1 = A_oC_2 = A_oC_3 \\
L_3 &= C_1D_1 = C_2D_2 = C_3D_3 \\
L_4 &= B_oD_1 = B_oD_2 = B_oD_3 \\
CP &= C_1P_1 = C_2P_2 = C_3P_3 \\
DP &= D_1P_1 = D_2P_2 = D_3P_3
\end{aligned}
$$

11. Measure the three orientation angles for link 2 (θ_{21}, θ_{22}, and θ_{23}) so that you can verify that point P goes through the three prescribed points (P_1, P_2, and P_3).
12. Draw the mechanism in all three positions to see if it goes through the prescribed points P_1, P_2, and P_3. (Always check your work!)

Example 3.4 Design a 4-bar linkage whose coupler point goes through three specified locations

Problem: Design a 4-bar linkage whose input is driven by a motor and whose coupler point goes through three points located at $P_1 = (5.00'', 3.38'')$, $P_2 = (6.50'', 2.38'')$, and $P_3 = (6.68'', 1.25'')$.

Solution: The procedure is listed above. After locating the three points labeled P_1, P_2, and P_3 in your design area, select a location for bearing A_o. Try to pick a location that makes sense relative to the specified points. For this example, bearing A_o is located at $(1.00'', 0.75'')$ (see Figure 3.12).

Figure 3.13 shows the design after steps 3, 4, and 5 are completed. In the design 1, the change in link 2's angular position is very small when the coupler point moves from P_2 to P_3. This will cause a very quick change in location of the coupler and generate high inertia forces in the coupler link. Design 2 has the coupler link moving from P_1 to P_2 in about the same amount of time that it takes to move it from P_2 to P_3. Remember that link 2 is rotating at a constant rate. Design 2 is a better design since a quick motion between points P_2 and P_3 is not specified.

In step 6, we randomly create the size of the coupler link, link 3. There are an infinite number of solutions here so if the one you pick does not work well, come back to this step and try a different shape.

Next, we randomly select the length of link 3 from C_1 to D_1 and its orientation from the C_1P_1 line. Then duplicate this information to locate D_2 and D_3 (see Figure 3.14).

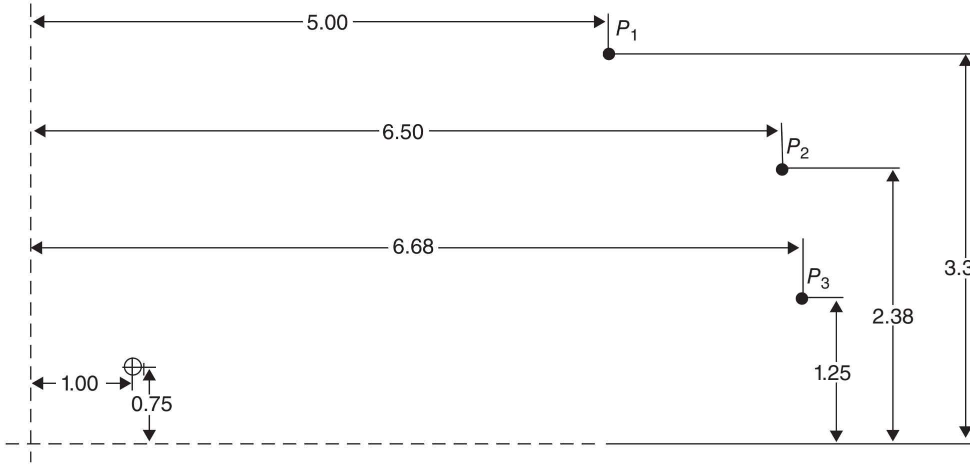

Figure 3.12 Three points and bearing A_o

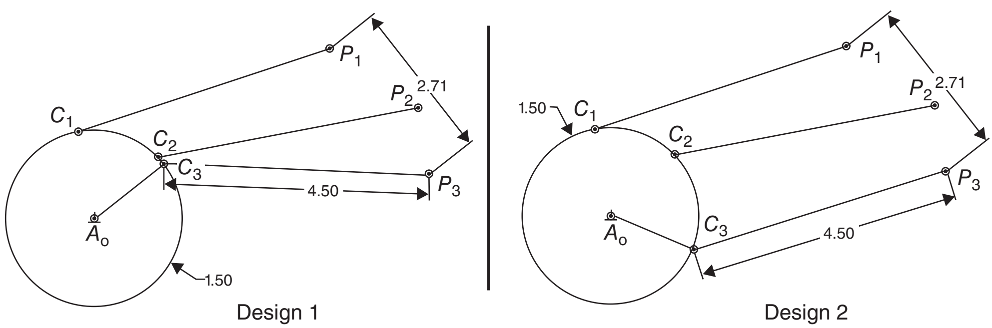

Figure 3.13 Two possible initial designs

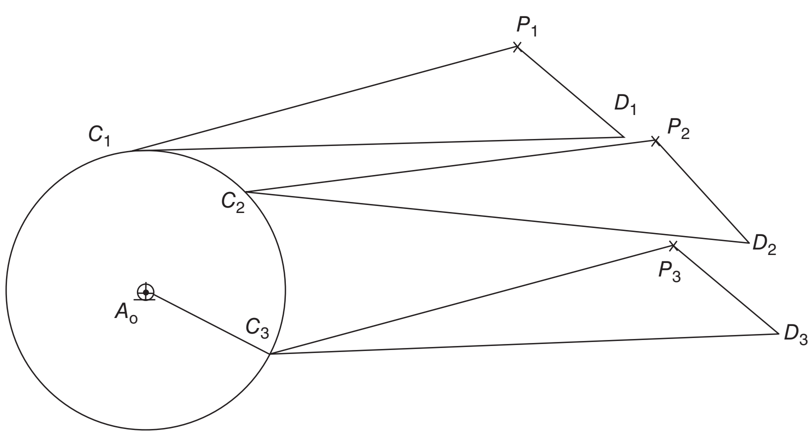

Figure 3.14 Adding the coupler link

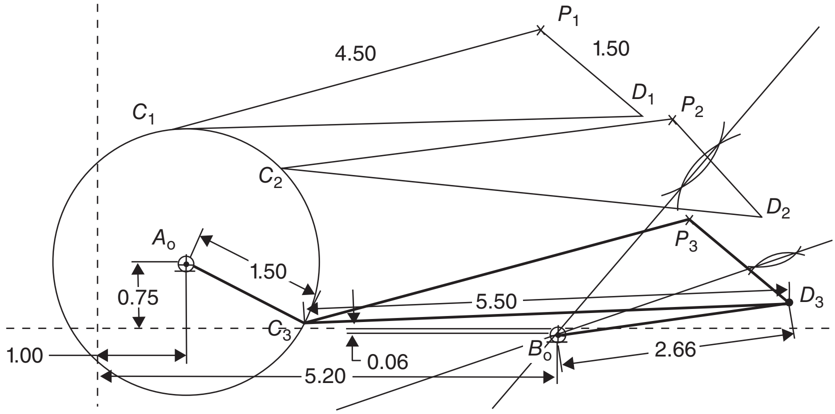

Figure 3.15 Final layout of linkage

By bisecting the imaginary lines between D_1 and D_2 and between D_2 and D_3, we can locate a point that is equal distance from all three "D" points. This is the location for bearing B_o. The final step in the layout is to determine the lengths of the links and the locations of the bearings (see Figure 3.15).

