

Mechanism Synthesis

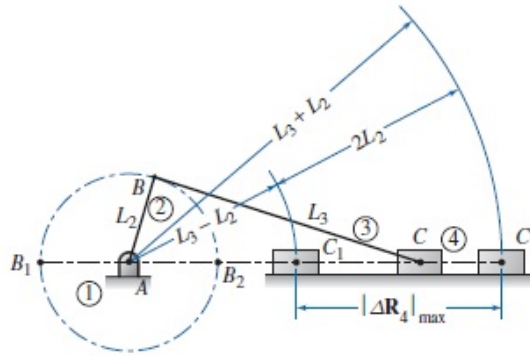
Introduction:

Mechanism synthesis is the procedure by which upon identification of the desired motion a specific mechanism (synthesis type), and appropriate dimensions of the linkages are identified (dimensional synthesis). Mechanism synthesis can be viewed as a two-fold iterative process, on one end the dimensions of the linkages are selected based on kinematic and geometrical constraints, on another end dynamical consideration which affects the mechanical efficiency of the mechanism is also taken into account. Mechanism synthesis helps in achieving this goal in fewer steps. Several procedures were devised for selected mechanisms to help designers in the synthesis process, this includes the slider crank, four-bar, crank shaper, two-point and three-point mechanism which will be discussed in this chapter.

Design of a slider-crank mechanism

In-line slider crank mechanism

The kinematic diagram of an in-line slider crank mechanism is shown in the figure below, the imbalance angle is ($\beta = 0$) and the time ratio is ($Q = 1$), as the crank rotates with constant angular velocity it takes the same amount of time to execute the forward/backward motions, we say that the in-line slider crank mechanism is a balanced linkage.



Design problem: Determine the appropriate lengths L_2 and L_3 of the crank and coupler respectively to achieve the desired stroke $|\Delta R_4|_{max}$

Mechanism Synthesis:

From the kinematic diagram in the figure we conclude that the center of the crank rotation is on the constrained path of the slider and that

$$|\Delta R_4|_{max} = 2 L_2$$

L_3 can take on any value since it does not affect the stroke, however a shorter connecting arm yields greater velocities and accelerations for the slider, therefore its length should be as large as possible. A

detailed acceleration analysis should be completed to determine the inertial load (in a future chapter), however a rule of thumb where the connecting arm is no less than three times the crank length is preferred. Inertial forces increase the risk for permanent damage as the slider oscillates between the extreme positions.

Example:

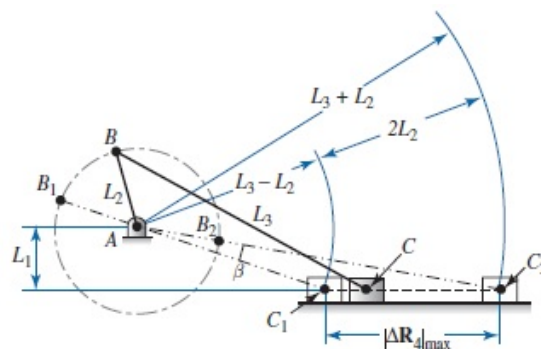
Design a slider crank mechanism with a time ratio $Q = 1.0$, a stroke of $|\Delta R|_{max} = 2.0 \text{ in}$ and time per cycle $t = 1.2 \text{ s}$,

- Specify the lengths L_2 , L_3 , and the crank constant speed, with the condition that the coupler length is no less than three times the length of the crank
- What is the range of the transmission angle
- Can you optimize your mechanism to achieve higher force transmission if so how

Offset slider crank mechanism

Because of the offset between the constrained path of the slider and the axis of rotation of the crank, an imbalance angle ($\beta > 0$) exists; accordingly the time it takes to complete the forward stroke is different than that of the backward stroke. From the kinematic diagrams depicted below we conclude the following relation

$$L_1 < L_3 - L_2$$



Design problem: Determine the appropriate lengths L_1 , L_2 and L_3 to achieve the desired stroke $|\Delta R_4|_{max}$ and imbalance angle β

Mechanism Synthesis: The dimensions of the different links can be identified using one of two ways, the first is geometrical, the second is analytical

