The Dynamic Characteristic Analysis of Four-bar Linkages with Joint Clearances Based on the Computer Simulation

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Abstract

Considering the influence of mechanism with joint clearance meanwhile based on the collision-hinge model, this paper established four-bar linkages dynamic equations of mechanism with three joint clearances. Then the hybrid contact model considering joint clearance was incorporated into the mechanical dynamics analysis software ADAMS. Targeting the four-bar linkages, this paper built up dynamic models which contains clearance and also set up dynamics simulation which analyzing how clearance affects the motion deviation of mechanical output. This simulation could predict how joint clearance affects dynamic characteristics of mechanism, and thus provide reference and basis for mechanical design and research.

Keywords: *Four-Bar Linkages, Joint Clearance, Computer Simulation*

1. Introduction

In most cases, clearances occur in the joints of the mechanism due primarily to manufacturing tolerances, material deformations and wearing after a certain working period. There will generate collision since every movable joints exists clearance which causes losing contact during the operation of the mechanism. The clearance will lead to deviating from ideal movements of the mechanism, meanwhile reducing the accuracy of mechanical motion, arousing concussion dynamic load easily thus impact system load transferring and tend to cause damage and failure of joints. With the development of precision mechanical engineering and aerospace engineering, the requirement on accurate prediction of system dynamics behavior becomes more and more urgent. At present, some designers have been focused on studying how joint clearance affects different mechanical systems. Flores etc [1] established the calculation model of joint with clearance, which analyzed and calculated a slider-crank mechanism with a joint clearance between piston and connecting rod. Tsai and Lai [2] considered existing joint clearance of planar linkage mechanism by adopting closed-loop method to derive position vector equation of each component and analyses the position parameter and transmission performance. Guo huixin etc [3] put forward a robustness design method of planar linkage mechanism considering the influence of joint clearance，it analyzed the eccentric distance of rotary pair and the impact of planar mechanical position caused by joint, described the planar mechanical position considering joint clearance by using position vector equation.

At present, the continuous contact hypothesis of two components is commonly used in the research of joint, while, compared with that, the hybrid contact model take into account the internal contact surface physics and geometrical characteristics of joint, thus, it’s more accurate to simulate the impact and friction between two components and describe mechanical movement with clearance. Based on hybrid contact model, this paper established four-bar linkage dynamics equations with three joint clearances and analyzed how the joint clearance affects the dynamic characteristics of the mechanism by means of ADAMS to simulate[4] the dynamic characteristics of mechanism.

2. The model of joint with clearance

The pin of joint in the hole is not restricted until two components contact each other thus generates collision because of the mechanism joint clearance. As Figure 1 shows, clearance r_{ci} is represented by the distance between the rotary center of the hole and the pin, pointing to the potential contact points of two components during relative motion. The size of clearance, which is limited within the circle that the center of the pin is the circle center and the radius difference between the pin and the hole, can reflect whether these two joints are contacted [5].

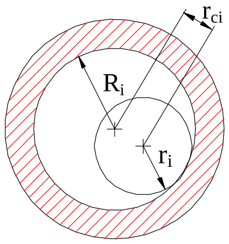

Figure 1 . Revolute joint with clearance

The clearance r_{ci} is represented by the hole radius R_i and the roller radius r_i,

$$r_{ci} \le R_i - r_i \tag{1}$$

Dubowsky etc. [6] proposed a rotary clearance collision model to describe the mechanical model of the joint with clearance, and divided the movement of a pin relative to a hole into "free motion" and "contact deformation" these two conditions. The contact collision model of the pin and the hole is given by

$$F_{\mathrm{n}} = K_{\mathrm{n}}\delta_{\mathrm{n}}^{\mathrm{m}} + C_{\mathrm{n}}\delta_{\mathrm{n}}\dot{\delta}_{\mathrm{n}} \tag{2}$$

In formula (2), F_{n} consists of two parts, the first part represents elastic deformation force of the collision process and the second part represents the energy lost in the collision process, while K_{n} is equivalent contact stiffness, C_{n} is damping coefficient and δ_{n} is negative element normal deformation.