Answers: $\boxed{A_o = (1.00, 0.75)}$, $\boxed{B_o = (5.20, -0.06)}$, $\boxed{L_2 = 1.50}$, $\boxed{L_3 = 5.50}$, $\boxed{L_4 = 2.66}$, $\boxed{CP = 4.5}$, and $\boxed{DP = 1.5 \text{ in.}}$

Although the design layout is complete, there are still some issues to address. For example, the transmission angle between the coupler link, link 3, and the output link, link 4, is very small when the linkage is in the third position. This will cause excessive forces, thus this is a bad design. Checking Grashof's condition shows that the input link, link 2, cannot rotate through a complete 360° cycle, and thus a dyad needs to be added to force link 2 to oscillate between its two extreme positions C_1 and C_3.

$$L_1 = \sqrt{(B_{ox} - A_{ox})^2 + (B_{oy} - A_{oy})^2} = 4.25 \text{ in.}$$

$$\text{shortest + longest} < \text{other\_two\_links}$$

$$1.50 + 5.50 < 2.66 + 4.25 \text{ (NOT True!!)}$$

A second attempt at a linkage design leads to the following, Figure 3.16. Bearing $A_o = (1.80, 5.00)$, $B_o = (4.86, 1.54)$, $L_1 = 4.62$, $L_2 = 1.50$, $L_3 = 4.80$, $L_4 = 1.88$, $CP = 4.80$, and $DP = 0.00$. The minimum transmission angle is slightly less than 40°. Grashof's condition states that link 2 will rotate through 360°. This is an acceptable design. Note that point P was selected as one end of the coupler link, link 3. There are many other acceptable designs for this problem.

3.7 Coupler Point Goes Through Three Points with Fixed Pivots and Timing

There are cases where the coupler point must go through three specified points along with some prescribed timing. That is, the input link must move through 40° as the coupler point moves

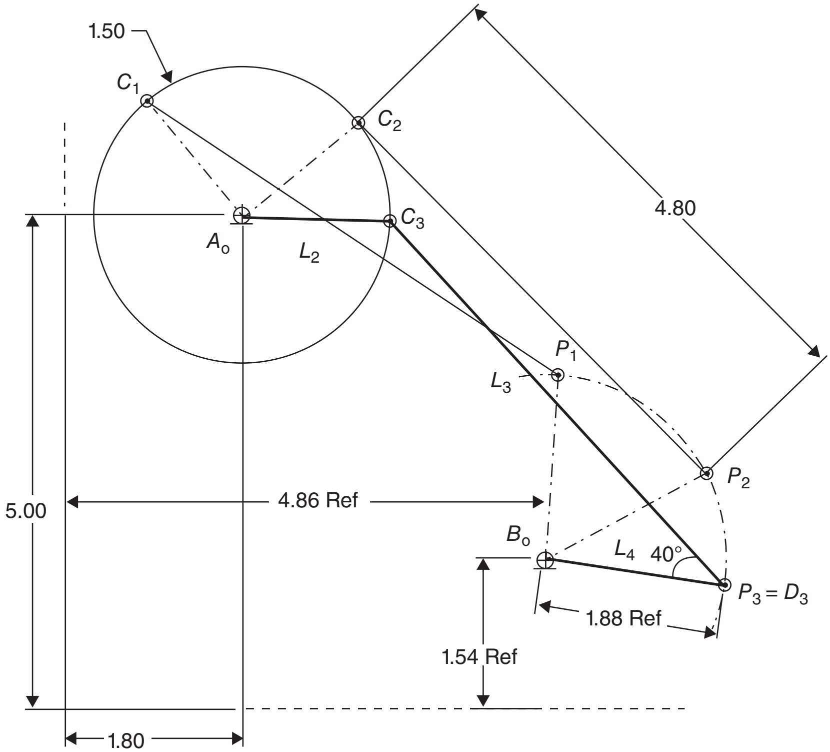

Figure 3.16 Acceptable design through three points

from point 1 to point 2 and through 90° as the coupler point moves from point 1 to point 3. Because of certain restrictions, the fixed bearings must be located in a specified location as well. If these conditions are present, then the following procedure can be used to design a 4-bar linkage that meets all the requirements. The procedure is outlined in the following example (see Figure 3.17).

Example 3.5 Design a 4-bar linkage whose coupler link goes through three specific points with prescribed timing and fixed pivot points

Problem: Design a 4-bar linkage whose coupler point goes through the following three points: $P_1 = (72, 147)$ mm, $P_2 = (128, 152)$ mm, and $P_3 = (163, 132)$ mm. In addition, the input crank must rotate through 40° clockwise as the coupler point moves from point 1 to point 2 and through 90° clockwise as the coupler point goes from point 1 to point 3. The bearing for the input crank must be located at the origin at (0, 0) mm. The output link's bearing, B_o, must be located approximately 172 mm and 10° above the horizontal from bearing A_o.

Solution: Procedure follows.

1. Locate the three prescribed positions (P_1, P_2, and P_3).
2. Locate the fixed pivot points, bearing A_o and B_o.
3. Draw a construction line from P_2 to A_o.