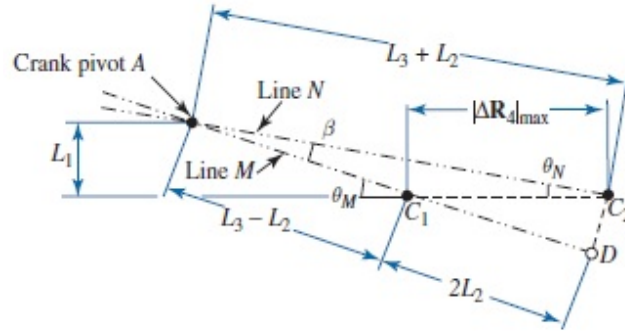
a. Geometrical Synthesis:

- Locate the axis of the pin joint on the sliding link. This joint is labeled as point C in the figure above
- Draw the extreme positions of the sliding link, separated by the stroke, $|\Delta R_4|_{max}$.

3. At one of the extreme positions, construct any line M through the sliding link pin joint, inclined at an angle θ_M . This point is labeled C_1 in the figure below.
4. At the other extreme position, draw a line N through the sliding link pin joint, labeled C_2 in the figure below and inclined at an angle β from line M. Note that $\theta_N = \theta_M - \beta$.
5. The intersection of lines M and N defines the pivot point for the crank, point A. The offset distance, L_1 , can be scaled from the construction. From the figure we also conclude

$$L_2 = \frac{1}{2}(AC_2 - AC_1)$$

$$L_3 = \frac{1}{2}(AC_2 + AC_1)$$



b. Analytical Synthesis:

Analytical synthesis makes use of trigonometry of the geometrical construction thus developed from which the following relations are derived

$$L_1 = |\Delta R_4|_{max} \left[\frac{\sin \theta_M \sin(\theta_M - \beta)}{\sin \beta} \right]$$

$$L_2 = |\Delta R_4|_{max} \left[\frac{\sin \theta_M - \sin(\theta_M - \beta)}{2 \sin \beta} \right]$$

$$L_3 = |\Delta R_4|_{max} \left[\frac{\sin \theta_M + \sin(\theta_M - \beta)}{2 \sin \beta} \right]$$

The lengths of the different links depend on a choice of the angle (θ_M), in general the design that produces the longest connecting arm is preferred since it is associated with the lowest acceleration of the slider. A detailed acceleration analysis should be completed to determine the inertial load (in a future chapter), however a rule of thumb the connecting arm should be at least three times greater than the length of the crank. When large forces are required to push the slider, transmission angles (as described in the previous set of notes) of value close to 90° are preferred during the entire rotation of the crank.

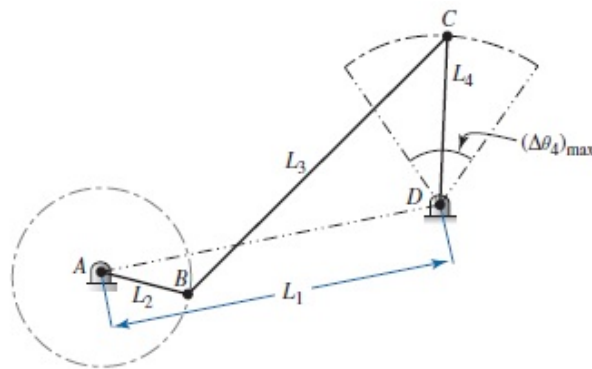
Example:

Design a slider crank mechanism with a time ratio $Q = 1.10$, a stroke of $|\Delta R|_{max} = 0.625 \text{ in}$ and time per cycle $t = 0.033 \text{ s}$,

- d. Specify the lengths L_2 , L_3 , L_1 and the crank speed, with the condition that L_3 is no less than three times the length of the crank
- e. What is the range of the transmission angle
- f. Can you optimize your mechanism to achieve higher force transmission if so how

Four-bar crank rocker mechanism

A four-bar crank rocker mechanism uses the continuous rotation of a motor at the input to produce a limited angular displacement of the follower as depicted in the diagram below.