Tangential friction model can be used as rotating clearance hinge tangential contact force model [7], this paper only considers the condition of dry friction rather than lubrication case. By using modified coulomb mechanical model, friction coefficient is a dynamic value rather than a constant, and this is a function of tangential slip velocity. The calculation formulas of tangential contact force and dynamic friction coefficient are expressed as:

$$F_t = -\mu(v_t)F_n\frac{v_t}{|v_t|} \tag{3}$$

$$\mu(v_t) = \begin{cases} -\mu_d sign(v_t) & |v_t| > v_d \\ -\left\{\mu_d + (\mu_s - \mu_d)\left(\dfrac{|v_t| - v_s}{v_d - v_s}\right)^2\left[3 - 2\dfrac{|v_t| - v_s}{v_d - v_s}\right]\right\} sign(v_t) & v_s \le v_s \le v_d \\ \mu_s - 2\mu_s\left(\dfrac{v_t + v_s}{2v_s}\right)\left(3 - \dfrac{v_t + v_s}{v_s}\right) & |v_t| < v_s \end{cases} \tag{4}$$

Among which, v_t is the the speed tangential component of collision contact point between shaft and hole, μ_d is kinetic friction coefficient, μ_s is static friction coefficient, v_s is critical velocity of static friction and v_d is the critical velocity of the biggest dynamic friction.

The function curve of dynamic friction coefficient is shown in Figure 2.

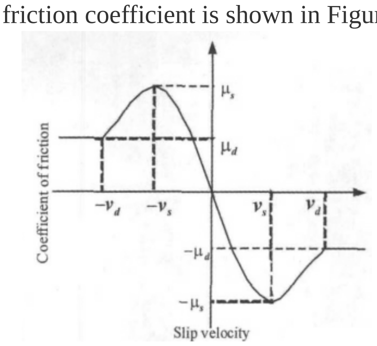

Figure 2 . Relation curve between coefficient of friction and slip velocity

3. The dynamics model of four-bar linkage with joint clearance

Figure 3 shows a four-bar linkage with three joint clearances that mainly composed by crank, connecting rod, rocker and frame. Where O, A, B and C are each link point, while link point A, B and C include clearance. L_1, L_2 and L_3 represent the length of crank, connecting rod and rocker, respectively. m_i and J_i represent each component's quality and the rotational inertia around the mass center. G_1, G_2 and G_3 represent mass center position of each component. θ_1, θ_2 and θ_3 are the angles between crank, connecting rod, rocker and forward x axis. L_4 is the length of the frame.

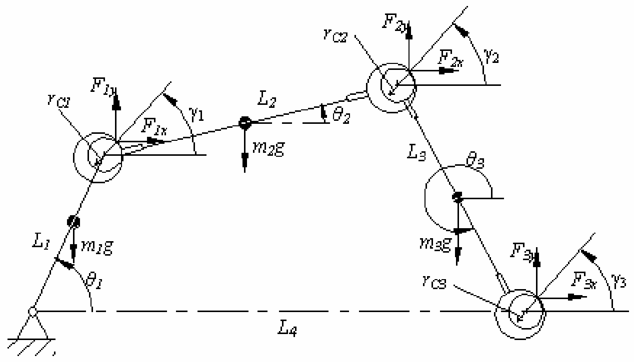

Figure 3 . Schematic diagram of four-bar linkage

Kinematical equation of the mechanism in ideal state is shown as

$$\begin{cases} L_1 \sin\theta_1 + L_2 \sin\theta_2 - L_3 \sin\theta_3 = 0 \\ L_1 \cos\theta_1 + L_2 \cos\theta_2 - L_3 \cos\theta_3 - L_4 = 0 \end{cases} \tag{5}$$

The centroid of each component can be expressed as

$$\begin{aligned} &\begin{bmatrix} x_{G_1} \\ y_{G_1} \end{bmatrix} = \frac{L_1}{2}\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix} \\ &\begin{bmatrix} x_{G_2} \\ y_{G_2} \end{bmatrix} = L_1\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix} + \frac{L_2}{2}\begin{bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{bmatrix} \\ &\begin{bmatrix} x_{G_2} \\ y_{G_2} \end{bmatrix} = L_1\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix} + L_2\begin{bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{bmatrix} + \frac{L_3}{2}\begin{bmatrix} \cos\theta_3 \\ \sin\theta_3 \end{bmatrix} \end{aligned} \tag{6}$$