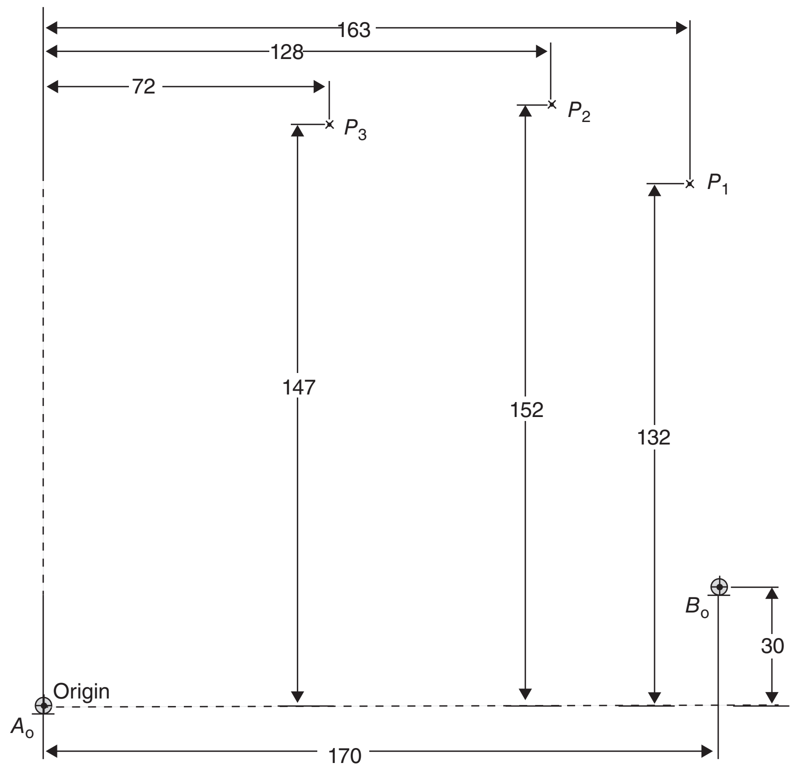

Figure 3.17 Three points and fixed pivots

4. We will be inverting the motion to fix the length of the unknown input link. Draw a second same length construction line rotated 40° counterclockwise (opposite direction from specification) about bearing A_o. Label its endpoint as P_2'.
5. Draw a construction line from P_3 to A_o.
6. Draw a second same length construction line rotated 90° counterclockwise (opposite direction from specification) about bearing A_o. Label its endpoint as P_3'.
7. Construct a perpendicular bisector between P_1 and P_2'.
8. Construct a perpendicular bisector between P_1 and P_3'.
9. The intersection of these two bisectors is the end of the crank, link 2. Label this point as C_1. The distance between C_1 and A_o is link 2's length. Draw a circle around bearing A_o to represent the path of point C (see Figure 3.18).
10. With the distance between C_1 and P_1 fixed, locate C_2 the same distance from P_2, and C_3 the same distance from P_3. Note that C_3 can be in two different locations on the circle and be the correct length. However, only one of those locations is 90° from link 2 in its original position at C_1 (see Figure 3.19).
11. Now we are ready to deal with bearing B_o.
12. The position of D is found by means of kinematic inversion. This is done by fixing the coupler link in position 1. The rest of the mechanism including the frame must move so that the same relative motion exists between all links in this inversion as well as the

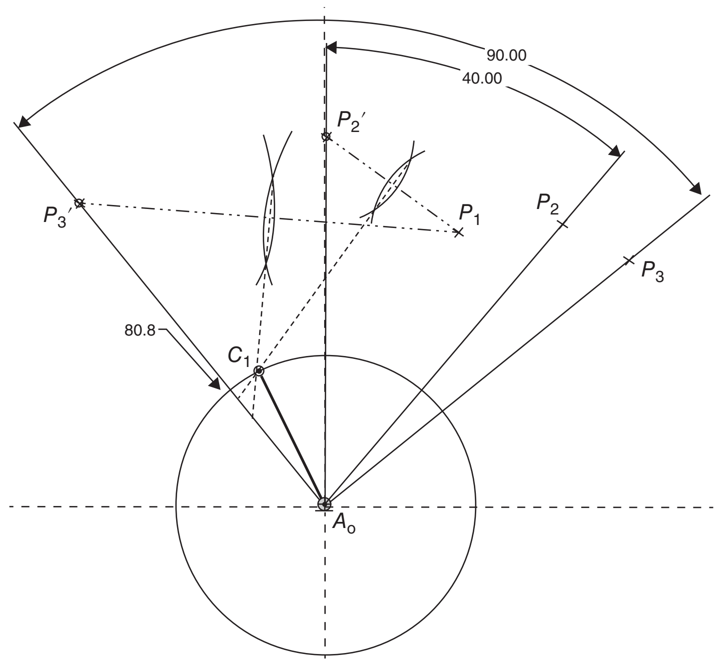

Figure 3.18 Locate end of crank link, C_1

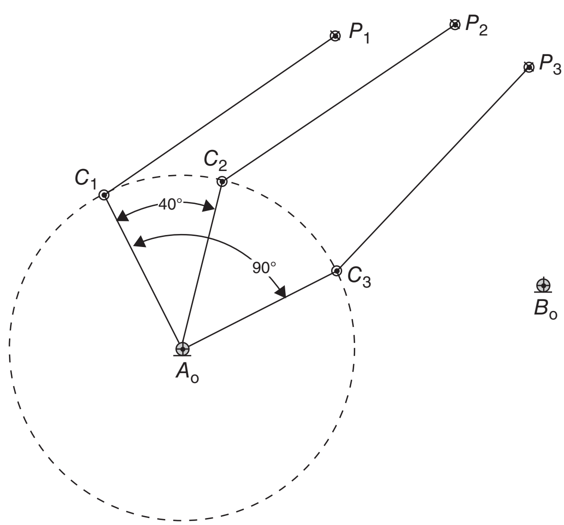

Figure 3.19 Crank in its three required positions

original configuration. The relative positions of B_o with respect to position 1 of the coupler are obtained by construction.

13. Rotate A_o about C_1 by $(\beta_2 - \beta_1)$ where $\beta_1 = \angle A_oC_1P_1$ and $\beta_2 = \angle A_oC_2P_2$. Label the endpoint A_o'.
14. Draw an arc around A_o' with a radius equal to the length of link 1, or $\overline{A_oB_o}$.
15. Draw an arc around P_1 with a radius equal to $\overline{P_2B_o}$. The intersection of these two arcs is the location of B_o'.
16. Rotate A_o about C_1 by $(\beta_3 - \beta_1)$ where $\beta_1 = \angle A_oC_1P_1$ and $\beta_3 = \angle A_oC_3P_3$. Label the endpoint A_o''.
17. Draw an arc around A_o'' with a radius equal to the length of link 1, or $\overline{A_oB_o}$.
18. Draw an arc around P_1 with a radius equal to $\overline{P_3B_o}$. The intersection of these two arcs is the location of B_o''.
19. Construct perpendicular bisector to the lines B_oB_o' and $B_o'B_o''$. The intersection locates D_1, which is the other end of the coupler link.
20. Draw the linkage in all three positions to check your design. If the design is not acceptable, these steps can be repeated with a different location for bearings A_o and B_o.

Answers: $\boxed{\text{All graphical designs will be different}}$.