Design problem: Identify the lengths L_1 , L_2 , L_3 , L_4 , given the angular stroke $(\Delta\theta)_{max}$ and imbalance angle β

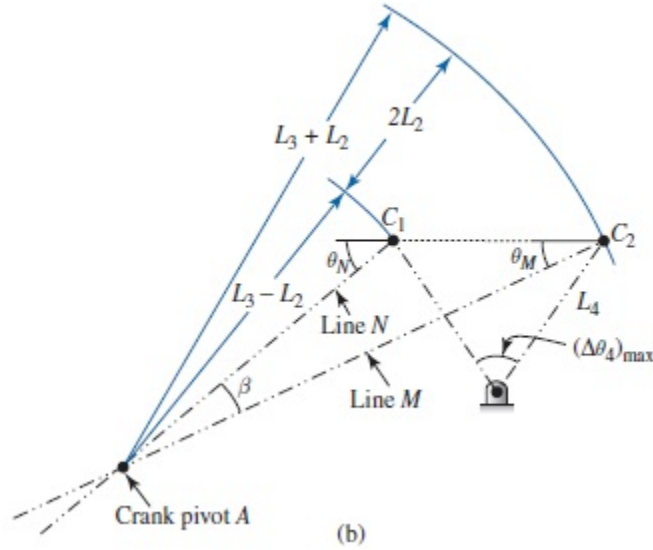
Mechanism Synthesis: The dimensions of the different links can be identified using one of two methods, the first is geometrical, the second is analytical

a. Geometrical Synthesis:

1. Locate the pivot of the rocker link
2. Choose any feasible rocker length, L_4 (this length is typically constrained by the spatial allowance for the mechanism).
3. Draw the two positions of the rocker, separated by the throw angle, $(\Delta\theta)_{max}$.
4. At one of the extreme positions, construct any line M through the end of the rocker link, inclined at an angle θ_M (this point is labeled C_2 in figure).
5. At the other extreme position, draw a line N through the end of the rocker link, which is inclined at angle β from line M.
6. The intersection of lines M and N defines the pivot point for the crank, point A.
7. If the motion of the mechanism is balanced ($\beta = 0$), lines M and N are collinear and the pivot point of the crank can be chosen anywhere along that line.
8. The length between the two pivots, L_1 , can be scaled from the construction, we also conclude that

$$L_2 = \frac{1}{2}(AC_2 - AC_1)$$

$$L_3 = \frac{1}{2}(AC_2 + AC_1)$$



b. Analytical Synthesis:

The following relations can be derived from the laws of sine and cosine to find the lengths of the different links

$$L_3 = \frac{L_4 \sin\left(\frac{(\Delta\theta)_{max}}{2}\right)}{\sin\beta} [\sin\theta_M + \sin(\theta_M + \beta)]$$

$$L_2 = \frac{L_4 \sin\left(\frac{(\Delta\theta)_{max}}{2}\right)}{\sin\beta} [-\sin\theta_M + \sin(\theta_M + \beta)]$$

$$L_1 = \sqrt{L_4^2 + (L_3 - L_2)^2 + 2 L_4 (L_3 - L_2) \sin\left(\beta + \theta_M - \frac{(\Delta\theta)_{max}}{2}\right)}$$

The lengths of the different links depend on a choice of the angle (θ_M) . In general the design that produces the longest coupler is preferred since it is associated with the lowest follower accelerations. A detailed acceleration analysis should be completed to determine the inertial load (in a future chapter), large inertial forces can induce damage to the mechanism. The transmission angle affects the force transmission to the coupler especially when larger forces are transmitted, the closer the transmission angle to 90° the better.

In some cases one of the lengths must be a specific size, most commonly L_1 , in a case such as this the length of the other links can be scaled to attain the desired length.