The i-th joint clearance is r_{ci}, where i=1, 2, 3, when consider the mechanism include clearance. γ_i is the azimuth. the projection $r_{ci\cdot x}$ and $r_{ci\cdot y}$ of clearance vector in direction x and y are shown as

$$\begin{cases} r_{c1,x} = L_1 \cos\theta_1 + L_2 \cos\theta_2/2 - x_{G_2} \\ r_{c1,y} = L_1 \sin\theta_1 + L_2 \sin\theta_2/2 - y_{G_2} \\ r_{c2,x} = x_{G_2} + L_2 \cos\theta_2/2 - L_3 \cos\theta_3/2 - x_{G_3} \\ r_{c2,y} = y_{G_2} + L_2 \sin\theta_2/2 - L_3 \sin\theta_3/2 - y_{G_3} \\ r_{c3,x} = x_{G_3} - L_4 - L_3 \cos\theta_3/2 \\ r_{c3,y} = y_{G_3} - L_3 \sin\theta_3/2 \end{cases} \tag{7}$$

Azimuth γ_i can be expressed as

$$\gamma_i = \arctan\left(r_{ci,y}/r_{ci,x}\right) \qquad \left(i = 1,2,3\right) \tag{8}$$

The force components of the pin with clearance to the hole depend on direction x and y can be expressed by

$$\begin{cases} F_{ix} = F_{in}\cos\gamma_i + F_{it}\sin\gamma_i \\ F_{iy} = F_{in}\sin\gamma_i - F_{it}\cos\gamma_i \end{cases} \qquad (i = 1,2,3) \tag{9}$$

Negative elements normal deformation when joints are in collision are given by

$$\delta_i = \begin{cases} r_{ci} - r_{\mathrm{m}} & r_{ci} > r_{\mathrm{m}} \\ 0 & r_{ci} \le r_{\mathrm{m}} \end{cases} \qquad (i = 1,2,3) \tag{10}$$

According to Fig.2, this paper establishes a kinetics equation of four-bar linkage with clearance is as crack：

$$\begin{aligned} &M_c + L_1\left(F_{1x}\sin\theta_1 - F_{1y}\cos\theta_1\right) + r_1\left(F_{1x}\sin\gamma_1 - F_{1y}\cos\gamma_1\right) \\ &-m_1 g L_1\cos\theta_1/2 = J_c\ddot{\theta}_1 \end{aligned}$$

connecting rod：

$$\begin{cases} F_{1x} - F_{2x} = m_2\ddot{x}_{G_2} \\ F_{1y} - F_{2y} - m_2 g = m_2\ddot{y}_{G_2} \\ r_2\left(F_{2x}\sin\gamma_2 - F_{2y}\cos\gamma_2\right) + L_2/2\left[\left(F_{1x}+F_{2x}\right)\sin\theta_2 - \left(F_{1y}+F_{2y}\right)\cos\theta_2\right] \\ \quad - \left(r_1 + r_{c1}\right)\left(F_{1x}\sin\gamma_1 - F_{1y}\cos\gamma_1\right) = J_{G_2}\ddot{\theta}_2 \end{cases}$$

rocker：

$$\begin{cases} F_{2x} - F_{3x} = m_3\ddot{x}_{G_3} \\ F_{2y} - F_{3y} - m_3 g = m_3\ddot{y}_{G_3} \\ r_3\left(F_{3x}\sin\gamma_3 - F_{3y}\cos\gamma_3\right) + L_3/2\left[\left(F_{2x}+F_{3x}\right)\sin\theta_3 - \left(F_{2y}+F_{3y}\right)\cos\theta_3\right] \\ \quad - \left(r_2 + r_{c2}\right)\left(F_{2x}\sin\gamma_2 - F_{2y}\cos\gamma_2\right) = J_{G_3}\ddot{\theta}_3 \end{cases}$$

Where M_c is the driving moment effect on the crack, J_c is the rotational inertia of the crack around rotation center, dynamic differential equations of four-bar linkage with clearance can be obtained by all the above equations [8].

4. Dynamic characteristic analysis of four-bar linkage with joint clearance

Based on joint models and kinetic equation of four-bar linkage with clearance established above, the hybrid contact model considering joint clearance was incorporated into the mechanical dynamics analysis software ADAMS to generate the kinetic model[9] of the mechanism of crack and rocker with clearance, meanwhile, the dynamic characteristics of the mechanism with clearance was analyzed[10].

The mechanism of crack and rocker can achieve the conversion and/or transfer power between the motion form of rotation and swing. The variable parameters of the angle, angular velocity, angular acceleration, the displacement deviation of the follower (connecting rod, rocker), and the dynamic characteristic of every components, those are very necessary while designing new machinery and understanding the existing mechanical transmission performance, and also can provide essential basis on studying motion performance and dynamic performance.