3.8 Two-Position Synthesis of Slider-Crank Mechanism

The inline slider-crank mechanism shown in Figure 3.20 has a stroke equal to twice the crank length. It has a forward stroke time equal to its reverse stroke time for a constant rotational speed of the crank. The extreme positions D_1 and D_2 are also known as the limiting positions. In general, the connecting link, link 3, must be larger than the crank, link 2. Good design practices assume the connecting link to three to four times the crank length. Shorter lengths of the connecting link gives rise to higher velocity and acceleration vectors for link 3.

In Figure 3.20, the desired stroke is 5.00 in. and therefore the crank is 2.50 in. and the connecting link is 7.50 in.

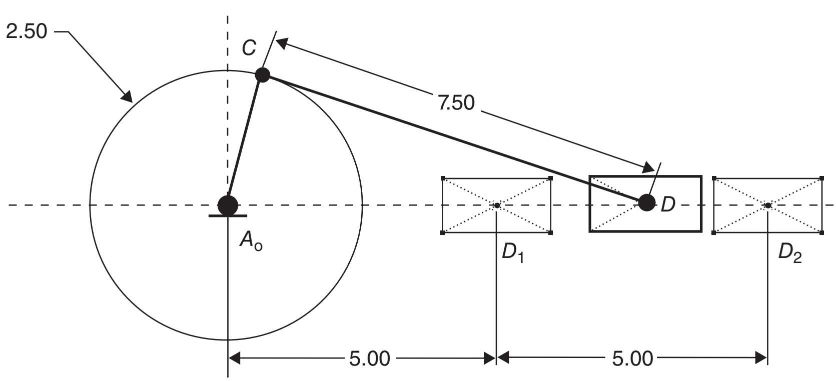

Figure 3.20 Inline slider-crank mechanism

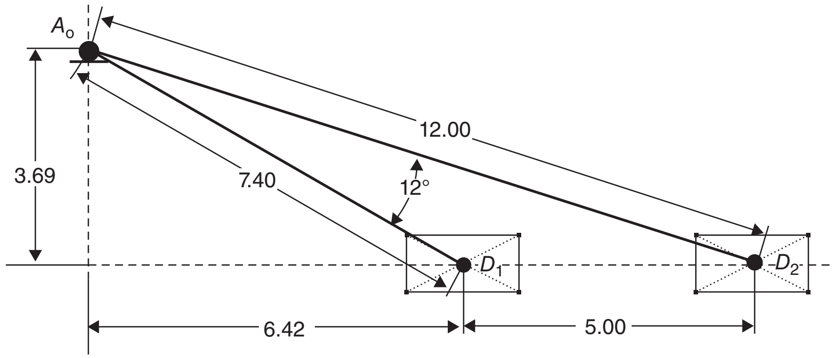

Figure 3.21 Offset slider-crank mechanism

The offset slider-crank mechanism shown in Figure 3.21 has a different time for its forward and reverse strokes if the crank is rotating at a constant speed. This feature can be used to synthesis a quick return mechanism where a slower working stroke is desired. In addition, the stroke D_1D_2 is always greater than twice the crank length. Once again, the connecting link should be greater than three times the crank length.

The offset slider-crank in Figure 3.21 will create a forward motion of the slider for 0.08 s and a reverse motion for 0.07 s if the crank makes a complete rotation in 0.15 s. The slider's stroke was specified as 5.00 in. The crank will be (12.00 − 7.40)/2 = 2.30 in. The connecting link will be (12.00 − 2.30) = 9.70 in. The crank's bearing will be 3.69 in. above the line of action of the slider and 6.42 in. to the left of the extreme left position of the slider.

Example 3.6 Design a slider-crank mechanism that will move the slider to the right slower than it moves it to the left

Problem: Design a slider-crank mechanism that will move the slider to the right in 1.1 s and return in approximately 0.90 s with a stroke of 6.00 in.

Solution: The procedure follows.

1. Determine the timing ratio and then the connector link's delta angle between the two extreme limits of the slider.

$$Q = \frac{\text{slow\_time}}{\text{fast\_time}} = \frac{1.1}{0.9} = 1.222$$

$$\beta = \left(\frac{Q-1}{Q+1}\right)\cdot 180^\circ = \left(\frac{1.222-1}{1.222+1}\right)\cdot 180^\circ = 18.0^\circ$$

2. Locate the two extreme positions of the slider and label them D_1 and D_2.
3. Draw a random slanted line starting at D_1.
4. Draw a line through D_2 that has the correct delta angle calculated in step 1. The intersection of these two lines is the location of the fixed bearing, A_o. If you do not like the location of A_o, then go back to step 3 and repeat.
5. Measure the distance between A_o and D_1.

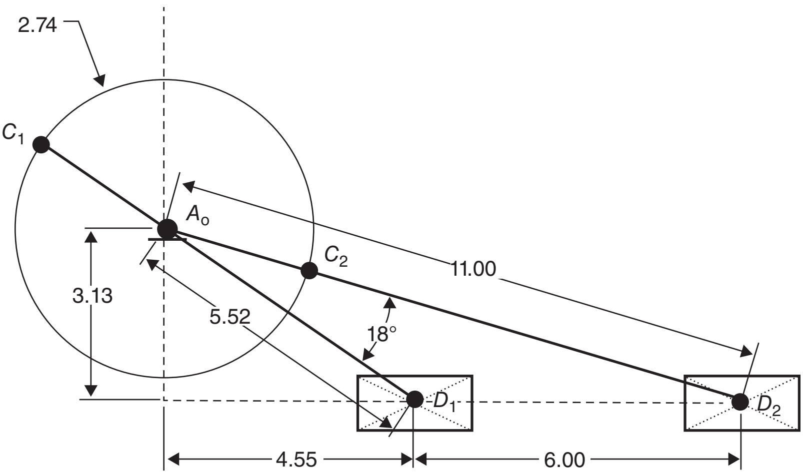

Figure 3.22 Offset slider-crank design

6. Measure the distance between A_o and D_2.
7. Calculate the crank length and the connecting link's length. If you do not like the values generated, then go back to step 3 and repeat.
8. Locate the A_o relative to one of the extreme positions of the slider (see Figure 3.22).
9. Construct the slider-crank mechanism and verify the design.

$$L_2 = \frac{11.00 - 5.52}{2} = 2.74''$$
$$L_3 = 11.00 - 2.74 = 8.26''(>3L_2)$$

Answers: $\boxed{L_1 = 3.13''}$, $\boxed{L_2 = \text{crank} = 2.74''}$, $\boxed{L_3 = \text{connecting link} = 8.26''}$, and $\boxed{X_{\text{AoD1}} = 4.55 \text{ in.}}$

3.9 Designing a Crank-shaper Mechanism

The crank-shaping mechanism is used to machine flat metal surfaces especially where a large amount of metal has to be removed. The reciprocating motion of the mechanism inside the crank-shaping mechanism can be seen in Figure 3.23. As the input link (disc) rotates, the top of the machine moves forward and backward, pushing a cutting tool. The cutting tool removes the material from the work piece, which is held firmly.