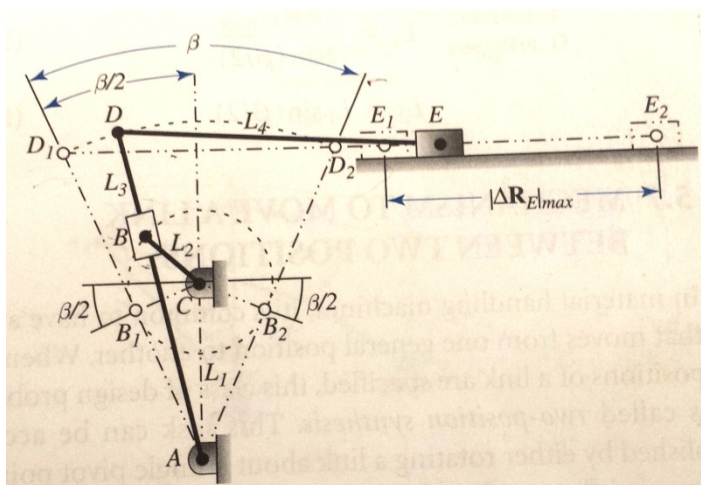
Example:

Design a crank rocker crank mechanism with a time ratio $Q = 1.08$, a stroke of $(\Delta\theta_4)_{max} = 88^\circ$, time per cycle $t = 0.75\text{ s}$, and $L_1 = 100\text{ mm}$

- Specify the lengths L_2 , L_3 , L_4 and the crank speed
- What is the range of the transmission angle

Crank shaper mechanism

The kinematic diagram of a crank shaper mechanism is shown in the figure below; this mechanism is capable of achieving higher time ratios



Design problem: Identify the lengths L_1 , L_2 , L_3 , and L_4 given the linear stroke $|\Delta R_4|_{max}$ and imbalance angle β

Mechanism Synthesis: The dimensions of the different links can be identified using one of two methods, the first is geometrical, the second is analytical

a. Geometrical Synthesis:

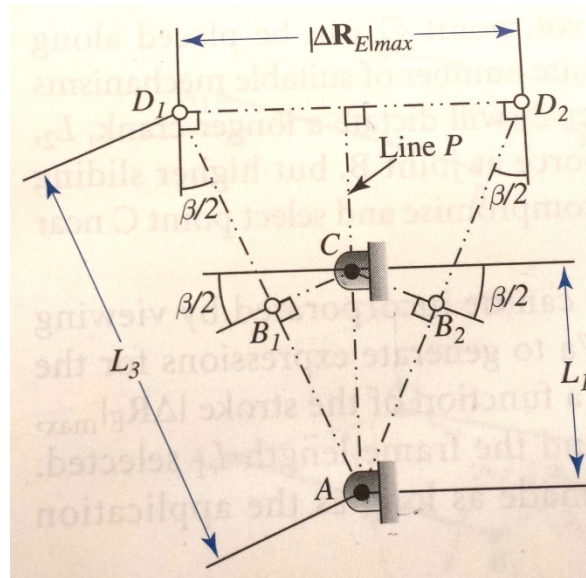
- Construct a line whose length is equal to the desired stroke, the end points are labeled D_1 and D_2 in the figure below.
- Construct a line from D_1 and another from D_2 at an angle equal to $\beta/2$
- The intersection of the two lines locates the rocker pivot, labeled A in the figure. The line between A and D_1 or between A and D_2 represents the rocker and its length is L_3
- Draw a line perpendicular to the line connecting D_1 and D_2 through A , the crank pivot (C) can be placed anywhere along this line. The distance between points A and C represents the frame and its length is L_1 .
- Draw a line perpendicular to AD_1 or AD_2 and passing through the crank pivot C , this represents the crank and its length is L_2 .
- The length L_4 , can take an appropriate value to fit the application. Note that longer length will reduce the maximum acceleration of the slider.

b. Analytical Synthesis:

The following relations can be derived from the laws of sine and cosine to find the lengths of the different links

$$L_3 = \frac{|\Delta R_E|_{max}}{2 \sin(\beta/2)}$$

$$L_2 = L_1 \sin(\beta/2)$$



The lengths of the different links depend on a choice of L1, a larger value of L1 will lead to larger values of the crank and for the same angular velocity of the motor, the speed and acceleration of slider at B will increase which is not preferred. The transmission angle affects the force transmission to the coupler especially when larger forces are transmitted, the closer the transmission angle to 90° the better.