Virtual prototype model shown in Figure 4 is established in ADAMS[11], parameters are: L_1=150 mm, L_2 =403.1 mm, L_3=450 mm, L_4=500 mm, m_1 = 0.6967 kg，J_1 =1.931×10^{-3} kg·m^2，m_2 = 1.5473 kg，J_2=2.233×10^{-4} kg·m^2，m_3 =1.7964 kg，J_2=3.431×10^{-4} kg·m^2, the radius of three clearances of the shaft and the hole are 7.5 mm and 8 mm, respectively. Contact constraint is defined between shaft and hole of joint clearance.

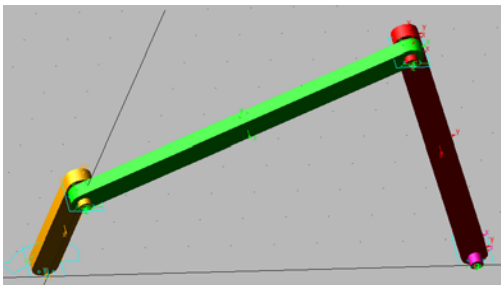

Figure 4 . Four-bar linkage model with clearances

For studying how joint clearances affect the kinematic characteristics of the mechanism, the dynamic simulation for mechanism with clearance and ideal mechanism without clearance is carried out, define the clearance rci= 0.5 mm.

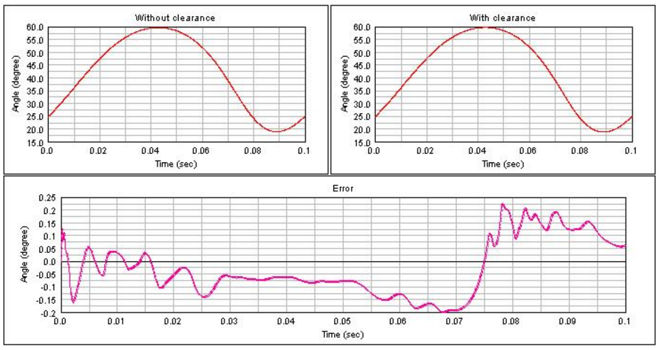

Figure 5 . The curve and the deviation curve of rocker angle in two conditions

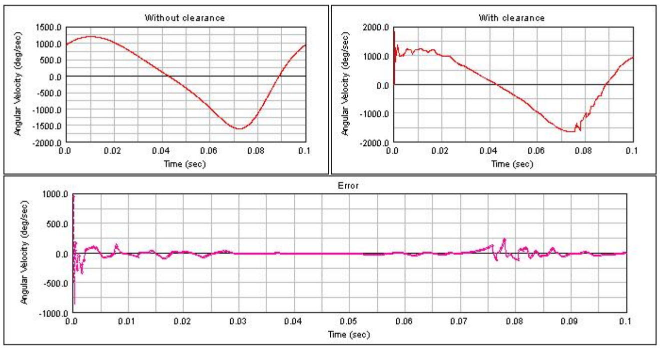

Figure 6 . The curve and the deviation curve of rocker angle velocity in two conditions

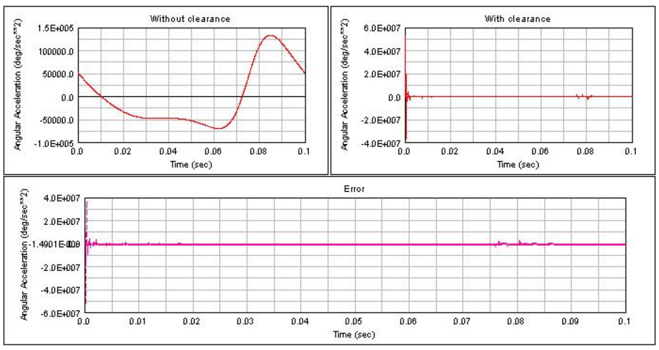

Figure 7 . The curve and the deviation curve of rocker angle acceleration in two conditions

Figure 5, Figure 6 and Figure 7 show the kinematics characteristic curve of the rocker. The upper-left view of each figure is the motion curve of ideal mechanism without joint clearance, while the upper-right view is the motion curve of mechanism with joint clearance, and underside view is the deviation curve. From Figure 5, the angle of rocker with clearance and the ideal mechanism curve are both smooth and have little difference between −0. 197°~0. 228°. From Figure 6, when considering joint clearance, the angular velocity of rocker compared with the ideal mechanism without clearance appear small fluctuation in overall trend, and the biggest deviation appears in the incipient stage. From Figure 7, the angular acceleration of rocker of real mechanism is quite smooth, whereas, it has large deviation when considering joint clearance and it will generate great influence to kinematic accuracy of the mechanism.