The crank-shaping machine is a simple and yet extremely effective mechanism. It is used to remove material, usually steel or aluminum, to produce a flat surface. However, it can also be used to manufacture gear racks and other complex shapes.

Figure 3.24 shows the kinematic diagram for a crank-shaper. Link 2 is the input crank and link 6 is the tool cutter.

Procedure assuming the slider, link 6, moves horizontally:

1. Draw the slider in its two extreme positions and label them F_1 and F_2.

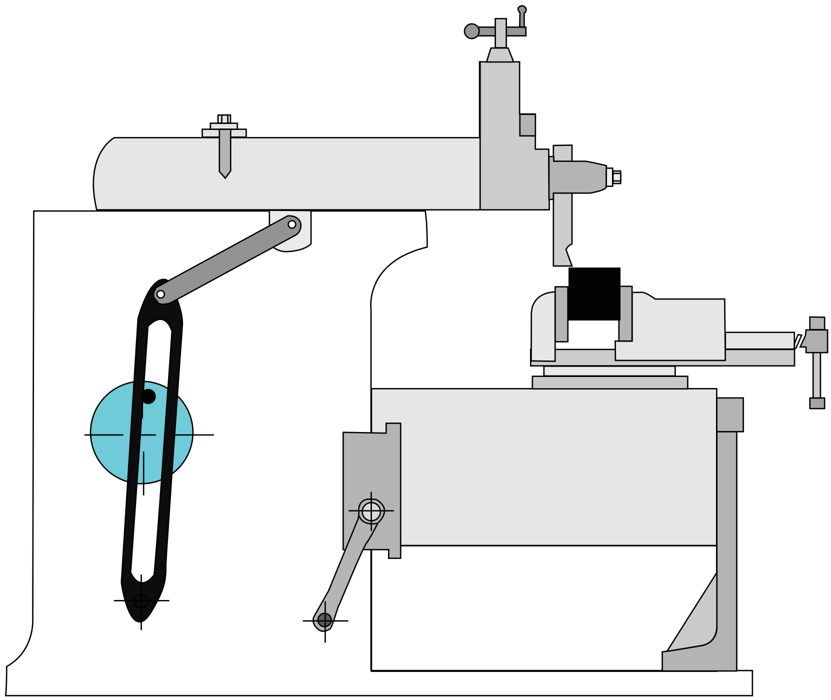

Figure 3.23 Crank-shaper machine

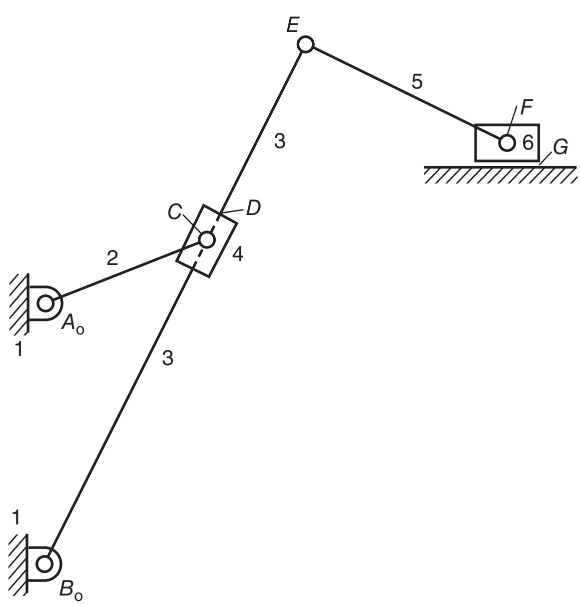

Figure 3.24 Kinematic diagram for crank-shaper

2. Draw link 5 at some angle and pick its length. Label its endpoints E_1 and E_2. Maximum force can be transmitted to the slider, link 6, if link 5 is nearly horizontal in this step.
3. Calculate the timing ratio, Q, as before.

$$Q = \frac{\text{Slower stroke time}}{\text{Faster stroke time}} \geq 1.00$$

4. Calculate the sweep angle for link 3.

$$\beta = \left(\frac{Q-1}{Q+1}\right) 180^\circ$$

5. Draw a line going through E_1 at an angle of $(-90^\circ - \tfrac{1}{2}\beta)$.
6. Draw a line going through E_2 at an angle of $(-90^\circ + \tfrac{1}{2}\beta)$.
7. To check your work, draw a perpendicular bisector through the imaginary line between E_1 and E_2. It should intersect at the same location where steps 5 and 6 above meet. This intersection point is fixed bearing B_0.
8. Locate on this vertical perpendicular bisector a point where you want to place fixed bearing A_o. The distance from B_o to A_o must be greater than the length of the crank, link 2. If it is not, move farther up the perpendicular bisector when locating A_o.
9. Calculate the crank-shaper mechanism link sizes (see Figure 3.25).

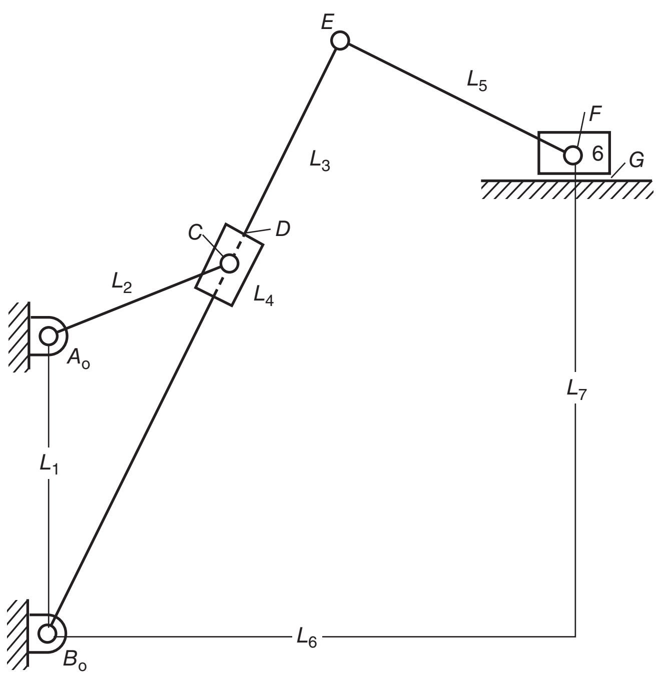

Figure 3.25 Crank-shaper lengths

$$L_1 = \overline{A_o B_o} L_2 = L_1 \sin\left(\frac{1}{2}\beta\right)$$

$$L_3 = B_o E_1 = B_o E_2$$

$$L_4 = B_o C$$

$$L_5 = E_1 F_1 = E_2 F_2$$

L_6 = horizontal distance from B_o to F

L_7 = vertical distance from B_o to F

Example 3.7 Design a crank-shaper with a specified timing ratio and stroke length

Problem: Design a crank-shaper with a timing ratio of 1.2 and stroke length of 150 mm. The slower stroke takes 0.140 s as the slider moves from left to right. What is the input crank speed? What are the link lengths as defined in the procedure above? Assume the slider, link 6, moves horizontal.