Example:

Design a crank shaper mechanism with a time ratio $Q = 1.150$, a stroke of $|\Delta R_E|_{max} = 2.75 \text{ in}$ and time per cycle $t = 0.6 \text{ s}$,

- Specify the lengths L2, L3, L1 and the crank speed, with the condition that L3 is no less than three times the length of the crank
- What is the range of variation of the transmission angle

Mechanism to move a link between two positions

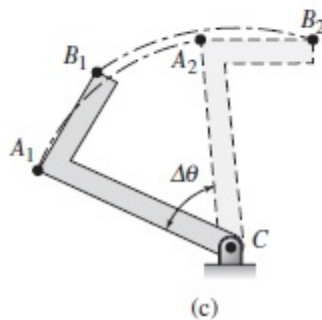
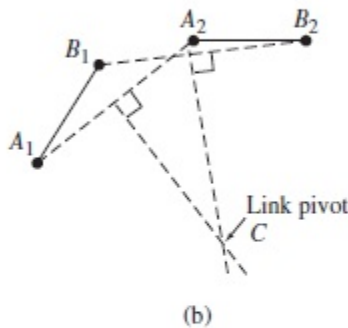
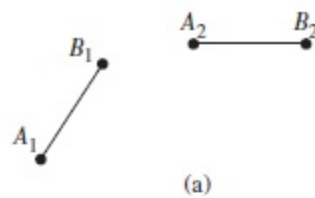
Some applications require a link to move between two fixed positions, when these positions are specified the design problem is termed two point synthesis, it can be achieved using a single pivot or by using the coupler in a four bar mechanism.

Tow point synthesis using a pivot

Design problem: Identify the location of the pivot and angle of rotation given the initial and final locations of two points on the link

a. Geometrical Synthesis:

1. Draw the lines that connect the initial and final locations of each point
2. Construct the perpendicular bisectors to these lines
3. The intersection is the location of the pivot
4. The angle between the lines connecting the pivot and the end locations of one of the points is the angle of rotation.



Example

The coordinates of the end points are A₁: (0, 9 in), A₂: (6.36, 6.36 in), B₁: (5, 9 in), B₂: (9.9005, 2.8295 in).

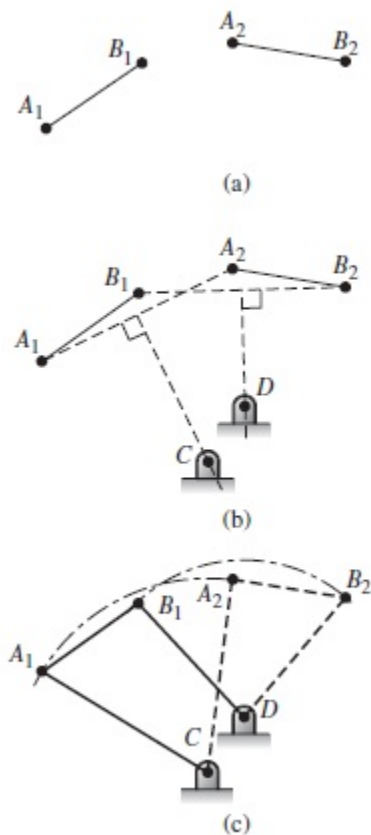
Tow point synthesis using a four bar mechanism

Design problem: Identify the location of the pivots and the lengths of all four links

Geometrical Synthesis:

5. Draw the lines that connect the initial and final locations of each point
6. Construct the perpendicular bisectors to these lines
7. The pivots can be placed anywhere on the perpendicular bisectors
8. The lengths can then be measured graphically

Note that longer pivoting lengths will rotate at smaller angles, this produces larger transmission angles and reduces the forces required to drive the linkage.



Mechanism to move a link between three positions

Some applications require a link to move between three fixed positions, when these positions are specified the design problem is termed three point synthesis and can be achieved using the coupler in a four bar mechanism.

Design problem: Identify the location of the pivots and the lengths of all four links

Geometrical Synthesis:

1. Draw the lines that connect the initial and final locations of each point
2. Construct the perpendicular bisectors to these lines
3. The intersections of two of the perpendicular bisectors are the pivot locations.
4. The lengths can then be measured graphically