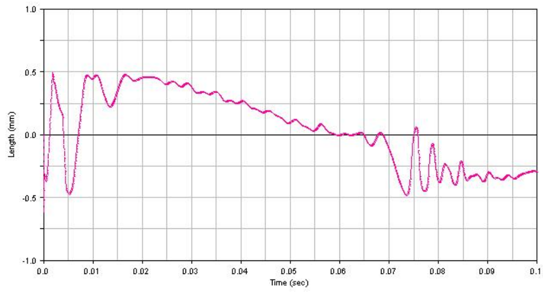

Figure 8 . Displacement deviation curve in direction x of clearance joint A

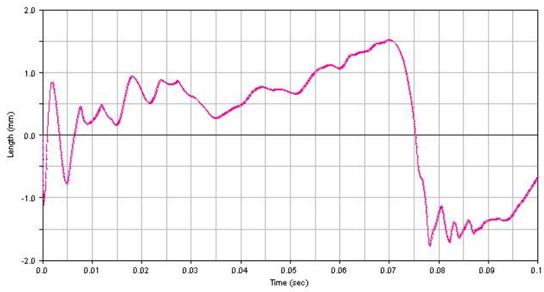

Figure 9 . Displacement deviation curve in direction x of clearance joint B

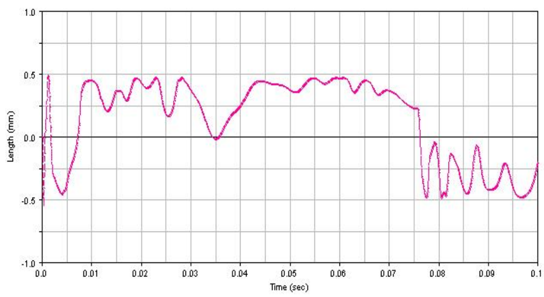

Figure 10 . Displacement deviation curve in direction x of clearance joint C

Figure 8, Figure 9 and Figure 10 represent the displacement deviation curve in direction x of clearance mechanism joint A, B, and C. The displacement deviation of the clearance joint A is within ±0.5mm, and deviation within clearance range is as Figure 8 shows. From Figure 9, the displacement deviation of clearance joint B is within ±1.5mm, it is because that the deviation depends on the clearance vector of three joints. As Figure 10 shows, because the link points of the frame are fixed, the displacement deviation of clearance joint C is also within ±0.5mm.

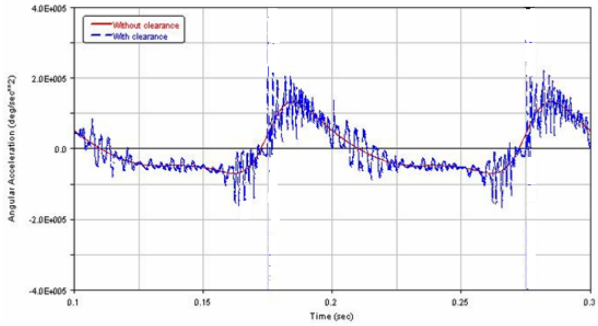

Figure 11 . Angle acceleration of the rocker

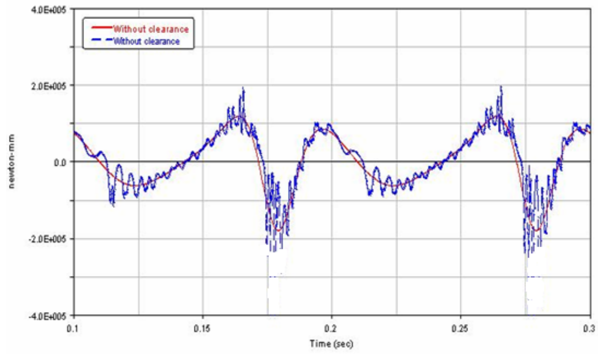

Figure 12 . Driving moment

The angular acceleration curve of rocker in stable state is shown in Figure 11, when considering joint clearance, there are obvious fluctuation and many peaks, those amplitudes are bigger than that of ideal mechanisms, this is because the existence of clearance which make pulsed contact collision power which reflects the characteristic of high-frequency oscillation in joints. The driving moment curve is shown in Figure 12, which reflects the same characteristic. Therefore, joint clearance has great influence on kinematic accuracy of the mechanism and the transmission efficiency [10].