Solution:

1. Draw the slider in its two extreme positions and label them F_1 and F_2.
2. Draw link 5, 5° from horizontal, and pick its length to be 200 mm. Label the endpoints E_1 and E_2.
3. Timing ratio was given as 1.20.

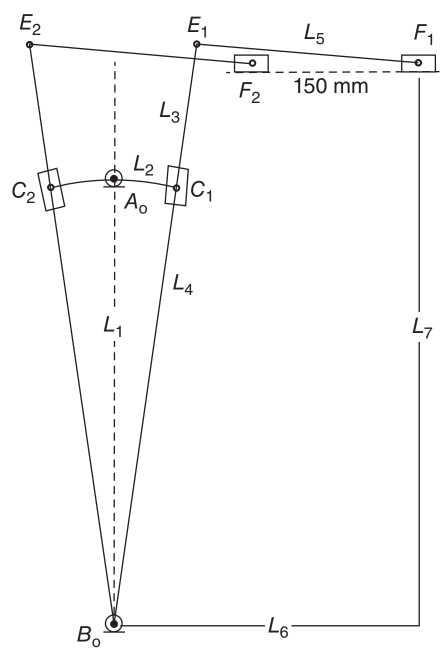

Figure 3.26 Designed crank-shaper mechanism

4. Calculate the sweep angle for link 3. $B = 16.4°$.
5. Draw a line going through E_1 at an angle of $-98.2°$.
6. Draw a line going through E_2 at an angle of $-81.8°$.
7. Construct perpendicular bisector to check your accuracy, thus locating fixed bearing B_o.
8. Locate A_o on the perpendicular bisector.
9. Determine all of the vector lengths for the crank-shaper (see Figure 3.26).

Slow stroke goes through 196.4° and fast stroke goes through 163.6°, thus $Q = 1.2$.
Complete cycle happens in 0.2566 s, thus the $\boxed{\text{crank rotates at 234 rpm.c.w.}}$

Answers: $\boxed{L_7 = 503 \text{ mm}}$, $\boxed{L_5 = 200 \text{ mm}}$, $\boxed{L_3 = 526 \text{ mm}}$, $\boxed{L_1 = 400 \text{ mm}}$, and $\boxed{L_2 = 57 \text{ mm}}$.

Problems

3.1 Design a 4-bar linkage that will cause the 3.00-in. output link to oscillate through 72° with equal forward and backward motion.

3.2 Design a 4-bar linkage that will cause the 80-mm output link to oscillate through 72° with equal forward and backward motion.

3.3 Design a 4-bar linkage that will cause the 2.50-in. output link to oscillate through 60°. The clockwise motion should take 0.5 s while the counterclockwise motion should take 0.6 s. What is the required angular velocity vector of the input crank?

3.4 Design a 4-bar linkage that will cause the 50-mm output link to oscillate through 60°. The clockwise motion should take 1.2 s while the counterclockwise motion should take 1.0 s. What is the required angular velocity vector of the input crank?

3.5 Design a 4-bar linkage that will cause the 3.50-in. output link to oscillate through 48°. The clockwise motion should take 0.4 s while the counterclockwise motion should take 0.5 s. What is the required angular velocity vector of the input crank?

3.6 Design a 4-bar linkage that will cause the 90-mm output link to oscillate through 48°. The clockwise motion should take 0.5 s while the counterclockwise motion should take 0.4 s. What is the required angular velocity vector of the input crank?

3.7 The output link must oscillate between the two positions shown in Figure 3.27. Determine the required pivot point, and then add a dyad so a motor with constant rotation can move this link between its two extreme positions with a cycle time of 0.75 s.

3.8 The output link must oscillate between the two positions shown in Figure 3.28. Determine the required pivot point and then add a dyad so a motor with constant rotation can move this link between its two extreme positions with a cycle time of 0.80 s.

3.9 Design a 4-bar linkage that will move the coupler link through the three positions shown in Figure 3.29. Can an electric continuously rotating motor drive this linkage or must a dyad be added to this linkage? The coupler link must travel through the three positions shown, but it does not have to be limited to this range.

3.10 Design a 4-bar linkage that will move the coupler link through the three positions shown in Figure 3.30. Can this linkage be driven by an electric continuously-rotating motor or must a dyad be added to this linkage? The coupler link must travel through the three positions shown, but it does not have to be limited to this range.

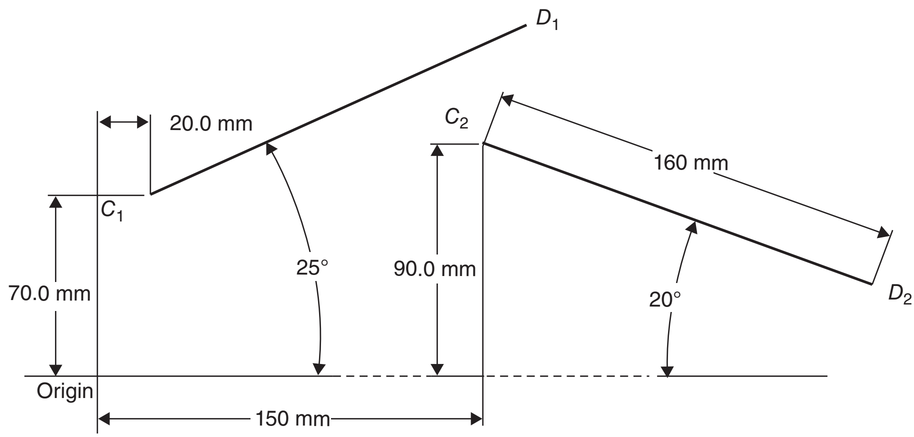

Figure 3.27 Problem 3.7

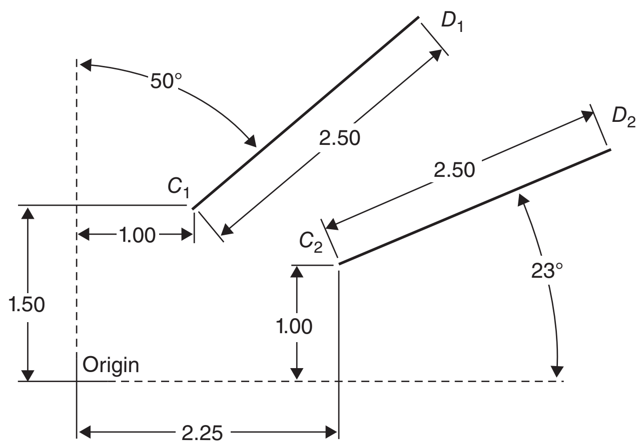

Figure 3.28 Problem 3.8

3.11 Design a 4-bar linkage that will move the coupler link through the three positions shown in Figure 3.31. Can this linkage be driven by an electric continuously-rotating motor or must a dyad be added to this linkage? The coupler link must travel through the three locations shown, but it does not have to be limited to this range.