This paper discusses how joint clearance affect the mechanism dynamitic characteristic based on establishing four-bar linkage model for dynamic simulation analysis with clearance 0.25mm, 0.1mm and 0.01mm. The simulation results of rocker angular acceleration are shown as Figure 13, Figure 14 and Figure 15 .

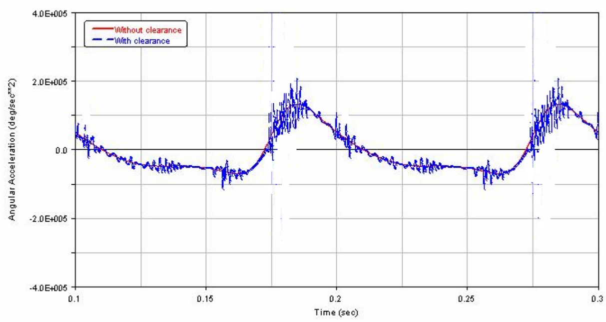

Figure 13 . Angle acceleration of the rocker with clearance 0.25mm

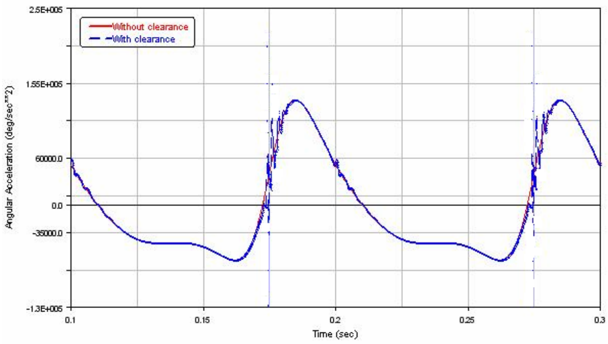

Figure 14 . Angle acceleration of the rocker with clearance 0.1mm

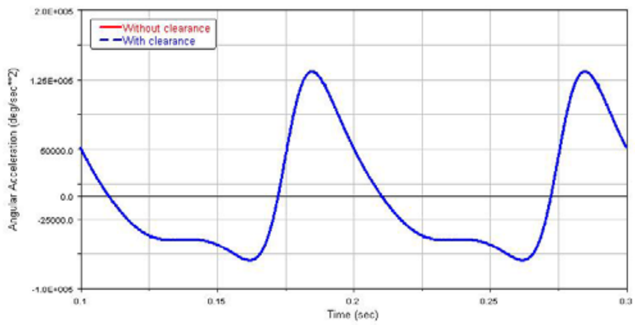

Figure 15 . Angle acceleration of the rocker with clearance 0.01mm

According to Figure 13, Figure 14 and Figure 15, the dynamic characteristic of rocker have large difference on different clearance. The fact is that the smaller the clearance is, the lower the impact on rocker angular acceleration; the smaller the wave amplitude, and the wave peak of clearance collision power is also smaller. Therefore, the mechanisms dynamic behavior curve becomes smoother when the clearance is smaller, and has smaller wave peak, moreover, the smaller the clearance is, the more ideal mechanism the dynamic characteristic of the system tends to be.

5. Conclusions

This paper focuses upon joints with clearance, adopts the hybrid contact model to study how joint clearance affect the dynamic characteristic of four-bar linkage mechanism and use software ADAMS to simulate the dynamic characteristics of the mechanism. It can be seen clearly from the displacement deviation curve of each joint, the deviation at the joint clearance between the connecting rod and the rocker is naturally affected by three joint clearances, that is, the kinematics contribution of joint A and B is superposed to the kinematics of joint C. The deviation curve of angular velocity and angular acceleration show that there is a great error in motions initial stage, which means the mechanism is quite instable in this stage. When the mechanism is stable, the angular acceleration curve of rocker has obvious wave and a great peak. The peak of the angular acceleration curve has an important influence on mechanisms inertia force and its direction, and these influences increase vibration and noise of the mechanism, which leads the mechanical transmissions performance going down. When the joint clearance get smaller, the dynamic behavior curve of mechanism become smoother and have a smaller wave peak, therefore, the smaller clearance is, the more ideal the dynamic characteristic of the system is.

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7. References

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