3.12 Design a 4-bar linkage that will move the coupler link through the three positions shown in Figure 3.32. Can this linkage be driven by an electric continuously-rotating motor or must a dyad be added to this linkage? The coupler link must travel through the three locations, but it does not have to be limited to this range.

3.13 Design a 4-bar linkage that will move the coupler link through the three positions shown in Figure 3.33. Can this linkage be driven by an electric continuously-rotating motor or must a dyad be added to this linkage? The coupler link must travel through the three locations shown, but it does not have to be limited to this range.

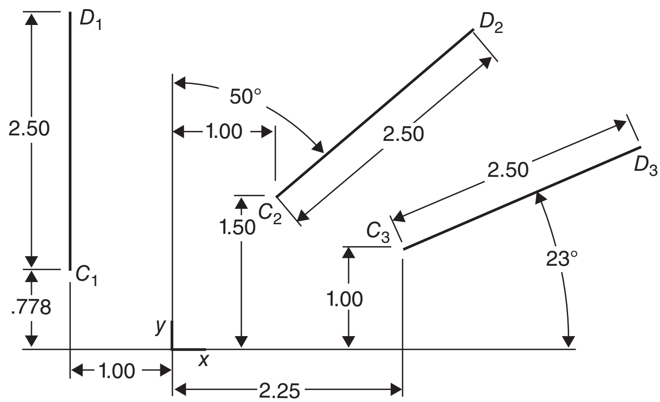

Figure 3.29 Problem 3.9

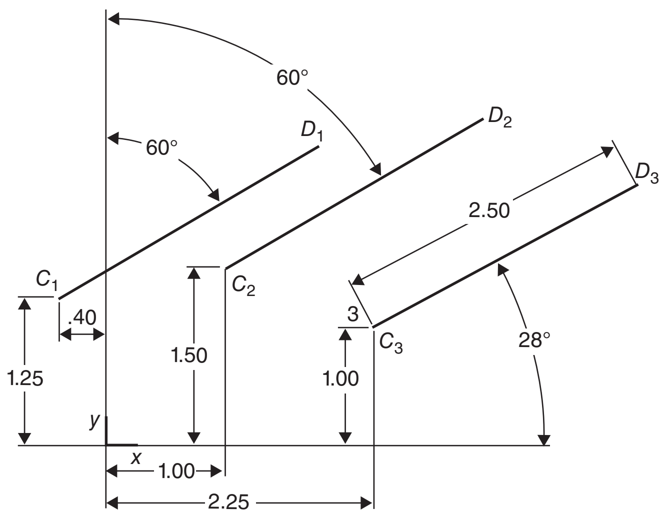

Figure 3.30 Problem 3.10

3.14 Design a 4-bar linkage that will move the coupler link through the three positions shown in Figure 3.34. Can this linkage be driven by an electric continuously-rotating motor or must a dyad be added to this linkage? The coupler link must travel through the three locations shown, but it does not have to be limited to this range.

3.15 Design a slider-crank mechanism with equal forward and reverse strokes and having a stroke length of 4.00 in.

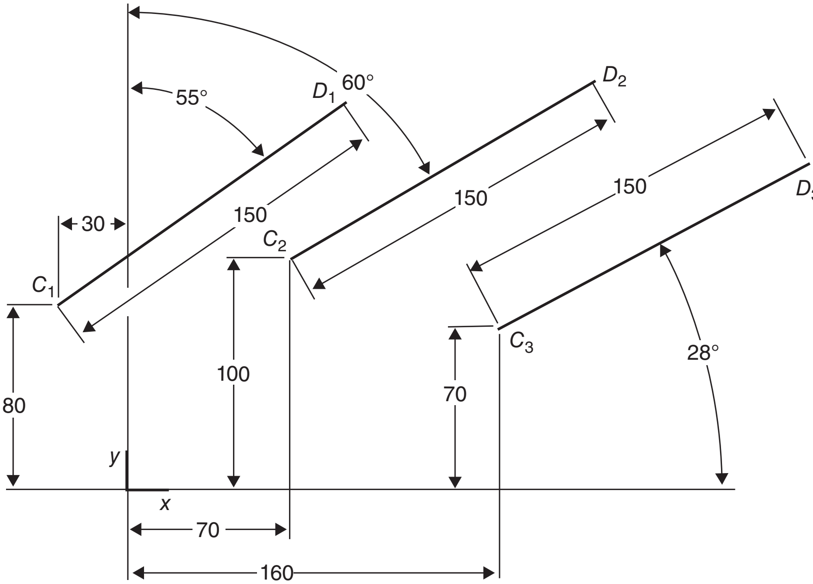

Figure 3.31 Problem 3.11

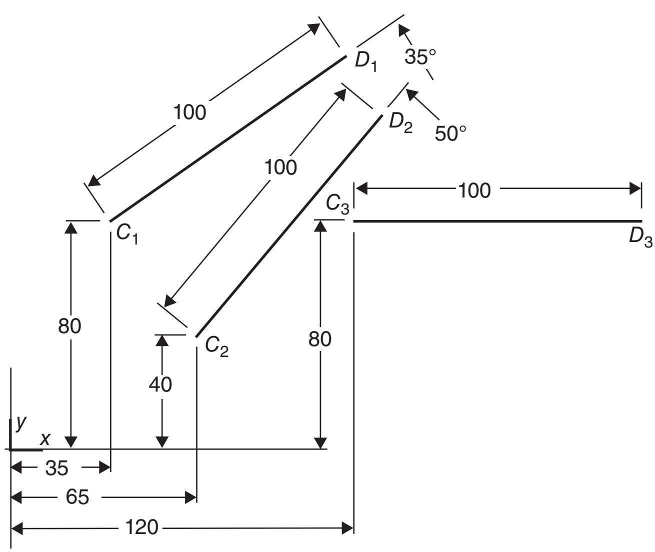

Figure 3.32 Problem 3.12

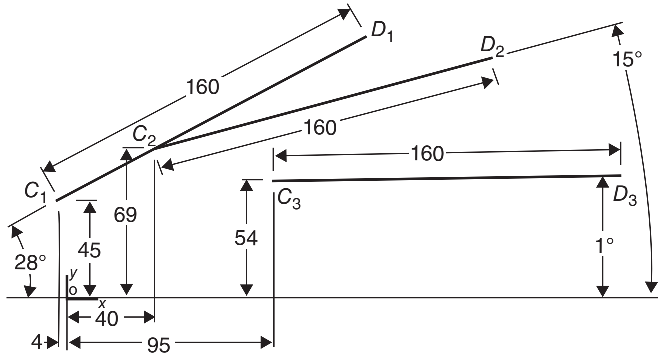

Figure 3.33 Problem 3.13

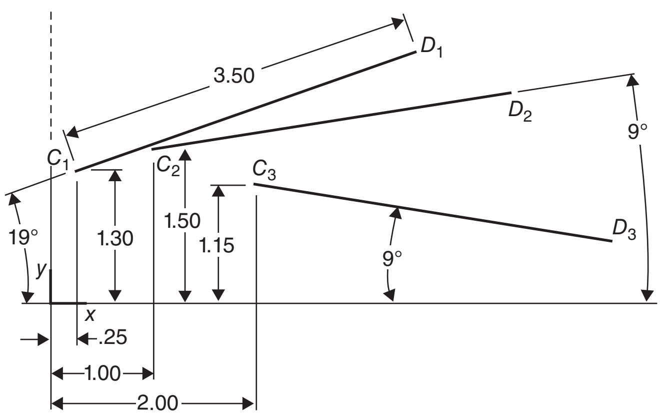

Figure 3.34 Problem 3.14

3.16 Design a slider-crank mechanism with equal forward and reverse strokes and having a stroke length of 150 mm.

3.17 Design an offset slider-crank mechanism with a stroke length of 4.00 in. and a timing ratio of 1.15.

3.18 Design an offset slider-crank mechanism with a stroke length of 150 mm and a timing ratio of 1.20.

3.19 Design a 4-bar linkage that goes through three precision points with a fixed pivot for link 2 at the origin. $P_1 = (5.5, 85)$ mm, $P_2 = (24, 91)$ mm, and $P_3 = (68, 85)$ mm. Let link 2 be equal to 50 mm. After designing the 4-bar, be sure to define all variables shown in Figure 3.35. What are the three values for θ_2 so that the 4-bar's point P is located at P_1, P_2, and P_3?

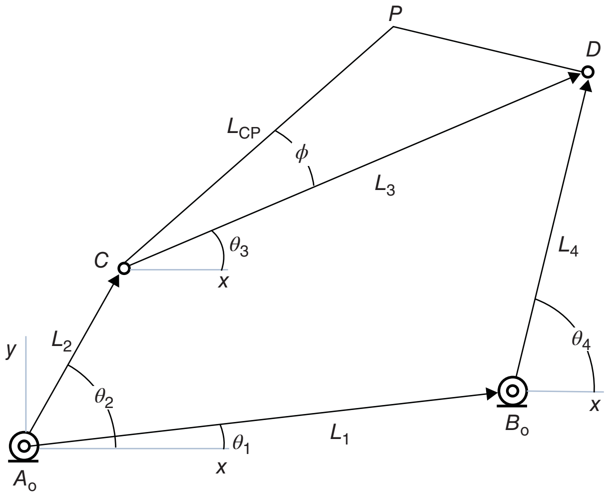

Figure 3.35 Problems 3.19 and 3.23 concept

3.20 Design a 4-bar linkage that goes through three precision points with a fixed pivot for link 2 at the origin. $P_1 = (0.0, 3^1/_8)$ inches, $P_2 = (2.0, 3¾)$ inches, and $P_3 = (3.0, 3^3/_8)$ inches. Let link 2 be equal to 2.5 in. After designing the 4-bar, be sure to define all variables shown in Figure 3.35. What are the three values for θ_2 so that the 4-bar's point P is located at P_1, P_2, and P_3?

3.21 Design a 4-bar linkage that goes through the three precision points. The locations for the fixed bearing A_o and B_o can be anywhere. $P_1 = (0, 55)$ mm, $P_2 = (20, 65)$ mm, and $P_3 = (40, 75)$ mm. After designing the 4-bar, be sure to define all variables shown in Figure 3.35.What are the three values for θ_2 so that the 4-bar's point P is located at P_1, P_2, and P_3?

3.22 Design a 4-bar linkage that goes through the three precision points. The locations for the fixed bearing A_o and B_o can be anywhere. $P_1 = (1, 3½)$ inches, $P_2 = (2½, 4)$ inches, and $P_3 = (4, 3¾)$ inches. After designing the 4-bar, be sure to define all variables shown in Figure 3.35.What are the three values for θ_2 so that the 4-bar's point P is located at P_1, P_2, and P_3?

3.23 Design a crank-shaper mechanism with a timing ratio of 1.5 and a stroke length of 4.00 in. (see Figure 3.36).

3.24 Design a crank-shaper mechanism with a timing ratio of 1.4 and a stroke length of 100 mm (see Figure 3.36).

3.25 Design a crank-shaper mechanism with the slower stroke going left to right and taking 0.25 s. The return stroke takes 0.15 s. Its stroke length must be 6.00 in. What is the speed and direction of the input crank? (see Figure 3.36).

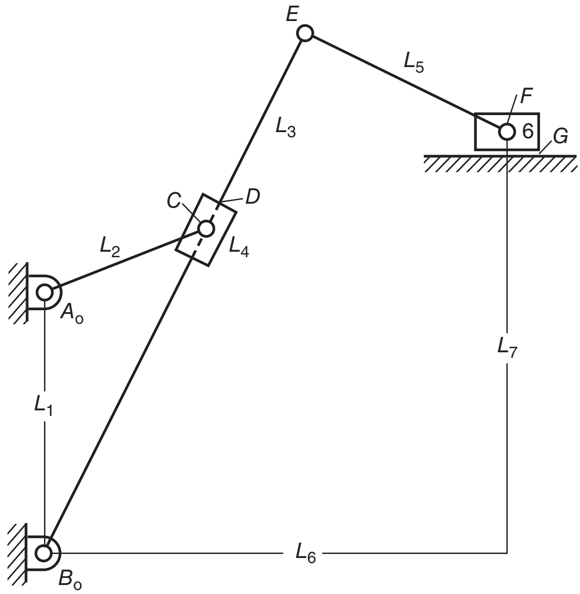

Figure 3.36 Crank-shaper lengths